2.) $E_{agg}(x) = E\left[\left\{ \frac{1}{M} \sum_{i=1}^{M} \epsilon_i(x) \right\}^2 \right]$ $\quad E_{avg} = \frac{1}{M} \sum_{i=1}^{M} E(\epsilon_i(x)^2)$

$$E_{agg}(x) = E\left(\frac{1}{M^2} \left(\epsilon_1(x) + \epsilon_2(x) + \ldots + \epsilon_M(x) \right)^2 \right)$$

$$= \frac{1}{M} E\left(\frac{1}{M} \left(\epsilon_1(x) + \epsilon_2(x) + \ldots \epsilon_M(x) \right)^2 \right)$$

$$\leq \frac{1}{M} E\left(\epsilon_1^2(x) + \epsilon_2^2(x) + \ldots \epsilon_M^2(x) \right)$$

$$= \frac{1}{M} \left(E\left(\sum_{i=1}^{M} \epsilon_i(x)^2 \right) \right) \leq \frac{1}{M} \sum_{i=1}^{M} E(\epsilon_i(x)^2)$$

$$\leq \quad E_{agg} \quad \leq \quad E_{avg}$$

Proof $\quad 2ab \leq a^2 + b^2$

$$a^2 - 2ab + b^2 \Rightarrow (a-b)^2 \geq 0$$ squares can't be negative

$$\frac{1}{M} \left(\sum_{i=1}^{M} \epsilon_i(x) \right)^2 = \frac{1}{M} \left(\sum_{i=1}^{M} \epsilon_i(x) \right) \left(\sum_{i=1}^{M} \epsilon_i(x) \right) = \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{M} \epsilon_i(x) \epsilon_j(x)$$

$$\frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{M} 2\epsilon_i(x)\epsilon_j(x) \leq \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{M} \left(\epsilon_i(x) + \epsilon_j(x) \right)$$

$$\boxed{M \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_M \end{pmatrix} + M \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_M \end{pmatrix} = 2M(\epsilon_1 + \epsilon_2 + \ldots \epsilon_M)}$$

$$\frac{2}{M} \sum_{i=1}^{M} \sum_{j=1}^{M} \epsilon_i(x) \epsilon_j(x) \leq \frac{2\cancel{M}}{\cancel{M}} \sum_{i=1}^{M} \epsilon_i(x)$$

$$\frac{1}{M} \left(\sum_{i=1}^{M} \epsilon_i(x) \right)^2 \leq \sum_{i=1}^{M} \epsilon_i(x)$$

OR

use cauchy-schwarz $\quad |\langle u, v \rangle|^2 \leq \langle u, u \rangle \cdot \langle v, v \rangle$

$$\left| \left\langle \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_M \end{bmatrix}, \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \right\rangle \right|^2 \leq (\overbrace{1 + 1 + \ldots 1}^{M}) (\epsilon_1^2 + \epsilon_2^2 + \ldots \epsilon_M^2)$$

$u = \langle \epsilon_1, \epsilon_2, \ldots \epsilon_M \rangle$
$V = \langle 1, 1, \ldots, 1 \rangle$

$$(\epsilon_1 + \epsilon_2 + \ldots + \epsilon_M)^2 \leq M (\epsilon_1^2 + \epsilon_2^2 + \ldots \epsilon_M^2)$$

$$\frac{1}{M} \left(\sum_{i=1}^{M} \epsilon_i(x) \right)^2 \leq \sum_{i=1}^{M} \epsilon_i(x)$$