# Math 2605 Project

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## 1 Hilbert Matrix

### 1.1 Why is it justified to use the LU or QR factorizations as opposed of calculating an inverse matrix?

It is justified to use the LU or QR factorizations as opposed to calculating an inverse matrix since calculating the inverse matrix is more time consuming than solving the problem using LU or QR factorizations. The range of the problem entails that we only solve for b, which LU/QR does without as much calculations as finding the inverse of the matrix. Essentially, we are not trying to find every single x value possible for every single b value, we are trying to find a single x value for a single b value, which LU/QR factorization is more efficient at. If the range of the problem was to find all solutions x provided a b vector, then calculating an inverse matrix would have been more efficient than factorization using the LU or QR factorization methods.

### 1.2 What is the benefit of using LU or QR-factorization in this way?

The benefit of using LU or QR-factorization in this way is that it saves time and energy by utilizing these factorization methods as opposed to manipulating a matrix B and deriving the inverse through the elimination of matrix A, which can take much longer whether done cognitively by a human or computationally by a machine.

# 2 Convolution Codes

## 2.1 Compare the results of the two methods above and discuss the number of iterations required to obtain the desired precision.

The Jacobi method substitutes the values of $x_i$ into the right hand side of the rearranged equations to get the appropriate approximation. These approximations can lead to iterations. The repeated iterations will form approximations that converges to the solution.

The Gauss-Seidel method can use the new values of $x_i$ as soon as they are known, and this $x_i$ can be determined to find $x_i + 1$.

For both Jacobi and Gauss-Seidel, we are trying to rearrange n equations and $n$ variables into the form $x = Tx + c$.

For convolution codes, the Gauss-Seidel and the Jacobi code generates a random x input. And this input is encoded into a y, which is decoded back into x. For example, when we provided the input y = 00, 00, 11, 01, 10, 11, 00, 00, 00, the Gauss-Seidel method decodes the given y with a mere 2 iterations, whereas for the Jacobi method, given the same input it took 6 iterations to converge to the solution. So clearly, based on the numbers, we see that the Gauss-Seidel is much more efficient than Jacobi, due to the fact that Gauss-Seidel utilizes its new values as soon as they are known, taking less iterations to approximately get to the solution than Jacobi. Therefore, the Gauss-Seidel method converges several times faster than the Jacobi iteration method.

## 2.2 Is the length of the initial stream $n$ important?

The length of the initial stream $n$ is important due to the fact that with the Gauss-Seidel method, the longer the initial stream $n$ is, the faster it converged to the solution.

### 2.3 Does $n$ have an effect on the number of iterations required to achieve the error tolerance?

As for the effect on the number of iterations, $n$ indeed has an effect on the number of iterations required to achieve the error tolerance, because the more iterations we have the more precise the answer will be, since with each iteration the program converges closer and closer to the correct value. Therefore, we will have less error tolerance with a larger $n$.

# 3 Urban Population Dynamics

## 3.1 Interpret the data in the matrix, and discuss the social factors that influence those numbers.

Given a certain age bracket, each value in the first row represents the reproduction rate of each age group for the female gender. The first value in the first row and first column, 0, is such due to the social tendency of young children aged 0-9 to not have children. The next 4 values represent the reproduction rates for the respective age groups 10-19, 20-29, 30-39, 40-49. The following 4 values are 0, because after the age of 50, human beings tend to stop reproducing and/or die.

The values on the diagonal represent the survival rate of each age groupin percentageto the next age group. For example, the value in the second row, first column, is 0.7, which reads 70 percent of the 0-9 year old age group make it to the 10-19 years age group. The social factors that yield this result is due to the fact that children are very vulnerable to environmental factors such as diseases, and are more likely to die.

## 3.2 What will the population distribution be in 2010? 2020? 2030? 2040? 2050? Calculate also the total population in those years, and by what fraction the total population changed each year.

### 3.2.1 In the year 2010

The population distribution is $(6.35, 1.47, 1.62, 1.62, 1.89, 1.76, 1.36, 0.92, 0.36) * 10^5$ and the total population is $17.25 * 10^5$. That is 1.22 times greater than the 2000 population.

### 3.2.2 In the year 2020

The population distribution is $(5.19, 4.45, 1.25, 1.45, 1.46, 1.66, 1.41, 1.05, 0.37) * 10^5$ and the total population is $18.28 * 10^5$. That is 1.05 times greater than the 2010 population.

### 3.2.3 In the year 2030

The population distribution is $(8.16, 3.63, 3.78, 1.12, 1.31, 1.28, 1.33, 1.08, 0.42) * 10^5$ and the total population is $22.12 * 10^5$. That is 1.21 times greater than the 2020 population.

### 3.2.4 In the year 2040

The population distribution is $(9.66, 5.71, 3.09, 3.40, 1.01, 1.15, 1.03, 1.02, 0.43) * 10^5$ and the total population is $26.51 * 10^5$. That is 1.20 times greater than the 2030 population.

### 3.2.5 In the year 2050

The population distribution is $(13.41, 6.76, 4.86, 2.78, 3.06, 0.89, 0.92, 0.79, 0.41) * 10^5$ and the total population is $33.88 * 10^5$. That is 1.28 times greater than the 2040 population.

## 3.3 Use the power method to calculate the largest eigenvalue of the Leslie matrix A. The iteration of the power method should stop when you get 8 digits of accuracy.

### 3.3.1 What is the largest eigenvalue of matrix A

The largest eigenvalue of matrix A is 1.2886562.

### 3.3.2 What does the largest eigenvalue tell you?

The population that the Leslie matrix represents will increase exponentially by a factor of the largest eigenvalue, which in our case is 1.28886562

### 3.3.3 Will the population go to zero, become stable, or be unstable in the long run? You might want to investigate the convergence of $||A_k||$.

The population will become unstable, because the population will become so large, due to exponential growth, that the world will become overpopulated since the population will increase by a factor of about 1.29 every ten years.

## 3.4 Suppose we are able to decrease the birth rate of the second age group by half in 2020. What are the predictions for 2030, 2040 and 2050? Calculate again the largest eigenvalue of A with your program and discuss its meaning regarding the population in the long run.

If the birth rate of the second age group was decreased by half in 2020 and stayed halved for the remainder of the period, the results would be as follows:

### 3.4.1 For the year 2030

The population distribution is $(8.16, 3.63, 1.19, 1.12, 1.31, 1.28, 1.33, 1.08, 0.42) * 10^5$ and the total population is $20.23 * 10^5$. That is 1.11 times greater than the 2020 population.

### 3.4.2 For the year 2040

The population distribution is $(7.58, 5.71, 1.54, 1.70, 1.01, 1.15, 1.03, 1.02, 0.43) * 10^5$ and the total population is $21.28 * 10^5$. That is 1.05 times greater than the 2030 population.

### 3.4.3 For the year 2050

The population distribution is $(10.19, 5.30, 2.43, 1.39, 1.53, 0.89, 0.92, 0.79, 0.41) * 10^5$ and the total population is $23.85 * 10^5$. That is 1.13 times greater than the 2040 population.

### 3.4.4 Calculate the largest eigenvalue of A and discuss its meaning regarding the population in the long run.

The largest eigenvalue of the modified Leslie matrix is 1.1500418. This means that the population will continually increase by a factor of around 1.15 every 10 years.