lab_taylorexp

August 28, 2017

1 Taylor approximation

For analytic function equivalence between value of a function at each point or value of all derivatives at a single point. Really?

We consider approximating the function

$$h(t) = t^2 e^{3t}$$

in the neighbourhood of some value $t = t_0$. The first-order Taylor approximation to $h(t)$ is

$$h_1(t) = h(t_0) + h'(t_0)(t - t_0),$$

where $h'(t)$ is the derivative of $h(t)$.

We define the symbolic function $h(t)$ and find its symbolic derivative $h'(t)$:

```
In [2]: import sympy as sp

        t = sp.symbols('t');
        h = sp.exp(3*t)*t**2;  # h(t)
        hp = h.diff(t);  # h'(t)
        hp
```

```
Out[2]: 3*t**2*exp(3*t) + 2*t*exp(3*t)
```

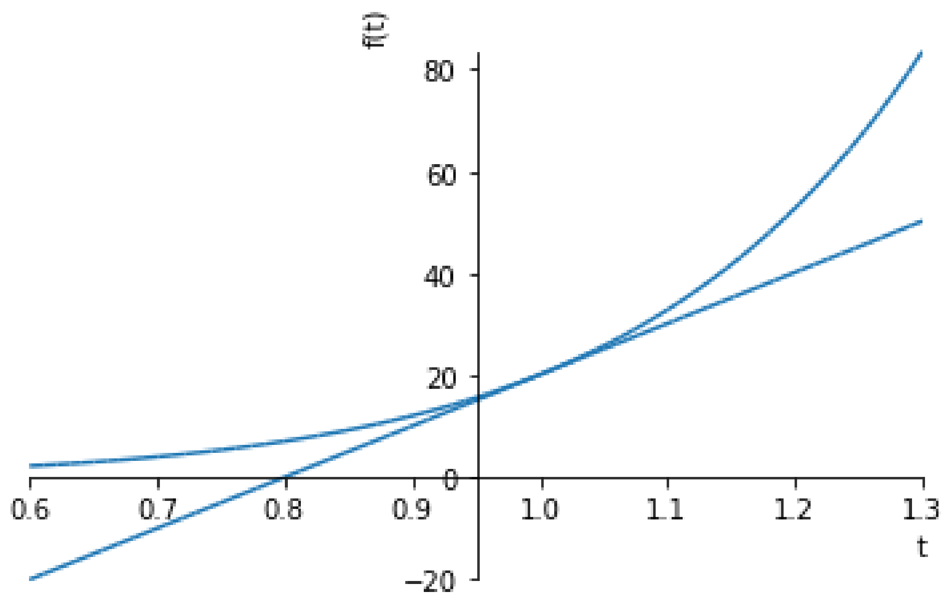
We want to evaluate $h(t_0)$ and $h'(t_0)$ so that we can use the approximation. We don't yet have to put in a value so we just substitute the symbol `t0`. The result will then be a symbolic expression in `t` and `t0`:

```
In [4]: t0 = sp.symbols('t0');
        h0 = h.subs(t,t0);  # h(t0)
        h1 = h0 + hp.subs({t:t0})*(t-t0);  # h1(t) Taylor approximation at t0
        print(h1);
```

```
t0**2*exp(3*t0) + (t - t0)*(3*t0**2*exp(3*t0) + 2*t0*exp(3*t0))
```

For a fixed value for `t0` we can plot the symbolic function $h(t)$ and the first-order Taylor expansion $h'(t)$ at that point:

```
In [10]: h1s = h1.subs(t0, 1);  # h1(t) for value t0=1
         sp.plot(h, h1s, (t,0.6,1.3));
```



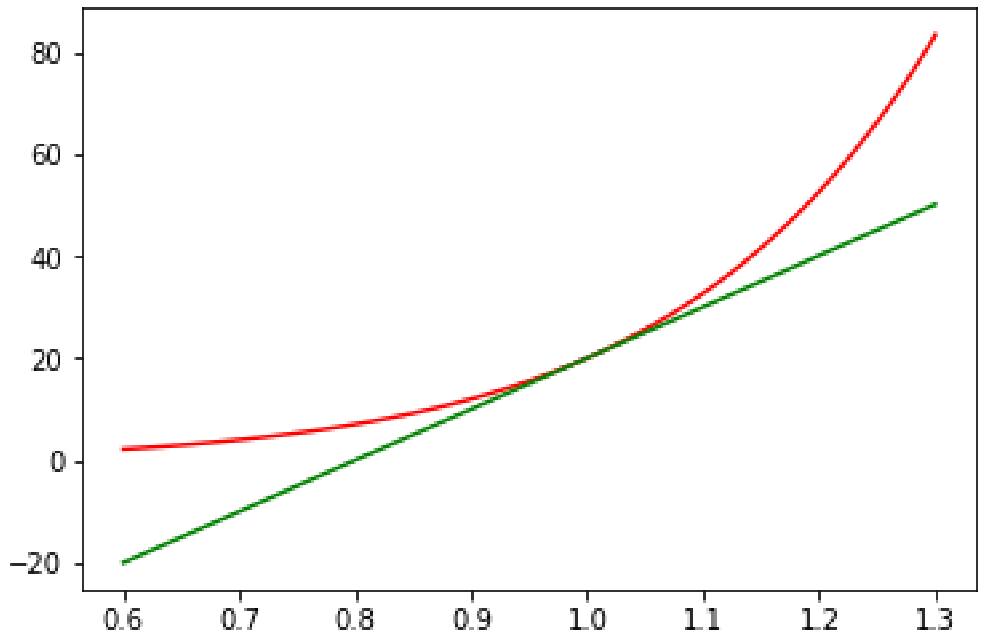
The previous cell used a symbolic plotting function. We could instead convert the symbolic expressions to functions that can be evaluated numerically using

```
In [11]: lam_h = sp.lambdify(t, h, modules=['numpy'])
         lam_h1s = sp.lambdify(t, h1s, modules=['numpy'])
```

and then plot from samples of `tv`:

```
In [12]: import numpy as np
         import matplotlib.pyplot as plt

         tv = np.linspace(0.6, 1.3, 100);
         h_vals = lam_h(tv);  h1s_vals = lam_h1s(tv)
         fig, ax = plt.subplots(1,1);
         ax.plot(tv, h_vals, 'r-', tv, h1s_vals, 'g-');
```




```
In [1]: %run src/labX_preamble.py  # For internal notebook functions
```

```
In [7]: %%writefileexec src/lab_taylorexp-1.py -s  # dump cell to file before execute

        import sympy as sp
        import numpy as np
        import matplotlib.pyplot as plt
        %matplotlib inline

        # Symbolic function and derivatives
        h, t = sp.symbols('h t');
        h = sp.exp(3*t)*t**2;
        hp = h.diff(t);
        #hpp = hp.diff(t);

        # Taylor expansions around point t0
        t0 = 1;
        h0 = h.subs(t,t0);
        h1 = h0 + hp.subs({t:t0})*(t-t0);
        #h2 = h0 + hp.subs(t,t0)*(t-t0) + 1/2*hpp.subs(t,t0)*(t-t0)**2;

        # Direct Taylor expansion using sympy
        h5s = sp.series(h, t, t0, 6).removeO();
        print("Taylor 5: ", h5s);
```

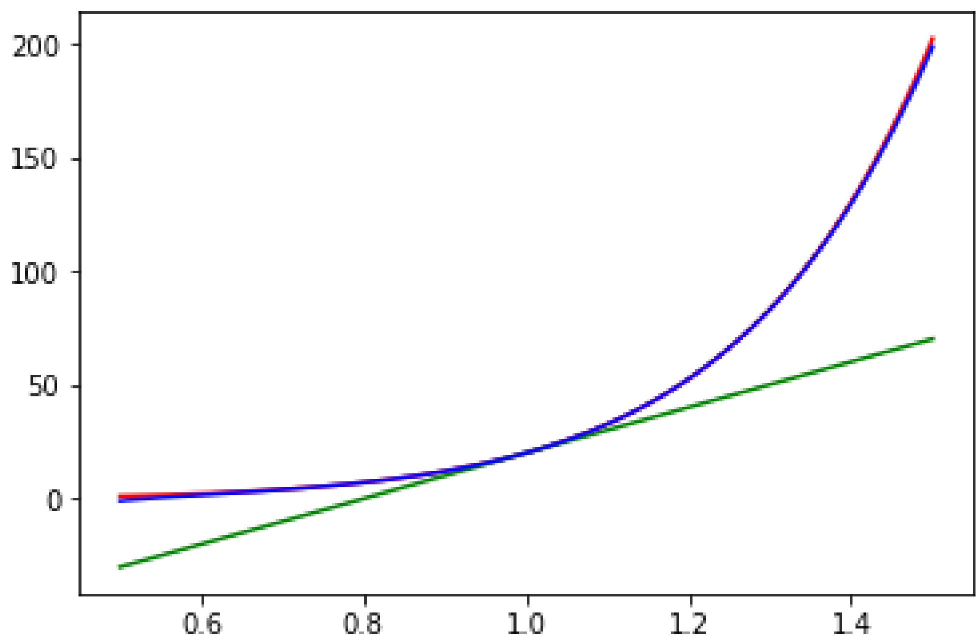
```
# Convert symbolic to functions that can be evaluated
lam_h = sp.lambdify(t, h, modules=['numpy']);
lam_h1 = sp.lambdify(t, h1, modules=['numpy']);
lam_h5s = sp.lambdify(t, h5s, modules=['numpy']);

# Plots
fig, ax = plt.subplots(1,1);
t_vals = np.linspace(0.5, 1.5, 100);
ax.plot(t_vals, lam_h(t_vals), 'r');
ax.plot(t_vals, lam_h1(t_vals), 'g');
#ax.plot(t_vals, lam_h5s(t_vals), 'b');

# Symbolic plotting also probably works
#sp.plot(h, h1, (t, 0, 1.5))
```

```
Taylor 5:  531*(t - 1)**5*exp(3)/40 + 135*(t - 1)**4*exp(3)/8 + 33*(t - 1)**3*exp(3)/2 + 23*(t
```

```
Out[7]: [<matplotlib.lines.Line2D at 0x10794dc50>]
```



2 Tasks

These tasks involve writing code, or modifying existing code, to meet the objectives described.

1. Generate Taylor series expansions of order 5, 9, and 13 centered around the point $t = 0$ for the function $x(t) = \sin(t)$. Plot all of these functions on the same set of axes over the domain $t = -2\pi$ to $t = 2\pi$, with a range from -1.5 to 1.5.
2. xxx

```
In [ ]:
```