Simulation And Analysis Of Radioactive Decay

By

Aafiya

Class Instructor:

Professor Christopher Rogan

Department of Physics and Astronomy
The University of Kansas, Lawrence, KS, USA

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Chapter 1

Introduction

Poisson Distribution (named after the French mathematician Simeon Denis Poisson) is a discrete probability distribution that gives the probability of a given number of events occurring in a fixed interval of time (or space) if these events occur with a known constant mean rate and independently of the time since the last event. For example: the number of text messages per hour, the number of patients arriving in an emergency room between 4 and 5 pm, the rate of decay of radioactive nuclei, the number of calls a call center receives 24 hours a day and so on.

For this project, I shall discuss the case of radioactive decay of a nuclei. Suppose, from the theoretical studies, we know that the rate of decay of nuclei is λ_1 but in the experiments it is found to be λ_2. We can develop two hypotheses using these two rates in the Poisson distribution. To make it a little more realistic, we can think of those rates to be dynamic i.e. rates (fig.1) are the pieces of another distribution (here I have used Gaussian distribution in my code [2].

The code that I used for this purpose has a function named normal() from numpy.random in Python. The function is then used to get the distribution of rates and among those rates,two numbers are chosen randomly and used in Poisson distribution (fig.2).

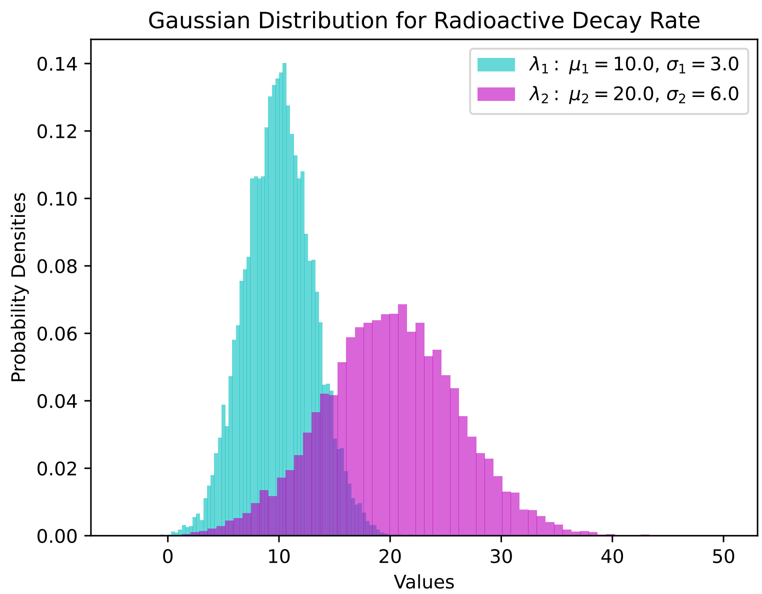


Figure 1: An example of histograms of Gaussian Distribution of radioactive decay rates λ_1 and λ_2.

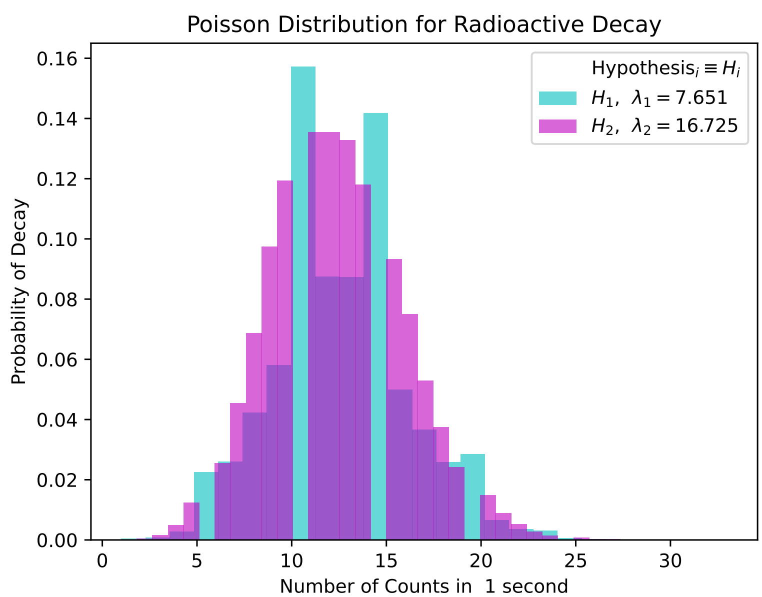


Figure 2: Histogram of Poisson Distribution for radioactive decay.

Chapter 2

Code and Algorithm Analysis

For this project, I decided to write a python code which includes code for the following cases: code for the distribution of rate and distribution of radioactive decay [2], likelihood of the two hypotheses [3] and log likelihood ratio for two different hypotheses [4].

```
decay_constant1 = np.random.normal(10.0, 3.0, 10000)
rate1 = np.random.choice(decay_constant1)
distrubuion1 = random.poisson(rate1, decay_number)
np.savetxt("distribution1.txt", dist1, fmt='%u')


decay_constant2 = np.random.normal(20.0, 6.0, 10000)
rate2 = np.random.choice(decay_constant2)
distribution2 = random.poisson(rate2, decay_number)
np.savetxt("distribution2.txt", dist2, fmt='%u')
```

Figure 3: Algorithm for Poisson distribution which takes arguments from Gaussian distribution. Complete code is in [2].

Above figure (fig.3) is a piece of code for the Poisson distribution which takes arguments from Gaussian distribution. Here emission constant i (i = 1, 2) are the distribution of rates i.e.

$$\lambda(x) = \lambda(x|\mu, \sigma) \tag{1}$$

is a normal distribution and distribution i (i = 1, 2) are radioactive decay given by Poisson distribution:

$$P(k) = P(k|\lambda) = P(k|\lambda(x|\mu, \sigma)) \tag{2}$$

```
for k in range(0, len(hypothesis1)):

    Liklihood1 = (np.exp( - rate1))*(rate1**hypothesis1[k])/np.math.factorial(hypothesis1[k])
    probability1.append(Liklihood1)
    Liklihood2 = (np.exp( - rate2))*(rate2**hypothesis1[k])/np.math.factorial(hypothesis1[k])
    probability2.append(Liklihood1)

    H1_LLR.append(np.log10(Liklihood1/Liklihood2))
    H2_LLR.append(np.log10(Liklihood2/Liklihood1))



w1 = np.ones_like(H1_LLR) / len(H1_LLR)
w2 = np.ones_like(H2_LLR) / len(H2_LLR)
```

Figure 4: Algorithm for Log Likelihood Ratio (LLR) for two hypotheses. Complete code is in [3].

Above figure (fig.4) is a piece of code for the calculation of Log Likelihood Ratio (LLR) for two hypotheses. It is given as:

$$H1(LLR) = log\frac{L_{H1}}{L_{H2}}$$

and

$$H2(LLR) = log\frac{L_{H2}}{L_{H1}}$$

Also, wi (i = 1, 2) are the weights of the LLRs for hypothesis 1 and hypothesis 2.

Chapter 3

Output Interpretation

Likelihoods

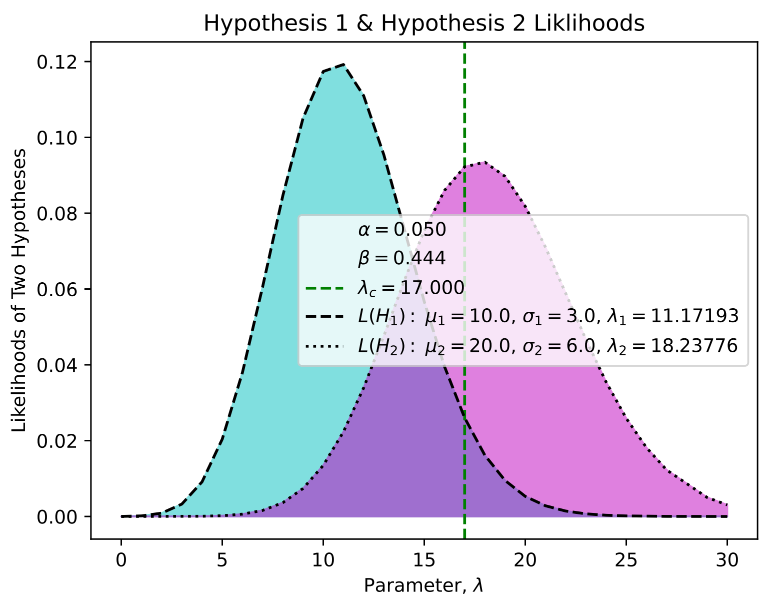


Figure 5: Likelihoods of Hypothesis 1 and Hypothesis 2. Complete code is in [3].

The confidence level for the test is 95% and the power of the test is $\beta = 0.444$. Also, we can see that the critical value is $\lambda_c = 17.0$, rate for hypothesis 1 (rate given by theory),$\lambda_1 = 11.17193$ and rate for hypothesis 2 (rate calculated in experiments), $\lambda_2 = 18.23776$. As $\lambda_c < \lambda_2$ hypothesis 2 can be rejected with 95% CL.

A few other likelihood plots are shown in (fig.6) below:

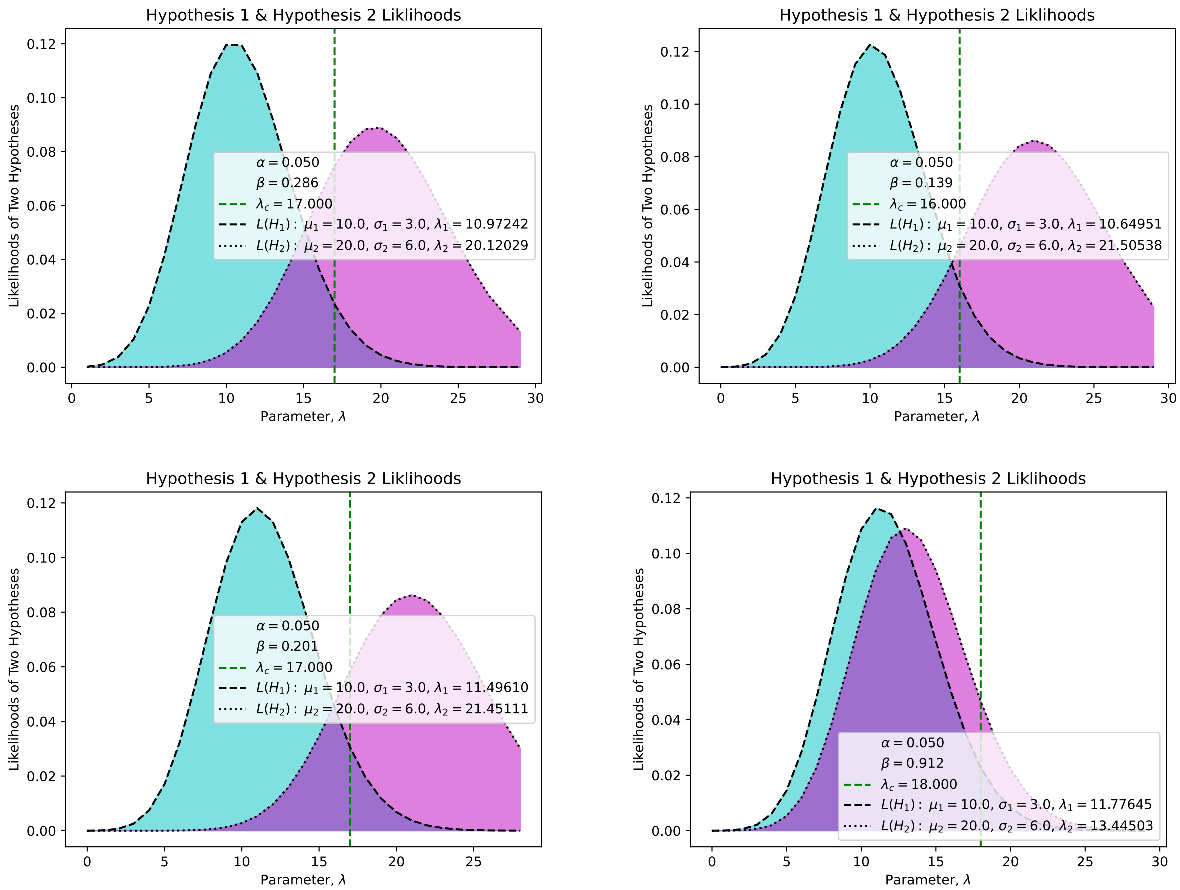


Figure 6: Likelihoods of Hypothesis 1 and Hypothesis 2. It can be noticed that last plot actually favors both the hypotheses.

Log Likelihood Ratio (LLR)

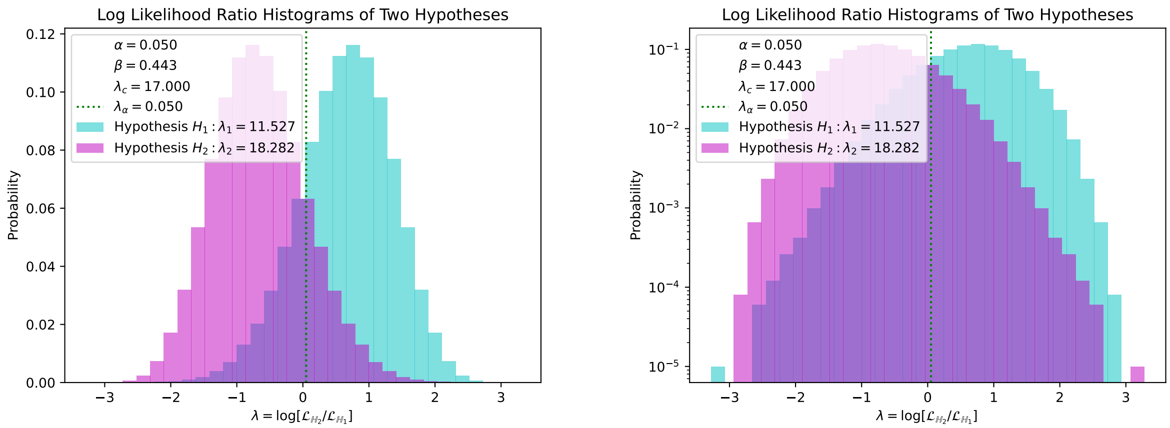
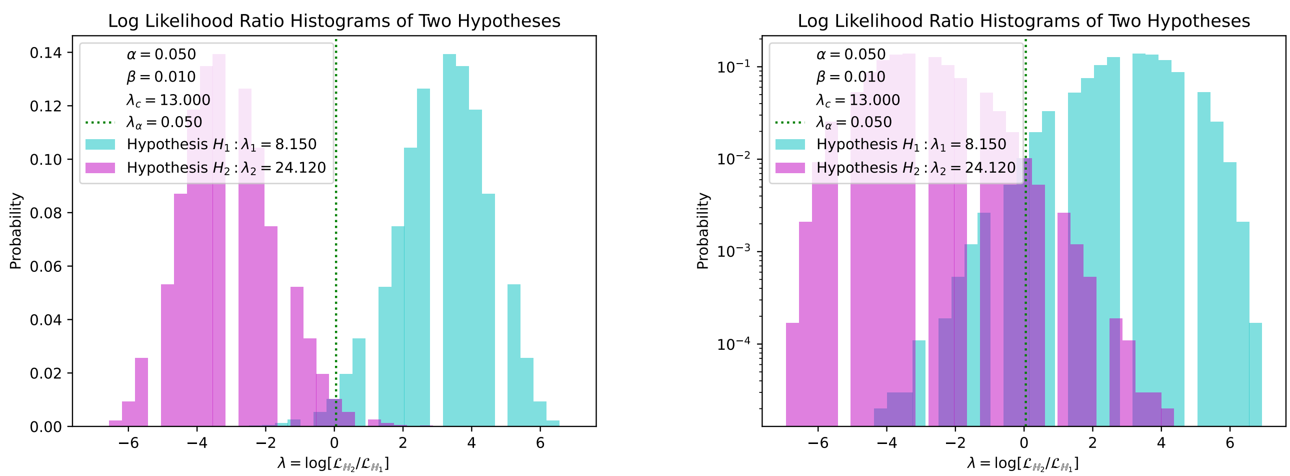


Figure 7: Histograms of Log Likelihood Ratio of two hypotheses. Code is given in [4]

In the above figure (7), the LLR plots supports hypothesis 1. Using more values of rate from the Normal distribution, we can show a few more LLR plots in normal scale and in log scale below (fig.8)



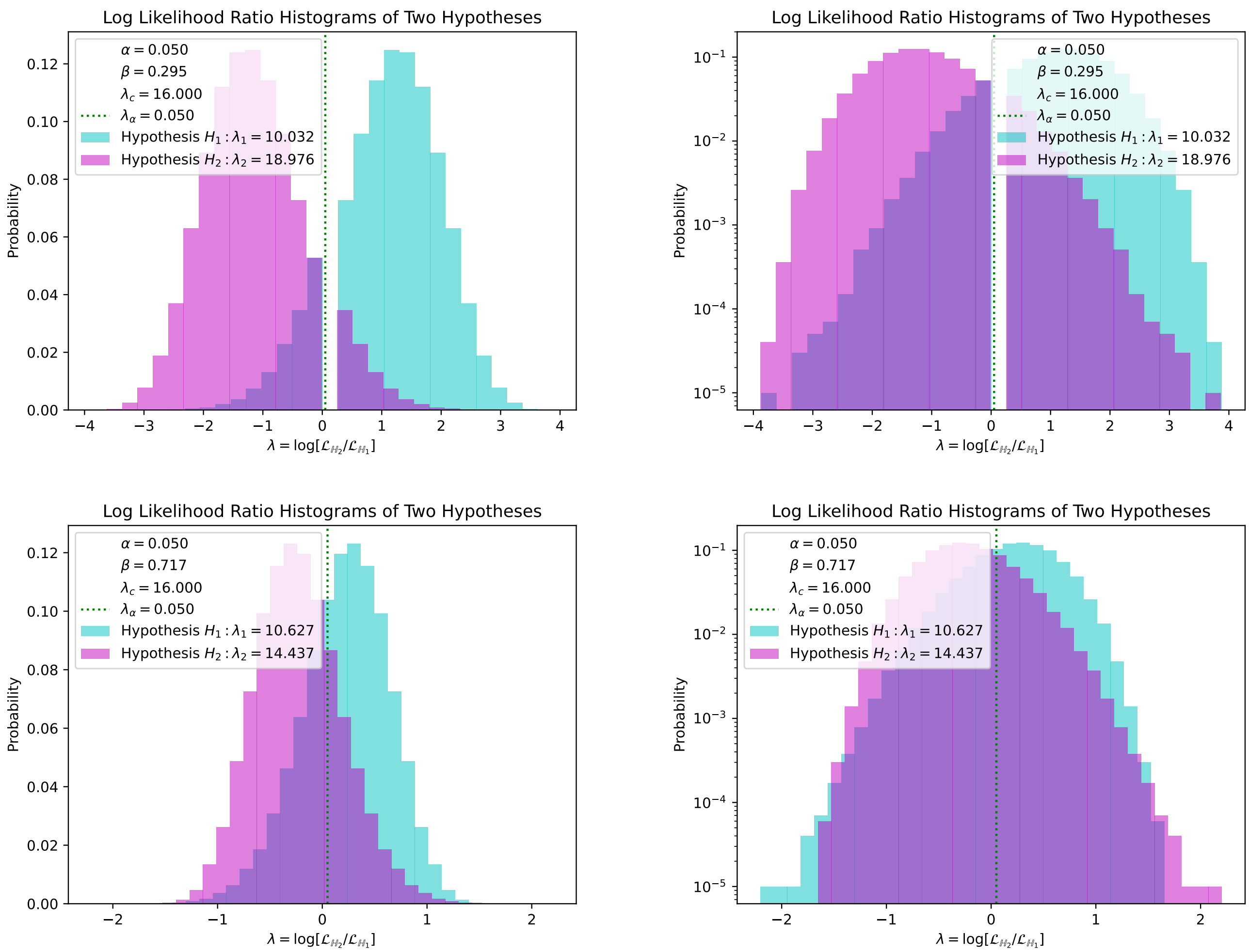


Figure 8: More Histograms of LLR. The rates are randomly chosen from Gaussian distribution. It can be noticed that last row LLR plots actually favor both the hypotheses.

Summary

In this project, Radioactive Decay has been studied using Poisson Distribution that help us understand the dependence of probability of decay on its rate. Besides the discussion on the hypotheses testing, this report gives a statistical analysis of the data obtained using them. It further discusses the probabilities obtained by selecting the rate paramters from a Gaussian distribution. By studying and comparing the histograms and plots, it can be concluded that given the confidence level, $(1-\alpha) \times 100\%$, the power of the test, β, and critical value of rate parameter,λ_c depend on the distribution and its rate.

Bibliography

[1] Poisson Distribution Wikipedia

[2] PHSX815˙Project2 Radioactive Decay

[3] PHSX815 Project 2 Likelihood

[4] PHSX815 Project 2 Log-Likelihood

[5] PHSX815_Project 2