Number Uncertainities In β^+ Decay Of ^{10}C Nuclei

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Chapter 1

Introduction

The fundamental law of radioactive decay states that the decay of a radioactive nucleus is a statistical process that occurs randomly and does not depend on time. This means that the likelihood of a particular nucleus decaying during a given time interval remains constant and is independent of the duration that the nucleus has existed.

This law is mathematically expressed through the exponential decay formula:

$$N(t) = N_0 e^{-\lambda t} \tag{1}$$

$$\lambda = \frac{\ln 2}{t_{1/2}} \tag{2}$$

Here, N(t) represents the number of radioactive nuclei present at a specific time t, N_0 represents the initial number of radioactive nuclei, λ represents the decay constant, and $t_{1/2}$ is the half life.
As the parent nuclei decays, daughter nuclei forms and the expression for the number of daughter nuclei is:

$$^{10}_{6}\mathrm{C} \rightarrow {}^{10}_{5}\mathrm{B} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$$

This isotope of carbon is not natural and is available in trace amount. Back in 1962, the half lives of 10C and 19Ne were not well known [1]. Only two measurements of the half-life of carbon-10 have been reported; the first of 8.8 sec [2] was in error due to an impurity in the boron powder used; the second, due to Sherr et al. [3], was 19.1 ± 0.8 sec. In the case of Neon-19, values ranging from 17.4 ± 0.2 sec [4] up to 20.3 ± 0.5 sec. have been reported.

Later the dead-time corrected data were fit and yielded a half-life of 19.3009 ± 0017 sec. Considering the lifetime of, the daughter and parent nuclei are plotted in the following 1 & 2

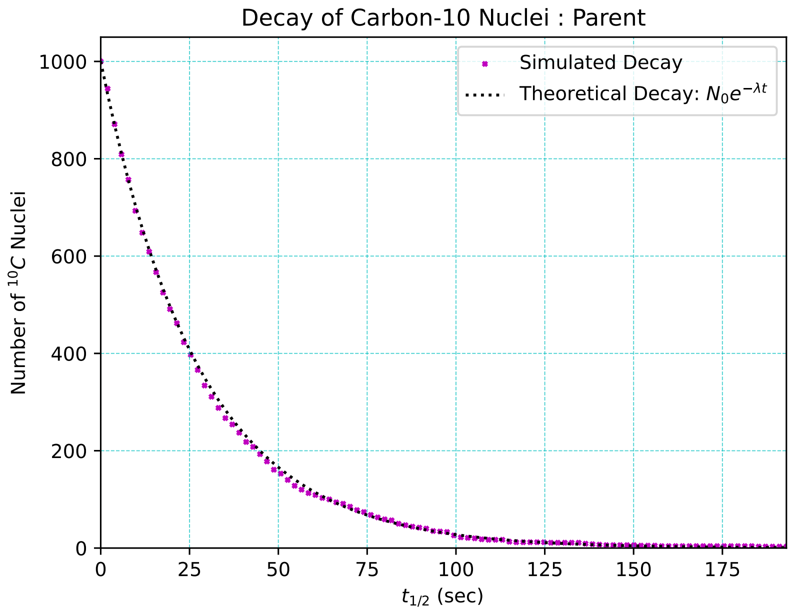

Figure 1: Decay of Carbon 10 Parent nuclei.

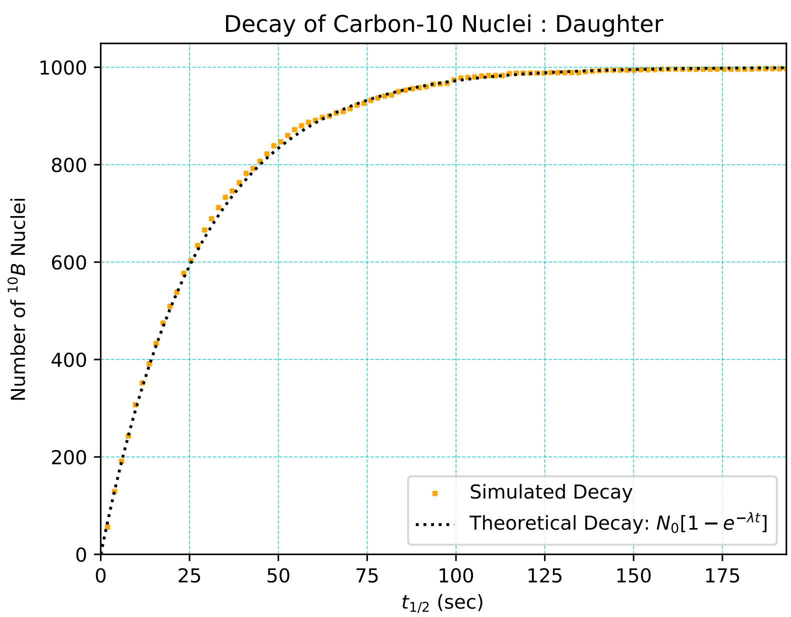

Figure 2: Decay of Carbon 10 daughter nuclei

Chapter 2

Code and Algorithm Analysis

In this project, I have developed three separate codes, labeled as [5], [6], and [7]. These codes are designed to generate distributions of parent and daughter nuclei decays and formation, as well as calculate the errors in both the simulated and theoretical counts. In addition, the codes also calculate the likelihoods and log likelihood ratios for two different hypotheses.

To simulate radioactive decay in the algorithm, random numbers (labeled as ”z”) are generated within the range of 0 to 1. The transformation function $x = \frac{1}{\lambda}\log(1-z)$ is then applied to the generated random numbers to obtain a new set of values for x. By using the relationship $z = 1 - e^{-\lambda x}$, these new values of x can be used to simulate the decay rates of radioactive nuclei in a defined function. The function returns the value of $-\frac{1}{\lambda}\log(1-z)$, which can be used later for modeling decay rates at different rates.

```
    z = random.random()
    return -np.log(1 - z)/rate

N0 = 1000
nuclei_life = []
t_half = 19.3009 # half life of C-10 nuclei
rate = np.log(2)/t_half

for n in range(N0):
    nuclei_life.append(my_ran(rate))

nuclei_life = np.sort(np.array(nuclei_life))
n_half_lives = 10
times = np.linspace(0.0, n_half_lives*t_half, 100)

N_decayed = []
N_undecayed = []

for t in times:
    try: q = np.argwhere(nuclei_life <= t)[-1][0]
    except: q = 0
    N_decayed.append(q)
N_undecayed = N0 - np.array(N_decayed)

residue_daughter = abs(np.array(N_decayed) - np.array(N0 - N0*np.exp(-times*rate)))
residue_parent = abs(np.array(N_undecayed) - np.array(N0*np.exp(-times*rate)))
```

Figure 3: Algorithm to calculate the simulated and theoretical decays of both parent and daughter 10C nuclei, while also accounting for number errors given by residues.

The algorithm is designed to ensure that the total number of decayed and undecayed nuclei at any point in time always adds up to N_o, whether using simulation or theoretical calculations. This means that the algorithm is formulated to maintain the conservation of nuclei in the system throughout the decay process.
The residues in daughter and parent nuclei are equal and is given as:

$$R = N_d - N_0(1 - e^{-\lambda t}) = N_u - N_0 e^{-\lambda t} \tag{3}$$

where R is the residue in both parent and daughter nuclei, N_u and N_d are number of nuclei undecayed and decayed respectively. For the two different hypotheses, it is considered that the first hypothesis has rate λ_1 (pure 10C nuclei) and the second one has a mixed rate of:

$$\lambda_{\text{mix}} = 0.8\lambda_1 + 0.2\lambda_2 \tag{4}$$

with 80% of ^{10}C and 20% of ^{19}Ne impurity.

For the calculation of Log Likelihood Ratio (LLR) for two hypotheses shown in code [7], the following was used:

$$H1(LLR) = log\frac{L_{H1}}{L_{H2}}$$

and

$$H2(LLR) = log\frac{L_{H2}}{L_{H1}}$$

Chapter 3

Output Interpretation

Uncertainties in Radioactive Decay

There are different types of uncertainties that can arise during the positron decay of ^{10}C nuclei. These uncertainties can be classified into conservative systematic uncertainty, uncertainties that are limited by data, and impurity uncertainty. The project considers only two types of uncertainties, namely impurity and statistical uncertainties. The statistical uncertainty is caused by the difference between simulated and theoretical outputs. On the other hand, impurity uncertainty is introduced by the presence of various radioactive elements in the sample.

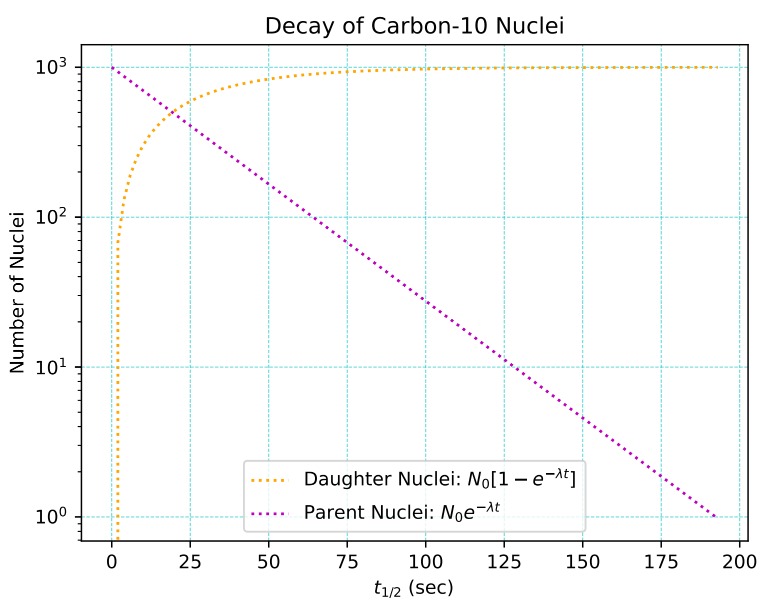

Figure 4: (Log scale) decay of 10C nuclei.

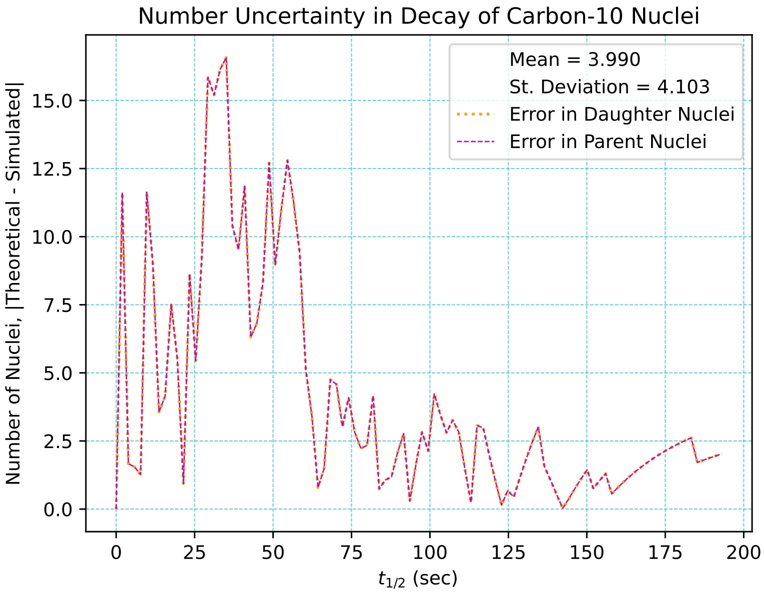

Figure 5: Coincided residues for parent and daughter nuclei. Complete code is in [5].

Number Uncertainty Independent of Number of Radioactive Nuclei

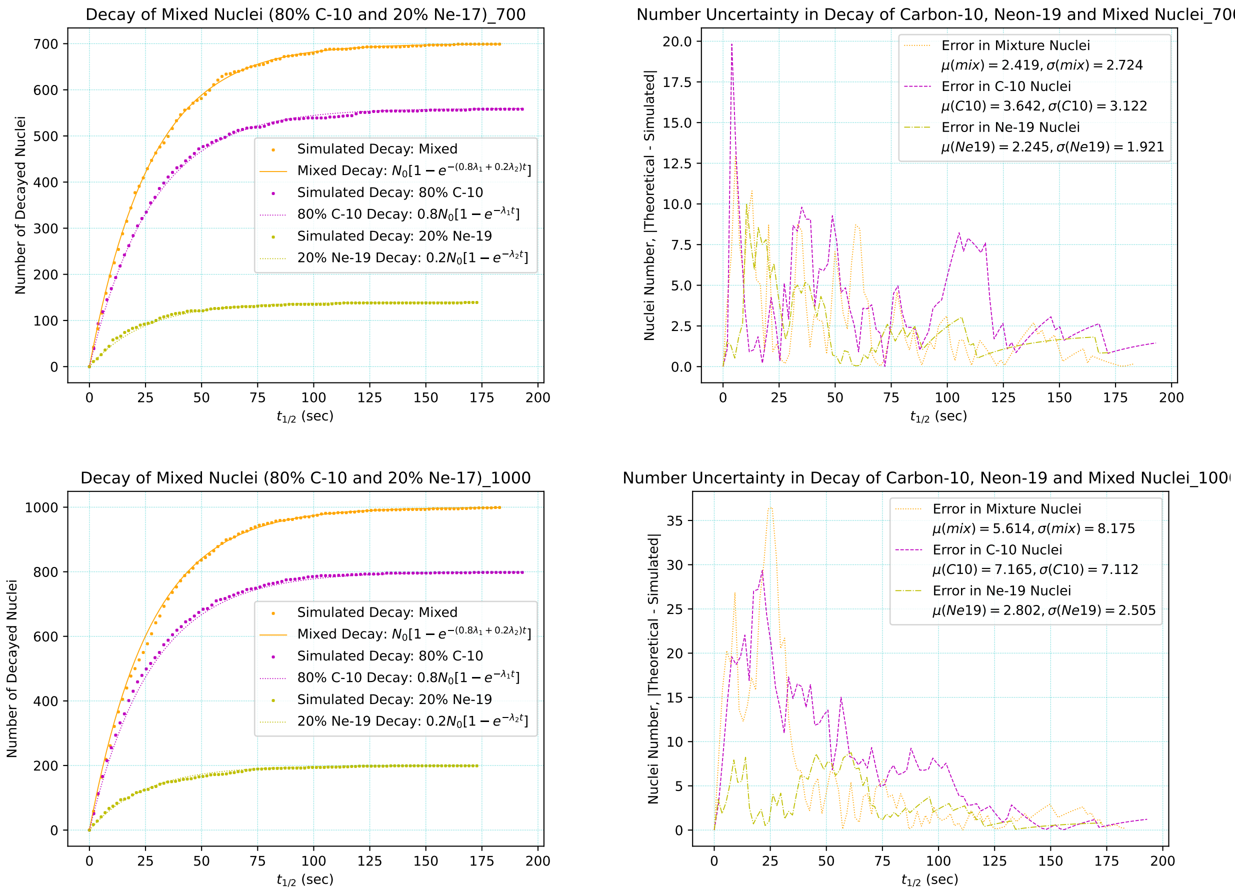

Figure 6: Number uncertainties in the decay of ^{10}C, ^{19}Ne, and mixed nuclei. Code is given in [6]

The figure 6 illustrates that the uncertainties in the simulated and theoretical calculations are independent of the number of radioactive nuclei. This means that the maximum standard deviation of 10C in the top two plots is not dependent on the number of 10C nuclei, and similarly, the maximum uncertainties in the bottom two plots are not dependent on the number of nuclei in the mixture. The reason for this is that the simulation is based on random numbers, which can cause the differences to fluctuate significantly.

Likelihood Ratio of Two Hypotheses

In this particular project, the first hypothesis (H1) refers to the decay rate of ^{10}C alone, while the second hypothesis (H2) is a combination of decay rates as given by Equation 4

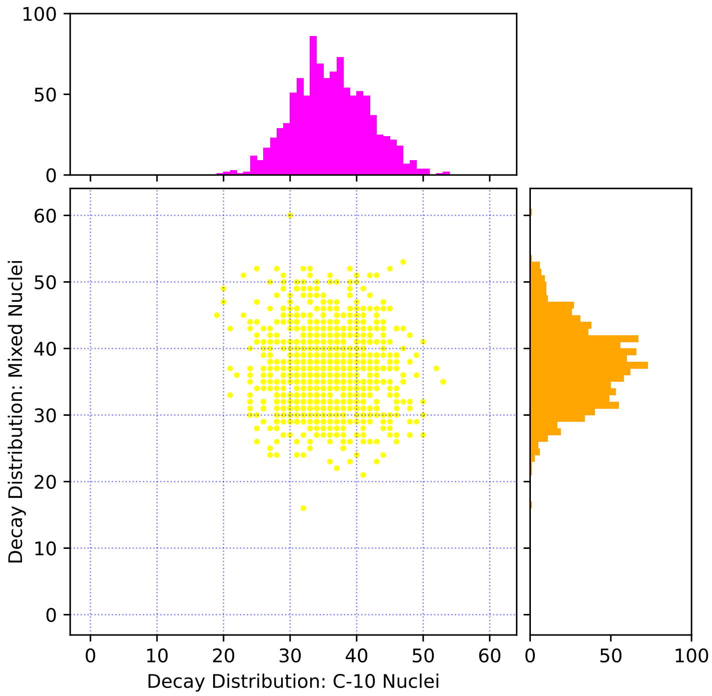

Figure 7: Comparison of two hypothesis using scatter plots and histograms of Poisson distributions. Code is given in [6]

The hypotheses testing may not be very informative when the value of lambda is low. To make the results more noticeable, λ_1 and λ_2 are multiplied by 1000.

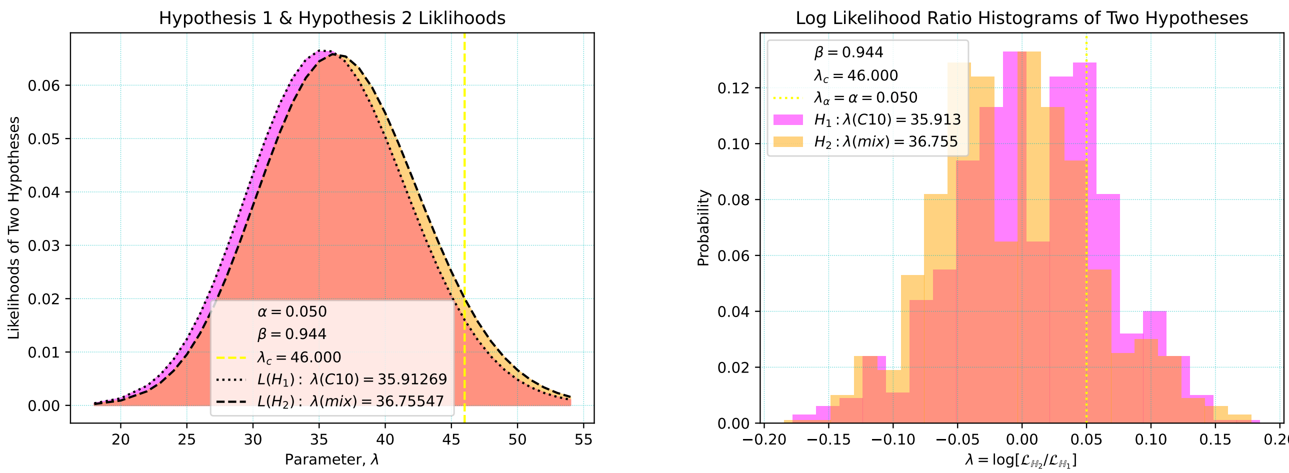

Figure 8: Likelihoods **Left** and Log Likelihood Ratio **Right** of two hypotheses.Complete Code is in [7]

As shown in above figure, both decay rates are smaller than the critical rate parameter, λ_c. Therefore, none of the hypotheses can be thrown away using this testing. The main reason for this is that there is no significant difference in the decay rates of the mixture of radioactive nuclei from pure ^{10}C nuclei.

Summary

This report is based on the discussion of two hypotheses: rate of pure ^{10}C nuclei and mixed (^{10}C & ^{19}Ne) nuclei, and gives a statistical analysis of the data obtained using them. The analysis of scatter plots, histograms, and plots indicates that the number uncertainties between theory and simulation remain consistent regardless of the number of radioactive nuclei used.

Bibliography

[1] L. G. Earwaker, J. G. Jenkin & E. W. Titterton, “Half Lives of Carbon-10 and Neon-19”, Nature volume 195, pages271-272(1962).

[2] Delsasso, L. A., White, M. G., Barkas, W., & Creutz, E. C., Phys. Rev., 58, 586 (1940).

[3] Sherr, R., Meuther, H. R., & White, M. G., Phys. Rev., 75, 282 (1949).

[4] Herrmannsfeldt, W. B., Burman, R. L., Stahelin, P., Allen, J. S., & Braid, T. H., Bull. Amer. Phys. Soc., 4, 77 (1959).

[5] PHSX815_Project 4

[6] PHSX815_Project 4

[7] PHSX815_Project 4