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21-670A1, Homework-2

1 To find the determinant of the matrix.

$$\det\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 3 \\ -1 & 2 & 4 & 0 & 2 \\ 5 & 1 & 3 & 1 & 7 \\ 2 & 0 & 5 & 0 & 2 \end{bmatrix}$$

Taking the firstrow

$$2\begin{vmatrix} 0 & 1 & 0 & 3 \\ 2 & 4 & 0 & 2 \\ 1 & 3 & 1 & 7 \\ 0 & 5 & 0 & 2 \end{vmatrix}$$

Taking the third column

$$2.1.\begin{vmatrix} 0 & 1 & 3 \\ 2 & 4 & 2 \\ 0 & 5 & 2 \end{vmatrix}$$

Taking the first column

$$2.1.-2.\begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = -4[2-15] = -4[-13]$$

$=52$

2 Making an upper triangular matrix

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 2 & 1 \\ 3 & 1 & 3 & 1 \\ 4 & 3 & 1 & 0 \end{vmatrix}$$

$R_2 - 2R_1 \rightarrow R_2$

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3 & 1 & 3 & 1 \\ 4 & 3 & 1 & 0 \end{vmatrix}$$

$R_3 - 3R_1 \rightarrow R_3$

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -5 & 0 & 1 \\ 4 & 3 & 1 & 0 \end{vmatrix}$$

$R_4 - 4R_1 \rightarrow R_4$

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -5 & 0 & 1 \\ 0 & -5 & -3 & 0 \end{vmatrix}$$

$R_2 \leftrightarrow R_4$

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -5 & -3 & 0 \\ 0 & -5 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$R_3 - R_2 \to R_3$

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -5 & -3 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$1 . -5 . 3 . 1 = -15$

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3. $A = \begin{bmatrix} 2 & a & 3 \\ a & 1 & 0 \\ -1 & 3 & 6 \end{bmatrix}$

To find the value of a such that A is not invertible
that means $\det(A) = 0$

$$\begin{vmatrix} 2 & a & 3 \\ a & 1 & 0 \\ -1 & 3 & 6 \end{vmatrix} = 0$$

$$2\begin{vmatrix} 1 & 0 \\ 3 & 6 \end{vmatrix} - a\begin{vmatrix} a & 0 \\ -1 & 6 \end{vmatrix} + 3\begin{vmatrix} a & 1 \\ -1 & 3 \end{vmatrix} = 0$$

$$2(6-0) - a(6a-0) + 3(3a+1) = 0$$

$$12 - 6a^2 + 9a + 3 = 0$$

$$-6a^2 + 9a + 15 = 0$$

$$2a^2 - 3a - 5 = 0$$

$$2a^2 + 2a - 5a - 5 = 0$$

$$2a(a+1) - 5(a+1) = 0$$

$$(2a-5)(a+1) = 0$$

$2a - 5 = 0$ or $a + 1 = 0$

$a = \frac{5}{2} = 2.5$ or $a = -1$

4(a) $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix}$

$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 1 \\ 2 & 3-\lambda \end{vmatrix}$

$= (2-\lambda)(3-\lambda) - 2$
$= 6 - 5\lambda + \lambda^2 - 2$
$= \lambda^2 - 5\lambda + 4$

4(b) $A - \lambda I$ is not invertible when the determinant is equal to 0. Taking the characteristic polynomial from the previous equation

$\det(A - \lambda I) = 0$
$\lambda^2 - 5\lambda + 4 = 0$
$\lambda^2 - 4\lambda - \lambda + 4 = 0$
$\lambda(\lambda - 4) - (\lambda - 4) = 0$
$(\lambda - 1)(\lambda - 4) = 0$
$\lambda = 1$, or $\lambda = 4$

$A - \lambda I$ is not invertible when $\lambda = 1$ or $\lambda = 4$

(2c) A Finding the value of $\vec{v}$ in the equation $(A-\lambda I)\vec{v} = 0$ where $\vec{v} \neq 0$

$\vec{v} \neq 0$ only if $(A-\lambda I)$ is not invertible.
We know that λ has values of 1 and 4

Let $\lambda = 1$

$$\left[\begin{array}{cc|c} 2-1 & 1 & 0 \\ 2 & 3-1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 2 & 0 \end{array}\right]$$

$R_2 - 2R_1 \to R_2$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Let $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

Let $v_2 = t$

$v_1 + v_2 = 0$
$v_1 = -v_2$
$v_1 = -t$

$\therefore \vec{v} = \begin{bmatrix} -t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} -t$

Taking t as -1

$\therefore \vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Let $\lambda = 4$ and $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$$\left[\begin{array}{cc|c} 2-4 & 1 & 0 \\ 2 & 3-4 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} -2 & 1 & 0 \\ 2 & -1 & 0 \end{array}\right]$$

$R_2 - 2R_1 \rightarrow R_2$

$$\left[\begin{array}{cc|c} -2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Let $v_2 = t$

$-2v_1 + v_2 = t$

$2v_1 = v_2$

$2v_1 = t \quad \Rightarrow \quad v_1 = t/2$

$\therefore \vec{v} = \begin{bmatrix} t/2 \\ t \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} t$

Take $t = 2$

$\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

5(a) $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$

$P_A(\lambda) = det(A - \lambda I)$

$P_A(\lambda) = \begin{vmatrix} 5-\lambda & -1 \\ 1 & 3-\lambda \end{vmatrix}$

$= (5-\lambda)(3-\lambda) - (-1)(1)$
$= 15 - 5\lambda - 3\lambda + \lambda^2 + 1$
$P_A(\lambda) = \lambda^2 - 8\lambda + 16$

(b) $A - \lambda I$ is not invertible when $det(A - \lambda I) = 0$

$\lambda^2 - 8\lambda + 16 = 0$
$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$
$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$
$(\lambda - 4)^2 = 0$
$\cancel{A}\ \lambda - 4 = 0 \Rightarrow \lambda = 4$
$\therefore A - \lambda I$ is not invertible when $\lambda = 4$

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(c) To find a corresponding vector $\vec{v} \neq 0$ such that $(A - \lambda I)\vec{v} = 0$

Let $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

Let $\lambda = 4$

$$\left[\begin{array}{cc|c} 5-4 & -1 & 0 \\ 1 & 3-4 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & -1 & 0 \end{array}\right]$$

$R_2 - R_1 \rightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Let $v_2 = t$

$v_1 - v_2 = 0$

$v_1 = v_2$

$v_1 = t$

$\vec{v} = \begin{bmatrix} t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t$

Let $t = 1$ $\therefore \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$