# GPU optimization of base form 820 - collective artificial spouse retirement plan

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## 1 Abstract

Abstract

## 2 Introduction

- Why is the project relevant, what is the problem at hand
Scope? [2]
Hvad regner vi med at lseren har af kompetencer

## 3 Background

SCOPE: Vi krer kun med een kunde ikke flere

- Related work, projects that precedes this project, why do we use the technology described, what is this new technology

### 3.1 Scope

## 4 The Math

The algorithm in this project is used to determine the lump sum of money the insurance company needs to possess to be able to pay the insurance holder's spouse in the case of his or her death. The payment to the spouse will be in the form of a life interest which are disbursed from the time of death of the insurance holder unless the death occurs before the pension age. In that case the money will be disbursed after a grace period determined by the insurance company at the time the insurance was taken out.

We assume that the spouse of the insurance holder is of the opposite gender. If the insurance holder does not have a spouse at the time of his or her death the insurance is forfeited.

The algorithm used here is a 4th order Runge-Kutta solution with a fixed stepsize (indst reference) where we use a series of constants determined before any calculation begins:

Constants

| **Name** | **Meaning** |
|---|---|
| $\tau$ | The time of death of the insurance holder |
| $r$ | The pension age |
| $g$ | The grace period |
| $x$ | The age of the insurance holder at calculation time ($t = 0$) |
| $t$ | The time of calculation |
| $h$ | The Stepsize of the Runge-Kutta solution |

$\tau$ is expressed as $x + t$. Apart from this there is also a constant $k$ which are determined by $\tau, r$ and $g$ in the following manner:

How $k$ is defined

| **If this statement holds** | **then $k$ equals** |
|---|---|
| $\tau < r$ | $g$ |
| $r \leq \tau < r + g$ | $r + g - \tau$ |
| $r + g \leq \tau$ | 0 |

The algorithm can be described as a combination of three models; an outer model that describes the life/death state of the insurance holder, a middle model that describes the married/unmarried state of the insurance holder and an inner model that describes the life/death state of the potential spouse.

NOTE: The description in the three model sections is loosely based on a describtion for Edlund[1] and some passages are directly translated from this document.

## 4.1 Outer model

The outer model is expressed as the following equation:

$\frac{d}{dt}f(t) = r(t)f(t) - \mu_t(x+t)(S^d_{x+t}(t) - f(t))$

Where $S^d_\tau$is the DDSFALDSSUM that for a $\tau$ year old at the time $t$ is needed to cover the payment from the insurance company, $\mu_t(x+t)$ is the mortality rate for a $(x+t)$ year old and $r(t)]$ is the interest rate function. In this project the interest rate function returns the constant 0.05.

The differential equation is solved from $t = 120 - x$ to $t = 0$ with the boundary condition $f(120 - x) = 0$

## 4.2 Middle model

The middle model is used to calculate $S^d_{x+t}(t)$ and is expressed with the following equation:

$$S^d_\tau(t) = \begin{cases} g_\tau \int f(\eta|\tau) a^I_{[\eta]+g}(t) d\eta & \tau \leq r \\ g_\tau \int f(\eta|\tau) a^I_{[\eta]+r+g-\tau}(t) d\eta & r \leq \tau \leq r+g \\ g_\tau \int f(\eta|\tau) a^I_{[\eta]}(t) d\eta & r+g \leq \tau \end{cases}$$

Where $g_\tau$ is the propability that a $\tau$ year is married and $f(\eta|\tau)$ is the probability distribution for, that a $\tau$ year old is married to a $\eta$ year provided that the $\tau$ year old is married. This equation can be rewritten to this form:

$\frac{d}{dn}f(\eta) = -g_\tau f(\eta|\tau) a^I_{[\eta]+k}(t)$

here the DET-DER-TAL-SOM-LIGNER-EN-EKSPONENT-MEN-ER-NEDE-I-STEDET-FOR-OPPE for $a$ is rewritten to $[\eta]+k$ where $k$ can fall into three different categories as shown in THE-TABLE-CONSTANTS.

This differential equation is solved from $\eta = 120$ to $\eta = 1$ with the boundary condition $f(120) = 0$

## 4.3 Inner model

The inner model is used to calculate $a^I_{[\eta]+k}(t)$ and is expressed with the following equation:

$\frac{d}{ds}f(s) = r(t+s)f(s) - 1_{s \geq k} - \mu_{t+s}(\eta + s)(0 - f(s))$

Where $t$, $\eta$ and $k$ are constants and $\mu_{t+s}(\eta + s)$ is the mortality rate for a $(\eta + s)$ year old.

This differential equation is solved from $s = 120 - \eta$ to $s = 0$ with the boundary condition $f(120 - \eta) = 0$

### 4.4 Runge-Kutta 4th order

The Runge-Kutta method is a method for approximating differential equations that builds on Eulers method (INDST REFERENCE) and the midpoint method (INDST REFERENCE). When given a start point and a differential equation one can choose a stepsize and approximate the graph. When given a point $(x_n, y_n)$, a differential equation $f$ and a stepsize $h > 0$ one can use the Runge-Kutta method to approximate the next point in the following way:

$k1 = hf(x_n, y_n)$
$k2 = hf(x_n + \frac{h}{2}, y_n + \frac{k1}{h})$
$k3 = hf(x_n + \frac{h}{2}, y_n + \frac{k2}{h})$
$k4 = hf(x_n + h, y_n + k3)$

$x_n + 1 = x_n + h$
$y_{n+1} = y_n + \frac{k1}{6} + \frac{k2}{3} + \frac{k3}{3} + \frac{k4}{6} + O(h^5)$ HVORFOR KAN VI IGNORERE $O(h^5)$??

BESKRIV HVORDAN EN NORMAL KRSEL SERUD = EKSEMPEL kig evt p en simpson integration nr vi kigger p middle da det egentlig ikke er en differential ligning men et integrale

## 5 CUDA

en trd skal lave et vist stykke arbejde fr det kan betale sig at oprette den. Der er overhead for at lave context for hver trd s hvis den arbejde trden laver er mindre end at stte overheaden op, giver det ingen mening.

- A more elaborate description of GPGPUs, architecture and CUDA C.

# 6 Implementation

- A description of the new implementation and design choices  I den kode der er nu er insurance holder kvinde og spouse mand ALTID skriv noget om memory bound og compute bound i rapporten

# 7 Testing

test p forskellige maskiner hvis vi deler steps op i blocks af 32 trde kan der i worst case scenario vre en sidste block med 1 aktiv og 31 inaktive trde. Lav en graf over rest trdene

- A test to ensure that the new implementation produces the same output as the current implementation

# 8 Benchmarks and Comparison

- Benchmarking and comparison on speed between the current and new implementation

# 9 Conclusion

Conclusion

# 10 Reflection / Discussion / Future improvements

improvements: det kan optimeres til at kunne kre flere kunder. Simpson lsning i middle?

KUNNE DET VRE INTERESSANT AT BRUGE DYNAMIC PROGRAMMING? KUNNE MAN GEMME HVILKE KALD TIL INNER DER ER LAVET OG HVIS INNER S KALDES IGEN MED SAMME VRDIER, KAN MAN SLIPPE FOR AT REGNE DET UD OG BARE HENTE RESULTATET. DOG SKAL MAN VRE OPMRKSOM P AT MAN KAN KOMME I EN SITUATION HVOR EN BEREGNING SER AT DER IKKE ER NOGET RESULTAT FOR ET ST PARAMETRE OG DERFOR GR I GANG

MED AT REGNE DET UD, MENS DEN FRST BEREGNING KRER KAN ANDRE BEREGNINGER MED SAMME PARAMETRE OG DE NYE BEREGNINGER VIL OGS GERNE GEMME RESULTATET NR DE ER FRDIGE HVILKET KAN GIVE CONFLICTS (MSKE). eller Det kan vre vi skal dokumentere en del om at man kan gemme alle resultater i en tabel og efter et vist antal kunder kan man ende med nok til bare at have opslag i stedet for beregninger. skal denne tabel et ligge p GPU s den kan sls op i konstant tid.

skriv evt noget om autotuning, hvor programmet frst tester med forskellige mngder af trde og blokke og vlger den hurtigste lsning.

- Did we achieve what we wanted? what did we discover during the project? What can be changed in future implementations? Threats to validity??? kan det betale sig at kre dele af programmet p CPU'en?

# 11 Glossary

- Forsikringstager - insurance holder
- gteflle - Spouse
- livrente - life interest
- ddfaldssum
- pause periode - grace period
- randbetingelse - boundary condition

# References


[1] EDLUND DOKUMENTET

[2] David B. Kirk, Wen-mei W. Hwu - Programming Massively Parallel Proccesors, A Hands-on Approach - Elsevier Inc. - 2010