Assignment - 1

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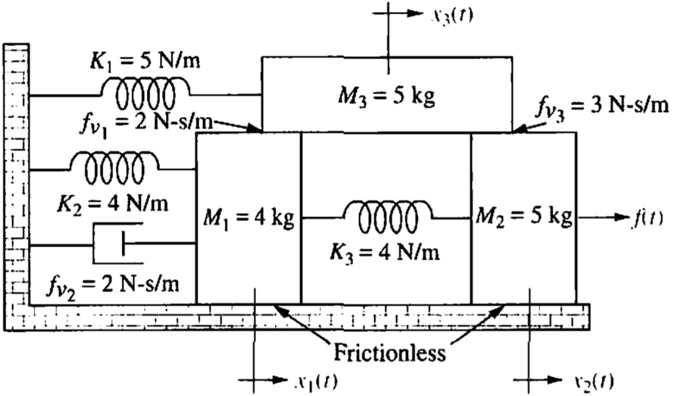
Problem Statement

Write, but do not solve, the equations of motion for the translational mechanical system shown in Figure.



(1.1)

Solution

The given system has three degree of freedoms, as all masses can be moved independently. We can form equation similar to electrical mesh equations. So, for M1 :

$$\begin{pmatrix} \textit{Sum of} \\ \textit{applied} \\ \textit{forces} \\ \textit{at } x1 \end{pmatrix} = \begin{pmatrix} \textit{Sum of} \\ \textit{impedances} \\ \textit{connected to} \\ \textit{the motion} \\ \textit{at } x1 \end{pmatrix} \mathrm{X}_1(s) - \begin{pmatrix} \textit{Sum of} \\ \textit{impedances} \\ \textit{between} \\ x1 \textit{ and } x2 \end{pmatrix} X_2(s) - \begin{pmatrix} \textit{Sum of} \\ \textit{impedances} \\ \textit{between} \\ x1 \textit{ and } x3 \end{pmatrix} X_3(s)$$

$$\implies 0 = [M_1 s^2 + f_{v_1} s + f_{v_2} s + K_2 + K_3] X_1(s) - [K_3] X_2(s) - [f_{v_1} s] X_3(s)$$

Substituting the values of variables in above equation from given figure:

$$0 = [4\mathrm{s}^2 + 2s + 2s + 4 + 4] X_1(s) - [4] X_2(s) - [2s] X_3(s)$$

$$0 = [4s^2 + 4s + 8] X_1(s) - [4] X_2(s) - 2s X_3 \tag{2.1}$$

Similarly for M2 :

$$\begin{pmatrix} Sum\ of \\ applied \\ forces \\ at\ x2 \end{pmatrix} = \begin{pmatrix} Sum\ of \\ impedances \\ connected\ to \\ the\ motion \\ at\ x2 \end{pmatrix} \mathrm{X}_2(s) - \begin{pmatrix} Sum\ of \\ impedances \\ between \\ x2\ and\ x1 \end{pmatrix} X_1(s) - \begin{pmatrix} Sum\ of \\ impedances \\ between \\ x2\ and\ x3 \end{pmatrix} X_3(s)$$

$$\implies F(s) = [M_2 s^2 + f_{v_3} s + K_3] X_2(s) - [K_3] X_1(s) - [f_{v_3} s] X_3(s)$$

Substituting the values of variables in above equation from given figure:

$$\mathrm{F(s)} = [5\mathrm{s}^2 + 3s + 4] X_2(s) - [4] X_1(s) - [3s] X_3(s)$$

$$F(s) = [5s^2 + 3s + 4] X_2(s) - [4] X_1(s) - 3s X_3 \tag{2.2}$$

Similarly for M3 :

$$\begin{pmatrix} Sum\ of \\ applied \\ forces \\ at\ x3 \end{pmatrix} = \begin{pmatrix} Sum\ of \\ impedances \\ connected\ to \\ the\ motion \\ at\ x3 \end{pmatrix} \mathrm{X}_3(s) - \begin{pmatrix} Sum\ of \\ impedances \\ between \\ x3\ and\ x1 \end{pmatrix} X_1(s) - \begin{pmatrix} Sum\ of \\ impedances \\ between \\ x3\ and\ x2 \end{pmatrix} X_2(s)$$

$$\implies F(s) = [M_3 s^2 + f_{v_3} s + f_{v_1} s + K_1] X_3(s) - [f_{v_1} s] X_1(s) - [f_{v_3} s] X_2(s)$$

Substituting the values of variables in above equation from given figure:

$$\mathrm{F(s)} = [5\mathrm{s}^2 + 2s + 3s + 4] X_3(s) - [2s] X_1(s) - [3s] X_2(s)$$

$$0 = [5s^2 + 5s + 4] X_3(s) - [2s] X_1(s) - 3s X_2(s) \tag{2.3}$$

So, the equations of motion for translational system are:
$0 = (4s^2 + 4s + 8)X_1(s) - 4X_2(s) - 2sX_3(s)$
$F(s) = (5s^2 + 3s + 4)X_2(s) - 4X_1(s) - 3sX_3(s)$
$0 = (5s^2 + 5s + 4)X_3(s) - 2sX_1(s) - 3sX_2(s)$

here, F(s) : Function of applied force on M2 in s-domain
$X_1(s)$, $X_2(s)$ *and* $X_3(s)$ *relates to motion of*
M1, M2 and M3 respectively.