

ASSIGNMENT

MODEL QUESTION PAPER-II

MODULE-2

1 a) A random variable x has the following probability function for various values of x

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i) Find the value of k .

ii) Evaluate $P[x < 6]$, $P[0 < x < 5]$ and $P[x \geq 6]$

\Rightarrow W.K.T $P(x_i) = 0$ and $\sum P(x_i) = 1$

i) To find k

$$\sum P(x_i) = 1$$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$10k^2 + 9k - 1 = 0$$

$$(10-1)(k+1) = 0$$

$$\boxed{k = \frac{1}{10}}$$

$k = -1 \rightarrow$ ignore as it is negative

considering $k = \frac{1}{10} = 0.1$ The probability distribution becomes

x	0	1	2	3	4	5	6	7
$P(x)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

ii) probability x less than 6.

$$P[x < 6] = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$$

$$0 + 0.1 + 0.2 + 0.2 + 0.3 + 0.01$$

$$\boxed{P[x < 6] = 0.81}$$

probability of x between 0 and 5.

$$P[0 < x < 5] = P(x=1) + P(x=2) + P(x=3) + P(x=4)$$

$$= 0.1 + 0.2 + 0.2 + 0.3$$

$$\boxed{P[0 < x < 5] = 0.8}$$

probability of x greater than equal to 6

$$P[x \geq 6] = 1 - P[x < 6]$$

$$1 - 0.81$$

$$P[x \geq 6] = 0.19$$

1b Find the mean and variance of poisson's distribution

$$\Rightarrow \text{Mean} = \mu = \sum_{x=0}^{\infty} x \cdot p(x)$$

Poisson's distribution is given by $p(x) = \frac{m^x e^{-m}}{x!}$

$$\therefore \mu = \sum_{x=0}^{\infty} \frac{x \cdot m^x e^{-m}}{x!}$$

$$\mu = \sum_{x=0}^{\infty} \frac{x \cdot m^x e^{-m}}{x(x-1)!}$$

$$\mu = \sum_{x=0}^{\infty} \frac{m^x e^{-m}}{(x-1)!}$$

$$\mu = \sum_{x=1}^{\infty} \frac{m \cdot m^{x-1} e^{-m}}{(x-1)!}$$

$$\mu = \sum_{x=1}^{\infty} \frac{m^{x-1} \cdot m e^{-m}}{(x-1)!}$$

$$\mu = m e^{-m} \sum_{x=1}^{\infty} \frac{m^{x-1}}{(x-1)!}$$

$$\mu = m e^{-m} \left[\frac{m^0}{0!} + \frac{m^1}{1!} + \frac{m^2}{2!} + \dots \right]$$

$$\mu = m e^{-m} \left[1 + m + \frac{m^2}{2!} + \dots \right]$$

$$\mu = m e^{-m} \cdot e^m$$

$$\boxed{\mu = m} \rightarrow \text{mean of poisson's distribution}$$

W.K.T. Variance $V = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$

$$V = \sum_{x=0}^{\infty} (x^2 + x - x) p(x) - \mu^2$$

$$V = \sum_{x=0}^{\infty} (x^2 - x) p(x) + \sum_{x=0}^{\infty} x p(x) - \mu^2$$

$$V = \sum_{x=0}^{\infty} x(x-1) p(x) + \mu - \mu^2$$

$$V = \sum_{x=0}^{\infty} x(x-1) \frac{m^x e^{-m}}{x!} + \mu - \mu^2$$

$$V = \sum_{x=0}^{\infty} x(x-1) \frac{m^x e^{-m}}{x(x-1)(x-2)!} + \mu - \mu^2$$

$$V = \sum_{x=0}^{\infty} \frac{m^x e^{-m}}{(x-2)!} + \mu - \mu^2$$

$$V = \sum_{x=2}^{\infty} \frac{m^{x-2} m^2 e^{-m}}{(x-2)!} + \mu - \mu^2$$

$$V = m^2 e^{-m} \cdot \sum_{x=2}^{\infty} \frac{m^{x-2}}{(x-2)!} + \mu - \mu^2$$

$$V = m^2 e^{-m} \left[\frac{m^0}{0!} + \frac{m^1}{1!} + \frac{m^2}{2!} + \dots \right] + \mu - \mu^2$$

$$V = m^2 e^{-m} e^m + \mu - m^2 \quad (\because \mu = m)$$

$$V = m^2 + m - m^2$$

$\boxed{V = m} \rightarrow$ variance of poisson's distribution.

Standard deviation $= \sqrt{V} = \sqrt{m}$

1c. In a certain town, the duration of a shower is exponentially distributed with the mean 5 minutes. What is the probability that a shower will last for?

- 10 minutes or more.
- Less than 10 minutes.
- Between 10 and 12 minutes.

Exponential distribution is given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{1}{\alpha}$$

Given mean = $\mu = 5$ minutes.

$$\therefore \boxed{\alpha = \frac{1}{5}} \Rightarrow f(x) = \left\{ \frac{1}{5} e^{-\frac{x}{5}} \right\}$$

- i) probability that shower will last 10 minutes or more

$$P(x \geq 10) = \int_{10}^{\infty} f(x) dx = \int_{10}^{\infty} \frac{1}{5} e^{-\frac{x}{5}} dx$$

$$= \frac{1}{5} \left[\frac{e^{-\frac{x}{5}}}{-\frac{1}{5}} \right]_{10}^{\infty}$$

$$= -\frac{5}{5} \left[e^{-\frac{x}{5}} \right]_{10}^{\infty}$$

$$= -(e^{-\frac{\infty}{5}} - e^{-\frac{10}{5}})$$

$$\boxed{P(x \geq 10) = 0.1353}$$

- ii) probability that shower will last for less than 10 min

$$P(x < 10) = \int_{-\infty}^{10} f(x) dx = \int_{-\infty}^{10} \frac{1}{5} e^{-\frac{x}{5}} dx$$

$$= -(e^{-\frac{10}{5}} - e^0) = \boxed{0.8647}$$

$$\text{or } P(x < 1) = 1 - P(x \geq 10) = 1 - 0.1353 = \boxed{0.8647}$$

probability that shower will last between 10 and 12 minutes

$$P(10 < x < 12) = \int_{-\infty}^{\infty} f(x) dx = \int_{10}^{12} \frac{1}{5} e^{-1/5 x} dx$$

$$= -[e^{-x/5}]_{10}^{12}$$

$$= -(e^{-12/5} - e^{-2})$$

$$P(10 < x < 12) = 0.0446$$

determine the value of k , so that the function $f(x) = k(x^2 + 4)$ for $x = 0, 1, 2, 3$ can serve as a probability distribution of the discrete random variable x :

Also, find (i) $P(0 < x \leq 2)$ and (ii) $P(x \geq 1)$

Given $f(x) = k(x^2 + 4)$

$$\Rightarrow x=0, f(x) = 4k \quad x=2, f(x) = 8k$$

$$x=1, f(x) = 5k \quad x=3, f(x) = 13k$$

\therefore probability distribution table.

x	0	1	2	3
$f(x)$	$4k$	$5k$	$8k$	$13k$

To find k , w.k.T $\sum P(x_i) = 1$

$$\therefore 4k + 5k + 8k + 13k = 1$$

$$k = \frac{1}{30}$$

$$k = 0.33$$

\therefore probability distribution table.

x	0	1	2	3
$f(x)$	$\frac{4}{30}$	$\frac{5}{30}$	$\frac{8}{30}$	$\frac{13}{30}$

$$\begin{aligned} \text{i) } P[0 < x \leq 2] &= P(x=1) + P(x=2) \\ &= 5/30 + 8/30 \\ &= 13/30 \end{aligned}$$

$$P[0 < x \leq 2] = 0.433$$

$$\begin{aligned} \text{ii) } P[x \geq 1] &= 1 - P(x < 1) \\ &= 1 - P(x=0) \\ &= 1 - \frac{4}{30} \\ &= \frac{26}{30} \end{aligned}$$

$$P[x \geq 1] = 0.8667$$

2 b. out of 800 families with 5 children each how many would you expect to have (i) 3 boys (ii) Atleast one boy (iii) Atmost two boys, assuming equal probabilities for boys and girls.

⇒ Given the probabilities for boys and girls are equal,

Then, let p = probability of boys = $\boxed{\frac{1}{2} = p}$

$$p + q = 1$$

$$q = 1 - p$$

$$\therefore \boxed{q = \frac{1}{2}}$$

Here $\boxed{n=5}$

w.k.T $P(x) = {}^nC_x p^x q^{n-x}$ (Binomial Distribution)

$$P(x) = {}^5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x}$$

$$= \left(\frac{1}{2}\right)^{5-x+x} {}^5C_x$$

$$= \left(\frac{1}{2}\right)^5 {}^5C_x$$

$$\boxed{P(x) = \frac{1}{32} {}^5C_x}$$

$$\begin{aligned} \text{Let } F(x) &= 800 \times P(x) \\ &= 800 \times \frac{1}{32} {}^5C_x \end{aligned}$$

$$\boxed{F(x) = 25 {}^5C_x}$$

Let x be the variable to denote number of boys.

i) Families with 3 boys.

$$F(x=3) = {}^{25}C_3 = 25 \times 10 = \boxed{250}$$

= 250 families will have 3 boys.

ii) Family with atleast one boy.

$$P(x \geq 1) = 800 \times (1 - P(x < 1)) = 800 (1 - P(x=0)) = 800 (1 - \frac{1}{32})$$
$$= 800 \times \frac{31}{32}$$

\therefore 775 families will have atleast one boy. $\boxed{775}$

iii) Families with atmost two boys.

$$\text{i.e., } F(x \leq 2) = F(x=0) + F(x=1) + F(x=2)$$
$$= 25 \{ {}^5C_0 + {}^5C_1 + {}^5C_2 \}$$
$$= 25 \{ 1 + 5 + 10 \}$$

$$\boxed{F(x \leq 2) = 400}$$

\therefore 400 Families will have atmost two boys.

2. c) In a Normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution.

\Rightarrow Given:- Probability of items under 45 = $P(x < 45) = 31\% = 0.31$
Probability of items under 64 = $P(x < 64) = 8\% = 0.08$

WKT, Standard Normal Variate (SNV) = $z = \frac{x - \mu}{\sigma}$

case 1, $P(x < 45)$, Let $x = 45$

$$\therefore z = \frac{45 - \mu}{\sigma} = z_1 \text{ (say)}$$

case 2, $P(x > 64)$, Let $x = 64$ $\therefore z = \frac{64 - \mu}{\sigma} = z_2 \text{ (say)}$

$$\therefore P(X < 45) = 0.31$$

$$P(Z < Z_1) = 0.31$$

$$Z(-\infty \text{ to } 0) + Z(0 \text{ to } Z_1) = 0.31$$

$$0.5 + \phi(Z_1) = 0.31$$

$$\phi(Z_1) = -0.19$$

$$\Rightarrow \phi(0.5) = 0.1915 \quad \Rightarrow \boxed{Z_1 = -0.5}$$

$$P(X > 64) = 0.08$$

$$P(Z > Z_2) = 0.08$$

$$= (0 \text{ to } \infty) - Z(0 \text{ to } Z_2) = 0.08$$

$$0.5 - \phi(Z_2) = 0.08$$

$$\phi(Z_2) = 0.42$$

$$\phi(1.4) = 0.4192$$

$$\boxed{Z_2 = 1.4}$$

We have.

$$Z_1 = \frac{45 - \mu}{\sigma} \quad Z_2 = \frac{60 - \mu}{\sigma}$$

$$Z_1 = -0.5 \quad Z_2 = 1.4$$

$$-0.5 = \frac{45 - \mu}{\sigma}$$

$$-0.5\sigma = 45 - \mu$$

$$\mu = 0.5\sigma + 45 \rightarrow \text{①}$$

on solving we get

$$\boxed{\mu = 50} \quad \boxed{\sigma = 10}$$

mean of distribution = 50

standard deviation of distribution = 10

MODULE-2

If the joint probability distribution of x and y are given by $f(x, y) = \frac{x+y}{30}$, for $x=0, 1, 2, 3$; $y=0, 1, 2$ find

- (i) $P[x \leq 2, y=1]$, (ii) $P[x > 2, y \leq 1]$ (iii) $P[x > y]$

$\Rightarrow f(x, y) = \frac{x+y}{30}$ given $x=0, 1, 2, 3$ $y=0, 1, 2$

$f(0,0) = 0$ $f(0,1) = \frac{1}{30}$ $f(0,2) = \frac{2}{15}$

$f(1,0) = \frac{1}{30}$ $f(1,1) = \frac{2}{15}$ $f(1,2) = \frac{3}{10}$

$f(2,0) = \frac{2}{15}$ $f(2,1) = \frac{3}{10}$ $f(2,2) = \frac{4}{15}$

$f(3,0) = \frac{3}{10}$ $f(3,1) = \frac{4}{15}$ $f(3,2) = \frac{5}{15}$

\therefore Joint probability distribution

$x \backslash y$	0	1	2	$f(x)$
x_1 0	0	$\frac{1}{30}$	$\frac{2}{15}$	$\frac{1}{10}$
x_2 1	$\frac{1}{30}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{1}{5}$
x_3 2	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{4}{15}$	$\frac{3}{10}$
x_4 3	$\frac{3}{10}$	$\frac{4}{15}$	$\frac{5}{15}$	$\frac{2}{5}$
$g(y)$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{7}{15}$	1

\therefore distribution in x

x	0	1	2	3
f	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

\therefore distribution in y

y	0	1	2
$g(y)$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{7}{15}$

i) $P[x \leq 2, y=1]$

$x = \{0, 1, 2\}$ $y = \{1\}$

$= (0, 1) (1, 1) (2, 1)$

$= (x_1, y_2) (x_2, y_2) (x_3, y_2)$

$= J_{12} + J_{22} + J_{32}$

$= \frac{1}{30} + \frac{1}{15} + \frac{1}{10}$

$P[x \leq 2, y=1] = \frac{1}{5}$

$$\text{ii) } P[X > 2, Y \leq 1]$$

$$X = \{3\} \quad Y = \{0, 1\}$$

$$= (3, 0) (3, 1)$$

$$= J_{41} + J_{42}$$

$$= \frac{1}{10} + \frac{2}{15}$$

$$= \frac{7}{30} = P[X > 2, Y \leq 1]$$

$$\text{iii) } P[X > Y]$$

$$= (1, 0) (2, 0) (2, 1) (3, 0) (3, 1) (3, 2)$$

$$= J_{21} + J_{31} + J_{32} + J_{41} + J_{42} + J_{43}$$

$$= \frac{1}{30} + \frac{1}{15} + \frac{1}{10} + \frac{1}{10} + \frac{2}{15} + \frac{1}{6}$$

$$P[X > Y] = \frac{3}{5}$$

b. Find the unique fixed probability vector for the regular Stochastic matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

\Rightarrow Let $v = (x, y, z)$ be the unique fixed probability vector.
i.e., $x + y + z = 1$ and $vA = v$

$$vA = v$$

$$(x, y, z) \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix} = (x, y, z)$$

$$\left[\frac{y}{6}, x + \frac{y}{2} + \frac{2z}{3}, \frac{y}{3} + \frac{z}{3} \right] = (x, y, z)$$

$$x = \frac{y}{6} \rightarrow (1) \quad y = x + \frac{y}{2} + \frac{2z}{3} - (2) \quad z = \frac{y}{3} + \frac{2z}{3} - (3)$$

W.K.T $x + y + z = 1$

from (1) from (3) $3z = y + z$

$$y = 6x \rightarrow (4)$$

$$y = 2z \rightarrow (5)$$

Subs (5) in (1)

$$x = \frac{2z}{6} \Rightarrow z = 3x \rightarrow (6)$$

$$\therefore x + y + z = 1$$

$$x + 6x + 3x = 1$$

$$10x = 1$$

$$x = \frac{1}{10}$$

Subs x in (4)

$$y = \frac{6}{10}$$

Subs x in (6)

$$z = \frac{3}{10}$$

$$\therefore V = (x, y, z) = \left(\frac{1}{10}, \frac{6}{10}, \frac{3}{10} \right)$$

c) A gambler's luck follows a pattern. If he wins a game the probability of winning the next game is 0.6. However, if he loses a game, the probability of losing the next game is 0.7. There is an even chance of the gambler winning the first game.

- What is the probability of he winning the second game?
- What is the probability of he winning the third game?
- In the long run, how often he will win?

⇒ State space = {win, lose} = {W, L}

Associated Transition probability matrix

$$P = P = \begin{matrix} & \begin{matrix} W & L \end{matrix} \\ \begin{matrix} W \\ L \end{matrix} & \begin{Bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{Bmatrix} \end{matrix} = \frac{1}{10} \begin{bmatrix} 6 & 4 \\ 3 & 7 \end{bmatrix}$$

Given:- probability of winning first game is even

i.e., $P = \frac{1}{2} \Rightarrow Q = \frac{1}{2}$

∴ Initial probability $P^{(0)} = \left[\frac{1}{2}, \frac{1}{2} \right]$

here $P^{(0)} \rightarrow$ first game $P^{(1)} \rightarrow$ second game $P^{(2)} \rightarrow$ Third game

i) probability of winning second game.

$$P^{(1)} = P^{(0)} \cdot P = \left[\frac{1}{2} \quad \frac{1}{2} \right] \cdot \frac{1}{10} \begin{bmatrix} 6 & 4 \\ 3 & 7 \end{bmatrix}$$
$$= \frac{1}{20} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 6 & 4 \\ 3 & 7 \end{bmatrix}$$

$$P^{(1)} = [P^{(W)}, P^{(L)}] = \left[\frac{9}{20}, \frac{10}{20} \right]$$

∴ probability of winning second game = $\boxed{\frac{9}{20}}$

ii) probability of winning Third game.

$$P^{(2)} = P^{(0)} \cdot P^2 = \left[\frac{1}{2} \quad \frac{1}{2} \right] \left[\frac{1}{10} \begin{bmatrix} 6 & 4 \\ 3 & 7 \end{bmatrix} \right] \left[\frac{1}{10} \begin{bmatrix} 6 & 4 \\ 3 & 7 \end{bmatrix} \right]$$

$$= \left[\frac{1}{2} \quad \frac{1}{2} \right] \cdot \frac{1}{100} \begin{bmatrix} 48 & 52 \\ 39 & 61 \end{bmatrix}$$

$$= \frac{1}{200} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 48 & 52 \\ 39 & 61 \end{bmatrix}$$

$$P^{(2)} = [P^{(W)}, P^{(L)}] = \left[\frac{87}{200}, \frac{113}{200} \right]$$

∴ probability of winning Third game = $\frac{87}{200}$

probability of he winning in the long run, i.e. $n \rightarrow \infty$

Find unique fixed probability vector

Let $v = (x, y)$ be unique fixed probability vector

$$x + y = 1 \quad VP = v$$

$$(x, y) \frac{1}{10} \begin{bmatrix} 6 & 4 \\ 3 & 7 \end{bmatrix} = (x, y)$$

$$\frac{1}{10} [6x + 3y, 4x + 7y] = (x, y)$$

$$6x + 3y = 10x \rightarrow \textcircled{1} \quad 4x + 7y = 10y \rightarrow \textcircled{2}$$

$$3y = 4x$$

$$4x = 3y$$

$$4x = 3(1-x) \quad (\because x+y=1)$$

$$4x = 3 - 3x$$

$$\boxed{x = \frac{3}{7}}$$

Sub x in $\textcircled{3}$

$$3y = 4\left(\frac{3}{7}\right)$$

$$21y = 12$$

$$y = \frac{12}{21}$$

$$\boxed{y = \frac{4}{7}}$$

$$v = (x, y) = \left(\frac{3}{7}, \frac{4}{7}\right)$$

In the long run he wins $\frac{3}{7}$ times i.e. $\approx 40\%$

a) determine the value of k , so that the function.

$f(x, y) = k[x \cdot y]$, for $x = -2, 0, 2$; $y = -2, 3$, represents joint probability distribution of the random variables x and y .

Also determine $\text{Cov}(x, y)$

$$f(x, y) = k |x - y|$$

\therefore joint probability distribution of x and y

$x \backslash y$	-2	3	$f(x)$
-2	0	$5k$	$5k$
0	$2k$	$3k$	$5k$
2	$4k$	k	$5k$
$g(y)$	$6k$	$4k$	$15k$

$$f(-2, -2) = 0k$$

$$f(-2, 3) = 5k$$

$$f(0, -2) = 2k$$

$$f(0, 3) = 3k$$

$$f(2, -2) = 4k$$

$$f(2, 3) = k$$

To find k $P(x \geq 0)$ and $\sum P(x) = 1$

$$\therefore 15k = 1$$

$$k = \frac{1}{15}$$

\therefore joint probability distribution

$x \backslash y$	-2	3	$f(x)$
-2	0	$\frac{1}{3}$	$\frac{1}{3}$
0	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{3}$
2	$\frac{4}{15}$	$\frac{1}{15}$	$\frac{1}{3}$
$g(y)$	$\frac{6}{15}$	$\frac{3}{5}$	1

Distribution of x

x	-2	0	2
$f(x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Distribution of y

y	-2	3
$g(y)$	$\frac{6}{15}$	$\frac{3}{5}$

To find $\text{cov}(x, y)$

$$\text{cov}(x, y) = E(xy) - E(x) \cdot E(y)$$

$$\therefore E(x) = \sum x f(x)$$

$$E(y) = \sum y g(y)$$

$$= -\frac{2}{3} + 0 + \frac{2}{3}$$

$$= -\frac{12}{15} + \frac{9}{5}$$

$$E(XY) = \sum x_i y_i + \sum f_{ij}$$

$$= 0 + (-2) + 0 + 0 + \left(-\frac{16}{15}\right) + \left(\frac{6}{15}\right)$$

$$E(XY) = -\frac{8}{3}$$

$$\therefore \text{COV}(XY) = \sum (x_i y_i) - E(X) \cdot E(Y)$$

$$= \left(-\frac{8}{3}\right) - (0)(1)$$

$$\text{COV}(XY) = -\frac{8}{3}$$

$$\text{COV}(XY) = -2.6667$$

b) Show that the matrix $\begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$ is a regular stochastic matrix.

$$\Rightarrow P = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \quad P^2 = P \cdot P = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$P^3 = P \cdot P^2 = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$P^4 = P \cdot P^3 = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

$$P^5 = P \cdot P^4 = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{2} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

$\therefore P$ is a regular stochastic matrix.

C Three boys A, B and C are throwing a ball to each other. A is just as likely to throw the ball to B as to C, B always throws the ball to A, and C is just as likely to throw the ball to A as to B. Find the probability that C has the ball after three throws if now A has the ball.

State space = $\{A, B, C\}$

Transition probability matrix = $P = P_{ij} = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \end{matrix}$

Initial state \rightarrow A has the ball $p^{(0)} = (1, 0, 0)$

probability after three throws $p^{(3)} = p^{(0)} \cdot P^{(3)}$

$$p^3 = [1, 0, 0] \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$$p^{(3)} = [1, 0, 0] \begin{bmatrix} \frac{1}{4} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{3}{8} & \frac{3}{8} & \frac{1}{4} \end{bmatrix}$$

$$p^{(3)} = \begin{bmatrix} \frac{1}{4} & \frac{3}{8} & \frac{3}{8} \end{bmatrix} = p^{(A)} \cdot p^{(B)} \cdot p^{(C)}$$

probability that C has the ball after three throws

$$= \boxed{\frac{3}{8}}$$

a) Explain The following MODULE-3
terms.

- i) Standard Error
- ii) Statistical Hypothesis
- iii) critical Region of a Statistical test.
- iv) Test of Significance.

⇒ i) Standard Error:- Standard Error of a statistic is the approximate standard deviation of a statistical sample population. The reciprocal of statistical sample is called precision. (or) Accuracy with which a sample distribution represents a population by using standard deviation is called standard Error.

ii) Statistical Hypothesis:- It is the statement that can be tested by scientific research. If we want to test a relationship between two or more things, we need to write hypothesis before we start experiment or data collection.
Example:- Eating an Apple every day leads to visit the doctor fewer times.

iii) Critical Region of a statistical test:- It is the region which amounts to the rejection of null hypothesis. (or) It is the set of values for which we want to reject the null hypothesis.

iv) Test of significance:- The process which helps us to decide about the acceptance or rejection of the hypothesis is called the Test of significance
i.e., To decide whether hypothesis is True or False.

5b) In 324 throws of a six faced die, an odd number turned up 181 times. Is it reasonable to think that the die is unbiased one at 5% level of significance?

⇒ Assume Null Hypothesis H_0 = The die is unbiased.

Given:- Sample size $n = 324$

Observed no. of Success $x = 181$

p = probability of getting odd number = $\frac{3}{6} = \frac{1}{2} = p$

$$q + p = 1 \Rightarrow q = \frac{1}{2}$$

Expected Success $\mu = np \Rightarrow \mu = 324 \times \frac{1}{2} \Rightarrow \mu = 162$

$$\begin{aligned} \text{Test of significance } Z &= \frac{x - \mu}{\sqrt{npq}} = \frac{181 - 162}{\sqrt{324 \times \frac{1}{2} \times \frac{1}{2}}} \\ &= \frac{181 - 162}{\sqrt{81}} \\ \boxed{Z = 2.11} \end{aligned}$$

At 5% level of significance $Z_{\alpha} = Z_{0.05} = 1.96$

$$|Z| > Z_{\alpha}$$

$$2.11 > 1.96$$

∴ Reject the Null Hypothesis.

∴ The die is unbiased.

50) In an examination given to students at a large number of different schools the mean grade was 74.5 and S.D. grade was 8. At one particular school where 200 students took the examination the mean grade was 75.9. Discuss the significance of this result at both 5% and 1% level of significance.

⇒ Given :- $\bar{x}_1 = 74.5$
 $\bar{x}_2 = 75.9$ } Be the two mean.

Sample size $n_1 = 200$ and $n_2 = 200$

Standard deviation $\sigma = 8$

Assume Null Hypothesis H_0 : The results are significant at 5% and 1% level of significance.

Test of significance
$$Z = \frac{\bar{x}_2 - \bar{x}_1}{\sigma \sqrt{\frac{1}{n_2} + \frac{1}{n_1}}} = \frac{75.9 - 74.5}{8 \sqrt{\frac{1}{200} + \frac{1}{200}}} = \frac{1.4}{8 \sqrt{\frac{1}{100}}} = \frac{1.4}{8 \times \frac{1}{10}} = \frac{1.4}{0.8} = 1.75$$

At 1% Level of significance

$$Z_{\alpha} = Z_{0.01} = 2.58 \Rightarrow |Z| < Z_{\alpha} \Rightarrow 1.75 < 2.58$$

At 5% Level of significance.

$$Z_{\alpha} = Z_{0.05} = 1.96 \Rightarrow |Z| < Z_{\alpha} \Rightarrow 1.75 < 1.96$$

∴ The Null Hypothesis is accepted at both 5% and 1% level of significance.

6 a) Define (i) Alternative Hypothesis (ii) A Statistic (iii) Level of Significance and (iv) Two-Tailed Test.

⇒ (i) Alternative Hypothesis:- It predicts that there is a relationship between two variables. Denoted by H_1 or H_2 followed by $\neq, >, <$ sign. The statement that directly contradicts the null hypothesis. H_1 or H_2 are independent variables that has relationship with dependent variables.

Example:- New drug reduce the no. of days to recover from a disease compared to standard drug.

(ii) A Statistic:- It deals with collection, analysis, Interpretation and presentation of numeric data.

(iii) Level of Significance:- It is a parameter used in Hypothesis testing to determine the threshold at which the null hypothesis is rejected. denoted by α (alpha). It represents the probability of rejecting or accepting the hypothesis. The commonly used significance levels are 1% and 5%.

(iv) Two-Tailed Test:- In our acceptance or rejection of null hypothesis, we consider the value of z and if the value of z is concentrated on both sides of the mean (normal area curve), it is called Two-Tailed Test.

A coin is tossed 1000 times and head turns up 540 times. decide on the hypothesis that the coin is unbiased at 1% level of significance.

Sample size $n = 1000$.

observed no. of heads on 1000 times tossing of coin $x = 540$

Let us Assume Null hypothesis H_0 = The coin is unbiased at 1% level of significance.

probability of getting head $p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$

Expected number of heads (success) $= \mu = np = 1000 \times \frac{1}{2} \Rightarrow$

$$\boxed{\mu = 500}$$

Test of significance $Z = \frac{x - \mu}{\sigma} = \frac{x - \mu}{\sqrt{npq}}$

$$\frac{540 - 500}{\sqrt{1000 \times \frac{1}{2} \times \frac{1}{2}}} = \frac{540 - 500}{\sqrt{250}}$$

$$\boxed{Z = 2.52}$$

At 1% level of significance, $Z_{0.01} = Z_{\alpha} = 2.58$

$$|Z| < Z_{\alpha}$$

$$2.52 < 2.58$$

\therefore Accept the Null Hypothesis.

\therefore The coin is unbiased.

6c) One type of aircraft is found to develop engine trouble in 5 flights out of a total of 100 and another type in 7 flights out of a total of 200 flights. Is there a significant difference in the two types of aircraft's so far as engine defects are concerned? Test at 5% level of significance.

⇒ Given :- Sample size $n_1 = 100$ Sample size $n_2 = 200$

$$\text{Let } P_1 = \frac{5}{100} = 0.05 \quad P_2 = \frac{7}{200} = 0.035$$

[proportion of engine trouble in 5 and 7 flights out of 100 and 200 flights]

Assume Null Hypothesis H_0 = There is a significant difference in the two types of aircrafts.

$$\text{Test of significance } Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \rightarrow \textcircled{1}$$

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

$$P = \frac{100 \times 0.05 + 200 \times 0.035}{100 + 200}$$

$$P = 0.04 \Rightarrow Q = 0.96$$

$$\therefore Z = \frac{0.05 - 0.035}{\sqrt{(0.04 \times 0.96)\left(\frac{1}{100} + \frac{1}{200}\right)}} = Z = 0.625$$

∴ For 5% level of significance $Z_{\alpha} = Z_{0.05} = 1.96$

$$|Z| < Z_{\alpha} \quad 0.625 < 1.96$$

∴ Accept the null hypothesis at 5% level of significance

MODULE - 4

- 7 a) An unknown distribution has a mean of 90 and a standard deviation of 15. Samples of size $n=25$ are drawn randomly from the population. Find the probability that the sample mean is between 85 and 92.

$$n = 25$$

$$\mu = 90$$

$$\sigma = 15$$

$$Z = \bar{x} \sim N\left(90, \frac{15}{\sqrt{25}}\right)$$

$$Z = \bar{x} \sim N(90, 3)$$

$$Z = \frac{\bar{x} - 90}{\frac{15}{\sqrt{25}}}$$

$$Z = \frac{\bar{x} - 90}{3}$$

$$\bar{x} = 85 \Rightarrow Z = \frac{85 - 90}{3} = \frac{-5}{3} = -1.66$$

$$\bar{x} = 92 \Rightarrow Z = \frac{92 - 90}{3} = \frac{2}{3} = 0.66$$

$$P(85 < \bar{x} < 92) = P(-1.66 < Z < 0.66)$$

$$\begin{aligned} P(-1.66 < Z < 0.66) &= P(0 < Z < 1.66) + P(0 < Z < 0.66) \\ &= 0.4515 + 0.2454 \end{aligned}$$

$$P(-1.66 < Z < 0.66) = 0.6965$$

7b) The heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Construct a 99% confidence interval for the mean height of all college students.

$$\Rightarrow n = 50$$

$$\mu = 174.5 \text{ cm}$$

$$\sigma = 6.9 \text{ cm}$$

confidence level of 99%, corresponding value is 2.576.

confidence interval $C.I = \text{mean} \pm z \left(\frac{\sigma}{\sqrt{n}} \right)$

$$C.I = 174.5 \pm \left(2.576 \times \frac{6.9}{\sqrt{50}} \right)$$

$$C.I \quad 174.5 \pm 2.5136$$

$$\begin{aligned} \text{Lower end of the confidence interval} &= 174.5 - 2.5136 \\ &= 171.9864 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Higher end of the confidence interval} &= 174.5 + 2.5136 \\ &= 177.0136 \text{ cm} \end{aligned}$$

Mean height is between 171.9864 and 177.0136 cm.

7c) A die was thrown 60 times and the following frequency distribution was observed:

Faces	1	2	3	4	5	6
Frequency	15	6	4	7	11	17

test whether the die is unbiased at 5% significance level.

⇒ Assuming die is unbiased.

Expected frequency for numbers: 1, 2, 3, 4, 5, 6 to appear on face is $\frac{60}{6} = 10$ each.

x	1	2	3	4	5	6
O_i	15	6	4	7	11	17
E_i	10	10	10	10	10	10

$$\chi^2 = \sum_i \left\{ \frac{O_i - E_i}{E_i} \right\}^2$$

$$\chi^2 = \frac{(15-10)^2}{10} + \frac{(6-10)^2}{10} + \frac{(4-10)^2}{10} + \frac{(7-10)^2}{10} + \frac{(11-10)^2}{10} + \frac{(17-10)^2}{10}$$

$$\chi^2 = 31.10$$

8a) In a recent study reported on the Flurry Blog, The mean age of tablet users is 34 years. Suppose the standard deviation is 15 years. Take a sample of size $n=100$ using central limit theorem, Find the probability that the sample mean age is more than 30 years.

$$\Rightarrow n = 100$$

$$\mu = 34$$

$$\sigma = 15$$

$$\bar{x} \sim N(34, \frac{15}{\sqrt{100}})$$

$$\bar{x} \sim N(34, 1.5)$$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$Z = \frac{\bar{x} - 34}{15/\sqrt{100}}$$

$$Z = \frac{\bar{x} - 34}{1.5}$$

$$\text{At } \bar{x} = 30$$

$$Z = \frac{30 - 34}{1.5} = \frac{-4}{1.5} = -2.66$$

$$P(\bar{x} > 30) = P(Z > -2.66)$$

$$P(Z < 2.66) = 0.5 + A(2.66)$$

$$= 0.9961$$

suppose that 10, 12, 16, 19 is the sample taken from a normal population with variance 6.25. find a 95 percent confidence interval for the population mean.

Samples are 10, 12, 16, 19.

$$n = 4$$

$$\bar{X} = 14.25$$

$$\sigma^2 = 6.25$$

$$\sigma = \sqrt{6.25} = 2.5.$$

confidence interval of 95%,

Z corresponds to 1.96

$$C.I = \mu = \text{Mean} \pm Z \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$C.I = 14.25 \pm \left(1.96 \times \left(\frac{2.5}{\sqrt{4}} \right) \right)$$

$$C.I = (14.25 - 2.45, 14.25 + 2.45)$$

$$C.I = (11.80, 16.70)$$

90 A random sample of 10 boys had the following I.Q.:
 70, 120, 110, 101, 88, 83, 95, 98, 107, 100.
 Do these data support the assumption of a population
 mean I.Q. of 100 (at 5% level of significance)
 \Rightarrow I.Q. of 10 boys

$$x: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100$$

$$\bar{x} = \frac{1}{n} \sum x = \frac{972}{10} = 97.2$$

$$S^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$$

$$S^2 = \frac{1}{9} \times 1833.6$$

$$S^2 = 203.7333$$

$$S = 14.2735$$

$$\mu = 100$$

$$t = \frac{\bar{x} - \mu}{S / \sqrt{n}}$$

$$t = \frac{97.2 - 100}{\left(\frac{14.2735}{\sqrt{10}} \right)}$$

$$t = -\frac{2.8}{4.5136} \approx -0.6203 < 2.262.$$

MODULE - 5

Three types of fertilizers are used on three groups of plants for 5 weeks. We want to check if there is a difference in the mean growth of each group. Using the data given below apply a one-way ANOVA test at 0.05 significant level.

Fertilizer - 1.	6	8	5	3	4
Fertilizer - 2.	8	12	9	11	6
Fertilizer - 3.	13	9	11	8	7

F(x)	F(y)	F(z)	x^2	y^2	z^2
6	8	13	36	64	169
8	12	9	64	144	81
4	9	11	16	81	121
5	11	8	25	121	64
3	6	7	9	36	49
4	8	12	16	64	144

$$\Sigma x = 30 \quad \Sigma y = 54 \quad \Sigma z = 60 \quad \Sigma x^2 = 166 \quad \Sigma y^2 = 570 \quad \Sigma z^2 = 628$$

$$n_1 = 6 \quad n_2 = 6 \quad n_3 = 6$$

$$n = 18$$

$$\Sigma x = 26 + 54 + 60$$

$$\Sigma x = 140$$

$$\Sigma x^2 = \Sigma x^2 + \Sigma y^2 + \Sigma z^2$$

$$\Sigma x^2 = 1304$$

$$SSB = \sum \frac{T_i^2}{n_i} - \frac{\sum x^2}{n}$$

$$SSB = 84$$

$$SSN = \sum x^2 - \frac{\sum T_i^2}{n} = 1304 - 1236$$

$$SSN = 68$$

$$MSB = \frac{SSB}{k-1} = \frac{84}{3-1} = 42$$

$$MSW = \frac{SSN}{n-k} = \frac{68}{18-3} = 4.53$$

$$SST = SSB + SSN = 84 + 68 = 152$$

$$F_{ratio} = \frac{MSB}{MSW} = \frac{42}{4.53} = 9.27$$

Source of variation	Sum of squares	d.f	Mean square	F-ratio
B/w samples	84	2	42	$F = \frac{MSB}{MSW} = 9.27$ $F(2, 15)$
Within sample	68	15	4.53	
Total	152			

The hypothesis is rejected.

9b) present your conclusions after doing analysis of variance to the following results of the Latin-Square design experiment conducted in respect of five fertilizers which were used on plots of different fertility.

A	B	C	D	E
16	10	11	9	9
E	C	A	B	D
10	9	14	12	11
B	D	E	C	A
15	8	8	10	18
D	E	B	A	C
12	6	13	13	12
C	A	A	E	B
13	11	10	7	14

⇒

A	B	C	D	E	T	T ²
16	10	11	9	9	5	25
E	C	A	B	D	6	36
10	9	14	12	11		
B	D	E	C	A	9	81
15	8	8	10	18		
D	E	B	A	C	6	36
12	6	13	13	12		
C	A	D	E	B	5	25
13	11	10	7	14		
16	-6	6	1	14		
856	36	36	1	196		

P
P²

A	B	C	D	E
36	0	1	1	1
E	C	A	B	D
0	1	16	4	1
B	D	E	C	A
4	16	9	9	4
C	A	D	E	B
9	1	0	9	16
74	22	30	23	86

$$= 235$$

$$C.F = \frac{T^2}{N} = \frac{(31)^2}{25} = \frac{961}{25} = 38.44$$

$$TSS = 235 - 38.44$$

$$TSS = 196.56$$

$$SSR = \frac{\sum_i T_i^2}{n_i} - CF$$

$$SSR = \frac{25}{5} + \frac{36}{5} + \frac{81}{5} + \frac{36}{5} + \frac{25}{5} - 38.44$$

$$SSR = 2.16$$

$$SSC = \frac{256}{5} + \frac{36}{5} + \frac{36}{5} + \frac{1}{5} + \frac{196}{5} - 38.44$$

$$SSC = 66.56$$

$$SST = \frac{485}{5} + \frac{196}{5} + \frac{25}{5} + \frac{0}{5} + \frac{100}{5} - 38.44$$

$$SST = 161 - 38.44$$

$$SST = 122.56$$

$$SSE = TSS - SSR - SSC - SST$$

$$SSE = 196.56 - 2.16 - 66.56 - 122.56$$

$$SSE = 5.28$$

Sources of Variation	d.f	SSR	MSS	F-Ratio
Rows	$5-1=4$	2.16	$MSR=0.54$	$F_R = \frac{0.54}{0.44} = 1.227$
columns	$5-1=4$	66.56	$MSC=16.64$	$F_C = \frac{16.64}{0.44} = 37.81$
Treatments	$5-1=4$	$SST=122.56$	$MST=30.64$	$F_T = \frac{30.64}{0.44} = 69.63$
Error	$4 \times 3 = 12$	$SSE=5.28$	$MSE=0.44$	
Total	$25-1=24$			

$F_R < F(4, 12) \rightarrow \text{Accepted } H_0$

$F_C > F(4, 12) \rightarrow H_0 \text{ Rejected}$

$F_T > F(4, 12) \rightarrow H_0 \text{ Rejected}$

10a) A trial was run to check the effects of different diets. Positive numbers indicate weight loss and negative numbers indicate weight gain. Check if there is an average difference in the weight of people following different diets using an ANOVA Table.

Low fat	Low calorie	Low protein	Low carbohydrate
8	2	3	2
9	4	5	2
6	3	4	-1
7	5	2	0
3	1	3	3

⇒ set null hypothesis: $H_0: \mu_1 = \mu_2 = \mu_3$

x	y	z	a
8	2	3	2
9	4	5	2
6	3	4	-1
7	5	2	0
3	1	3	3
Σ 33	15	17	6
Σx^2 1089	225	289	36

x^2	y^2	z^2	a^2
64	4	9	4
81	16	25	4
36	9	16	1
49	25	4	0
9	1	9	9
$\Sigma x^2 = 239$	$\Sigma y^2 = 55$	$\Sigma z^2 = 63$	$\Sigma a^2 = 18$

71

-

= 375

$$\text{correction factor} = \frac{T^2}{N} = \frac{(71)^2}{20} = \frac{5041}{20} = 252$$

$$TSS = 375 - 25$$

\therefore Total sum of squares $TSS = 123$.

Sum of Squares of between treatments SST

$$= \sum_i \frac{T_i^2}{n_i} - CF$$

$$SST = \frac{1089}{5} + \frac{225}{5} + \frac{289}{5} + \frac{36}{5} - 252$$

$$SST = 75.80$$

\therefore Sum of squares due to error

$$SSE = TSS - SST$$

$$SSE = 123 - 75.80$$

$$SSE = 47.2$$

Sources of variation	d.f	SS	MSS	F-ratio
B/W Treatments	$4-1=3$	$SST = 75.80$	$MST = \frac{75.80}{3}$ $MST = 25.26$	$F = \frac{25.26}{2.95}$ $F = 8.56$
Error	$20-4=16$	$SSE = 47.20$	$MSE = \frac{47.20}{16}$ $MSE = 2.95$	
Total	$20-1=19$			

$8.56 > 3.24$ for $F(3,16)$ at 5% level of significance.

Hence null hypothesis is rejected.

10 b) The following data show the number of worms quarantined from the GI areas of four groups of muskrats in a carbon tetrachloride anthelmintic study. conduct a two-way ANOVA

I	II	III	IV
33	41	12	38
32	38	35	43
26	40	46	25
14	23	22	13
30	21	11	26

Subtract 30 from all observations, we get.

	I	II	III	IV
A	3	11	-18	8
B	2	8	5	13
C	-4	10	16	-5
D	-16	-7	-8	-17
E	0	-9	-19	-4

I	II	III	IV	P	P ²
3	11	-18	8	4	16
2	8	5	13	28	784
-4	10	16	-5	17	289
-16	-7	-8	-17	-48	2304
0	-9	-19	-4	-32	1024
T	-15	13	-24	-5	-31
T ²	225	169	576	25	

<u>I</u>	<u>II</u>	<u>III</u>	<u>IV</u>
9	121	324	64
4	64	25	169
16	100	256	25
256	49	64	289
0	81	361	16
285	415	1030	563

2293

set null hypothesis $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_n$

$$C.F = \frac{T^2}{N} = \frac{(-31)^2}{20} = \frac{961}{20} = 48$$

Total sum of squares $T_{SS} = \sum_i \sum_j x_{ij}^2 - C.F$

$$T_{SS} = 2293 - 48$$

$$T_{SS} = 2245$$

$$SSC = \frac{\sum_i T_i^2}{n_i} - C.F$$

$$SSC = \frac{225}{5} + \frac{169}{5} + \frac{576}{5} + \frac{25}{5} - 48$$

$$SSR = \frac{\sum_i p_i^2}{n_i} - CF$$

$$SSR = \frac{16}{4} + \frac{784}{4} + \frac{289}{4} + \frac{2304}{4} + \frac{1024}{4} - 48$$

$$SSR = 1056.25$$

$$SSE = TSS - SSC - SSR$$

$$SSE = 2245 - 151 - 1056.25$$

$$SSE = 1037.75$$

$$F(4,12) = 3.2, 6 \text{ and } F(3,12) = 3.49$$

Sources of variation	d.f	SS	mss
Rows	5-1=4	SSR 1056.2	$MSR = \frac{1056.25}{4} = 264.062$
columns	4-1=3	SSC 151	$MSC = \frac{151}{3} = 50.33$
Errors.	4x3=12	SSE 1037.75	$MSE = \frac{1037.75}{12} = 86.48$

$$F_R = \frac{264.06}{86.48}$$

$$F_R < F(4,12)$$

$H_0 \rightarrow$ Accepted

$$F_R = 3.058$$

$$F_C = \frac{86.48}{50.33}$$

$$F_C < F(3,12)$$

$H_0 \rightarrow$ Accepted.

$$F_C = 1.718$$