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CS 4375 Assignment 3

Part 1

1) $X \to y$

Error:

$$\epsilon_i(x) = f(x) - h_i(x)$$

$$E(\epsilon_i(x)^2) = E[(f(x) - h_i(x))^2]$$

$$E_{avg} = \frac{1}{M}\sum_{i=1}^{M} E(\epsilon_i(x)^2)$$

Aggregate model:

$$h_{agg}(x) = \frac{1}{M}\sum_{i=1}^{M} h_i(x)$$

$$E_{agg}(x) = E\left[\left\{\frac{1}{M}\sum_{i=1}^{M} h_i(x) - f(x)\right\}^2\right]$$

To prove: $E_{agg} = \frac{1}{M} E_{avg}$

Assumption: 0 mean errors $\Rightarrow E(\epsilon_i(x)) = 0$ for all i

$E(\epsilon_i(x)\epsilon_j(x)) = 0$ for all $i \neq j$

Ans: $E_{avg} = \frac{1}{M}\sum_{i=1}^{M} E(\epsilon_i(x)^2) = \frac{1}{M}\sum_{i=1}^{M} E\left((f(x) - h_i(x))^2\right)$

$$= \frac{1}{M}\sum_{i=1}^{M} E\left(f(x)^2 + h_i(x)^2 - 2f(x)\,h_i(x)\right)$$

$$= \frac{1}{M}\sum_{i=1}^{M}\left(E(f(x)^2) + E(h_i(x)^2) - 2E(f(x)\,h_i(x)\right.$$

$$= E(f(x)^2) + \frac{1}{M}\sum_{i=1}^{M} E(h_i(x)^2) - \frac{2}{M}\sum_{i=1}^{M} E(\quad h_i$$

Since, $E(\epsilon_i(x)) = 0$, $E(f(x)) = E(h_i(x))$

$$\therefore E_{avg} = E(f(x)^2) + \frac{1}{M}\sum_{i=1}^{M}\left(E(h_i(x)^2) - 2E(h_i(\right.$$

Ans: $E_{agg} = E\left(\left(\frac{1}{M}\sum_{i=1}^{M} \epsilon_i(x)\right)^2\right)$

$$= E\left(\frac{1}{M^2}\left(\sum_{i=1}^{M}\epsilon_i(x)\right)\left(\sum_{j=1}^{M}\epsilon_j(x)\right)\right)$$

$$= E\left(\frac{1}{M^2}\sum \epsilon_i(x)^2\right)$$ ($\because$ since $E(\epsilon_i(x)\,\epsilon_j($ $=($

$$= \frac{1}{M}\left(\frac{1}{M}\sum_{i=1}^{M} E(\epsilon_i(x)^2)\right)$$

$$= \frac{1}{M} E_{avg}$$