Normal forms:-

There are two types of normal forms

i) CNF (Chomsky normal form)

ii) GNF (Gribach normal form)

i) CNF (Chomsky normal form):-

Let $G=(V,T,P,S)$ be a CFG. The grammar G is said to be in CNF if all the productions are of the form

$A \rightarrow BC$

$A \rightarrow a$, where A, B, C are variables and 'a' is a terminal. To get the CFG into CNF form the prerequisites are

i) eliminate the useless symbols.

ii) eliminate the ϵ-productions

iii) eliminate the unit productions.

i) Elimination of useless symbols:-

'X' is a useful grammar for $G=(V,T,P,S)$, it should satisfy the following two conditions

i) X should generate some terminal

ii) X is reachable from start symbol

Eliminate useless symbols from the following

1. $S \rightarrow aAa|aBC$

$A \rightarrow aS|bb$

$B \rightarrow aBa|b$

$C \rightarrow abb|DD$

$D \rightarrow aDa$

D is not generating a terminal. So, it is a useless symbol, eliminate D.

Eliminate D:-

$S \rightarrow aAa|aBC$

$A \rightarrow aS$

$B \rightarrow aBa|bb$

$C \rightarrow abb$

2. S → aA | bB
A → aA | a
B → bB
~~D → ab | Ea~~
E → ac | d
D is not reachable from start symbol, so eliminate D
Eliminate D:-
S → aA | bB
A → aA | a
B → bB
~~E → ac | d~~
E is dependent on D and is not reachable from start symbol, so eliminate E
S → aA | ~~bB~~
A → aA | a
~~B → bB~~
B is not generating a terminal, so eliminate B

S → aA
A → aA | a

3. S → aA | a | Bb | ~~∈~~
A → aB
B → a | Aa
~~C → cCD~~
D → ddd
C is not generating a terminal, so eliminate C
S → aA | a | Bb
A → aB
B → a | Aa
~~D → ddd~~
Eliminate D as D is dependent on C and is not reachable from start symbol

S → aA | a | Bb
A → aB
B → a | Aa

2) Eliminating ϵ - productions:-
A production of the form $A \rightarrow \epsilon$ is undesireable in CFG unless an empty string is derived from the start symbol (i.e, don't eliminate from start symbol)

Example:- $S \rightarrow Aab$
$A \rightarrow aA|\epsilon$

Eliminate ϵ -productions

$S \rightarrow Aab|ab$
$A \rightarrow aA|a$

Eliminate ϵ productions from the following grammar

1. $S \rightarrow AB$
$A \rightarrow aB|\epsilon$
$B \rightarrow bB|\epsilon$

Eliminate ϵ - productions

$S \rightarrow AB|A|B|\epsilon$
$A \rightarrow aB|a$
$B \rightarrow bB|b$

2. $S \rightarrow ABCa|bD$
$A \rightarrow BC|b$
$B \rightarrow b|\epsilon$
$C \rightarrow c|\epsilon$
$D \rightarrow d$

Eliminate ϵ - productions

$S \rightarrow ABCa|bD|a|ABa|ACa|BCa|Ca|Aa|Ba$
$A \rightarrow BC|b|C|B$
$B \rightarrow b$
$C \rightarrow C$
$D \rightarrow d$

3. $S \rightarrow ABC$
$A \rightarrow BC|a$
$B \rightarrow bAC|\epsilon$
$C \rightarrow CAB|\epsilon$

Eliminate ϵ -productions

$S \longrightarrow ABC|A|B|C|AB|AC|BC|\epsilon$

$A \longrightarrow BC|a|B|C$

$B \longrightarrow bAC|bA|bC|b$

$C \longrightarrow CAB|C|A|B|CA|CB|AB$

4. $S \longrightarrow BAAB$

$A \longrightarrow 0A2|2A0|\epsilon$

$B \longrightarrow AB|1B|\epsilon$

Eliminate ϵ-productions

$S \longrightarrow BAAB|AAB|BAB|BAA|AA|BB|A|B|\epsilon|BA|AB$

$A \longrightarrow 0A2|2A0|02|20$

$B \longrightarrow AB|1B|A|B|1$

3) Eliminating unit productions:-

Consider the production $A \longrightarrow B$, the LHS of the production and RHS of the productions contain only one variable, such productions are called unit productions.

Eliminate unit productions from the following grammar

1. $A \longrightarrow B$

$B \longrightarrow aB|b$

$A \longrightarrow aB|b$

$B \longrightarrow aB|b$

2. $S \longrightarrow AB$

$A \longrightarrow a$

$B \longrightarrow C|b$

$C \longrightarrow D$

$D \longrightarrow E|bC$

$E \longrightarrow d|Ab$

$S \longrightarrow AB$

$A \longrightarrow a$

$B \longrightarrow d|Ab|bC|b$

$C \longrightarrow d|Ab|bC$

$D \longrightarrow d|Ab|bC$

$E \longrightarrow d|Ab$

3. $S \longrightarrow A0 \mid B$
$B \longrightarrow A \mid 11$
$A \longrightarrow 0 \mid 12 \mid B$

$S \longrightarrow A0 \mid 0 \mid 12 \mid 11$
$B \longrightarrow 0 \mid 12 \mid 11$
$A \longrightarrow 0 \mid 12 \mid 11$

Eliminate useless symbol and unit production

1. $S \longrightarrow Aa \mid B \mid Ca$
$B \longrightarrow aB \mid b$
$C \longrightarrow Db \mid D$
$D \longrightarrow E \mid d$
$E \longrightarrow ab$

Eliminate A as A is not generating terminal, it is useless symbol

$S \longrightarrow B \mid Ca$
$B \longrightarrow aB \mid b$
$C \longrightarrow Db \mid D$
$D \longrightarrow E \mid d$
$E \longrightarrow ab$

Eliminate unit productions

$S \longrightarrow aB \mid b \mid Ca$
$B \longrightarrow aB \mid b$
$C \longrightarrow Db \mid ab \mid d$
$D \longrightarrow ab \mid d$
$E \longrightarrow ab$

2. $S \longrightarrow aAa \mid bBb \mid \epsilon$
$A \longrightarrow C \mid a$
$B \longrightarrow C \mid b$
$C \longrightarrow CDE \mid \epsilon$
$D \longrightarrow A \mid B \mid ab$

Eliminate E as E is not generating terminal. It is useless symbol

$S \longrightarrow aAa|bBb|\epsilon$

$A \longrightarrow C|a$

$B \longrightarrow C|b$

$C \longrightarrow \epsilon$

$D \longrightarrow A|B|ab$

Eliminate D as it is not reachable from start state

$S \longrightarrow aAa|bBb|\epsilon$

$A \longrightarrow C|a$

$B \longrightarrow C|b$

$C \longrightarrow \epsilon$

Eliminate ϵ productions

$S \longrightarrow aAa|bBb|\epsilon|aa|bb$

$A \longrightarrow a$

$B \longrightarrow b$

Convert

1. $S \longrightarrow AB|a$

$A \longrightarrow aab$

$B \longrightarrow Ac$

No useless, ϵ-production and unit productions.

$S \longrightarrow AB|a$

$A \longrightarrow A_0A_0B_0$

$B \longrightarrow AC_0$

$A_0 \longrightarrow a$

$B_0 \longrightarrow b$

$C_0 \longrightarrow c$

$S \longrightarrow AB|a$

$A \longrightarrow A_1B_0$

$B \longrightarrow AC_0$

$A_0 \longrightarrow a$

$B_0 \longrightarrow b$

$C_0 \longrightarrow c$

$A_1 \longrightarrow A_0A_0$

2. $S \longrightarrow AB1 \mid 0$
$A \longrightarrow 00A \mid 11$
$B \longrightarrow 1A1$

No useless, ϵ and unit productions

$S \longrightarrow ABB_0 \mid 0$
$A \longrightarrow A_0A_0A \mid B_0B_0$
$B \longrightarrow B_0AB_0$
$A_0 \longrightarrow 0$
$B_0 \longrightarrow 1$

$S \longrightarrow AB_1 \mid 0$
$A \longrightarrow A_1A \mid B_0B_0$
$B \longrightarrow B_2B_0$
$A_0 \longrightarrow 0$
$B_0 \longrightarrow 1$
$A_1 \longrightarrow A_0A_0$
$B_1 \longrightarrow BB_0$
$B_2 \longrightarrow B_0A$

3. $S \longrightarrow aSb \mid ab \mid Aa$
$A \longrightarrow aab$

No useless, ϵ and unit productions

$S \longrightarrow A_0SB_0 \mid A_0B_0 \mid AA_0$
$A \longrightarrow A_0A_0B_0$
$A_0 \longrightarrow a$
$B_0 \longrightarrow b$

$S \longrightarrow A_1B_0 \mid A_0B_0 \mid AA_0$
$A \longrightarrow A_2B_0$
$A_0 \longrightarrow a$
$B_0 \longrightarrow b$
$A_1 \longrightarrow A_0S$
$A_2 \longrightarrow A_0A_0$

4. $S \longrightarrow aB \mid bA$

$A \longrightarrow a \mid aS \mid bAA$

$B \longrightarrow b \mid aS \mid aBB$

No useless, ϵ and unit productions

$S \longrightarrow A_0B \mid B_0A$

$A \longrightarrow a \mid A_0S \mid B_0AA$

$B \longrightarrow b \mid A_0S \mid A_0BB$

$A_0 \longrightarrow a$

$B_0 \longrightarrow b$

$S \longrightarrow A_0B \mid B_0A$

$A \longrightarrow a \mid A_0S \mid B_1A$

$B \longrightarrow b \mid A_0S \mid A_1B$

$A_0 \longrightarrow a$

$B_0 \longrightarrow b$

$B_1 \longrightarrow B_0A$

$A_1 \longrightarrow A_0B$

5. $S \longrightarrow AaA \mid CA \mid BaB$

$A \longrightarrow aaBa \mid CDa \mid aa \mid DC$

$B \longrightarrow bB \mid bAB \mid bb \mid aS$

$C \longrightarrow Ca \mid bC \mid D$

$D \longrightarrow bD \mid \epsilon$

Eliminate ϵ -productions:-

$S \longrightarrow AaA \mid CA \mid BaB$

$A \longrightarrow aaBa \mid CDa \mid aa \mid DC \mid Ca \mid Da \mid C \mid D$

$B \longrightarrow bB \mid bAB \mid bb \mid aS$

$C \longrightarrow Ca \mid bC \mid a \mid b$

$D \longrightarrow bD \mid b$

Eliminating unit productions:-

$S \longrightarrow AaA \mid CA \mid BaB$

$A \longrightarrow aaBa \mid CDa \mid aa \mid DC \mid Ca \mid bC \mid a \mid b \mid Da \mid bD \mid b$

$B \longrightarrow bB \mid bAB \mid bb \mid aS$

$C \longrightarrow Ca \mid bC \mid a \mid b$

$D \longrightarrow bD \mid b$