21-671: Computational Linear Algebra
Homework Assignment Due 11:59pm on 09/05/24

Instructions: Write or type solutions to the following problems and create a PDF file of your work. Then upload that PDF to Gradescope before the due date and time.

1. (15 pts.) For each of the following functions $f(n)$, find a big-O estimate in terms of n:

   (a) $f(n) = \log n! + n2^n + 5n^4$

   (b) $f(n) = n\cos(n^3) + n^2 + n + 1$

   (c) $f(n) = \frac{1}{100}n^3 + 100n^2 + 1000n$

2. (10 pts.) Find formulas to compute the forward and backward error incurred when approximating the function $y = f(x) = e^x$ using the truncated series

$$\hat{f}(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}.$$

   Use these formulas to estimate the actual forward and backward error when $x = 1$ and $x = 10$. Do your results support the estimation of the condition number of $f(x) = e^x$ that we arrived at in class?

3. (10 pts.) Estimate the condition number for the mathematical problem of evaluating the function $f(x) = \cos(x^2) + (1 + 2x)^3$ (in terms of x). Then use that to give an estimate for the condition number of evaluating $f(10)$.

4. (25 pts.) (Programming) **Include the code and any results from the code to solve this problem in your single PDF file. Your code should be commented.**

   Write a function, in MATLAB or Python, to compute the product of two matrices A and B. (Of course, functions/operations already exist in both platforms to compute AB, but I want you to get some practice writing code involving matrices before we move on to more sophisticated algorithms.) The basic algorithm for computing the product of two matrices is given below. Note that it first checks to see if the matrices are compatible for multiplication. You will need to use some built-in functions to get the dimensions of the factors A and B (such as MATLAB's `size()` command or Python's `numpy.shape()` function).

   Once you have implemented the algorithm, check to see that it works by performing some tests - pick two random matrices that are compatible for matrix multiplication, multiply them together, and see if the result matches the result from the built-in functions (`*` in MATLAB and `numpy.linalg.matmul()` in Python. Include at least one test in your homework write-up.

Algorithm 1: Finding the Product of Two Matricess A, B

Input : Matrices $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times p}$
Output: The Matrix $C = AB \in \mathbb{R}^{m \times p}$

```
1  product (A, B){
2      if (A, B are not compatible for matrix multiplication) then
3          print warning message and exit
4      end
5      for i = 1 to m do
6          for j = 1 to p do
7              c_ij = 0
8              for k = 1 to n do
9                  c_ij = c_ij + a_ik b_kj
10             end
11         end
12     end
13     return C
14 }
```