Volume computation of Convex Polytopes in high dimensions.

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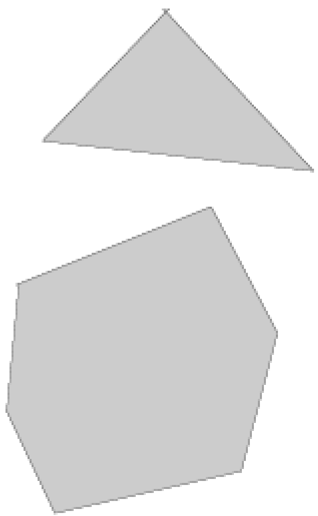
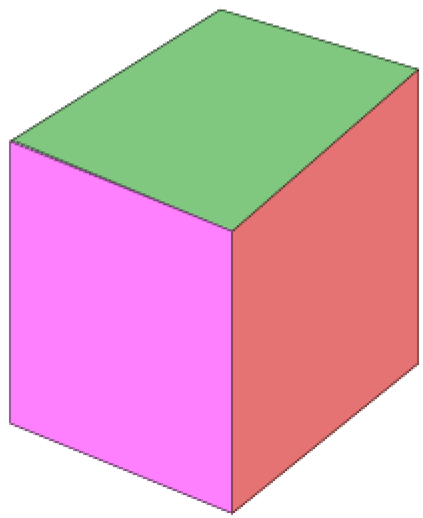
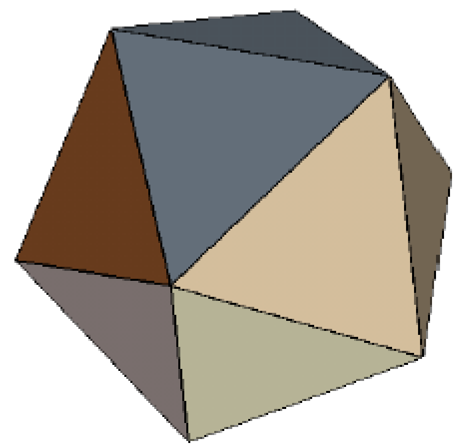




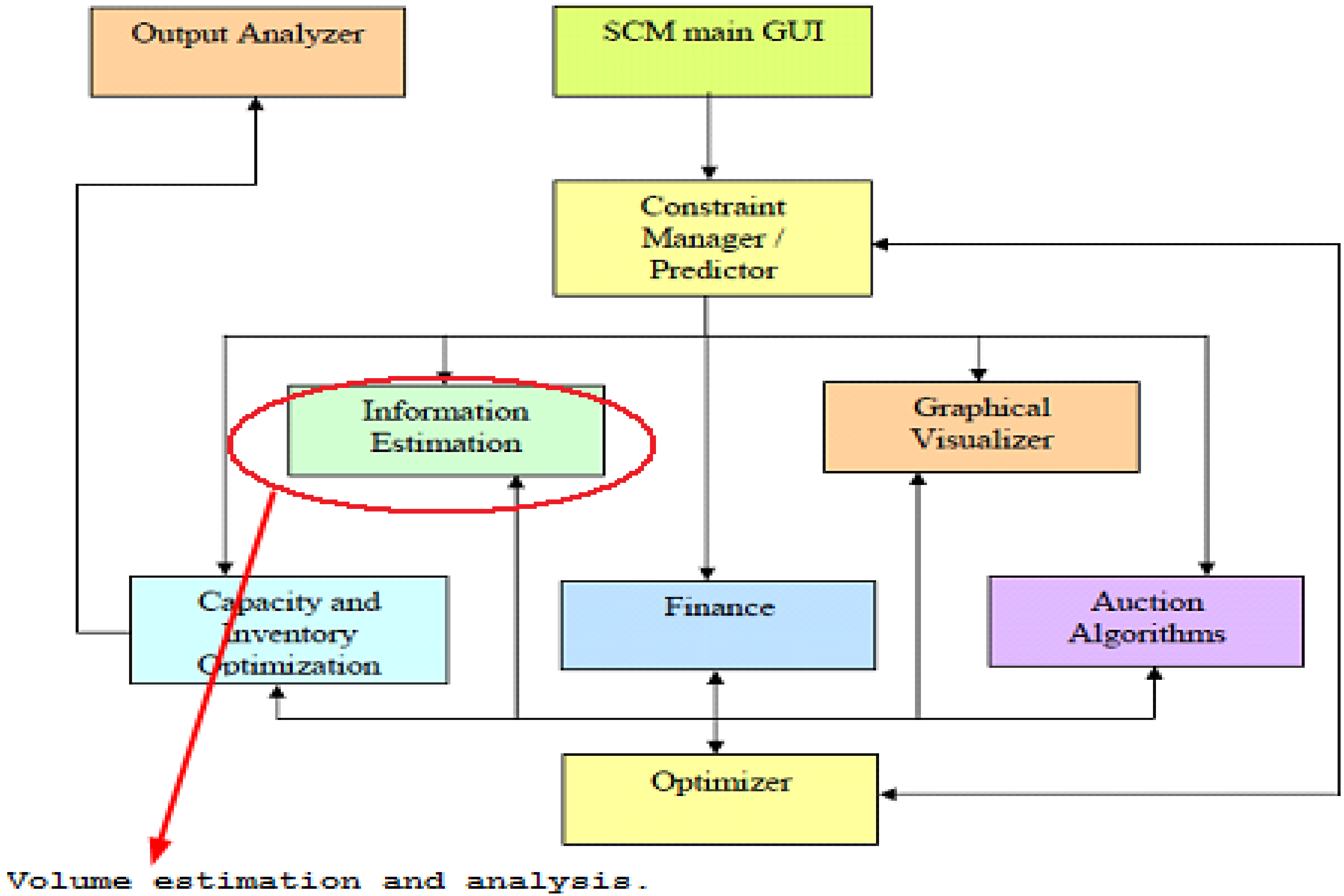
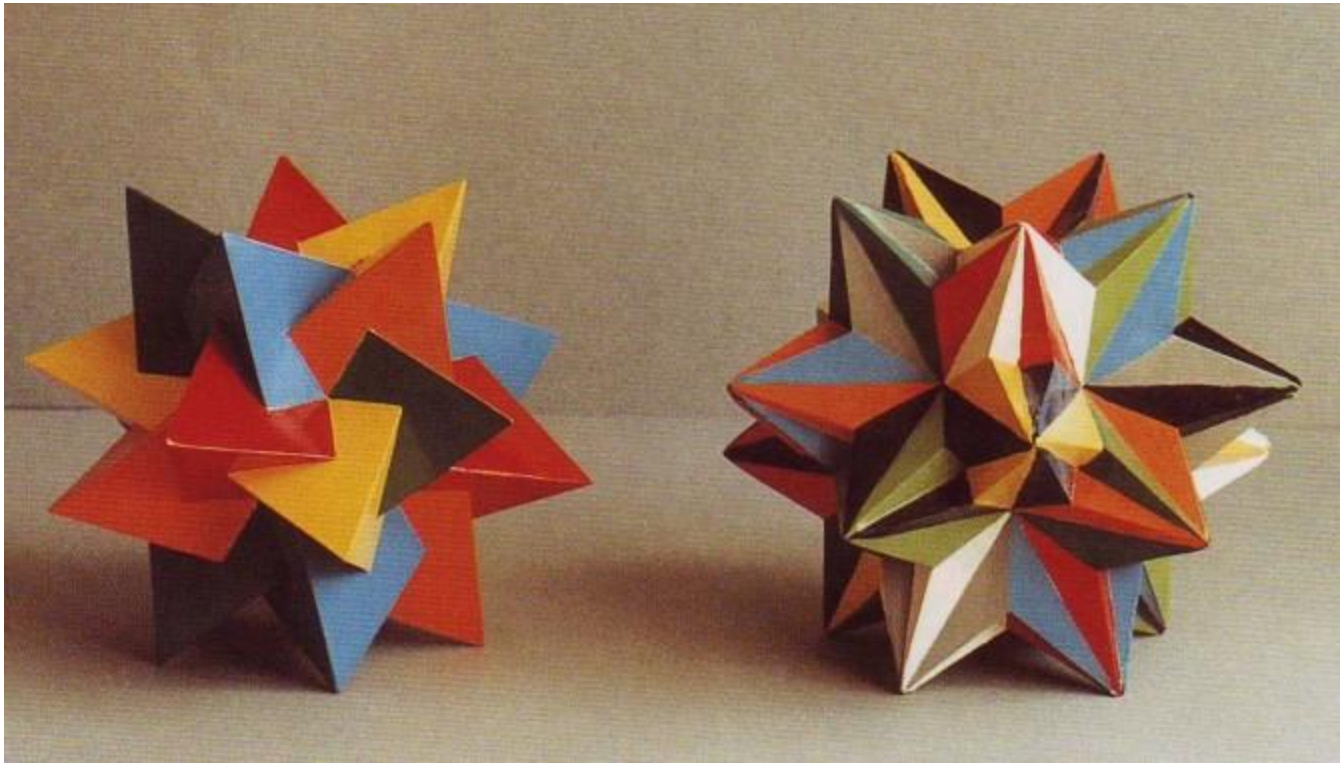


Figure 1: SCM software architecture

What is a Convex Polytope?

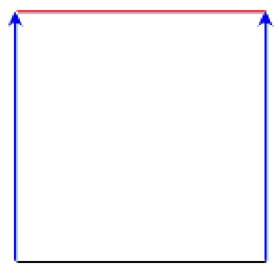
Polytopes which are not Convex..



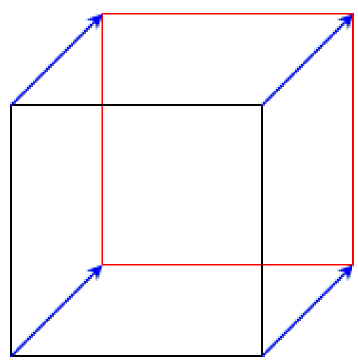
Thinking in Higher dimensions…

The 0-dimension cube (point)

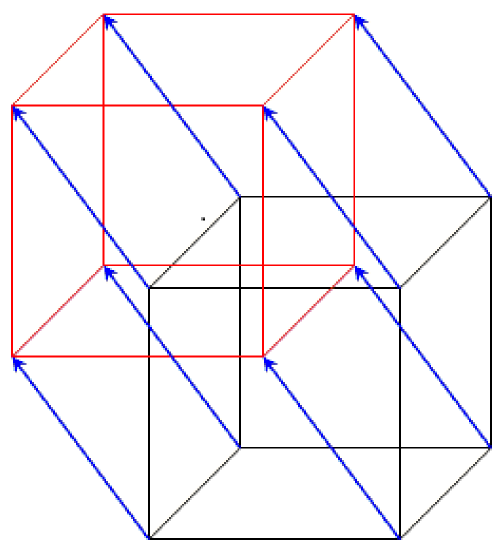
The 1-dimension cube (line)



The 2-dimension cube (square)



The 3-dimension cube



The 4-dimension cube (tesseract)

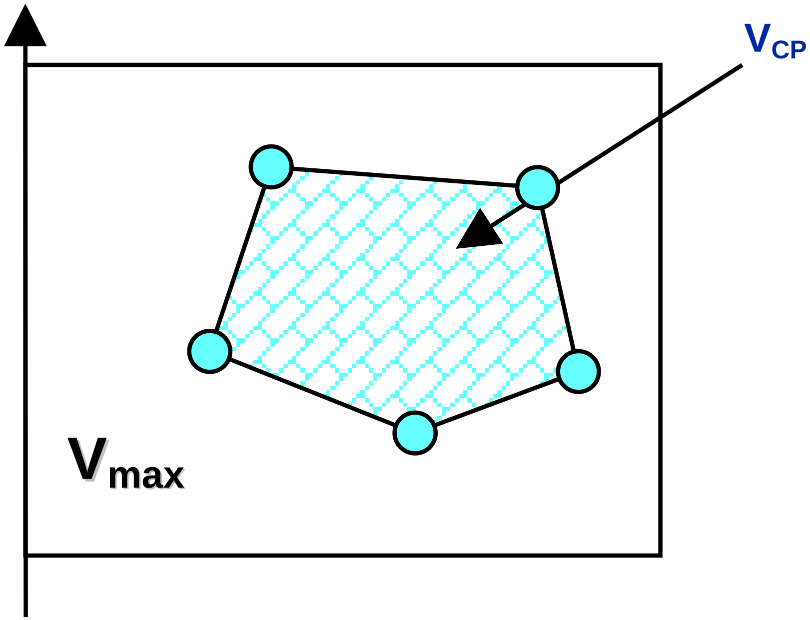
Volume of Tetrahedra.. Is easy!

- To compute the volume of a polyhedron divide it as a disjoint union of tetrahedra.
- Calculate volume for each tetrahedron (an easy determinant)and then add them up!

Computing Exact volume is Hard!

- Existing softwares like Vinci, Qhull, etc.
- Stops giving volume after 9 dimensions.

Information Theory – Shannon's Theorem



$$I = \log_2\left(\frac{V_{\max}}{V_{CP}}\right)$$

Figure illustrating Information Content in a Polyhedron, of volume V_{CP}, relative to a total volume (not necessarily rectangular) V_{max}

Generating Constraint sets..

$171.43\ dem\_M0\_p0 + 128.57\ dem\_M1\_p0 \leq 79285.71$

$171.43\ dem\_M0\_p0 + 128.57\ dem\_M1\_p0 \geq 42857.14$

$57.14\ dem\_M0\_p0 + 42.86\ dem\_M1\_p0 \leq 26428.57$

$57.14\ dem\_M0\_p0 + 42.86\ dem\_M1\_p0 \geq 14285.71$

$175.0\ dem\_M0\_p0 + 25.0\ dem\_M1\_p0 \leq 65000.0$

$175.0\ dem\_M0\_p0 + 25.0\ dem\_M1\_p0 \geq 22500.0$

$0.51\ dem\_M0\_p0 - 0.39\ dem\_M1\_p0 \leq 237.86$

$0.51\ dem\_M0\_p0 - 0.39\ dem\_M1\_p0 \geq 128.57$

$300.0\ dem\_M0\_p0 \leq 105000.0$

$300.0\ dem\_M0\_p0 \geq 30000.0$

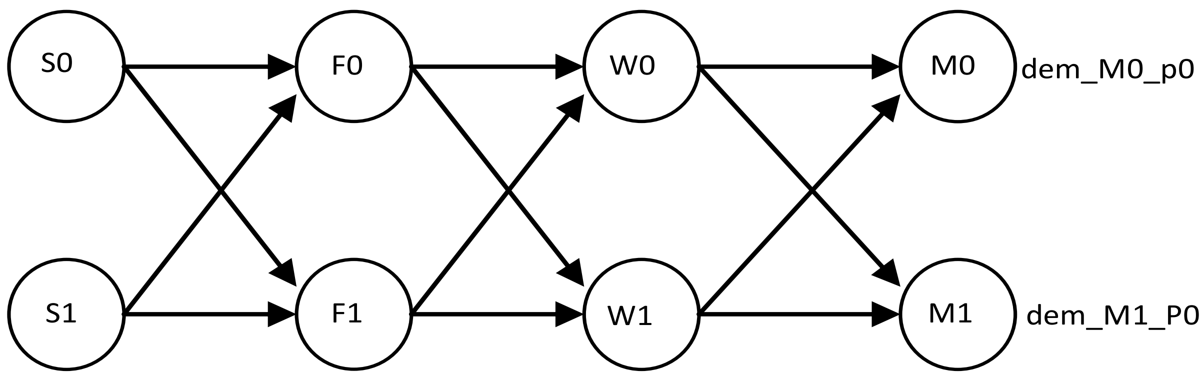
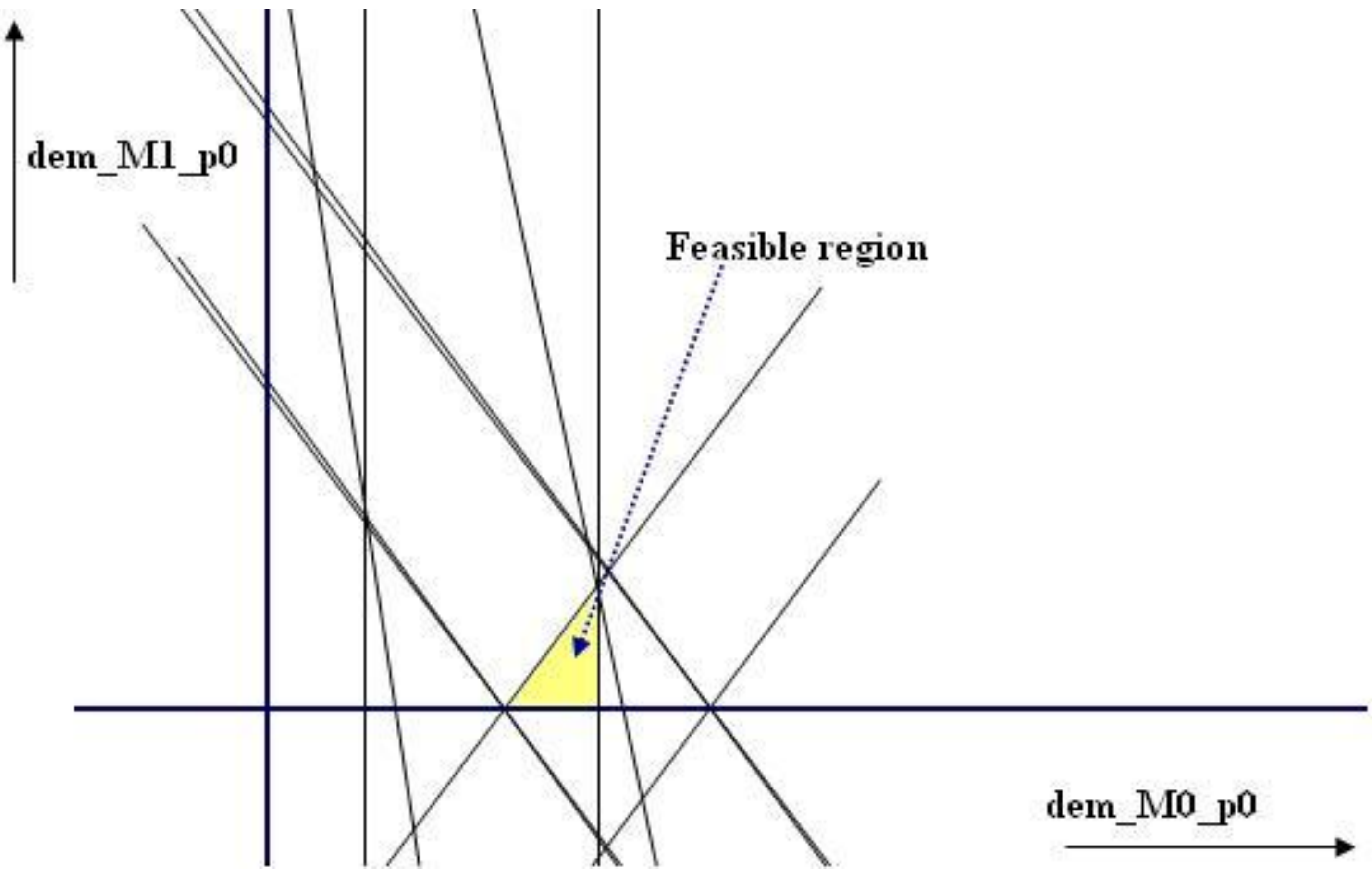
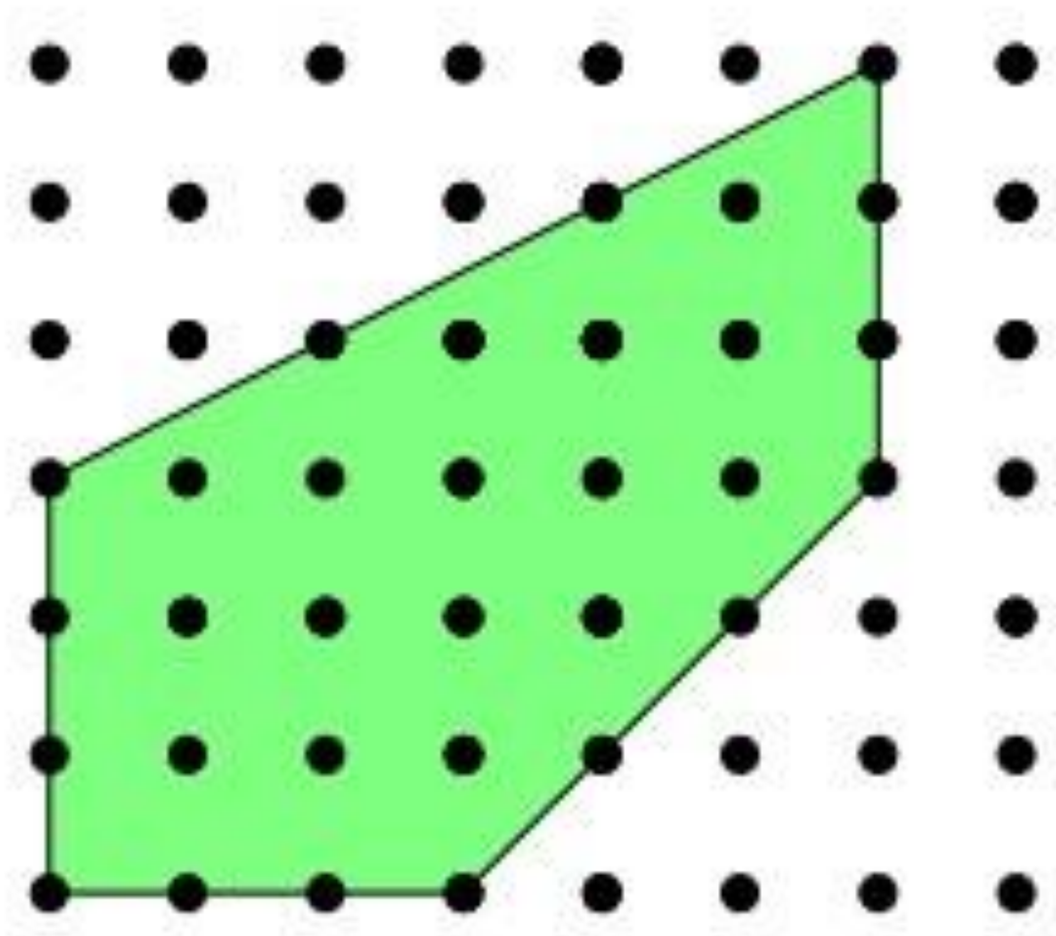


Figure 2: Model of a small supply chain

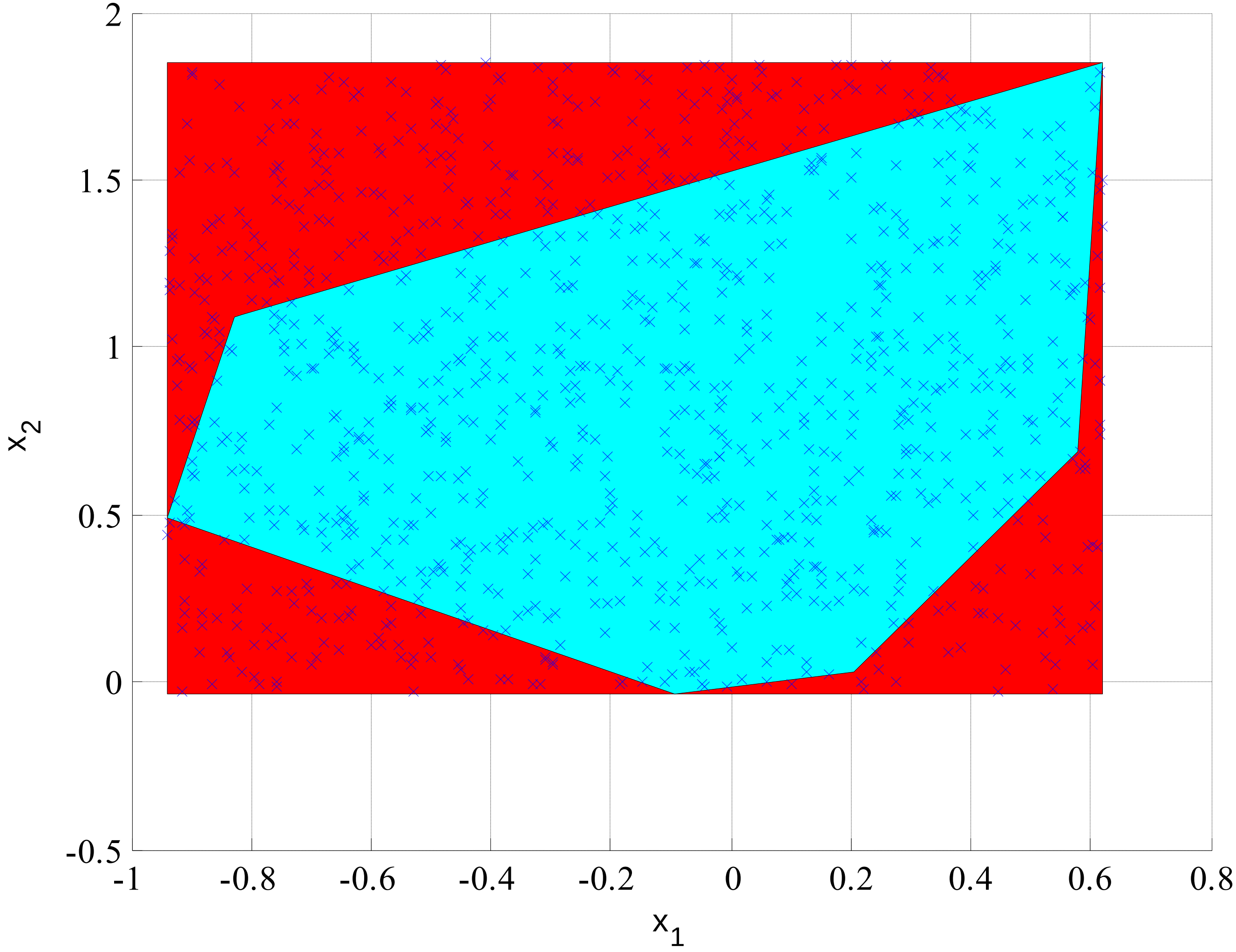
Feasible region



We can approximate the volume!
- Sampling



Our Implementation - Direct Monte Carlo

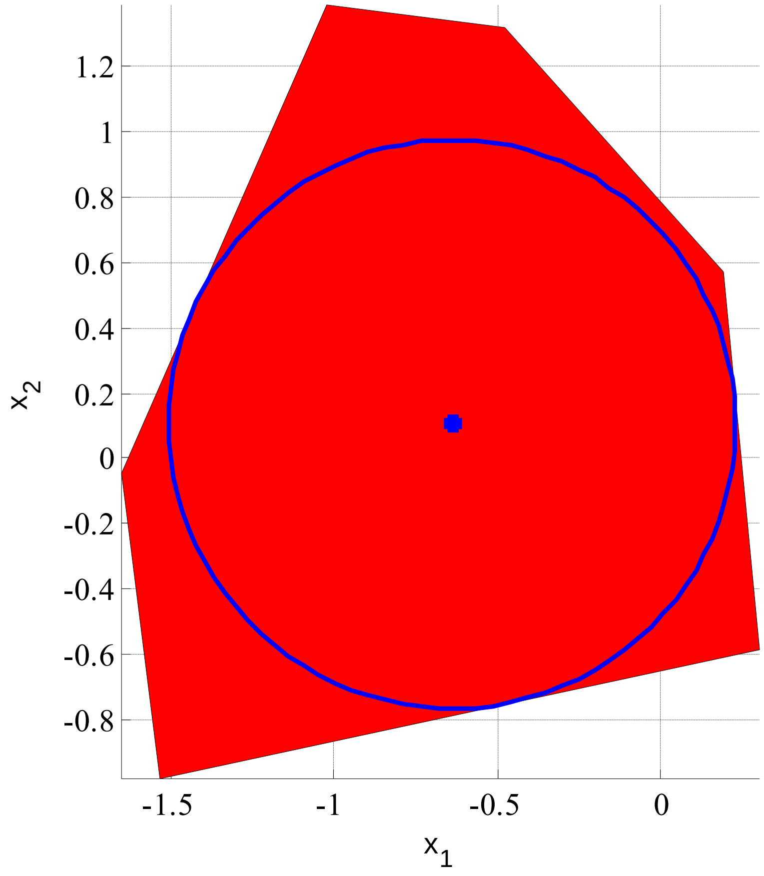


cont.>

- Number of samples = 1000
- Volume of bounding box = 2.9448
- Exact volume of polytope = 1.8424
- Estimated volume of polytope (Direct Monte Carlo) = 1.8113
- The estimated volume is found by counting the hits inside the polytope (light blue).

$$estimated\ vol = \frac{hits}{\#of\ samples} \times vol\ of\ bounding\ box$$

Direct Monte Carlo – Chebyball variant



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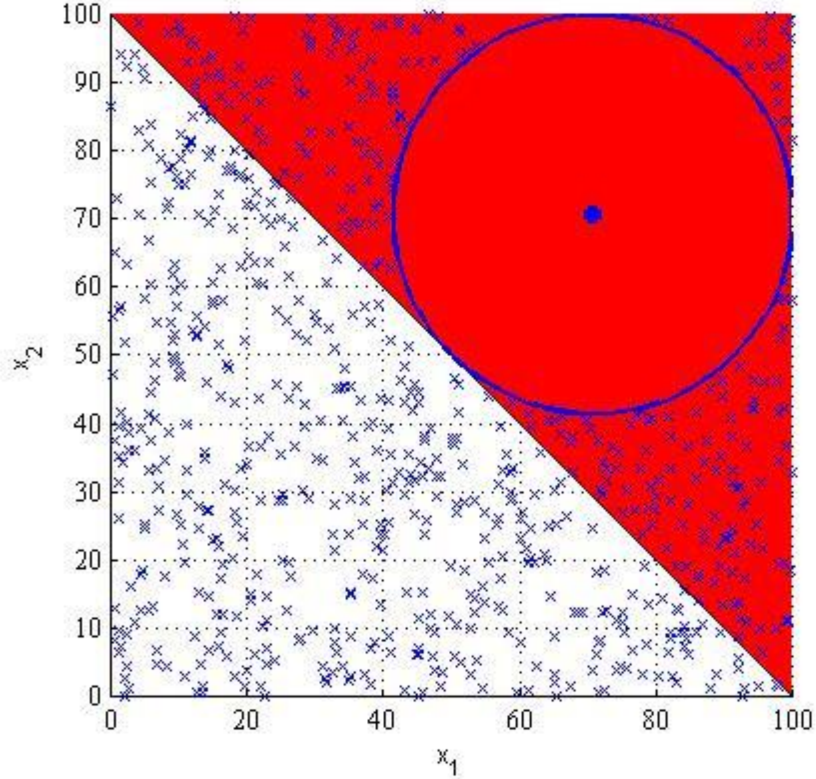
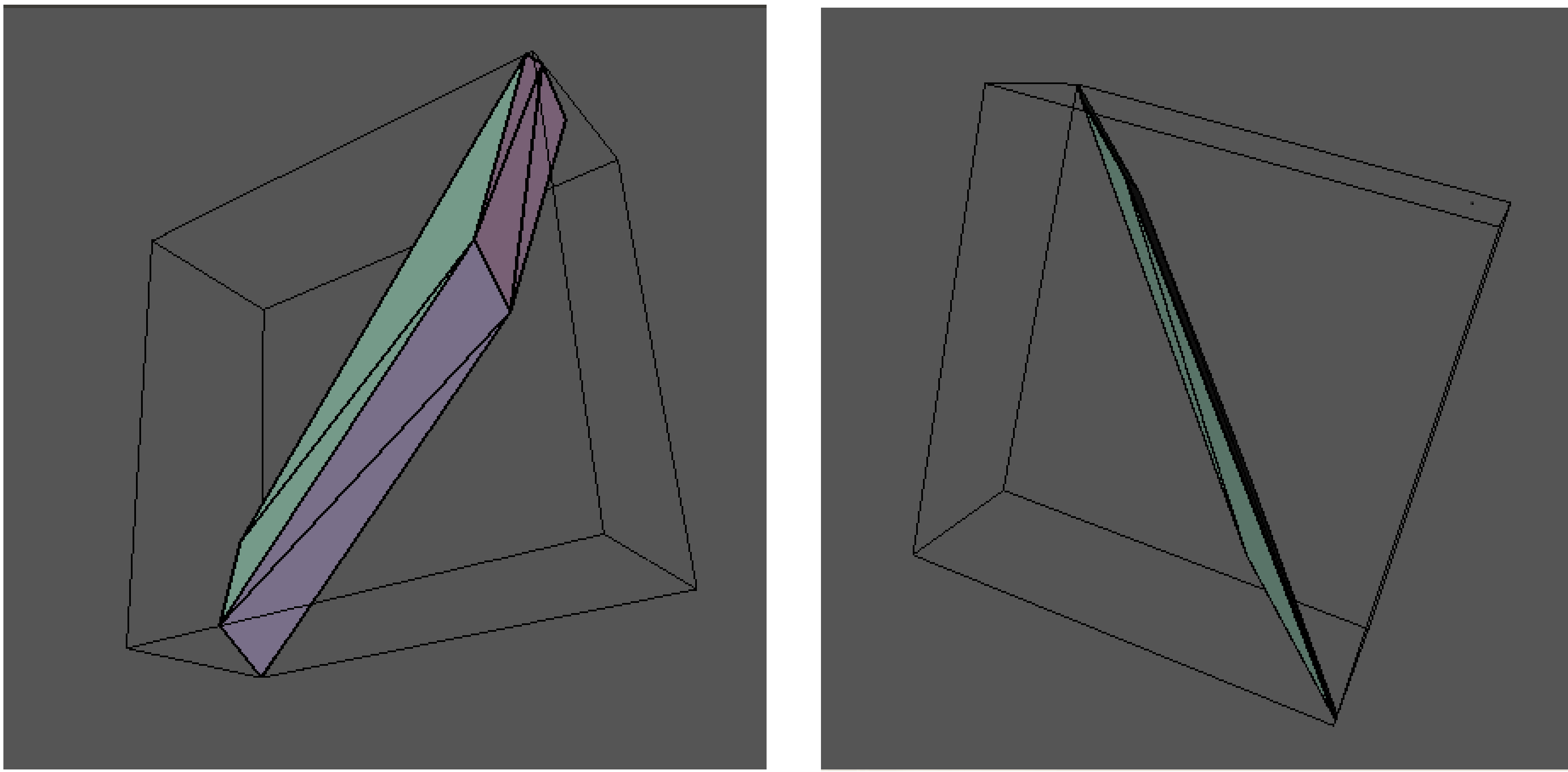
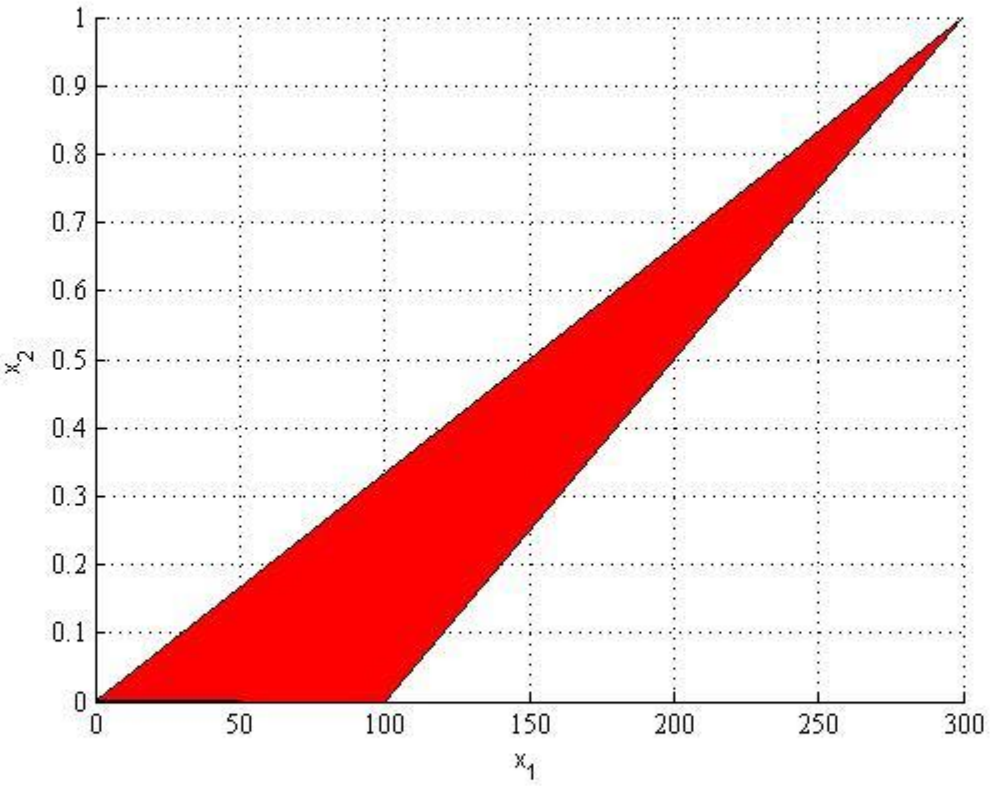


$$estimated\ vol = \frac{hits}{\#of\ samples} \times (vol\ of\ bounding\ box - cheby\ vol) + cheby\ vol$$

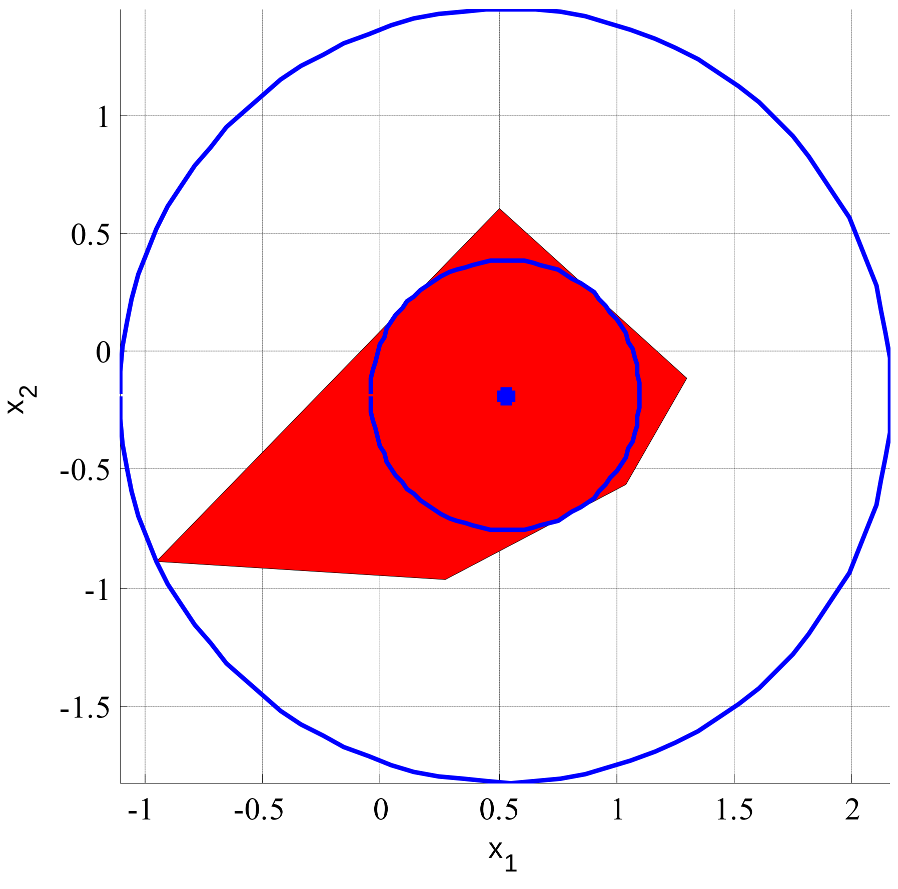
Ill-conditioned Polytopes



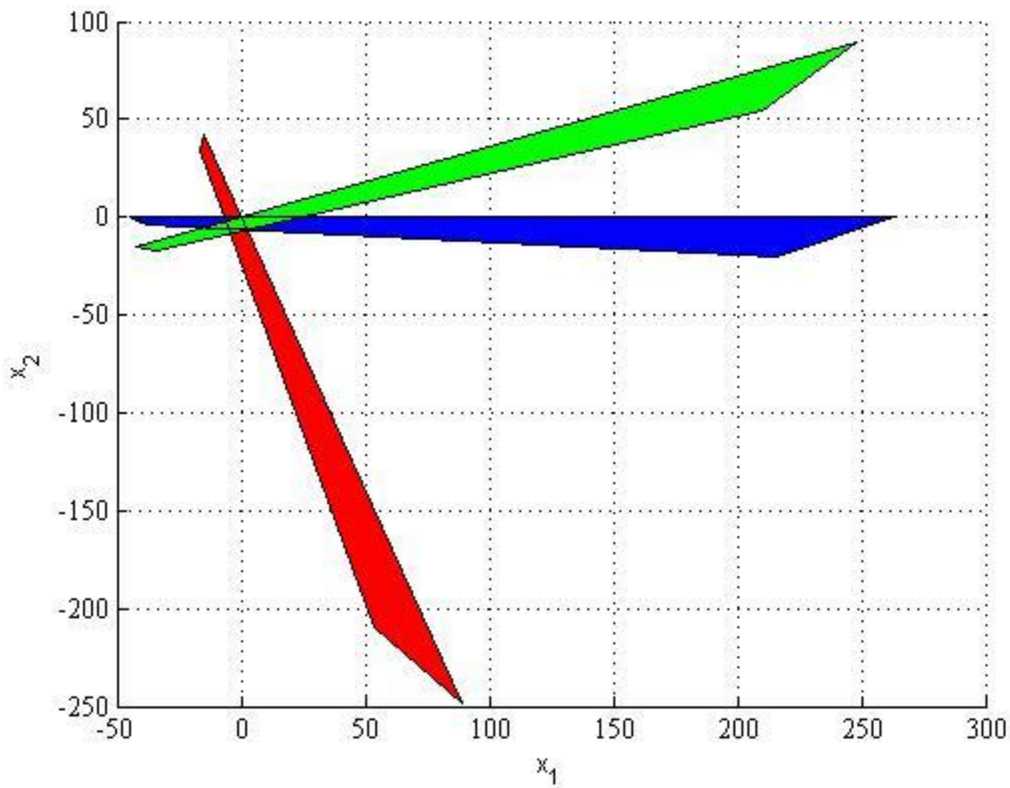
Condition number 481



Sandwiching – How bad is the Polytope?

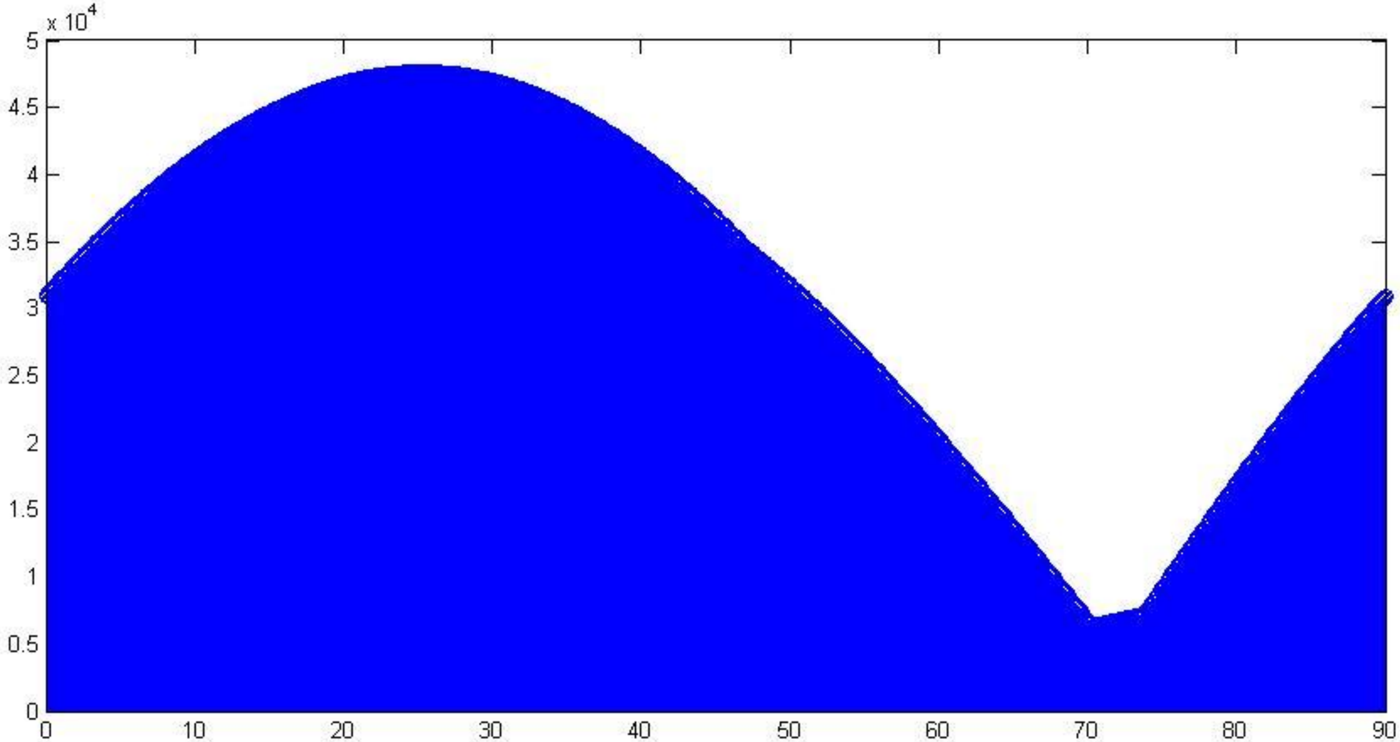


Rotation along axis – minimize bounding box volume



- The example above shows a very thin polytope with condition number 50.
- The original polytope is shown in red and the final one after rotating by 90 degrees is shown in green.
- The polytope with minimum bounding box volume is shown in blue.

This minimum bounding box of the polytope occurs at 70.3 degrees.



Further work…

- Importance sampling.
- MCMC.
- Parallel Monte Carlo.
- Feature Extraction (machine learning).

References

[1] *Prasanna G.N.S., et al.,* Decision Support Methods under Uncertainty, International patent application PCT/IN2009/000390 (filed July 10, 2008).

[2] Miklós *Simonovits*. How to compute the volume in high dimension? (2003).

[3] *Aswal. Abhilasha*, Information Theory Application in Supply Chain Management, Whitepaper