Assignment 1

DSECL ZC416 - Mathematical Foundations for Data Science

Instructions

1. Use Octave or Python for coding questions.
2. By random entries, I mean a system generated random number. No marks would be awarded for deterministic entries.
3. Submissions beyond 24th of June, 2022 23.59 hrs would not be graded
4. Assignments sent via email would not be accepted
5. This is a group assignment. There should be only one submission per group.
6. Copying is strictly prohibited. Adoption of unfair means would lead to disciplinary action.
7. Assignment Q2 should be **handwritten**. Scan this handwritten solution. Combine this scan and results of programming question Q1 in a single doc file. Convert the doc file to a pdf file. Group member names should be mentioned in the pdf file and upload the file in CANVAS. Hence the final upload should be a fully self contained single pdf file.
8. Assignment Q2 should be written in NEAT and LEGIBLE handwriting. Violation will invite marks penalty.
9. Left MARGINS must be clearly drawn.
10. Scans must be clear and should not be out of focus. Out of focus assignments will be rejected and no marks given.
11. Avoid photographs of handwritten sheets and use only scanning.

Answer all the questions

Q1) Gauss Seidel and Gauss Jacobi (5 marks)

i Write a function to check whether a given square matrix is diagonally dominant. Test the function on a randomly generated 4×4 matrix.
Deliverable(s) : Code that performs the check and the results obtained for the matrix (1)

ii Write a function to generate Gauss Seidel Iteration for a given square matrix. The function should also return the values of $1, \infty$ and $Frobenius$ norms of iteration matrix. Generate a random 4×4 matrix. Report the Iteration matrix and its norm values returned by the function along with the input matrix.
Deliverable(s): The input matrix, iteration matrix and the three norms obtained (1)

iii Repeat part (ii) for the Gauss Jacobi iteration
Deliverable(s): The input matrix, iteration matrix and the three norms obtained (1)

iv Write a function that perform Gauss Seidel iterations. Generate a random 4×4 matrix A and generate a random $b \in \mathbb{R}^4$. Report the results of passing this matrix to function written in (i). Solve linear system $Ax = b$ by using function in (ii). Generate a plot of $\|x_{k+1} - x_k\|_2$ for first 100 iterations. Does it converge ? or Is it diverging? Specify your observation. Take a screenshot of plot and paste it in the assignment document.
Deliverable(s): The input matrix and the vector, the 10 successive iterates and the plot (1)

v Repeat part (iv) for the Gauss Jacobi iteration
Deliverable(s): The input matrix and the vector, the 10 successive iterates and the plot (1)

Q2) LU Decomposition,Vector Spaces and LT (5 marks)

i Find the LU decomposition of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ a & a & a \\ b & b & a \end{bmatrix}$ when it exists. For which real numbers a and b does it exist? (1.5)

ii Find the dimension of the vector space spanned by the vectors $\{[1, 1, -2, 0, 1], [1, 2, 0, -4, 1], [0, 1, 3, -3, 2], [2, 3, 0, -2, 0]\}$ and find a basis for the space. (1)

iii Suppose that A is a matrix such that the complete solution to

$$Ax = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 1 \end{bmatrix}$$

is of the form :

$$x = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, c \in \mathcal{R}$$

(a) What can be said about the columns of matrix A? (0.5)

(b) Find the dimension of null space and rank of matrix A. (2)