SMART SKILLS

2020-2021

MATHS

CONTENTS

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FIRST TERM

March - May

UNIT-VI: PROBABILITY

Probability:

Multiplication theorem on probability. Conditional probability, independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of random variable. Repeated independent (Bernoulli) trials and Binomial distribution.

UNIT-II: ALGEBRA

Matrices:

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence ofnon-zero matrices whose product is the zero matrix (restrict to square matrices of order2).Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

Determinants:

Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle.

Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix

UNIT I. RELATIONS AND FUNCTIONS

Relations and Functions :

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function.

July

Inverse Trigonometric Functions:

Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

UNIT-III: CALCULUS

Continuity and Differentiability:

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit function. Concept of exponential and logarithmic functions and their derivative. Logarithmic differentiation. Derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.

August

Applications of Derivatives:

Applications of derivatives: rate of change, increasing/decreasing functions, tangents& normals, approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems(that illustrate basic principles and understanding of the subject as well as real-lifesituations).

Integrals:

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, only simple integrals of the type $\int\frac{dx}{x^2\pm a^2}$,$\int\frac{dx}{a^2-x^2}$, $\int\frac{dx}{\sqrt{x^2\pm a^2}}$,$\int\frac{dx}{\sqrt{a^2-x^2}}$,$\int\frac{dx}{ax^2+bx+c}$,$\int\frac{(px+q)dx}{ax^2+bx+c}$

$\int\frac{dx}{\sqrt{ax^2+bx+c}}$,$\int\frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$, $\int\sqrt{a^2-x^2}dx$, $\int\sqrt{x^2\pm a^2}dx$, $\int\sqrt{ax^2+bx+c}dx$,

$\int(px+q)\sqrt{ax^2+bx+c}dx$ to be evaluated.

SECOND TERM

October

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (withoutproof).Basic properties of definite integrals and evaluation of definite integrals.

Applications of the Integrals:

Applications in finding the area under simple curves, especially lines, areas of circles/Parabolas/ellipses (in standard form only), area between the two above said curves(the region should be clearly identifiable).

Differential Equations:

Definition, order and degree, general and particular solutions of a differentiale quation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx}+p(x)y=q(x)$, where p(x) and q(x) are functions of x.

$\frac{dx}{dy}+p(y)x=q(y)$, where p(y) and q(y) are functions of y.

November

UNIT-IV: VECTORS AND THREE-DIMENSIONAL GEOMETRY

Vectors:

Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of Scalar (dot) product of vectors, Vector (cross) product of vectors, scalar triple product of vectors.

Three-Dimensional Geometry:

Direction cosines and direction ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesianand vector equation of a plane. Angle between (i) two lines, (ii)two planes. (iii) a line and a plane. Distance of a point from a plane.

UNIT-V: LINEAR PROGRAMMING

Linear Programming:

Introduction, definition of related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Assignment No. 1

Relations and Functions

Note:Q1-7are very short and short answer type questions

1. If the function $f: R \to A$ given by $f(x)=\dfrac{x^2}{x^2+1}$ is surjective, then find A.
2. Let A = $\{2,3,4,5\}$.Define a relation on A which is reflexive and symmetric but not transitive.
3. If $f: R \to R$ and $g: R \to R$ are defined by $f(x)=8x^3;\ g(x)=x^{\frac{1}{3}}$, then find $f \circ g$ & $g \circ f$.
4. If $f(x)=\left(a-x^n\right)^{1/n}$, then find $(f \circ f)(x)$
5. Let $f(x)=x^2-2;\ g(x)=x+2,\ x \in R,$ find $(g \circ f)(1)$
6. If $f(x)=\dfrac{x-1}{x+1}$, find $(fof^{-1})(2)$, assuming that f^{-1} exists.
7. Let f be the greatest integer function and g be the absolute value function, find the value of $(gof)\left(\dfrac{5}{3}\right)-(fog)\left(\dfrac{-5}{3}\right)$.
8. Show that $f: R-\{-1\} \to R-\{1\}$ given by $f(x)=\dfrac{x}{x+1}$ is invertible. Also find its inverse.
9. Let $A=\{1,2,3,\ldots,9\}$ and R be the relation on $A \times A$ defined by (a,b)R(c,d) if a+d=b+c for all $(a,b),(c,d) \in A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class [(2,5)].
10. Let N be the set of all natural numbers and R be a relation on $N \times N$, defined as $(a,b)R(c,d) \Leftrightarrow ad=bc$, for all $(a,b),(c,d) \in N \times N$. Show that "R" is an equivalence relation.
11. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a,b)R(c,d)$ if $ad(b+c)=bc(a+d)$. Show that R is an equivalence relation.

12. If $f, g : R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$ for all $x \in R$. Then, find fog and gof. Hence, find $fog(-3)$, $fog(5)$ and $gof(-2)$.

13. If $f : R \to R$ is a function defined by $f(x) = x^3 + 27 \;\; \forall \;\; x \in R$. Show that f is bijective and hence find f^{-1}.

14. (i) Let R be the equivalence relation in the set $A = \{0,1,2,3,4,5\}$ given by $R = \{(a,b) : 2 \text{ divides } (a-b)\}$. Write the equivalence class $[0]$.

    (ii) Let R be a relation on set of natural numbers N as follows. R= $\{(x,y) : x \in N, y \in N \text{ and } 2x + y = 24\}$. Find the domain and range of the relation R. Is R an equivalence relation or not?

15. Consider $f : R_+ \to (-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \dfrac{\sqrt{54 + 5y} - 3}{5}$

16. If the function $f : R \to R$ be defined by $f(x) = 2x - 3$ and $g : R \to R$ by $g(x) = x^3 + 5$ then show that $f \circ g$ is invertible. Also find $(f \circ g)^{-1}(x)$ hence find $(f \circ g)^{-1}(9)$.

Assignment No. 2
Inverse Trigonometric Functions

Q 1 – 7 are very short and short answer type questions

Q1. For the principal values, evaluate $\tan^{-1}(-1)+\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$.

Q2. Find the value of $\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$

Q3. Simplify: $\cot^{-1}\frac{1}{\sqrt{x^2-1}}$ for x < -1

Q4. If $4\sin^{-1}x+\cos^{-1}x=\pi$, find the value of x.

Q5. Solve for x: $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$

Q6. If $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$, find x.

Q7. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1}x$, prove that $x=\frac{a+b}{1-ab}$. , $|a|\le 1, |b|\le 1$

Q8. Solve for x: $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

Q9. If $\left(\tan^{-1}x\right)^2+\left(\cot^{-1}x\right)^2=\frac{5\pi^2}{8}$, then find x.

Q10. Prove that if $\frac{1}{2}\le x\le 1$ then $\cos^{-1}x+\cos^{-1}\left(\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right)=\frac{\pi}{3}$

Q11. Simplify each of the following :

a. $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x\neq 0$

b. $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), 0<x<\frac{\pi}{2}$

c. $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right), 0\le x\le 1$

Q12. Prove that : $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\begin{cases}\frac{\pi}{4}+\frac{x}{2}, 0<x<\frac{\pi}{2}\\ \frac{\pi}{4}-\frac{x}{2}, \pi<x<\frac{3\pi}{2}\end{cases}$

Q13. Prove: $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$

Q14. Solve for x: $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

Q15. Solve for x: $\sin^{-1}(1-x)+\sin^{-1}x=\cos^{-1}x$

Assignment No. 3
MATRICES

Questions 1-5 are very short and short answer type questions

1. Find the values of x, y, if $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$
2. If A is a square matrix , such that $A^2 = A$, then find $(I+A)^2 - 3A$
3. If A=$\lfloor a_{ij} \rfloor$ where $a_{ij} = \begin{cases} i+j & if \;\; i \geq j \\ i-j & if \;\; i < j \end{cases}$ then construct a 2×3 matrix A.
4. Prove that the diagonal elements of a skew symmetric matrix are all zero.
5. Find the matrix P satisfying the matrix equation $\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}P\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}=\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$
6. If $A=\begin{bmatrix}-1\\2\\3\end{bmatrix}, B=\begin{bmatrix}-2 & -1 & -4\end{bmatrix}$, verify that $(AB)^T = B^T A^T$
7. Express $\begin{bmatrix}3 & 2 & 7\\1 & 4 & 3\\-2 & 5 & 8\end{bmatrix}$ as the sum of symmetric and skew symmetric matrix.
8. Find the value of "x" which satisfy $\begin{bmatrix}x & 4 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix}=0$.
9. If $A=\begin{bmatrix}\alpha & 1\\0 & \alpha\end{bmatrix}$,then prove that $A^n=\begin{bmatrix}\alpha^n & n\alpha^{n-1}\\0 & \alpha^n\end{bmatrix}$ by principle of Mathematical Induction
10. If $A=\begin{bmatrix}1 & 2 & 0\\3 & -4 & 5\\0 & -1 & 3\end{bmatrix}$, and $f(x)=x^2-4x+3$, then find $f(A)$
11. An institute conducts classes in two batches I and II and fees for rich and poor children are different. In batch I it has 20 poor and 5 rich children and total monthly collection is Rs 9,000. Whereas in batch II it has 5 poor and 25 rich children and total monthly collection is Rs 26,000. Using matrix method, find monthly fees paid by each child of two types.
12. Find the inverse of each of the following matrices by using elementary transformations:

    (i) $\begin{bmatrix}2 & 5\\1 & 3\end{bmatrix}$ (ii) $\begin{bmatrix}-1 & 1 & 2\\1 & 2 & 3\\3 & 1 & 1\end{bmatrix}$

Assignment No. 4

Determinants

Questions 1-8 are very short and short answer type questions

1. If for a matrix A , $|A| = 4$, find $|3A|$, where matrix A is of order 2×2

2. A is a non singular matrix of order 3 and $|A| = -5$, find $|adjA|$.

3. If $\left[a_{ij}\right]$ is a matrix of order 3×3, find the value of

   $a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33}$, where C_{ij} = the cofactor of a_{ij}.

4. If A is a square matrix of order 3 and $|4A| = k|A|$, find k

5. If $\left[a_{ij}\right]$ is a matrix of order 2×2 such that $|A| = -15$, then find

   $a_{21}C_{21} + a_{22}C_{22}$, where C_{ij} = the cofactor of a_{ij}.

6. If A is a square matrix of order 3, then find $|adjA|$ if $|A| = 6$.

7. Without expanding the determinant at any stage, prove that $\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix} = 0$

8. Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where a, b, c are in A.P.

9. Using matrix method solve the following system of linear equations :

a) $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

b) $5x + 3y + z = 16, 2x + y + 3z = 19, x + 2y + 4z = 25$

10. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB and hence solve the following system of equations: $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$

11. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, Find A^{-1}, and hence solve $\begin{matrix} 3x + 4y + 7z = 14 \\ 2x - y + 3z = 4 \\ x + 2y - 3z = 0 \end{matrix}$

12. If $A=\begin{bmatrix}5 & 3\\12 & 7\end{bmatrix}$, then verify that $A^2-12A-I=0$, where I is a unit matrix of order 2 and hence find A^{-1}.

13. Find the matrix "A" for which $\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

14. Area of the triangle with vertices $(-2,4),(2,k)(5,4)$ is 35 square units. Find "k".

15. Using properties of determinants prove that $\begin{vmatrix}y+z & z & y\\z & z+x & x\\y & x & x+y\end{vmatrix}=4xyz$.

16. Using properties of determinants prove that
$$\begin{vmatrix}a+b+2c & a & b\\c & b+c+2a & b\\c & a & c+a+2b\end{vmatrix}=2(a+b+c)^3.$$

17. If $p\neq 0, q\neq 0$ and $\begin{vmatrix}p & q & p\alpha+q\\q & r & q\alpha+r\\p\alpha+q & q\alpha+r & 0\end{vmatrix}=0$ then using the properties of determinants, prove that at least one of the following statements is true (a) p,q,r are in G.P. (b) α is a root of the equation $px^2+2qx+r=0$

18. Using properties of determinants prove that
$$\begin{vmatrix}(b+c)^2 & a^2 & a^2\\b^2 & (c+a)^2 & b^2\\c^2 & c^2 & (a+b)^2\end{vmatrix}=2abc(a+b+c)^3$$

19. Using properties of determinants prove that $\begin{vmatrix}a & a+b & a+b+c\\2a & 3a+2b & 4a+3b+2c\\3a & 6a+3b & 10a+6b+3c\end{vmatrix}=a^3$.

20. Using properties of determinants prove that $\begin{vmatrix}b^2+c^2 & ab & ac\\ab & c^2+a^2 & bc\\ca & bc & a^2+b^2\end{vmatrix}=4a^2b^2c^2$

Assignment No. 5

Continuity and Differentiability

Questions 1 – 10 are very short and short answer type questions

1. If $f(1)=4,\ f'(1)=2$, find the value of derivative of $\log f\left(e^{x}\right)$ w.r.t. "x" at $x=0$

2. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(\frac{2^{x+1}.3^{x}}{1+(36)^{x}}\right), x<0, 0<6^{x}<1$

3. If $y=e^{x+e^{x+e^{x+e^{x+\cdots}}}}$, Prove that $\frac{dy}{dx}=\frac{y}{1-y}$.

4. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^{2}}\right)$ with respect to "x", $|x|<1$.

5. State the points of discontinuity for the function $f(x)=[x]$ in $-3<x<3$

6. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^{2}}-1}{x}\right)$ with respect to "x".

7. Differentiate $\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{x}}{1+\sqrt{a}\sqrt{x}}\right)$ w.r.t x.

8. Differentiate $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ with respect to $\tan^{-1}x$, $0<x<\pi$

9. If "f" is a differentiable function at "x"=1 such that $f(1)=5,\ f'(1)=\frac{1}{5}$ & $g=f^{-1}$, then find $g'(5)$

10. If $y=\tan^{-1}\frac{5x}{1-6x^{2}}, \frac{-1}{\sqrt{6}}<x<\frac{1}{\sqrt{6}}$, then prove that $\frac{dy}{dx}=\frac{2}{1+4x^{2}}+\frac{3}{1+9x^{2}}$

11. For what value of "k" the function $f(x)=\begin{cases}\frac{\sqrt{5x+2}-\sqrt{4x+4}}{x-2}; & x\neq 2\\ k & x=2\end{cases}$ is continuous at $x=2$.

12. Determine the value of "a","b" and "c" if the following function is continuous at x=0 $f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x}; & \text{when } x \prec 0\\ c & \text{when } x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{\mathrm{b}x^{\frac{3}{2}}} & \text{when } x \succ 0\end{cases}$

13. Discuss the derivability of the function $f(x)=\begin{cases}x-1, & x<2\\ 2x-3, & x\geq 2\end{cases}$ at $x=2$

14. Find "a" and "b" , if the function given by $f(x)=\begin{cases}x^2, & x\leq 1\\ ax+b, & x>1\end{cases}$ is differentiable at $x=1$

15. Discuss the continuity of the function $f(x)=|x-1|-|x-2|$.

16. Verify Rolle's theorem for the function $f(x)=e^x\left(sinx-\cos x\right)$ on $\left[\frac{\pi}{4},\frac{5\pi}{4}\right]$

17. If $y=x^{\sin x}+\sin^{-1}\sqrt{x}$, find $\frac{dy}{dx}$.

18. If $x^p.y^q=\left(x+y\right)^{p+q}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

19. If $y=\sin\left(m\sin^{-1}x\right)$, prove that $\left(1-x^2\right)y_2-xy_1+m^2y=0$.

20. If $x=\sin t,\ y=\sin pt$, prove that $\left(1-x^2\right)y_2-xy_1+p^2y=0$.

21. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a\left(x-y\right)$, Show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

22. If $x=a\left(\cos\theta+\log\tan\left(\frac{\theta}{2}\right)\right)$ and $y=a\sin\theta$ find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{4}$

23. If $x=a\left(1-\cos^3\theta\right)$and $\mathrm{y}=\mathrm{a}\left(\sin^3\theta\right)$. Find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{6}$

24. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ $w.r.t.\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ where $x\in\left(\frac{1}{\sqrt{2}},1\right)$

25. If $y=x^x$, prove that $\frac{\mathrm{d}^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0$

Application of Derivatives

1. Water is dripping out from a conical funnel at a uniform rate of 4cucm/sec. When the slant height is 3cm , find the rate of decrease of slant height of the water cone, given the vertical angle of the funnel is 120°.
2. The total cost C(x) associated with production of x units is given by $C(x) = 0.0005x^3 - 0.002x^2 + 30x + 3000$. Find the marginal cost when 3 units are produced.
3. A balloon which always remains spherical, has a variable diameter 3(2x+5) . Determine the rate of change of volume w.r.t x.
4. Find the equation of the tangent to the curve $x = \theta + sin\theta$, $y = 1 + cos\theta$ at $\theta = \frac{\pi}{4}$
5. Find equation of all lines of slope 0 and that are tangent to the curve
$$y = \frac{1}{x^2 - 2x + 3}$$
6. Find equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $y = 4x - 5$
7. Find the intervals in which the following functions are strictly increasing or strictly decreasing:

a) $f(x) = 10 - 6x - 2x^2$
b) $f(x) = 2x^3 - 12x^2 + 18x + 15$
c) $f(x) = 5 + 36x + 3x^2 - 2x^3$
d) $f(x) = 5x^3 - 15x^2 - 120x + 3$
e) $f(x) = -2x^3 - 9x^2 - 12x + 1$
f) $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$
g) $f(x) = \log(1 + x) - \frac{x}{1+x}$

8. Find all the points of local maxima and minima of the following functions:

a) $f(x) = x^3 - 3x$
b) $f(x) = x^3(x - 1)^2$
c) $f(x) = (x - 1)(x + 2)^2$
d) $f(x) = (x - 1)^3(x + 1)^2$
e) $f(x) = \sin x - cosx$, $0 < x < 2\pi$
f) $f(x) = \sin x + \cos x$, $0 < x < \frac{\pi}{2}$
g) $f(x) = x^4 - 62x^2 + 120x + 9$
h) $f(x) = x^3 - 6x^2 + 9x + 15$

9. Find the absolute maximum and minimum values of the following functions in the given intervals:

a) $f(x) = 4x - \frac{x^2}{2}$ in [-2, 4.5]
b) $f(x) = 3\,x^4 - 8x^3 + 12x^2 - 48x + 25$ in [0,3]
c) $f(x) = cos^2x + sinx$, $x \in \left[0, \frac{\pi}{2}\right]$

d) $f(x) = (x-2)\sqrt{x-1}$ in [1,9]

10. Show that of all rectangles with given perimeter, the square has largest area.
11. Show that of all rectangles of given area, the square has the least perimeter.
12. Show that of all rectangles inscribed in a given circle, the square has maximum area.
13. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius 'a' is a square. Also find the side of the square.
14. If the sum of the lengths of the hypotenuse and side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.
15. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.
16. Prove that the area of a right angled triangle of given hypotenuse is maximum when the triangle is isosceles.
17. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.
18. A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter of the figure, find the dimensions in order that the area may be maximum.
19. Find the volume of the largest cylinder that can be inscribed in a sphere of radius 'r' cm.
20. Show that a cylinder of given volume which is open at the top, has maximum total surface area, when the height of the cylinder is equal to the radius of the base.
21. Show that the height of the closed cylinder of given surface area and maximum volume is equal to the diameter of the base.
22. Show that the height of a cylinder which is open at the top having a given surface area and greatest volume is equal to the radius of its base.
23. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius 'a' is $\frac{2a}{\sqrt{3}}$.
24. An open box with a square base is to be made of a given quantity of cardboard of area c^2sq units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$cu.units.

Assignment No. 6

Application of Derivatives

1. Water is dripping out from a conical funnel of semi vertical angle $\frac{\pi}{4}$ at a uniform speed of 2 $cm^3/\sec$ through a tiny hole at the vertex of the bottom. When the slant height of water is 4cm , find the rate of decrease of slant height of the water.
2. A man is moving away from a tower 49.6 m high at the rate of 2 m/s. Find the rate at which the angle of elevation of the top of the tower is changing, when he is at a distance of 36 m from the foot of the tower. Assume that the eye level of the man is 1.6 m from the ground.
3. Evaluate following up to three decimal places using differentiation:
   $\sqrt{25.2}$, $\sqrt[3]{29}$, $\sqrt{0.037}$
4. Find the intervals in which the function $f(x)=\log(1+x)-\frac{2x}{2+x}$ increasing or decreasing.
5. Find the intervals in which the function $f(x)=(x+1)^3(x-3)^3$ is increasing or decreasing. Also find the points at which the function has local maxima, local minima and the point of inflexion.
6. Find all the points of local maximum and minimum and the corresponding maximum and minimum values of the following function $\frac{3}{4}x^4-8x^3+\frac{45}{2}x^2+105$.
7. Find the point on the curve $y^2=4x$ which is nearest to the point $(2,-8)$
8. Find the equation of the tangent to the curve $y=(x^3-1)(x-2)$ at the points where the curve cuts the x –axis.
9. Find the intervals in which the function $f(x)=2x^3-9x^2+12x+15$ is increasing and decreasing.
10. Separate $\left[0,\frac{\pi}{2}\right]$ into sub intervals in which $f(x)=\sin^4 x+\cos^4 x$ is increasing or decreasing.

11. Find the points of local maxima and local minima and also the local maximum and local minimum values of the following functions : $(i)\ f(x)=2\cos x+x, x\in(0,\pi)$ $(ii)\ f(x)=2\sin x-x, x\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$
12. Find the equation of the tangent and normal to the curve $x=1-\cos\theta;\ y=\theta-\sin\theta$ at $\theta=\frac{\pi}{4}$
13. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of cone.
14. An open box with a square base is to be made of given iron sheet of area 27 sq.m. Show that the maximum volume of the box is 13.5 cu. m.
15. Find the coordinates of a point on the parabola $y=x^2+7x+2$ which is closest to the line $y=3x-3$
16. Find the equation of the normal to the curve $2y=x^2$, which passes through (2,1).

Indefinite integral

Some generalized results of the Method of substitution:

$\int f(x)\times f'(x)dx=\int t dt$, where $f(x)=t$, $\int f(g(x))\times g'(x)dx=\int f(t)dt$, where $g(x)=t$

Formula: $\int \frac{f'(x)}{f(x)}dx=\log\left|f(x)\right|+c$

This formula may be derived by using the method of substitution.

Results based on the above formula:

1. $\int \tan x dx=\log\left|\sec x\right|+c$ 2. $\int \cot x dx=\log\left|\sin x\right|+c$

3. $\int \sec x dx=\log\left|\sec x+\tan x\right|+c=\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|+c$

4. $\int co\sec x dx=\log\left|co\sec x-\cot x\right|+c=\log\left|\tan\frac{x}{2}\right|+c$

Direct application of the above formula in finding the following integrals:

1. $\int \frac{2x}{1+x^2}dx$ 2. $\int \frac{x}{9-4x^2}dx$ 3. $\int \frac{e^{2x}-1}{e^{2x}+1}dx$ 4. $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$ 5. $\int \frac{\sin x}{1+\cos x}dx$ 6. $\int \frac{2\cos x-3\sin x}{6\cos x+4\sin x}dx$

Formula: If $\int f(x)dx=g(x)+c$, then $\int f(ax+b)dx=\frac{1}{a}g(ax+b)+c$

This formula may be derived either by direct differentiation or by using the method of substitution.

Direct application of the above formula in finding the following integrals:

7. $\int \sec^2(7-4x)dx$ 8. $\int \tan^2(2x-3)dx$ 9. $\int \sqrt{ax+b}dx$ 10. $\int e^{2x+3}dx$

11. $\int \sec 2x dx$ 12. $\int \sin(3x-1)dx$

The application of method of substitution in finding some basic integrals:

13. $\int \frac{(\log x)^2}{x} dx$ 14. $\int \frac{1}{x + x\log x} dx$ 15. $\int x\sqrt{1+2x^2} dx$ 16. $\int (x^3 - 1)^{\frac{1}{3}} x^5 dx$ 17. $\int \frac{x}{e^{x^2}} dx$ 18. $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx$

Integrals of the type: $\int \frac{a\sin x + b\cos x}{c\sin x + d\cos x} dx$

19. $\int \frac{2\sin x + 3\cos x}{5\sin x + 4\cos x} dx$ 20. $\int \frac{1}{1+\cot x} dx$ 21. $\int \frac{1}{1-\tan x} dx$

Integration using trigonometric identities:

22. $\int \sin^2 x dx$ 23. $\int \cos^2 x dx$ 24. $\int \sin^3 x dx$ 25. $\int \cos^3 x dx$ 26. $\int \sin^2 (2x+5) dx$

27. $\int \sin^3 (2x+1) dx$ 28. $\int \sin 3x \cos 4x dx$ 29. $\int \sin 4x \sin 8x dx$ 30. $\int \cos 2x \cos 4x \cos 6x dx$ 31. $\int \sin^4 x dx$ 32. $\int \cos^4 (2x) dx$ 33. $\int tan^4 x dx$ 34. $\int \frac{1}{\sin x \cos^3 x} dx$ 35. $\int \sin^2 x \cos^2 x dx$ 36. $\int \sin^3 x \cos^3 x dx$

Problems which require multiplication and division by $\sin(a \pm b)$ ***or by*** $\cos(a \pm b)$

37. $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$ 38. $\int \frac{1}{\sin(x+a)\cos(x+b)} dx$

Integrals of the type: $\int \sin^m x \cos^n x dx$

If m is odd, then put cosx = t and if n is odd, then put sinx = t. If both are odd, then put either of them = t. If both are even, then use suitable trigonometric identities.

39. $\int \sin^3 x \cos^2 x dx$ 40. $\int \sin^4 x \cos^4 x dx$ 41. $\int \sin^5 x dx$

Integrals of the types: $\int \frac{1}{x^2 - a^2} dx$, $\int \frac{1}{a^2 - x^2} dx$, $\int \frac{1}{x^2 + a^2} dx$, $\int \frac{1}{\sqrt{x^2 - a^2}} dx$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$, $\int \frac{1}{\sqrt{x^2 + a^2}} dx$

42. $\int \frac{3x^2}{x^6+1}dx$ 43 $\int \frac{1}{\sqrt{1+4x^2}}dx$ 44. $\int \frac{1}{\sqrt{(2-x)^2+1}}dx$ 45. $\int \frac{1}{\sqrt{9-25x^2}}dx$

46. $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx$ 47. $\int \frac{\cos x}{\sqrt{\sin^2 x-2\sin x-3}}dx$

Integrals of the types: $\int \frac{1}{ax^2+bx+c}dx$, $\int \frac{px+q}{ax^2+bx+c}dx$, $\int \frac{1}{\sqrt{ax^2+bx+c}}dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$

48. $\int \frac{1}{x^2-2x-5}dx$ 49. $\int \frac{1}{9x^2+6x+5}dx$ 50. $\int \frac{5x-2}{3x^2+2x+1}dx$ 51. $\int \frac{x+3}{x^2-2x-5}dx$

52. $\int \frac{1}{\sqrt{x^2+2x+2}}dx$ 53. $\int \frac{1}{\sqrt{7-6x-x^2}}dx$ 54. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$ 55. $\int \frac{x+2}{\sqrt{4x-x^2}}dx$ 56. $\int \frac{2\sin 2\phi-\cos\phi}{6-\cos^2\phi-4\sin\phi}d\phi$

Integrals of the type: $\int \frac{\sin x \pm \cos x}{f(\sin 2x)}dx$

If the numerator is sinx + cosx, then put sinx – cosx = t. If the numerator is sinx - cosx, then put sinx+cosx = t.

57. $\int \frac{\cos x-\sin x}{1+\sin 2x}dx$ 58 $\int \frac{\cos x+\sin x}{\sqrt{\sin 2x}}dx$ 59. $\int \frac{\cos x+\sin x}{9+16\sin 2x}dx$

Integrals of the type: $\int \frac{1}{x^{\frac{1}{a}}+x^{\frac{1}{b}}}dx$

Put $x = t^{\text{the lcm of a and b}}$

60. $\int \frac{1}{x-\sqrt{x}}dx$ 61. $\int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx$

Integration using partial fractions:

Integrals of the type: $\int \frac{f(x)}{(x-a)(x-b)\ldots}dx$ ***(Denominator has non repeating linear factors)***

62. $\int \frac{x}{(x+1)(x+2)}dx$ 63. $\int \frac{1}{x^2-9}dx$ 64. $\int \frac{3x-1}{(x-1)(x-2)(x-3)}dx$ 65. $\int \frac{1-x^2}{x(1-2x)}dx$ 66. $\int \frac{2x-3}{(x^2-1)(2x+3)}dx$ 67. $\int \frac{x^3+x+1}{x^2-1}dx$

Integrals of the type: $\int \frac{f(x)}{(x-a)^m(x-b)...}dx$ ***(Denominator has repeating linear factors and/or non repeating linear factors)***

68. $\int \frac{x}{(x-1)^2(x+2)}dx$ 69. $\int \frac{3x-1}{(x+2)^2}dx$ 70. $\int \frac{3x-1}{(x-1)^3(x-2)}dx$

Integrals of the type: $\int \frac{f(x)}{(ax^2+bx+c)(x-d)^m(x-e)...}dx$ ***(Denominator has a quadratic factor which is not resolvable into linear factors and/or hasrepeating linear factors and/or non repeating linear factors)***

71. $\int \frac{2}{(1-x)(1+x^2)}dx$ 72. $\int \frac{x}{(x-1)(1+x^2)}dx$ 73. $\int \frac{1}{x^4-1}dx$ 74. $\int \frac{2x^2+1}{x^2(4+x^2)}dx$

Integrals of the type: $\int \frac{1}{x(x^n \pm 1)}dx$

75. $\int \frac{1}{x(x^n+1)}dx$ 76. $\int \frac{1}{x(x^4-1)}dx$

Integrals of the type: $\int \frac{(x^2+a)(x^2+b)...}{(x^2+c)(x^2+d)...}dx$

77. $\int \frac{(x^2+1)}{(x^2+2)(2x^2+1)}dx$ 78. $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx$

Some indirect problems of partial fractions:

79. $\int \frac{2x}{(x^2+1)(x^2+3)}dx$ 80. $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$ 81. $\int \frac{\sin x}{\sin 4x}dx$ 82. $\int \frac{\tan x+\tan^3 x}{1+\tan^3 x}dx$ 83. $\int \frac{1}{\sin x+\sin 2x}dx$

Integration by parts:

84. $\int x\sin x dx$ 85. $\int x^2e^x dx$ 86. $\int x^2 \log x dx$ 87. $\int x\sin^{-1} x dx$

88. $\int x\tan^{-1} x dx$ 89. $\int \sin^{-1} x dx$ 90. $\int (\sin^{-1} x)^2 dx$ 91. $\int \log x dx$ 92. $\int (\log x)^2 dx$ 93. $\int x(\log x)^2 dx$ 94. $\int tan^{-1}x dx$ 95. $\int \sec^3 x dx$ 96. $\int co\sec^3 x dx$

Integrals based on the result: $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)+c$

97. $\int e^x(\sin x+\cos x)dx$ 98. $\int \frac{xe^x}{(1+x)^2}dx$ 99. $\int e^x \frac{(1+\sin x)}{(1+\cos x)}dx$ 100. $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$

101. $\int \frac{(x-3)e^x}{(x-1)^3}dx$ 102. $\int \frac{(x^2+1)e^x}{(1+x)^2}dx$ 103. $\int \left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}\right\}dx$ 104. $\int \left\{\log(\log x)+\frac{1}{(\log x)^2}\right\}dx$ 105. $\int \left(\log x+\frac{1}{x^2}\right)e^x dx$

Integrals of the types: $\int e^{ax}\sin bx dx, \int e^{ax}\cos bx dx$

106. $\int e^{2x}\sin x dx$ 107. $\int e^{-x}\cos x dx$ 108. $\int e^x \sin^2 x dx$ 109. $\int e^{2x}\cos^2 x dx$

Integrals of the types: $\int \sqrt{a^2-x^2}dx, \int \sqrt{a^2+x^2}dx, \int \sqrt{x^2-a^2}dx, \int \sqrt{ax^2+bx+c}dx,$ $\int (px+q)\sqrt{ax^2+bx+c}dx$

110. $\int \sqrt{4-x^2}dx$ 111. $\int \sqrt{1-4x^2}dx$ 112. $\int \sqrt{(x+2)^2-9}dx$ 113. $\int \sqrt{x^2+4x+6}dx$

114. $\int \sqrt{1+3x-x^2}dx$ 115. $\int (x+3)\sqrt{3-4x-x^2}dx$

Integrals of the types: $\int \frac{1}{a+b\sin x}dx$, $\int \frac{1}{a+b\cos x}dx$, $\int \frac{1}{a\sin x+b\cos x}dx$, $\int \frac{1}{a\sin x+b\cos x+c}dx$

116. $\int \frac{1}{1-2\sin x}dx$ 117. $\int \frac{1}{5-4\cos x}dx$ 118. $\int \frac{1}{3\sin x+\cos x}dx$ 119. $\int \frac{1}{\sin x+\cos x+2}dx$

Some specific Integrals:

120. $\int \frac{x^2+1}{x^4+1}dx$ 121. $\int \frac{x^2-1}{x^4+1}dx$ 122. $\int \frac{x^2}{x^4+1}dx$ 123. $\int \frac{1}{x^4+1}dx$ 124. $\int \frac{1}{x^4+3x^2+1}dx$ 125. $\int \frac{x^2+1}{x^4+7x^2+1}dx$ 126. $\int \sqrt{\tan x}\,dx$, 127. $\int \sqrt{\cot x}\,dx$

Integrals of the types: $\int \frac{1}{a\sin^2 x+b\cos^2 x}dx$, $\int \frac{1}{a+b\cos^2 x}dx$, $\int \frac{1}{a+b\sin^2 x}dx$, $\int \frac{1}{(a\sin x+b\cos x)^2}dx$, $\int \frac{1}{a\sin^2 x+b\cos^2 x+c}dx$

128. $\int \frac{1}{3+2\cos^2 x}dx$ 129. $\int \frac{1}{1+3\sin^2 x}dx$ 130. $\int \frac{1}{4\sin^2 x+5\cos^2 x}dx$ 131. $\int \frac{\cos x}{\cos 3x}dx$

132. $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$ 133. $\int \frac{1}{(2\sin x+\cos x)(\sin x-2\cos x)}dx$ 134. $\int \frac{1}{2-3\cos 2x}dx$ 135. $\int \frac{1}{(2\sin x+3\cos x)^2}dx$

Miscellaneous problems which require some specific steps in order to reduce them to some known form:

136. $\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx$ 137. $\int \sqrt{\frac{a-x}{a+x}}dx$ 138. $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}}dx$ 139. $\int \sqrt{\sec x-1}\,dx$ 140. $\int \frac{1}{\sin x+\sec x}dx$

141. $\int \sqrt{\frac{x}{a^3-x^3}}dx$ 142. $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}dx$ 143. $\int \frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}dx$ 144. $\int \frac{\sqrt{x^2+1}\{\log(x^2+1)-2\log x\}}{x^4}dx$ 145. $\int \left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

Assignment No. 7(a)

Indefinite Integrals-1

(Method of Substitution, Trigonometric Identities, Special Integrals)

Q1-10 are very shortand short answer type questions

1. Evaluate : $\dfrac{x-2}{\sqrt{x^2+1}}$

2. Evaluate : $\int x^{1/2}(1+x^{3/2})\ dx$

3. Write a value of $\int \dfrac{1}{\sqrt{3}\sin x+\cos x}\ dx$

4. Write a value of $\int \dfrac{1-\tan x}{x+\log\cos x}dx$

5. If $f'(x)=\dfrac{4}{x^2}$ and $f(1)=6$, find $f(2)$

6. Write a value of $\int \left(\cos(\log x)+\sin(\log x)\right) dx$

7. If $\int \dfrac{2^x}{\sqrt{1-4^x}}\ dx = k\sin^{-1}(2^x)+c$, then what is the value of k?

8. Find a value of $\int \dfrac{1}{9x^2-16}dx$

9. Evaluate $\int \dfrac{x}{e^{x^2}}dx$

10. Evaluate: $\int \dfrac{dx}{e^x+e^{-x}}$

11. $\int \dfrac{1}{(\sin x-2\cos x)(2\sin x+\cos x)}\ dx$

12. Evaluate : $\int \dfrac{1}{\sqrt{\sin^3 x(\sin x+2\cos x)}}\ dx$

13. Evaluate : $\int \operatorname{cosec}^8 x\ dx$

14. Evaluate : $\int 5^{5^{5^x}}.5^{5^x}.5^x\ dx$

15. Evaluate : $\int \dfrac{x^2}{\sqrt{x^6-a^6}}\ dx$

16. Evaluate : $\int \dfrac{x^3+x^2+2x+1}{x^2-x+1}\ dx$

17. Evaluate : $\int \sqrt{\dfrac{3-x}{x-2}}\ dx$

18. Evaluate : $\int \dfrac{e^x}{2e^{2x}+3e^x+1}\ dx$

19. Evaluate : $\int \dfrac{6x+5}{\sqrt{6+x-2x^2}}\ dx$

20. Evaluate : $\int \sqrt{1+\operatorname{cosec} x}\ dx$

21. $\dfrac{1}{(3\sin x + \cos x)^2}$

22. $\dfrac{1}{x\log x\log(\log x)}$

23. $\sqrt{\tan\theta} + \sqrt{\cot\theta}$

24. $\dfrac{1}{\sin^2 x + \sin 2x}$

25. $\dfrac{\log x^2}{x}$

26. $\sqrt{\sec x - 1}$

27. $\dfrac{1}{5 + 7\cos x + \sin x}$

28. $\dfrac{x^2 + 9}{x^4 + 81}$

29. $\dfrac{1}{(a^2\sin^2 x + b^2\cos^2 x)^2}$

Assignment No. 7(b)

Indefinite Integrals-2

(By Parts, Partial Fraction& three more special integrals)

Q: Integrate following functions with respect to x:

1. $\dfrac{x^2}{(x-1)(x+1)^2}$

2. $\cos^{-1}(4x^3-3x)$

3. $e^x\left[\dfrac{x-1}{(x+1)^3}\right]$

4. $\dfrac{\tan x+\tan^3 x}{1+\tan^3 x}$

5. $\left[\dfrac{\sqrt{1-\sin x}}{1+\cos x}\right]e^{-x/2}$

6. $\dfrac{x\tan^{-1}x}{(1+x^2)^{3/2}}$

7. $\dfrac{\log(x+2)}{(x+2)^2}$

8. $\dfrac{1}{\sin x(5-4\cos x)}$

9. $e^{\sqrt{x}}$

10. $\cos^3\sqrt{x}$

11. $e^{-3x}\cos 2x$

12. $e^x(\tan x-\log\cos x)$

13. $e^{2x}\dfrac{\sin 4x-2}{1-\cos 4x}$

14. $\dfrac{x^3-1}{x^3+x}$

15. $(x+1)\sqrt{3-x-x^2}$

16. $x^2\operatorname{cosec}^{-1}x$

17. $\left[\log(\log x)+\dfrac{1}{(\log x)^2}\right]$

18. $(3x+1)\sqrt{x^2+2x-3}$

Practice Questions

Integrate following functions w.r.t. x

1. $\dfrac{x}{x^4+x^2+1}$
2. $\dfrac{\sin x}{\sqrt{\cos^2 x-2\cos x-3}}$
3. $\dfrac{\sin^3 x}{\sqrt{\cos x}}$
4. $\dfrac{1}{1+\sqrt{x}}$
5. $\dfrac{1}{x\log x\log(\log x)}$
6. $\dfrac{1}{x(x^n+1)}$
7. $\dfrac{1}{\sqrt{x}+\sqrt[3]{x}}$
8. $\dfrac{1}{(3\sin x+\cos x)^2}$
9. $\dfrac{1}{sinx+\sqrt{3}\cos x}$
10. $\cos^7 x$
11. $\dfrac{1-x^2}{x(1-2x)}$
12. $\dfrac{\tan x}{\sqrt{\sin^4 x+\cos^4 x}}$
13. $\dfrac{\cos 2x}{\sin x}$
14. $\dfrac{\sin x+\cos x}{\sqrt{\sin 2x}}$
15. $\dfrac{1}{\sqrt{2x-x^2}}$
16. $\dfrac{x^2+5x+3}{x^2+3x+2}$
17. $\dfrac{2\sin 2x-\cos x}{6-\cos^2 x-4\sin x}$
18. $\dfrac{1}{4\sin^2 x+5\cos^2 x}$
19. $\dfrac{1}{3\sin^2 x+8\cos^2 x+1}$
20. $\dfrac{1}{2-3\cos 2x}$
21. $\dfrac{\cos x}{\cos 3x}$
22. $\dfrac{1}{\sec x+\cos ecx}$
23. $\dfrac{1}{(e^x+e^{-x})^2}$
24. $\dfrac{\sin 2x}{\sin^4 x+\cos^4 x}$
25. $\dfrac{1}{1+\cot x}$
26. $\sqrt{\cot\theta}$
27. $\dfrac{ax^3+bx}{x^4+c^2}$
28. $\sqrt{\dfrac{1+x}{x}}$
29. $\left(\sin^{-1} x\right)^3$
30. $\dfrac{x^2+x+1}{(x+1)^2(x+2)}$
31. $\dfrac{1}{\sin x(2+3\cos x)}$
32. $\dfrac{\cos x}{1+\cos x}$
33. $e^x\dfrac{(1-x)^2}{\left(1+x^2\right)^2}$
34. $\dfrac{1}{1+x+x^2+x^3}$
35. $\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}$
36. $\dfrac{1}{a+b\tan x}$

37. $\sqrt{1+x-2x^2}$

38. $\dfrac{\log(1-x)}{x^2}$

39. $e^x\left[\dfrac{1-\sin x}{1-\cos x}\right]$

40. $e^x\left[\dfrac{x-1}{(x+1)^3}\right]$

41. $(1-2x)\sqrt{4-3x-3x^2}$

42. $\left[\dfrac{\sqrt{1-\sin x}}{1+\cos x}\right]e^{-x/2}$

43. $\dfrac{x^3-1}{x^3+x}$

44. $(\log x)^2$

45. $\sin x\sin 2x\sin 3x$

46. $\dfrac{1}{\sin x(5-4\cos x)}$

47. $\dfrac{1}{(x-1)^2(x^2+2)}$

48. $\dfrac{\sin x}{(2+\cos x)(1-3\cos x)}$

Assignment No. 8

Definite Integrals

Q1 - 8 are very shortand short answer type questions.

1. Evaluate, $\int_0^{1.5}[x]\,dx$ (where [x] is greatest integer function)

2. If $\int_0^1\left(3x^2+2x+k\right)dx=0$, then find "$k$"

3. If $\int_0^a 3x^2dx=8$, then find the value of "a"

4. Evaluate $\int_{-1}^1|1-x|dx$

5. Evaluate $\int_0^1 e^{|x|}dx$

6. If $\int_0^k\frac{dx}{2+8x^2}=\frac{\pi}{16}$, find the value of k.

7. Evaluate $\int_0^{\pi/2}\frac{\sqrt{\cot x}}{1+\sqrt{\cot x}}\,dx$

8. If $\int_a^b x^3dx=0$ and $\int_a^b x^2\,dx=\frac{2}{3}$, find the value of a and b.

9. Evaluate the following definite integrals:

a) $\int_0^{\pi/4}2\tan^3 x\,dx$

b) $\int_0^{\pi/2}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

c) $\int_0^{\pi}\frac{dx}{6-\cos x}$

d) $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\cot^{3/2}x}$

e) $\int_0^{\pi}e^{\cos^2 x}\cos x\,dx$

f) $\int_0^{\pi/2}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}\,dx$

g) $\int_0^1\cot^{-1}\left(1-x+x^2\right)dx$

h) $\int_0^1\frac{\log(1+x)}{1+x^2}\,dx$

i) $\int_1^3\left|x^2-2x\right|dx$

j) $\int_0^{\pi}\frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$

k) $\int_{-5}^{0} \left(|x| + |x+2| + |x+5|\right) dx$

l) $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$

m) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$

10. Evaluate following definite integrals as limit of a sum:

i. $\int_{0}^{1} \left(x^2 - 3x\right) dx$

ii. $\int_{1}^{3} e^{2x} dx$

iii. $\int_{-2}^{2} \left(3x^2 - 2x + 4\right) dx$

iv. $\int_{0}^{4} \left(x + e^{x}\right) dx$

Assignment No. 9

Applications of Integrals-Area of the bounded regions

1. Find the area of the region bounded by the curve $y^2 = x-2, x=4, x=6$ and the x axis in the first quadrant using integration.

2. Sketch the region common to the circle $x^2+y^2=16$ and the parabola $x^2=6y$.Also find the area of the region using integration.

3. Find the area of the region bounded between the parabolas $y^2 = 4ax, x^2 = 4ay$, where a>0.
4. Using Integration find the area of the triangle ABC whose vertices has coordinates given by $A(2,5), B(4,7), C(6,2)$
5. Compute the area bounded by the lines $x+2y=2,\ y-x=1\ \&\ 2x+y=7$

6. Find the area of the region bounded by the curve $y^2 = 2y - x$ and the y axis.

7. Using integration find the area of the region $\{(x,y):|x| \le y \le \sqrt{4-x^2}\ \}$
8. Find the area bounded by the triangle whose vertices are $(0,0),(2,4),(4,-2)$.
9. Sketch the graph of $f(x) = \begin{cases} |x-2|+2, x \le 2 \\ x^2-2, x>2 \end{cases}$.Evaluate $\int_0^4 f(x)dx$, what does the value of this integral represent on the graph?
10. Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$
11. Find the area of the region $\{(x,y): x^2+y^2 \le 1 \le x+y\}$
12. Find the area of the region enclosed between the circles $x^2+y^2=4, (x-2)^2+y^2=1$.
13. Using integration, find the area of the triangle formed by the tangent and the normal to the curve $y^2 = -4x$ at the point (-1, 2) and the x-axis.
14. Using integration find the area of the region bounded by the parabola $y^2 = 2x$ and the line $x - y = 4$
15. Using integration find the area of the region $\{(x,y): |x-1| \le y \le \sqrt{5-x^2}\}$

Differential Equations

Find the general solution:	Answer:
Q1. $\sqrt{a+x}\,\dfrac{dy}{dx} = -xy$	$\log y + \dfrac{2}{3}(a+x)^{3/2} - 2a\sqrt{a+x} = c$
Q2. $\left[x\sqrt{x^2+y^2} - y^2\right]dx + xydy = 0$	$\sqrt{x^2+y^2} = x\log\left\lvert\dfrac{c}{x}\right\rvert$
Q3. $\left(y - x\dfrac{dy}{dx}\right)x = y$	$\log\left\lvert\dfrac{x}{y}\right\rvert + \dfrac{1}{x} = c$
Q4. $\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\dfrac{x}{y}\right)dy = 0$	$e^{x/y} + \dfrac{x}{y} = \dfrac{c}{y}$
Q5. $y - x\dfrac{dy}{dx} = a\left(y^2 + \dfrac{dy}{dx}\right)$	$(x+a)(1-ay) = cy$
Q6. $\dfrac{dy}{dx} = e^{x+y} + x^2e^{x^3+y}$	$e^{-y} = e^x + \dfrac{1}{3}e^{x^3} + c$
Q7. $\dfrac{dy}{dx} + \dfrac{y^2+y+1}{x^2+x+1} = 0$	$\sqrt{3}(x+y-1) = c(1-x-y-2xy)$
Q8. $\sqrt{1+x^2+y^2+x^2y^2} + xy\dfrac{dy}{dx} = 0$	$\sqrt{1+x^c} + \sqrt{1+y^2} = \log\dfrac{1+\sqrt{1+x^2}}{x} + c$
Q9. $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$	$\log\dfrac{y}{x} = cx$
Q10. $\left(x\cos\dfrac{y}{x} + y\sin\dfrac{y}{x}\right)y - \left(y\sin\dfrac{y}{x} - x\cos\dfrac{y}{x}\right)x\dfrac{dy}{dx} = 0$	$\sec\dfrac{y}{x} = cxy$
Q11. $\dfrac{dy}{dx} = \dfrac{x+y+1}{x+1}$	$\dfrac{y}{x+1} = \log(x+1) + c$
Q12. $x\dfrac{dy}{dx} - y = (x-1)e^x$	$y = e^x + cx$
Q13. $(1+y^2) + \left(x - e^{-\tan^{-1}y}\right)\dfrac{dy}{dx} = 0$	$xe^{\tan^{-1}y} = \dfrac{1}{2}e^{2\tan^{-1}y} + c$

Q14. $x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)=1$	$y=\frac{\sin x}{x}+\frac{c\cos x}{x}$
Q15. $ydx+(x-y^2)dy=0$	$x=\frac{y^2}{3}+\frac{c}{y}$
Q16. $\frac{dy}{dx}=1-x+y-xy$	$\log\lvert 1+y\rvert=x-\frac{x^2}{2}+c$
Q17. $(x+y+1)\frac{dy}{dx}=1$	$(x+y+1)-\log\lvert x+y+2\rvert=x+c$
Q18. $\frac{dy}{dx}+1=e^{x-y}$	$\frac{-1}{2}\log\lvert 2e^{y-x}-1\rvert=x+c$
Q19. $(x\log x)\frac{dy}{dx}+y=\frac{2}{x}\log x$	$y\log x=\frac{-2}{x}(\log x+1)+c$
Q20. $\log(\frac{dy}{dx})=ax+by$	$-\frac{1}{b}e^{-by}=\frac{1}{a}e^{ax}+c$
Q21. $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dy}{dx}=1$	$ye^{2\sqrt{x}}=2\sqrt{x}+c$
Q22. $xdy-ydx=\sqrt{x^2+y^2}dx$	$y+\sqrt{x^2+y^2}=cx^2$
Q23. $\frac{dy}{dx}+1=e^{x+y}$	$(x+C)e^{x+y}+1=0$
Q24. $y+\frac{d}{dx}(xy)=x(\sin x+\log x)$	$y=-\cos x+\frac{2\sin x}{x}+\frac{2\cos x}{x^2}+\frac{x}{3}\log x-\frac{x}{9}+cx^{-2}$
Q25. Form the differential equation having $y=(\sin^{-1}x)^2+A\cos^{-1}x+B$, where A and B are arbitrary constants, as its general solution	$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0$
Q26. Form the differential equation of system of concentric circles with centre (1,2)	$(y-2)\frac{dy}{dx}+(x-1)=0$

Find the particular solution:

Question		Answer:
Q27.	$\left(1+y^2\right)(1+\log x)dx + xdy = 0, y(1)=1$	$\log\lvert x\rvert + \frac{(\log\lvert x\rvert)^2}{2} + \tan^{-1} y = \frac{\pi}{4}$
Q28.	$\left(1+\sin^2 x\right)dy + (1+y^2)\cos x dx = 0, y(\frac{\pi}{2}) = 0$	$\tan^{-1} y + \tan^{-1}(\sin x) = \frac{\pi}{4}$
Q29.	$x\frac{dy}{dx}\sin(\frac{y}{x}) + x - y\sin(\frac{y}{x}) = 0, \quad y(1) = \frac{\pi}{2}$	$\cos(\frac{y}{x}) = \log\lvert x\rvert$
Q30.	$\frac{dy}{dx} = \frac{2x+y-1}{4x+2y+5}, y(0)=0$	$\frac{2}{5}(2x+y) + \frac{7}{25}\log\lvert 10x+5y+9\rvert = x + \frac{7}{25}\log 9$
Q31.	$\left(3x^2+y\right)\frac{dx}{dy} = x > 0, y(1)=1$	$y = 3x^2 - 2x$
Q32.	$x\frac{dy}{dx} + \frac{y}{\log x} = 1, y(1)=1$	$y = \frac{1}{2}\log\lvert x\rvert$
Q33.	$dy = \cos x(2 - y\operatorname{cosec} x)dx$; $y\left(\frac{\pi}{2}\right) = 2$	$y\sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$
Q34.	$\left(x+2y^2\right)\frac{dy}{dx} = y, y(2)=1$	$x = 2y^2$
Q35.	$\sqrt{1-y^2}\,dx = \left(\sin^{-1} y - x\right)dx, y(0)=0$	$x + 1 - \sin^{-1} y = e^{-\sin^{-1} y}$

Assignment No. 10

Differential Equations

Q1-4 and each part of Q5&6 are very short and short answer type question.

1. Find the differential equation corresponding to $y = Ae^{x} + Be^{-x}$

2. Find a solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

3. Show that the differential equation $x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$ is homogenous.

4. Determine the order and degree (if defined) of each of the following differential equations:

   i. $\left(\frac{d^2x}{dt^2}\right)^4 - 7t\left(\frac{dx}{dt}\right)^3 = \log t$

   ii. $\left(\frac{d^2y}{dx^2}\right)^2 + \sin\left(\frac{dy}{dx}\right) = 0$

   iii. $1 + \left(\frac{dy}{dx}\right)^2 = 2x - \frac{dy}{dx}$

   iv. $\left[1 + \left(\frac{dy}{dx}\right)^2\right] = 5\left(\frac{d^2y}{dx^2}\right)^3$

5. Write the integrating factor of the following differential equations:

   i. $\left(1 + x^2\right)\frac{dy}{dx} + y = \tan^{-1} x$

   ii. $x\frac{dy}{dx} - y = \log x$

   iii. $\frac{dx}{dy} - \frac{2x}{y} = 3y^3 - 5y + 1$

   iv. $\frac{dy}{dx} + 2y = xe^{4x}$

6. Solve the following differential equations:

   i. $\left(x - 1\right)\frac{dy}{dx} = 2xy,$ given that $x = 2, y = 1$

   ii. $\left(1 + x\right) ydx + \left(1 + y\right) xdy = 0$

   iii. $\frac{dy}{dx} + 2y = e^{-2x} \sin x,$ given $x = 0, y = 0$

   iv. $ydx - (x + 2y^2)dy = 0$, given that x=2 when y = 1.

   v. $\frac{dy}{dx} = \cos\left(x + y\right) + \sin\left(x + y\right)$

   vi. $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$

   vii. $(x + y + 1)\frac{dy}{dx} = 1$

   viii. $\left(1 + e^{2x}\right)dy + \left(1 + y^2\right)e^{x}dx = 0,$ given that $x = 0, y = 1$

   ix. $\frac{dy}{dx} = 1 - x + y - xy$

Vectors

1. If $\left|\vec{a}\right| = 3$ and $-2 \le k \le 1$, then what can you say about $\left|k\vec{a}\right|$?
2. For what values of 'a' the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.
3. Find a vector of magnitude 11 units in the direction opposite to $\overrightarrow{PQ}$ where P and Q are the points $(1,3,2)$ and $(-1,0,8)$ respectively.
4. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{2}$ with y-axis and an acute angle θ with z-axis.
5. A vector $\vec{r}$ has magnitude 14 units and direction ratios $2,3,-6$. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with x-axis.
6. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find unit vectors parallel to the vector $\vec{a} + \vec{b}$.
7. Prove that the points with position vectors $\hat{i} - \hat{j}, 4\hat{i} - 3\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + 5\hat{k}$ are the vertices of a right angled triangle.
8. Find the position vectors of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - 3\vec{b}\right)$ externally in the ratio 1:2. Also, show that P is the mid-point of the line segment RQ.
9. If the vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC, then find the length of median through A.
10. If the points $A(-1,-1,2), B(2,m,5)$ and $C(3,11,6)$ are collinear, then find the value of m by vector method.
11. If $\vec{a}$ is a unit vector and $(2\vec{x} - 3\vec{a}).(2\vec{x} + 3\vec{a}) = 91$, then find the value of $\left|\vec{x}\right|$.
12. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left(\vec{a} + \vec{b}\right)$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.
13. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, then show that the vectors $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ are perpendicular to each other.
14. The vectors $\vec{a} = 3\hat{i} + x\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular. If $\left|\vec{a}\right| = \left|\vec{b}\right|$, then find the value of y.
15. Find λ when the scalar projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.
16. If A,B,C,D are points with position vectors $\hat{i} + \hat{j} - \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$, $2\hat{i} - 3\hat{k}$, $3\hat{i} - 2\hat{j} + \hat{k}$ respectively, find the projection of $\overrightarrow{AB}$ along $\overrightarrow{CD}$.

17. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $(\vec{b} + \vec{c})$.
18. Three vertices of a triangle are $A(0,-1,-2)$, $B(3,1,4)$ and $C(5,7,1)$. Show that it is a right angled triangle. Also, find the other two angles.
19. If $\vec{a}$ and $\vec{b}$ are two non-zero, non-collinear vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that $(2\vec{a} + \vec{b})$ is perpendicular to $\vec{b}$.
20. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $(\vec{a} + \vec{b} + \vec{c}) = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.
21. Find vectors of magnitude $10\sqrt{3}$ units that are perpendicular to the plane of vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$.
22. Find a unit vector perpendicular to the plane of triangle ABC where the coordinates of its vertices are $A(3,-1,2)$, $B(1,-1,-3)$ and $C(4,-3,1)$.
23. Find the angle between the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ if $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$ and hence find a vector perpendicular to both $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$.
24. Find the value of $\vec{a}.\vec{b}$, ,if $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = 16$.
25. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that $(\vec{a} - \vec{d})$ is parallel to $(\vec{b} - \vec{c})$ where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.
26. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.
27. Prove the following:
    (i) $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$
    (ii) $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}] = 0$
28. Show that the four points A,B,C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}, -\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively are coplanar.
29. Prove that : $\vec{a}.(\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}]$.
30. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\dfrac{\pi}{6}$, then prove that:
    (i) $\vec{a} = \pm 2(\vec{b} \times \vec{c})$
    (ii) $[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = \pm 1$

Assignment No.11 (a)

VECTORS

Very short and short answer type questions

1. Write down a unit vector in XY plane making an angle of 60^0 with the positive direction of x-axis.
2. Find the distance of the point (a, b, c) from the z- axis.
3. Give an example of two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}|$ but $\vec{a} \neq \vec{b}$.
4. If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 6\hat{i} + \lambda\hat{j} + 3\hat{k}$, such that they are collinear vectors, then find λ
5. Write the direction cosines of the vector $3\hat{i} - 6\hat{j} + 2\hat{k}$.
6. If the magnitude of the position vector of the point(3 , -2, p) is 7 units , find all possible values of p.
7. A vector is inclined at $\frac{\pi}{4}, \frac{\pi}{3}$ with the "x" and "y" axes respectively. Find the angle it makes with the "z" axis.
8. If $\vec{a} = p\hat{i} + 3\hat{j}$ and $\vec{b} = 4\hat{i} + p\hat{j}$, find the values of p so that $\vec{a}$ and $\vec{b}$ may be parallel.
9. Write the position vector of the point dividing the line segments joining the points with position vectors $\vec{a}$ and $\vec{b}$ in the ratio 1 : 4 externally, where $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$
10. Find the angle at which the following vectors are inclined to each of the coordinate axes: (i) $\hat{i} + \hat{j} - \hat{k}$ (ii) $-\hat{i} - \hat{j}$
11. Show that $\cos\alpha\cos\beta\,\hat{i} + \cos\alpha\sin\beta\,\hat{j} + \sin\alpha\,\hat{k}$ is a unit vector.
12. Find a unit vector perpendicular to the vectors $\hat{i} - 3\hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$.
13. Find a vector of magnitude 3 units, which is orthogonal to the vectors $3\hat{i} + \hat{j} - 4\hat{k}$ and $6\hat{i} + 5\hat{j} - 2\hat{k}$.
14. If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ & $\vec{b} = -\hat{i} + 3\hat{j} - 2\hat{k}$, then find $|\vec{a} - 2\vec{b}|$
15. Find the scalar and vector projection of $2\hat{i} - \hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$.
16. Show that the three points A (3, -5 , 1) B (-1 , 0 , 8) and C (7 , -10 ,-6) are collinear.

17. For what value of 'p', the vectors $\vec{a} = 3\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = p\hat{i} + 3\hat{j} + 3\hat{k}$ are perpendicular to each other?

18. If $\vec{a} = x\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + y\hat{k}$, find the value of x and y so that $\vec{a}$ and $\vec{b}$ may be collinear.

19. Find the position vector of the midpoint of the line segment joining the points $A\left(5\hat{i} + 3\hat{j}\right)$ and $B\left(3\hat{i} - \hat{j}\right)$

20. If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, find $\vec{a}.\vec{b}$.

21. A line makes angles of 45° and 45° with x and y axis respectively. Find the angle it makes with z-axis.

22. Evaluate: $\hat{i}.\left(\hat{j} \times \hat{k}\right) + \hat{j}.\left(\hat{k} \times \hat{i}\right) + \hat{k}.\left(\hat{j} \times \hat{i}\right)$

Assignment No.11(b)

VECTORS

1. The vectors $\vec{a}$ and $\vec{b}$ are non - zero non- collinear and $\vec{c}=(p+4q)\vec{a}+(2p+q+1)\vec{b}$, and $\vec{d}=(-2p+q+2)\vec{a}+(2p-3q-1)\vec{b}$ then find the values of p and q so that $3\vec{c}=2\vec{d}$

2. If $\vec{a}$ and $\vec{b}$ are unit vectors , then what is the angle between $\vec{a}$ and $\vec{b}$ so that $\vec{a}$ -$\sqrt{2}\ \vec{b}$ is a unit vector?

3. If $|\vec{a}|=3,\ |\vec{b}|=5$ and $\vec{a}.\vec{b}=-8,$ find $|\vec{a}+\vec{b}|$.

4. The adjacent sides of a parallelogram are represented by the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=-2\hat{i}+\hat{j}+2\hat{k}$.Find unit vectors parallel to the diagonals of the parallelogram.

5. Compute area of a parallelogram whose diagonals are the vectors $2\hat{i}-3\hat{j}+6\hat{k}$ and $2\hat{i}-2\hat{j}-\hat{k}$.

6. If A, B, C have position vectors (2, 0 ,0), (0, 1 , 0) , (0, 0 , 2), show using vectors that triangle ABC is isosceles.

7. Determine λ and μ if $\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=\vec{0}$

8. Dot product of a vector with vectors $\hat{i}+\hat{j}+\hat{k},\hat{i}+2\hat{j}+3\hat{k}$ and $\hat{i}+3\hat{j}+4\hat{k}$ is respectively 7, 16 and 22. Find the vector.

9. If $\vec{a},\vec{b},\vec{c}$ are unit vectors s.t. $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$,prove that $\vec{a}=\pm 2(\vec{b}\times\vec{c})$.

10. Find the volume of the parallelopiped whose sides are given by $-3\hat{i}+7\hat{j}+5\hat{k}$, $-5\hat{i}+7\hat{j}-3\hat{k}$ & $7\hat{i}-5\hat{j}-3\hat{k}$

11. Show that the vectors $-2\hat{i}-2\hat{j}+4\hat{k}$, $-2\hat{i}+4\hat{j}-2\hat{k}$ and $4\hat{i}-2\hat{j}-2\hat{k}$ are coplanar.

12. Find "λ", for which the four points with position vectors $-\hat{j}-\hat{k}$, $4\hat{i}+5\hat{j}+\lambda\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

13. Prove that $\left(\vec{a}-\vec{b}\right)\cdot\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\}=0$

14. Evaluate : $\left[2\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{i}\ \hat{k}\ \hat{j}\right]+\left[\hat{k}\ \hat{j}\ 2\hat{i}\right]$

15. For any three vectors $\vec{a},\ \vec{b},\ \vec{c}$, prove that $\left[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}\right]=2\left[\vec{a},\vec{b},\vec{c}\right]$

16. Determine "α" such that a vector $\vec{r}$, is at right angles to each of the three vectors $\vec{a}=\alpha\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}$ and $\vec{c}=-2\hat{i}+\hat{j}+3\hat{k}$

17. If $\vec{a},\vec{b},\vec{c}$ are three vectors s.t. $\vec{a}\times\vec{b}=\vec{c},\vec{b}\times\vec{c}=\vec{a}$. Prove that $\vec{a},\vec{b},\vec{c}$ are mutually at right angles and $\left|\vec{b}\right|=1,\left|\vec{a}\right|=\left|\vec{c}\right|$.

18. *If* $\vec{a}\times\vec{b}=\vec{c}\times\vec{d},\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, Prove that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, provided $\vec{a}\neq\vec{d}$, and $\vec{b}\neq\vec{c}$.

19. Find two vectors of unit length which make angles of $45°$ with the position vector of(1 , 0, 0) and are at right angles to the position vector of (0, 0 , 1).

20. Find vector $\vec{c}$ such that $\vec{c}.\hat{i}=\vec{c}.\hat{j}=\vec{c}.\hat{k}$ and $\left|\vec{c}\right|=100$.

Three Dimensional Geometry

1. A straight line makes angles $60°$ and $45°$ with the positive directions of X-axis and Y-Axis respectively. What angle does it make with the Z-Axis?
2. Find the direction cosines of the line passing through the two points $(-2,4,-5)$ and $(1,2,3)$.
3. For what values of p and q will the line joining the points $A(3,2,5)$ and $B(p,5,0)$ be parallel to the line joining points $C(1,3,q)$ and $D(6,4,-1)$.
4. Find the coordinates of the foot of perpendicular drawn from the point $A(-1,8,4)$ to the line joining the points $B(0,-1,3)$ and $C(2,-3,1)$. Hence find the image of the point A in the line BC.
5. The equations of a line are $5x-3=15y+7=3-10z$. Write the direction cosines of the line.
6. The Cartesian equations of a line are $2x-3=3y+1=5-6z$. Find the direction ratios of the line and write down the vector equation of the line through $(7,-5,0)$ which is parallel to the given line.
7. The points $A(1,2,3), B(-1,-2,-1)$ and $C(2,3,2)$ are three vertices of a parallelogram ABCD. Find vector and Cartesian equations of the sides AB and BC. Also find the coordinates of D.
8. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point $P(1,3,3)$.
9. Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}, z+1=0$ and $\frac{x-4}{2}=\frac{z+1}{3}, y=0$ intersect. Also, find their point of intersection.
10. Find the angle between the lines $2x=3y=-z$ and $6x=-y=-4z$.
11. Find the value of p, so that the lines $l_1: \frac{1-x}{3}=\frac{7y-14}{p}=\frac{z-3}{2}$ and $l_2: \frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other. Also, find the equations of a line passing through the point $(3,2,-4)$ and parallel to the line l_1.
12. Find the image of the point $P(1,6,3)$ in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.
13. A line passing through the point A with position vector $\vec{a}=4\hat{i}+2\hat{j}+2\hat{k}$ is parallel to the vector $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$. Find the length of perpendicular drawn on this line from a point P with position vector $\vec{r_1}=\hat{i}+2\hat{j}+3\hat{k}$.

14. Find the vector and cartesian equations of a line through the point $(1,-1,1)$ and perpendicular to the lines joining the points $(4,3,2)$, $(1,-1,0)$ and $(1,2,-1)$, $(2,1,1)$.
15. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angles of $\frac{\pi}{3}$.
16. Find the shortest distance between the lines $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{3-x}{-1}=\frac{y-5}{-2}=\frac{z-7}{1}$.
17. Find the shortest distance between the following lines: $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and $\vec{r}=2\hat{i}+4\hat{j}+5\hat{k}+\mu(4\hat{i}+6\hat{j}+8\hat{k})$
18. Find the equations of the line which intersects the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and passes through the point $(1,1,1)$.
19. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i}-3\hat{j}+6\hat{k}$.
20. Find the vector and cartesian equations of the plane which bisects the line joining the points $(3,-2,1)$ and $(1,4,-3)$ at right angles.
21. Show that the line $\vec{r}=4\hat{i}-7\hat{k}+\lambda(4\hat{i}-2\hat{j}+3\hat{k})$ is parallel to the plane $\vec{r}.(5\hat{i}+4\hat{j}-4\hat{k})=7$.
22. Show that the line $\vec{r}=2\hat{i}+3\hat{j}+\lambda(7\hat{i}-5\hat{k})$ lies in the plane $\vec{r}.(5\hat{i}-3\hat{j}+7\hat{k})=1$.
23. Find the distance of the point $A(-1,-5,-10)$ from the point of intersection of the line $\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$.
24. Find the coordinates of the point where the line $\vec{r}=(-\hat{i}-2\hat{j}-3\hat{k})+\lambda(3\hat{i}+4\hat{j}+3\hat{k})$ meets the plane which is perpendicular to the vector $\vec{n}=\hat{i}+\hat{j}+3\hat{k}$ at a distance of $\frac{4}{\sqrt{11}}$ from origin.
25. Find the distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured along a line parallel to the line $\frac{x-1}{2}=\frac{y-3}{3}=\frac{z+2}{-6}$.
26. Find the distance of the point $A(-2,3,-4)$ from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$.

27. Find the length and the foot of the perpendicular from the point $\left(1,\frac{3}{2},2\right)$ to the plane $2x-2y+4z+5=0$.
28. Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i}+3\hat{j}+4\hat{k}$ to the plane $\vec{r}.(2\hat{i}+\hat{j}+3\hat{k})-26=0$. Also find image of P in the plane.
29. Find the equation of the plane passing through the points $(1,2,3),(0,-1,0)$ and parallel to the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}$.
30. Find the equation of the plane which is parallel to X-Axis and has intercepts 5 and 7 on y-axis and z-axis respectively.
31. Find the equation of the plane passing through the point $2\hat{i}-\hat{k}$ and parallel to the lines $\frac{x}{-3}=\frac{y-2}{4}=z+1$ and $x-4=\frac{1-y}{2}=2z$.
32. Show that the lines $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ are coplanar. Also find the equation of the plane containing these lines.
33. Find the equation of the plane containing two parallel lines $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{3}$ and $\frac{x}{4}=\frac{y-2}{-2}=\frac{z+1}{6}$. Also find if the plane thus obtained contains the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{5}$ or not.
34. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of k and hence find the equation of the plane containing these lines.
35. Find the equation of the plane passing through the points $(3,4,2),(2,-2,-1)$ and $(7,0,6)$.
36. Find the coordinates of the point P where the line through $A(3,-4,-5)$ and $B(2,-3,1)$ crosses the plane passing through three points $L(2,2,1), M(3,0,1)$ and $\mathrm{N}(4,-1,0)$. Also find the ratio in which P divides the line segment AB.
37. Find the equation of the plane which cuts off intercepts $3,-4$ and 6 from the axes. Reduce it to normal form and hence find the length of perpendicular from origin to the plane.
38. If a plane meets the coordinate axes in points A,B,C and the centroid of the triangle ABC is (α,β,γ), find the equation of the plane.
39. Find the equations of the two planes passing through the points $(0,4,-3)$ and $(6,-4,3)$, if the sum of their intercepts on the three axes is zero.

40. Find the equation of the plane passing through the line of intersection of the planes $x+2y+3z-5=0$ and $3x-2y-z+1=0$ and cutting off equal intercepts on x-axis and z-axis.
41. Find the angle between the planes whose vector equations are $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=5$ and $\vec{r}.(3\hat{i}-3\hat{j}+5\hat{k})=3$.
42. Find the angle between the line $\frac{x-2}{3}=\frac{y-3}{5}=\frac{z-4}{4}$ and the plane $2x-2y+z-5=0$.
43. Find the coordinates of the point where the line through the points $A(3,4,1)$ and $B(5,1,6)$ crosses the XZ plane. Also find the angle which this line makes with the XZ plane.
44. Find the direction ratios of a normal to the plane, which passes through the points $(1,0,0)$ and $(0,1,0)$ and makes angle $\frac{\pi}{4}$ with the plane $x+y=3$. Also find the equation of the plane.
45. Find the vector equation of the line passing through $(1,2,3)$ and parallel to the planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}.(3\hat{i}+\hat{j}+\hat{k})=6$.
46. Find the equation of the plane through the point $(4,-3,2)$ and perpendicular to the line of intersection of the planes $x-y+2z-3=0$ and $2x-y-3z=0$. Find the point of intersection of the line $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(\hat{i}+3\hat{j}-9\hat{k})$ and the plane obtained above.
47. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})+5=0$ which is perpendicular to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.
48. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$ and $5x-3y+4z-9=0$ and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{5-z}{-5}$.
49. Find the distance of the point $(2,5,-3)$ from the plane $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=4$.
50. Find the Cartesian as well as vector equations of the planes through the intersection of the planes $\vec{r}.(\hat{i}+3\hat{j})-6=0$ and $\vec{r}.(3\hat{i}-\hat{j}+4\hat{k})=0$ which are at a unit distance from the origin.
51. Find the distance between the planes $\vec{r}.(2\hat{i}-\hat{j}+2\hat{k})=6$ and $\vec{r}.(6\hat{i}-3\hat{j}-6\hat{k})=27$.
52. Find the equation of the plane mid parallel to the planes $2x-3y+6z+21=0$ and $2x-3y+6z-14=0$.

Assignment No. 12

Three Dimensional Coordinate Geometry

Q1-10 are very short and shortanswer type questions.

1. Find the perpendicular distance of the plane $\vec{r}.(5\hat{i}-3\hat{j}+4\hat{k})+9=0$ from origin.
2. Find vector equation of the plane which is at a distance of 3 units from origin and has $\hat{j}$ as the unit normal.
3. Write the equation of the plane passing through the point $(2,-1,1)$ and parallel to the plane $3x+2y-z=7$
4. If the lines $\frac{x-1}{-3}=\frac{y-2}{2p}=\frac{z-3}{2}$ and $\frac{x-1}{3p}=\frac{y-1}{1}=\frac{z-6}{-5}$ are perpendicular to each other, then find the value of p.
5. Find the equation of the line passing through the point $2\hat{i}-3\hat{j}+4\hat{k}$ parallel to the line $\vec{r}=\hat{i}-3\hat{j}-5\hat{k}+\lambda(2\hat{i}+5\hat{k})$.
6. Write position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ & $\vec{b}$ externally in the ratio 1: 4, where $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}+\hat{k}$
7. The cartesian equations of a line are $3x+1=6y-2=1-z$. Find the fixed point through which it passes and its direction ratios. Also find the vector equation.
8. Find the angle between $\frac{x-1}{2}=\frac{2-y}{-1}=\frac{-z-3}{2}$ and $x+y+4=0$
9. Find the intercepts cut off by the plane $3x+2y+z=7$.
10. Write the equation of the plane parallel to XOY plane passing through the point $(1,-2,5)$
11. Find the foot of the perpendicular from P (1, 2, 3) on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Also obtain the equation of the plane containing the line and the point (1, 2 , 3).
12. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)=3$ and $\vec{r}.\left(5\hat{i}-3\hat{j}+4\hat{k}\right)+9=0$ and parallel to the line $\vec{r}=\hat{i}+3\hat{j}+5\hat{k}+\lambda(2\hat{i}+4\hat{j}+5\hat{k})$.

13. Find the equation of the plane through the points (1 , 0 , -1) , (3, 2, 2) and parallel to the line $\frac{x-1}{1}=\frac{y-1}{-2}=\frac{z-2}{3}$.

14. Find the shortest distance between the lines:

a) $\vec{r}=(-4\hat{i}+4\hat{j}+\hat{k})+\lambda(\hat{i}+\hat{j}-\hat{k})$
$\vec{r}=(-3\hat{i}-8\hat{j}-3\hat{k})+\mu(2\hat{i}+3\hat{j}+3\hat{k})$

b) $\vec{r}=(3+\lambda)\hat{i}+(5-2\lambda)\hat{j}+(7+\lambda)\hat{k})$
$\vec{r}=(7\mu-1)\hat{i}+(-1-6\mu)\hat{j}+(\mu-1)\hat{k}$

15. Find the equation of the plane through the points (2,1, -1) , (-1, 3, 4) and perpendicular to the plane $x-2y+4z=10$

16. Show that the four points (0, -1 , -1) , (4, 5 , 1) , (3, 9, 4) and (-4, 4, 4) are coplanar. Also find the equation of the plane containing them.

17. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i}+3\hat{j})+6=0$ and $\vec{r}.(3\hat{i}-\hat{j}-4\hat{k})=0$, which is at a unit distance from the origin.

18. Find the equation of the plane passing through the line of intersection of the planes $4x-y+z=10$ and $x+y-z=4$ and parallel to the line with direction ratios proportional to 2, 1, 1. Find also the perpendicular distance of (1, 1, 1) from this plane.

19. Show that the lines $\vec{r}=2\hat{j}-3\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$ and $\vec{r}=2\hat{i}+6\hat{j}+3\hat{k}+\mu(2\hat{i}+3\hat{j}+4\hat{k})$ are coplanar. Also find the equation of the plane containing them.

20. Prove that the image of the point (3,-2,1) in the plane $3x-y+4z=2$ lies on the plane $x+y+z+4=0$.

21. Find the distance of the point (3,4,5) from the plane $x+y+z=2$ measured parallel to the line $2x=y=z$

22. Show that the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ is parallel to the plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$ .Also, find the distance between them.

23. Find the coordinates of the point where the line through (5, 1 ,6) and (3, 4, 1) crosses the ZX plane .

24. Find the equations of the planes parallel to the plane $x-2y+2z-3=0$ and which are at a unit distance from the point (1,1,1).

25. Find the equation of the line which intersects the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passes through the point (1, 1, 1).
26. Find the equation of the plane through the point (4, -3, 2) and perpendicular to the line ofintersection of the planes $x-y+2z-3=0$ and $2x-y-3z=0$. Find the point of intersection of the line $\vec{r}=\hat{i}+2\hat{j}-\hat{k}+\lambda(\hat{i}+3\hat{j}-9\hat{k})$ and the plane obtained above.

Probability

1. A die is thrown three times. If the first throw is a four, find the chances of getting 12 as the sum.
2. Two numbers are selected at random from numbers 1 to 11. If the sum is even, then find the probability that both numbers are odd.
3. A coin is tossed, then a die is thrown. Find the probability of obtaining a "6" given that head came up.
4. If P(A) = 3/8, P(B)=1/2, $P(A \cap B)$= ¼, find $P(\overline{A}/\overline{B})$
5. If A and B are two events associated with same random experiment such that $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$ then find $P(A/B)$, $P(A \cup B)$ and $P(\overline{B}/\overline{A})$.
6. If $P(E) = \frac{7}{13}$, $P(F) = \frac{9}{13}$, $P(E \cap F) = \frac{4}{13}$, evaluate $P(E/F)$, $P(\overline{E}/F)$, $P(E/\overline{F})$ & $P(\overline{E}/\overline{F})$
7. A coin is tossed once. If it shows head, it is tossed again and if it shows tail, then a die is tossed. Let E: the first throw shows a tail and F: the die shows a number greater than 4. Find $P(F/E)$.
8. Two balls are drawn one after another(without replacement) from a bag containing 2 white, 3 red and 5 blue balls. What is the probability that at least one ball is red?
9. A bag contains 50 tickets numbered 1,2,3,…..,50 of which five are drawn at random and arranged in ascending order of the numbers appearing on the tickets $x_1 < x_2 < x_3 < x_4 < x_5$. Find the probability that $x_3 = 30$.
10. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, find the probability that neither fails.
11. A problem in mathematics is given to three students A, B and C. Their chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{6}$ respectively. Find the probability (i) that the problem will be solved (ii) that exactly one of them solves the problem (iii) problem is not solved.
12. If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$, then find P(A) and P(B).

13. A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6. Find the probability that B is selected.
14. A , B and C in order throw a die in succession till one gets a "six" and wins the game. Find their respective probabilities of winning.
15. A and B throw a pair of dice alternately. A wins if he throws a 6 before B throws a 7 and B wins if he throws a 7 before A throws 6. If A begins, then show that his chances of winning are 30/61.
16. Two thirds of the students of a class are boys and the rest are girls. It is known that the probability of a girl getting A1 grade in Board Exam is 0.4 and a boy getting A1 grade is 0.35. Find the probability that a student chosen at random will get A1 grade in Exam.
17. There are three urns containing 3 white & 2 black balls, 2 white & 3 black balls and 1 black & 4 white balls. There is equal probability of each urn being chosen. One ball is drawn from an urn chosen at random. What is the probability that a white ball is drawn.
18. One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.
19. There are two bags. Bag X contains 5 white and 3 black balls and bag Y contains 3 white and 5 black balls. Two balls are drawn from bag A and put into bag B and then two balls are drawn from bag B. Find the probability that the balls drawn from bag B are white and black
20. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the sum of the numbers obtained is noted. If the result is a tail, a ticket is drawn from a pack of 15 tickets numbered 1,2,3,...14,15and the number on the ticket is noted. What is the probability that the noted number is either 12 or 13?
21. Bag A contains 1 white, 2 black and 3 red balls; bag B contains 2 white, 1 black and 1 red ball and bag C contains 4 white, 5 black and 3 red balls. A bag is chosen at random and two balls are drawn. What is the probability that the balls are white and red?

22. A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A containing 3 red and 2 white balls. If 2 or 3 turn up, a ball is picked up from bag B containing 3 red and 4 white balls. And if 4,5 or 6 turn up a ball is picked up from bag C containing 4 red and 5 white balls. The die is rolled, a bag is picked and a ball drawn. (a) what are the chances of drawing a red ball? (b) If the ball drawn is red, what are the chances that bag B was picked up.
23. A bag contains 4 balls. Two balls are drawn at random, and are found to be white. What is the probability that all balls are white? (Ans : 3/5)
24. 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag A, two balls are drawn at random and then transferred to bag B. A ball is then drawn from bag B and is found to be black ball. What is the probability that the transferred balls were 1 white and 1 black? (Ans: 3/5)
25. A letter is known to have come either from "RANIGANJ" or "RANGANAPUR". On the envelope just two consecutive letters "AN" are visible. What is the probability that the letter is from (i) Raniganj (ii) Ranganapur
26. A letter is known to have come either from TATANAGAR or CALCUTTA. On the envelope just first two consecutive letters TA are visible. What is the probability that the letter has come from (i) Calcutta (ii) Tatanagar ?
27. Each of three identical has two drawers. In each drawer of the first box there is a gold watch. In one drawer of the third box there is a gold watch while in the other there is a silver box. In each drawer of the second box there is a silver watch. If we select a box at random, open one of the drawers and find it to contain a silver watch, what is the probability that the other drawer has the gold watch?
28. A fair coin is tossed until a head or five tails occur. If X denotes the number of tosses of the coin, find the mean and variance of X.
29. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times, find the probability distribution of number of tails. Also find the mean of the number of tails.
30. Two cards are drawn simultaneously (or, successively, without replacement) from a well shuffled pack of cards. Find the mean, variance and standard deviation of the number of kings.

31. A bag contains 5 white and 3 black balls. Four balls are drawn one at a time with replacement. Find the probability that the balls drawn are alternately of different colours.
32. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%? (Ans : 3 times)
33. An experiment succeeds twice as often as it fails. Find the probability that in the next 6 trials there will be at least 4 successes.
34. What is the probability of throwing doublets at least twice in four throws of a pair of dice?

Assignment No. 13
Probability

(Q1-Q5 are very short and short answer questions)

1. If $P(A)=\frac{1}{4}, P(B)=\frac{1}{2}, P(A\cap B)=\frac{1}{8}$, then find P(not A and not B).
2. If $P(A\cup B)=0.8$ and $P(A\cap B)=0.3$ then find $P(\bar{A})+P(\bar{B})$
3. A couple has 2 children. Find the probability that both are boys given that the older child is a boy.
4. Given that the events A and B are such that $P(A)=\frac{1}{2}$, $P\left(A\bigcup B\right)=\frac{3}{5}$ & $P\left(B\right)=K$, Then find "K" if A and B are independent.
5. If A and B are two independent events and $P\left(A\right)=0.3, P(B)=0.6$, then find $P(A \ \& \ notB)$
6. A car producing company knows from past experience that the probability of an order for cars will be ready for shipment on time is 0.85, and the probability that an order for cars will be ready for shipment and will be delivered on time is 0.75. What is the probability that an order for cars will be delivered on time given that it was ready for shipment on time?
7. A pair of dice is thrown and the product of the numbers is observed to be even. What is the probability that both dice have come up with even numbers?
8. A speaks truth in 55 percent cases and B speaks truth in 75 percent cases. Determine the percentage of cases in which they are likely to contradict each other in stating the same fact.
9. A bag contains 4 yellow and 5 red balls and another bag contains 6 yellow and 3 red balls. Two balls are drawn at random from the first bag and are transferred to the second bag. Then a ball is drawn from the second bag. Find the probability that it is yellow in colour.
10. Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be white and red. What is the probability that they came from Bag III.
11. There are two bags, one of which contains 3 black and 4 white balls, while the other contains 4 black and 3 white balls. A fair die is cast, if the face 1 or 3 turns up, a ball is

taken from the first bag, and if any other face turns up a ball is chosen from the second bag. Find the probability of choosing a black ball.

12. A bag contains 3 green and 7 white balls. Two balls are selected at random without replacement. If the second selected ball is given to be green what is the probability that the first selected ball is also green.
13. A letter is known to have come either from LONDON or CLIFTON. On the envelope just two consecutive letters ON are visible. What is the probability that the letter has come from (i) LONDON (ii) CLIFTON?
14. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn successively without replacement. Also find the mean, variance and standard deviation of the distribution.
15. A bag contains 4 red and 5 black marbles. Find the probability distribution of number of red marbles in a random draw of three marbles. Also find the mean and standard deviation of the distribution.
16. In four throws of a pair of dice, what is the probability of throwing doublets (i) at least twice (ii) at most twice (iii) at least thrice (iv) not more than once.
17. If the probability of "success" in each trial of an experiment is $\frac{1}{4}$, then how many trials are necessary so that the probability of getting at least one success is greater than $\frac{2}{3}$?

Practice Assignment-I

Very Short and short Answer Type Questions

1. At the point $(2,1)$, find the slope of the curve $x^6y^6=64$.
2. Find the derivative of $\sin^{-1}\left(x^3\right)$.
3. Evaluate $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.
4. If "c" is a number that satisfies the conclusions of the Mean Value theorem for x^3-2x^2 on the interval $[0,2]$, find the value of "c".
5. If $f(x)=\sqrt{9-x};\ g(x)=x^3+1$, find $f\circ g(x)$.
6. If $f(x)=(x+1)e^x$, find the intervals in which the function is increasing..
7. Write the equation of the tangent to the curve x^3-3x+2 at the point $(2,4)$.
8. Find the stationary points of the function $f(x)=(x-2)^{\frac{2}{3}}(2x+1)$
9. Find the maximum value of the function $f(x)=\sin 2x$ on the interval $\left[0,\frac{\pi}{2}\right]$.
10. If $f(x)=x^4$, defined from $R\to R$, is this function one – one ?
11. If given that $f(x)=16x^2+8x-14$, is an invertible function, find its inverse.
12. Differentiate $\cos\left(x^x\right)$ with respect to x^x.
13. Find the slope of the tangent to the curve represented by $x=t^2+3t-8;\ y=2t^2-2t-5$ at $(2,-1)$.
14. If $y=\tan^{-1}\frac{4x}{1+5x^2}-\tan^{-1}\frac{2-3x}{3+2x}$, show that $\frac{dy}{dx}=\frac{5}{1+25x^2}$.
15. Differentiate $\log x$ with respect to e^x.
16. Differentiate $\tan^{-1}\frac{2x}{1-x^2}$ with respect to $\sin^{-1}\frac{2x}{1+x^2}$.
17. If $y=e^{x+e^{x+e^{x+e^{x+\ldots\ldots\infty}}}}$, prove that $\frac{dy}{dx}=\frac{y}{1-y}$.

18. If $y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x+....\infty}}}$, prove that $\frac{dy}{dx}=\frac{\sin x}{1-2y}$.

19. If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, show that $2x\frac{dy}{dx}+y=2\sqrt{x}$.

20. If $y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$, show that $\frac{dy}{dx}=0$.

21. Differentiate $\tan^{-1}\left(\frac{x^{\frac{1}{3}}+a^{\frac{1}{3}}}{1-x^{\frac{1}{3}}a^{\frac{1}{3}}}\right)$ with respect to "x"

22. If $y=\sin^2 x^2$, find $\frac{dy}{dx}$.

23. If $y=\sqrt{x+y}$, prove that $\frac{dy}{dx}=\frac{1}{2y-1}$.

24. Find $\frac{dy}{dx}$, if $x=a\log t; y=b\sin t$.

25. Find $\frac{dy}{dx}$, if $x=\sqrt{\sin 2\theta}; y=\sqrt{\cos 2\theta}$.

26. If $x=at^2, y=2at$ find $\frac{d^2y}{dx^2}$.

27. Show that the function $f(x)=2x+3$ is continuous at $x=-4$.

28. Show that the function $|x-4|$ is a continuous function.

29. Show that the function $f(x)=\begin{cases}\frac{x}{\sin 3x}, & x\neq 0\\ 3, & x=0\end{cases}$ is discontinuous at x=0

30. If the function $f(x)=\begin{cases}\frac{\sin^2 kx}{x^2}, & x\neq 0\\ 1, & x=0\end{cases}$, is continuous at x=0, find "k".

31. Show that the function $f(x)=\sin|x|$ is a continuous function.

32. Show that the function $f(x)=\frac{1}{x-5}$ is a continuous function.

33. If $\tan^{-1}3+\tan^{-1}x=\tan^{-1}8$, then find x.

34. Show that the function $f(x)=\sin^2 x+x^2-2x$ is continuous at x=0.

35. Evaluate a) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ b) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

36. Find the Principal value of $\cot^{-1}\left(-\sqrt{3}\right)$.

37. Simplify $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$.

38. Find the value of a) $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$ b) $\cos\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right), |x|\geq 1$

39. Find the value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$.

40. The function $f(x)=\begin{cases}\frac{\sin 3x}{x}, & x\neq 0\\ \frac{k}{2}, & x=0\end{cases}$ is continuous at "x = 0". Find "k"

41. Differentiate $\cos^{-1}\left(\frac{2x}{1+x^2}\right), -1<x<1$ with respect to "x"

42. Differentiate $\tan^{-1}\left(\sqrt{1+x^2}-x\right),\ x\in R$ with respect to "x"

43. Differentiate with respect to "x" : $\tan^{-1}\left(\frac{a+x}{1-ax}\right)$

44. Differentiate with respect to "x" : $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

45. If $\sin y = x\sin(a+y)$, find $\frac{dy}{dx}$.

Practice assignment -II

Very Short and short Answer Type Questions

Q1 Find the integrating factor for the following differential equation : $\frac{dx}{dy}-\frac{2x}{y}=3y^3-5y+1$

Q2 Show that the following differential equation is homogenous: $x\frac{dy}{dx}-y=\sqrt{x^2+y^2}$

Q3 If A, B are symmetric matrices and AB= BA, then show that AB is symmetric.

Q4 If A, B and AB are all symmetric matrices, then show that AB= BA.

Q5 If A, B are skew symmetric matrices and AB = BA, then show that AB is symmetric.

Q6 If A, B are square matrices of equal order and B is a skew symmetric matrix, then show that ABA' is also skew symmetric.

Q7 If a matrix is both symmetric and skew symmetric, then show that it is a null matrix.

Q8 What is the number of all possible matrices of order 3×3 with each entry 0 or 1?

Q9 If A, B are square matrices of equal order and B is a symmetric matrix, then show that $A'BA$ is also symmetric.

Q10 Give an example of two non-zero matrices A and B such that AB = O.

Q11 Give an example of two non-zero matrices A and B such that AB = O but BA $\neq$ O.

Q12 What is the order of AB + CB, where A, B and C are matrices of order $3\times4, 4\times2, 3\times4$ respectively.

Q13 Give an example of symmetric and skew symmetric matrix.

Q14 If A= $[a_{ij}]$ is 3×3 matrix and A_{ij}'s denote the cofactors of the corresponding elements a_{ij}'s, then write the value of $a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$.

Q15 If A= $[a_{ij}]$ is 3×3 matrix and A_{ij}'s denote the cofactors of the corresponding elements a_{ij}'s, then write the value of $a_{21}A_{11}+a_{22}A_{12}+a_{23}A_{13}$.

Q16 If A= $[a_{ij}]$ is 3×3 matrix and A_{ij}'s denote the cofactors of the corresponding elements a_{ij}'s, then write the value of $a_{11}A_{13}+a_{21}A_{23}+a_{31}A_{33}$.

Q17 If A is a square matrix of order 2 and $|A|=-5$, find the value of $|3A|$.

Q18 If A is a square matrix of order 3 and $|A|=-2$, find the value of $|5A|$.

Q19 If $x\in I$ and $\begin{vmatrix}2x & 3\\ -1 & x\end{vmatrix}=\begin{vmatrix}3 & 1\\ x & -1\end{vmatrix}$, find the value(s) of x.

Q20 Evaluate without expanding: $\begin{vmatrix} 2 & 2 & 2 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$

Q21 If A is a square matrix of order 3 such that $|adjA| = 100$, find $|\mathrm{A}|$

Q22 If A, B and C are all non-zero square matrices of the same order , then find the condition on A such that AB = AC implies B = C.

Q23 If A is a skew symmetric matrix of order 3, then show that $|A| = 0$.

Q24 Examine whether the following system of equations is consistent : $2x - y = -2, 2y - z = -1, 3x - 5y = 3.$

Q25 Prove that the diagonal elements of a skew symmetric matrix are zero.

Q26 Find the values of x, y and z from the following equation: $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Q27 Evaluate without expanding: $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

Q28 If A=$\{$-1,1,3$\}$, then what is the number of relations on A?

Q29 Show that the function $f : N \to N$, defined by $f(x) = 2x - 1$ is not onto.

Q30 Is the function f defined by $f(x) = \begin{cases} 2x-1, x<0 \\ x+2, x \geq 0 \end{cases}$ continuous at x = 0.

Q31 A four digit number is formed using the digits 1, 2, 3, 5 with no repetition. Find the probability that the number is divisible by 5.

Q32 Find the rate of change of the area of a circle with respect to its radius when the radius is 5 cm.

Q33 If A is a matrix of order 3×4 then what should be the order of the matrix B such that $A'B$ and BA' are both defined?

Q34 Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

Q35 Show that $\cot^{-1} x + x$ is an increasing function on R.

Q36 Differentiate $\cot(x^{\cos x})$ w.r.t $x^{\cos x}$.

Q37 Find the maximum and minimum value of $2\sin x + 3\cos x$

Q38 If $y = \sqrt{\cos x + y}$, find $\frac{dy}{dx}$.

Q39 Find the value of x if $\begin{vmatrix} 2 & x \\ 3 & 7 \end{vmatrix} = \begin{vmatrix} 2 & 8 \\ -1 & 7 \end{vmatrix}$

Q40 Each side of an equilateral triangle is increasing at the rate of 8cm/hr. Find the rate of increase of its area when side is 2cm.

Q41 Find a, for which $f(x) = a(x + \sin x) + a$ is increasing.

Q42 What is the approximate change in the volume V of a cube of side x cm caused by increasing the side by 2%?

Q43 The diameter of a circle is increasing at the rate of 1cm/sec. Find the rate of increase of its area when its radius is π .

Q44 If the tangent to the curve $x = at^2, y = 2at$ is perpendicular to x-axis, then find its point of contact.

Q45 Evaluate: $\tan^{-1}\left(x/y\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$

Q46 Differentiate $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ w.r.t. x.

Q47 If Rolle's theorem is applicable to $f(x) = e^x \sin x$ in $[0,\pi]$, then find the 'c' in Rolle's theorem.

Q48 If $\cos(x-y) = \log(x-y)$, then find $\frac{dy}{dx}$.

Q49 If the curve $ay + x^2 = 7$ and $x^3 = y$ cut orthogonally at (1, 1) then show that a = 6.

Q50 Write in the simplest form: $\tan^{-1}\left(\frac{2\sqrt{x}}{1-x}\right)$

Practice Assignment-III

Very Short and shortAnswer Type Questions

Q1 If a line makes angles α, β, γ with the x,y,z axes respectively, find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

Q2 Evaluate, $\int_0^{1.5} [x]\, dx$ (where [x] is greatest integer function)

Q3 Evaluate, $\int_0^{1.5} [x^2]\, dx$ (where [x] is greatest integer function)

Q4 Write the order and degree of the differential equation, $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Q5 If $f(1) = 4; f'(1) = 2$, find the value of the derivative of $\log f(e^x)$ w.r.t x at the point $x = 0$.

Q6 Let $f : R - \left\{-\frac{3}{5}\right\} \to R$ be a function defined as $f(x) = \frac{2x}{5x+3}$, find f^{-1} : Range of $f \to R - \left\{-\frac{3}{5}\right\}$

Q7 Let $f(x) = \sin x, g(x) = 2x$ and $h(x) = \cos x$, show that $f \circ g = g \circ (fh)$

Q8 If $y = f(x) = \frac{1-x}{1+x}$, show that $x = f(y)$

Q9 Let S=$\{1,2,3\}$ Find whether the function $f : S \to S$ defined as $f = \{(1,3),(3,2),(2,1)\}$ has inverse. If yes, find f^{-1}.

Q10 Let A = $\{0,3,5\}$, define a relation on A which is reflexive and transitive but not symmetric.

Q11 Is signum function $f : R \to R$, given by $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0 & , x = 0 \end{cases}$ an onto function? Justify your answer.

Q12 Let $f(x) = 1/x$ and $g(x) = 0$ be two real valued functions. Is $f \circ g$ defined? Justify.

Q13 Given that $f : R \to A$ given by $f(x) = \frac{x^2}{x^2+1}$ is a surjection , find A.

Q14 $f : A \to A,$ where A=$[-1,1]$ given by $f(x) = x/3$. Is f bijective?

Q15 Check if the following functions are one – one, many – one, onto or into

a) $f:R\to R$; $f(x)=|x|+x$ b) $g:R\to R$; $g(x)=x^3$.

Q16 For any three vectors $\vec{a},\vec{b},\vec{c}$, write the value of $\vec{a}\times\left(\vec{b}+\vec{c}\right)+\vec{b}\times\left(\vec{c}+\vec{a}\right)+\vec{c}\times\left(\vec{a}+\vec{b}\right)$.

Q17 Find vector equation of line through points with position vectors $2\hat{i}-3\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

Q18 What should be the angle between vectors $\vec{a}$ and $\vec{b}$ such that $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}\right|+\left|\vec{b}\right|$.

Q19 Write a value of $\int\frac{1+\cot x}{x+\log\sin x}dx$

Q20 Show that the differential equation $2ye^{x/y}dx+(y-2xe^{x/y})dy=0$ is homogenous.

Q21 If $f'(x)=\frac{4}{x^2}$ and $f(1)=6$, find $f(2)$

Q22 If $f(x)=e^x g(x), g(0)=2$ and $g'(0)=1$, then find $f'(0)$.

Q23 What are the maximum and minimum values of 3sinx + 4 cosx .

Q24 Let f and g be differentiable functions satisfying

$g'(a)=2, g(a)=b$ and $f\circ g=I$ (identity function), show that $f'(b)=\frac{1}{2}$.

Q25 If $y=\cos^{-1}\left(\frac{2\cos x-3\sin x}{\sqrt{13}}\right)$, then show that $\frac{dy}{dx}=1$.

Q26 Write a value of $\int\left(\cos(\log x)+\sin(\log x)\right)dx$

Q27 If $f(x)=\frac{|x|}{x}, x\neq 0$, show that $\left|f(\alpha)-f(-\alpha)\right|=2$, where $\alpha\neq 0$

Q28 If $f(x)=\left(a-x^n\right)^{1/n}$, then find $(f\circ f)(x)$

Q29 Write the order and degree of differential equation $x\frac{dy}{dx}+\frac{3}{\frac{dy}{dx}}=y^2$

Q30 Let $f:R\to R$ be a mapping defined by $f(x)=x^3+5$, find $f^{-1}:R\to R$

Q31 If $f(x)=\sin x$ and $g(x)=\cos x$, find $(2f)\left(\pi/2\right)$ and $(f-g)\left(\pi/2\right)$

Q32 Write a vector of magnitude 15 units in the direction of $2\hat{i}+4j-5k$

Q33 For any $\vec{r}$, find $\left(\vec{r}.\hat{i}\right)\hat{i}+\left(\vec{r}.\hat{j}\right)\hat{j}+\left(\vec{r}.\hat{k}\right)\hat{k}$

Q34 If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with j , and an acute angle θ with k , then find the value of θ.

Q35 If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 3 and 2/3 respectively such that $\vec{a}\times\vec{b}$ is a unit vector, find the angle between $\vec{a}$ and $\vec{b}$.

Q36 Find $|\vec{x}|$ if $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=75$, where $\vec{a}$ is a unit vector.

Q37 Find a solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}$

Q38 Find the integrating factor of differential equation $\cos x\frac{dy}{dx}+y\sin x=1$

Q39 Write the order and degree of differential equation $5\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^3\right]^{3/2}$

Q40 Find the differential equation corresponding to $y=Ae^{x}+Be^{-x}$

Q41 Let $f:R\to R$ be given by $f(x)=x^2-3$. Find $f^{-1}:R\to R$.

Q42 Find the slope of the tangent to the curve $x=t^2+3t-8, y=2t^2-2t-5$ at the point (2,-1).

Q43 Write the position vector of a point dividing the line segment joining points A and B whose position vectors are $2\hat{i}-3\hat{j}+5\hat{k}$ and $3\hat{i}+2\hat{j}-\hat{k}$ in the ratio 1:4 externally.

Q44 If $\theta=\sin^{-1}(\sin(-600^\circ))$, then find one of the possible values of θ.

Q45 If $f:[2,\infty[\to X$ defined by $f(x)=4x-x^2$ is given to be invertible , then find X.

Q46 Evaluate: $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right\}$

Q47 If $y=\log\sqrt{\tan x}$, then find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

Q48 If $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ then find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

Q49 If $x=a\cos nt-b\sin nt$, then find $\frac{\mathrm{d}^2x}{dt^2}$.

Q50 If the line y = x touches the curve $y=x^2+bx+c$ at (1, 1), then show that b = -1 and c = 1.

Q51 If the curves $y=ae^{x}$ and $y=be^{-x}$ cut orthogonally , then show that $ab=1$.

Q52 If $\tan^{-1}(\cot\theta)=2\theta$, then find a possible value of θ.

Q53 Evaluate $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

Q54 Find a unit vector parallel to the sum of $5\hat{i}-\hat{j}+2\hat{k}$ and $-\hat{i}+6\hat{j}+\hat{k}$

Q55 Find the vector projection of $3\hat{i}-\hat{j}+5\hat{k}$ on $-2\hat{i}+3\hat{j}+\hat{k}$

Q56 Find a vector which is equally inclined to the axes.

Q57 Find the value λ of if $22\hat{i}-3\hat{j}+5\hat{k}$ is perpendicular to $2\hat{i}+\lambda\hat{j}+\hat{k}$

Q58 If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left|\vec{a}+\vec{b}\right|=1$, then find $\left|\vec{a}-\vec{b}\right|$

Q59 If $\sin^{-1}x-\cos^{-1}x=\pi/6$, then find *x*.

Practice Test-1

Relations and Functions

Q1. Give an example of a relation which is symmetric and reflexive but not transitive.

Q2. Check the injectivity of $f: Z \to Z$ given by $f(x) = x^2$

Q3. If $f = \{(1,3),(2,7),(8,6)\}$ and $g = \{(7,11),(6,0),(3,5)\}$, find gof.

Q4. If f :R→R defined by $f(x) = 3x - 4$ is invertible then write $f^{-1}(x)$.

Q5. Let f :R→R be given by $f(x) = \left(3 - x^3\right)^{\frac{1}{3}}$, then find $fof(x)$

Q6. Consider $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible. Find the inverse of f.

Q7. Let R be a relation on the set A of ordered pairs of positive integers defined by $(x, y) R (u, v)$ if and only if $xy = uv$. Show that R is an equivalence relation.

Q8. Show that the function $f: R \to R$ defined by $f(x) = \dfrac{3x+1}{2}$ is one- one and onto. Hence find the inverse of the function.

Q9. Show that the function $f: A \to B$ defined as $f(x) = \dfrac{3x+4}{5x-7}$, where $A = R - \left\{\dfrac{7}{5}\right\}, B = R - \left\{\dfrac{3}{5}\right\}$ is invertible and hence find f^{-1}.

Q10. Show that the relation R on the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a,b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1 i.e. equivalence class [1].

Practice Test-2

Inverse Trigonometric Functions

Q1. Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$.

Q2. Write the value of $2\cos^{-1}\frac{1}{2}+3\sin^{-1}\frac{1}{2}$

Q3. Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}\left(\cos\left(\frac{-\pi}{3}\right)\right)$

Q4. Solve for x: $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$

Q5. Evaluate : $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

Q6. Evaluate : $\cot^{-1}\left[2\sin\left(2\cos^{-1}\frac{1}{2}\right)\right]$

Q7. Solve for x : $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

Q8. Prove that : $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2}$, $x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

Q9. Prove that: $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

Q10. Prove that : $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1+x}}{\sqrt{1+x}+\sqrt{1+x}}\right)=\left(\frac{\pi}{2}-\frac{1}{2}\cos^{-1}x\right)$

Q11. Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

Q12. Show that : $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$

Practice Test 3

Matrices and Determinants

Q1. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, find the value of x.

Q2. Find **B** if A= $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and 2A+B= $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Q3. If A= $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = \lambda A$, find the value of λ

Q4. If A is a square matrix of order 3 and $|A| = -4$, find the value of $|4A|$.

Q5. Express $\begin{bmatrix} 2 & 5 & -9 \\ 3 & 0 & -1 \\ 4 & 2 & 5 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrices.

Q6. If $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = O$.

Q7. Using elementary transformations find the inverse of $\begin{bmatrix} 3 & 1 \\ 6 & 4 \end{bmatrix}$.

Q8. By using properties of determinants show that :

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & xz \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

Q9. By using properties of determinants show that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

Q10. An amount of Rs 600 crores is spent by the government in three schemes. Scheme A is for saving girl child.Scheme B is for saving of newlywed girls from death due to dowry. Scheme C is planning for good health for senior citizen. Twice the amount spent on Scheme C together with amount spent on Scheme A is Rs 700 crores. And three times the amount spent on Scheme A together with amount spent on Scheme B and Scheme C is Rs 1200 crores. Find the amount spent on each Scheme using matrices? What is the importance of saving girl child? Suggest any one measure that you will do to save girl child.

Practice Test 4
Differentiation

Q1. If $f(x)=\begin{cases}\dfrac{x^2-2x-3}{x+1}, & x\neq -1\\ \lambda & ,x=-1\end{cases}$ is continuous at x= -1, find the value of λ.

Q2. Differentiate $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ w.r.t. $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

Q3. If $\log\left(\sqrt{1+x^2}-x\right)=y\sqrt{1+x^2}$ show that $\left(1+x^2\right)\dfrac{dy}{dx}+xy+1=0$.

Q4. Find the value of "a " for which the function defined as

$$f(x)=\begin{cases}a\sin\dfrac{\pi}{2}(x+1), x\leq 0\\ \dfrac{\tan x-\sin x}{x^3}, x>0\end{cases}$$ is continous at x = 0

Q5. Differentiate $x^{x\cos x}+\dfrac{x^2+1}{x^2-1}$ w.r.t x .

Q6. If $x^y=y^x$, show that $\dfrac{dy}{dx}=\dfrac{y(x\log y-y)}{x(y\log x-x)}$

Q7. Find $\dfrac{dy}{dx}$ if $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$.

Q8. If $\cos^{-1}\left[\dfrac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right]$ find $\dfrac{dy}{dx}$.

Q9. If $x=a\left(\cos t+t\sin t\right)$ and $y=b\left(\sin t-t\cos t\right)$, find $\dfrac{d^2y}{dx^2}$ at $t=\dfrac{\pi}{3}$.

Q10. Verify Lagrange's Mean Value Theorem for $f(x)=x(x-1)(x-2)$ in $\left[0,\dfrac{1}{2}\right]$.

Q11. If $x=\sin t$ and $y=\sin pt$,prove that $\left(1-x^2\right)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+p^2y=0$.

Q12. Differentiate $\cos^{-1}\left[\dfrac{\sqrt{1-x^2}+x}{\sqrt{2}}\right]$ w.r.t $\tan^{-1}\left[\dfrac{x}{\sqrt{1-x^2}+1}\right]$.

Q13. If $\cos y=x\cos(a+y)$,prove that $\dfrac{dy}{dx}=\dfrac{\cos^2(a+y)}{\sin a}$.

Q14. If $y=\left(\tan^{-1}x\right)^2$,prove that $\left(1+x^2\right)^2\dfrac{d^2y}{dx^2}+2x\left(1+x^2\right)\dfrac{dy}{dx}=2$.

Practice Test 5

Application of Derivatives

Q1. Use differential to approximate $\sqrt{0.0037}$.

Q2. Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 1%.

Q3. Find the intervals in which $(x-1)^3(x-2)^2$ is increasing and decreasing.

Q4. Find the equation of tangent and normal to the curve $x = a\sin^3 t, y = b\cos^3 t$ at the point 't".

Q5. Find equation(s) of tangent drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1,2).

Q6. Find the intervals in which $f(x) = 2\log(x-2) - x^2 + 4x + 1$ is increasing and decreasing.

Q7. Find all points of local maxima and minima and corresponding maximum and minimum values of the function $f(x) = \frac{-3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$.

Q8. Find points of local maxima and minima of $f(x) = \sin x + \frac{1}{2}\cos 2x$ where $0 \le x \le \frac{\pi}{2}$. Also find the absolute maximum and minimum values of the function.

Q9. Water is dripping out from a conical funnel at a uniform rate of 4 $cm^3/\sec$ through a tiny hole at the vertex in the bottom. When the slant height of water is 3 cm, find the rate of decrease of the slant height of the water cone given that the vertical angle of the funnel is 120° .

Q10. Sand is being poured into a conical pile at a constant rate of 50 cm^3/sec such that the height of the cone is always one half of the radius of the base. How fast is the height of the pile increasing when the sand is 5cm deep/

Q11. If the sum of the lengths of the hypotenuse and a side of a right triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

Q12. Show that the volume of the largest cone that can be inscribed in a sphere of radius 'r' is $\frac{8}{27}$ of the volume of the sphere.

Q13. An open box with a square base is to be made out of a given quantity of cardboard of area c^2 sq.units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cu. units.

Practice Test 6

Integration

Evaluate the following:

Q1. $\int \frac{x^2-3x}{(x-1)(x-2)}dx$

Q2. $\int \frac{\sin x}{\sin 4x}dx$

Q3. $\int e^x \cos^2 x \, dx$

Q4. $\int \frac{2\sin 2x-\cos x}{6-4\cos^2 x-4\sin x} dx$

Q5. $\int_0^{3/2} |x\cos \pi x| dx$

Q6. $\int \frac{x-\sin x}{1-\cos x}dx$

Q7. $\int \frac{dx}{5+7\cos x+\sin x}$

Q8. $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}dx$

Q9. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}dx$

Q10. $\int_0^1 (3x^2+2x)dx$ as a limit of sums.

Q11. $\int \frac{dx}{\sin x(5-4\cos x)}$

Practice Test 7

Application of Integration and Differential Equations

Q1. Find the differential equation of all circles touching the x axis at the origin.

Q2. Find the differential equation of all circles in the first quadrant which touch the coordinate axes.

Q3. Solve : $e^{\frac{dy}{dx}} = x+1$, $y(0)=5$

Q4. Solve : $x(xdy - ydx) = ydx$

Q5. Solve the differential equation : $(x+y)^2 \dfrac{dy}{dx} = a^2$

Q6. Solve the differential equation : $\left(x^3 - 3xy^2\right)dx = \left(y^3 - 3x^2 y\right)dy$

Q7. Solve the differential equation : $\left(1+y^2\right)dx = \left(\tan^{-1} y - x\right)dy$

Q8. Find the particular solution of the equation : $ye^y dx = \left(y^3 + 2xe^y\right)dy$, $y(0)=1$

Q9. Find the area of the region $\left\{(x,y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\right\}$

Q10. Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and x=0.

Q11. Find area of the region common to $x^2 + y^2 = 16$ and $6y = x^2$

Practice Test 8

Vectors and 3- Dimensional Geometry

Q1. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$; find a vector of magnitude 6 units which is parallel to $2\vec{a} - \vec{b} + 3\vec{c}$.

Q2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors of magnitude 3,4,5 respectively such that each is perpendicular to the sum of the other two , prove that $\left|\vec{a} + \vec{b} + \vec{c}\right| = 5\sqrt{2}$.

Q3. Find the equation of the plane passing through the points (3,2,1) and (0,1,7) and parallel to the line $\dfrac{x-2}{1} = \dfrac{y+1}{-1} = \dfrac{1-z}{-1}$

Q4. Find equation of the perpendicular drawn from the point (2,4,-1) to the line $\dfrac{x+5}{1} = \dfrac{y+3}{4} = \dfrac{z-6}{-9}$.

Q5. The dot products of a vector with the vectors $\hat{i} - 3\hat{k}$, $\hat{i} - 2\hat{k}$ and $\hat{i} + \hat{j} + 4\hat{k}$ are 0,5 and 8 respectively. Find the vector.

Q6. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{a} \cdot \hat{b} = \hat{a} \cdot \hat{c} = 0$ and angle between $\hat{b}$ and $\hat{c}$ is $\dfrac{\pi}{6}$ prove that , $\hat{a} = \pm 2\left(\hat{b} \times \hat{c}\right)$.

Q7. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = 3\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - \hat{k}$; find a vector $\vec{d}$ which is perpendicular to both $\hat{a}$ and $\hat{b}$ and $\vec{c} \cdot \vec{d} = 1$.

Q8. Find the coordinates of the point where the line through (3, -4, -5) and (2, -3, 1) crosses the plane determined by the points (1,2,3) , (2, 2,1) and (-1, 3,6).

Q9. Find the equation of the plane through the intersection of the planes $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$, $\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$.

Q10. Find the distance of the point (-1,-5,-10) from the point of intersection of the line $\vec{r} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and plane $\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) - 5 = 0$.

Q11. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (3,2,1) from the plane $2x - y + z + 1 = 0$. Also find the image of the point in the plane.

Practice Test 9

Probability

Q1. A bag contains 4 red and 4 black balls. Another bag contains 2 red and 6 black balls. One of the two bags is selected and a ball is drawn. If it is found to be red, find the probability the second bag was chosen.

Q2. The probability of A , B and C solving a problem independently is ½ ,¾ and 1/3. If all of them try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly two of them will solve (iii) atmost two of them solve.

Q3. There are three urns A , B and C. A contains 10red and 4 green marbles. B contains 9 red and 5 green marbles. C contains 8 red and 6 green marbles. One ball is drawn from each of these urns. What is the probability that, out of these three balls drawn, two are red and one is green?

Q4. Rohan and Sid throw a die alternatively till one of them gets a 6 and wins the game. Find the probability of Sid winning the game if Rohan starts the game.

Q5. There are two bags. The first bag contains 5 white and 6 black marbles. The second bag contains 4 white and 7 black marbles. Two balls are drawn from the first bag and without noticing their colour,are put into the second bag. Then two balls are drawn from the second bag. What is the probability that the balls drawn are white?

Q6. A man is known to speak the truth 3 out of 5 times. He throws a die and reports that a 4 has occurred. What is the probability that a 4 has actually occurred?

Q7. A card from a deck of 52 cards is lost. From the remaining cards two cards are drawn and found to be diamonds. What is the probability that a spade card is lost?

Q8. The probability of a man hitting a target is ½. How many times must he fire so that the probability of hitting the target at least once is more than 90%/

Q9. Find the probability distribution of number of sixes in three tosses of a die.

Q10. From a lot of 10 items containing 3 defective, a sample of 4 items is drawn. Find the probability distribution of X, where X denotes the number of defective items drawn. Also find the mean and variance of the distribution.

Q11. In a bulb factory, machines A, B and C manufacture 60%, 30 % and 10 % bulbs respectively. 1 %, 2 % and 3% of the bulbs produce respectively by A , B and C are found to be defective. A bulb is picked up at random from the total production and is found to be defective. Find the probability that the bulb was produced by the machine A.

Academic Session: 2019-20
First Term Examination
Subject: Mathematics

Time : 3 hr **Max Mks : 80**

General Instructions:

1. *All questions are compulsory*
2. *This question paper consists of 36 questions divided into four sections A, B C and D. Section A contains 20 questions of 1 mark each. Section B contains 6 questions of 2 marks each. Section C contains 6 questions of 4 marks each. Section D contains 4 questions of 6 marks each.*
3. *Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.*
4. *If you wish to attempt any question again, then cancel out the previous answer.*
5. *This paper has 5 printed sides.*

SECTION A

State True/False (Q1-Q5)

Q1. Every differentiable function is continuous.

Q2. The function $f(x) = |\cos x|$ is a continuous function.

Q3. $\int \sec^2(7-4x)dx = \tan(7-4x) + C$

Q4. The relation S defined on the set R of all real numbers by the rule aSb iff $a \geq b$ is symmetric,transitive but not reflexive.

Q5. If $f : [a,b] \to R$ is continuous on $[a,b]$ and differentiable on (a,b). Then there is always a , c in (a,b) such that $f'(c) = 0$.

Choose the correct option (Q6-Q10)

Q6. If two events are independent, then

(a) They must be mutually exclusive
(b) The sum of their probabilities must be equal to 1
(c) (a) and (b) both are correct
(d) None of the above is correct

Q7. If E and F are events such that P(E/F)= P(F/E), then

(a) $E \cap F = \phi$ (b) E=F (c) P(E)=P(F) (d) $E \subset F$

Q8. The approximate change in the volume of a cube of side x m caused by increasing the side by 3% is :

(a) $0.9x^3m^3$ (b) $0.09x^3m^3$ (c) $0.6x^3m^3$ (d) $0.06x^3m^3$

Q9. Let $f : R \to R$ be a function defined by $f(x) = 2x + 3$. Then $f^{-1}(x) =$

(a) $2x - 3$ (b) $\frac{x-3}{2}$ (c) $\frac{x-2}{3}$ (d) $\frac{1}{2x+3}$

Q10. If A and B are two events such that $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A) =$

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$

Fill in the blanks (Q11-Q20)

Q11. $\int e^x(\tan x + \sec^2 x)dx =$ .... +C

Q12. The rate of change of the area of a circle with respect to its radius r when r=5 cm is ……………………………

Q13. If $f : R \to R$ and $g : R \to R$ are two functions defined by $f(x) = |x|$ and $g(x) = [x]$ respectively. Then $(gof)(-\sqrt{2})$ =……………………………

Q14. The value of c in Rolle's Theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$ is …………………………………….

Q15. $\int e^{2x+3}dx = f(x) + C$, then $f(x) =$ ....

Q16. Derivative of $sin^2(x^2)$ with respect to x^2 is ……………………….

Q17. The principal value of $\cos^{-1}(\cos\frac{13\pi}{6})$ is =…………………………

Q18. The derivative of $\sin^{-1}\left(x^3\right)$ with respect to x is ……………………….

Q19. $\int_0^1 [x]dx$ =………………………

Q20. For the curve $y = 3x^2 + 4x$, the slope of the tangent to the curve at the point whose x- coordinate is -2 =………………………………….

SECTION B

Q21. If P(A) = $\frac{2}{5}, P(B) = \frac{1}{3}, P(A \cap B) = \frac{1}{5}, then find P(\bar{A}/B)$.

OR

The probability of student A passing an examination is 5/7 and of student B passing is 3/7. Assuming the two events "A passes", "B passes" as independent, find the probability of only one of them passing the examination.

Q22. If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$ then find the value of a.

Q23. If $y = x^{\sin x}$, find $\frac{dy}{dx}$.

Q24. Find the point(s) on the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ where tangent is parallel to the y-axis.

Q25. Evaluate: $\int \frac{x}{(1+x)^2} e^x dx$

OR

Evaluate $\int \frac{(x^4 - x)^{\frac{1}{4}}}{x^5}$dx

Q26. Discuss the differentiability of f(x) = $|x-2|$at x = 2.

SECTION C

Q27. Determine the intervals where $f(x) = \sin x - \cos x, 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing. Also find the absolute maximum and absolute minimum values of function in this interval.

OR

Find the intervals in which the function $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$ is increasing or decreasing. Also find all the points of local maxima and local minima of the function f.

Q28. Prove: $cot^{-1}\left(\frac{\sqrt{(1+\sin x)}+\sqrt{(1-\sin x)}}{\sqrt{(1+\sin x)}-\sqrt{(1-\sin x)}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$.

Q29. Evaluate :$\int_0^4 (x^2 + 2)$ dx as the limit of a sum.

Q30. If $x = a\sin pt$ and $y = b\cos pt$, then find $\frac{d^2y}{dx^2}$ at $t = 0$

Q31. A relation R on the set of real numbers is defined as $R = \{(a,b) : a, b \in R; a \le b^3\}$. Is the relation R reflexive, symmetric and transitive? Justify your answer.

OR

Show that the function f on A= $R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1} (4).

Q32. Evaluate: $\int \frac{(6e^x + 7)e^x}{\sqrt{(e^x - 5)(e^x - 4)}} dx$

OR

Evaluate: $\int \frac{(\sin\theta + 2)\cos\theta}{7 - 2\cos^2\theta + 6\sin\theta} d\theta$

Section D

Q33. Find the area of the region $\{(x, y) : x^2 + y^2 \le 4, \ x + y \ge 2\}$, using the method of integration.

OR

Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0, \quad x = 4, \ y = 4$ and $y = 0$ into three equal parts.

Q34. Evaluate: $\int \frac{x^4}{(x-1)(x^2+1)} dx$

Q35. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Q36. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1,2,3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1,2,3 or 4 with the die ?

Class – XII

MATHEMATICS- 041
SAMPLE QUESTION PAPER 2019-20

Time: 3 Hrs. **Maximum Marks: 80**

General Instructions:

- All questions are compulsory.
- The question paper consists of 36 questions divided into four sections A, B, C and D.
- Section – A contains 20 questions of 1 mark each.
- Section – B contains 6 questions of 2 marks each.
- Section – C contains 6 questions of 4 marks each.
- Section – D contains 4 questions of 6 marks each.
- Use of calculator is not permitted

Q.No	**SECTION A**	**Marks**
	Q1 to Q5 are multiple choice type questions. Select the correct option	
Q1	If A is any square matrix of order 3×3 such that $\|A\| = 3$, then the value of $\|adjA\|$ is ? (a) 3 (b) $\frac{1}{3}$ (c) 9 (d) 27	1
Q2	Suppose P and Q are two different matrices of order $3 \times n$ and $n \times p$, then the order of the matrix $P \times Q$ is ? (a) $3 \times p$ (b) $p \times 3$ (c) $n \times n$ (d) 3×3	1
Q3	If $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$,then the values of p and q are ? (a) p= 6 ,q=27(b)p=3,q=$\frac{27}{2}$ (c) p=6,q= $\frac{27}{2}$ (d) p=3 ,q=27	1
Q4	If A and B are two events such that P(A)=0.2 , P(B)=0.4 and $P(A \cup B)$=0.5 , then value of P(A/B) is ? (a)0.1 (b)0.25 (c)0.5 (d) 0.08	1

Q5 The point which does not lie in the half plane 1

$2x + 3y - 12 \leq 0$ is

(a) (1,2) (b) (2,1) (c) (2,3) (d)$(-3, 2)$

Q6 to Q10 are fill in the blank type questions

Q6 If be the greatest integer function defined as $f(x) = [x]$ and g be the modulus function defined as $g(x) = |x|$, then the value of $gof\left(-\frac{5}{4}\right)$ is________________________ 1

Q7 If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then the value of 1

$\cos^{-1} x + \cos^{-1} y$ is ________________________

Q8 If the function $f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{when} \quad x \neq 1 \\ k & \text{when} \quad x = 1 \end{cases}$ is given 1

to be continuous at $x = 1$, then the value of k is ____________

Q9 If tangent to the curve $aty^2 + 3x - 7 = 0$ at the point (h, k) is parallel to line $x - y = 4$, then value of k is ___? 1

Q10 The magnitude of projection of $(2\hat{\imath} - \hat{\jmath} + \hat{k})$ on $(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$is________ 1

Q11 to Q20 are very short answer type questions

Q11 If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$, then write the values of x and y. 1

Q12 Check whether (l + m + n) is a factor of the 1

determinant $\begin{vmatrix} l+m & m+n & n+l \\ n & l & m \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{vmatrix}$ or not.

Give reason.

Q13 Evaluate 1

$\int_{-2}^{2}(x^3+1)dx.$

Q14 Evaluate $\int \frac{1+\cos x}{x+\sin x}dx.$ 1

Q15 Evaluate $\int xe^{(1+x^2)}dx.$ 1

Q16 Evaluate $\int \frac{dx}{\sqrt{9-25x^2}}$ 1

Q17 What is general solution of $\frac{dy}{dx}=e^{x+y}$? 1

Q18 What is the distance between the two planes 1

$3x + 5y + 7z = 3$ and $9x + 15y + 21z = 9$?

Q19 Write the equation of the line in vector form passing through the point $(-1,3,5)$ and parallel to line 1

$\frac{x-3}{2}=\frac{y-4}{3}, z=2.$

Q20 An urn contains 6 balls of which two are red and four are Black. Two balls are drawn at random. What is the probability that they are of the different colours ? 1

SECTION – B

Q21 Express $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$;where$-\frac{\pi}{4}<x<\frac{\pi}{4}$,in the simplest form. 2

OR

Express $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ in the simplest form.

Q22 If $= ae^{2x} + be^{-x}$, then show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0.$ 2

Q23 A particle moves along the curve $x^2 = 2y$. At what point, ordinate increases at the same rate as abscissa increases? 2

Q24 For three vectors $\vec{a}, \vec{b}, \vec{c}$, prove that $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}] = 0.$ 2

OR

If $\vec{a} + \vec{b} + \vec{c} = 0$ *and* $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then find the value of $\vec{a} . \vec{b} + \vec{b}.\vec{c} + \vec{c} . \vec{a}$. 2

Q25 Find the angle between the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. 2

Q26 A speaks truth in 80% cases and B speaks truth in 90%cases. In what percent of cases are they likely to agree with each other in stating the same fact? 2

SECTION – C

Q27 Let $f: A \to B$ be a function defined as $f(x) = \frac{2x+3}{x-3}$, where $A = R - \{3\}$ and $B = R - \{2\}$. Is the function f one –one and onto? Is f invertible? If yes, then find its inverse. 4

Q28 If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$. 4

OR

If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and

$y = a(\sin 2\theta - 2\theta \cos 2\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$.

Q29 Solve the differential equation 4

$x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$.

Q30 Find the value of $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ 4

OR

Evaluate $\int_1^3 |x^2 - 2x|\,dx$.

Q31 Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X. Also, find mean of the distribution. 4

OR

There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up head 75% the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. It shows a head. What is probability that it was the two headed coin ?

Q32 Two tailors A and B earn Rs.150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. Form a L.P.P to minimize the labour cost to produce at least 60 shirts and 32 pants and solve it graphically. 4

SECTION D

Q33 Using the properties of determinants, prove that

$$\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (z+x)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^2 .$$ 6

OR

If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations

$$x - y = 3 \;;$$
$$2x + 3y + 4z = 17;$$
$$y + 2z = 7$$

Q34 Using integration, find the area of the region 6

$\{(x, y) : x^2 + y^2 \leq 1 , x + y \geq 1, x \geq 0, y \geq 0 \}$

Q35 A given quantity of metal is to cast into a half cylinder with a rectangular base and semi-circular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diameter of semi-circular ends is $\pi : \pi + 2$. 6

OR

Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

Q36 Find the equation of a plane passing through the points $A(2,1,2)$ and $B(4,-2,1)$ and perpendicular to plane $\vec{r}.(\hat{\imath} - 2\hat{k}) = 5$. Also, find the coordinates of the point, where the line passing through the points (3,4,1) and (5,1,6) crosses the plane thus obtained. 6

Academic Session: 2019-20
Pre-board Examination
Subject: Mathematics

Time: 3 Hrs. **Maximum Marks: 80**

General Instructions:

- All questions are compulsory.
- The question paper consists of 36 questions divided into four sections A, B, C and D.
- Section – A contains 20 questions of 1 mark each.
- Section – B contains 6 questions of 2 marks each.
- Section – C contains 6 questions of 4 marks each.
- Section – D contains 4 questions of 6 marks each.
- Use of calculator is not permitted

SECTION A

Q1 to Q5 are multiple choice type questions. Select the correct option.

Q1.If A is any square matrix of order 3 such that $|adjA| = 64$, then the value(s) of $|A|$ is/are

(a) ± 4 (b) ± 8 (c) 64 (d) 16

Q2.If A and B are matrices such that AB = O, then

(a) it is not necessary that either A = O or B = O (b) A = O or B = O

(c) A = O and B = O (d) $A^{-1} = B, B^{-1} = A$

Q3. If the points $A(60\hat{i} + 3\hat{j}), B(40\hat{i} - 8\hat{j}), C(a\hat{i} - 52\hat{j})$ are collinear, then a is equal to

(a) 40 (b) -40 (c) 20 (d) -20

Q4. If A and B are two events such that $P(A) \neq 0$, $P(B) \neq 1$, then $P(\bar{A}/\bar{B}) =$

(a) $1 - P(A/B)$ (b) $1 - P(\bar{A}/B)$ (c) $\dfrac{1 - P(A \cup B)}{P(\bar{B})}$ (d) $\dfrac{P(\bar{A})}{P(\bar{B})}$

Q5. The region represented by the system of inequalities $x \geq 0, y \geq 0, x+y \geq 3$ is

(a) unbounded in the first quadrant

(b) unbounded in the first and the second quadrants

(c) bounded in the first quadrant (d) bounded in the second quadrant

Q6 to Q10 are fill in the blank type questions. Fill in the blanks to make the statement correct.

Q6. If $f(x)=\sin^2 x$ and the composite function $gof(x)=|\sin x|$, then $g(x)=$ ______________

Q7. If $\sin^{-1} x + 4\cos^{-1} x = \pi$, then x = _______________

Q8. If the function $f(x) = \begin{cases} \frac{1-\sin x}{(\pi-2x)^2} & \text{when } x \neq \frac{\pi}{2} \\ k & \text{when } x = \frac{\pi}{2} \end{cases}$, then f(x) will be

continuous at $x = \frac{\pi}{2}$, if k is ______________

Q9. Let $y = \log x$. Then, when x = 3 and $\Delta x = 0.03, dy =$ ___________

Q10. The magnitude of the vector projection of the vector $(3\hat{i} - \hat{j} + \hat{k})$ on the vector

$(\hat{i} + 5\hat{j} - \hat{k})$is__________

Q11 to Q15 are True/False type questions.(State whether the statement is true or False)

Q11. If A is a symmetric matrix, then A^T is a symmetric matrix.

Q12. The sum of the products of the elements of any row (or column) of a determinant and their corresponding cofactors is the value of the determinant.

Q13. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (sinx - cosx)^2 dx = 2\int_0^{\frac{\pi}{2}} (sinx - cosx)^2 dx$

Q14. $\int \frac{1}{e^x+e^{-x}} dx = tan^{-1}(e^x) + c$

Q15. $\int e^{x}\left(\frac{1}{x}-\frac{1}{x^{2}}\right)dx = e^{x}\frac{1}{x^{2}}+c$

Q16 to Q20 are very short answer type questions.

Q16. Evaluate $\int \frac{x^{2}}{\sqrt{x^{6}+1}}dx$

Q17. Write the I. F. of the differential equation $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$

Q18. Find the point of intersection of the line $\frac{x}{2}=\frac{y}{1}=\frac{z}{3}$ with the plane x + y + z = 6.

Q19. Find the value of a if the planes ax + y – 3z = 1 and 2x + y + 2z = 5 are perpendicular to each other.

Q20. A die is tossed thrice. Find the probability of getting odd number at least once.

SECTION B

Q21. Prove that $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$;

OR

Simplify $\cos^{-1}x+\cos^{-1}\left(\frac{x}{2}+\frac{\sqrt{3-3x^{2}}}{2}\right), \frac{1}{2}\le x\le 1$

Q22. If $e^{y}(x+1)=1$, then show that $\frac{d^{2}y}{dx^{2}}=(\frac{dy}{dx})^{2}$.

Q23. Find the equation of the tangent to the curve $y=\sqrt{x-2}$, which is parallel to the line 4y = 2x - 5.

Q24. Prove that $[\vec{a}\quad \vec{b}\quad \vec{a}\times\vec{b}]+(\vec{a}.\vec{b})^{2}=|\vec{a}|^{2}|\vec{b}|^{2}$

OR

If $|\vec{a}|=2, |\vec{b}|=3$, the angle between the vectors $\vec{a},\vec{b}=\frac{\pi}{3}$, then find $|\vec{a}+\vec{b}|$.

Q25. Find the distance between the lines

$\vec{r}=\hat{i}-\hat{j}+2\hat{k}+\lambda(2\hat{i}+\hat{j}+4\hat{k}),$

$\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+4\hat{k})$

Q26. Prove that if E and F are independent events, then so are the events $\overline{E}$ and $\overline{F}$.

SECTION C

Q27. Let $f: R - \left\{-\frac{4}{3}\right\} \rightarrow R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that $f: R - \left\{-\frac{4}{3}\right\} \rightarrow \text{Range}(f)$

is one-one and onto. Hence, find f^{-1}.

Q28. If $y = \left\{\log(x + \sqrt{x^2+1})\right\}^2$, then show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$

OR

If $y = (\sin x)^{\tan x} + x^{\log x}$, then find $\frac{dy}{dx}$. .

Q29. Find the general solution of the differential equation

$$(x\cos\frac{y}{x} + y\sin\frac{y}{x})y - (y\sin\frac{y}{x} - x\cos\frac{y}{x})x\frac{dy}{dx} = 0.$$

Q30. Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\sin x}dx$

OR

Evaluate $\int_{-1}^{\frac{3}{2}} |x\sin(\pi x)|dx$

Q31. A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed three times, then find the probability distribution of the number of tails. Also, find the mean of the number of tails.

OR

From a pack of 52 playing cards, a card is lost. From the remaining pack of cards, two cards are drawn at random and are found to be both diamonds. Find the probability that the lost card is a spade card.

Q32. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as an LPP and solve it graphically.

SECTION D

Q33. If x, y, z are different and

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$, then using the properties of determinants, prove that

$1 + xyz = 0$

OR

If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Hence, solve the following system of equations

$3x + 4y + 7z = 14,$

$2x - y + 3z = 4,$

$x + 2y - 3z = 0$

Q34. Using integration, find the area of the triangle with vertices A(0, 0), B(2, 3), C(4, -1)

Q35. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

OR

Determine the points on the curve $x^2 = 4y$, which are nearest to the point (0, 5).

Q36. Find the vector as well as the cartesian equation of the plane passing through the point $(4, -3, 2)$ and perpendicular to the line of intersection of the plane x – y + 2z – 3 = 0 and 2x – y – 3z = 0. Find, also, the intercepts on the axes cut off by the plane so obtained.

Answers

Assignment 1

1. $[0,1[$

3. $g \circ f(x) = 2x$

$f \circ g(x) = 8x$

4. x 5. 1

6. 27. 0

8. $f^{-1}(x) = \dfrac{x}{1-x}$

9. $[(2,5)] = \{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)\}$

12. $fog(x) = \begin{cases} 0, x \geq 0 \\ -4x, x < 0 \end{cases}$

$gof(x) = 0,$ *for* all x

$fog(-3) = 12$

$fog(5) = 0, gof(-2) = 0$

13. $f^{-1}(x) = (x-27)^{\frac{1}{3}}$

14.(i) $\{0,2,4\}$

(ii) R is not an equivalence relation

$f^{-1}(0) = -3$

16. $\left(\dfrac{x-7}{2}\right)^{\frac{1}{3}}$,1

Assignment 2:

1. $\dfrac{\pi}{2}$ 2. $\dfrac{-\pi}{10}$

14. $x = \dfrac{1}{4}$

3. $\pi - \sec^{-1} x$

15. $0, \dfrac{1}{2}$

4. $\dfrac{1}{2}$ 5. $\dfrac{1}{\sqrt{3}}$ 6. $\dfrac{\sqrt{3}}{2}$ *8. 0,-1 9. -1*

11.a) $\dfrac{1}{2}\tan^{-1} x$

b) $\frac{x}{2}$ / $\frac{\pi}{2}-\frac{x}{2}$

c) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$

Assignment No. 3

1. $x=3; y=3$ 2. I 3. $\begin{bmatrix} 2 & -1 & -2 \\ 3 & 4 & -1 \end{bmatrix}$ 4. 0

5. $\begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$

6. $\begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$ 7. $\begin{bmatrix} 3 & 3/2 & 5/2 \\ 3/2 & 4 & 4 \\ 5/2 & 4 & 8 \end{bmatrix}+\begin{bmatrix} 0 & 1/2 & 9/2 \\ -1/2 & 0 & -1 \\ -9/2 & 1 & 0 \end{bmatrix}$ 8. $x=-1; x=-2$

10. $\begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$

11. x=200,y=1000 12. $(i)\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$

Assignment No. 4

1. 36 2. 25 3.0 4.64 5. -15 6. 216

9. a) $x=2, y=3, z=5$ 9.b) $x=1, y=2, z=5$ 10. $x=2, y=-1, z=4$

11. $x=1, y=1, z=1$ 12. $\begin{bmatrix} -7 & 3 \\ 12 & -5 \end{bmatrix}$ 13. $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ 14. $k=-6$ or 14

Assignment No. 5

1. $\frac{1}{2}$ 2. $\frac{2\left(6^{x}\log 6\right)}{1+6^{2x}}$ 4. $\frac{2}{1+x^{2}}$ 5. Discontinuous at $x=-2,-1,0,1,2$ 6. $\frac{1}{2\left(1+x^{2}\right)}$

7. $\frac{-1}{(1+x)2\sqrt{x}}$ 8. $\frac{1+x^{2}}{2}$

9. 5 11. $\frac{1}{2\sqrt{12}}$ 12. $a=-\frac{3}{2},c=\frac{1}{2}$ & $b\in R-\{0\}$

13. Not derivable

14. $a=2,\ b=-1$ 15. Continuous

16. $x=\pi$ 17. $x^{\sin x}\left[\frac{\sin x}{x}+\cos x\log x\right]+\frac{1}{\sqrt{1-x}}\frac{1}{2\sqrt{x}}$ 22. $\frac{2\sqrt{2}}{a}$ 23. $\frac{32}{27a}$ 24. $\frac{-1}{2}$

Assignment No. 6

1. $\frac{\sqrt{2}}{4\pi}\,cm/\sec$ 2. $-\frac{2}{75}\,radian/\sec$ 3. 5.02, 3.074, 0.1925 4. $(-1,\infty)$ function is increasing.

5. $(-\infty,-1)\,\&\,(-1,1)$ function decreasing, $(1,3)\,\&\,(3,\infty)$ function increases ; point of minima is 1, points of inflexion are $-1,3$. 6. $x=3$ point of maxima, $x=0,5$ are points of minima. Minimum values : $f(0)=105$; $f(5)=\frac{545}{4}$; Maximum value: $f(3)=\frac{609}{4}$.

7. $(4,-4)$ 8. $y+3x=3$ & $y=7x-14$ 9. Increasing in the interval $(-\infty,1)\,\&\,(2,\infty)$ and decreasing in the interval $(1,2)$ 10. $\left(0,\frac{\pi}{4}\right)$ decreasing and $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ increasing 11.a) $x=\frac{\pi}{6}$ is a point of maxima and the maximum value $\sqrt{3}+\frac{\pi}{6}$. Point of minima is $x=\frac{5\pi}{6}$ and the minimum value is $-\sqrt{3}+\frac{5\pi}{6}$. 11(b) $x=\frac{\pi}{3}$ is a point of maxima and the maximum value is $\sqrt{3}-\frac{\pi}{3}$; $x=-\frac{\pi}{3}$ is a point of minima and the minimum value is $-\sqrt{3}+\frac{\pi}{3}$

12. $y-\frac{\pi}{4}+\frac{1}{\sqrt{2}}=\left(\sqrt{2}-1\right)\left(x-\frac{\sqrt{2}-1}{\sqrt{2}}\right)$ & $y-\frac{\pi}{4}+\frac{1}{\sqrt{2}}=\frac{-1}{\left(\sqrt{2}-1\right)}\left(x-\frac{\sqrt{2}-1}{\sqrt{2}}\right)$

15. (-2,-8) 16. $x+2^{\frac{2}{3}}y=2+2^{\frac{2}{3}}$

Answers of Indefinite Integral questions

1. $\log\left(1+x^2\right)+c$ 2. $-\frac{1}{8}\log\left|9-4x^2\right|+c$ 3. $\log\left|e^x+e^{-x}\right|+c$ 4. $\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+c$ 5. $-\log\left|1+\cos x\right|+c$ 6. $\frac{1}{2}\log\left|2\sin x+3\cos x\right|+c$ 7. $-\frac{1}{4}\tan\left(7-4x\right)+c$ 8. $\frac{1}{2}\tan\left(2x-3\right)+c$ 9. $\frac{2}{3a}\left(ax+b\right)^{\frac{3}{2}}+c$ 10. $\frac{e^{2x+3}}{2}+c$ 11. $\frac{1}{2}\log\left|\sec 2x+\tan 2x\right|+c$ 12. $-\frac{1}{3}\cos\left(3x-1\right)+c$ 13. $\frac{1}{3}\left(\log|x|\right)^3+c$ 14. $\log\left|1+\log x\right|+c$ 15. $\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}+c$ 16. $\frac{1}{7}\left(x^3-1\right)^{\frac{7}{3}}+\frac{1}{4}\left(x^3-1\right)^{\frac{4}{3}}+c$ 17. $-\frac{1}{2e^x}+c$ 18. $-\frac{1}{4}\cos\left(\tan^{-1}x^4\right)+c$ 19. $\frac{9}{82}\log\left|5\sin x+4\cos x\right|+\frac{22}{41}x+c$ 20. $\frac{x}{2}-\frac{1}{2}\log\left|\cos x+\sin x\right|+c$ 21. $\frac{x}{2}-\frac{1}{2}\log\left|\cos x-\sin x\right|+c$ 22. $\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+c$ 23. $\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)+c$ 24. $-\frac{3}{4}\cos x+\frac{\cos 3x}{12}+c$ 25. $\frac{1}{4}\left(\frac{\sin 3x}{3}+3\sin x\right)+c$ 26. $\frac{x}{2}-\frac{1}{8}\sin\left(4x+10\right)+c$ 27. $-\frac{1}{2}\cos\left(2x+1\right)+\frac{1}{6}\cos^3\left(2x+1\right)+c$ 28. $-\frac{1}{14}\cos 7x+\frac{1}{2}\cos x+c$ 29. $\frac{1}{2}\left(\frac{1}{4}\sin 4x-\frac{1}{12}\sin 12x\right)+c$ 30. $\frac{1}{4}\left(\frac{1}{12}\sin 12x+x+\frac{1}{8}\sin 8x+\frac{1}{4}\sin 4x\right)+c$ 31. $\frac{3x}{8}-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+c$ 32. $\frac{3x}{8}+\frac{1}{8}\sin 4x+\frac{1}{64}\sin 8x+c$ 33. $\frac{1}{3}\tan^3 x-\tan x+x+c$ 34. $\log\left|\tan x\right|+\frac{1}{2}\tan^2 x+c$ 35. $\frac{1}{8}\left(x-\frac{\sin 4x}{4}\right)+c$ 36. $\frac{1}{16}\left(-\frac{3}{4}\cos 2x+\frac{1}{12}\cos 6x\right)+c$ 37. $\frac{1}{\sin\left(a-b\right)}\log\left|\frac{\cos\left(x-a\right)}{\cos\left(x-b\right)}\right|+c$ 38. $\frac{1}{\cos\left(a-b\right)}\log\left|\frac{\sin\left(x+a\right)}{\cos\left(x+b\right)}\right|+c$ 39. $-\frac{\cos^3 x}{3}+\frac{\cos^5 x}{5}+c$ 40. $\frac{1}{128}\left(3x-\sin 4x+\frac{1}{8}\sin 8x\right)+c$ 41. $-\left(\cos x+\frac{1}{5}\cos^5 x-\frac{2}{3}\cos^3 x\right)+c$ 42. $\tan^{-1}x^3+c$ 43 $\frac{1}{2}\log\left|2x+\sqrt{1+4x^2}\right|+c$ 44. $\log\left|\frac{1}{2-x+\sqrt{x^2-4x+5}}\right|+c$ 45. $\frac{1}{5}\sin^{-1}\frac{5x}{3}+c$ 46. $\log\left|\tan x+\sqrt{\tan^2 x+4}\right|+c$ 47. $\log\left|\sin x-1+\sqrt{\sin^2 x-2\sin x-3}\right|+c$ 48. $\frac{1}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|+c$ 49. $\frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right)+c$

50. $\frac{5}{6}\log\left|3x^2+2x+1\right|-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+c$ 51. $\frac{1}{2}\log\left|x^2-2x-5\right|+\frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|+c$

52. $\log\left|x+1+\sqrt{x^2+2x+2}\right|+c$ 53. $\sin^{-1}\left(\frac{x+3}{2}\right)+c$

54. $6\sqrt{x^2-9x+20}+34\log\left|x-\frac{9}{2}+\sqrt{x^2-9x+20}\right|+c$ 55. $-\sqrt{4x-x^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)+c$

56. $2\log\left|\sin^2\phi-4\sin\phi+5\right|+7\tan^{-1}\left(\sin\phi-2\right)+c$ 57. $-\frac{1}{\cos x+\sin x}+c$

58. $\sin^{-1}\left(\sin x-\cos x\right)+c$ 59. $\frac{1}{40}\log\left|\frac{5+(\sin x-\cos x)}{5-(\sin x-\cos x)}\right|+c$ 60. $2\log\left|\sqrt{x}-1\right|+c$

61. $2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+c$ 62. $\log\frac{(x+2)^2}{|x+1|}+c$ 63. $\frac{1}{6}\log\left|\frac{x-3}{x+3}\right|+c$

64. $\log|x-1|-5\log|x-2|4\log|x-3|+c$ 65. $\frac{x}{2}+\log|x|-\frac{3}{4}\log|1-2x|+c$

66. $\frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{12}{5}\log|2x+3|+c$ 67. $\frac{x^2}{2}+\frac{1}{2}\log|x+1|+\frac{3}{2}\log|x-1|+c$

68. $\frac{2}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3(x-1)}+c$ 69. $3\log|x-2|+\frac{7}{x+2}+c$

70. $-5\log|x-1|+\frac{5}{x-1}+\frac{1}{(x-1)^2}+5\log|x-2|+c$ 71. $-\log|x-1|+\frac{1}{2}\log\left(1+x^2\right)+\tan^{-1}x+c$

72. $\frac{1}{2}\log|x-1|-\frac{1}{4}\log\left(1+x^2\right)+\frac{1}{2}\tan^{-1}x+c$ 73. $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+c$

74. $-\frac{1}{4x}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)+c$ 75. $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+c$ 76. $\frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right|+c$

77. $\frac{1}{3\sqrt{2}}\left\{\tan^{-1}\frac{x}{\sqrt{2}}+\tan^{-1}\left(\sqrt{2}x\right)\right\}+c$ 78. $x+\frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}-3\tan^{-1}\frac{x}{2}+c$ 79. $\frac{1}{2}\log\left(\frac{x^2+1}{x^2+3}\right)+c$ 80. $\log\left|\frac{2-\sin x}{1-\sin x}\right|+c$ 81. $-\frac{1}{8}\log\left|\frac{1+\sin x}{1-\sin x}\right|+\frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right|+c$

82. $-\frac{1}{3}\log|1+\tan\theta|+\frac{1}{6}\log\left|\tan^2\theta-\tan\theta+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan\theta-1}{\sqrt{3}}\right)+c$

83. $\frac{1}{6}\log|1-\cos x|+\frac{1}{2}\log|1+\cos x|-\frac{2}{3}\log|1+2\cos x|+c$ 84. $-x\cos x+\sin x+c$

85. $e^x\left(x^2-2x+2\right)+c$ 86. $\frac{x^3}{3}\log x-\frac{x^3}{9}+c$ 87. $\frac{1}{4}\left(2x^2-1\right)\sin^{-1}x+\frac{x\sqrt{1-x^2}}{4}+c$

88. $\frac{x^2}{2}\tan^{-1}x-\frac{x}{2}+\frac{1}{2}\tan^{-1}x+c$ 89. $x\sin^{-1}x+\sqrt{1-x^2}+c$

90. $\left(\sin^{-1}x\right)^2x+2\sqrt{1-x^2}\sin^{-1}x-2x+c$ 91. $x\log x-x+c$ 92. $x\left(\log x\right)^2-2\left(x\log x-x\right)+c$

93. $\frac{x^2}{2}(\log x)^2-\frac{x^2}{2}\log x+\frac{x^2}{4}+c$ 94. $x\tan^{-1}x-\frac{1}{2}\log\left(1+x^2\right)+c$

95. $\frac{1}{2}\left(\sec x\tan x+\log|\sec x+\tan x|\right)+c$ 96. $\frac{1}{2}\left(-\operatorname{cosec}x\cot x+\log|\operatorname{cosec}x-\cot x|\right)+c$

97. $e^x sinx+c$ 98. $\frac{e^x}{1+x}+c$ 99. $e^x\tan\frac{x}{2}+c$ 100. $e^x\frac{1}{x}+c$

101. $\frac{e^x}{(x-1)^2}+c$ 102. $e^x-\frac{2e^x}{x+1}+c$ 103. $\frac{x}{\log x}+c$ 104. $x\log(\log x)-\frac{x}{\log x}+c$ 105. $e^x\log x-\frac{e^x}{x}+c$

106. $\frac{e^{2x}}{5}(2\sin x-\cos x)+c$ 107. $\frac{e^{-x}}{2}(\sin x-\cos x)+c$ 108. $\frac{e^x}{2}-\frac{e^x}{10}(\cos 2x+2\sin 2x)+c$ 109. $\frac{e^{2x}}{4}+\frac{e^{2x}}{8}(\cos 2x+2\sin 2x)+c$ 110. $\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+c$ 111. $\frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}(2x)+c$

112. $\frac{1}{2}\left\{x\sqrt{(x+2)^2-9}-9\log\left|x+2+\sqrt{(x+2)^2-9}\right|\right\}+c$

113. $\frac{1}{2}\left\{(x+2)\sqrt{x^2+4x+6}+2\log\left|x+2+\sqrt{x^2+4x+6}\right|\right\}+c$

114. $\frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+c$

115. $-\frac{1}{3}\left(3-4x-x^2\right)^{\frac{3}{2}}+\frac{1}{2}\left(x+2\right)\sqrt{3-4x-x^2}+\frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}}+c$

116. $\frac{1}{\sqrt{3}}\log\left|\frac{\tan\frac{x}{2}-2-\sqrt{3}}{\tan\frac{x}{2}-2+\sqrt{3}}\right|+c$ 117. $\frac{2}{3}\tan^{-1}\left(3\tan\frac{x}{2}\right)+c$ 118. $\frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right|+c$

119. $\sqrt{2}\tan^{-1}\left(\frac{\tan\frac{x}{2}+1}{\sqrt{2}}\right)+c$ 120. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$ 121. $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+c$

122. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right)+\frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+c$

123. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right)-\frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+c$

124. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)+c$ 125. $\frac{1}{3}\tan^{-1}\left(\frac{x^2-1}{3x}\right)+c$

126. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right|+c$

127. $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\cot x-1}{\sqrt{2\cot x}}\right)-\frac{1}{2\sqrt{2}}\log\left|\frac{\cot x-\sqrt{2\cot x}+1}{\cot x+\sqrt{2\cot x}+1}\right|+c$

128. $\frac{1}{\sqrt{15}}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{5}}\right)+c$ 129. $\frac{1}{2}\tan^{-1}\left(2\tan x\right)+c$ 130. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+c$

31. $\frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right|+c$ 132. $\tan^{-1}\left(\tan^2 x\right)+c$ 133. $\frac{1}{5}\log\left|\frac{\tan x-2}{2\tan x+1}\right|+c$ 134.
$\frac{1}{2\sqrt{5}}\log\left|\frac{\sqrt{5}\tan x-1}{\sqrt{5}\tan x+1}\right|+c$ 135. $-\frac{1}{2\left(2\tan x+3\right)}+c$ 136. $2\sqrt{\tan x}+c$ 137. $a\sin^{-1}\frac{x}{a}+\sqrt{a^2-x^2}+c$

138. $-\cos\alpha\sin^{-1}\left(\frac{\cos x}{\cos\alpha}\right)-\sin\alpha\log\left|\sin x+\sqrt{\sin^2 x-\sin^2\alpha}\right|$

139. $-\log\left|\cos x+\frac{1}{2}+\sqrt{\cos^2 x+\cos x}\right|+c$

140. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\left(\sin x-\cos x\right)}{\sqrt{3}-\left(\sin x-\cos x\right)}\right|+\tan^{-1}\left(\sin x+\cos x\right)+c$

141. $-\frac{1}{4}\sin^{-1}\left(\frac{\cos^2 2x}{3}\right)+c$ 142. $2\tan^{-1}\sqrt{x+\frac{1}{x}+1}+c$ 143. $\frac{4}{15}\left(1-\frac{1}{x^3}\right)^{\frac{5}{4}}+c$

144. $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+c$ 145. $\sqrt{2}\sin^{-1}\left(\sin x-\cos x\right)+c$

<u>Assignment No. 7 (a)</u>

1. $\sqrt{x^2+1}-2\log\left|x+\sqrt{x^2+1}\right|+c$ 2. $\frac{2}{3}x^{\frac{3}{2}}+\frac{x^3}{3}+c$ 3. $\frac{1}{2}\log\left|\cos ec(x+\frac{\pi}{6})-\cot(x+\frac{\pi}{6})\right|$

4. $\log\left|x+\log\cos x\right|$ 5. 8 6. $x\sin(\log x)$ 7. $k=\frac{1}{\log 2}$ 8. $\frac{1}{24}\log\frac{3x-4}{3x+4}$

9. $-\frac{1}{2}e^{-x^2}+c$ 10. $\tan^{-1}e^x+c$ 11. $\frac{1}{3}\log\frac{2\tan x+1}{\tan x-2}+C$ 12. $-\sqrt{1+2\cot x}+c$

13. $\cot x+\frac{1}{7}\cot^7 x+\frac{3}{5}\cot^5 x+\cot^3 x+c$ 14. $\frac{1}{(\log 5)^3}5^{5^{5x}}+c$

15. $\frac{1}{3}\log\left|x^3+\sqrt{x^6-a^6}\right|+c$ 16. $\frac{x^2}{2}+2x+\frac{3}{2}\log\left|x^2-x+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}}+c$

17. $\sqrt{-x^2+5x+6}+\frac{1}{2}\sin^{-1}\left(2x-5\right)+c$ 18. $\log\left|\frac{2e^x+1}{2e^x+2}\right|+c$

19. $-3\sqrt{6+x-2x^2}+\frac{13}{2\sqrt{2}}\sin^{-1}\frac{4x-1}{7}+c$ 20. $\sin^{-1}(2\sin x-1)+c$

21. $\frac{-1}{3(3\tan x+1)}+C$ 22. $\log(\log(\log x))+C$ 23. $\sqrt{2}(\sin^{-1}(\sin\theta-\cos\theta))+C$

24. $\frac{1}{2}\log\left|\frac{\tan x}{\tan x+2}\right|+C$ 25. $(\log x)^2+C$ 26. $\log\left|\cos x+\frac{1}{2}+\sqrt{\cos^2 x+\cos x}\right|+C$

27. $\frac{-2}{\sqrt{23}}\tan^{-1}\frac{2\tan\frac{x}{2}+1}{\sqrt{23}}+c$ 28. $\frac{1}{3\sqrt{2}}\tan^{-1}\frac{x^2-9}{3\sqrt{2}x}+c$

29. $\frac{1}{2a^3b^3}\left\{\left(a^2+b^2\right)\tan^{-1}\left(\frac{a\tan x}{b}\right)+\frac{(a^2-b^2)ab\tan x}{a^2\tan^2 x+b^2}\right\}+c$

<u>Assignment 7 (b)</u>

1. $\frac{1}{4}\log|x-1|+\frac{3}{4}\log|x+1|+\frac{1}{2(x+1)}+c$ 2. $3\left[x\cos^{-1}x-\sqrt{1-x^2}\right]+c$ 3. $\frac{e^x}{(x+1)^2}+c$

4. $-\frac{1}{3}\log|\tan x+1|+\frac{1}{6}\log\left|\tan^2 x-\tan x+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x-1}{\sqrt{3}}+c$

5. $e^{\frac{-x}{2}}\sec\frac{x}{2}+c$ 6. $-\frac{\tan^{-1}x}{\sqrt{1+x^2}}+\frac{x}{\sqrt{1+x^2}}+c$ 7. $-\frac{\log(x+2)}{x+2}-\frac{1}{x+2}+c$

8. $-\frac{1}{2}\log|1-\cos x|+\frac{1}{18}\log|1+\cos x|+\frac{4}{9}\log|5-4\cos x|+c$

9. $2\left[\sqrt{x}e^{\sqrt{x}}-e^{\sqrt{x}}\right]+c$ 10. $\frac{3}{2}\sqrt{x}\sin\sqrt{x}+\frac{3}{2}\cos\sqrt{x}+\frac{1}{6}\sqrt{x}\sin 3\sqrt{x}+\frac{1}{18}\cos 3\sqrt{x}+c$

11. $\frac{9}{13}\left[-\frac{1}{3}e^{-3x}\cos 2x+\frac{2}{9}e^{-3x}\sin 2x\right]+c$ 12. $e^x\left[-\log\cos x\right]+c$

13. $\frac{1}{2}e^{2x}\cot 2x+c$ 14. $x-\log x+\frac{1}{2}\log\left|x^2+1\right|-\tan^{-1}x+c$

15. $-\frac{1}{3}\left(3-x-x^2\right)^{\frac{3}{2}}+\frac{1}{2}\left[\frac{2x+1}{4}\sqrt{3-x-x^2}+\frac{13}{8}\sin^{-1}\frac{2x+1}{\sqrt{13}}\right]+c$

16. $\frac{x^3}{3}\operatorname{cosec}^{-1}x+\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2-1}-\frac{1}{2}\log\left|x+\sqrt{x^2-1}\right|\right]+\frac{1}{3}\log\left|x+\sqrt{x^2-1}\right|+c$

17. $x\log(\log x)-\frac{x}{\log x}+c$

18. $\left(x^2+2x-3\right)^{\frac{3}{2}}-2\left[\frac{x+1}{2}\sqrt{x^2+2x-3}-2\log\left|x+1+\sqrt{x^2+2x-3}\right|\right]+c$

Assignment 8

$1.\ 0.5\quad 2.\ -2\quad 3.\ 2$
$4.\ 2\quad 5.\ e-1$
$6.\ \frac{1}{2}\quad 7.\ \frac{\pi}{4}$
$8.\ a=-1, b=1\quad 9.\ (a)\ 1-\log 2\quad (b)\ \sqrt{2}\pi$
$(c)\ \frac{\pi}{\sqrt{35}}\quad (d)\ \frac{\pi}{12}\quad (e)\ 0\quad (f)\ \frac{\pi^2}{16}\quad (g)\ \frac{\pi}{2}-\log 2$
$(h)\ \frac{\pi}{8}\log 2\quad (i)\ 2\quad (j)\ \frac{\pi}{2}\quad (k)\ \frac{63}{2}\quad (l)\ \pi^2\quad (m)\ \frac{\pi^2}{6\sqrt{3}}$
$10.\ (i)\ \frac{-7}{6}\quad (ii)\ \frac{e^2\left(e^4-1\right)}{2}\quad (iii)\ 32$
$(iv)\ e^4+7$

Assignment 9

1. $\left(\frac{16}{3}-\frac{4}{3}\sqrt{2}\right)$ *sq units* 2. $\frac{4\sqrt{3}+16\pi}{3}$ *sq units*
3. $\frac{16a^2}{3}$ *sq units* 4. 7 *sq units* 5. 6 *sq units*
6. $\frac{4}{3}$ *sq units* 7. (π) *sq units*
8. 10 *sq units* 9. $\frac{62}{3}$ *sq units* 10. $\frac{64}{3}$ *sq units*
11. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ *sq units* 12. $\left(\frac{5\pi}{2}-\sin^{-1}\frac{1}{4}-4\sin^{-1}\frac{7}{8}-\frac{\sqrt{15}}{2}\right)$ *sq units*
13. 4 *sq units* 14. 18 *sq units* 15. $\left(\frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}}\right)-\frac{1}{2}\right)$ *sq units*

Assignment10

1.$\frac{d^2y}{dx^2} - y = 0$ 2.$y = x$ 5.$(i)\,2,4\ (ii)\,2, n.d.$

$(iii)1,2\ (iv)2,3$

6.$(i)e^{\tan^{-1}x}\ (ii)\frac{1}{x}\ (iii)\frac{1}{y^2}\ (iv)e^{2x}$ 7.$(i)\log|y| = 2x + 2\log|x-1| - 4$

$(ii)\log|x| + x = -\log|y| - y + c\ (iii)e^{2x}y = -\cos x + 1$

$(iv)x = 2y^2\ \ (v)\log\left|1 + \tan\frac{x+y}{2}\right| = x + c$

$(vi)c - \sqrt{1+y^2} = \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right|$

$(vii)x = y + 1 - \log|x+y+2| = x + c$

$(viii)x^2\sin x + 2x\cos x - 2\sin x - x^2y + c = 0$

$(ix) - \tan^{-1}y = \tan^{-1}e^x - \frac{\pi}{2}$

$(x)\log|1+y| = x - \frac{x^2}{2} + c$

Answers to Vectors

1. $0 \le |k\vec{a}| \le 6$
2. $a = -4$
3. $\frac{11}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$
4. $5\hat{i} + 5\hat{k}$
5. $\left\langle \frac{2}{7}, \frac{3}{7}, -\frac{6}{7} \right\rangle$, *Scalar* components : 4,6, -12 and vector components : $4\hat{\mathrm{i}}, 6\hat{\mathrm{j}}, -12\hat{\mathrm{k}}$
6. $\pm\left(\frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}\right)$
7. $|\overrightarrow{AB}| = \sqrt{14}, |\overrightarrow{BC}| = \sqrt{21}, |\overrightarrow{CA}| = \sqrt{35}$
8. $(3\vec{a} + 5\vec{b})$
9. $\frac{\sqrt{34}}{2}$
10. 8
11. 5
12. $120°$
13. Prove dot product to be zero
14. $\pm 2\sqrt{10}$

15. 5
16. $\sqrt{21}$
17. $\lambda = 1,\ \frac{1}{7}\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right)$
18. $45°, 45°$
19. Prove
20. $60°$
21. $\pm 10\left(\hat{i} - \hat{j} + \hat{k}\right)$
22. $\pm \frac{1}{\sqrt{165}}\left(-10\hat{i} - 7\hat{j} + 4\hat{k}\right)$
23. $\frac{\pi}{2}$, $2\hat{i} - 26\hat{j} - 10\hat{k}$
24. ± 12
25. Prove
26. $\left(\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$

Assignment11(a)

1. $\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}$ 2. $\sqrt{a^2+b^2}$ 3. $\hat{i}+\hat{j},-\hat{i}-\hat{j}$

4. 5 5. $\frac{3}{7},\frac{-6}{7},\frac{2}{7}$

6. ± 6 7. $\frac{\pi}{3},\frac{2\pi}{3}$ 8. $\pm 2\sqrt{3}$ 9. $\frac{9\hat{i}+11\hat{j}+15\hat{k}}{3}$

10. $(i)\cos^{-1}\frac{1}{\sqrt{3}},\cos^{-1}\frac{1}{\sqrt{3}},\cos^{-1}\frac{-1}{\sqrt{3}}$

$(ii)\frac{3\pi}{4},\frac{3\pi}{4},\frac{\pi}{2}$ 12. $\frac{-2\hat{i}+\hat{j}+5\hat{k}}{\sqrt{30}}$ 13. $2\hat{i}-2\hat{j}+\hat{k}$

14. $\sqrt{114}$ 15. $\frac{5}{\sqrt{6}},\frac{5}{6}\left(\hat{i}-2\hat{j}+\hat{k}\right)$

17. -3 18. $x=-6, y=2$

19. $4\hat{i}+\hat{j}$ 20. ± 6 21. 90° 22. 1

Assignment11(b)

1. $2,-1$ 2. $\frac{\pi}{4}$ 3. $3\sqrt{2}$

4. $\frac{-\hat{i}+2\hat{j}+\hat{k}}{\sqrt{6}},\frac{\hat{i}-\hat{k}}{\sqrt{2}}$ 5. $5\sqrt{17}\,sq\,units$

7. $3\frac{27}{2}$ 8. $\hat{i}+3\hat{j}+3\hat{k}$

10. 264 *cubicunits*

12. $\lambda=1$ 13. $\alpha=-2,-3$

14. -1

19. $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j},\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}$

20. $\pm\frac{100}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

Answers to Three Dimensional Geometry

1. 60° or 120°
2. $\left\langle \frac{3}{\sqrt{77}}, -\frac{2}{\sqrt{77}}, \frac{8}{\sqrt{77}} \right\rangle$
3. $p = 18, q = \frac{2}{3}$
4. $(-2,1,7)$ and $(-3,-6,10)$
5. $\left\langle \frac{6}{7}, \frac{2}{7}, -\frac{6}{7} \right\rangle$
6. $\langle 3,2,-1 \rangle$, $\vec{r} = 7\hat{i} - 5\hat{j} + \lambda(3\hat{i} + 2\hat{j} - \hat{k})$
7. $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$, $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$

   $\vec{r} = -\hat{i} - 2\hat{j} - \hat{k} + \mu(3\hat{i} + 5\hat{j} + 3\hat{k})$, λ and μ are parameters , $\frac{x+1}{3} = \frac{y+2}{5} = \frac{z+1}{3}$

   $D(4,7,6)$
8. $(-2,-1,3)$ and $(4,3,7)$
9. $(4,0,-1)$
10. 90°
11. $p = 7$, $\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$
12. $(1,0,7)$
13. $\sqrt{10}$
14. $\vec{r} = \hat{i} - \hat{j} + \hat{k} + \lambda(10\hat{i} - 4\hat{j} - 7\hat{k})$, $\frac{x-1}{10} = \frac{y+1}{-4} = \frac{z-1}{-7}$
15. $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ and $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$
16. $2\sqrt{29}$
17. $\frac{5}{\sqrt{29}}$
18. $\frac{x-1}{3} = \frac{y-1}{10} = \frac{z-1}{17}$
19. $\vec{r}.(2\hat{i} - 3\hat{j} + 6\hat{k}) = 35$
20. $\vec{r}.(\hat{i} - 3\hat{j} + 2\hat{k}) + 3 = 0$, $x - 3y + 2z + 3 = 0$

23. 13
24. $(2,2,0)$
25. 1 unit.

26. $\frac{17}{2}$

27. $\sqrt{6}$, $\left(0,\frac{5}{2},0\right)$

28. $3\hat{i}+\frac{7}{2}\hat{j}+\frac{11}{2}\hat{k}$, $\frac{\sqrt{14}}{2}$, $(4,4,7)$

29. $6x-3y+z-3=0$

30. $\vec{r}.(\ 7\hat{j}+5\hat{k}\ )=35$

31. $\vec{r}.(\ 8\hat{i}+5\hat{j}+4\hat{k}\ )-12=0$

32. $x+y+z=0$

33. $8x+y-5z-7=0$, It contains the given line.

34. $k=\frac{9}{2}$, $5x-2y-z-6=0$

35. $9x+2y-7z-21=0$

36. $(1,-2,7), 2:1$

37. $4x-3y+2z=12, \frac{12}{\sqrt{29}}$ *units*

38. $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$

39. $6x+3y-2z=18$ and $2x-3y-6z=6$

40. $5x+2y+5z-9=0$

41. $\cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$

42. $0°$

43. $\left(\frac{17}{3},0,\frac{23}{3}\right)$, $\sin^{-1}\left(\frac{3}{\sqrt{38}}\right)$

44. $\langle 1,1,\pm\sqrt{2}\rangle$, $x+y+\sqrt{2}z-1=0$ and $x+y-\sqrt{2}z-1=0$

45. $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(\ -3\hat{i}+5\hat{j}+4\hat{k}\ )$

46. $\vec{r}.(\ 5\hat{i}+7\hat{j}+\hat{k}\ )=1$, $(0,-1,8)$

47. $\vec{r}.(\ 33\hat{i}+45\hat{j}+50\hat{k}\ )=41$

48. $7x+9y-10z-27=0$

49. $\frac{13}{7}$

50. $2x+y+2z-3=0$ & $x-2y+2z+3=0$ $\vec{r}.(2\hat{i}+\hat{j}+2\hat{k})-3=0$
$\vec{r}.(\hat{i}-2\hat{j}+2\hat{k})+3=0$. 51. 1 unit 52. $4x-6y+12z+7=0$

Assignment12

1. $\frac{9}{\sqrt{50}}$ 2. $\vec{r}.\hat{j}=3$ 3. $3x+2y-z-3=0$

4. $-\frac{10}{7}$ 5. $\vec{r}=2\hat{i}-3\hat{j}+4\hat{k}+\mu\left(2\hat{i}+5\hat{k}\right)$

6. $\frac{9\hat{i}+11\hat{j}+15\hat{k}}{3}$ 7. $\left(-\frac{1}{3},\frac{1}{3},1\right),\vec{r}=\frac{-1}{3}\hat{i}+\frac{1}{3}\hat{j}+\hat{k}+\lambda\left(\frac{1}{3}\hat{i}+\frac{1}{6}\hat{j}-\hat{k}\right)$

8. $acute\ angle=\sin^{-1}\frac{1}{3\sqrt{2}}$ 9. $\frac{7}{3},\frac{7}{2},7$

10. $z=5$ 11. $(3,5,9),-18x+22y-5z=11$ 12. $\vec{r}.\left(7\hat{i}+9\hat{j}-10\hat{k}\right)=27$

13. $\vec{r}.\left(4\hat{i}-\hat{j}-2\hat{k}\right)=6$ 14. $(a)\sqrt{62}units\ (b)\frac{84}{29\sqrt{2}}units$

15. $\vec{r}.\left(18\hat{i}+17\hat{j}+4\hat{k}\right)=49$ 17. $\vec{r}.\left(\hat{i}+3\hat{j}-4\hat{k}\right)=-6,\vec{r}.\left(-2\hat{i}+4\hat{j}+4\hat{k}\right)=-6$

18. $5y-5z-6=0,\frac{6}{5\sqrt{2}}units$ 21. $6units$ 22. $\frac{10}{3\sqrt{3}}units$

23. $\left(\frac{17}{3},0,\frac{23}{3}\right)$ 24. $x-2y+2z+2=0,x-2y+2z-4=0$ 25. $\frac{x-1}{3}=\frac{y-1}{10}=\frac{z-1}{17}$

26. $\vec{r}.\left(5\hat{i}+7\hat{j}+\hat{k}\right)=1,-\hat{j}+8\hat{k}$

Assignment13

1. $\frac{3}{8}$ 2. 0.9 3. $\frac{1}{2}$ 4. $\frac{1}{5}$ 5. 0.12 6. $\frac{15}{17}$ 7. $\frac{1}{3}$ 8. 47.5% 9. $\frac{62}{99}$ 10. $\frac{64}{199}$ 11. $\frac{11}{21}$

12. $\frac{2}{9}$ 13. $(i)\frac{12}{17}(ii)\frac{5}{17}$

14. Mean = 2/5 , Variance= 144/475 , SD= $\frac{12}{5\sqrt{19}}$

X	0	1	2
P(X)	12/19	32/95	3/95

15. Mean = 4/3 , SD = $\frac{\sqrt{5}}{3}$

X	0	1	2	3
P(X)	5/42	20/42	15/42	2/42

16. (i) 19/144 (ii) $1275/6^4$ (iii) 7/432 (iv) $1125/6^4$

17. The minimum number of trials required is 4.

Answers to Practice Tests

Practice Test 1

2. Not injective 3. {(1,5), (2,11), (8,0)}4. $f^{-1}(x)=\frac{x+4}{3}$ 5. x6. $\frac{\sqrt{y+6}-1}{3}$

8. $\frac{2x-1}{3}$ 9. $\frac{7y+4}{5y-3}$10.{1,5,9}

Practice Test 2

1. $\frac{5}{12}$ 2. $\frac{7\pi}{6}$ 3. $\frac{7\pi}{12}$ 4. $\frac{\sqrt{3}}{2}$

5. $\frac{7\pi}{12}$ 6. $\frac{\pi}{6}$ 7. $\frac{1}{4}$ 11. $\frac{1}{6}$

Practice Test 3

1. x = 2 2. B= $\begin{bmatrix}-5 & -4\\-5 & -6\end{bmatrix}$ 3. λ = 8

2. 4. -256 7. $\begin{bmatrix}2/3 & -1/6\\-1 & 1/2\end{bmatrix}$ 10. 300, 100, 200

Practice Test 4

1. $\lambda=-4$ 2. 1 4. $a=\frac{1}{2}$

5. $x^{\cos x}\left[\cos x - x\log\sin x+\cos x\log x\right]-\frac{4x}{\left(x^2-1\right)^2}$

6. $\frac{y}{x}\left(\frac{x\log y-x}{y\log x-x}\right)$ 7. $\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$

8. $\frac{-1}{\sqrt{1-x^2}}$ 9. $\frac{8b\pi}{3a^2}$ 10. $1-\frac{\sqrt{21}}{6}$ 12. 2

Practice Test 5

1. 0.0608 2. $.03x^3 cm^3$ 3. Increasing in $(-\infty,1)\cup\left(1,\frac{8}{5}\right)\cup(2,\infty)$, decreasing in $\left(\frac{8}{5},2\right)$

4. $\frac{y-b\cos^3 t}{x-a\sin^3 t}=\frac{-b}{a}\cot t$ and $\frac{y-b\cos^3 t}{x-a\sin^3 t}=\frac{a}{b}\tan t$

5. $y-\left(2+2\sqrt{3}\right)=2\sqrt{3}(x-2)$ and $y-\left(2-2\sqrt{3}\right)=-2\sqrt{3}(x-2)$

6. Increasing in $(-\infty,1)\cup(1,3)$ and decreasing in $(3,\infty)$

7. local max.at -5 and 0 , local min.at -3

8. local max. at $\frac{\pi}{2}$ and local min. at $\frac{\pi}{6}$

9. $\frac{-32}{27\pi}$ 10. $\frac{1}{2\pi} cm/s$

Practice Test 6

$Q1.\ x+2\log\left|\frac{x-1}{x-2}\right|+c$

$Q2.\ \frac{1}{8}\log|1-\sin x|-\frac{1}{8}\log|1+\sin x|-\frac{1}{4\sqrt{2}}\log\left|1-\sqrt{2}\sin x\right|+\frac{1}{4\sqrt{2}}\log\left|1+\sqrt{2}\sin x\right|+c$

$Q3.\ \frac{1}{4}e^{2x}+\frac{1}{8}e^{2x}(\sin 2x+\cos 2x)+c$ $Q4.\ \frac{-2}{5}\tan^{-1}\frac{x}{2}+\frac{3}{5}\tan^{-1}\frac{x}{3}+c$

$Q6.\ \frac{-1}{3}\left(1+x-x^2\right)^{3/2}+\frac{1}{2}\left[\frac{1}{2}\left(x-\frac{1}{2}\right)\sqrt{1+x-x^2}+\frac{5}{8}\sin^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)\right]+c$

$Q7.\ \frac{1}{5}\log\left|\frac{\tan x/2+2}{\tan x/2-3}\right|+c$ $Q8.\ \pi a$ $Q9.\ \frac{-1}{\sqrt{2}}\log\left(\sqrt{2}-1\right)$ $Q10.\ 3$

$Q11.\ \frac{1}{18}\log|1+\sin x|-\frac{1}{2}\log|1-\sin x|+\frac{4}{9}\log|5-4\sin x|+c$

Practice Test 7

$Q1. \left(x^2 - y^2\right)\frac{dy}{dx} - 2xy = 0$ $Q2. \left(x-y\right)^2 \left\{1 + \left(\frac{dy}{dx}\right)^2\right\} = \left(x + y\frac{dy}{dx}\right)^2$

$Q3.\ y = (x+1)\log|x+1| - x + 5$ $Q4.\ cx = ye^{1/x}$

$Q5.\ y = c + a\tan^{-1}\frac{(x+y)}{a}$ $Q6.\ x^4 + 6x^2y^2 + y^4 = 8$ $Q7.\ xe^{\tan^{-1}y} = e^{\tan^{-1}y}\left(\tan^{-1}y - 1\right) + c$

$Q8.\ x = y^2\left(e^{-1} - e^{-y}\right)$ $Q9. \left(\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}\right) sq. units$ $Q10. \frac{11}{6}$ $Q11. \frac{4\sqrt{3}}{3} + \frac{16\pi}{3}\ sq\ units$

Practice Test 8

$Q1. 2\hat{i} - 4\hat{j} + 4\hat{k}$ $Q3.\ 5x + 9y + 4z - 37 = 0$ $Q4. (-4, -1, -3)$ $Q5. 15\hat{i} - 27\hat{j} + 5\hat{k}$

$Q6. \vec{d} = \frac{1}{4}\hat{i} + \frac{1}{4}\hat{j} + \frac{3}{4}\hat{k}$ $Q8. (1, -2, 7)$ $Q9. \vec{r}.\left(33\hat{i} + 45\hat{j} + 50\hat{k}\right) - 41 = 0$ $Q10.$ distance $= 13$

$Q11.$ Foot of perpendicular is $(1,3,0)$, perpendicular distance $= \sqrt{6}$, image is $(-1,4,1)$

Practice Test 9

$Q1. \frac{1}{3}$ $Q2. (i)\frac{11}{12}$ $(ii)\frac{5}{12}$ $(iii)\frac{7}{8}$ $Q3. \frac{307}{686}$ $Q4. \frac{5}{11}$ $Q5. \frac{18}{143}$ $Q6. \frac{3}{13}$ $Q7. \frac{13}{50}$ $Q8.\ no.$ of trials ≥ 4

$Q9.$

X	0	1	2	3
P(X)	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

$Q10.$

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$Mean = \frac{6}{5}$, Variance $= \frac{14}{25}$ $Q11. \frac{2}{5}$