Motor Trend Data Regression Analysis

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Executive Summary

This analysis was done for Motor Trend, a magazine about the automobile industry. The task was to look at the mtcars data set of a collection of cars. They are interested in exploring the relationship between a set of variables and miles per gallon (MPG) (outcome).

They are particularly interested in the following two questions:

- "Is an automatic or manual transmission better for MPG"
- "Quantify the MPG difference between automatic and manual transmissions"

After concluding the analysis, the following points can be made:

- Manual transmission y for MPG, based on the evidence from the box plot, as well as the simple linear regression model.
- With a 95% confidence interval, we estimate the true difference between automatic and manual cars to be between 3.2 and 11.3
- Our simple linear model with transmission type as predictor shows us that the manual transmission cars have 7.24 (+/- 3.60) MPG more in fuel efficiency than automatic cars.
- As for a multivariate linear regression model, using a stepwise selection method, we found that the predictor variables wt, qsec and am best predict mpg, explaining roughly 85% of it's variation. This model was further tested with anova, the variance inflation factor, along with residual diagnostic plots.
- This multivariate regression model tells us that, while adjusting for the other predictor variables wt and qsec, manual transmission cars on average have 2.94 MPG (+/- 2.89) more in fuel efficiency than automatic cars.

Importing libraries

```
library(GGally)
library(dplyr)
library(ggplot2)
library(car)
library(broom)
library(printr)
library(pander)
library(kableExtra)

theme_set(theme_classic())
```

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21.0	6	160	110	3.90	2.620	16.46	0	1	4	4
Mazda RX4 Wag	21.0	6	160	110	3.90	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108	93	3.85	2.320	18.61	1	1	4	1
Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1
Hornet Sportabout	18.7	8	360	175	3.15	3.440	17.02	0	0	3	2
Valiant	18.1	6	225	105	2.76	3.460	20.22	1	0	3	1

Exploring Data Analysis

To know more about the data, you can look at the appendix section with title 'About the data'

```r
head(mtcars) %>% kbl() %>% kable_styling(bootstrap_options = c("striped", "hover"))
```

Since vs and am are factor variables, we'll be factorizing them to get more interpretable outputs in regression.

The question focuses on the am variable, which is transmission type - automatic or manual. To answer the question, we can plot a box plot to see the difference between automatic and manual.

Based on the box plot in the appendix, we can form a hypothesis that the manual cars have higher miles per gallon, which means it has higher fuel efficiency as compared to automatic cars. o test for this claim, we can use a statistical test such as the t test.

Two sample t test

```r
panderOptions('table.split.table', '50')
pander(t.test(mtcars$mpg ~ mtcars$am))
```

Table 1: Welch Two Sample t-test: `mtcars$mpg` by `mtcars$am`

Test statistic	df	P value	Alternative hypothesis	mean in group automatic	mean in group manual
-3.767	18.33	0.001374 * *	two.sided	17.15	24.39

From the t test, we get a significant p-value, this means we can reject the null that there is no difference between auto and manual cars. In other words, the probability that the difference in these two groups appeared by chance is very low. Observing the confidence interval, we are 95% confident that the true difference between automatic and manual cars are between 3.2 and 11.3.

Since we'll be fitting regression models on this data, it's useful to look pairwise scatter plots as this gives us a quick look into the relationship between variables. This plot can be observed at the appendix.

Regression Model and Hypothesis testing

Simple Linear Regression Model

Since Motor Trends is more interested in the am variable, we'll be fitting it to the model and observe the results.

```r
fit_am <- lm(mpg ~ am, mtcars)
summary(fit_am)
```

```
##
## Call:
## lm(formula = mpg ~ am, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.3923 -3.0923 -0.2974  3.2439  9.5077
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   17.147      1.125  15.247 1.13e-15 ***
## ammanual       7.245      1.764   4.106 0.000285 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.902 on 30 degrees of freedom
## Multiple R-squared:  0.3598, Adjusted R-squared:  0.3385
## F-statistic: 16.86 on 1 and 30 DF,  p-value: 0.000285
```

The reference variable follows an alphabetical order, so interpreting the coefficients, note that the reference variable in this case is automatic transmission.

- The intercept here shows us that 17.15 is mean mpg for automatic transmission.
- The slope coefficient shows us that 7.24 is the change in mean between the automatic and manual transmission (this can be observed from the box plot previously)
- The p-value for the slope coefficient tells us that the mean difference between auto and manual transmission is significant, and thus we can conclude that manual transmission is more fuel efficient as compared to automatic.
- The r squared for our model is low, with only 36% of variation explained by the model. This makes sense because models with only one variable usually isn't enough.

Simple linear regression is usually insufficient in terms of creating a good model that can predict mpg because there are other predictor variables or regressors that can help explain more variation in the model. Thus, this is where multivariate linear regression can help us fit more variables to produce a better model.

Multivariate Linear Regression Model

The goal is to create a model that best predicts mpg, or ultimately fuel efficiency. This means that each of the predictors variables should have a statistically significant p-value and are not correlated in any way (this will be tested with the Variance Inflation factor later on). Model fit can also be tested with anova, where you can observe whether adding a variable explains away a significant portion of variation (or looking at the p-value).

The challenge with Multivariate regression is which variables you should include or remove. Here we see what happens if we include all the variables in the data.

```r
full.model <- lm(mpg ~ ., data = mtcars)
summary(full.model)
```

cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
15.37383	21.62024	9.832037	3.37462	15.16489	7.527958	4.965873	4.648487	5.357452	7.908747

```
##
## Call:
## lm(formula = mpg ~ ., data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.4506 -1.6044 -0.1196  1.2193  4.6271
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 12.30337   18.71788   0.657   0.5181
## cyl         -0.11144    1.04502  -0.107   0.9161
## disp         0.01334    0.01786   0.747   0.4635
## hp          -0.02148    0.02177  -0.987   0.3350
## drat         0.78711    1.63537   0.481   0.6353
## wt          -3.71530    1.89441  -1.961   0.0633 .
## qsec         0.82104    0.73084   1.123   0.2739
## vs1          0.31776    2.10451   0.151   0.8814
## ammanual     2.52023    2.05665   1.225   0.2340
## gear         0.65541    1.49326   0.439   0.6652
## carb        -0.19942    0.82875  -0.241   0.8122
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.65 on 21 degrees of freedom
## Multiple R-squared:  0.869,  Adjusted R-squared:  0.8066
## F-statistic: 13.93 on 10 and 21 DF,  p-value: 3.793e-07
```

You can see that almost all (besides wt) the variables have p-values that are not significant

An issue with multivariate regression is certain variables may be correlated with each other, which can increase the standard error of other variables. To assess colinearity, we can use the Variance inflation factor, which r has a nifty function (vif) that does so.

```r
rbind(vif(full.model))  %>%
  kbl() %>%
  kable_styling(bootstrap_options = c("striped", "hover"))
```

We see some of the variables have really high VIF (more than 10) which shows signs of co linearity.

Stepwise regression model

There are many ways to test for different variables to choose the best model, here I will be using the stepwise selection method to help find the predictor variables that can best explain MPG.

```r
bestModel <- step(full.model, direction = "both",
                  trace = FALSE)
summary(bestModel)
```

	VIF of bestmodel
wt	2.482951
qsec	1.364339
am	2.541437

```
##
## Call:
## lm(formula = mpg ~ wt + qsec + am, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.4811 -1.5555 -0.7257  1.4110  4.6610
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   9.6178     6.9596   1.382 0.177915
## wt           -3.9165     0.7112  -5.507 6.95e-06 ***
## qsec          1.2259     0.2887   4.247 0.000216 ***
## ammanual      2.9358     1.4109   2.081 0.046716 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.459 on 28 degrees of freedom
## Multiple R-squared:  0.8497, Adjusted R-squared:  0.8336
## F-statistic: 52.75 on 3 and 28 DF,  p-value: 1.21e-11
```

Using the stepwise method, we end up with 3 predictor variables, wt, qsec and am.

- all three variables have significant p-values, which suggest that they are all important addition to the model for predicting mpg.
- note that our am variable has a less significant p-value after adjusting for variabes wt and qsec
- after adjusting for other predictor variables, our coefficient for am went down to 2.94, and our pvalue became less significant.
- The r squared value denotes how much of the variation in mpg is explained. Our best model explains around 84% of the variation, which indicates it's a good model.

Regression Diagnostics

```
vif <- cbind(vif(bestModel))
colnames(vif) <- "VIF of bestmodel"
vif  %>%
  kbl() %>%
  kable_styling(bootstrap_options = c("striped", "hover"), full_width = F)
```

The variance inflation factor of all three of our variables are small, which means they are not highly correlated.

##Anova test on nested models

Anova is a useful statistical tool to use on nested models. With it, we can interpret what the effects of adding a new variable are on the coefficients and the p-values

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
30	720.8966	NA	NA	NA	NA
29	278.3197	1	442.57690	74.9945513	0.0000000
28	169.2859	1	109.03377	18.4757461	0.0002140
27	166.0099	1	3.27607	0.5551293	0.4629119
26	153.4378	1	12.57205	2.1303314	0.1563873

```
fit0 <- lm(mpg ~ am, mtcars)
fit1 <- update(fit0, mpg ~ am + wt)
fit2 <- update(fit1, mpg ~ am + wt + qsec)
fit3 <- update(fit2, mpg ~ am + wt + qsec + disp)
fit4 <- update(fit3, mpg ~ am + wt + qsec + disp + hp)

anova(fit0, fit1, fit2, fit3, fit4)  %>%
  kbl() %>%
  kable_styling(bootstrap_options = c("striped", "hover"))
```

Looking at the results, we see how our best fit gives us a significant result (consistent with our stepwise selection model), but adding the variables disp and hp gives us p-values that are not significant, thus a worse model.

Residual diganostic plots

To diagnose a regression model it's also important to look at the residual diagnostics, which can be seen at the appendix.

- From our residual vs fitted plot, we don't see any distinct patterns, which is a good sign
- Our normal Q-Q plot shows that our standardized residuals are considerably normal, and doesn't deviate that much from the line.
- scale-location is compares standardized residuals with fitted values, and we don't see any patterns as well
- Our residual vs leverage plot don't contain any systematic patterns.

#Appendix

##Code

```
library(GGally)
library(dplyr)
library(ggplot2)
library(car)
library(broom)
library(printr)
library(pander)
library(kableExtra)

theme_set(theme_classic())
```

```
head(mtcars) %>%
  kbl() %>%
  kable_styling(bootstrap_options = c("striped", "hover"))
```

```r
# factoring categorical variables for regression
mtcars <- mtcars %>%
    mutate(am = factor(am, labels = c("automatic", "manual"))) %>%
    mutate(vs = factor(vs))
```

```r
panderOptions('table.split.table', '50')
pander(t.test(mtcars$mpg ~ mtcars$am))
```

```r
fit_am <- lm(mpg ~ am, mtcars)
summary(fit_am)
```

```r
full.model <- lm(mpg ~ ., data = mtcars)
summary(full.model)
```

```r
rbind(vif(full.model))  %>%
  kbl() %>%
  kable_styling(bootstrap_options = c("striped", "hover"))
```

```r
bestModel <- step(full.model, direction = "both",
                  trace = FALSE)
summary(bestModel)
```

```r
vif <- cbind(vif(bestModel))
colnames(vif) <- "VIF of bestmodel"
vif  %>%
  kbl() %>%
  kable_styling(bootstrap_options = c("striped", "hover"), full_width = F)
```

```r
fit0 <- lm(mpg ~ am, mtcars)
fit1 <- update(fit0, mpg ~ am + wt)
fit2 <- update(fit1, mpg ~ am + wt + qsec)
fit3 <- update(fit2, mpg ~ am + wt + qsec + disp)
fit4 <- update(fit3, mpg ~ am + wt + qsec + disp + hp)

anova(fit0, fit1, fit2, fit3, fit4)  %>%
  kbl() %>%
  kable_styling(bootstrap_options = c("striped", "hover"))
```

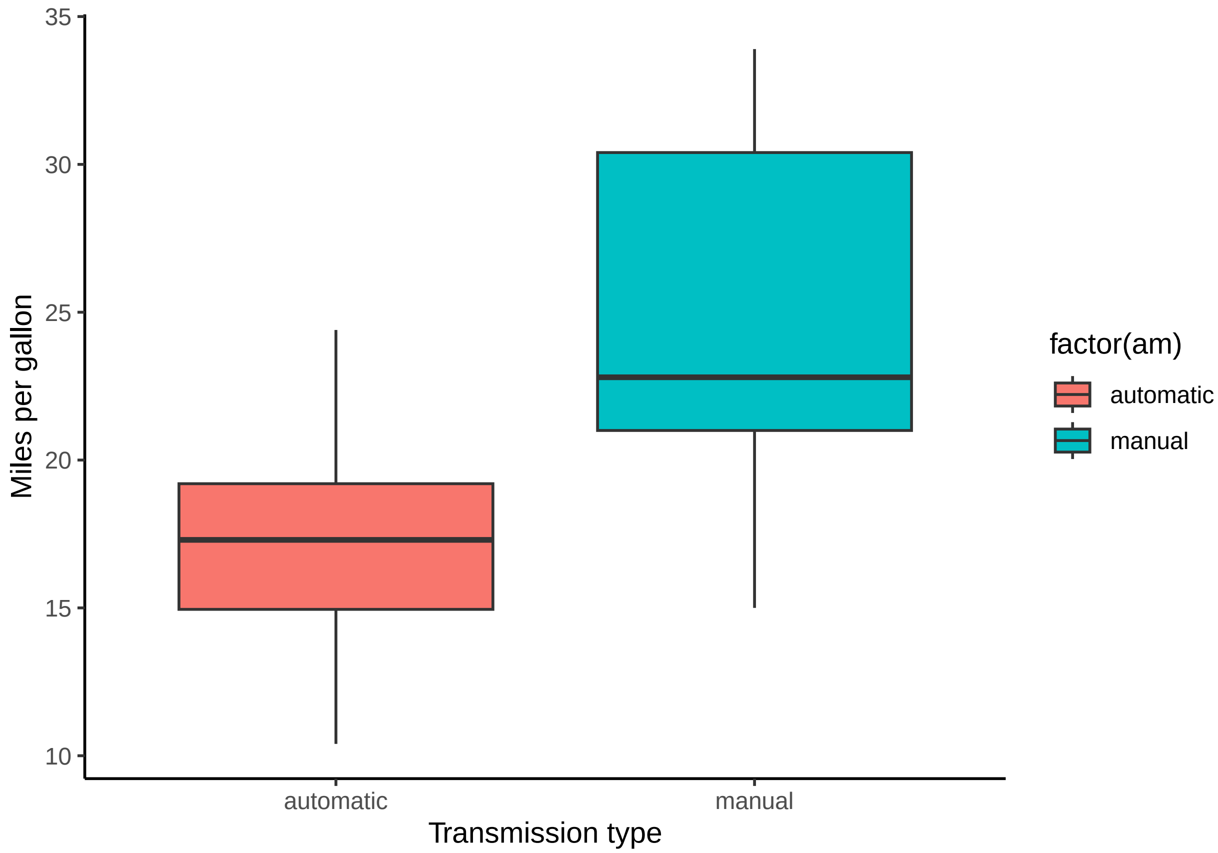
About the data

A data frame with 32 observations on 11 (numeric) variables.

[, 1] mpg Miles/(US) gallon [, 2] cyl Number of cylinders [, 3] disp Displacement (cu.in.) [, 4] hp Gross horsepower [, 5] drat Rear axle ratio [, 6] wt Weight (1000 lbs) [, 7] qsec 1/4 mile time [, 8] vs Engine (0 = V-shaped, 1 = straight) [, 9] am Transmission (0 = automatic, 1 = manual) [,10] gear Number of forward gears [,11] carb Number of carburetors

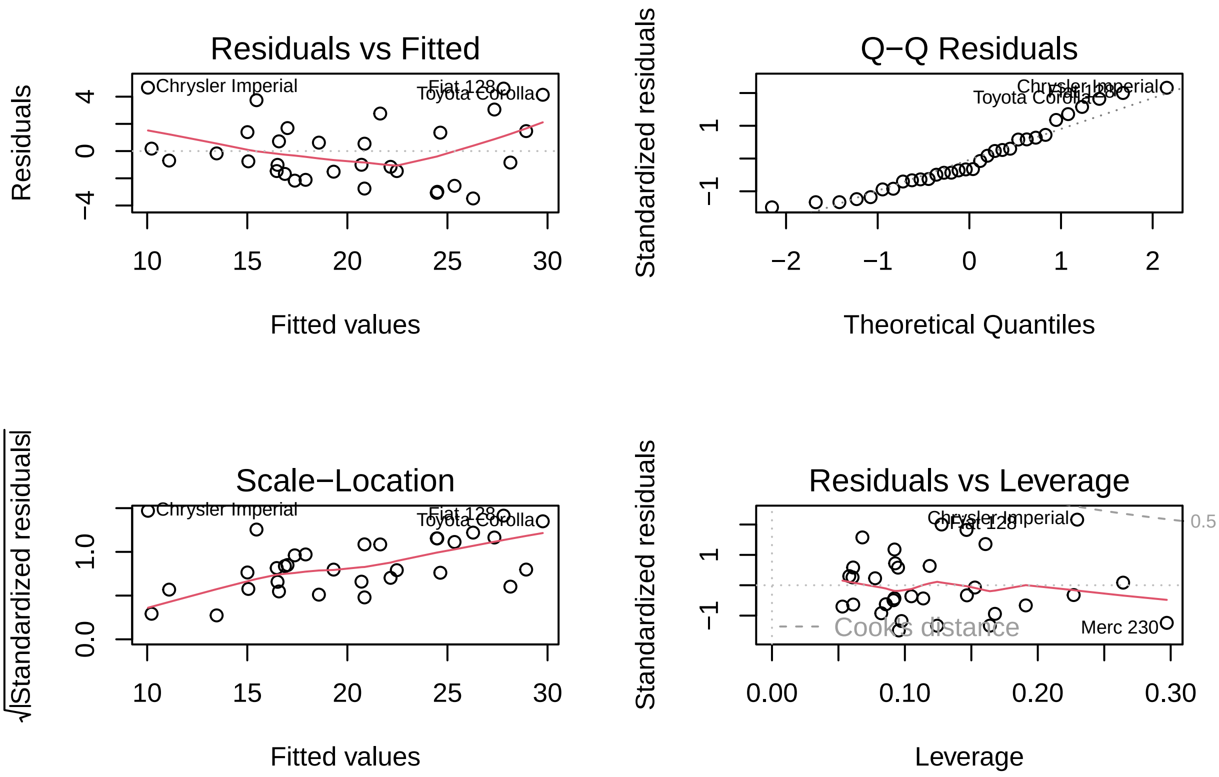
Box plot

```r
ggplot(mtcars, aes(factor(am, labels = c(
    "automatic", "manual"
)), mpg, fill = factor(am))) +
    geom_boxplot() +
    labs(x = "Transmission type", y="Miles per gallon")
```



Regression diagnostics plot

```r
par(mfrow = c(2, 2))
plot(bestModel)
```



Session info

```r
sessionInfo()
```

```
## R version 4.4.1 (2024-06-14 ucrt)
## Platform: x86_64-w64-mingw32/x64
## Running under: Windows 10 x64 (build 19045)
##
## Matrix products: default
##
##
## locale:
## [1] LC_COLLATE=English_India.utf8  LC_CTYPE=English_India.utf8
## [3] LC_MONETARY=English_India.utf8 LC_NUMERIC=C
## [5] LC_TIME=English_India.utf8
##
## time zone: Asia/Calcutta
## tzcode source: internal
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] kableExtra_1.4.0 pander_0.6.5     printr_0.3       broom_1.0.7
```

```
## [5] car_3.1-3        carData_3.0-5    dplyr_1.1.4      GGally_2.2.1
## [9] ggplot2_3.5.1
##
## loaded via a namespace (and not attached):
##  [1] utf8_1.2.4         generics_0.1.3     tidyr_1.3.1        xml2_1.3.6
##  [5] stringi_1.8.4      digest_0.6.37      magrittr_2.0.3     evaluate_1.0.0
##  [9] grid_4.4.1         RColorBrewer_1.1-3 fastmap_1.2.0      plyr_1.8.9
## [13] backports_1.5.0    Formula_1.2-5      purrr_1.0.2        fansi_1.0.6
## [17] viridisLite_0.4.2  scales_1.3.0       abind_1.4-8        cli_3.6.3
## [21] rlang_1.1.4        munsell_0.5.1      withr_3.0.1        yaml_2.3.10
## [25] tools_4.4.1        colorspace_2.1-1   ggstats_0.7.0      vctrs_0.6.5
## [29] R6_2.5.1           lifecycle_1.0.4    stringr_1.5.1      pkgconfig_2.0.3
## [33] pillar_1.9.0       gtable_0.3.5       glue_1.8.0         Rcpp_1.0.13
## [37] systemfonts_1.1.0  highr_0.11         xfun_0.49          tibble_3.2.1
## [41] tidyselect_1.2.1   rstudioapi_0.17.1  knitr_1.48         farver_2.1.2
## [45] htmltools_0.5.8.1  labeling_0.4.3     rmarkdown_2.28     svglite_2.1.3
## [49] compiler_4.4.1
```