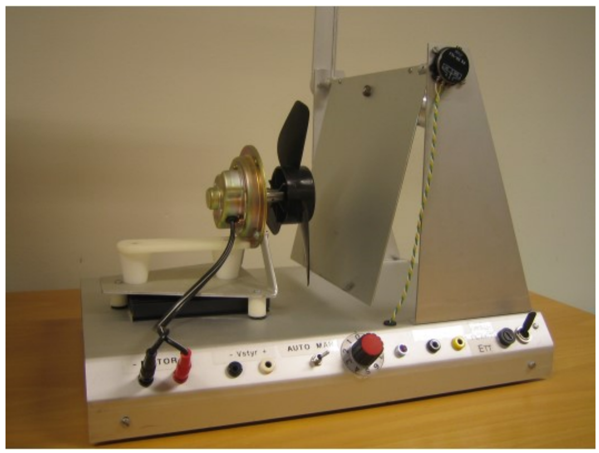
TSRT92 System identification of a weather vane

Andreas Nordström, andno773

October 10, 2024



Contents

1 Description of the task

The goal of this task is to identify a mathematical model for a simple weather vane through system identification. The weather vane is seen as a ”black box” which means that no information about the internal workings of the system are known. Only inputs and outputs are known. The weather vane consists of a small fan that blows air on a metal plate that can move. This is done with a voltage to the fan which is considered the input. The output is a voltage which is proportional to the plate angle. Data collection will be conducted that will be used later to find a good model.

2 Identification experiment

2.1 Sample Time

To find a proper sampling time, the time constant for the system is needed. The sampling time can be approximated by

$$\omega_S = 20\omega_B \approx \frac{20}{\tau} \,\text{unit}\,\text{rad/s} \tag{1}$$

where τ is the time constant of the system. The time constant is the time it takes for the system to reach 63% of its final value. Since the ω_S is not in the correct unit, the sampling time is $T_s = \frac{2\pi}{\omega_S}\,\text{unit}\,\text{s}$. It was calculated to be 0.05.

2.2 Input

Different inputs will be used during the lab. All of them have to be between 0 and 10 volts in order for the weather vane to not break down. Since there are some nonlinear properties for certain voltages, the input is also restricted to 4 volts with a deviation of 2 volts. The final input interval is 2-6 volts.

The first type of input that was used was a pulse train. This was used to get a step response from the system to be able to calculate the time constant.

The second type of input was white noise. Since the spectrum of white noise is constant for all frequencies, this is an excellent signal to use in order to obtain which frequencies that are intersting to look more into. This frequency band was found to be between 0 and 1.25 Hertz. This is important when we use our third type of input because we do not want to waste energy on frequencies that are outside this interval.

The third and final type of input that was used was a telegraph signal. This signal contains a switch probability which means that for each sample time, the signal have some probability of changing between 2 and 6 volts. The switch probability was calculated as:

$$\frac{1}{T_s} \cdot P_s = 0.6250Hz \Leftrightarrow P_s = 0.6250Hz \cdot T_s = 0.0313 \tag{2}$$

This signal was used to generate the data that was used in the toolbox.

2.3 Data collection

Once a good input signal had been identified, the simulation time, intially set to 10 seconds, was increased to 200 seconds. This generated 4000 samples which were saved in to a variable z in MATLAB. The variable z was then split in half, creating two new variables: Zestimation and Zvalidation, each containing 2000 samples. Both datasets were preprocessed in the toolbox by removing their means. Additionally, the first 30 samples in the estimation dataset were removed due to nonlinear behaviour of the weather vane during startup. Zestimation was used for model estimation, while Zvalidation was used for model validation.

3 Identification from data

3.1 Spectral Analysis

Spectral analysis is used to understand the frequency content of the system. It provides insights into how different frequencies contribute to the overall behaviour of the system. This approach is similar to when the white-noise input was used. By counting the peaks in the frequency response, one can determine the value of **na**. This is because **na** represents the order of the system, which in turn means that it is connected to the number of poles of the system. Since a pole gives a rise in frequency, it is easy to see how large **na** should be. From 2 there are three peaks. Later when comparing the best ARX model to a ARX model with **na** $= 3$, it will be seen that the increasing the value of **na**, it will only give slight improvements but the system will be more complicated. Comparing figure 1 and figure 2, one can see that they booth have a peak at around 1-1.5Hz.

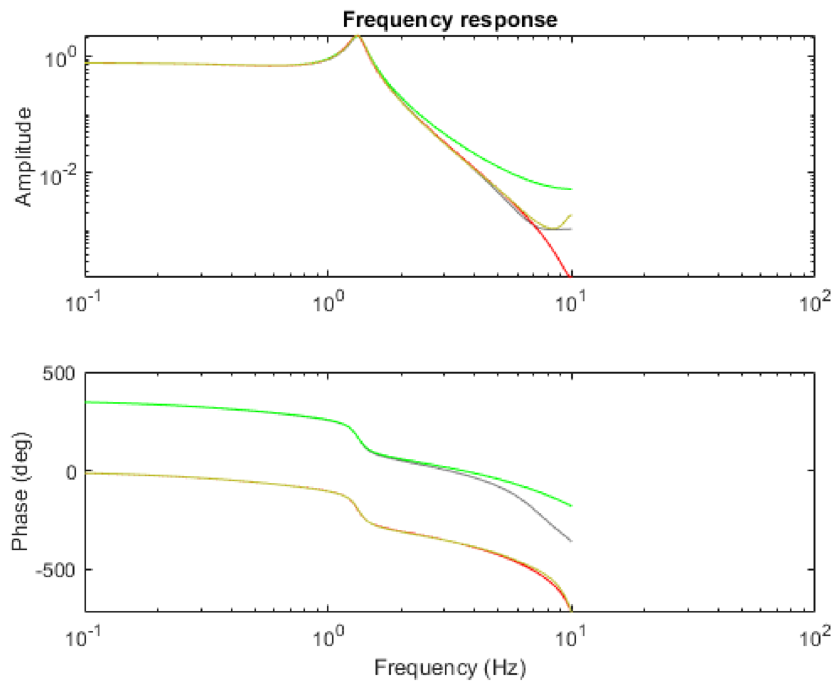


Figure 1: Frequency response for all models

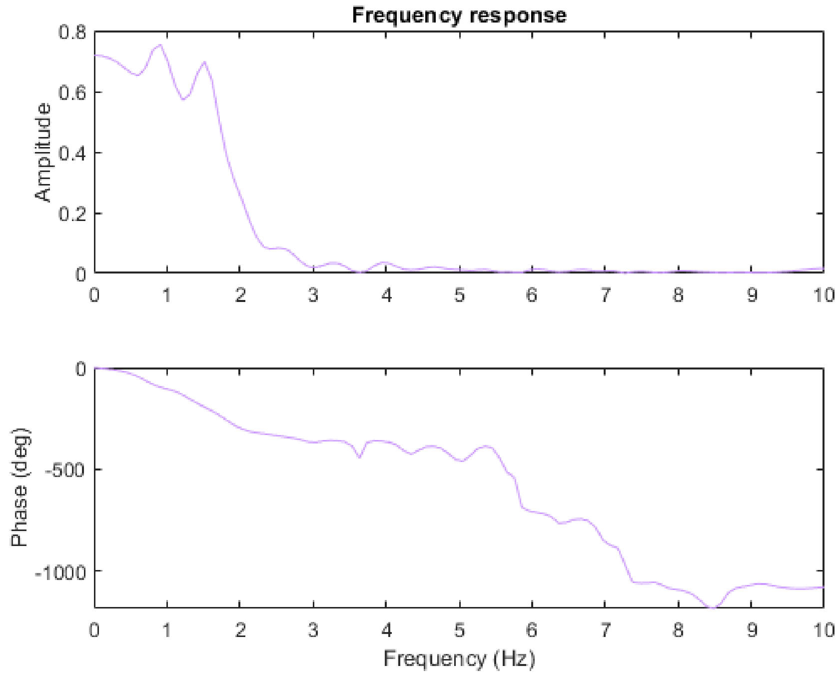


Figure 2: SPA

3.2 Correlation Analysis

Correlation analysis is based on calculating the correlations between input and output. It is used to determine the value of **nk**, the delay.

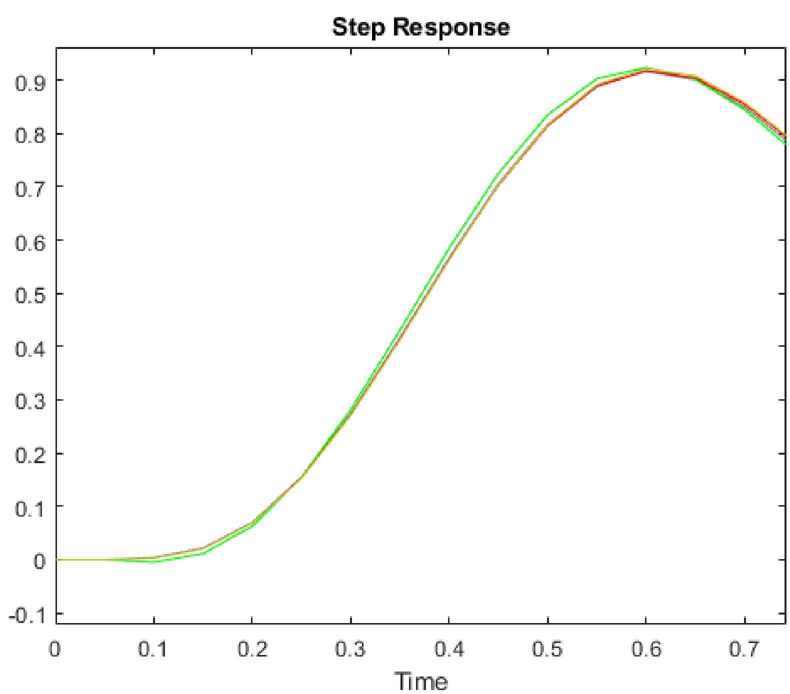


Figure 3: Impulse response, all models

It is easily seen that the delay is about 0.2 seconds which correspond to 4 samples, therefore **nk** $= 4$.

This value didnt make a significant difference in any of the models so it was kept to **nk** $= 2$.

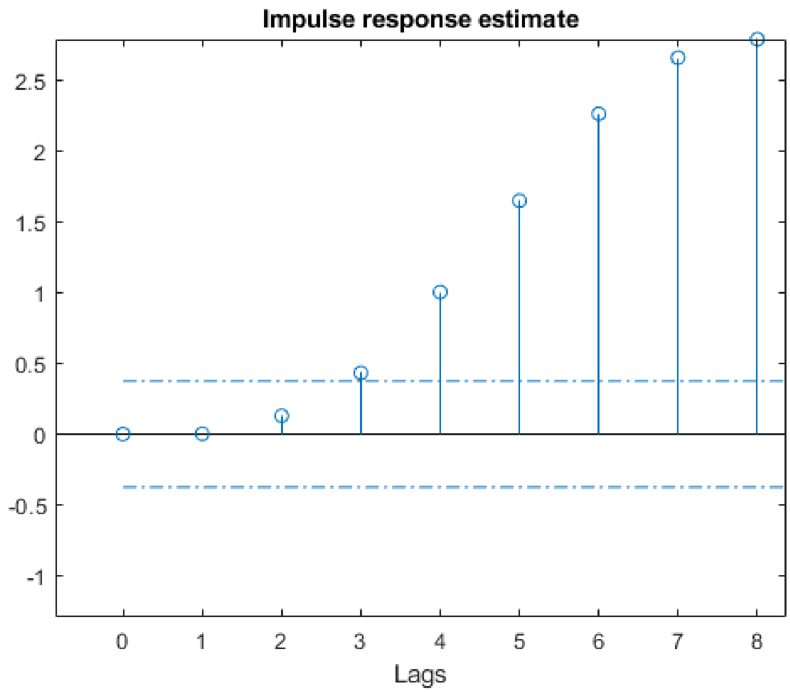


Figure 4: CRA

The delay is essentialy the same.

3.3 Parametric Models

The parametric models used in this lab are so called black-box models which means that they do not require any physical knowledge of the system beforehand. They are linear parametric models. They were produced by using the two datasets, Zestimation and Zvalidation. The models used were ARX, ARMAX, OE and BJ which all have their strength and weaknesses. The best fits for each model with the parameter and parameter values are listed below:

- arx762, ARX-model, **na** $= 7$, **nb** $= 6$, **nk** $= 2$
  $\mathbf{A(z)} = 1 - 1.809z^{-1} + 0.5254z^{-2} + 0.7375z^{-3} - .4453z^{-4} + 0.2398z^{-5} - 0.2593z^{-6} + 0.06953z^{-7}$
  $\mathbf{B(z)} = 0.002739z^{-2} + 0.01269z^{-3} + 0.0177z^{-4} + 0.008194z^{-5} + 0.003355z^{-6} - 8.658e - 05z^{-7}$

- amx3332, ARMAX-model, **na** $= 3$, **nb** $= 3$,**nc** $= 3$ **nk** $= 2$
  $\mathbf{A(z)} = 1 - 2.606z^{-1} + 2.417z^{-2} - 0.7818z^{-3}$
  $\mathbf{B(z)} = 0.002822z^{-2} + 0.01043z^{-3} + 0.008689z^{-4}$
  $\mathbf{C(z)} = 1 - 0.8163z^{-1} + 0.4369z^{-2} - 0.1991z^{-3}$

- oe332, OE-model, **nb** $= 3$, **nf** $= 3$, **nk** $= 2$,
  $\mathbf{B(z} = -0.004777z^{-2} + 0.02856z^{-3} - 0.001801z^{-4}$
  $\mathbf{F(z)} = 1 - 2.601z^{-1} + 2.408z^{-2} - 0.7783z^{-3}$

- bj3332, BJ-model, **nb** $= 3$, **nc** $= 3$, **nd** $= 3$, **nf** $= 3$, **nk** $= 2$
  $\mathbf{B(z)} = 0.003923z^{-2} + 0.007695z^{-3} + 0.01108z^{-4}$
  $\mathbf{C(z)} = 1 - 0.8639z^{-1} + 0.4339^{-2} - 0.2365z^{-3}$
  $\mathbf{D(z)} = 1 - 2.65z^{-1} + 2.484z^{-2} - 0.8131z^{-3}$
  $\mathbf{F(z)} = 1 - 2.599z^{-1} + 2.407z^{-2} - 0.7785z^{-3}$

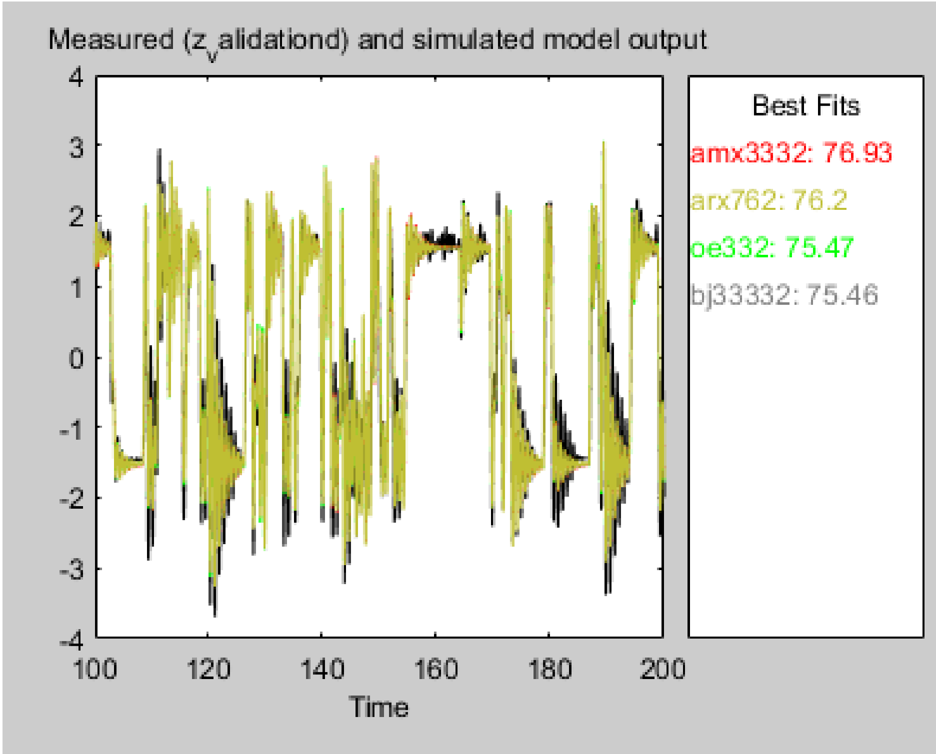


Figure 5: The best fitting models

4 Model validation

4.1 Cross validation

One of the key questions in model validation is: How accurately can the model predict data it hasn't encountered before? Cross-validation can be used to determine how the data from the simulated model compares to the validation data. This is what is seen in figure 5 [Ljung].

4.2 Residual analysis

Residuals are the differences between the model's one-step-ahead predicted output and the actual measured output from the validation data set. They represent the part of the validation data that the model fails to account for. Ideally, the cross-correlation should be zero, but this is hard to achieve. If a model is outside the confidence interval, like oe332(green) in figure6, then the output is not correctly described by the model [3] [2]

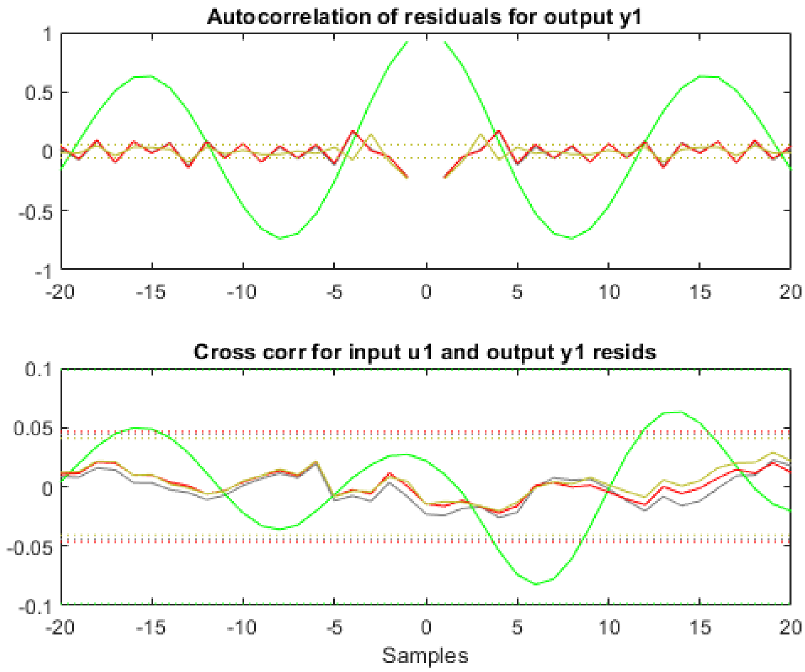


Figure 6: Residuals for the models, green=oe332

4.3 Pole/zero diagram

The diagram of pole sand zeros can be used to describe the dynamics of the system. In this case, if poles are inside the unit circle, the system is stable. If poles exists outside the unit circle then the system is unstable. If the confidence interval for a pole-zero pair overlap it indicates a possible pole-zero cancellation. When this occurs it might be useful to try to reduce the model order [4].

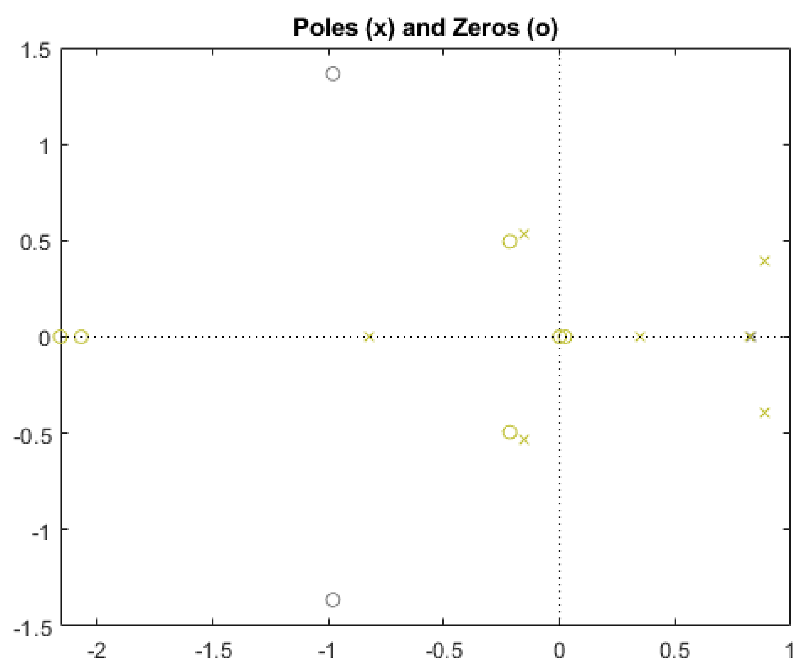


Figure 7: Pole/Zero diagram for arx762

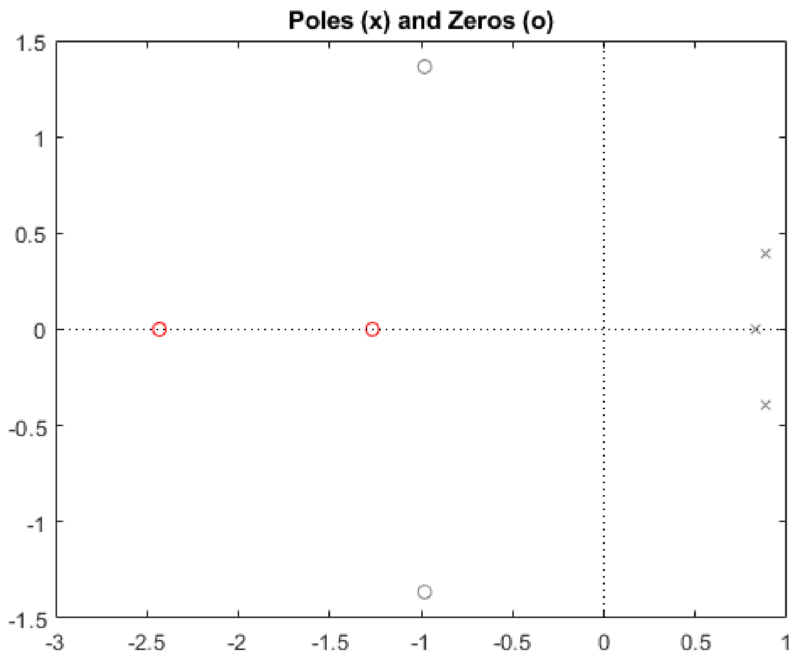


Figure 8: Pole/Zero diagram for amx3332

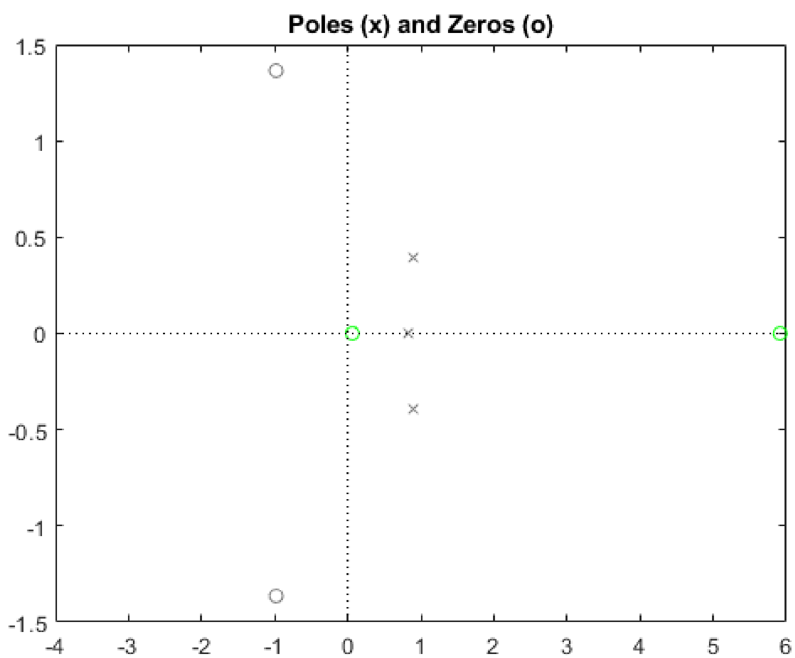


Figure 9: Pole/Zero diagram for oe332

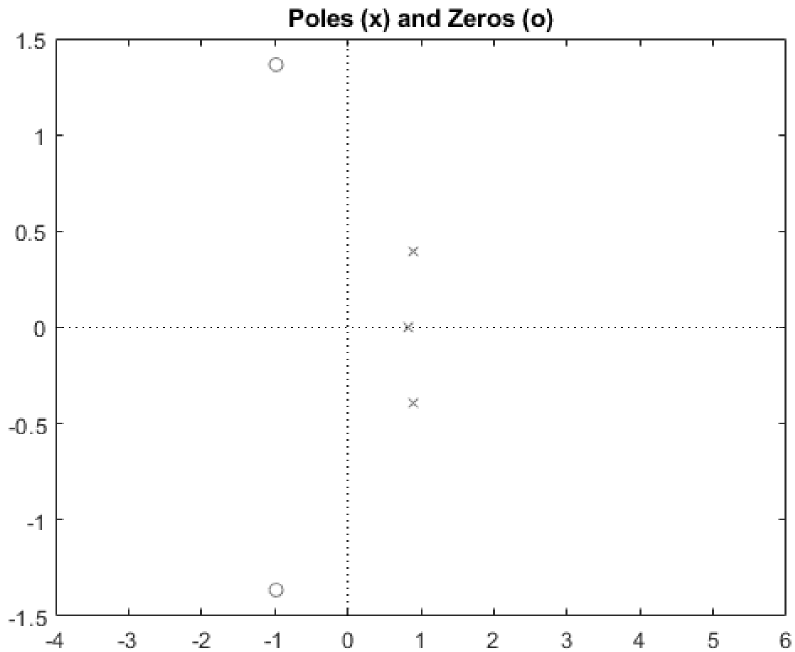


Figure 10: Pole/Zero diagram for bj33332

4.4 Confidence interval for the parameters

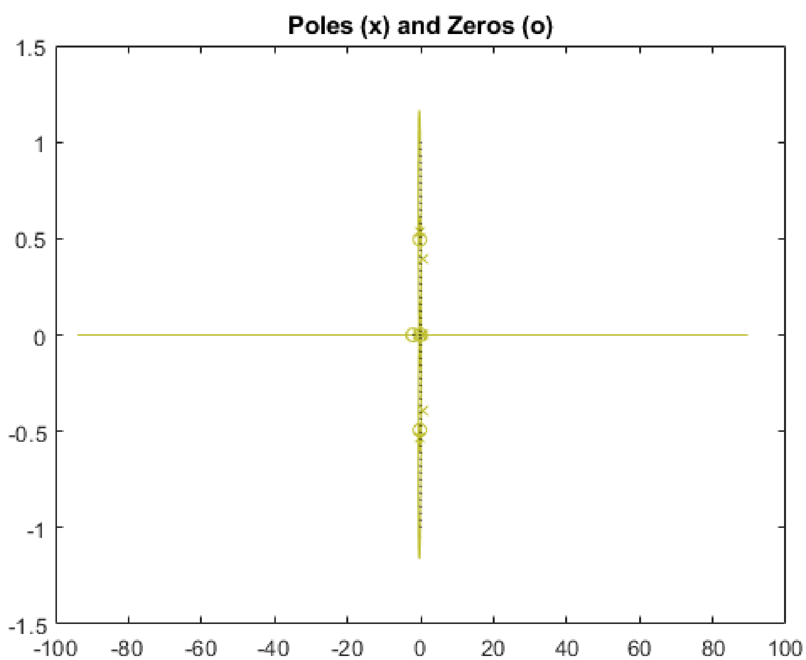


Figure 11: Pole/Zero diagram with confidence intervals for arx762

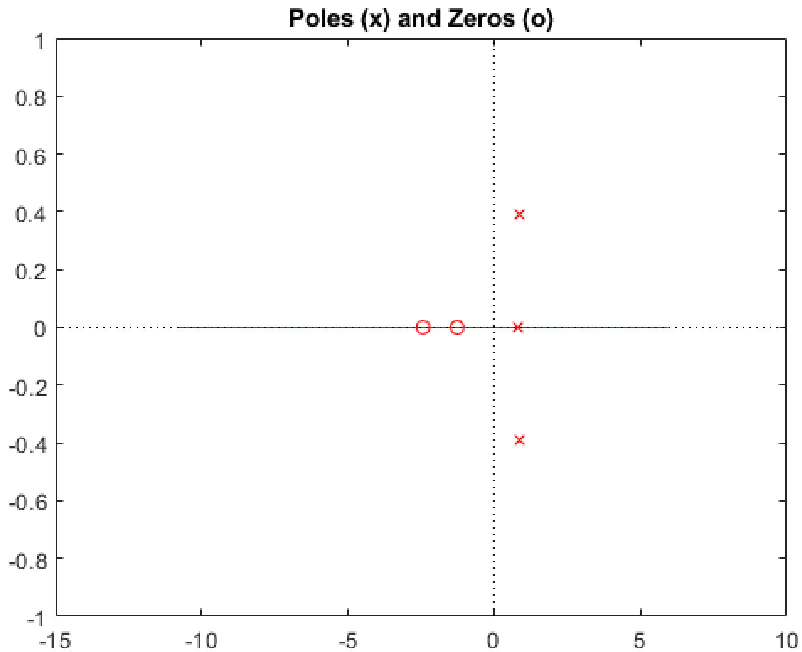


Figure 12: Pole/Zero diagram with confidence intervals for amx3332

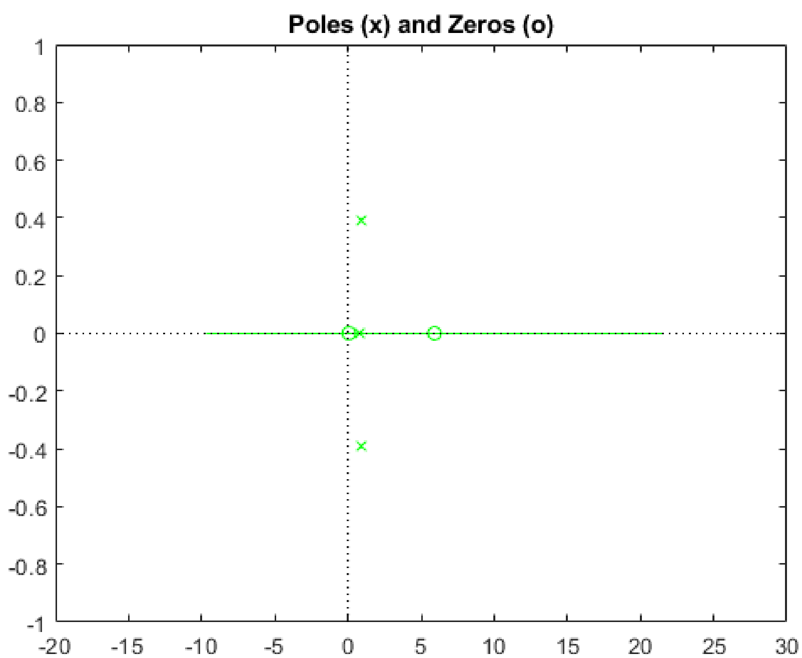


Figure 13: Pole/Zero diagram with confidence intervals for oe332

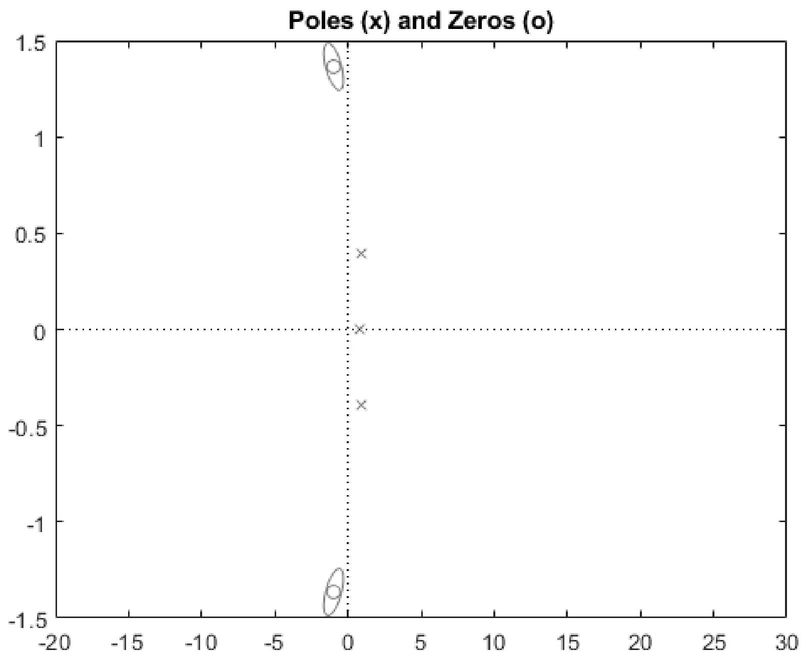


Figure 14: Pole/Zero diagram with confidence intervals for bj33332

4.5 Strength/weaknesses of the models

4.5.1 ARX

Among all the models used, the ARX model is the simplest to estimate, and it's generally a good practice to start with simpler approaches. ARX is also a universal approximator for any linear system. However, due to its simplicity, achieving highly accurate fits often requires a large number of parameters, leading to a high polynomial order.

4.5.2 ARMAX

The ARMAX-model is a more flexible model compared to ARX since it can also model disturbance. Since it is capable of modeling disturbances, it is also more complex to estimate.

4.5.3 OE

If the system does not contain any feedback when collecting the data, then the OE-model can produce the correct transfer function regardless of the disturbance. One has to keep in mind that the OE-model cannot model the disturbance, only add it to the model.

4.5.4 BJ

A BJ model can effectively represent the disturbances in a system, handling them separately from the system's dynamics. This approach makes it highly capable of modeling the entire system. However, because BJ models are more complex than models like OE, they may require greater computational resources for estimation.

4.6 Model recommendation

Since all the models showed similar fits, as indicated in Figure 5, it is necessary to consider other factors. The ARX and ARMAX models are closely related, but the ARMAX model (amx3332) required only 11 parameters compared to 15 for the ARX model (arx762). Additionally, the ARMAX model is better equipped to handle disturbances, making it the superior choice between the two. The OE model, as seen in Figure 6, exhibited peaks outside the confidence interval, which leaves the BJ model. While both the OE and ARMAX models can manage disturbances, the OE model is more complex to estimate numerically and required more parameters. Therefore, the ARMAX model is the best option for this system.

5 References

References

[1] Lennart Ljung, Torkel Glad, and Andreas Hansson, *Modeling and Identification of Dynamic Systems*, Studentlitteratur 2021.

[2] Lennart Ljung, *Introduction to System Identification*, https://www.youtube.com/watch?v=u7hJ1aF-JrU&t=1600s&ab_channel=MATLAB

[3] Mathworks, *What Is Residual Analysis?*, https://se.mathworks.com/help/ident/ug/what-is-residual-analysis.html

[4] Mathworks, *Reducing Model Order Using Pole-Zero Plots*, https://se.mathworks.com/help/ident/ug/reducing-model-order-using-pole-zero-plots.html