# MODULE – I
# FUNDAMENTALS OF LOGIC

1. Explain the following with one example each: i) Disjunction ii) Conjunction iii) Negation.
2. Define logical connectives, conjunction and disjunction. **(June2016, Dec 2011)**
3. Define the following with an example each: i) Proposition ii) Tautology iii) Contraction iv) Dual of a statement **(June 2017)**
4. Determine the truth value of each of the following
   i) If 3 + 4 = 12 then 3 + 2 = 6 ii) If 4 + 4 = 8 then 5 + 4 = 10
   iii) If Dr. Radhakrishnan was the first president of India then 3 + 4 = 7. **(Dec 2007)**
5. Let p, q be primitive statements for which the implication $p \to q$ is false. Determine the truth values for each of the following: i) $\wedge q$ii)$\neg p \vee q$iii) $q \to p$ iv)$\neg q \to \neg p$. **(Dec 2014, 2013, June 2008)**
6. Let p, q, r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound propositions: i) $(p \wedge q) \to r$ ii) $p \to (q \wedge r)$ iii) $p \wedge (r \to q)$ iv) $p \to (q \to \neg r)$. **(Dec 2016)**
7. Find the possible truth values of p, q and r if i)$p \to (q \vee r)$ is FALSE
   ii) $\wedge (q \to r)$ is TRUE. **(June 2017,Dec 2007)**
8. Show that $(p \wedge (p \to q)) \to q$ is independent of its components. **(June 2017)**
9. If the statement q has the truth value 1, determine all truth value assignments for the primitive statements p. r, and s for which the truth value of the statement: $(q \to [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \to (\neg r \wedge q)]$ is 1. **(June 2019, Dec 2017, 2007)**
10. Define a proposition, a tautology and a contradiction. Prove that, for any propositions p, q, r the compound propositions,
    i) $\{(p \to q) \wedge (q \to r)\} \to \{(p \to r)\}$ **(Dec 2019, 2012, June 2019, 2015, 2010)**
    ii)$\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ **(June 2019, 2018, 2012)**
    iii) $(p \to (q \vee r)) \leftrightarrow ((p \wedge \neg q) \to r)$ **(June2016)**
    iv)$[(p \vee q) \to r] \leftrightarrow [\neg r \to \neg(p \vee q)]$ **(Dec2018, 2015)**
    v) $[(p \to r) \wedge (q \to r)] \to [(p \vee q) \to r]$
    vi) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$are tautologies. **(Dec 2016, 2011, 2009, 2007)**
11. Show that the truth values of the following statements are independent of their components:
    i) $[p \wedge (p \to q)] \to q$ ii) $(p \to q) \leftrightarrow [\neg p \vee q]$ **(June 2017)**
12. Use truth tables to verify,
    i) $[p \to (q \wedge r) \Leftrightarrow (p \to q) \wedge (p \to r)$ **(Dec 2017)**
    ii)$[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$ **(Dec 2019, 2009)**
    iii)$[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \Leftrightarrow [(p \to q) \wedge (q \to r) \wedge (r \to p)]$ **(June 2008)**
    iv) $\neg[p \wedge q] \Leftrightarrow [\neg p \vee \neg q]$
    v) $(p \leftrightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg[q \vee p]$ **(Dec 2015)**
    vi)$[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(Dec 2015,2009)**
    vii)$(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \Leftrightarrow p \wedge q$ **(Dec 2013)**
13. Construct the truth table for i) $[p \wedge (p \to q)] \to q$ii)$\{(p \to q) \wedge (q \to r)\} \to \{(p \to r)\}$ **(Dec 2014, June 2014)**
14. By constructing the truth table, show that the compound propositions $p \wedge (\neg q \vee r)$ and $p \vee (\quad \neg r)$ are not logically equivalent. **(Dec 2014)**
15. Prove the following using laws of logic:
    i) $p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$**(Dec 2016, Dec 2009)**ii) $[(\sim p \vee \sim q) \wedge (F_0 \vee p) \wedge p] \Leftrightarrow p \wedge \sim q$ **(Dec 2018)**
16. Define logical equivalence of two propositions. Prove the following logical equivalences using laws of logic:
    i)$p \vee [p \wedge (p \vee q)] \Leftrightarrow p$ii)$[(\neg p \vee \neg q) \longrightarrow (p \wedge q \wedge r) \Leftrightarrow p \wedge$ **(June 2019, 2013, Dec 2013)**
    iii) $p \vee q \vee (\sim p \wedge \sim q \wedge r) \equiv p \vee q \vee r$ **(June 2019)**
    iv) $(p \to q) \wedge (\neg q \wedge (r \vee \neg q)) \Leftrightarrow \neg[q \vee p]$ **(Dec 2018, 2012, June 2018, 2015)**
    v)$[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(June 2018, Dec 2016)**

17. Define dual of logical statement. Write the dual of the following logical statements:
 i)$(p \vee T_0) \wedge (q \vee r) \vee (r \wedge s \wedge T_0)$**(Dec 2012)**ii)$(p \wedge q) \vee T_0$iii) $[\neg(p \vee q) \wedge \{p \vee \neg(q \wedge \neg s)\}]$
 iv) $p \to (q \to r)$ v)$[(p \vee T_0) \wedge (q \vee F_0)] \vee [(r \wedge s) \wedge T_0]$ **(Dec 2018)**
18. Verify the principle of duality for the logical equivalence: $\sim(p \wedge q) \to \sim p \vee (\sim p \vee q) \Leftrightarrow \sim p \vee q$.
 **(Dec 2016, 2009, June 2014)**
19. Write the negation of the statement: "If$x$ is not a real number, then it is not a rational number and not anirrational number".
20. Express each of the following statements in symbols, negate them and write in smooth English. **(Dec 2007)**
 i) Vimala will get a good education if she puts her studies before her interest in cheer leading.
 ii) Nirmala is doing her homework and Kamala is practicing her music lessons.
21. Show that $[p \wedge (p \to q) \wedge r] \Rightarrow (p \vee q) \to r$. **(Dec 2009)**
22. Define $(p \uparrow q) \Leftrightarrow \sim(p \wedge q)$. Represent $p \vee q$ and $p \to q$ using NAND only. **(June 2012)**
23. For any statements p, q, prove that i) $\neg(p \downarrow q) \Leftrightarrow (\neg p \uparrow \neg q)$ii)$\neg(p \uparrow q) \Leftrightarrow (\neg p \downarrow \neg q)$
24. Define inverse, converse and contra positive of a conditional statement with truth table. Write down the contrapositive of $[p \to (q \to r)]$ with i) only one occurrence of the connective→ ii)no occurrence of the connective→ **(June 2014)**
25. Define converse, inverse and contrapositive of an implication. Hence find converse, inverse and contrapositive for "$\forall x, (x > 3) \to (x^2 > 9)$"where universal set is the set of real numbers R. **(Dec 2018)**
26. Write dual, negation, converse, inverse and contra positive of the statements given below:
 i) If Kabir wears brown pant then he will wear white shirt. **(June 2012)**
 ii) If a triangle is not isosceles then it is not equilateral. **(June 2013)**
 iii) If Ram can solve the puzzle then Ram can solve the problem. **(June2016)**
27. Define an argument and valid argument. Give examples to each. Determine all truth value assignments for the primitive statements for the valid argument $p \wedge (q \wedge r) \to (s \vee t)$.
28. Show that$r \vee s$follows from$c \vee d,\ c \vee d \to \neg h, \neg h \to a \wedge \neg b, a \wedge \neg b \to r \vee s$. **(June 2019, 2009, Dec 2014, 2013)**
29. Show that $s \vee r$ is tautology implied by $p \vee q, p \to rakd\ \ q \to s$.
30. Prove that $r \wedge (p \vee q)$ is valid conclusion from the premises $p \vee q, q \to r, p \to makd\ \neg m$.
31. Test whether the following is a valid argument: If Sachin Tendulkar hits a century, then he gets a free car. Sachin Tendulkar hits a century. ∴ Sachin Tendulkargets a free car.
32. Establish the validity of the following argument using rules of inference. If the band could not play rock music or the refreshments were not served on time, then the new year party could have been cancelled and Alica would have been angry. If the party were cancelled, then refunds would have been made. No refunds were made. ∴The band could play rock music. **(June 2017)**
33. Translate into symbolic form and test the validity of the argument
 If I work, then I cannot study. Either I study or I pass my Mathematics class. I work.
 ∴ I pass my mathematics class.
34. Test the validity of the following statements:
 i) If there is a strike by students, the examination will be postponed.The exam was not postponed.
 ∴There was no strike by students. **(Dec 2018)**
 ii) If Ravi studies, then he will pass DMS. If Ravi does not play cricket, then he will study. Ravi failed in DMS.

$\therefore$ Ravi played cricket. **(Dec 2018)**

iii) I will become famous or I will not become a musician. I will become a musician

$\therefore$ I will become famous.

iv) If I study, then I will not fail in the examination. If I do not watch T.V. in the evenings, then I will study. I failed in the examination.

$\therefore$ I must have watched T.V. in the evenings. **(Dec 2019, 2018)**

v) If I like mathematics, then I will study. Either I study or I fail. $\therefore$ If I fail then I do not like mathematics. **(Dec 2011)**

35. Consider the following argument:

I will get grade A in this course or I will not graduate.
If I do not graduate, I will join army.
<u>I got grade A</u>

$\therefore$ I will not join the army. Is this a valid argument? Prove using rules of inference.

36. Write the following in symbolic form and establish if the argument is valid:
If A gets the supervisors position and works hard, then he will get a rise. If he gets a rise, then he will buy a new car.
$\therefore$ A did not get supervisors position or he did not work hard. **(Dec 2011, 2007)**
37. Test whether the following argument is valid:
If interest rates fall then stock market will rise. The stock market will not rise. $\therefore$ The interest rates will not fall.
38. Show that the hypothesis "if you do not send me an e-mail message, then I will finish writing the program", "if you do not send me an e-mail message, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed". **(June2015)**
39. When a conclusion q is said to follow from the premises $H_1, H_2, H_3, \dots\dots, H_n$?
Let p,q,r be the primitive statements,
p: Raghu studies
q: Raghu plays tennis
r: Raghu passes in discrete mathematics.
Let $H_1, H_2, H_3$ be the premises.
$H_1$: If Raghu studies then he will pass in discrete mathematics.
$H_2$: If Raghu does not play tennis then he will study.
$H_3$: Raghu failed in discrete mathematics
show that $q$ follows from $H_1, H_2$ and $H_3$. **(Dec 2009)**
40. Verify the following without using the truth tables. $[(p \to q) \wedge (s \vee \sim r)] \;\therefore\; \sim q \to s$
41. Show that the following argument is invalid by giving a counter example :
$[\,(p \vee \sim q) \wedge (p \to (q \to r)] \to \sim r$
42. Establish the validity of the following Argument

| | | | | | |
|---|---|---|---|---|---|
| i) $p$<br>$p \to q$ $s \vee r$<br>$r \to \sim q$<br>$\therefore s \vee t$<br>**(Dec2016, June2011,2010,2008)** | ii) $p \vee q$<br>$\sim(p \vee r)$<br>$\underline{\sim r}$<br>$\therefore q$<br>**(June 2012)** | iii) $(\neg p \vee q) \to r$<br>$r \to (s \vee t)$<br>$\neg s \wedge \neg u$<br>$\underline{\neg u \to \neg t}$<br>$\therefore p$<br>**(June 2018)** | iv) $(q \vee \sim r) \vee s$<br>$\underline{\sim q \vee (r \wedge \sim q)}$<br>$\therefore r \to s$<br>**(June2016)** | v) $p$<br>$p \to r$<br>$p \to (q \vee \sim r)$<br>$\underline{\sim q \vee \sim s}$<br>$\therefore s$<br>**(June 2012)** | vi) $(\neg p \vee \neg q) \to (r \wedge s)$<br>$r \to t$<br>$\underline{\neg t}$<br>$\therefore p$<br>**(June 2019, 2018)** |

43. Establish the validity of the following argument: **(Dec 2017)**

$$\begin{array}{c} p \to q \\ q \to (r \wedge s) \\ \sim r \vee (\sim t \vee u) \\ \underline{\quad p \wedge t \quad} \\ \therefore u \end{array}$$

44. Establish the validity of the following argument: **(June 2019)**

$$\begin{array}{c} u \to r \\ (r \wedge s) \to (p \vee t) \\ q \to (u \wedge s) \\ \sim t \\ \underline{q} \\ \therefore p \end{array}$$

45. Prove that, the following are valid arguments: **(Dec 2019)**

(i) $$\begin{array}{c} p \to (q \to r) \\ \sim q \to \sim p \\ \underline{\quad p \quad} \\ \therefore r \end{array}$$

(ii) $$\begin{array}{c} \sim p \leftrightarrow q \\ q \to r \\ \underline{\quad \sim r} \\ \therefore p \end{array}$$

46. Define: (i) Open sentences (ii) Quantifiers (iii) Free variables (iv) Bound variables. **(Dec 2016, June 2015)**

47. Define the rule of universal specification and rule of universal generalization. Also write their symbolic form. **(Dec 2011)**

48. For the universe of all integers, let p(x), q(x), r(x), s(x) and t(x) denote the following open statements: p(x): x>0, q(x): x is even, r(x): x is a perfect square, s(x): x is divisible by 3, t(x): x is divisible by 7. Write the following statements in symbolic form: i) At least one integer is even. ii) There exists a positive integer that is even. iii) If x is even, then x is not divisibleby 3. iv) No even integer is divisible by 7. v) There exists even integer divisible by 3. **(Dec 2009)**

49. Let p(x) denotes the sentence "$x + 2 > 5$". State whether or not p(x) is a proportional function on each of the following sets: i) N, the set of positive integers ii)C, the set of complex numbers. **(Dec 2011)**

50. For the universe of all integers, define the following open statements: p(x): x>0, q(x): x is even, r(x): x is a perfect square, s (x): x is divisible by 4 and t(x): x is divisible by 5. Write the following statements in symbolic form and determine whether each of the statements is true or false. For each false statement, provide a counter example.
i) At least one integer is even. ii) There exists a positive integer that is even. iii) If x is even, then x is not divisible by 5. iv) If x is even and a perfect square, then x is divisible by 4. **(Dec 2008)**

51. Let p(x), q(x) and r(x) denote the following open statements: p (x): $x^2 - 7x + 10 = 0$, q(x): $x^2 - 2x + 3 = 0$, r(x) : x<0. Determine the truth or falsity of the following statements when the universe contains only the integers 2 and 5. If a statement is false, provide a counter example or explanation.

i) $\forall x, [p(x) \to \neg r(x)]$ ii) $\forall x, [q(x) \to r(x)]$

iii) $\exists x, [(x) \to (x)]$ iv) $\exists x, [p(x) \to r(x)]$. **(Dec 2019, 2015, June 2019)**

52. Let$(x)$: $x \geq 0$, q$(x)$: $x^2 \geq 0$and $r(x)$: $x^2 - 3x - 4 = 0$. Then for the universe comprising of all real numbers, find the truth values of,

    (i) $\exists x, [(x) \wedge (x)]$ (ii) $\forall x, [p(x) \rightarrow q(x)]$ (iii)$\exists x, [p(x) \wedge r(x)]$

    (iv) $\forall x, [(x) \rightarrow (x)]$ (v) $\forall x, [r(x) \rightarrow p(x)]$(vi) $\forall x, [r(x) \vee q(x)]$ **(Dec 2018, 2012, June 2009)**

53. Consider the following open statements with the set of all real numbers as the universe, $p(x)$: $x \geq 0$, $q(x) : x^2 \geq 0$, $(x)$: $x^2 - 3x - 4 = 0$ and $(x)$: $x^2 - 3 > 0$, then find the truth values of

    (i) $\exists x, [(x) \wedge (x)]$ (ii) $\forall x, [p(x) \rightarrow q(x)]$ (iii) $\forall x, [q(x) \rightarrow s(x)]$ **(Dec 2007)**

54. Let p(x) be the open statement "$x^2 = 2x$", where the universe consists of all integers. Determine whether each of the following statements is true or false:i) p(0) ii) p(-2) iii) $\exists x, p(x)$. **(June 2008)**

55. Write the following sentences in symbolic form, and find its negation: (i) If all triangles are right angled, then no triangle is equiangular.(ii) All integers are rational numbers and some rational numbers are not integers. **(Dec 2018, June 2013)**

56. Write the negation of each of the following statements for (i) and (ii) the universe consists of all integers and for (iii) the universe consists of all real numbers.

    (i) For all integers n, if n is not divisible by 2, then n is odd.

    (ii) If k, m, n are any integers where k – m and m – n are odd, then k – n is even. **(Dec 2007)**

    (iii) For all real numbers x, if$|x - 3| < 7$, then –4 < x < 10.

57. Write the following open statement in symbolic form and find its negation.

    "Some straight lines are parallel or all straight lines intersect". **(June2016)**

58. Negate and simplify:

    i) $\forall x, [p(x) \wedge \neg q(x)]$ ii) $\exists x, [(p(x) \vee q(x)) \rightarrow r(x)]$

    iii) $\forall x, [(x) \rightarrow (x)]$iv) $\forall x, [p(x) \vee q(x)]$ **(Dec 2014, 2012, 2011, June 2014, 2010)**

59. For the universe of all polygons with 3 or 4 sides defined by the following open statements:

    i(x): all interior angles of x are equal h(x): all sides of x are equal

    s(x): x is a squaret(x): x is a triangle,

    translate each of the following into an English sentence and determine its truth value:

    i) $\forall x, [s(x) \leftrightarrow (i(x) \wedge h(x))]$ ii) $\exists x, [t(x) \rightarrow (i(x) \leftrightarrow h(x))]$

    Write the following statements symbolically and determine their truth values.

    iii) Any polygon with three or four sides is either a triangle or a square

    iv) For any triangle if all the interior angles are not equal, then all its sides are not equal. **(June 2012)**

60. For the following statement state the converse, inverse and contra positive. Determine the truth values of the given statement and the truth values of its converse, inverse and contra positive. The universe consists of all integers: "if m divides n and n divides p then n divides p ". **(Dec 2007)**

61. Given R (x, y): x + y is even and the variables x and y represent integers. Write an English sentence corresponding to each of the following (i) $\forall x, \exists y, (x, y)$ (ii) $\exists x, \forall y, (x, y)$. **(Dec 2012)**

62. What are the bound variables and free variables? Identify the same in each of the following expressions:

(i) $\forall y, \exists z, [\cos(x+y) = \sin(z-x)]$ (ii) $\exists x, \exists y, [x^2 - y^2 = z]$ **(Dec 2018)**

63. Write down the converse,inverse and contrapositive of:

i) $\forall x, [x^2 + 4x + 21 > 0] \to [(x > 3) \vee (x < -7)]$ **(Dec2015)**

ii) $\forall x, [(x > 3) \to (x^2 > 9)]$ **(Dec 2014, 2013)**

64. Let the universe comprise of all integers

i) Given p (x): x is odd, q (x): $x^2 - 1$ is even. Express the statement “If x is odd then $x^2 - 1$ is even” in symbolic form using quantifiers and negate it. **(Dec 2013)**

ii) If r (x): 2x +1 = 5, s (x): $x^2 = 9$ are open sentences, obtain the negation of the quantified statement $\exists x, [r(x) \wedge s(x)]$ **(June 2015, Dec 2011)**

65. Define rule of universal specification and rule of universal generalization. Also write their symbolic notation forms. **(Dec 2011, 2007)**

66. Negate the following statements. i) $\forall x, p(x) \wedge \exists y, q(y)$ ii) $\exists x, p(x) \vee \forall y, q(y)$

67. Let $(x, y)$ denote the open statement $x$ divides $y$ where the universe consists of all integers. Determine the truth value of the following statements. Justify your answer.

i) $\forall x, \forall y [p(x,y) \wedge p(y,x) \to (x = y)]$ ii) $\forall x, \forall y [p(x,y) \vee p(y,x)]$ **(June 2012)**

68. For the universe of all people, find whether the following is a valid argument:All mathematics professors have studied calculus. Ramanujan is a mathematics professor. Therefore Ramanujan has studied calculus.

69. Find whether the following argument is a valid argument for which the universe is the set of all students:

No engineering student is bad in studies. Ram is not bad in studies. Therefore, Ram is an engineering student.

70. Determine if the argument is valid or not. All people who are concerned about the environment, recycle their plastic containers. B is not concerned about the environment. Therefore B does not recycle his plastic containers.**(Dec 2011)**

71. Write in symbolic form and establish the validity of the argument. All human beings are mortal. Socrates is a human being. Therefore, Socrates is mortal.

72. Over the universe of all quadrilaterals in plane geometry, verify the validity of the argument, “since every square is a rectangle, and every rectangle is a parallelogram, it follows that every square is a parallelogram”. **(June 2015)**

73. Find whether the following argument is a valid argument: No engineering student of $1^{st}$ or $2^{nd}$ semester studies logic. Anil Raj is an engineering student who studies logic. ∴ Anil Raj is not in $2^{nd}$ semester. **(June 2018, 2013, Dec 2014)**

74. Find whether the following argument is a valid argument: “All employers pay their employees. Anil is an Employer. Therefore Anil pays his employees. **(June2016)**

75. Find whether the following argument is valid or not for which the universe is the set of all triangles. If a triangle has two equal sides, then it is isosceles. If a triangle is isosceles, then it has two equal angles. A certain triangle ABC does not have two equal angles. ∴triangleABC does not have two equal sides. **(Dec 2018, 2017)**

76. Provide the steps and reasons to establish the validity of the argument: $\forall x, [p(x) \to (q(x) \wedge r(x))]$

$$\underline{\forall x, [p(x) \wedge s(x)]}$$

$$\therefore \forall x, [(x) \wedge (x)]$$ **(June 2008)**

77. Verify if the following argument is valid: **(June 2019, 2010, Dec2018, 2016, 2015, 2011)**

$$\forall x, [p(x) \vee q(x)]$$

$$\exists x, \neg p(x)$$
$$\forall x, [\neg q(x) \vee r(x)]$$
$$\underline{\forall x, [s(x) \rightarrow \neg r(x)]}$$
$$\therefore \exists x, \neg s(x);$$

78. Establish the validity of the following argument: **(June 2014, 2011, Dec 2012)**

$$\forall x, [p(x) \vee q(x)]$$
$$\underline{\forall x, [(\neg p(x) \wedge q(x)) \rightarrow r(x)]}$$
$$\therefore \forall x, [\neg r(x) \rightarrow p(x)]$$

79. Check whether following argument is valid**.** **(June 2019, Dec 2014, 2013)**

$\forall x, [p(x) \rightarrow q(x)]$

$\underline{\forall x, [q(x) \rightarrow r(x)]}$

$\therefore \forall x, [p(x) \rightarrow r(x)]$

80. Determine the truth value of each of the following quantified statements for the set of all non-zero integers:

i) $\exists x, \exists y [xy = 1]$ ii) $\forall x, \exists y [xy = 1]$ iii) $\exists x, \exists y [(2x + y = 5) \wedge (x - 3y = -8)]$

iv) $\exists x, \exists y, [(3x - y = 17) \wedge (2x + 4y = 3)]$ v) $\exists x, \forall y, [xy = 1]$ **(Dec 2017, 2016, 2015, June 2010)**

81. Determine the truth value of each of the following quantified statements for the set of all non-zero integers:

i) $\exists x, \exists y, [xy = 2]$ ii) $\forall x, \exists y, [xy = 2]$ iii) $\exists x, \forall y, [xy = 2]$

iv) $\exists x, \exists y, [(3x + y = 8) \wedge (2x - y = 7)]$v) $\exists x, \exists y, [(4x + 2y = 3) \wedge (x - y = 1)]$ **(June 2017)**

82. Give : (i) A direct proof (ii) An indirect proof and (iii) Proof by contradiction, for the following statement:

"If n is an even integer, then (n + 7) is an even integer". **(Dec 2014)**

83. Give a direct proof for each of the following.

(i) For all integers $k$and $l$, if $k$, $l$ are both even, then $k + l$ is even.

(ii) For all integers $k$and $l$, if $k$, $l$ are even, then $k.l$ is even. **(June 2019, 2017, 2015, Dec 2018)**

(iii) For all integers $k$ and $l$, if $k$ and $l$ are both odd, then $k + l$ is even and $kl$ is odd. **(Dec 2017)**

84. For each of the following statements, provide an indirect proof by stating and proving the contra positive:

(i) For all integers $k$and $l$, if $k.l$ is odd, then both $k$ and$l$ are odd.

(ii) For all integers $k$and $l$, if $k + l$ is even, then $k$ and $l$ are both even or both odd. **(June 2008)**

85. Prove that for every integer n, $n^2$ is even if and only if n is even. **(June 2012)**

86. Give a direct proof of the statement "the square of an odd integer is an odd integer". **(June 2019, Dec2013, 2009)**

87. Provide proof by contradiction for "For every integer n, if $n^2$ is odd, then n is odd". **(June 2016)**

88. Prove that for all real numbers x and y, if $x + y > 100$ , then $x > 50$or$y > 50$. **(June 2016, Dec 2011)**

89. Let n be an integer. Prove that n is odd if and only if 7n+8 is odd. **(June 2011)**

90. Give i) direct proof ii) indirect proof iii) proof by contradiction for the following statement:

" if n is an odd integer then n+9 is an even integer". **(Dec 2019, 2016, 2008, June 2013)**

91. Prove that every even integer $k$ with $2 \leq k \leq 26$ can be written as a sum of at most three perfect squares. **(June 2014)**

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# MODULE II
# PROPERTIES OF INTEGERS

## Mathematical Induction:

1. Define the following: i)Well ordering Principle ii) Principle of Mathematical Induction. **(Dec 2015, 2011, June 2015)**
2. By the principle of M.I. prove that, $1^2 + 2^2 + 3^2 + \cdots . + k^2 = \frac{(n+1)(2n+1)}{6}$. **(Dec 2019)**
3. By the principle of M.I. prove that $1 + 2 + 3 + \cdots .. + k = \frac{(n+1)}{2}$. **(June 2019)**
4. Prove that $1^2 + 3^2 + 5^2 + \cdots . + (2k-1)^2 = \frac{n(2n+1)(2n-1)}{3}$ by Mathematical Induction. **(June 2019, 2018, Dec 2016)**
5. Prove by M.I., $1.3 + 2.4 + 3.5 + \cdots . + \text{n}(\text{n}+2) = \frac{n(n+1)(2n+7)}{6}$. **(June 2019, Dec 2018)**
6. By using M.I, Prove that $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{7.11} + \cdots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$ . **(Dec 2015)**
7. Establish the following by Mathematical Induction: $\sum_{i=1}^{n} i2^i = 2 + (k-1)2^{n+1} \; \forall \; k \in z^+$. **(Dec2018, 2014, 2011)**
8. Prove $\sum_{i=1} \frac{1}{(i+1)} = \frac{}{n+1}$ for all $k \in Z$. **(Dec 2014, June 2012)**
9. Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(June 2017, Dec 2013, 2012)**
10. Prove that $2^n > n^2$ for all positive integers $n > 4$.
11. Prove that $2^n < k!$ for all positive integers $n > 3$. **(June 2012)**
12. Prove by M.I. that for every positive integer n, $k! \geq 2^{n-1}$.
13. By M.I Prove that, for any positive integer n, the number $A_n = 5^n + 2 \times 3^{n-1} + 1$ is a multiple of 8.

    (OR) Prove by M.I, for every positive integer $k$, 8 divides $5^n + 2 \times 3^{n-1} + 1$. **(Dec 2017, June 2017)**
14. By M.I Prove that, for any positive integer n, $6^{n+2} + 7^{2n+1}$ is divisible by 43. **(June 2016)**
15. By M.I Prove that, for every integer n, 3 divides $k^3 - k$.
16. State Induction Principle. Prove the following result by Mathematical Induction: "For every positive integer n, 5 divides $k^5 - k$." **(June 2018, 2013, Dec 2017)**
17. For all $k \in Z^+$, Show that if $k \geq 24$ then n can be written as a sum of 5's and/or 7's. **(Dec 2019, 2016, 2012, June 2015)**
18. For all $k \in Z^+$, Show that if $k \geq 14$ then n can be written as a sum of 3's and /or 8's.
19. $H_1 = 1, H_2 = 1 + \frac{1}{2}, \ldots . H_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$ are Harmonic numbers, then Prove that for all $k \in z^+$, $\sum_{i=}^{n} H_i = (k+1) - k$. **(Dec 2014)**

## Principles of Counting:

1. State Sum and Product rule of counting. Give one example for each. **(June 2012)**
2. Maruthi cars come in 4 models, 12 colors, 3 engine types and 2 transmission types. How many distinct Maruthi cars can be manufactured? Of these how many have the same color? **(Dec 2012, June 2008)**
3. A bit is either 0 or 1.A byte is a sequence of 8 bits. Find i) the number of bytes. ii) the number of bytes that begin with 11 and end with 11. iii) the number of bytes that begin with 11and do not end with 11. iv) the number of bytes that begin with 11 or end with 11. **(Dec2014)**
4. Find the number of proper divisors of 441,000.

5. How many four digit numbers can be formed with 10 digits 0,1,2,3,4,5,6,7,8,9 if i) repetitions are allowed ii) Repetitions are not allowed iii) Last digit must be 0 and repetitions are not allowed.
6. Determine the number of 6 digit integers (no leading 0) in which, i) no digit is repeated ii) no digit is repeated and it is even iii) no digit is repeated and it is divisible by 5 **(Dec 2011)**
7. In how many possible ways a student answer a 10 question TRUE/FALSE test? **(Dec 2012)**
8. A computer science professor has 7 different programming books on a book shelf. Three of the books deal with C++and the other four with Java. In how many ways he can arrange these books on shelf i) if there are no restrictions ii) if the languages should alternate iii) if all the C++ books must be next to each other iv) if all the C++ books must be next to each other and all the Java books must be next to each other.
9. How many distinct four digit integers can one make from the digits 1, 3, 3, 7,7and 8?
10. How many positive integers n can we form using the digits 3, 4,4,5,5,6,7 if we want n to exceed 5,000,000? **(Dec 2018, 2017, June 2009, 2008)**
11. How many numbers greater than 1000000 can be formed by using the digits 1, 2, 2, 2, 4, 4, 0? **(June 2018)**
12. In how many ways 3 men and 3 women can be arranged at a round table if i) no restriction is imposed ii) two particular women must not sit together iii) each woman is to be between two men.
13. In how many ways 8 men and 8 women be seated in a row if
    i) any person may sit next to any other.
    ii) men and women must not occupy alternate seats
    iii) generalize this result for $k$ men and $k$ women. **(Dec2014)**
14. A woman has 11 close relatives and she wishes to invite five of them to dinner. In how many ways she can invite them in the following situations. i) There is no restriction on the choice. ii) Two particular persons will not attend separately. iii) Two particular persons will not attend together. **(Dec2019, 2016, 2013, June 2019)**
15. A woman has 11 colleagues in her office of which 8 are men. She wishes some of her colleagues to dinner. Find the number of her choices if she decides to invite i) at least 9 of them. ii) All her women colleagues and sufficient men colleagues to make the number of men and women equal.
16. How many arrangements are there for all the letters of the word 'SOCIOLOGICAL'? In how many of the arrangements i) A and G are adjacent ii) all the vowels are adjacent. **(June 2019, 2016, 2015, 2014, Dec 2016)**
17. How many 9 letter words can be formed using the letters of the word 'DIFFICULT'? **(Dec 2010)**
18. How many arrangements of the letters in MISSISSIPPI have no consecutive S's? **(Dec2014)**
19. A certain question paper contains two parts each containing four questions. How many different ways can a student answer 5 questions by selecting at least two from each part? **(June 2018, Dec 2010)**
20. How many bytes contain i) exactly 2 one's ii) exactly 4 one's iii) exactly 6 one's iv) at least 6 one's.
21. A committee of 12 is to be selected from 10 men and 10 women. In how many ways a selection can be carried out if i) There are no restrictions. ii) There must be 6 men and 6 women.iii) There must be an even number of women. **(Dec 2018)**
22. Find the number of arrangements of all the letters in the word 'TALLAHASSEE'. How many of these arrangements have no adjacent A's. **(Dec 2018, June 2013)**
23. Find the number of permutations of the letters of the word 'MASSASAUGA'. In how many of these, all four A's are together? How many of them begin with S? **(Dec 2019)**
24. From seven consonants and five vowels how many sets consisting of four different consonants and three different vowels can be formed? **(Dec 2013, June 2013)**
25. Prove the following: i) $C(k+1,r) = C(k,r-1) + C(k,r)$ ii) $C(m+k,2) - C(m,2) - C(k,2) = mk$ . **(June 2011)**

26. A certain question paper contains three parts A, B, C with four questions in part A, five questions in part B and six questions in part C. It is required to answer seven questions in selecting at least two questions from each part. In how many ways can a student select his seven questions for answering? **(Dec 2017)**
27. Find the coefficient of $x^9y^3$ in the expansion of $(2x-3y)^{12}$ **(June 2019, 2016, 2015, 2013, Dec 2017, 2016)**
28. Find the coefficient of $x^{12}$ in the expansion of $x^3(1-2x)^{10}$ **(June 2019, Dec 2017)**
29. Find the coefficient of $x^0$ in the expansion of $3(x^2-\frac{2}{x})^{15}$ **(Dec 2014, June 2013)**
30. Determine the coefficient of $xyz^2$ in the expansion of $(2x-y-z)^4$ **(Dec 2019, 2013, 2012, June 2018, 2015, 2011, 2009)**
31. Determine the coefficient of $x^2y^2z^3$ in the expansion of $(3x-2y-4z)^7$ **(Dec 2019, 2014, June 2015, 2014,2008)**
32. Find the coefficient of $x^{11}y^4z^2$in the expansion of $(2x^3-3xy^2+z^2)^6$ **(Dec 2013)**
33. Find the coefficient of $w^3x^2yz^2$ in the expansion of $(2w-x+3y-2z)^8$. **(Dec 2018)**
34. State and explain the meaning of the binomial theorem. Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$ . **(June 2019, 2017, 2016, 2011, Dec 2018, 2016, 2012)**
35. In how many ways 10 identical dimes can be distributed among 5 children if i) there are no restrictions ii) each child gets at least one dime. iii) The oldest child gets at least 2 dimes. **(June 2009)**
36. Find the number of ways of giving 10 identical gift boxes to six persons A, B, C, D, E, F in such a way that the total number of boxes given to A and B together does not exceed 4. **(Dec2012)**
37. In how many ways one can distribute 8 identical marbles in 4 distinct containers so that i) no container is empty ii) the fourth container has an odd number of marbles in it. **(Dec 2019, 2012, June 2016, 2014, 2008)**
38. In how many ways one can distribute 10 identical marbles in 6 distinct containers? **(June 2011, Dec 2010)**
39. In how many ways we can distribute 7 apples, 6 oranges, 6 bananas among 4 so that each child receives at least one apple?
40. In how many ways we can distribute 7 apples and 6 oranges among 4 children so that each child receives at least one apple? **(Dec 2018, 2014)**
41. A bag contains coins of 7 different denominations, with at least one dozen of coins in each denomination. In how many ways can we select a dozen coins from the bag?
42. Find the number of integer solutions of the equation $x_1+x_2+x_3+x_4+x_5=30$ where
$$x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 2, x_5 \geq 0.$$
43. Determine the number of integer solution for $x_1+x_2+x_3+x_4+x_5<40$ where i) $x_i \geq 0, 1 \leq i \leq 5$ ii) $\geq -3, 1 \leq i \leq 5$. **(June 2015)**
44. Find the number of integer solutions of $x_1+x_2+x_3+x_4=32$ where $x_i \geq 0, 1 \leq i \leq 4$. **(Dec 2018)**
45. Find the number of distinct terms in the expansion of $(w+x+y+z)^{12}$ **(June 2017, Dec 2011)**
46. A total amount of Rs.1500 is to be distributed to 3 poor students A, B and C of a class in how many ways distribution can be made in multiples of Rs.100 such that i) every one of these must get at least Rs.300 ii) if A must get at least Rs.500 and B and C must get at least Rs.400 each.
47. A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to those 12 symbols, the transmitter will also send a total of 45 blank spaces between the symbols with at least 3 spaces between each pair of consecutive symbols. In how many ways can the transmitter send such a message? **(June 2019, 2008, Dec 2013)**
48. Assuming PASCAL language is case insensitive, an identifier consists of a single letter followed by upto seven symbols which may be letters or digits (26 letters, 10 digits). There are 36 reserved words. How many distinct identifiers are possible in this version of PASCAL? **(June 2017)**

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# MODULE - III
# RELATIONS & FUNCTIONS

**Functions:**

1. Define a i) function ii) 1 to 1 function iii) onto function. Give an example for each. **(Dec 2013)**
2. If A and B are finite sets with $|A| = m, |B| = k$ and if there are 2187 functions from $A$ to $B$ and $|B| = 3$, then determine $|A|$.
3. If $f$ is a real valued function defined by $(x) = x^2 + 1 \ \ \forall x \in R$. Find the images of the following:
   i) $A_1 = \{2, 3\}$ ii) $A_2 = \{-2, 0, 3\}$ iii) $A_3 = \{0, 1\}$ iv) $A_4 = \{-6, 3\}$. **(Dec 2016, 2015)**
4. If $f : A \to B$ and $B_1, B_2 \subseteq B$, then prove the following: i) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$
   ii) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$ iii) $f^{-1}(\overline{B_1}) = \overline{f^{-1}(B_1)}$. **(June 2017)**
5. Let f: R → R be defined by, $f(x) = \begin{cases} 3x - 5 & \text{if } x > 0 \\ 1 - 3x & \text{if } x \le 0 \end{cases}$. Find $f\left(\frac{5}{3}\right)$, $f^{-1}(0), f^{-1}(1), f^{-1}(-1), f^{-1}(3), f^{-1}(-3), f^{-1}(6)$, $f^{-1}([-6, 5])$ and $f^{-1}([-5, 5])$ **(Dec 2019, 2017, 2015, 2014, June 2019, 2018)**
6. Let $f$: Z → Z be defined by $(a) = a + 1$ for all $a \in Z$. Show that $f$ is a Bijection. **(June 2013)**
7. In each of the following cases sets A and B and a function f from A to B are given. Determine whether f is one to one or onto or both or neither. i) A = {a, b, c}, B = {1, 2, 3, 4}, f = {(a, 1), (b, 1), (c, 3)}
   ii) A = {1, 2, 3}, B = {1, 2, 3, 4, 5}, f = {(1, 1), (2, 3), (3, 4)}
   iii) A = {1, 2, 3, 4}, B = {1, 2, 3, 4} f = {(1, 1), (2, 3), (3, 4), (4, 2)}
8. Let A = R, B = $\{x | x \text{ is real akd } x \ge 0\}$. Is the function $f: A \to B$ defined by $(a) = a^2$ an onto function? Is it a one to one function? **(Dec 2018, June 2016)**
9. Let f, g: $Z^+ \to Z^+$, where $\forall x \in Z^+, f(x) = x + 1$ and $g(x) = \max(1, x - 1)$
   i) What is range of f? ii) Is f onto function? iii) Is f one to one function?
   iv) What is range of g? v) Is g onto function? vi) Is g one to one function? **(June 2017)**
10. For each of the following functions, determine whether it is 1 – 1 and determine its range.
    i): Z → Z, $(x) = 2x + 1$ ii) $f$: Z → Z, $f(x) = x^3 - x$ **(Dec 2011)**
11. Let A= {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6} **(June 2019, 2017)**
    i) How many functions are there from A to B? ii) How many of these are one to one? iii) How many are onto?
    iv) How many functions are there from B to A? v) How many of these are onto? vi) How many are one to one?
12. If $A = \{1, 2, 3, 4, 5\}$ and there are 6720 injective functions $f: A \to B$, what is $|B|$? **(Dec 2018)**
13. Define Stirling's number of 2nd kind. If $|A| = 7, |B| = 4$ find the number of onto functions from A to B. Hence find S(7, 4).
14. Define Stirling's number of 2nd kind and evaluate S (8, 6). **(Dec 2012)**
15. Evaluate S (5, 4). **(June 2018)**
16. Define Stirling's number of 2nd kind. Find the number of ways of distributing 6 objects among 4 identical containers with some container(s) possibly empty. **(Dec 2014, June 2014, 2013)**
17. Let $m, k$ be positive integers with $1 < k \le m$, then prove that $(m + 1, k) = (m, k - 1) + k\, s(m, k)$. **(Dec 2018)**

18. Let $: R \to R$ , $g: R \to R$ be defined by $f(x) = x^2$ and $g(x) = x + 5$. Determine f $\circ\, g$ and g $\circ$ . Show that the composition of two functions is not commutative. **(Dec 2013)**

19. Let f, g, h be functions from Z to Z defined by $f(x) = x - 1, g(x) = 3x, h(x) = \begin{cases} 0, & if\ x\ is\ evek \\ 1, & if\ x\ is\ odd \end{cases}$.Show that f ∘ (g ∘ h) = (f ∘ g) ∘ h. **(Dec 2019, 2016, 2012)**

20. Let f, g, h be functions from R to R defined by $(x) = x + 2, (x) = x - 2, h(x) = 3x$ for all $x \in R$. Find f$\circ\, g$, g$\circ\, f$, f$\circ\, f$ , $h$ ∘ g and $f$ ∘ h. **(June 2018)**

21. If $f: A \to B, g: B \to C, h: C \to D$ are three functions, then Prove that h∘ (g ∘ f) = (h ∘ g) ∘ f. **(June 2014, 2013)**

22. Let $f, g, h : R \to R$ where $(x) = x^2$, $(x) = x + 5$, $h(x) = \sqrt{x^2 + 2}$. Show that (h ∘ g) ∘ f = h ∘ (g ∘ f). **(June 2019)**

23. Let $f$ and $g$ be functions from R to R defined by $(x) = ax + b$ and $(x) = 1 - x + x^2$. If (g ∘ f)(x) $= 9x^2 - 9x + 3$. Determine $a$ and $b$. **(Dec 2017)**

24. Define a function. Prove that the function f: A → B is invertible iff it is one to one and onto. **(Dec 2018, 2012, June 2015)**

25. What is Invertible function? If f: A → B, g: B → C are invertible functions, then prove that g ∘ f: A → C is an invertible function and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. **(Dec 2017, 2015, June 2017)**

26. Consider the function $f: R \to R$ defined by $(x) = 2x + 5$. Let a function $g: R \to R$ be defined by $g(x) = \frac{1}{2}(x - 5)$. Prove that $g$ is an inverse of$f$. **(June 2019, Dec 2018)**

27. Let $A = \{1, 2, 3, 4\}$f and g be functions from A to A given by: $f = \{(1,4)(2,1)(3,2)(4,3)\}$ $g = \{(1,2)(2,3)(3,4)(4,1)\}$. Prove that $f$ and $g$ are inverses of each other. **(June 2016)**

28. Let A = B = R be the set of the real numbers. The functions $f : A \to B$ and $g : B \to A$ be defined by $f(x) = 2x^3 - 1$, $\forall x \in A$; $g(y) = \{\frac{1}{2}(y + 1)\}^{1/3} \forall y \in B$. Show that each of $f$ and $g$ is the inverse of the other. **(Dec 2016)**

29. Let A = B = C = R, $f: A \to B, (a) = 2a + 1$; $g: B \to C, (b) = b/2$. Compute g$\circ\, f$ and show that it is invertible. **(Dec 2011)**

30. If $f: R \to R$ defined by $(x) = x^3$, determine whether $f$ is invertible. If so, determine $f^{-1}$. **(Dec 2018)**

31. State the pigeonhole principle. Let ABC be an equilateral triangle with AB = 1. Show that if we select five points in the interior of this triangle, there must be at least two whose distance apart is less than$1/2$. **(June 2019, Dec 2016)**

32. State the pigeonhole principle. Let ABC be an equilateral triangle with AB = 1. Show that if we select ten points in the interior of this triangle, there must be at least two whose distance apart is less than $1/3$. **(Dec 2011)**

33. Show that if $k + 1$ numbers are chosen from 1 to $2k$ then at least one pair add to $2k + 1$. **(Dec 2012)**

34. Show that if any 14 integers are selected from the set $S = \{1, 2, 3, \dots \dots, 25\}$ there are atleast two integers whose sum is 26.

35. State Pigeon hole principle. An office employs 13 clerks. Show that at least 2 of them will have birthdays during the same month of the year. **(Dec 2013)**

36. State Pigeon hole principle. Prove that in any set of 29 persons; at least 5 persons have been born on the same day of the week. **(Dec 2015)**

37. Prove that if 30 dictionaries in a library contain a total of 61,327 pages, then at least one of the dictionaries must have at least 2045 pages. **(Dec 2017, 2014)**
38. State Pigeon hole principle. Show that if any seven numbers from 1 to 12 are chosen, then two of them will add to 13. **(Dec 2018)**
39. Shirts numbered consecutively from 1 to 20are worn by 20 students of a class. When any 3 of these students are chosen to be a debating team from the class, the sum of their shirt numbers is used as the code number of the team. Show that if any eight of the 20 are selected, then from these eight, we may form at least two different teams having the same code number. **(June 2015)**

**Relations**:

40. Find $A \times B, A \times (B \cup C), (A \cap B) \times C$ for A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {2, 4, 6}.
41. For any sets A and B prove that, i) A × (B∩C) = (A×B) ∩ (A×C) ii)$A \times (B \cup C) = (A \times B) \cup (A \times C)$.
iii) $A \times (B - C) = (A \times B) - (A \times C)$. **(Dec 2019, 2013, 2011)**
42. Define Cartesian product of two sets. For $A, B, C \subseteq U$, prove that i) A × (B – C) = (A×B) – (A×C).
ii) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ **(Dec 2018, June 2016, 2013)**
43. Suppose $A, B, C \subseteq Z \times Z$ with $A = \{(x, y)|y = 5x - 1\}$; $B = \{(x, y)|y = 6x\}$; $C = \{(x, y)|3x - y = -7\}$. Find
i) $A \cap B$ii) $B \cap C$ iii) $\overline{\bar{A} \cup \bar{C}}$ iv) $\bar{B} \cup \bar{C}$. **(June 2014)**
44. Let A= {1, 2, 3} and B= {2, 4, 5}. Determine the following: i) $|A \times B|$
ii) Number of relations from A to B
iii) Number of binary relations on A iv) Number of relations from A to B that contain (1, 2) and (1, 5)
v) Number of relations from A to B that contain exactly 5 ordered pairs
vi) Number of binary relations on A that contains at least 7 ordered pairs. **(Dec 2016, June 2015)**
45. Define a matrix of a relation and digraph of a relation with example.
46. Let A = {1, 2, 3, 4} and let $R$ be the relation on $A$ defined by "$xRy$ iff $x$ divides $y$",
i) Write down R as a set of ordered pairs. ii) Write down the relation matrix $M(R)$ and draw the digraph of $R$
iii) Determine the in-degrees and out -degrees of the vertices in the digraph.
47. Let $A = \{1, 2, 3, 4, 6\}$ and $R$ be a relation on $A$ defined by $aRb$ if and only if $a$ is a multiple of $b$. i) Write $R$ as a set of ordered pairs ii) Represent $R$ as a matrix iii) Draw the digraph of $R$ iv)Determine the in-degrees and out-degrees of the vertices in the digraph. **(Dec 2019, 2015, June 2015, 2013)**
48. Let A = {1, 2, 3, 4}. Let R be a relation on A defined by $xRy$ iff $x | y$ and $y = 2x$. Write i) R as a set of ordered pairs
ii) Draw the digraph of R iii) Determine in – degree and out – degree of every vertex of digraph.
49. Let R = {(1, 2), (1, 3), (2, 4), (3, 2)} be a relation on A = {1, 2, 3, 4}.Find M(R) and $[M(R)]^2$. Hence find $R^2$.
50. If A = {1, 2, 3, 4} and R is a relation on A defined by R = {(1, 2), (1, 3), (2, 4), (3, 2), (3,3), (3, 4)}. Find $R^2$, $R^3$
Draw their digraphs. **(June 2016)**

51. Let R = {(1, 1),(1, 2), (2, 3), (3, 3), (3, 4)} be a relation on A = {1, 2, 3, 4}. i)Draw the digraph of R. ii) Obtain $R^2$, $R^3$ and draw the digraph of $R^2$,$R^3$iii) FindM($R^2$), M($R^3$) **(Dec 2011)**
52. For A = {1, 2, 3, 4}, let R and S be the relations on A defined by R={(1, 2), (1, 3), (2, 4), (4, 4)} and S = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4)}. Find R ∘ S, S ∘ R, $R^2$, $S^2$ and write down their matrices. **(June 2019)**
53. For $A = \{1, 2, 3, 4\}$, let $R = \{(1, 1)(1, 2)(2, 3)(3, 3)(3, 4)\}$ be a relation on $A$. Draw the directed graph $G$ on $A$ that is associated with $R$. Do likewise for $R^2$, $R^3$. **(Dec 2018)**
54. Let A = {1, 2, 3, 4}, B = {w, x, y, z} and C={p, q, r, s}. Consider $R_1 = \{(1, x), (2, \mathrm{w}), (3, z)]$, a relation from A to B, $R_2 = \{(\mathrm{w}, p), (z, q), (y, s), (x, p)\}$, a relation from B to C.
   i) What is the composite relation $R_1 \circ R_2$ from A to C
   ii) Write the relation matrices $M(R_1)$, $M(R_2)$ $akd$ $M(R_1 \circ R_2)$
   iii) Verify$M(R_1) \cdot M(R_2) = M(R_1 \circ R_2)$ **(June 2012)**
55. Let A = {1, 2, 3, 4}, B = {w, x, y, z} and C = {5, 6, 7}. Let $R_1$ be a relation from A to B defined by $R_1 = \{(1, x), (2, x), (3, y), (3, z)\}$ and $R_2$ $akd$ $R_3$ be relations from B to C defined by $R_2 = \{(\mathrm{w}, 5), (x, 6)\}$, $R_3 = \{(\mathrm{w}, 5), (\mathrm{w}, 6)\}$. Find $R_1 \circ R_2$and $R_1 \circ R_3$. **(June 2014, Dec 2012)**
56. Let R = {(1, 2), (3, 4), (2, 2)} and S = {(4, 2), (2, 5), (3, 1), (1, 3)} be relations on the set A = {1, 2, 3, 4, 5}. Find the following: i) R ∘ (R ∘ S) ii) R ∘ (S ∘ R) iii) S ∘ (R ∘ S) iv) S ∘ (S ∘ R) .
57. Define the following with an example each: i) Reflexive ii) Irreflexive iii) Antisymmetric iv) Transitive v) Partition set.
58. Given a set A with $|A| = k$ and a relation R on A, let M denote the relation matrix for R, then Prove that: i) R is symmetric if and only if $M = M^1$. ii) R is transitive if and only if$M.M = M^2 \le M$. **(Dec 2014)**
59. Define equivalence relation and equivalence class with one example. **(Dec 2015)**
60. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}. On this set define the relation R by $(x, y) \in R$ if and only if $(x - y)$ is a multiple of 5. Verify that R is an equivalence relation. **(Dec 2012)**
61. For each of the following relations, determine if the relation R is reflexive, symmetric, anti-symmetric or transitive: i) On the set of all lines in the plane$l_1 R l_2$ if line $l_1$ is perpendicular to line $l_2$ii) On Z, $xRy$ if $x - y$ is even. **(Dec 2018, 2011)**
62. Find the number of equivalence relations that can be defined on a finite set A with $|A| = 6$. **(June 2016, 2014)**
63. LetA= $\{1, 2, 3, 4\}$, R= $\{(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)\}$ be the relation on A. Determine whether the relation R is reflexive, irreflexive, symmetric, antisymmetric or transitive. **(June 2017)**
64. LetA= $\{1, 2, 3, 4\}$, and let R be the relation defined by $R = \{(x, y)|x, y \in A, \ x \le y\}$. Determine whether R is reflexive, symmetric, antisymmetric or transitive. **(Dec 2013)**
65. Define equivalence relation R on a set A. For a fixed integer $k > 1$, prove that the relation "Congruent modulo $k$" is an equivalence relation on the set of all integers Z. **(June 2015)**
66. Define partition set. List all partitions ofP = {1, 2, 3}.
67. For $A = \{a, b, c, d, x, y, z\}$.Define equivalence relation. Hence find equivalence class. Also find partition of A.

68. Define a partition of a set. Prove that the relation R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 3), (3, 3), (4, 4)}is an equivalence relation on set A = {1, 2, 3, 4}. Also determine the partition induced by R on A. **(Dec 2018, 2013)**

69. Let A = {a, b, c, d, e}. Consider partition P = {{a, b}, {c, d}, {e}} of A. Find the equivalence relation inducing this partition. **(June 2018)**

70. Let A = {1, 2, 3, 4, 5, 6, 7} and R be the equivalence relation on A that induces the partition $A = \{1, 2\} \cup \{3\} \cup \{4, 5, 7\} \cup \{6\}$. Find R. **(June 2016)**

71. Let S be the set of all non-zero integers and $A = S \times S$ on $A$. Define the relation $R$ by $(a, b)R(c, d)$ if and only if $ad = bc$. Show that 'R' is an equivalence relation. **(June 2018)**

72. Let A = {1, 2, 3, 4, 5} and define a relation R on A×A by $(x_1, y_1)$ R $(x_2, y_2)$ iff $x_1 + y_1 = {}_2 + y_2$.

    i) Verify that R is an equivalence relation on A.

    ii) Determine the equivalence classes [(1, 3)], [(2, 4)], [(1, 2)], [(2, 5)] and [(1, 1)].

    iii) Determine the partition of $A \times A$ induced by R. **(Dec 2019, 2018, 2017, 2016, 2015, 2014, June 2019, 2012)**

73. Define an equivalence relation. Let N be the set of all natural numbers. On $N \times N$, the relation R is defined as $(a, b)\ R\ (c, d)$ if and only if $a + d = b + c$. Show that R is an equivalence relation. Find the equivalence class of the element (2, 5). **(June 2013)**

74. Let R be an equivalence relation on set A and $a, b \in A$. Then prove the following are equivalent:

    i) $a \in [a]$ ii) $a\ R\ b$ iff$[a] = [b]$ iii) If $[a] \cap [b] \neq \emptyset$, then$[a] = [b]$ **(Dec 2018)**

75. Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 2), (2, 4), (1, 3), (3, 3), (3, 4), (1, 4), (4, 4)}. Draw the Hasse diagram for the Poset (A, R).

76. Define Partial Order. If R is a relation on A = {1, 2, 3, 4} defined by $xRy$iff $x|y$. Prove that (A, R) is a Poset. Draw its Hasse diagram. **(Dec 2013)**

77. Let A = {1, 2, 3, 4, 6, 12, 18}. On A, define the relation R by xRy iff $x|y$. Prove that R is a partial order on A. Draw the Hasse diagram for this relation.

78. Let A = {1, 2, 3, 4, 6, 12}. On A define the relation R by aRb iff a divides b. i) Prove that R is a partial order on A

    ii) Draw the Hasse diagram iii) Write down the matrix of relation. **(Dec 2017)**

79. Let A = {1, 2, 3, 6, 9, 12, 18}. On A, define the relation R by aRb iff a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation. **(June 2012)**

80. Let A = {1, 2, 3, 6, 9, 18}and define $R$on $A$, by $xRy$ if $x|y$ . Draw the Hasse diagram for the POSET $(A, R)$. Also write the matrix of relation. **(June 2019, Dec 2018)**

81. Let A = {1, 2, 3, 4, 6, 8, 12} and R be the partial ordering on A defined by aRb iff a divides b, then

    i) Draw the Hasse diagram of the Poset (A, R) ii) Determine the relation matrix for R. **(Dec 2014)**

82. Draw the digraph and Hasse diagram representing the positive divisors of 36. **(Dec 2019, June 2018, 2016)**

83. Draw the digraph and Hasse diagram representing the positive divisors of 72. **(June 2015, Dec 2011)**

84. Let $A = \{a, b, c\}, B = (A)$ where $(A)$ is the power set of $A$. Let $R$ be a subset relation on $A$. Show that $(B, R)$ is a POSET and draw its Hasse diagram. **(Dec 2018)**

85. Determine the matrix of the partial order whose Hasse diagram is as shown in fig(i).
86. For A = {a, b, c, d, e}, the Hasse diagram for the Poset (A, R) is as shown in fig (ii). Determine the relation matrix for R and construct the digraph for R. **(Dec 2016, June 2014)**
87. Let (A,R) be a Poset with B⊆A. Define i)lower bound of B ii) upper bound of B iii) least upper bound of B iv) greatest lower bound of B.
88. Let (A,R) be a Poset. Define i) maximal elementii) minimal element iii) greatest element iv) least element.
89. Let A = {2, 3, 4, 6, 8, 12, 24} and let ≤ denotes the partial order of divisibility that is $x \leq y$ means $x|$ . Let B = {4, 6, 12}. Determine i) All upper bounds of B ii) All lower bounds of B iii) Least upper bound of B iv) Greatest lower bound of B. **(Dec 2013)**
90. For the Hasse diagram given in fig (iii), write i) maximal ii) minimal iii) greatest and iv) least element (s). **(June 2017)**
91. Show that the set of positive divisors of 36 is a POSET and draw its Hasse diagram. Hence find its i) least element ii) greatest element. **(June 2019, Dec 2018)**
92. Consider the Hasse diagram of a Poset (A, R) given in fig (iv). If B = {c, d, e}, find i) all upper bounds of B ii) all lower bounds of B iii) the least upper bound of B iv) the greatest lower bound of B. **(Dec 2016, 2015, 2012)**
93. Consider the Poset whose Hasse diagram is given in fig (v). Consider B = {3, 4, 5}. Find i) all upper bounds of B ii) all lower bounds of B iii) the least upper bound of B iv) the greatest lower bound of B v) Is this a Lattice? **(June 2019, Dec 2017)**



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# MODULE - IV
# PRINCIPLES OF COUNTING - II

1. Determine the number of positive integers $k$ such that $1 \leq k \leq 100$ and $k$ is not divisible by 2, 3 or 5. **(Dec 2018, 2016, June 2017, 2014, 2008)**
2. Find the number of integers between 1 to 10000 (inclusive), which are divisible by none of 5, 6 or 8. **(June 2015, Dec 2014)**
3. How many integers between 1 and 300(inclusive) are divisible by (i) at least one of 5,6,8 ii) exactly one of 5,6,8 iii) none of 5,6,8? **(Dec 2019, 2014, 2013, 2012, June 2013, 2009)**
4. Determine the number of integers between 1 and 300(inclusive) which are divisible by (i) at least two of 5,6,8 ii) exactly two of 5,6,8 **(June 2019, Dec 2018)**
5. There are 30 students in a hostel. In that 15 study history, 8 study economics and 6 study geography it is known that 3 students study all these subjects. Show that there are 7 or more students studies none of these subjects. **(Dec 2018, 2017, June 2012, 2011)**
6. In a survey of 260 college students, the following data were obtained. 64 had taken Mathematics course, 94 had taken CS course, 58 had taken EC course, 28 had taken both Mathematics and EC course, 26 had taken both Mathematics and CS course, 22 had taken CS and EC course and 14 had taken all these three types of course. Determine how many of these students had taken none of the three subjects. **(June 2018)**
7. In how many ways can 4 a's, 3 b's and 2 c's be arranged so that all identical letters are not in a single block? **(Dec 2011)**
8. In how many ways can 3 x's 3y's and 3 z's be arranged so that no consecutive triple of the same letter arrives? **(Dec 2012)**
9. In how many ways the 26 letters of English alphabet are permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(Dec 2019, 2018, 2017, 2016, 2013, June 2019, 2016)**
10. Find the number of permutations of a, b, c… x, y, z in which none of the patterns spin, game, path or net occurs. **(June 2013)**
11. In how many ways one can distribute 10 distinct prizes among 4 students with i) exactly 2 students getting nothing ii) at least 2 students getting nothing? **(Dec 2012)**
12. In how many ways one can arrange the letters of the word **CORRESPONDENTS** so that there are i) exactly 2 pairs of consecutive identical letters? ii) at least 3 pairs of consecutive identical letters? iii) no pair of consecutive identical letters? **(June 2017, 2011, Dec 2016)**
13. Determine in how many ways the letters of the word **ARRANGEMENT** can be arranged so that there are exactly two pairs of consecutive identical letters. **(June 2015, Dec 2014, 2010)**
14. In how many ways one can arrange the integers $1, 2, 3, 4 \ldots 10$ in a line such that no even integer is in its natural place? **(Dec 2018, 2012, 2010, June 2016, 2013)**
15. Define Derangement. Find the number of derangements of the numbers 1, 2, 3 and 4. List all the derangements. **(June 2019, 2008, Dec 2018, 2014, 2013)**
16. Define Derangement. In how many ways can each of 10 people select a left glove and a right glove out of a total of 10 pairs of gloves so that no person selects a matching pair of glove? **(June 2013)**
17. Define derangement. Find the number of derangements of the numbers 1,2,3,4 and 5. List those derangements where the first 3 numbers are 1, 2, and 3 in some order. **(Dec 2018, 2011)**
18. How many permutations of 1,2,3,4,5,6,7, 8 are not derangements? **(June 2018)**

19. Define derangement. In how many ways one can arrange the numbers 1,2,3,4, ... . , 10 such that 1 is not in the 1st place, 2 is not in the 2nd place, 3 is not in the 3rd place,…., 10 is not in the 10th place? **(June 2011)**
20. For the positive integers $1, 2, 3, \dots \dots, k$ there are 11660 derangements where 1,2, 3, 4,5 appear in first five positions. What is the value of $k$? **(Dec 2013)**
21. Find the number of derangements of the integers from 1 to $2k$ satisfying the condition that the elements in first $k$ places are: i) $1, 2, 3, \dots \dots, k$ in some order ii) $k+1, k+2, k+3, \dots \dots, 2k$ in some order. **(June 2012)**
22. Define derangement. There are 8 letters to 8 different people to be placed in 8 different addressed envelopes. Find the number of ways of doing this so that at least one letter goes to the right person. **(Dec 2019, June 2019, 2014, 2009)**
23. Seven books distributed to seven students for reading. The books are collected and redistributed. In how many ways can the 2 distributions be made such that no student will get the same book in both distributions? **(June 2008)**
24. Sheela has seven books to review for ABC Company, so she hires7 people to review them. She wants two reviews per book, so the first week she gives each person one book to read and redistributes the books at the start of the second week. In how many ways can she make the two distributions so that she gets two reviews of each book? **(Dec 2012)**
25. Describe the expansion formula for Rook Polynomials. Find the rook polynomial for the $3 \times 3$ board using expansion formula. **(Dec 2018, 2013, June 2018, 2017, 2014, 2012, 2009)**
26. Find the rook polynomial for the chess board as shown in the figure (i). **(Dec 2011)**
27. Find the rook polynomial for the chess board as shown in the figure (ii). **(June 2008)**
28. Find the rook polynomial for the chess board as shown in the figure (iii). **(June 2015)**
29. Find the rook polynomial for the chess board as shown in the figure (iv). **(June 2013)**
30. Find the rook polynomial for the chess board as shown in the figure (v). **(Dec 2018)**

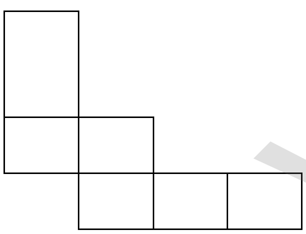


Fig (i)

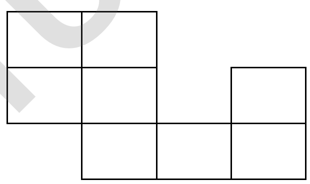


Fig (ii)

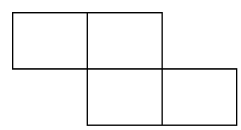
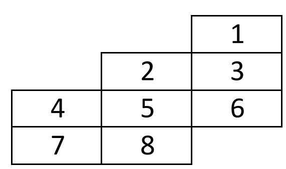


Fig (iii)

Fig (vi)



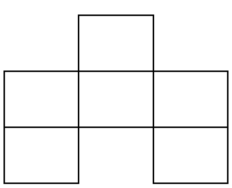



Fig (iv)

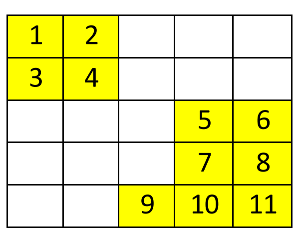



Fig (v)

31. By using the expansion formula, obtain the rook polynomial for the board C given in fig (vi). **(Dec 2019)**
32. Five teachers $T_1, T_2, T_3, T_4, T_5$ are to be made class teachers for five classes $C_1, C_2, C_3, C_4, C_5$, one teacher for each class. $T_1$and $T_2$ do not wish to become class teachers for $C_1$ $or$ $C_2$. $T_3$ and $T_4$ for $C_4$ $or$ $C_5$ and $T_5$ for $C_3$or $C_4$or $C_5$. In how many ways can the teachers be assigned to work without displeasing any teacher? **(June 2019, 2013, Dec 2017, 2014)**
33. A girl student has sarees of 5 different colors: blue, green, white, red and yellow. On Monday she does not wear green; on Tuesday, blue or red; on Wednesday, blue or green; on Thursday, red or yellow; on Friday red. In how many ways she can dress without repeating a color during a week from Monday to Friday?**(Dec 2016)**
34. Four persons $P_1, P_2, P_3, P_4$ who arrive late for a dinner party find that only one chair at each of the five tables $T_1, T_2, T_3, T_4, T_5$ is vacant. $P_1$will not sit at $T_1 or\ T_2$.$P_2$will not sit at $T_2$. $P_3$will not sit at $T_3$ or $T_4$ and $P_4$ will not sit at$T_4$or$T_5$. Find the number of ways they occupy the vacant chairs. **(June 2019, Dec 2013)**
35. An apple, a banana, a mangoand an orange are to be distributed among four boys $B_1, B_2, B_3,$and$B_4$. The boys $B_1 akd\ B_2$ do not wish to have the apple. The boy $B_3$ does not want banana or mango and $B_4$refuses orange. In how many ways the distribution is made so that no one is displeased? **(Dec2019, 2018,2016, 2010, June 2018, 2016)**

**RECURRENCE RELATIONS:**

36. Solve the recurrence relation: $6a_n - 7a_{n-1} = 0, k \geq 1$, given $a_3 = 343$. **(Dec 2014)**
37. Solve therecurrence relation: $a_n - 3a_{n-1} = 5 \times 3^n, k \geq 1, a_0 = 2$. **(Dec 2017)**
38. Find the recurrence relation and the initial condition for the sequence $2, 10, 50, 250$, ..... Hence find the general term of the sequence. **(Dec 2013)**
39. Find the recurrence relation and the initial conditions for the sequence $0, 2, 6, 12, 20, 30, 42$ .... Hence find the general term of the sequence. **(June 2019, 2017)**
40. The number of virus affected files in system is 1000(to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(June 2019, 2018, 2016, 2015, 2014, 2013, Dec 2018,2016, 2014, 2013)**
41. A bank pays 6% interest compound quarterly. If Laura invests $100, then how many months must she wait for her money to double? **(June 2015)**
42. Solve the recurrence relation: $a_n - 6a_{n-1} + 9a_{n-2} = 0, k \geq 2$. **(Dec 2017)**
43. Solve therecurrence relation: $a_n + a_{n-1} - 6a_{n-2} = 0, k \geq 2, a_0 = -1, a_1 = 8$. **(June 2018, Dec 2013)**
44. Solve therecurrence relation $a_n = 2(a_{n-1} - a_{n-2})$, given $a_0 = 1, a_1 = 2$and $k \geq 2$. **(June 2019)**
45. Solve therecurrence relation: $a_{n+2} - 3a_{n+1} + 2a_n = 0,\ a_0 = 1, a_1 = 6$. **(June 2012)**
46. Solve the recurrence relation: $C_n = 3C_{n-1} - 2C_{n-2}, for\ \ k \geq 2,$ given $C_1 = 5, C_2 = 3$.**(Dec 2016, June 2014)**
47. If $a_0 = 0, a_1 = 1, a_2 = 4$ and $a_3 = 37$ satisfy the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$ for $k \geq 0$, determine the constants $b$ and $c$ and then solve the relation for $a_n$. **(Dec 2019, June 2019)**
48. Solve therecurrence relation: $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n,\ \ k \geq 0,\ \ a_0 = 0, a_1 = 1, a_2 = 2$. **(Dec 2018)**
49. Solve therecurrence relation $F_{n+2} = F_{n+1} + F_n$, given $F_0 = 0, F_1 = 1$and $k \geq 0$. **(Dec 2018, 2012, June 2015,2012, 2011)**
50. Solve the recurrence relation: $D_n = bD_{n-1} - b^2 D_{n-2}$ , $\geq 3$, given $D_1 = b > 0$ and $D_2 = 0$. **(June 2017)**
51. Find the General solution of the equation $(k) + 3(k-1) - 4S(k-2) = 0$. **(June 2017)**
52. Let $a_n$ denote the number of n-letter sequences that can be formed using the letters A, B and C such that non-terminal A has to be immediately followed by a B. Find the recurrence relation for $a_n$and solve it. **(Dec 2017)**