FUNCTIONAL DEPENDENCY THEORY

Relational Database Design _Part 3b

Outline

Features of Good Relational Design

Atomic Domains and First Normal Form

Functional Dependencies

Normal Forms

Functional Dependency Theory

Decomposition Using Functional Dependencies

Algorithms for Decomposition using Functional Dependencies

Decomposition Using Multivalued Dependencies

More Normal Form

Canonical Cover

Sets of functional dependencies may have redundant dependencies that can be inferred from the others

- For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B, \ B \rightarrow C, A \rightarrow C\}$
- Parts of a functional dependency may be redundant
  - E.g.: on RHS: $\{A \rightarrow B, \ B \rightarrow C, \ A \rightarrow CD\}$ can be simplified to $\{A \rightarrow B, \ B \rightarrow C, \ A \rightarrow D\}$
  - E.g.: on LHS: $\{A \rightarrow B, \ B \rightarrow C, \ AC \rightarrow D\}$ can be simplified to $\{A \rightarrow B, \ B \rightarrow C, \ A \rightarrow D\}$

Intuitively, **a canonical cover of F** is a “minimal” set of functional dependencies equivalent to F, having no redundant dependencies or redundant parts of dependencies

Extraneous Attributes

Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.

- Attribute A is **extraneous** in α if $A \in \alpha$ and F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
- Attribute A is **extraneous** in β if $A \in \beta$ and the set of functional dependencies $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.

Note: implication in the opposite direction is trivial in each of the cases above, since a "stronger" functional dependency always implies a weaker one

Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$

- B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (I.e. the result of dropping B from $AB \rightarrow C$).

Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$

- C is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting C

Testing if an Attribute is Extraneous

Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.

To test if attribute $A \in \alpha$ is **extraneous** in α

1. compute $(\{\alpha\} - A)^+$ using the dependencies in F
2. check that $(\{\alpha\} - A)^+$ contains β; if it does, A is extraneous in α

To test if attribute $A \in \beta$ is **extraneous** in β

1. compute α^+ using only the dependencies in
   $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\},$
2. check that α^+ contains A; if it does, A is extraneous in β

Canonical Cover

Minimal set of functional dependencies

A **canonical cover** for F is a set of dependencies F_c such that

- F logically implies all dependencies in F_c, and
- F_c logically implies all dependencies in F, and
- No functional dependency in F_c contains an extraneous attribute, and
- Each left side of functional dependency in F_c is unique.

To compute a canonical cover for *F*:
repeat

Use the union rule to replace any dependencies in F
$\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$
Find a functional dependency $\alpha \rightarrow \beta$ with an
extraneous attribute either in α or in β
/* Note: test for extraneous attributes done using F_c, not
F*/

If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$
until F does not change

Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied

Computing a Canonical Cover

$R = (A, B, C)$
$F = \{A \rightarrow BC$
$B \rightarrow C$
$A \rightarrow B$
$AB \rightarrow C\}$

Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$

- Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$

A is extraneous in $AB \rightarrow C$

- Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
  - Yes: in fact, $B \rightarrow C$ is already present!
- Set is now $\{A \rightarrow BC, B \rightarrow C\}$

C is extraneous in $A \rightarrow BC$

- Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
  - Yes*:* using transitivity on $A \rightarrow B$ *and* $B \rightarrow$ C.
    - Can use attribute closure of A in more complex cases

The canonical cover is: $\boldsymbol{A \rightarrow B}$
$\boldsymbol{B \rightarrow C}$