POWER SPECTRUM ESTIMATION

Welch’s Non-Parametric Method

AIM:

To study and estimate the power spectrum of a random sequence x (n) using Welch’s Non-Parametric Method in MATLAB.

THEORY:

An improved estimator of the PSD is the one proposed by Welch. The method consists of dividing the time series data into (possibly overlapping) segments, computing a modified periodogram of each segment, and then averaging the PSD estimates. The result is Welch's PSD estimate.

Welch's method is implemented in the Signal Processing Toolbox by the pwelch function. By default, the data is divided into eight segments with 50% overlap between them. A Hamming window is used to compute the modified periodogram of each segment.

The averaging of modified periodograms tends to decrease the variance of the estimate relative to a single periodogram estimate of the entire data record. Although overlap between segments tends to introduce redundant information, this effect is diminished by the use of a non-rectangular window, which reduces the importance or weight given to the end samples of segments (the samples that overlap).

However, as mentioned above, the combined use of short data records and non-rectangular windows results in reduced resolution of the estimator. In summary, there is a trade-off between variance reduction and resolution. One can manipulate the parameters in Welch's method to obtain improved estimates relative to the periodogram, especially when the SNR is low.

Yule-Walker AR Parametric Method

AIM:

To study and estimate the power spectrum of a random sequence x (n) using Yule-Walker AR Parametric Method in MATLAB.

THEORY:

Parametric methods can yield higher resolutions than non-parametric methods in cases when the signal length is short. These methods use a different approach to spectral estimation; instead of trying to estimate the PSD directly from the data, they model the data as the output of a linear system driven by white noise, and then attempt to estimate the parameters of that linear system.

The most commonly used linear system model is the all-pole model, a filter with all of its zeroes at the origin in the z-plane. The output of such a filter for white noise input is an autoregressive (AR) process. For this reason, these methods are sometimes referred to as AR methods of spectral estimation.

The Yule-Walker AR method of spectral estimation computes the AR parameters by forming a biased estimate of the signal's auto correlation function, and solving the least squares minimization of the forward prediction error. This results in the Yule-Walker equations.

$$\begin{bmatrix} r(1) & r(2)^* & \cdots & r(p)^* \\ r(2) & r(1) & \cdots & r(p-1)^* \\ \vdots & \ddots & \ddots & \vdots \\ r(p) & \cdots & r(2) & r(1) \end{bmatrix} \begin{bmatrix} a(2) \\ a(3) \\ \vdots \\ a(p+1) \end{bmatrix} = \begin{bmatrix} -r(2) \\ -r(3) \\ \vdots \\ -r(p+1) \end{bmatrix}$$

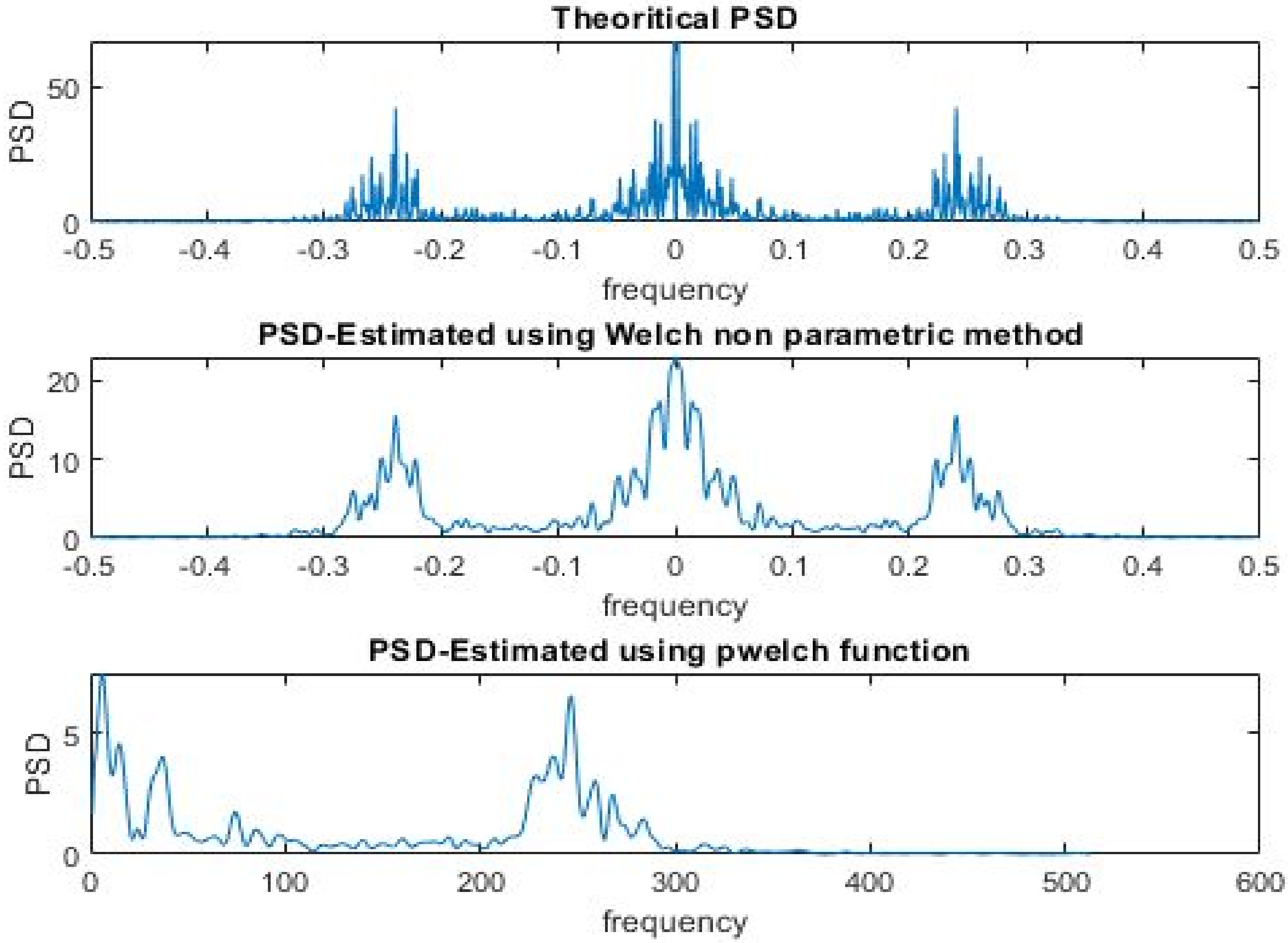
The use of a biased estimate of the auto correlation function ensures that the auto correlation matrix above is positive definite. Hence, the matrix is invertible and a solution is guarantee to exist. Moreover, the AR parameters thus computed always result in a stable all-pole model.

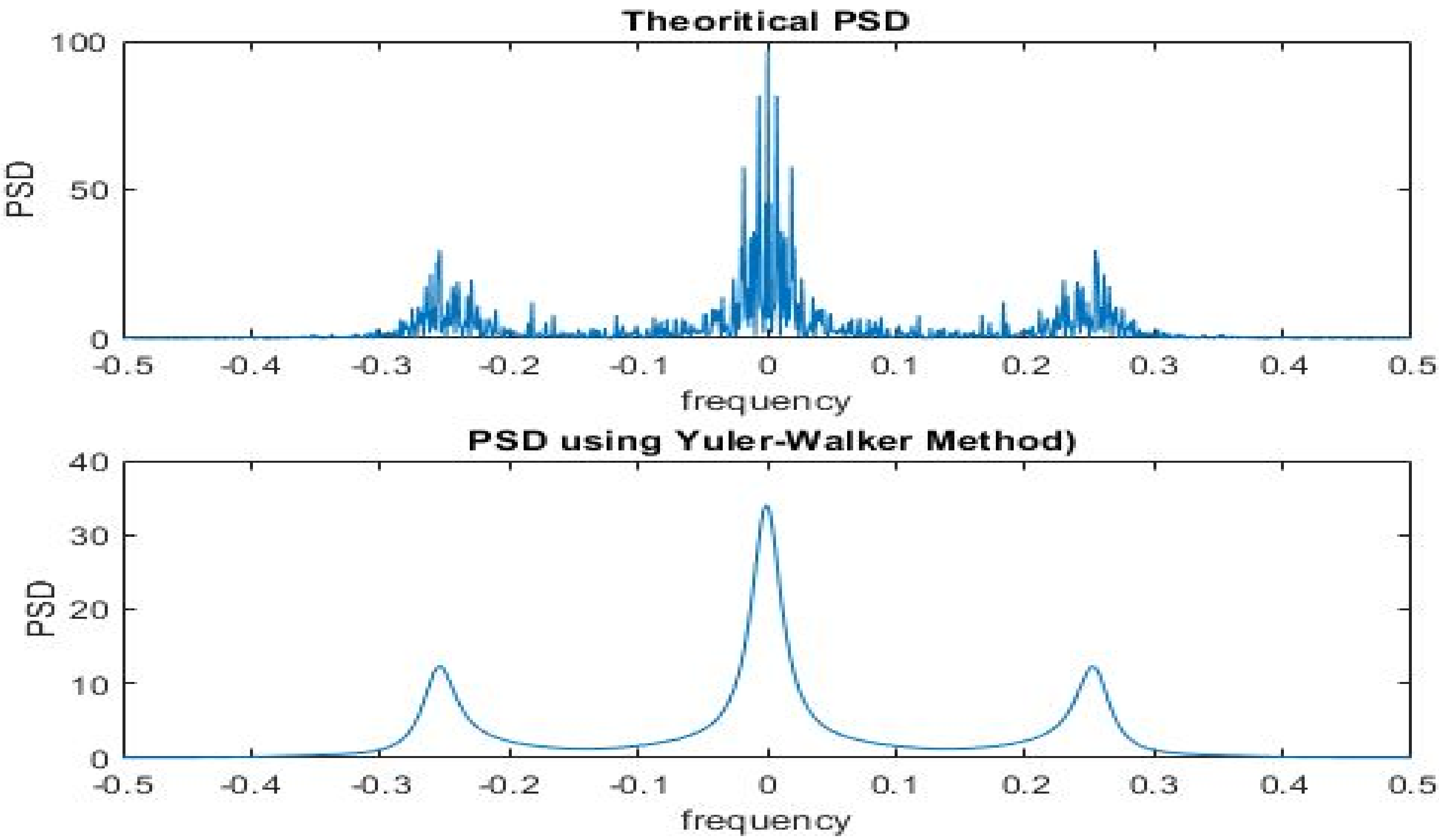
The toolbox function *pyulear* implements the Yule-Walker AR method.

SOURCE CODE AND RESULTS:

```
sigma = 1;
noise = sigma.*normrnd(0,1,1,N);
x = filter(1, [1, -0.9, 0.81, -0.729], noise).';
f = fftshift(fft(x, 1024));
f_hat=linspace(-1,1,1024)*1/2;

subplot(2,1,1);
plot(f_hat,abs(f).^2/N);
xlabel( "frequency");
ylabel("PSD");
title(" Theoritical PSD ")

rxx = zeros(p+1,1);

for m = 1:(p+1)
    for n = 1: N-m
        rxx(m,1)=rxx(m,1)+x(n)*x(n+m-1);
    end
    rxx(m,1)= (rxx(m,1)./N);
end

R = zeros(p+1,p+1);

for i=1:p+1
    for j=1:p+1
        R(i,j)= (rxx(abs(i-j)+1,1));
    end
end


AR = R(1:p,1:p);
C=R(2:p+1,1);

A=-inv(AR)*C;

var=AR(1,1)+ sum(A(:,1).*C(:,1));

Y = zeros(1024,1);

for f = 1:1024
    for k = 1:p
        e = exp(-1j*2*pi*f/1024*k);
        Y(f,1) = Y(f,1) + A(k,1)*e;
    end
    Y(f,1) = Y(f,1) +1;
end

P = var./(Y.*conj(Y));

subplot(2,1,2);
plot(f_hat, fftshift(P));
xlabel( "frequency");
ylabel("PSD");
title(" PSD using Yuler-Walker Method)");

print(gcf,'05b.png','-dpng','-r300');
```

```
b = 1;
a = [1, -0.9, 0.81, -0.729];
x = filter(b, a,noise);
f = fftshift(fft(x, 1024));
f_hat=linspace(-1,1,1024)*1/2;
subplot(3,1,1);
plot(f_hat,(abs(f).^2/N)*(sigma^2));
xlabel( "frequency");
ylabel("PSD");
title(" Theoritical PSD ")


L = 7;                            %number of blocks
D = 125;                          %overlap between blocks
M = (N + (L-1)*D)/L;              %size of block
y = [];

for block = 1:L
    y(:,block) = x((block-1)*D+1:(block -1)*D + M );
end

w = hamming(M);
sq_w = w.*w;
U = sum(sq_w)/M;

win_sig = [];
fft_sig = [];

for block = 1:L
   win_sig(:,block) = y(:,block).*w;
   fft_sig(:,block) = fft(win_sig(:,block),1024);
   power(:,block) = (abs(fft_sig (:,block)).^2)./(M*U);
end

p_welch = power(:,1);
for block = 2:L
    p_welch = p_welch + power(:, block);
end
p_welch = p_welch / L;


subplot(3,1,2);
plot(f_hat,fftshift(p_welch));
xlabel( "frequency");
ylabel("PSD");
title("PSD-Estimated using Welch non parametric method ");

subplot(3,1,3);
pxx = pwelch(x,w,0,1024);
plot(pxx);
xlabel( "frequency");
ylabel("PSD");
title("PSD-Estimated using pwelch function ");

print(gcf,'05a.png','-dpng','-r300');
```


Theoritical PSD
PSD
frequency
0
50
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
PSD-Estimated using Welch non parametric method
PSD
frequency
0
10
20
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
PSD-Estimated using pwelch function
PSD
frequency
0
5
0
100
200
300
400
500
600


Theoritical PSD
PSD
frequency
0
50
100
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
PSD using Yuler-Walker Method)
PSD
frequency
0
10
20
30
40
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

DISCUSSION:

- The power density estimation is done by two methods i.e. parametric and non-parametric. For parametric estimation we used Yule-Walker AR method and for non-parametric we used Welch’s Non-Parametric Method.
- In Yule Walker method we predicted the coefficients and then found the PSD.
- The Welch’s method consists of dividing the time series data into (possibly overlapping) segments, computing a modified periodogram of each segment, and then averaging the PSD estimates.
- For parametric estimation of power spectral density we have to model the source with help of coefficients obtained using auto correlation values.
- In Yule-Walker method, the processing of the random signal has its own limitations with increasing size of the signal.
- In Welch method, choosing the length of interval and percentage of overlap is trade off between frequency resolution and variance.