1. The sum of the first five positive integers and the sum of the first six positive integers are multiplied. What is the resulting product?

   (A) 315 **(B)** 335 **(C)** 355 **(D)** 375 **(E)** 395

2. Jack is reading a 100 page book. He reads two pages every minute. After every 12 pages he reads, he takes a one minute break. If Jack starts reading at 2:00, what time will it be when he finishes reading his book?

   (A) 2:32 **(B)** 2:56 **(C)** 2:58 **(D)** 3:05 **(E)** 3:18

3. In square $ABCD$, let M be the midpoint of side $\overline{CD}$, and let N be the reflection of M over side $\overline{AB}$. What fraction of $\triangle MND$ lies within $ABCD$?

   (A) $\frac{1}{2}$ **(B)** $\frac{5}{8}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{7}{8}$

4. Which of the following is closest to the value of $\sqrt{2020}$?

   (A) 44.6 **(B)** 44.7 **(C)** 44.8 **(D)** 44.9 **(E)** 45.0

5. Pedro currently has 2 quarters, 3 dimes and 2 pennies. If he can only obtain quarters, dimes, nickels, and pennies, what is the minimum number of coins he needs to earn in order to reach a total of exactly 1 dollar?

   (A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 10

6. 15 students are to be split into 5 groups of 3 to work on a project. Alice, Bob, and Cooper are three of the students. Given that Alice and Bob are in the same group, what is the probability that Cooper is not in that group?

   (A) $\frac{4}{7}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{8}{9}$ **(E)** $\frac{12}{13}$

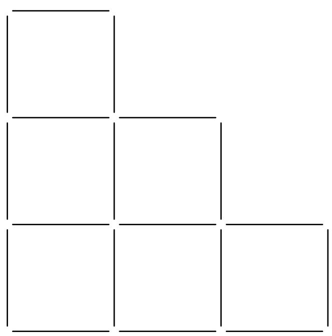
7. When all 9 diagonals of regular hexagon $AMCTEN$ are drawn, they partition the hexagon into some number of individual regions. What percent of these regions are triangular?

   (A) 25% **(B)** 37.5% **(C)** 50% **(D)** 62.5% **(E)** 75%

8. Mark wants to distribute all 100 pieces of his candy to his five children, Albert, Bob, Charlie, Diana and Ethan. Diana and Ethan insist on each having a prime number of candies whose sum is also a prime number. Charlie insists on having exactly 35 candies, exactly 1 more than Albert and Bob's amounts combined. Given that Ethan has the smallest number of candies, how many candies must Mark give to Diana?

(A) 3 **(B)** 7 **(C)** 17 **(D)** 23 **(E)** 29

9. A 3-step staircase is shown below, where each side of each square is of unit length.



Extend this pattern to create two 2021-step staircases. When these two staircases are fit together to form a 2021×2022 rectangle, the two staircases meet each other at a crease of length L. What is L? (In the resulting rectangle, a crease is defined as the total length of the segments touched by both of the original staircases.)

(A) 2022 **(B)** 2023 **(C)** 4041 **(D)** 4042 **(E)** 4043

10. Let the area of equilateral triangle ABC be 9. Let O denote its circumcenter. Let ω_1 and ω_2 be circles centered at A and B, respectively, such that they both pass through O. Let ω_1 and ω_2 intersect at $P \neq O$. What is the area of $CBPA$?

(A) $6\sqrt{3}$ **(B)** 12 **(C)** $8\sqrt{3}$ **(D)** $9\sqrt{3}$ **(E)** 15

11. How many integers n satisfy

$$|n^2 - 5n + 2|^{(n^2-9n+21)} = 2?$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

12. Kevin writes the first 10 positive perfect squares on a whiteboard. He then uses as many of the digits that he wrote as possible to create a multiple of 9. For example, with the digits 9, 8, 2, and 1, he can create the number 189. How many digits does Kevin use?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

13. A positive integer is called a *flake* if it has at least three distinct prime factors. Two flakes are said to be in a *snowflake* if there exists a prime such that the greatest common divisor of the two flakes evenly divides the prime. When two flakes in a snowflake are multiplied, what is the least possible number of positive divisors in the resulting number?

(A) 18 **(B)** 27 **(C)** 48 **(D)** 54 **(E)** 64

14. A function f is defined by a real-valued expression

$$f(x) = \frac{2020x+1}{x+2020} + \frac{\sqrt[4]{|x|-2021} - \sqrt[8]{2021-|x|}}{|x-2021|}.$$

For x in the domain of f, $f(x)$ can be simplified to a value N. What is the remainder when N is divided by 100?

(A) 19 **(B)** 21 **(C)** 41 **(D)** 59 **(E)** N is not an integer.

15. A right circular cone has base radius 3 and height 4. Then, the largest possible cube that can be inscribed inside the cone is inserted, such that two of its faces are parallel to the base of the cone. The side length of the cube can be written as $a - b\sqrt{c}$, where a, b, and c are positive integers, and c is not divisible by the square of any prime. What is $a + b + c$?

(A) 62 **(B)** 64 **(C)** 66 **(D)** 68 **(E)** 70

16. Bob and Bill are playing a card game with 7 red cards and 3 blue cards. Each of the cards are numbered from 1 to 10, inclusive, and are flipped over so that neither person can see the numbers. Bill knows that the blue cards are numbered 1, 8, and 9, but Bob does not know this. Bill chooses a card first, and then Bob chooses a different card. Using optimal strategy, what is the probability that Bill chooses a card with a larger number?

(A) $\frac{1}{2}$ **(B)** $\frac{5}{9}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{7}{10}$ **(E)** $\frac{25}{27}$

17. A sequence is defined recursively by $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all integers $n \geq 2$. Let S be the sum of all positive integers n such that $|F_n - n^2| \leq 3n$. What is the sum of the digits of S?

(A) 6 **(B)** 7 **(C)** 10 **(D)** 11 **(E)** 14

18. In right triangle ABC with right angle at B, $AB = 15$ and $BC = 20$. Point D is chosen on side $\overline{AC}$ and is reflected over sides $\overline{AB}$ and $\overline{BC}$ to create points M and N, respectively. What is the smallest possible value of MN?

(A) 12 **(B)** 15 **(C)** 24 **(D)** 30 **(E)** 32

19. Farmers A and B are mowing a rectangular grid of 21 rows and 23 columns of cells of grass. A starts from the top-left cell of the grid and mows right until he has mowed all the grass in the row. Then, B starts from where A stopped and mows down the same column until he has mowed all the grass in the column. They continue taking turns mowing, each time turning $90°$ clockwise, switching the farmer, and mowing only unmowed grass in the same direction until not possible. If A and B mow at rates of 7 and 3 cells per minute, respectively, find the number of minutes it will take them to finish mowing all the grass, assuming it takes no time to turn $90°$ clockwise and switch between the farmers.

(A) 108 **(B)** 109 **(C)** 110 **(D)** 111 **(E)** 112

20. A coin is painted such that one side is red and the other side is blue. A die is painted such that all 6 faces are blue. Each move, Daniel flips the coin and rolls the die. He then paints the face facing up on the die the color of the side facing up on the coin. The probability that the die is completely red after 7 moves can be expressed as $\frac{p}{12^q}$, where p and q are positive integers such that p is not divisible by 12. What is $p + q$?

(A) 40 **(B)** 41 **(C)** 42 **(D)** 43 **(E)** 44

21. In acute $\triangle ABC$ with circumcircle Γ, the tangents to Γ at points B and C intersect at point D. Segment $\overline{AD}$ intersects Γ at another point $E \neq A$ and $\overline{BC}$ at a point F. If $BF = 8$, $CF = 6$, and $EF = 4$, then the area of $\triangle BCD$ can be expressed as $a\sqrt{b}$, where a and b are positive integers, and b is not divisible by the square of any prime. What is $a + b$?

(A) 146 **(B)** 147 **(C)** 148 **(D)** 149 **(E)** 150

22. Andy and Aidan want to meet up to write a math problem. Before meeting, they each choose a random time between 12:00 PM and 1:30 PM to arrive. After arriving, Andy will wait 40 minutes before leaving, while Aidan will wait 10 minutes before leaving. Later, Andy learns that he has an unexpected recital he has to attend at 1:10 PM, so if Andy is waiting for Aidan to arrive and the time passes 1:10 PM, he will leave, and if Andy chooses a time after 1:10 PM, he will not come to the meeting. What is the probability that Andy meets Aidan?

(A) $\dfrac{37}{162}$ **(B)** $\dfrac{2}{9}$ **(C)** $\dfrac{53}{162}$ **(D)** $\dfrac{10}{27}$ **(E)** $\dfrac{4}{9}$

23. What is the sum of the digits of the remainder when

$$(4^2 - 9)(5^2 - 9)(6^2 - 9) \cdots (93^2 - 9)$$

is divided by 97?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

24. A right square pyramid $ABCDM$ with square base $ABCD$ and apex M has $AB = 1$ and $AM = 2$. Alex the ant moves an 8-unit path starting at vertex A such that from any vertex, Alex can move to any other vertex strictly along an edge of the pyramid. Given that Alex ends at a vertex, what is the probability that Alex ends at vertex A?

(A) $\dfrac{4}{15}$ **(B)** $\dfrac{97}{352}$ **(C)** $\dfrac{13}{44}$ **(D)** $\dfrac{53}{176}$ **(E)** $\dfrac{106}{345}$

25. Let a, b, c, d be real numbers with $a \geq b \geq c \geq d$ satisfying

$$\begin{aligned} a + b + c + d &= 0, \\ a^2 + b^2 + c^2 + d^2 &= 100, \\ a^3 + b^3 + c^3 + d^3 &= (a+b)(a+c)(b+c). \end{aligned}$$

The maximum possible value of $a^2 + b$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 201 **(B)** 203 **(C)** 205 **(D)** 207 **(E)** 209