Problem Sheet: 1

Section 1: Propositional Logic and Basics

1.1) Let p, q be atomic propositions.

(a) Express the following sentences using propositional logic:

- If p is true, then q must also be true.
- p is true if and only if q is false.

(b) Construct a truth table for the formula: $(p \to q) \land (\neg q)$.

1.2) **Murder Mystery: Logic Deduction**

Three suspects (A, B, C) are involved in a murder investigation.

- A: "I didn't do it. The victim knew B. But C hated him."
- B: "I didn't do it. I didn't know the victim. I was out of town."
- C: "I didn't do it. I saw A and B with the victim that day; one of them did it."

Assumption: Two innocent people tell the truth; the guilty one may lie.

(a) Define propositional variables for relevant facts (e.g., A_{did} = A did it).

(b) Encode the statements logically.

(c) Deduce who the murderer is.

Section 2: Regular Languages

2.3) Consider the alphabet $\Sigma = \{a, b\}$.

(a) Write a regular expression for the language of strings that contain exactly one a.

(b) Write a regular expression for strings that do not contain the substring "ab".

2.4) Construct a DFA for the regular expression: a^*b over $\Sigma = \{a, b\}$.

2.5) Construct a NFA for the language of strings over $\{a, b\}$ that end with "ab".

2.6) Construct a DFA that accepts all strings over $\{0, 1\}$ where the number of 1's is even.

2.7) For each of the following languages, determine whether it is:

(i) Regular(A language is regular if it can be recognized by a finite automaton)

(ii) FO-definable (first-order definable over words)

(a) The set of words over $\{a, b\}$ that have equal number of occurrences of 'ab' and 'ba'. Example: 'aba' is in the language, while 'abab' is not.

(b) The set of words over $\{a, b, \#\}$ with a single occurrence of '', and all symbols before it are 'a''s, and all symbols after it are 'b''s.

(c) The set of strings over $\{a, b\}$ that do not contain the substring 'ba'.

(d) The set of strings over $\{0, 1\}$ where the second symbol from both ends is '0'.

(e) Let $\Sigma = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \middle| a, b \in \{0, 1\} \right\}$. A string over Σ gives two rows of bits. Interpret each row as a binary number. The language is: $\{w \in \Sigma^* \mid \text{top row is greater than bottom row}\}$.