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ARTIFICIAL INTELLIGENCE(BCS515B)

MODULE 2

Problem-solving: Problem-solving agents, Example problems, Searching for Solutions Uninformed Search Strategies

Chapter 3 - 3.1, 3.2, 3.3, 3.4

Text book: Stuart J. Russell and Peter Norvig, Artificial Intelligence, 3rd Edition, Pearson, 2015

Data: Raw facts, unformatted information.

Information: It is the result of processing, manipulating and organizing data in response to a specific need. Information relates to the understanding of the problem domain.

Knowledge: It relates to the understanding of the solution domain – what to do?

Intelligence: It is the knowledge in operation towards the solution – how to do? How to applythe solution?

ARTIFICIAL INTELLIGENCE: Artificial intelligence is the study of how make computers to do thingswhich people do better at the moment. It refers to the intelligence controlled by a computer machine.

This chapter describes one kind of goal-based agent called a **problem-solving agent**. Problem-solving agents use **atomic** representations, that is, states of the world are considered as wholes, with no internal structure visible to the problem solving algorithms. Goal-based agents that use more advanced **factored** or **structured** representations are usually called **planning agents.**

Our discussion of problem solving begins with precise definitions of **problems** and their **solutions** and give several examples to illustrate these definitions. We then describe several general-purpose search algorithms that can be used to solve these problems. We will see several **uninformed** search algorithms—algorithms that are given no information about the problem other than its definition. Although some of these algorithms can solve any solvable problem, none of them can do so efficiently. **Informed** search algorithms, on the other hand, can do quite well given some guidance on where to look for solutions.

In this chapter, we limit ourselves to the simplest kind of task environment, for which the solution to a problem is always a *fixed sequence* of actions. Themore general case—where the agent's future actions may vary depending on future percepts.

3.1 PROBLEM-SOLVING AGENTS

Goal formulation, based on the current situation and the agent's performance measure, is the first step in problem solving. **Problem formulation** is the process of deciding what actions and states to consider, given a goal.

The process of looking for a sequence of actions that reaches the goal is called **search**. A search algorithm takes a problem as input and returns a **solution** in the form of an action sequence. Once a solution is found, the actions it recommends can be carried out. This is called the **execution** phase.

3.1.1 Well-defined problems and solutions

A **problem** can be defined formally by five components:

· The **initial state** that the agent starts in. A description of the possible **actions** available to the agent.

Given a particular state s, ACTIONS(s) returns the set of actions that can be executed in s. We say that each of these actions is **applicable** in s.

· A description of what each action does; the formal name for this is the **transition model**, specified by a function RESULT(s, a) that returns the state that results from doing action a in state s. We also use the term **successor** to refer to any state reachable from a given state by a single action. Together, the initial state, actions, and transition model implicitly define the **state space** of the problem—the set of all states reachable from the initial state by any sequence of actions. The state space forms a directed network or **graph** in which the nodes are states and the links between nodes are actions.

A **path** in the state space is a sequence of states connected by a sequence of actions. The **goal test**, which determines whether a given state is a goal state. Sometimes there is an explicit set of possible goal states, and the test simply checks whether the given state is one of them. The agent's goal in Romania is the singleton set {In(Bucharest)}.

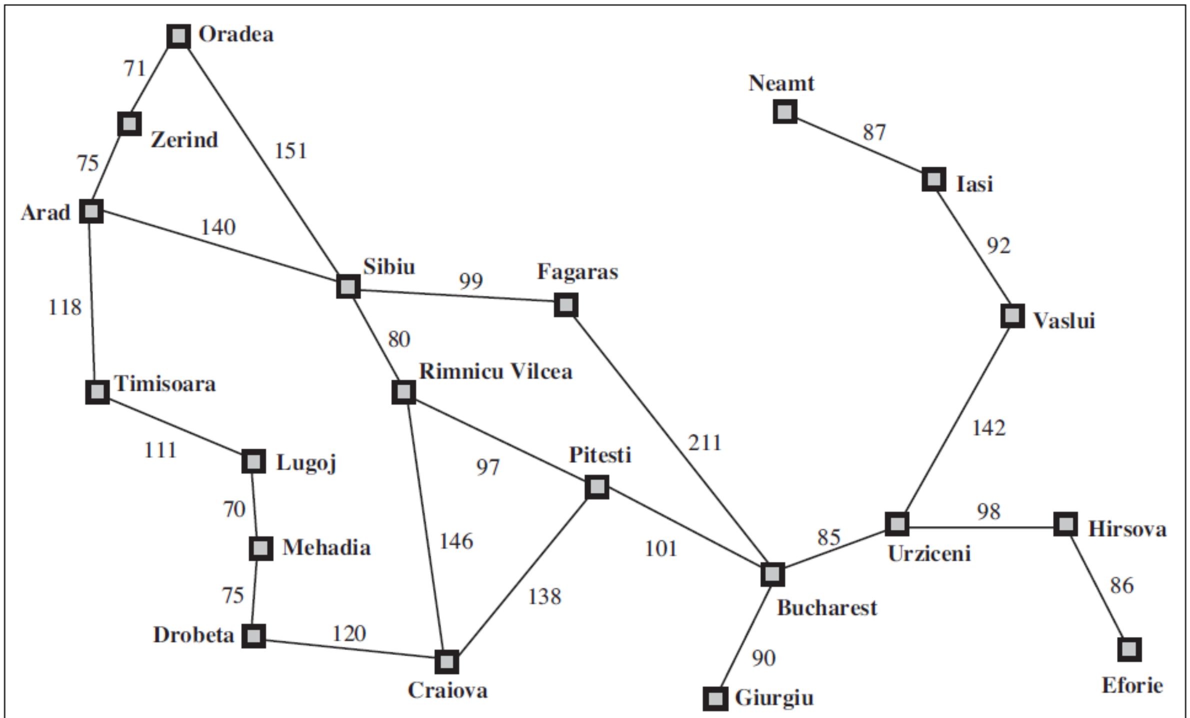

Figure 3.2 A simplified road map of part of Romania.

A **path cost** function that assigns a numeric cost to each path. The problem-solving agent chooses a cost function that reflects its own performance measure. For the agent trying to get to Bucharest, time is of the essence, so the cost of a path might be its length in kilometers. In this chapter, we assume that the cost of a path can be described as the *sum* of the costs of the individual actions along the path.

The **step cost** of taking action a in state s to reach state s is denoted by c(s, a, s). The step costs for Romania are shown in Figure 3.2 as route distances. We assume that step costs are nonnegative. The preceding elements define a problem and can be gathered into a single data structure that is given as input to a problem-solving algorithm. A **solution** to a problem is an action sequence that leads from the initial state to a goal state. Solution quality is measured by the path cost function, and an **optimal**

solution has the lowest path cost among all solutions.

3.1.2 Formulating problems

The process of removing detail from a representation is called **abstraction**.

3.2 EXAMPLE PROBLEMS

The problem-solving approach has been applied to a vast array of task environments. We list some of the best known here, distinguishing between *toy* and *real-world* problems. A **toy problem** is intended to illustrate or exercise various problem-solving methods. It can be given a concise, exact description and hence is usable by different researchers to compare the performance of algorithms. A **real-world problem** is one whose solutions people actually care about. Such problems tend not to have a single agreed-upon description, but we can give the general flavor of their formulations.

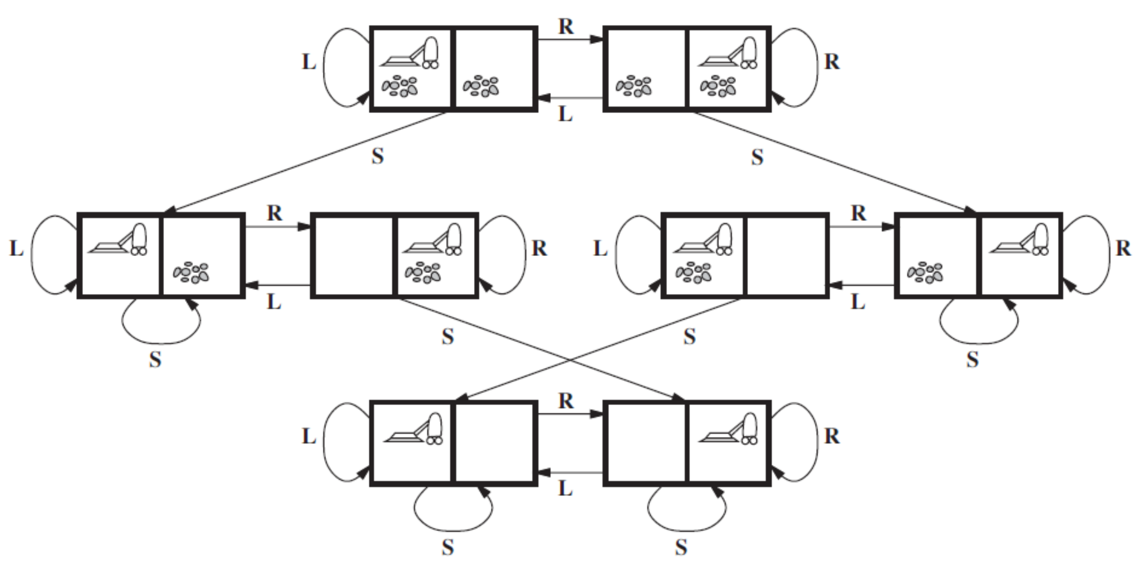

Figure 3.3 The state space for the vacuum world. Links denote actions: L = *Left*, R = *Right*, S = *Suck*.

3.2.1 Toy problems

The first example we examine is the **vacuum world** . (See Figure 2.2.) This can be formulated as a problem as follows:

· **States**: The state is determined by both the agent location and the dirt locations. The agent is in one of two locations, each of which might or might not contain dirt. Thus, there are 2 × 22 = 8 possible world states. A larger environment with n locations has n · 2n states.

· **Initial state**: Any state can be designated as the initial state.

· **Actions**: In this simple environment, each state has just three actions: *Left*, *Right*, and *Suck*. Larger environments might also include *Up* and *Down*.

· **Transition model**: The actions have their expected effects, except that moving *Left* in the leftmost square, moving *Right* in the rightmost square, and *Suck*ing in a clean square have no effect. The complete state space is shown in Figure 3.3.

· **Goal test**: This checks whether all the squares are clean.

· **Path cost**: Each step costs 1, so the path cost is the number of steps in the path.

The **8-puzzle**, an instance of which is shown in Figure 3.4, consists of a 3×3 board with eight numbered tiles and a blank space. A tile adjacent to the blank space can slide into the space. The object is to reach a specified goal state, such as the one shown on the right of the figure. The standard formulation is as follows:

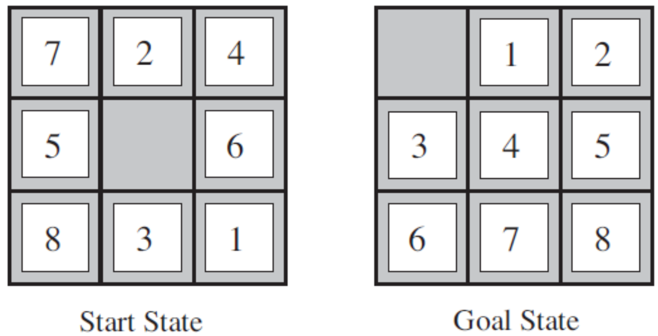

Figure 3.4 A typical instance of the 8-puzzle.

· **States**: A state description specifies the location of each of the eight tiles and the blank in one of the nine squares.

· **Initial state**: Any state can be designated as the initial state. Note that any given goal can be reached from exactly half of the possible initial states.

· **Actions**: The simplest formulation defines the actions as movements of the blank space *Left*, *Right*, *Up*, or *Down*. Different subsets of these are possible depending on where the blank is.

· **Transition model**: Given a state and action, this returns the resulting state; for example, if we apply *Left* to the start state in Figure 3.4, the resulting state has the 5 and the blank switched.

· **Goal test**: This checks whether the state matches the goal configuration shown in Figure 3.4. (Other goal configurations are possible.)

· **Path cost**: Each step costs 1, so the path cost is the number of steps in the path.

What abstractions have we included here? The actions are abstracted to their beginning and final states, ignoring the intermediate locations where the block is sliding. We have abstracted away actions such as shaking the board when pieces get stuck and ruled out extracting the pieces with a knife and putting them back again. We are left with a description of the rules of the puzzle, avoiding all the details of physical manipulations.

The 8-puzzle belongs to the family of **sliding-block puzzles**, which are often used as test problems for new search algorithms in AI. This family is known to be NP-complete, so one does not expect to find methods significantly better in the worst case than the search algorithms described in this chapter and the next. The 8-puzzle has 9!/2=181, 440 reachable states and is easily solved. The 15-puzzle (on a 4×4 board) has around 1.3 trillion states, and random instances can be solved optimally in a few milliseconds by the best search algorithms. The 24-puzzle (on a 5 × 5 board) has around 1025 states, and random instances take several hours to solve optimally. The goal of the **8-queens problem** is to place eight queens on a chessboard such than no queen attacks any other. (A queen attacks any piece in the same row, column or diagonal.) Figure 3.5 shows an attempted solution that fails: the queen in the rightmost column is attacked by the queen at the top left.

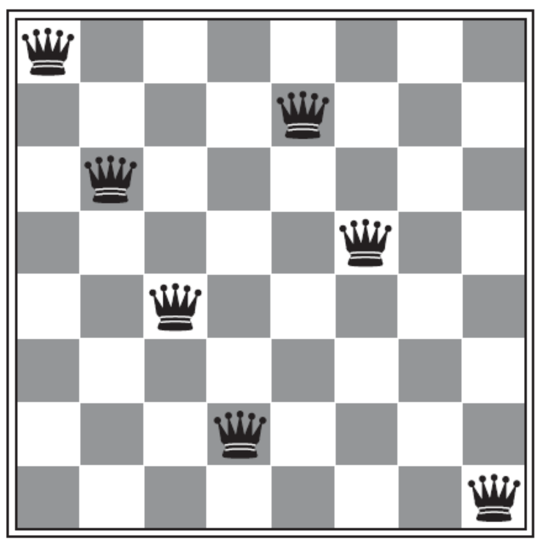

Figure 3.5 Almost a solution to the 8-queens problem. (Solution is left as an exercise.)

Although efficient special-purpose algorithms exist for this problem and for the whole n-queens family, it remains a useful test problem for search algorithms. There are two main kinds of formulation. An **incremental formulation** involves operators that *augment* the state description, starting with an empty state; for the 8-queens problem, this means that each action adds a queen to the state. A **complete-state formulation** starts with all 8 queens on the board and moves them around. In either case, the path cost is of no interest because only the final state counts. The first incremental formulation one might try is the following:

· **States**: Any arrangement of 0 to 8 queens on the board is a state.

· **Initial state**: No queens on the board.

· **Actions**: Add a queen to any empty square.

· **Transition model**: Returns the board with a queen added to the specified square.

· **Goal test**: 8 queens are on the board, none attacked.

In this formulation, we have 64 · 63 · · · 57 ≈ 1.8×1014 possible sequences to investigate. A better formulation would prohibit placing a queen in any square that is already attacked:

· **States**: All possible arrangements of n queens ($0 \leq n \leq 8$), one per column in the leftmost n columns, with no queen attacking another.

· **Actions**: Add a queen to any square in the leftmost empty column such that it is not attacked by any other queen.

This formulation reduces the 8-queens state space from 1.8×1014 to just 2,057, and solutions are easy to find. On the other hand, for 100 queens the reduction is from roughly 10400 states to about 1052 states (Exercise 3.5)—a big improvement, but not enough to make the problem tractable.

Our final toy problem was devised by Donald Knuth (1964) and illustrates how infinite state spaces can arise. Knuth conjectured that, starting with the number 4, a sequence of factorial, square root, and floor operations will reach any desired positive integer. For example, we can reach 5 from 4 as follows:

(4!)!_ = 5 .

The problem definition is very simple:

· **States**: Positive numbers.

· **Initial state**: 4.

· **Actions**: Apply factorial, square root, or floor operation (factorial for integers only).

· **Transition model**: As given by the mathematical definitions of the operations.

· **Goal test**: State is the desired positive integer.

To our knowledge there is no bound on how large a number might be constructed in the process of reaching a given target—for example, the number 620,448,401,733,239,439,360,000 is generated in the expression for 5—so the state space for this problem is infinite. Such state spaces arise frequently in tasks involving the generation of mathematical expressions, circuits, proofs, programs, and other recursively defined objects.

3.2.2 Real-world problems

Route-finding algorithms are used in a variety of applications. Some, such as Web sites and in-car systems that provide driving directions, are relatively straightforward extensions of the Romania example. Others, such as routing video streams in computer networks, military operations planning, and airline travel-planning systems, involve much more complex specifications. Consider the airline travel problems that must be solved by a travel-planning Web site:

· **States**: Each state obviously includes a location (e.g., an airport) and the current time. Furthermore, because the cost of an action (a flight segment) may depend on previous segments, their fare bases, and their status as domestic or international, the state must record extra information about these "historical" aspects.

· **Initial state**: This is specified by the user's query.

· **Actions**: Take any flight from the current location, in any seat class, leaving after the current time, leaving enough time for within-airport transfer if needed.

· **Transition model**: The state resulting from taking a flight will have the flight's destination as the current location and the flight's arrival time as the current time.

· **Goal test**: Are we at the final destination specified by the user?

· **Path cost**: This depends on monetary cost, waiting time, flight time, customs and immigration procedures, seat quality, time of day, type of airplane, frequent-flyer mileage awards, and so on.

3.3 SEARCHING FOR SOLUTIONS

A solution is an action sequence, so search algorithms work by considering various possible action sequences. The possible action sequences starting at the initial state form a search tree with the initial state at the root; the branches are actions and the nodes correspond to states in the state space of the problem. Figure 3.6 shows the first few steps in growing the search tree for finding a route from Arad to Bucharest. The root node of the tree corresponds to the initial state, *In(Arad)*. The first step is to test whether this is a goal state. (Clearly it is not, but it is important to check so that we can solve trick problems like "starting in Arad, get to Arad.") Then we need to consider taking various actions. We do this by expanding the current state; that applying each legal action to the current state, thereby generating a new set of states. In this case, we add three branches from the parent node *In(Arad)* leading to three new child nodes: *In(Sibiu)*, *In(Timisoara)*, and *In(Zerind)*. Now we must choose which of these three possibilities to consider further.

The set of all leaf nodes available for expansion at any given point is called the **frontier**. Many authors call it the **open list.** The process of expanding nodes on the frontier continues until either a solution is found or there are no more states to expand. The general algorithm is shown informally in Figure 3.7. Search algorithms all share this basic structure; they vary primarily according to how they choose which state to expand next—the so-**called search strategy**. The eagle-eyed reader will notice one peculiar thing about the search tree shown in Figure 3.6: it includes the path from Arad to Sibiu and back to Arad again! We say that *In(Arad)* is a **repeated state** in the search tree, generated in this case by a **loopy path**.

Loopy paths are a special case of the more general concept of **redundant paths**, which exist whenever there is more than one way to get from one state to another.

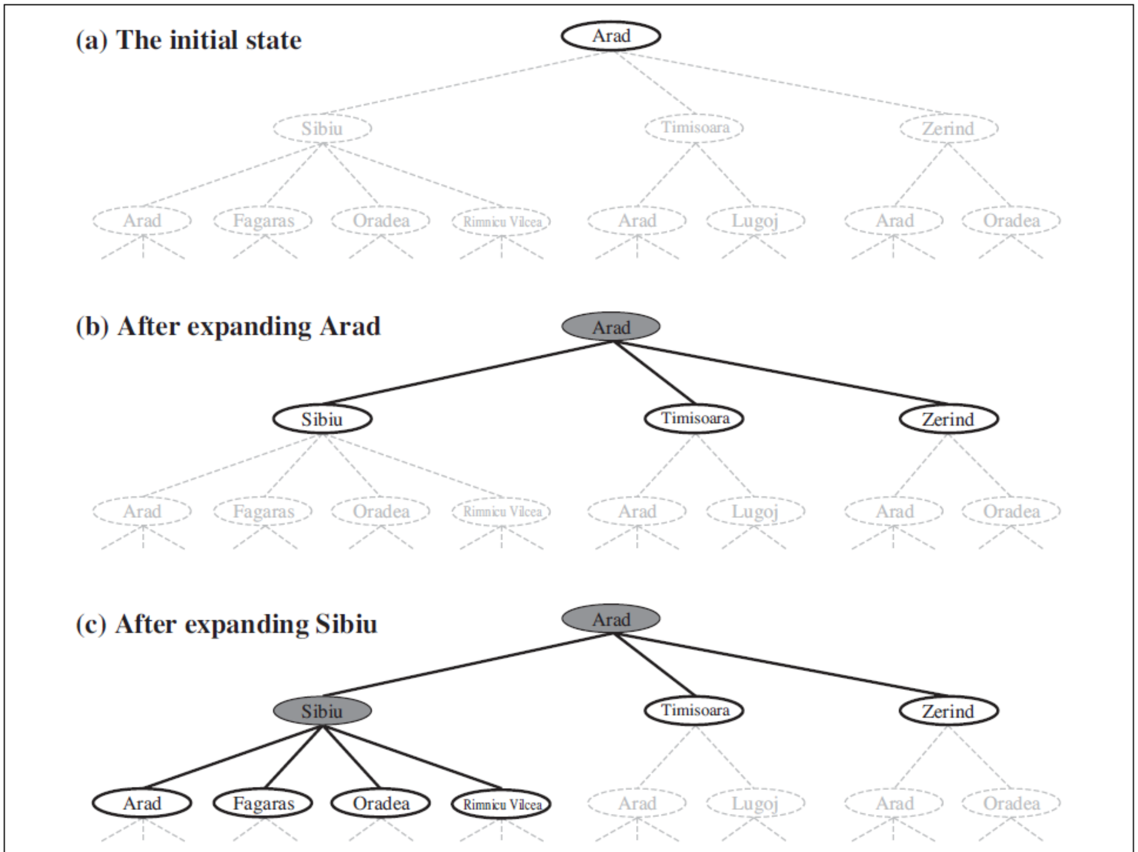

Figure 3.6 Partial search trees for finding a route from Arad to Bucharest. Nodes that have been expanded are shaded; nodes that have been generated but not yet expanded are outlined in bold; nodes that have not yet been generated are shown in faint dashed lines.

function TREE-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    expand the chosen node, adding the resulting nodes to the frontier

---

function GRAPH-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  initialize the explored set to be empty
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    add the node to the explored set
    expand the chosen node, adding the resulting nodes to the frontier
      only if not in the frontier or explored set

Figure 3.7 An informal description of the general tree-search and graph-search algorithms. The parts of GRAPH-SEARCH marked in bold italic are the additions needed to handle repeated states.

In other cases, redundant paths are unavoidable. This includes all problems where the actions are reversible, such as route-finding problems and sliding-block puzzles. Route finding on a **rectangular grid** (like the one used later for RECTANGULAR GRID Figure 3.9) is a particularly important example in computer games. In such a grid, each state has four successors, so a search tree of depth d that includes repeated states has 4^d leaves; but there are only about $2d^2$ distinct states within d steps of any given state. For d = 20, this means about a trillion nodes but only about 800 distinct states. Thus, following redundant paths can cause a tractable problem to become intractable. This is true even for algorithms that know how to avoid infinite loops. As the saying goes, *algorithms that forget their history are doomed to repeat it.* The way to avoid exploring redundant paths is to remember where one has been. To do this, we augment the algorithm with a data structure called the **explored set** (also known as the **closed list**), which remembers every expanded node. Newly generated nodes that match previously generated nodes—ones in the explored set or the frontier—can be discarded instead of being added to the frontier. The new algorithm, called, is shown informally in Figure 3.7. The specific algorithms in this chapter draw on this general design.

Clearly, the search tree constructed by the algorithm contains at most one copy of each state, so we can think of it as growing a tree directly on the state-space graph, as shown in Figure 3.8. The algorithm has another nice property: the frontier **separates** the state-space graph into the explored region and the unexplored region, so that every path from the initial state to an unexplored state has to pass through a state in the frontier. This property is illustrated in Figure 3.9.

As every step moves a state from the frontier into the explored region while moving some states from the unexplored region into the frontier, we see that the algorithm is *systematically* examining the states in the state space, one by one, until it finds a solution.

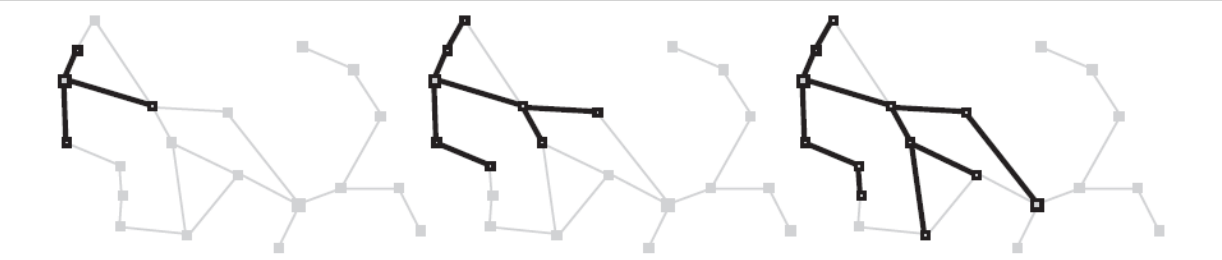

Figure 3.8 A sequence of search trees generated by a graph search on the Romania problem of Figure 3.2. At each stage, we have extended each path by one step. Notice that at the third stage, the northernmost city (Oradea) has become a dead end: both of its successors are already explored via other paths.

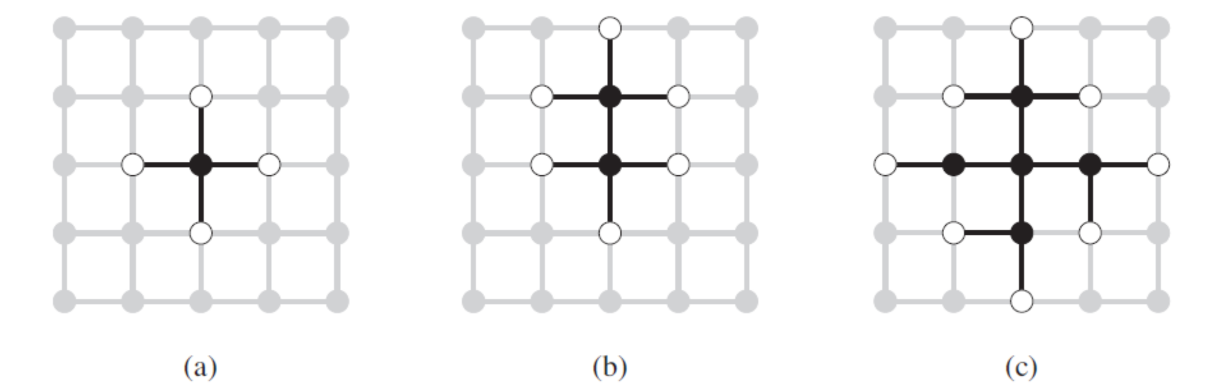

Figure 3.9 The separation property of GRAPH-SEARCH, illustrated on a rectangular-grid problem. The frontier (white nodes) always separates the explored region of the state space (black nodes) from the unexplored region (gray nodes). In (a), just the root has been expanded. In (b), one leaf node has been expanded. In (c), the remaining successors of the root have been expanded in clockwise order.

3.3.1 Infrastructure for search algorithms

Search algorithms require a data structure to keep track of the search tree that is being constructed.

For each node n of the tree, we have a structure that contains four components:

· n.STATE: the state in the state space to which the node corresponds;

· n.PARENT: the node in the search tree that generated this node;

· n.ACTION: the action that was applied to the parent to generate the node;

· n.PATH-COST: the cost, traditionally denoted by g(n), of the path from the initial state to the node, as indicated by the parent pointers

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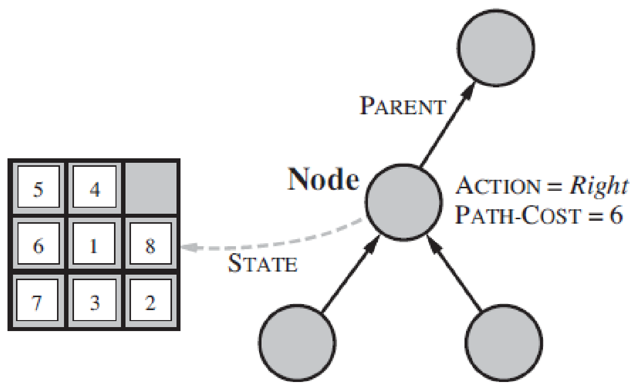


Figure 3.10 Nodes are the data structures from which the search tree is constructed. Each has a parent, a state, and various bookkeeping fields. Arrows point from child to parent.

Given the components for a parent node, it is easy to see how to compute the necessary components for a child node. The function CHILD-NODE takes a parent node and an action and returns the resulting child node:

```
function CHILD-NODE(problem, parent, action) returns a node
  return a node with
     STATE = problem.RESULT(parent.STATE, action),
     PARENT = parent, ACTION = action,
     PATH-COST = parent.PATH-COST + problem.STEP-COST(parent.STATE, action)
```

The node data structure is depicted in Figure 3.10. Notice how the pointers string the nodes together into a tree structure. These pointers also allow the solution path to be extracted when a goal node is found; we use the function to return the sequence of actions obtained by following parent pointers back to the root.

Up to now, we have not been very careful to distinguish between nodes and states, but in writing detailed algorithms it's important to make that distinction. A node is a bookkeeping data structure used to represent the search tree. A state corresponds to a configuration of the world. Thus, nodes are on particular paths, as defined pointers, whereas states are not. Furthermore, two different nodes can contain the same world state if that state is generated via two different search paths.

Now that we have nodes, we need somewhere to put them. The frontier needs to be stored in such a way that the search algorithm can easily choose the next node to expand according to its preferred strategy. The appropriate data structure for this is a **queue**. The operations on a queue are as follows:

• EMPTY?(queue) returns true only if there are no more elements in the queue.
• POP(queue) removes the first element of the queue and returns it.
• INSERT(element, queue) inserts an element and returns the resulting queue.

Queues are characterized by the *order* in which they store the inserted nodes. Three common variants are the first-in, first-out FIFO QUEUE or **FIFO queue**, which pops the *oldest* element of the queue; LIFO QUEUE the last-in, first-out or **LIFO queue** (also known as a **stack**), which pops the *newest* element of the queue; and the **priority queue**, which pops the element of the queue with the highest priority according to some ordering function.

The explored set can be implemented with a hash table to allow efficient checking for repeated states. With a good implementation, insertion and lookup can be done in roughly constant time no matter how many states are stored. One must take care to implement the hash table with the right notion of equality between states.

3.3.2 Measuring problem-solving performance

Before we get into the design of specific search algorithms, we need to consider the criteria that might be used to choose among them. We can evaluate an algorithm's performance in four ways:

- **Completeness**: Is the algorithm guaranteed to find a solution when there is one?
- **Optimality**: Does the strategy find the optimal solution?
- **Time complexity**: How long does it take to find a solution?

- **Space complexity**: How much memory is needed to perform the search?

Time and space complexity are always considered with respect to some measure of the problem difficulty. In theoretical computer science, the typical measure is the size of the state space graph, |V | + |E|, where V is the set of vertices (nodes) of the graph and E is the set of edges (links). This is appropriate when the graph is an explicit data structure that is input to the search program. (The map of Romania is an example of this.)

In AI, the graph is often represented *implicitly* by the initial state, actions, and transition model and is frequently infinite. For these reasons, complexity is expressed in terms of three quantities: b, the **branching factor** or maximum number of successors of any node; d, the **depth** of the shallowest goal node (i.e., the number of steps along the path from the root); and m, the maximum length of any path in the state space. Time is often measured in terms of the number of nodes generated during the search, and space in terms of the maximum number of nodes stored in memory. For the most part, we describe time and space complexity for search on a tree; for a graph, the answer depends on how "redundant" the paths in the state space are.

To assess the effectiveness of a search algorithm, we can consider just the **search cost**— which typically depends on the time complexity but can also include a term for memory usage—or we can use the **total cost**, which combines the search cost and the path cost of the solution found. For the problem of finding a route from Arad to Bucharest, the search cost is the amount of time taken by the search and the solution cost is the total length of the path.

3.4 UNINFORMED SEARCH STRATEGIES

This section covers several search strategies that come under the heading of uninformed search (also called blind search). The UNINFORMED term means that the strategies have no additional information about states beyond that provided in the problem definition. All they can do is generate successors and distinguish a goal state from a non-goal state. All search strategies are distinguished by the *order* in which nodes are expanded. Strategies that know whether one non-goal state is "more promising" than another are called informed search or heuristic search strategies.

3.4.1 Breadth-first search

Breadth-first search is a simple strategy in which the root node is expanded first, then all the successors of the root node are expanded next, then *their* successors, and so on. In general, all the nodes are expanded at a given depth in the search tree before any nodes at the next level are expanded. Breadth-first search is an instance of the general graph-search algorithm (Figure 3.7) in which the *shallowest* unexpanded node is chosen for expansion. This is achieved very simply by using a FIFO queue for the frontier. Thus, new nodes (which are always deeper than their parents) go to the back of the queue, and old nodes, which are shallower than the new nodes, get expanded first. There is one slight tweak on the general graph-search algorithm, which is that the goal test is applied to each node when it is *generated* rather than when it is selected for expansion. This decision is explained below, where we discuss time complexity. Note also that the algorithm, following the general template for graph search, discards any new path to a state already in the frontier or explored set; it is easy to see that any such path must be at least as deep as the one already found. Thus, breadth-first search always has the shallowest path to every node on the frontier. Pseudocode is given in Figure 3.11. Figure 3.12 shows the progress of the search on a simple binary tree. How does breadth-first search rate according to the four criteria from the previous section?We can easily see that it is *complete*—if the shallowest goal node is at some finite depth d, breadth-first search will eventually find it after generating all shallower nodes (provided the branching factor b is finite). Note that as soon as a goal node is generated, we know it is the shallowest goal node shallower nodes must have been generated already and failed the goal test. Now, the *shallowest* goal node is not necessarily the *optimal* one; technically, breadth-first search is optimal if the path cost is a nondecreasing function of the depth of the node. The most common such scenario is that all actions have the same cost.

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier ← a FIFO queue with node as the only element
  explored ← an empty set
  loop do
     if EMPTY?(frontier) then return failure
     node ← POP(frontier)  /* chooses the shallowest node in frontier */
     add node.STATE to explored
     for each action in problem.ACTIONS(node.STATE) do
        child ← CHILD-NODE(problem, node, action)
        if child.STATE is not in explored or frontier then
           if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
           frontier ← INSERT(child, frontier)
```

Figure 3.11 Breadth-first search on a graph.

Imagine searching a uniform tree where every state has b successors. The root of the search tree generates b nodes at the first level, each of which generates b more nodes, for a total of b^2 at the second level. Each of these generates b more nodes, yielding b^3 nodes at the third level, and so on. Now suppose that the solution is at depth d. In the worst , it is the last node generated at that level. Then the total number of nodes generated is

$b + b^2 + b^3 + \cdot \cdot \cdot + b^d = O(b^d)$.

(If the algorithm were to apply the goal test to nodes when selected for expansion, rather than when generated, the whole layer of nodes at depth d would be expanded before the goal was detected and the time complexity would be $O(b^{d+1})$.)

As for space complexity: for any kind of graph search, which stores every expanded node in the explored set, the space complexity is always within a factor of b of the time complexity. For breadth-first graph search in particular, every node generated remains in memory. There will be $O(b^{d-1})$ nodes in the explored set and $O(b^d)$ nodes in the frontier, so the space complexity is $O(b^d)$, i.e., it is dominated by the size of the frontier. Switching to a tree search would not save much space, and in a state space with many redundant paths, switching could cost a great deal of time. An exponential complexity bound such as $O(b^d)$ is scary. Figure 3.13 shows why. It lists, for various values of the solution depth d, the time and memory required for a breadth first search with branching factor b = 10. The table assumes that 1 million nodes can be generated per second and that a node requires 1000 bytes of storage. Many search problems fit roughly within these assumptions (give or take a factor of 100) when run on a modern personal computer.

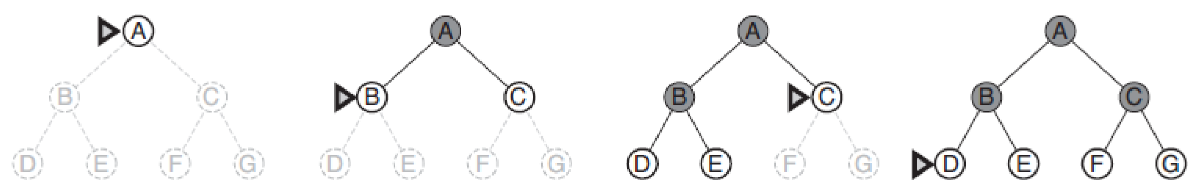


Figure 3.12 Breadth-first search on a simple binary tree. At each stage, the node to be expanded next is indicated by a marker.

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

Figure 3.13 Time and memory requirements for breadth-first search. The numbers shown assume branching factor $b = 10$; 1 million nodes/second; 1000 bytes/node.

First, *the memory requirements are a bigger problem for breadth-first search than is the execution time*. One might wait 13 days for the solution to an important problem with search depth 12, but no personal computer has the petabyte of memory it would take. Fortunately, other strategies require less memory. The second lesson is that time is still a major factor. If your problem has a solution at depth 16, then (given our assumptions) it will take about 350 years for breadth-first search (or indeed any uninformed search) to find it. In general, *exponential-complexity search problems cannot be solved by uninformed methods for any but the smallest instances.*

3.4.2 Uniform-cost search

When all step costs are equal, breadth-first search is optimal because it always expands the shallowest unexpanded node. By a simple extension, we can find an algorithm that is optimal with any step-cost function. Instead of expanding the shallowest node, **uniform-cost search** expands the node n with the lowest path cost g(n). This is done by storing the frontier as a priority queue ordered by g. The algorithm is shown in Figure 3.14. In addition to the ordering of the queue by path cost, there are two other significant differences from breadth-first search. The first is that the goal test is applied to a node when it is selected for expansion (as in the generic graph-search algorithm shown in Figure 3.7) rather than when it is first generated. The reason is that the first goal node that is generated may be on a suboptimal path. The second difference is that a test is added in case a better path is found to a node currently on the frontier. Both of these modifications come into play in the example shown in Figure 3.15, where the problem is to get from Sibiu to Bucharest. The successors of Sibiu are Rimnicu Vilcea and Fagaras, with costs 80 and 99, respectively. The least-cost node, Rimnicu Vilcea, is expanded next, adding Pitesti with cost 80 + 97=177. The least-cost node is now Fagaras, so it is expanded, adding Bucharest with cost 99+211=310. Now a goal node has been generated, but uniform-cost search keeps going, choosing Pitesti for expansion and adding a second path.

It is easy to see that uniform-cost search is optimal in general. First, we observe that whenever uniform-cost search selects a node n for expansion, the optimal path to that node has been found. (Were this not the case, there would have to be another frontier node n_ on the optimal path from the start node to n, by the graph separation property of Figure 3.9; by definition, n would have lower g-cost than n and would have been selected first.)

Then, because step costs are nonnegative, paths never get shorter as nodes are added. These two facts together imply that uniform-cost search expands nodes in order of their optimal path cost. Hence, the first goal node selected for expansion must be the optimal solution.

```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  frontier ← a priority queue ordered by PATH-COST, with node as the only element
  explored ← an empty set
  loop do
     if EMPTY?(frontier) then return failure
     node ← POP(frontier)  /* chooses the lowest-cost node in frontier */
     if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
     add node.STATE to explored
     for each action in problem.ACTIONS(node.STATE) do
        child ← CHILD-NODE(problem, node, action)
        if child.STATE is not in explored or frontier then
           frontier ← INSERT(child, frontier)
        else if child.STATE is in frontier with higher PATH-COST then
           replace that frontier node with child
```

Figure 3.14 Uniform-cost search on a graph. The algorithm is identical to the general graph search algorithm in Figure 3.7, except for the use of a priority queue and the addition of an extra check in case a shorter path to a frontier state is discovered. The data structure for *frontier* needs to support efficient membership testing, so it should combine the capabilities of a priority queue and a hash table.

Uniform-cost search does not care about the *number* of steps a path has, but only about their total cost. Therefore, it will get stuck in an infinite loop if there is a path with an infinite sequence of zero-cost actions—for example, a sequence of NoOp actions.6 Completeness is guaranteed provided the cost of every step exceeds some small positive constant.

Uniform-cost search is guided by path costs rather than depths, so its complexity is not easily characterized in terms of b and d. Instead, let C* be the cost of the optimal solution,7 and assume that every action costs at least _. Then the algorithm's worst-case time and space complexity is O(b ^{1}+floor(C)∗/epsilon), which can be much greater than b^d. This is because uniform cost search can explore large trees of small steps before exploring paths involving large and perhaps useful steps. When all step costs are equal, b ^{1}+floor(C)∗/epsilon is just b^d+1. When all step costs are the same, uniform-cost search is similar to breadth-first search, except that the latter stops as soon as it generates a goal, whereas uniform-cost search examines all the nodes at the goal's depth to see if one has a lower cost; thus uniform-cost search does strictly more work by expanding nodes at depth d unnecessarily.

3.4.3 Depth-first search

Depth-first search always expands the *deepest* node in the current frontier of the search tree. The progress of the search is illustrated in Figure 3.16. The search proceeds immediately to the deepest level of the search tree, where the nodes have no successors. As those nodes are expanded, they are dropped from the frontier, so then the search "backs up" to the next deepest node that still has unexplored successors. The depth-first search algorithm is an instance of the graph-search algorithm in Figure 3.7; whereas breadth-first-search uses a FIFO queue, depth-first search uses a LIFO queue. A LIFO queue means that the most recently generated node is chosen for expansion. This must be the deepest unexpanded node because it is one deeper than its parent—which, in turn, was the deepest unexpanded node when it was selected. As an alternative to the style implementation, it is common to implement depth-first search with a recursive function that calls itself on each of its children in turn. (A recursive depth-first algorithm incorporating a depth limit is shown in Figure 3.17.)

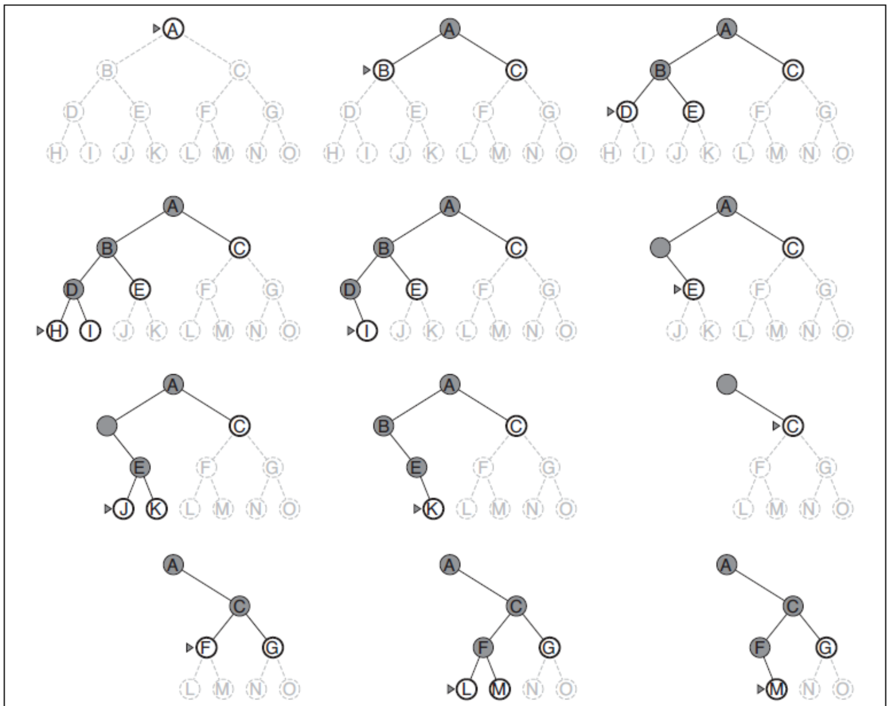


Figure 3.16 Depth-first search on a binary tree. The unexplored region is shown in light gray. Explored nodes with no descendants in the frontier are removed from memory. Nodes at depth 3 have no successors and M is the only goal node.

The properties of depth-first search depend strongly on whether the graph-search or tree-search version is used. The graph-search version, which avoids repeated states and redundant paths, is complete in finite state spaces because it will eventually expand every node. The tree-search version, on the other hand, is *not* complete—for example, in Figure 3.6 the algorithm will follow the Arad–Sibiu–Arad–Sibiu loop forever. Depth-first tree search can be modified at no extra memory cost so that it checks new states against those on the path from the root to the current node; this avoids infinite loops in finite state spaces but does not avoid the proliferation of redundant paths. In infinite state spaces, both versions fail if an infinite non-goal path is encountered. For example, in Knuth's 4 problem, depth-first search would keep applying the factorial operator forever.

For similar reasons, both versions are nonoptimal. For example, in Figure 3.16, depthfirst search will explore the entire left subtree even if node C is a goal node. If node J were also a goal node, then depth-first search would return it as a solution instead of C, which would be a better solution; hence, depth-first search is not optimal.

The time complexity of depth-first graph search is bounded by the size of the state space (which may be infinite, of course). A depth-first tree search, on the other hand, may generate all of the $O(b^m)$ nodes in the search tree, where m is the maximum depth of any node; this can be much greater than the size of the state space. Note that m itself can be much larger than d (the depth of the shallowest solution) and is infinite if the tree is unbounded.

So far, depth-first search seems to have no clear advantage over breadth-first search, so why do we include it? The reason is the space complexity. For a graph search, there is no advantage, but a depth-first tree search needs to store only a single path from the root to a leaf node, along with the remaining unexpanded sibling nodes for each node on the path. Once a node has been expanded, it can be removed from memory as soon as all its descendants have been fully explored. (See Figure 3.16.) For a state space with branching factor b and maximum depth m, depth-first search requires storage of only $O(b^m)$ nodes. Using the same assumptions as for Figure 3.13 and assuming that nodes at the same depth as the goal node have no successors, we find that depth-first search would require 156 kilobytes instead of 10 exabytes at depth d = 16, a factor of 7 trillion times less space. This has led to the adoption of depth-first tree search as the basic workhorse of many areas of AI, including constraint satisfaction , propositional satisfiability , and logic programming. For the remainder of this section, we focus primarily on the tree search version of depth-first search. A variant of depth-first search called **backtracking search** uses still less memory. In backtracking, only one successor is generated at a time rather than all successors; each partially expanded node remembers which successor to generate next. In this way, only O(m) memory is needed rather than O(bm). Backtracking search facilitates yet another memory-saving (and time-saving) trick: the idea of generating a successor by *modifying* the current state description directly rather than copying it first. This reduces the memory requirements to just one state description and O(m) actions. For this to work, we must be able to undo each modification when we go back to generate the next successor. For problems with large state descriptions, such as robotic assembly, these techniques are critical to success.

3.4.4 Depth-limited search

The embarrassing failure of depth-first search in infinite state spaces can be alleviated by supplying depth-first search with a predetermined depth limit. That is, nodes at depth are treated as if they have no successors. This approach is called **depth-limited search**. The depth limit solves the infinite-path problem. Unfortunately, it also introduces an additional source of incompleteness if we choose _ < d, that is, the shallowest goal is beyond the depth limit. (This is likely when d is unknown.) Depth-limited search will also be nonoptimal if we choose _ > d. Its time complexity is O(b_) and its space complexity is O(b_). Depth-first search can be viewed as a special case of depth-limited search with _=∞.Sometimes, depth limits can be based on knowledge of the problem. For example, on the map of Romania there are 20 cities. Therefore, we know that if there is a solution, it mustbe of length 19 at the longest, so _ = 19 is a possible choice. But in fact if we studied the map carefully, we would discover that any city can be reached from any other city in at most 9 steps. This number, known as the **diameter** of the state space, gives us a better depth limit, which leads to a more efficient depth-limited search. For most problems, however, we will not know a good depth limit until we have solved the problem. Depth-limited search can be implemented as a simple modification to the general tree or graph-search algorithm.

```
function DEPTH-LIMITED-SEARCH(problem, limit) returns a solution, or failure/cutoff
  return RECURSIVE-DLS(MAKE-NODE(problem.INITIAL-STATE), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns a solution, or failure/cutoff
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  else if limit = 0 then return cutoff
  else
     cutoff_occurred? ← false
     for each action in problem.ACTIONS(node.STATE) do
        child ← CHILD-NODE(problem, node, action)
        result ← RECURSIVE-DLS(child, problem, limit − 1)
        if result = cutoff then cutoff_occurred? ← true
        else if result ≠ failure then return result
     if cutoff_occurred? then return cutoff else return failure
```

Figure 3.17 A recursive implementation of depth-limited tree search.

Alternatively, it can be implemented as a simple recursive algorithm as shown in Figure 3.17. Notice that depth-limited search can terminate with two kinds of failure: the standard failure value indicates no solution; the cutoff value indicates no solution within the depth limit.

3.4.5 Iterative deepening depth-first search

Iterative deepening search (or iterative deepening depth-first search) is a general strategy, often used in combination with depth-first tree search, that finds the best depth limit. It does this by gradually increasing the limit—first 0, then 1, then 2, and so on—until a goal is found. This will occur when the depth limit reaches d, the depth of the shallowest goal node. The algorithm is shown in Figure 3.18. Iterative deepening combines the benefits of depth-first and breadth-first search. Like depth-first search, its memory requirements are modest: $O(b^d)$ to be precise. Like breadth-first search, it is complete when the branching factor is finite and optimal when the path cost is a nondecreasing function of the depth of the node. Figure 3.19 shows four iterations of on a binary search tree, where the solution is found on the fourth iteration. Iterative deepening search may seem wasteful because states are generated multiple times. It turns out this is not too costly. The reason is that in a search tree with the same (or nearly the same) branching factor at each level, most of the nodes are in the bottom level, so it does not matter much that the upper levels are generated multiple times. In an iterative deepening search, the nodes on the bottom level (depth d) are generated once, those on the next-to-bottom level are generated twice, and so on, up to the children of the root, which are generated d times. So the total number of nodes generated in the worst case is

N(IDS) = (d)b + (d − 1)b

2 + · · · + (1)b

d

,

which gives a time complexity of $O(b^d)$—asymptotically the same as breadth-first search. There is some extra cost for generating the upper levels multiple times, but it is not large. For example, if b = 10 and d = 5, the numbers are

N(IDS) = 50 + 400 + 3, 000 + 20, 000 + 100, 000 = 123, 450

N(BFS) = 10 + 100 + 1, 000 + 10, 000 + 100, 000 = 111, 110 .

If you are really concerned about repeating the repetition, you can use a hybrid approach that runs breadth-first search until almost all the available memory is consumed, and then runs iterative deepening from all the nodes in the frontier. *In general, iterative deepening is the preferred* *uninformed search method when the search space is large and the depth of the solution is not known.* Iterative deepening search is analogous to breadth-first search in that it explores a complete layer of new nodes at each iteration before going on to the next layer. It would seem worthwhile to develop an iterative analog to uniform-cost search, inheriting the latter algorithm's optimality guarantees while avoiding its memory requirements. The idea is to use increasing path-cost limits instead of increasing depth limits. The resulting algorithm, called **iterative lengthening search**, is explored in Exercise 3.17. It turns out, unfortunately, that iterative lengthening incurs substantial overhead compared to uniform-cost search.

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution, or failure
  for depth = 0 to ∞ do
    result ← DEPTH-LIMITED-SEARCH(problem, depth)
    if result ≠ cutoff then return result
```

Figure 3.18 The iterative deepening search algorithm, which repeatedly applies depth-limited search with increasing limits. It terminates when a solution is found or if the depth-limited search returns *failure*, meaning that no solution exists.

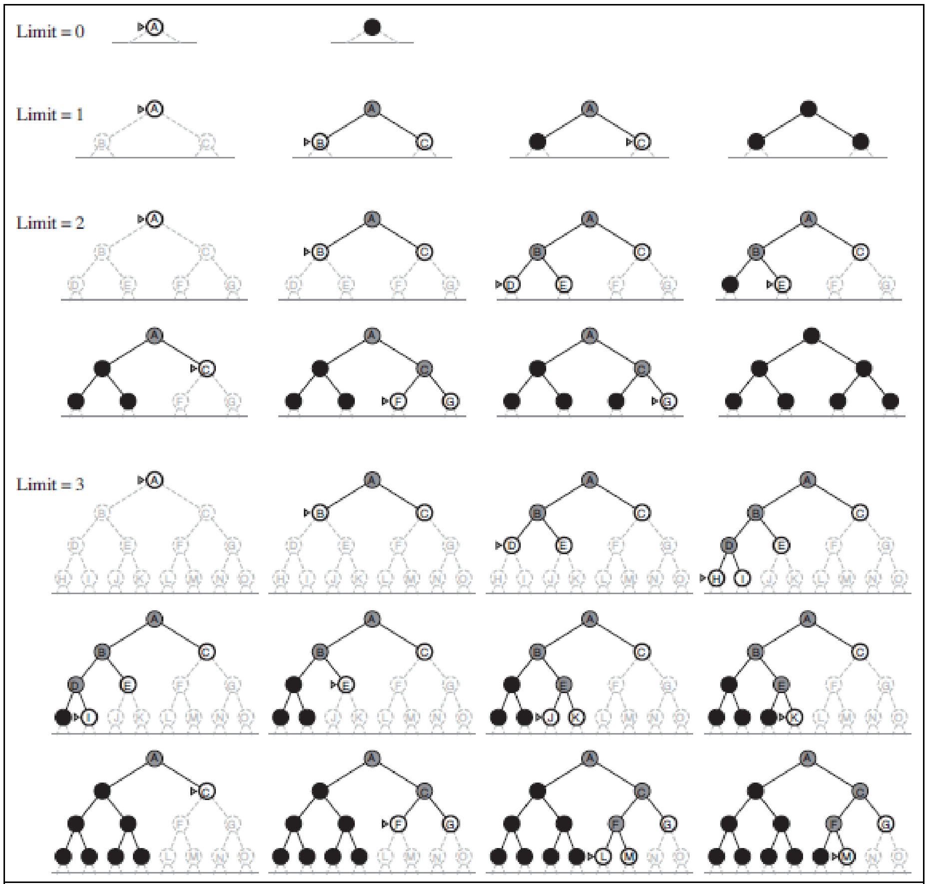


Figure 3.19 Four iterations of iterative deepening search on a binary tree.

3.4.6 Bidirectional search

The idea behind bidirectional search is to run two simultaneous searches—one forward from the initial state and the other backward from the goal—hoping that the two searches meet in the middle (Figure 3.20). The motivation is that $b^{d/2} + b^{d/2}$ is much less than b^d, or in the figure, the area of the two small circles is less than the area of one big circle centered on the start and reaching to the goal. Bidirectional search is implemented by replacing the goal test with a check to see whether the frontiers of the two searches intersect; if they do, a solution has been found. (It is important to realize that the first such solution found may not be optimal, even if the two searches are both breadth-first; some additional search is required to make sure there isn't another short-cut across the gap.) The check can be done when each node is generated or selected for expansion and, with a hash table, will take constant time. For example, if a problem has solution depth d=6, and each direction runs breadth-first search one node at a time, then in the worst case the two searches meet when they have generated all of the nodes at depth 3. For b=10, this means a total of 2,220 node generations, compared with 1,111,110 for a standard breadth-first search. Thus, the time complexity of bidirectional search using breadth-first searches in both directions is $O(b^{d/2})$. The space complexity is also $O(b^{d/2})$. We can reduce this by roughly half if one of the two searches is done by iterative deepening, but at least one of the frontiers must be kept in memory so that the intersection check can be done. This space requirement is the most significant weakness of bidirectional search.

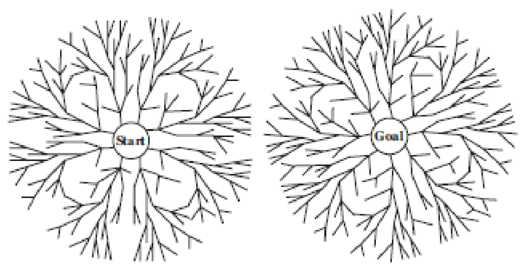


Figure 3.20 A schematic view of a bidirectional search that is about to succeed when a branch from the start node meets a branch from the goal node.

The reduction in time complexity makes bidirectional search attractive, but how do we search backward? This is not as easy as it sounds. Let the **predecessors** of a state x be all those states that have x as a successor. Bidirectional search requires a method for computing predecessors. When all the actions in the state space are reversible, the predecessors of x are just its successors. Other cases may require substantial ingenuity. Consider the question of what we mean by "the goal" in searching "backward from the goal." For the 8-puzzle and for finding a route in Romania, there is just one goal state, so the backward search is very much like the forward search. If there are several *explicitly listed* goal states—for example, the two dirt-free goal states in Figure 3.3—then we can construct a new dummy goal state whose immediate predecessors are all the actual goal states. But if the goal is an abstract description, such as the goal that "no queen attacks another queen" in the n-queens problem, then bidirectional search is difficult to use.

3.4.7 Comparing uninformed search strategies

Figure 3.21 compares search strategies in terms of the four evaluation criteria set forth in Section 3.3.2. This comparison is for tree-search versions. For graph searches, the main differences are that depth-first search is complete for finite state spaces and that the space and time complexities are bounded by the size of the state space.

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes[a]	Yes[a,b]	No	No	Yes[a]	Yes[a,d]
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes[c]	Yes	No	No	Yes[c]	Yes[c,d]

Figure 3.21 Evaluation of tree-search strategies. b is the branching factor; d is the depth of the shallowest solution; m is the maximum depth of the search tree; l is the depth limit. Superscript caveats are as follows: [a] complete if b is finite; [b] complete if step costs $\geq \epsilon$ for positive ϵ; [c] optimal if step costs are all identical; [d] if both directions use breadth-first search.

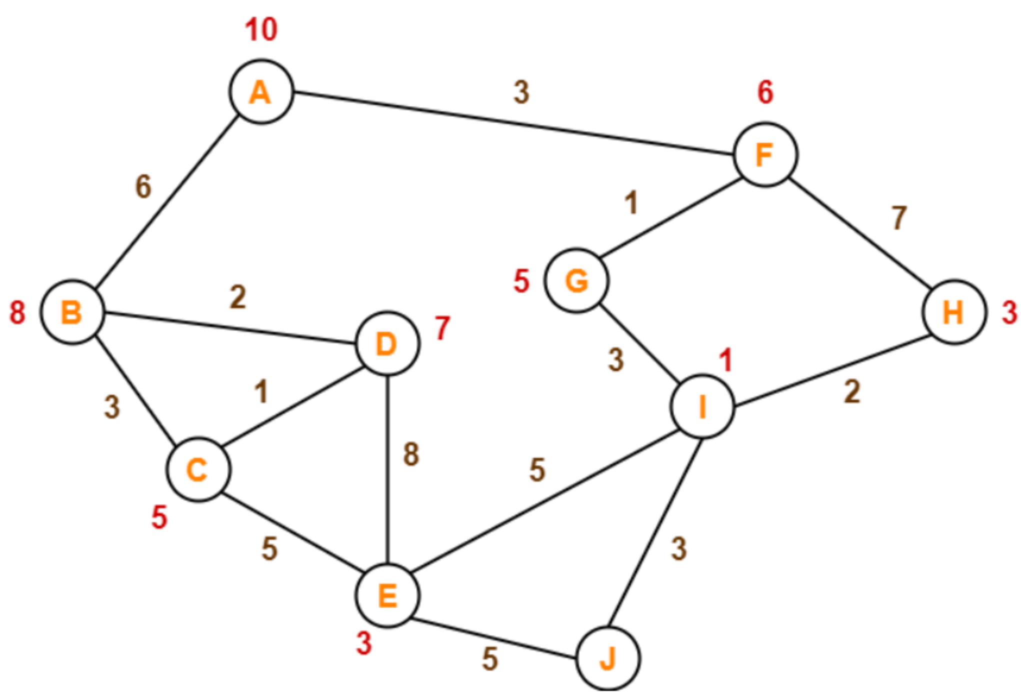
This section shows how an **informed search** strategy—one that uses problem-specific knowledge beyond the definition of the problem itself—can find solutions more efficiently than can an uninformed strategy.The general approach we consider is called **best-first search**. Best-first search is an instance of the general algorithm in which a node is selected for expansion based on an **evaluation function**, f(n). The evaluation function is construed as a cost estimate, so the node with the *lowest* evaluation is expanded first. The implementation of best-first graph search is identical to that for uniform-cost search (Figure 3.14), except for the use of f instead of g to order the priority queue. The choice of f determines the search strategy. (For example, as Exercise 3.21 shows, best-first tree search includes depth-first search as a special case.) Most best-first algorithms include as a component of f a **heuristic function**, denoted h(n): h(n) = estimated cost of the cheapest path from the state at node *n* to a goal state. (Notice that h(n) takes a *node* as input, but, unlike g(n), it depends only on the *state* at that node.) For example, in Romania, one might estimate the cost of the cheapest path from Arad to Bucharest via the straight-line distance from Arad to Bucharest. Heuristic functions are the most common form in which additional knowledge of the problem is imparted to the search algorithm. We study heuristics in more depth in Section 3.6. For now, we consider them to be arbitrary, nonnegative, problem-specific functions, with one constraint: if n is a goal node, then h(n)=0. The remainder of this section covers two ways to use heuristic information to guide search.

QUESTION BANK: MODULE 2

1. Explain a well-defined problem with its components.
2. Explain with a neat schematic diagrammatic example AI solution for the Toy problem.
3. Explain 8 puzzle problem with its solutions.
4. Give the algorithm of Graph-Tree search
5. Explain the algorithm of a simple problem-solving AI agent.
6. Explain the problem-solving agent components of a vacuum based agent.
7. Explain the problem-solving agent components of a Eight Puzzle problem.
8. Explain the problem-solving agent components of a path search problem.
9. Explain the components of problem-solving agent for an airline reservation application.
10. Explain the infrastructure for structure of AI agents.
11. Explain the parameters required for measuring the performance of any problem solving AI agent.
12. With an example tree/graph explain the algorithm of Breadth First Search and also mention the problem-solving performance measures.
13. With an example tree/graph explain the algorithm of Breadth First Search and also mention the problem-solving performance measures.
14. With an example tree/graph explain the algorithm of Depth First Search and also mention the problem-solving performance measures.
15. With an example tree/graph explain the algorithm of Depth Limited Search and also mention

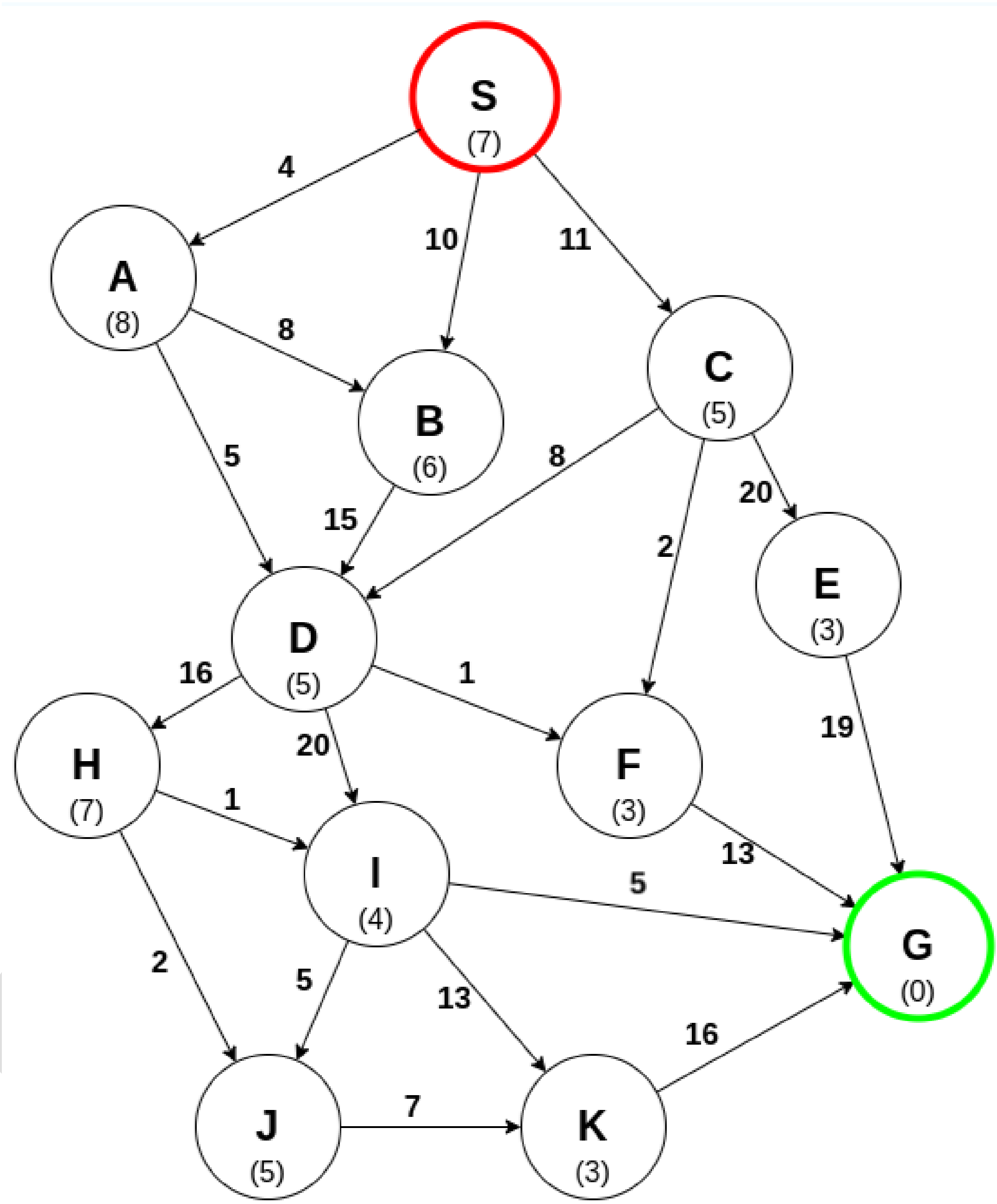
the problem-solving performance measures.

16. With an example tree/graph explain the algorithm of Iterative Deepening DFS and also mention the problem-solving performance measures.
17. With an example tree/graph explain the algorithm of Uniform Cost Search and also mention the problem-solving performance measures.
18. With an example tree/graph explain the algorithm of Bidirectional AI Search and also mention the problem-solving performance measures.
19. Explain what is an uninformed search strategy and the features of uninformed search strategies.
20. Compute the A star algorithm for the below tree



i.

ii.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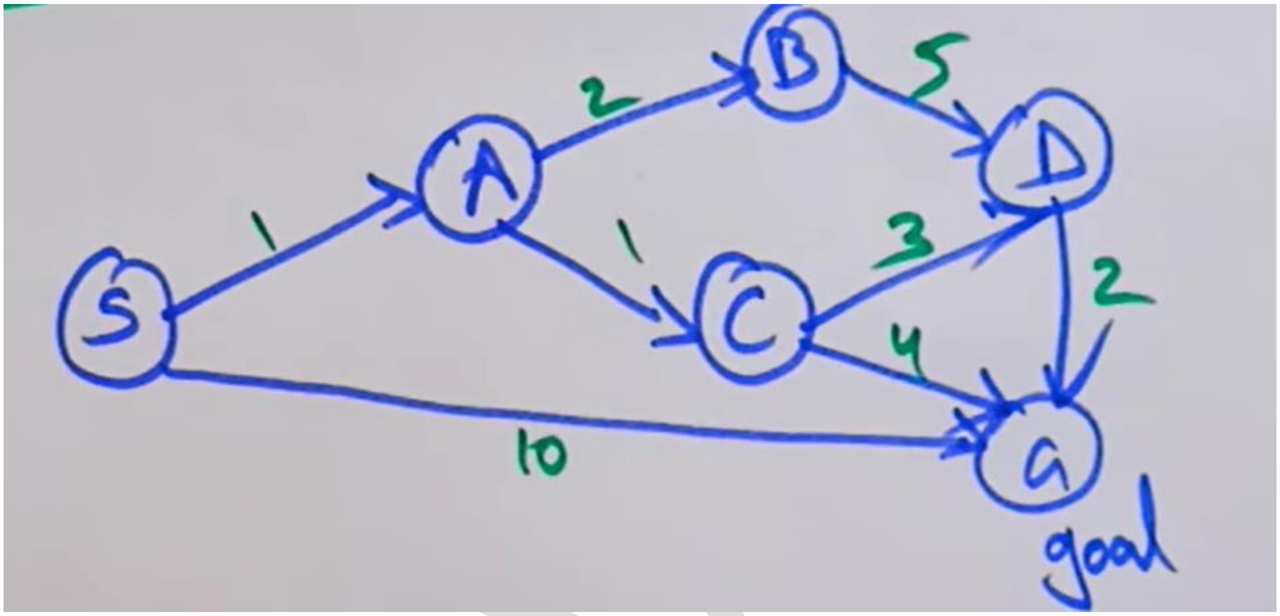
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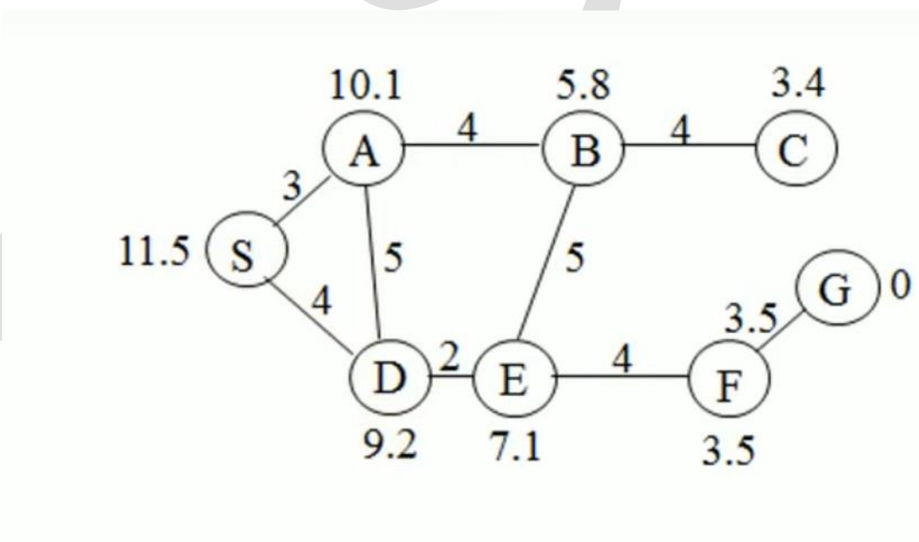
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F
9.2
7.1
3.5