Exercise 6.11

1. Find the particular integral of the following

(i) $y'' - y = 3e^{2x}$

Solution: Given that, $y'' - y = 3e^{2x}$ .... (i)

Here, $R = 3e^{2x}$. So, choose for y_p,

$y_p = c_1e^{2x}$ .... (ii)

Then, $y'_p = 2c_1e^{2x}$ and $y''_p = 4c_1e^{2x}$

So, equation (i) becomes,

$4c_1e^{2x} - c_1e^{2x} = 3e^{2x}$

$\Rightarrow 3c_1e^{2x} = 3e^{2x}$

Now, equating the coefficients from both sides we get, $c_1 = 1$.

Thus, equations (ii) becomes,

$y_p = e^{2x}$

This is required particular solution of (i).

(ii) $y'' + y' - 2y = 14 + 2x - 2x^2$

Solution: Given that, $y'' + y' - 2y = 14 + 2x - 2x^2$ .... (i)

Here, $R = 14 + 2x - 2x^2$. So, choose for y_p,

$y^p = c_1x^2 + c_2x + c_3$ ...... (ii)

Then, $y'_p = 2c_1x + c_2$ and $y''_p = 2c_1$

So, equation (i) becomes,

$2c_1 + 2c_1x + c_2 - 2c_1x^2 - 2c_2x - 2c_3 = 14 + 2x - 2x^2$

$(2c_1 + c_2 - 2c_3) + x(2c_1 - 2c_2) - 2c_1x^2 = 14 + 2x - 2x^2$

Comparing the coefficient on both side

$2c_1 + c_2 - 2c_3 = 14,\quad 2c_1 - 2c_2 = 2,\quad -2c_1 = -2$

Solving we get, $c_1 = 1$, $c_2 = 0$ and $c_3 = -6$.

Now equation (ii) becomes,

$y_p = x^2 - 6$

This is required particular solution of (i).

(iii) $y'' + 9y = 17e^{-5x}$

Solution: Given that, $y'' + 9y = 17e^{-5x}$ ...... (i)

Here, $R = 17e^{-5x}$. So, choose for y_p,

$y_p = c_1e^{-5x}$ ...... (ii)

Then, $y_p = -5c_1e^{-5x}$ and $y''_p = 25c_1e^{-5x}$

So, equation (i) becomes

$25c_1e^{-5x} + 9c_1e^{-5x} = 17e^{-5x}$

$\Rightarrow 34c_1e^{-5x} = 17e^{-5x} \Rightarrow c_1 = \frac{1}{2}$

Now, equation (ii) becomes,

$y_p \frac{1}{2}e^{-5x}$

This is required particular solution of (i).

(iv) $y'' - 6y' + 9y = 2e^{3x}$

Solution: Given that, $y'' - 6y' + 9y = 2e^{3x}$ ...... (i)

Here, $R = e^{3x}$. So, choose for y_p,

$y_p = c_1x^2e^{3x}$ ...... (ii)

Then, $y'_p = 3c_1x^2e^{3x} + 2c_1xe^{3x}$ and $y''_p = 9c_1x^2e^{3x} + 12c_1xe^{3x} + 2c_1e^{3x}$

So, equation (i) becomes

$9c_1x^2e^{3x} + 12c_1xe^{3x} + 2c_1e^{3x} - 6(3c_1x^2e^{3x} + 2c_1xe^{3x}) + 9(c_1x^2e^{3x}) = 2e^{3x}$

$\Rightarrow 2c_1e^{3x} = 2e^{3x} \Rightarrow c_1 = 1$.

Now, equation (ii) becomes,

$y_p = x^2e^{3x}$

This is required particular solution of (i).

(v) $y'' + 3y' + 4y = -6.8\sin x$

Solution: Given that, $y'' + 3y' + 4y = -6.8\sin x$ ...... (i)

Here, $R = -6.8\sin x$. So, choose for y_p,

$y_p = (c_1 \cos x + c_2\sin x)$ ...(ii)

Then, $y'_p = (-c_1 \sin x + c_2\cos x)$ and $y''_p = (-c_1\cos x - c_2\sin x)$

So, equation (i) becomes

$-c_1\cos x - c_2\sin x - 3c_1\sin x + 3c_2\cos x + 4_1\cos x + 4_2\sin x = -6.8\sin x$

$\Rightarrow \sin x(-c_2 - 3c_1 + 4c_2) + \cos x(-c_1 + 3c_2 + 4c_1) = -6.8 \sin x$

$\Rightarrow \sin x(-3c_1 + 3c_2) + \operatorname{cox}(3c_2 + 3c_1) = -6.8\sin x$

Comparing coefficient on both side,

$-3c_1 + 3c_2 = -6.8$ and $3c_1 + 3c_2 = 0$

$\Rightarrow 6c_2 = -6.8$ $\Rightarrow c_2 = -1.13$

$\Rightarrow c_1 = 1.3$

Thus, (ii) becomes,

$$y_p = 1.3(\cos x - \sin x)$$

This is required particular solution of (i).

(2) **Find the general solution of the following**

(i) $y'' - 4y' + 3y = 10e^{-2x}$ **[2009 Fall Q. No. 4(b)]**

Solution: Given that, $y'' - 4y' + 3y = 10e^{-2x}$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - 4m + 3 = 0 \Rightarrow m^2 - 3m - m + 3 = 0$$
$$\Rightarrow m(m-3) - 1(m-3) = 0$$
$$\Rightarrow (m-3)(m-1) = 0$$
$$\Rightarrow m = 1, 3$$

So, its general solutions is

$$y_h(x) = c_1e^x + c_2e^{3x}$$

And for the particular solution of (i), let,

$$y_p = c_3e^{-2x} \quad \text{...... (ii)}$$

Then, $y'_p = -2c_3e^{-2x}$ and $y''_p = 4c_3e^{-2x}$

So, equation (i) becomes

$$4c_3e^{-2x} + 8c_3e^{-2x} + 3c_3e^{-2x} = 10e^{-2x}$$
$$\Rightarrow 15c_3e^{-2x} = 10e^{-2x}$$
$$\Rightarrow c_3 = \frac{10}{15} = \frac{2}{3}$$

Thus, equation (ii) becomes, $y_p = \frac{2}{3}e^{-2x}$

Now, the solution of (i) is,

$$y(x) = y_h(x) + y_p(x) = c_1e^x + c_2e^{3x} + \frac{2}{3}e^{-2x}$$

(ii) $y'' + 4y = 8x^2$

Solution: Given that, $y'' + 4y = 8x^2$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 4 = 0 \Rightarrow m = \pm 2i$$

So, its general equation is

$$y_h(x) = e^0 (A \cos 2x + B \sin 2x)$$
$$\Rightarrow y_h(x) = A \cos 2x + B \sin 2x \quad \text{...(ii)}$$

And for the particular solution of (i), let,

$$y_p = c_1x^2 + c_2x + c_3 \quad \text{...(iii)}$$

Then, $y'_p = 2c_1x + c_2$ and $y''_p = 2c_1$

So, equation (i) becomes

$$2c_1 + 4(c_1x^2 + c_2x + c^3) = 8x^2$$
$$\Rightarrow 2c_1 + 4c_1x^2 + 4c_2x + 4c_1x^2 = 8x^2$$
$$\Rightarrow (2c_1 + 4c_3) + 4c_2x + 4c_1x^2 = 8x^2$$

Comparing the coefficient on the both side

$$2c_1 + 3c_3 = 0, \quad 4c_2 = 0 \quad \text{and} \quad 4c_1 = 8.$$

Solving we get, $c_1 = 2$, $c_2 = 0$ and $c_3 = 1$.

Thus, equation (iii) becomes

$$y_p = 2x^2 - 1$$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p(x)$$
$$= A \cos 2x + B \sin 2x + 2x_2 - 1$$

(iii) $y'' - y' - 2y = 10 \cos x$ **[2004 Spring; 2006 Fall Q. No. 4(b)]**

Solution: Given that, $y'' - y' - 2y = 10 \cos x$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - m - 2 = 0 \Rightarrow m^2 - 2m + m - 2 = 0$$
$$\Rightarrow m(m-2) + 1(m-2) = 0$$
$$\Rightarrow (m-2)(m+1) = 0 \Rightarrow m = 2, -1$$

So, its general equation is,

$$y_h(x) = c_1e^{2x} + c_2e^{-x} \quad \text{...... (ii)}$$

And for the particular solution of (i), let,

$$y_p = c_3 \cos x + c_4 \sin x \quad \text{...... (iii)}$$

Then, $y'_p = -c_3 \sin x + c_4 \cos x$ and $y''_p = -c_3 \cos x - c_4 \sin x$

So, equation is (i) becomes

$$-c_3\cos x - c_4\sin x - (-c_3\sin x + c_4\cos x) - 2(c_3\cos x + c_4\sin x) = 10\cos x.$$
$$\Rightarrow -c_3\cos x - c_4\sin x + c_3\sin x - c_4\cos - 2c_3\cos x - 2c_4\sin x = 10\cos x$$
$$\Rightarrow \cos x(-c_3 - c_4 - 2c_3) + \sin x(-c_4 + c_3 - 2c_4) = 10 \cos x$$
$$\Rightarrow \cos x(-3c_3 - c_4) + \sin x(-c_3 + 3c_4) = 10\cos x$$

Comparing the coefficient on both side, then,

$$-3c_3 - c_4 = 10, \quad c_3 - 3c_4 = 0.$$

Solving we get, $c_3 = -3$ and $c_4 = -1$.

Thus, equation (iii) becomes

$$y_p = -3\cos x - \sin x$$

Now, general equation of (i) is,

$y_p = 1.3(\cos x - \sin x)$

This is required particular solution of (i).

(2) **Find the general solution of the following**

(i) $y'' - 4y' + 3y = 10e^{-2x}$ **[2009 Fall Q. No. 4(b)]**

Solution: Given that, $y'' - 4y' + 3y = 10e^{-2x}$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - 4m + 3 = 0 \Rightarrow m^2 - 3m - m + 3 = 0$$
$$\Rightarrow m(m-3) - 1(m-3) = 0$$
$$\Rightarrow (m-3)(m-1) = 0$$
$$\Rightarrow m = 1, 3$$

So, its general solutions is

$$y_h(x) = c_1e^x + c_2e^{3x}$$

And for the particular solution of (i), let,

$$y_p = c_3e^{-2x} \quad \text{...... (ii)}$$

Then, $y'_p = -2c_3e^{-2x}$ and $y''_p = 4c_3e^{-2x}$

So, equation (i) becomes

$$4c_3e^{-2x} + 8c_3e^{-2x} + 3c_3e^{-2x} = 10e^{-2x}$$
$$\Rightarrow 15c_3e^{-2x} = 10e^{-2x}$$
$$\Rightarrow c_3 = \frac{10}{15} = \frac{2}{3}$$

Thus, equation (ii) becomes, $y_p = \frac{2}{3}e^{-2x}$

Now, the solution of (i) is,

$$y(x) = y_h(x) + y_p(x) = c_1e^x + c_2e^{3x} + \frac{2}{3}e^{-2x}$$

(ii) $y'' + 4y = 8x^2$

Solution: Given that, $y'' + 4y = 8x^2$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 4 = 0 \Rightarrow m = \pm 2i$$

So, its general equation is

$$y_h(x) = e^0 (A \cos 2x + B \sin 2x)$$
$$\Rightarrow y_h(x) = A \cos 2x + B \sin 2x \quad \text{...(ii)}$$

And for the particular solution of (i), let,

$$y_p = c_1x^2 + c_2x + c_3 \quad \text{...(iii)}$$

Then, $y'_p = 2c_1x + c_2$ and $y''_p = 2c_1$

So, equation (i) becomes

$$2c_1 + 4(c_1x^2 + c_2x + c^3) = 8x^2$$
$$\Rightarrow 2c_1 + 4c_1x^2 + 4c_2x + 4c_1x^2 = 8x^2$$
$$\Rightarrow (2c_1 + 4c_3) + 4c_2x + 4c_1x^2 = 8x^2$$

Comparing the coefficient on the both side

$$2c_1 + 3c_3 = 0, \quad 4c_2 = 0 \quad \text{and} \quad 4c_1 = 8.$$

Solving we get, $c_1 = 2$, $c_2 = 0$ and $c_3 = 1$.

Thus, equation (iii) becomes

$$y_p = 2x^2 - 1$$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p(x)$$
$$= A \cos 2x + B \sin 2x + 2x_2 - 1$$

(iii) $y'' - y' - 2y = 10 \cos x$ **[2004 Spring; 2006 Fall Q. No. 4(b)]**

Solution: Given that, $y'' - y' - 2y = 10 \cos x$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - m - 2 = 0 \Rightarrow m^2 - 2m + m - 2 = 0$$
$$\Rightarrow m(m-2) + 1(m-2) = 0$$
$$\Rightarrow (m-2)(m+1) = 0 \Rightarrow m = 2, -1$$

So, its general equation is,

$$y_h(x) = c_1e^{2x} + c_2e^{-x} \quad \text{...... (ii)}$$

And for the particular solution of (i), let,

$$y_p = c_3 \cos x + c_4 \sin x \quad \text{...... (iii)}$$

Then, $y'_p = -c_3 \sin x + c_4 \cos x$ and $y''_p = -c_3 \cos x - c_4 \sin x$

So, equation is (i) becomes

$$-c_3\cos x - c_4\sin x - (-c_3\sin x + c_4\cos x) - 2(c_3\cos x + c_4\sin x) = 10\cos x.$$
$$\Rightarrow -c_3\cos x - c_4\sin x + c_3\sin x - c_4\cos - 2c_3\cos x - 2c_4\sin x = 10\cos x$$
$$\Rightarrow \cos x(-c_3 - c_4 - 2c_3) + \sin x(-c_4 + c_3 - 2c_4) = 10 \cos x$$
$$\Rightarrow \cos x(-3c_3 - c_4) + \sin x(-c_3 + 3c_4) = 10\cos x$$

Comparing the coefficient on both side, then,

$$-3c_3 - c_4 = 10, \quad c_3 - 3c_4 = 0.$$

Solving we get, $c_3 = -3$ and $c_4 = -1$.

Thus, equation (iii) becomes

$$y_p = -3\cos x - \sin x$$

Now, general equation of (i) is,

$y(x) = y_h(x) + y_p$
$= c_1e^{2x} + c_2e^{-x} - 3\cos x - \sin x$

(iv) $y'' - 3y' + 2y = 4x + e^x$

Solution: Given that, $y'' - 3y' + 2y = 4x + e^x$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - 3m + 2 = 0 \Rightarrow m^2 - 2m - m + 2 = 0$$
$$\Rightarrow m(m-2) - 1(m-2) = 0$$
$$\Rightarrow (m-2)(m-1) = 0 \Rightarrow m = 2, 1$$

So, its general equation is,

$$y_h(x) = c_1e^{2x} + c_2e^x \quad \text{...... (ii)}$$

And for the particular solution of (i), let,

$$y_p = c_3x + c_4 + c_5e^x \quad \text{...... (iii)}$$

Then, $y'_p = c_3 + c_5e^x$ and $y''_p = c_5e^x$

(v) $y'' + 4y' + 4y = 18 \cosh x$

Solution: Given that, $y'' + 4y' + 4y = 18 \cosh x$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 4m + 4 = 0 \Rightarrow m^2 + 2m + 2m + 4 = 0$$
$$\Rightarrow (m+2)^2 = 0$$
$$\Rightarrow m = -2, -2$$

So, its general equation is,

$$y_h(x) = (c_1 + c_2x)e^{-2x} \quad \text{...... (ii)}$$

And for the particular solution of (i), let,

$$y_p = c_3\cosh x + c_4\sinh x \quad \text{...... (iii)}$$

Then, $y'_p = c_3\sinh x + c_4\cosh x$ and $y''_p = c_3\cosh x + c_4 \sinh x$

Then equations (i) becomes,

$$c_3\cosh x + c_4\sinh x + 4(c_3\sinh x + c_4\cosh x) + 4(c_4\cosh x + c_4\sinh x) = 18\cosh x$$
$$\Rightarrow c_3\cosh x + c_4\sinh x + 4c_3\sinh x + 4c_4\cosh x + 4c_3\cosh x + 4c_4\sinh x = 18\cosh x$$
$$\Rightarrow \cosh x(c_3 + 4c_4 + 4c_3) + \sinh x(c_4 + 4c_3 + 4c_4) = 18\cosh x$$
$$\Rightarrow \cosh x(5c_3 + 4c_4) + \sinh x(5c_4 + 4c_3) = 18 \cosh x$$

Comparing coefficient on both side then,

$$5c_3 + 4c_4 = 18 \quad \text{and} \quad 5c_4 + 4c_3 = 0$$

Solving we get, $c_3 = 10$ and $c_4 = -8$.

Then, the equation (iii) becomes

$$y_p = 10\cosh x - 8\sinh x$$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$
$$= (c_1 + c_2x)e^{-2x} + 10\cosh x - 8\sinh x$$
$$= (c_1 + c_2x)e^{-2x} + e^x + 9e^{-x}$$

(vi) $y'' - 2y' = e^x \sin x$

Solution: Given that, $y'' - 2y' = e^x \sin x$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - 2m = 0 \Rightarrow m(m-2) = 0 \Rightarrow m = 0, 2.$$

So, its general equation is,

$$y_h(x) = c_1e^{0x} + c_2e^{2x}$$
$$\Rightarrow y_h(x) = c_1 + c_2e^{2x}$$

And for the particular solution of (i), let,

$$y_p = e^x(c_1\sin x + c_2\cos x)$$
$$\Rightarrow y_p = c_1e^x\sin x + c_2e^x\cos x \quad \text{...... (ii)}$$

Then, $y'_p = c_1(e^x\cos x + e^x\sin x) + c_2(e^x\cos x - e^x\sin x)$

And, $y''_p = c_1(e^x\cos x - e^x\sin x + e^x\sin x + e^x\cos x) + c_2(e^x\cos x - e^x\sin x - e^x\cos x)$

$$\Rightarrow y''_p = 2c_1e^x\cos x - 2c_2e^x\sin x$$

Then, the equation (ii) becomes

$$2c_1e^x\cos x - 2c_2e^x\sin x - 2c_1e^x\cos x - 2c^1e^x\sin x - 2c_2e^x\cos x + 2c_2e^x\sin x = e^x\sin x$$
$$\Rightarrow -2c_1e^x\sin x - 2c_2e^x\cos x = e^x\sin x$$

Comparing coefficient on both side then,

$$-2c_1e^x = e^x \quad \text{and} \quad -2c_2e^x = 0$$
$$\Rightarrow c_1 = -\frac{1}{2} \qquad \Rightarrow c_2 = 0.$$

So the equation (2) becomes,

$$y_p = \frac{1}{2}e^x\sin x$$

Now, general equation of (i) is,

$$y(x) = y_p + y_h(x)$$
$$\Rightarrow y(x) = c_1 + c_2e^x - \frac{1}{2}e^x\sin x$$

(vii) $y'' + y' = x^2 + 2x + 4$ **[2004 Spring Q. No. 4(b)]**

Solution: Given that, $y'' + y' = x^2 + 2x + 4$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + m = 0 \Rightarrow m(m+1) = 0 \Rightarrow m = 0, -1$$

So, its solution is, $y_h(x) = c_1e^{0x} + c_2e^{-x}$

$$\Rightarrow \quad y_h(x) = c_1 + c_2e^{-x}$$

And for the particular solution of (i), let,

$$y_p = c_3x^3 + c_4x^2 + c_5x \quad \ldots\ldots (2)$$

Then, $y'_p = 3c_3x^2 + 2c_4x + c_5$ and $y''_p = 6c_3x + 2c_4$

Now equation (i) becomes,

$$6c_3x + 2c_4 + 3c_3x^2 + 2c_4x + c_5 = x^2 + 2x + 4$$

$$\Rightarrow \quad 3c_3x^2 + 6c_3x + 2c_4x + 2c_4 + c_5 = x^2 + 2x + 4$$

$$\Rightarrow \quad 3c_3x^2 + x(6c_3 + 2c_4) + (2c_4 + c_5) = x^2 + 2c + 4.$$

Comparing coefficient on both side then,

$$3c_3 = 1, \qquad 6c_3 + 2c_4 = 2, \qquad 2c_4 + c_5 = 4$$

Solving we get, $c_3 = \frac{1}{3}$, $c_4 = 0$ and $c_5 = 5$.

So, the equations (2) becomes, $y_p = \frac{1}{3}x^3 + 4x$

Now, general equation of (i) is,

$$y(x) = y_p + y_h(x)$$

$$= c_1 + c_2e^{-x} + \frac{1}{3}x^3 + 4x$$

(vii) $y''' + 2y'' - y' - 2y = 1 - 4c^3$

Solution: Given that, $y''' + 2y'' - y' - 2y = 1 - 4c^3 \quad \ldots\ldots (i)$

The auxiliary equation of homogeneous part of (i) is,

$$m^3 + 2m^2 - m - 2 = 0$$

$$\Rightarrow \quad m^2(m+2) - 1(m+2) = 0$$

$$\Rightarrow \quad (m^3 - 1)(m+2) = 0 \Rightarrow (m+1)(m-1)(m+2) = 0$$

$$\Rightarrow m = 1, -1, -2$$

So, its solution is

$$y_h(x) = c_1e^x + c_2e^{-x} + c_3e^{-2x}$$

And for the particular solution of (i), let,

$$y_p = c_4x^3 + c_5x^2 + c_6x + e_7 \quad \ldots(ii)$$

Then, $y'_p = 3c_4x^2 + 2c^5x + c_6$

$y''_p = 6c_4x + 2c_5$ and $y'''_p = 6c_4$

Therefore, equation (i) becomes,

$$6c_4 + 2(6c_4 + 2c_5) - (3c_4x^2 + 2c_5x + c_6) - 2(c_4x^3 + c_5x^2 + c_6x + c_7) = 1 - 4$$

$$\Rightarrow \quad 6c_4 + 12c_4x + 4c_5 - 3c_4x^2 - 2c_5x - c_6 - 2c_4x^3 - 2c_5x^2 - 2c_6x - 2c_7 = 1 - 4x^3$$

$$\Rightarrow -2c_4x^3 + x^2(-3c_4 - 2c_5) + (12c_4 - 2c_5 - 2c_6) + (6c_4 + 4c_5 - c_6 - 2c_7) = -4x^3 + 1$$

Comparing coefficient on both side then,

$$-2c_4 = -4, \qquad -3c_4 - 2c_5 = 0,$$

$$12c_4 - 2c_5 - 2c_6 = 0 \qquad 6c_4 + 4c_5 - c_6 - 2c_7 = 1.$$

Solving we get, $c_4 = 2$, $c_5 = -3$, $c_6 = 15$, $c_7 = -8$

So, equation (ii) becomes,

$$y_p = 2x^3 - 3x^2 + 15x - 8$$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$

$$\Rightarrow \quad y(x) = c_1e^x + c_2e^{-x} + c_3e^{-2x} + 2x^3 - 3x^2 + 15 - 8$$

(ix) $y'' + 4y = \sin 3x$

Solution: Given that, $y'' + 4y = \sin 3x \quad \ldots\ldots (i)$

The auxiliary equation of homogeneous part of (i) is,

$$m^2 = -4 \Rightarrow m = \pm 2i$$

Then, its solution is,

$$y_h(x) = e^{0x}(A\cos 2x + B\sin 2x)$$

$$\Rightarrow \quad y_h(x) = (A\cos 2x + B\sin 2x)$$

And for the particular solution of (i), let,

$$y_p = c_1\sin 3x + c_2\cos 3x \quad \ldots\ldots (ii)$$

Then, $y'_p = 3c_1\cos 3x - 3c_2\cos 3x$ and $y''_p = 9c_1\sin 3x + 9c_2\cos 3x$

So that, (i) becomes,

$$-9c_1\sin 3x - 9c_2\cos 3x + 4(c_1\sin 3x + c_2\cos 3x) = \sin 3x$$

$$\Rightarrow \quad -9c_1\sin 3x - 9c_2\cos 3x + 4c_1\sin 3x + 4c_2\cos 3x = \sin 3x$$

$$\Rightarrow \quad -5c_1\sin 3x - 5c_2\cos 3x = \sin 3x$$

Comparing coefficient on both side then,

$$-5c_1 = 1 \qquad -5c_2 = 0$$

Solving we get, $c_1 = -\frac{1}{5}$, $c_2 = 0$.

Therefore, (ii) becomes, $y_p = \frac{-1}{5}\sin 3x$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$

$$= A\cos 2x + B\sin 2x - \frac{1}{5}\sin 3x$$

(x) $y'' + 3y' = 28\cosh 4x$

Solution: Given that, $y'' + 3y' = 28\cosh 4x$ .... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 3m = 0 \Rightarrow m(m+3) = 0 \Rightarrow m = 0, -3$$

So, its solution is,

$$y_h(x) = c_1 + c_2e^{-3x}$$

And for the particular solution of (i), let,

$$y_p = c_3\cosh 4x + c_4\sinh x4 \quad \text{...... (ii)}$$

Then, $y'_p = 4c_3\sinh 4x + 4c_4\cosh 4x$

and $y''_p = 16c_3\cosh 4x + 16c_4\sinh 5x$

Therefore, (i) becomes,

$$16c_3\cosh 4x + 16c_4\sinh 4x + 12c_3\sinh 4x + 12c_4\cosh 4x = 28\cosh 4x$$

$$\Rightarrow \cosh 4x\,(16c_3 + 12c_4) + \sinh 4x\,(16c_4 + 12c_3) = 28\cosh 4x$$

Comparing coefficient on both side then,

$$16c_3 + 12c_4 = 28, \qquad 16c_4 + 12c_3 = 0$$

Solving we get, $c_4 = -3$, $c_3 = 4$

So that (ii) becomes,

$$y_p = 4\cosh 4x - 3\sinh 4x$$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p = c_1 + c_2e^{-3x} + 4\cosh 4x - 3\sinh 4x$$

$$= c_1 + c_2e^{-3x} + 4\left(\frac{e^{4x}+e^{-4x}}{2}\right) - 3\left(\frac{e^{4x}-e^{-4x}}{2}\right)$$

$$= c_1 + c_2e^{-3x} + 2e^{4x} + 2e^{-1/2} - 1.5e^{4x} + 1.5e^{-4x}$$

$$= c_1 + c_2e^{-3x} + \frac{1}{2}e^{4x} + \frac{7}{2}e^{-4x}$$

(xi) $y'' + 2y' + 10y = 25x^2 + 3$

Solution: Given that, $y'' + 2y' + 10y = 25x^2 + 3$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 2m + 10 = 0$$

$$\Rightarrow m = \frac{-2 \pm \sqrt{4-40}}{2} = \frac{-2 \pm 6i}{2} = (-1 \pm 3i).$$

So, its solution is,

$$y_h(x) = e^{-2x}(A\cos 3x + B\sin 3x)$$

And for the particular solution of (i), let,

$$y_p = c_1x^2 + c_2x + c_3 \quad \text{....... (ii)}$$

Then, $y'_p = 2c_1x + c_2$ and $y''_p = 2c_1$

So that (i) becomes,

$$2c_1 + 2(2c_1x + c_2) + 10(c_1x^2 + c_2x + c_3) = 25x^2 + 3$$

$$\Rightarrow 2c_1 + 4c_1x + 2c_2 + 10c_1x^2 + 10c_2x + 10c_3 = 25x^2 + 3$$

$$\Rightarrow 10c_1x^2 + x(4c_1 + 10c_2) + (2c_1 + 2c_2 + 10c_3) = 25x^2 + 3$$

Comparing coefficient on both side then,

$$10c_1 = 25, \qquad 4c_1 + 10c_2 = 0, \qquad 2c_1 + 2c_2 + 10c_3 = 3.$$

Solving we get, $c_1 = \frac{5}{2}$, $c_2 = -1$, $c_3 = 0$.

Therefore, (ii) becomes, $y_p = \frac{5}{2}x^2 - x$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$

$$\Rightarrow y(x) = e^{-2x}(A\cos 3x + B\sin 3x) + \frac{5}{2}x^2 - x$$

(xii) $y'' + y' - 6y = -6x^3 + 3x^2 + 6x$

Solution: Given that, $y'' + y' - 6y = -6x^3 + 3x^2 + 6x$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + m - 6 = 0 \Rightarrow m^2 + 3m - 2m - 6 = 0$$

$$\Rightarrow m(m+3) - 2(m+3) = 0$$

$$\Rightarrow (m+3)(m-2) = 0$$

$$\Rightarrow m = -3, 2$$

So, its solution is, $y_h(x) = c_1e^{-3x} + c_2e^{2x}$

And for the particular solution of (i), let,

$$y_p = c_3x^3 + c_4x^2 + c_5x + c_6 \quad \text{...... (ii)}$$

Then, $y'_p = 3c_3x^2 + 2c_4x + c_5$ and $y''_p = 6c_3x + 2c_4$

Terefore, (i) becomes,

$$6c_3x + 2c_4 + 3c_3x^2 + 2c_4x + c_5 - 6c_3x^3 - 6c_4x^2 - 6c_5x - 6c_6 = 6x^3 + 3x^2 + 6x$$

$$\Rightarrow -6c_3x^3 + x^2(3c_3 - 6c_4) + x(6c_3 + 2c_4 - 6c_5) + (2c_4 + c_5 - 6c_6) = 6x^3 + 3x^2 + 6x$$

Comparing coefficient on both side then,

$$-6c_3 = -6, \quad 3x_3 - 6c_4 = 3, \quad 6c_3 + 2c_4 - 6c_5 = 6, \quad 2c_4 + c_5 - 6c_6 = 0$$

Solving we get,

$$c_3 = 1, c_4 = 0, c_5 = 0, c_6 = 0.$$

So that (ii) becomes

$$y_p = x^3$$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$
$$= c_1e^{-3x} + c_2 3^{2x} + x^3$$

(xiii) $y'' + 2y' - 35y = 12e^{5x} + 37\sin 5x$

Solution: Given that, $y'' + 2y' - 35y = 12e^{5x} + 37\sin 5x$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 2m - 35 = 0 \Rightarrow m^2 = 7m - 5m - 35 = 0$$
$$\Rightarrow m(m+7) - 5(m+7) = 0$$
$$\Rightarrow (m+7)(m-5) = 0$$
$$\Rightarrow m = -7, 5$$

So, its solution is, $y_h(x) = c_1e^{-7x} + c_2e^{5x}$

And for the particular solution of (i), let,

$$y_p = c_3xe^{5x} + c_4\sin 5x + c_5\cos 5x \quad \text{....... (ii)}$$

Then, $y'_p = c_3(5xe^{5x} + e^{5x}) + 5c_4\cos 5x - 25c_5\cos 5x$

$y''_p = c_3(25xe^{5x} + 5e^{5x} + 5e^{5x}) - 25c_4\sin 5x - 25c_5\cos 5x$

$\Rightarrow \quad y''_p = 25c_3xe^{5x} + 10c_3e^{5x} - 25c_4\sin 5x - 25c_5\cos 5x$

So (i) becomes,

$25c_3xe^{5x} + 10c_3e^{5x} - 25c_4\sin^{5x} - 25c_5\cos^{5x} + 10c_3xe^{5x} + 2c_3e^{5x} + 10c_4\cos^{5x}$
$- 10c_5\sin^{5x} - 35xc_3e^{-5x} - 35c_4\text{sn}^{5x} - 35c_5\cos^{5x} = 12e^{5x} + 37\sin 5x$

$\Rightarrow \quad 12c_3e^{5x} - 60c_4\sin 5x - 60c_5\cos 5x + 10c_4\cos 5x - 10c_5\sin 5x = 12e^{5x} + 37\sin 5x$

$\Rightarrow \quad 12c_3e^{5x} + \sin 5x(-60c_4 - 10c_5) + \cos 5x(-60c_5 + 10c_4) = 12e^{5x} + 37\sin 5x$

Comparing coefficient on both side then,

$$12c_3 = 12, \qquad -60c_4 - 10c_5 = 37, \qquad -60c_5 + 10c_4 = 0$$

Solving we get, $c_3 = 1$, $c_4 = -\frac{3}{5}$ and $c_5 = -0.1$.

So (ii) becomes

$$y_p = xe^{5x} - 0.60\sin 5x - 0.1\cos 5x$$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$
$$= c_1e^{-7x} + c^2e^{5x} - 0.6\sin 5x - 0.10\cos 5x$$

(xiv) $y'' + 10y' + 25y = e^{-5x}$

Solution: Given that, $y'' + 10y' + 25y = e^{-5x}$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 10m + 25 = 0 \Rightarrow m^2 + 2(5)m + 5^2 = 0$$
$$\Rightarrow (m+5)^2 = 0$$
$$\Rightarrow m = -5, -5$$

So, its solution is, $y_h(x) = (c_1 + c_2x)e^{-5x}$

And for the particular solution of (i), let,

$$y_p = c_3x^2e^{-5x} \quad \text{......... (ii)}$$

Then, $y'_p = c_3(-5x^2e^{-5x} + 2xe^{-5x})$

And, $y''_p = -5c_3(-5x^2e^{-5x} + 2xe^{-5x}) + 2c_3(-5xe^{-5x} + e^{-5x})$
$= 25c_3x^2e^{-5x} - 10c_3xe^{-5x} - 10c_3xe^{-5x} + 2c_3e^{-5x}$
$= 25c_3x^2e^{-5x} - 20c_3xe^{-5x} + 2c_3e^{-5x}$

So (i) becomes,

$25c_3x^2e^{-5x} - 20c_3xe^{-5x} + 2c_3e^{-5x} - 50c_3x^2e^{-5x} + 20c_3xe^{-5x} + 25c_3x^2e^{-5x} = e^{-5x}$

$\Rightarrow \quad 2c_3e^{-5x} = e^{-5x}$

This gives, $c_3 = \frac{1}{2}$

Then (ii) becomes, $y_p = \frac{1}{2}x^2e^{-5x}$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p = (c_1 + c_2x)e^{-5x} + \frac{1}{2}x^2e^{-5x}$$

(xv) $y'' + 8y' + 16y = 64\cosh 4x$

Solution: Given that, $y'' + 8y' + 16y = 64\cosh 4x$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 8m + 16 = 0 \Rightarrow (m+4)^2 = 0$$
$$\Rightarrow m = -4, -4$$

So, its solution is, $y_h(x) = (c_1 + c_2x)e^{-4x}$

And for the particular solution of (i), we have,

$$R = 64\left(\frac{e^{4x} + e^{-4x}}{2}\right) \Rightarrow R = 32(e^{4x} + 3^{-4x})$$

So, let, $y_p = Ae^{4x} + Bx^2e^{-4x}$ ...... (ii)

Then, $y'_p = 4Ae^{4x} + B\{x^2(-4e^{-4x}) + e^{-4x} \times 2x\}$
$= 4Ae^{4x} - 4Bx^2e^{-4x} + 2Bxe^{-4x}$

And, $y''_p = 16Ae^{4x} + 16Bx^2e^{-4x} - 8Bxe^{-4x} + 2Be^{-4x} - 8Bxe^{-4x}$
$= 16Ae^{4x} - 16Bx^2e^{-4x} - 8Bxe^{-4x} + 2B^{-4x} - 8Bxe^{-4x}$
$= 16Ae^{4x} + 16Bx^2e^{-4x} - 16Bxe^{-4x} + 2Be^{-4x}$

So (i) becomes,

$$16Ae^{4x} + 16Bx^2e^{-4x} - 16Bxe^{-4x} + 2Be^{-4x} + 32Ae^{4x} - 32Bx^2e^{-4x} + 16Bxe^{-4x} + 16Ae^{4x} + 16Bx^2e^{-4x} = 32e^{4x} + 32e^{-4x}$$

$$\Rightarrow \quad 64Ae^{4x} + 2Be^{-4x} = 32e^{4x} + 32e^{-4x}$$

Comparing coefficient on both side then,

$$64A = 32 \Rightarrow A = \frac{1}{2} \qquad \text{and} \qquad 2B = 32 \Rightarrow B = 16.$$

Then, (ii) becomes, $\quad y_P = \frac{1}{2}e^{4x} + 16x^2e^{-4x}$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_P$$
$$= (c_1 + c_2x)\, e^{-4x} + \frac{1}{2}e^{4x} + 16x^2e^{-4x}$$

(xvi) $y'' + y' = x$ [2009 Spring Q. No. 4(b)]

Solution: Given that, $\quad y'' + y' = x$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + m = 0 \Rightarrow m(m + 1) = 0 \Rightarrow m = 0, -1$$

So, its solution is, $\quad y_h(x) = c_1 + c_2e^{-x}$

And for the particular solution of (i), let,

$$y_P = (c_3x + c_4)\, x = c_3x^2 + c_4x \qquad \text{..... (ii)}$$

Then, $y'_p = 2c_3x + c_4 \qquad$ and $\qquad y''_p = 2c_3$

So (i) becomes,

$$2c_3 + 2c_3x + c_4 = x$$
$$\Rightarrow \quad 2c_3x + (2c_3 + c_4) = x$$

Comparing coefficient on both side then,

$$2c_3 = 1 \qquad \text{and} \qquad 2c_3 + c_4 = 0$$

Solving we get, $\quad c_3 = \frac{1}{2}$ and $c_4 = -1$.

Then, (ii) becomes, $\quad y_p = \frac{1}{2}x^2 - x$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$
$$= c_1 + c_2e^{-x} + \frac{1}{2}x^2 - x$$

(xvii) $y'' + y = \sin x$

Solution: Given that, $\quad y'' + y = \sin x$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

So, its solution is

$$y_h(x) = e^{0x}(A\cos x + B\sin x) = A\cos x + B\sin x$$

And for the particular solution of (i), let,

$$y_p = x(c_1\sin x + c_2\cos x) \qquad \text{..... (ii)}$$

Then, $\quad y'_p = (c_1\sin x + c_2\cos x) + x(c_1\cos x - c_2\sin x)$

$$= c_1\sin x + c_2\cos x + c_1x\cos x - c_2x\sin x$$

And, $\quad y''_p = c_1\cos x - c_2\sin x + c_1\{x(-\sin x) + \cos x\} - c_2\{x\cos x + \sin x\}$

$$= c_1\cos x - c_2\sin x - c_1x\sin x + c_1\cos x - c_2x\cos x - c_2\sin x$$

So (i) becomes,

$$2c_1\cos x - 2c_2\sin x - c_1x\sin x - c_2x\cos x + c_1x\sin x + c_2x\cos x = \sin x$$
$$\Rightarrow \quad 2c_1\cos x - 2c_2\sin x = \sin x$$

Comparing coefficient of sinx and cosx from both side then,

$$2c_1 = 0 \qquad \text{and} \qquad -2c_2 = 1$$

Solving we get, $\quad c_1 = 0,\ c_2 = \frac{-1}{2}$

Then, (ii) becomes, $\quad y_p = -\frac{1}{2}x\cos x$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$
$$= A\cos x + B\sin x - \frac{1}{2}x\cos x$$

(xviii) $y'' + 2y' + y = e^{-x}$

Solution: Given that, $\quad y'' + 2y' + y = e^{-x}$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 2m + 1 = 0 \Rightarrow (m + 1)^2 = 0$$
$$\Rightarrow \quad m = -1, -1$$

So, its solution is, $\quad y_h(x) = (c_1 + c_2)\, e^{-x}$

And for the particular solution of (i), let,

$$y_p = c_3x^2e^{-x} \qquad \text{....... (ii)}$$

Then, $\quad y'_p = c_3(2xe^{-x} - x^2e^{-x})$

And, $\quad y''_p = c_3[2(-xe^{-x} + e^{-x}) - (2xe^{-x} + x^2e^{-x})]$

$$= c_3(2e^{-x} - 4xe^{-x} + x^2e^{-x})$$

So (i) becomes,

$$c_3(2e^{-x} - 4xe^{-x} + x^2e^{-x} + 4xe^{-x} - 2x^2e^{-x} + x^2e^{-x}) = e^{-x}$$
$$\Rightarrow \quad c_3e^{-x}(2 - 4x + x^2 + 4x - 2x^2 + x^2) = e^{-x}$$
$$\Rightarrow \quad 2c_3e^{-x} = e^{-x}$$

Comparing coefficient on both side then, $c_3 = \frac{1}{2}$

Then, (ii) becomes, $y_p = \frac{1}{2}x^2e^{-x}$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p = (c_1 + c_2)e^{-x} + \frac{1}{2}x^2e^{-x}$$

(xix) $y'' - y = e^x$

Solution: Given that, $y'' - y = e^x$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$m^2 - 1 = 0 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$

So, its solution is, $y_h(x) = c_1e^x + c_2e^{-x}$

And for the particular solution of (i), let,

$y_p = c_3xe^x$ .... (ii)

Then, $y'_p = c_3xe^x + c_3e^x$ and $y''_p = c_3xe^x + c_3e^x + c_3e^x = c_3e^x + 2c_3e^x$

So (i) becomes,

$c_3xe^x + 2c_3e^x - c_3xe^x = e^x$

$\Rightarrow 2c_3e^x = e^x$

Comparing coefficient on both side then, $c_3 = \frac{1}{2}$

Then, (ii) becomes, $y_p = \frac{1}{2}xe^x$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p = c_1e^x + c_2e^{-x} + \frac{1}{2}xe^x$$

(xx) $y'' + 4y' + 5y = 10$

Solution: Given that, $y'' + 4y' + 5y = 10$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

$m^2 + 4m + 5 = 0$

$$\Rightarrow \quad m = \frac{-4 \pm \sqrt{4^2 - 4.1.5}}{2/1} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 + 2i}{2} = (-2 \pm i)$$

So, its solution is, $y_h(x) = e^{-2x}(A \cos x + B \sin x)$

And for the particular solution of (i), here $R = 10$ so let,

$y_p = c_1$ ...... (ii)

Then, $y'_p = 0$ and $y''_p = 0$

So (i) becomes,

$0 + 0 + 5c_1 = 10 \Rightarrow c_1 = 2$

Then, (ii) becomes, $y_p = 2$.

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p = e^{-2x}(A \cos x + B \sin x) + 2$$

(xxi) $y'' - y' = e^x + e^{-x}$

Solution: Given that, $y'' - y' = e^x + e^{-x}$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

$m^2 - m = 0 \Rightarrow m(m-1) = 0 \Rightarrow m = 0, 1$

So, its solution is, $y_h(x) = c_1 + c_2e^x$

And for the particular solution of (i), here R = 10 so let,

$y_p = c_3xe^x + c_4e^{-x}$ ...... (ii)

Then, $y'_p = c_3(xe^x + e^x) + (-1)c_4e^{-x}$

And, $y''_p = c_3xe^x + c_3e^x + c_3e^x + c_4e^{-x}$

So (i) becomes,

$c_3xe^x + 2c_3e^x + c_4e^{-x} - c_3xe^x - c_3e^x + c_4e^{-x} = e^x + e^{-x}$

$\Rightarrow \quad c_3e^x + 2c_4e^{-x} = e^x + e^{-x}$

Comparing coefficient on both side then,

$c_3 = 1$ and $2c_4 = 1 \Rightarrow c_4 = \frac{1}{2}$

Then, (ii) becomes, $y_p = xe^x + \frac{1}{2}e^{-x}$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p = c_1 + c_2e^x + xe^x + \frac{1}{2}e^{-x}$$

(xxii) $y'' - 4y' - 5y = e^x + 4$

Solution: Given that, $y'' - 4y' - 5y = e^x + 4$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

$m^2 - 4m - 5 = 0$

$$\Rightarrow \quad m = \frac{4 \pm \sqrt{16 + 4.1.5}}{2} = \frac{4 \pm 6}{2} = 2 \pm 3$$

$\Rightarrow \quad m = 5, -1$

So, its solution is, $y_h(x) = c_1e^{5x} + c_2e^{-x}$

And for the particular solution of (i), here $R = e^x + 4$, so let,

$$y_p = c_3e^x + c_4 \qquad \text{...... (ii)}$$

Then, $y'_p = c_3e^x$ and $y''_p = c_3e^x$

So (i) becomes,

$$c_3e^x - 4c_3e^x - 5c^3e^x - 5c_4 = e^x + 4$$

$$\Rightarrow -8c_3e^x - 5c_4 = 4^x + 4$$

Comparing coefficient on both side then,

$$-8c_3 = 1 \Rightarrow c_3 = \frac{-1}{8} \quad \text{and} \quad -5c_4 = 4 \Rightarrow c_4 = \frac{-4}{5}$$

Then, (ii) becomes, $y_p = -\frac{1}{8}e^x - \frac{4}{5}$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$

$$= c_1e^{5x} + c_2e^{-x} - \frac{e^x}{8} - \frac{4}{5}$$

(xxiii) $y'' - y' - 6y = e^{-x} - 7\cos x$

Solution: Given that, $y'' - y' - 6y = e^{-x} - 7\cos x$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - m - 6 = 0 \Rightarrow m^2 - 3m + 2m - 6 = 0$$

$$\Rightarrow (m-3)(m+2) = 0$$

$$\Rightarrow m = 3, -2$$

So, its solution is, $y_h(x) = c_1e^{3x} + c_2e^{-2x}$

And for the particular solution of (i), here $R = e^{-x} - 7\cos x$, so let,

$$y_p = c_3e^{-x} - (c_4\cos x + c_5\sin x) \qquad \text{...... (ii)}$$

Then, $y'_p = -c_3e^{-x} - (c_4 - \sin x + c_5\cos x)$

And, $y''_p = c_3e^{-x} - (-c_4\cos x - c_5\sin x)$

$$= c_3e^{-x} + c_4\cos x + c_5\sin x$$

So (i) becomes,

$$c_3e^{-x} + c_4\cos x + c_5\sin x - (-c_3e^{-x} + c_4\sin x - c_5\cos x) - 6(c_3e^{-x} - c_4\cos x - c_5\sin x) = e^{-x} - 7\cos x$$

$$\Rightarrow c_3e^{-x} + c_4\cos x + c_5\sin x + c_3e^{-x} - c_4\sin x + c_5\cos x - 6c_3e^{-x} + 6c_4\cos x + 6c_5\sin x = e^{-x} - 7\cos x$$

$$\Rightarrow -4c_3e^{-x} + 7c_4\cos x + 7c_5\sin x - c_4\sin x + c_5\cos x = e^{-x} - 7\cos x$$

$$\Rightarrow -4c_3e^{-x} + \cos x\,(7c_4 + c_5) + \sin x\,(7c_5 - c_4) = e^{-x} - 7\cos x$$

Comparing coefficient on both side then,

$$-4c_3 = 1, \qquad 7c_4 + c_5 = -7 \qquad 7c_5 - c_4 = 0$$

Solving we get, $c_3 = -\frac{1}{4}$, $c_4 = \frac{-49}{50}$ and $c_5 = -7$

Then, (ii) becomes, $y_p = \frac{-1}{4} + \frac{7}{50}\cos x + \frac{49}{50}\sin x$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$

$$= c_1e^{3x} + c_2e^{-2x} - \frac{1}{4}e^{-x} + \frac{7}{50}\cos x + \frac{49}{50}\sin x$$

(xxiv) $y'' + 5y' = 15x^2$

Solution: Given that, $y'' + 5y' = 15x^2$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 5m = 0 \Rightarrow m(m+5) = 0 \Rightarrow m = 0, -5$$

So, its solution is, $y_h(x) = c_1 + c_2e^{-5x}$

And for the particular solution of (i), here $R = 15x^2$, so let,

$$y_p = (c_3x^2 + c_4x + c_5)x$$

$$\Rightarrow y_p = c_3x^3 + c_4x^2 + c_5x \qquad \text{...... (ii)}$$

Then, $y'_p = 3c_3x^2 + 2c_4x + c_5$ and $y''_p = 6c_3x + 2c_4$

So (i) becomes,

$$6c_3x + 2c_4 + 15c_3x^2 + 10c_4x + 5c_5 = 15x^2$$

$$\Rightarrow 15c_3x^2 + x(6c_3 + 10c_4) + (2c_4 + 5c_5) = 15x^2$$

Comparing coefficient on both side then,

$$15c_3 = 15, \qquad 6c_3 + 10c_4 = 0, \qquad 2c_4 + 5c_5 = 0$$

Solving we get, $c_3 = 1$, $c_4 = \frac{-3}{5}$ and $c_5 = \frac{5}{25}$

Then, (ii) becomes, $y_p = x^3 - \frac{3}{5}x^2 + \frac{6}{25}x$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$

$$= c_1 + c_2e^{-5x} + x^3 - \frac{3}{5}x^2 + \frac{6}{25}x$$

(xxv) $y'' - 3y' = e^{3x} - 12x$

Solution: Given that, $y'' - 3y' = e^{3x} - 12x$ ..... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - 3m = 0 \Rightarrow m(m-3) = 0 \Rightarrow m = 0, 3$$

So, its solution is, $y_h(x) = c_1 + c_2e^{3x}$

And for the particular solution of (i), here $R = e^{3x} - 12x$, so let,

$$y_p = c_3xe^{3x} - (c_4x + c_5)\,x$$

$$\Rightarrow\ y_p = c_3xe^{3x} - c_4x^2 - c_5x \qquad \text{...... (ii)}$$

Then, $y'_p = 3c_3xe^{3x} + c_3e^{3x} - 2c_4x - c_5$

And, $y''_p = 9c_3xe^{3x} + 3c_3e^{3x} + c_3e^{3x} - 2c_4$

$$= 9c_3xe^{3x} + 4c_3e^{3x} - 2c_4$$

So (i) becomes,

$$9c_3xe^{3x} + 4c_3e^{3x} - 2c_4 - 3(3c_3xe^{3x} + c_3e^{3x} - 2c_4 - 5) = e^{3x} - 12x$$

$$\Rightarrow\ 9c_3xe^{3x} + 4c_3e^{3x} - 2c_4 - 9c_3xe^{3x} - 3c_3xe^{3x} + 6c_4x + 3c_5 = e^{3x} - 12x$$

$$\Rightarrow\ c_3e^{3x} + 6c_4x + (-2c_4 + 3c_5) = e^{3x} - 12x$$

Comparing coefficient on both side then,

$c_3 = 1,\qquad 6c_4 = -12\qquad 4 + 3c_5 = 0$

Solving we get, $c_3 = 1,\ c_4 = -2,\ c_5 = \frac{3}{-4}$

Then, (ii) becomes, $y_p = xe^{3x} + 2x^2 + \frac{4}{3}x$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$

$$= c_1 + c_2e^{3x} + ce^{3x} + 8x^2 + \frac{4}{3}x$$

(xxvi) $y'' - y' = x^3$

Solution: Given that, $y'' - y' = x^2$ ...... (i)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - m = 0\ \Rightarrow\ m(m-1) = 0\ \Rightarrow\ m = 0, 1.$$

So, its solution is, $y_h(x) = c_1 + c_2e^x$

And for the particular solution of (i), here $R = x^3$ so let,

$$y_p = (c_3x^3 + c_4x^2 + c_5x + c_6) \times x$$

$$\Rightarrow\ y_p = c_3x^4 + c_4x^3 + c_5x^2 + c_6x \qquad \text{..... (ii)}$$

Then, $y'_p = 4c_3x^3 + 3c_4x^2 + 2c_5x + c_6$ and $y''_p = 12c_3x^2 + 6c_4x + 2c_5$

So (i) becomes,

$$12c_3x^2 + 6c_4x + 2c_5 - 4c_3x^3 - 3c_4x^2 - 2c_5x - c_6 = x^3$$

$$\Rightarrow\ -4c_3x^3 + x^2(12c_3 - 3c_4) + x(6c_4 - 2c_5) + (2c_5 - c_6) = x^3$$

Comparing coefficient on both side then,

$-4c_3 = 1,\qquad 12c_3 - 3c_4 = 0,\qquad 6c_4 - 2c_5 = 0,\qquad 2c_5 - c_6 = 0.$

Solving we get, $c_3 = \frac{-1}{4},\ c_4 = -1,\ c_5 = -3,\ c_6 = -6.$

Then, (ii) becomes,

$$y_p = \frac{-1}{4}x^4 - x^3 - 3x^2 - 6x$$

Now, general equation of (i) is,

$$y_p = y_h(x) + y_p$$

$$= c_1 + c_2e^x + \frac{-1}{4}x^4 - x^3 - 3x^2 - 6x$$

Solve the following initial value problems.

3\. (i) **$y'' - y' - 2y = 3e^{2x},\ y(0) = 0,\ y'(0) = -2.$** **[2006 Fall Q. No. 5(b)]**

Solution: Given that, $y'' - y' - 2y = 3e^{2x}$ .... (i)

$y(0) = 0,\ y'(0) = -2$ .... (ii)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - m - 2 = 0\ \Rightarrow\ m^2 - 2m + m - 2 = 0$$

$$\Rightarrow\ m(m-2) + 1(m-2) = 0$$

$$\Rightarrow\ (m-2)(m+1) = 0$$

$$\Rightarrow\ m = 2, -1.$$

So, its solution is, $y_h(x) = c_1e^{2x} + c_2e^{-x}$

And for the particular solution of (i), here $R = 3e^{2x}$ so let,

$$y_p = c_3xe^{2x} \qquad \text{....... (iii)}$$

Then, $y'_p = 2c_3xe^{2x} + c_3e^{2x}$ and $y''_p = 4c_3xe^{2x} + 2c_3e^{2x} + 2c_3e^{2x}$

$$= 4c_3xe^{2x} + 4c_3e^{2x}$$

So (i) becomes,

$$4c_3xe^{2x} + 4c_3e^{2x} - 2c_3xe^{2x} - c_3e^{2x} - 2c_3xe^{2x} = 3e^{2x}$$

$$\Rightarrow\ 3c_3e^{2x} = 3e^{2x}$$

$$\Rightarrow\ c_3 = 1$$

Then (iii) becomes, $y_p = xe^{2x}$

Now, general equation of (i) is,

$$y_p = y_h(x) + y_p$$

$$\Rightarrow\ y(x) = c_1e^{2x} + c_2e^{-x} + xe^{2x} \qquad \text{...... (iv)}$$

Since, by (ii), $y(0) = 0$ then

$$0 = c_1e^{2.0} + c_2e^{-0} + 0\ \Rightarrow\ c_1 + c_2 = 0 \qquad \text{— (A)}$$

Differentiating (iv), we get,

$$y'(x) = 2c_1e^{2x} - c_2e^{-x} + 2xe^{2x} + e^{2x}$$

Since, by (ii), $y'(0) = -2$ then

$$-2 = 2c_1 - c_2 + 1 \Rightarrow 2c_1 - c_2 + 3 = 0 \qquad \text{— (B)}$$

Solving (A) and (B) we get, $c_1 = -1$, $c_2 = 1$.

Now (iv) becomes

$$y(x) = -e^{2x} + e^{-x} + xe^{2x}$$

(ii) $\mathbf{y'' + y' - 2y = 14 + 2x - 2x^2, y(0) = 0, y'(0) = 0}$ **[2008 Spring Q. No. 5(b)]**

Solution: Given that, $y'' + y' - 2y = 14 + 2x - 2x^2$ .... (i)

$y(0) = 0, y'(0) = 0$ .... (ii)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + m - 2 = 0 \Rightarrow m^2 + 2m - m - 2 = 0$$
$$\Rightarrow m(m+2) - 1(m+2) = 0$$
$$\Rightarrow (m+2)(m-1) = 0$$
$$\Rightarrow m = -2, 1.$$

So, its solution is, $y_h(x) = c_1e^{-2x} + c_2e^x$

And for the particular solution of (i), here $R = 14 + 2x - 2x^2$ so let,

$$y_p = c_3x^2 + c_4x + c_5 \qquad \text{...... (iii)}$$

Then, $y'_p = 2c_3x + c_4$ and $y''_p = 2c_3$

So (i) becomes,

$$2c_3 + 2c_3x + c_4 - 2c_3x^2 - 2c_4x - 2c_5 = 14 + 2x - 2x^2$$
$$\Rightarrow -2c_3x^2 + x(2c_3 - 2c_4) + (2c_3 + c_4 - 2c_5) = 14 + 2x - 2x^2$$

And for the particular solution of (i), here $R = 3e^{2x}$ so let,

$$-2c_3 = -2, \quad 2c_3 - 2c_4 = 2, \quad 2c_3 + c_4 - 2c_5 = 14.$$

Solving we get, $c_3 = 1, c_4 = 0, c_5 = -6$.

Then (iii) becomes, $y_p = x^2 - 6$.

Now, general equation of (i) is,

$$y_p = y_h(x) + y_p$$
$$\Rightarrow y(x) = c_1e^{-2x} + c_2e^x + x^2 - 6 \qquad \text{...... (iv)}$$

Since, by (ii), $0 = c_1 + c_2 - 6 \Rightarrow c_1 + c_2 = 6$ ...... (A)

And differentiating (iv) w. r. t. x. we get,

$$y'(x) = -2c_1 + c_2$$

Since, by (ii), $0 = -2c_1 + c_2 \Rightarrow -2c_1 + c_2 = 0$ — (B)

Solving (A) and (B) we get, $c_1 = 2$ and $c_2 = 4$.

Now (iv) becomes,

$$y(x) = 2e^{-3x} + 4e^x + x^2 - 6.$$

(iii) $\mathbf{y'' + y' - 2y = -6\sin 2x - 18\cos 2x, y(0) = 2, y'(0) = 2.}$ **[2006 Spring Q. No. 5(b)]**

Solution: Given that, $y'' + y' - 2y = -6\sin 2x - 18\cos 2x$ ....(i)

$y(0) = 2, y'(0) = 2$ .... (ii)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + m - 2 = 0 \Rightarrow m^2 + 2m - m - 2 = 0$$
$$\Rightarrow m(m+2) - 1(m+2) \; 0$$
$$\Rightarrow (m+2)(m-1) = 0$$
$$\Rightarrow m = -2, 1.$$

So, its solution is, $y_h(x) = c_1e^{-2x} + c_2e^x$

And for the particular solution of (i), here $R = -6\sin 2x - 18\cos 2x$, so let,

$$y_p = c_3\sin 2x + c_4\cos 2x \qquad \text{...... (iii)}$$

Then, $y'_p = 2c_3\cos 2x - 2c_4\sin 2x$ and $y''_p = -4c_3\sin 2x - 4c_4\cos 2x$

So (i) becomes,

$$-4c_3\sin 2x - 4c_4\cos 2x + 2c_3\cos 2x - 2c_4\sin 2x - 2c_3\sin 2x - 2c_4\cos 2x = -6\sin 2x - 18\cos 2x$$
$$\Rightarrow -6c_3\sin 2x - 6c_4\cos 2x + 2c_3\cos 2x - 2c_4\sin 2x = -6\sin 2x - 18\cos 2x$$
$$\Rightarrow \sin 2x(-6c_3 - 2c_4) + \cos 2x(-6c_4 + 2c_3) = -6\sin 2x - 18\cos 2x$$

Comparing coefficient on both side then,

$$-6c_3 - 2c_4 = -6, \qquad -6c_4 + 2c_3 = -18.$$

Solving we get, $c_3 = 0$ and $c_4 = 3$.

Then (iii) becomes, $y_p = 3\cos 2x$

Now, general equation of (i) is,

$$y_p = y_h(x) + y_p$$
$$\Rightarrow y(x) = c_1e^{-2x} + c_2e^x + 3\cos 2x \qquad \text{...... (iv)}$$

Since, by (ii), $2 = c_1 + c_2 + 3 \Rightarrow c_1 + c_2 = -1$ ...... (A)

And differentiating (iv) w. r. t. x., we get,

$$y'(x) = -2c_1e^{-2x} + c_2e^x - 6\sin 2x$$

Since, by (ii), $2 = -2c_1 + c_2$ ...... (B)

Solving (A) and (B) we get, $c_2 = 0$ and $c_1 = -1$.

Now (iv) becomes

$$y(x) = -e^{-2x} + 3\cos 2x$$
$$\Rightarrow y(x) = 3\cos 2x - e^{-2x}$$

(iv) $\mathbf{y'' + 1.5y' - y = 12x^2 + 6x^3 - x^4, y(0) = 4, y'(0) = -8}$

Solution: Given that, $y'' + 1.5y' - y = 12x^2 + 6x^3 - x^4$ .... (i)

$y(0) = 4, y'(0) = -8$ .... (ii)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + \frac{3m}{2} - 1 = 0 \Rightarrow 2m^2 + 3m - 2 = 0$$

$$\Rightarrow 2m^2 + 4m - m - 2 = 0$$

$$\Rightarrow 2m(m+2) - 1(m+2) = 0$$

$$\Rightarrow (m+2)(2m-1) = 0$$

$$\Rightarrow m = -2, \frac{1}{2}$$

So, its solution is, $y_h(x) = c_1e^{-2x} + c_2e^{0.5x}$

And for the particular solution of (i), here $R = 12x^2 + 6x^3 - x^4$, so let,

$$y_p = c_3x^4 + c_4x^3 + c_5x^2 + c_6x + c_7 \qquad \text{...... (iii)}$$

Then, $y'_p = 4c_3x^3 + 3c_4x^2 + 2c_5x + c_6$ and $y''_p = 12c_3x^2 + 6c_4x + 2c_5$

So (i) becomes,

$$12c_3x^2 + 6c_4x + 2c_5 + 6c_3x^3 + \frac{9}{2}c_4x^2 + 3c_5x + \frac{3}{2}c_6 - c_3x^4 - c_4x^3 - c_5x^2 - c_6x - c_7 = 12x^2 + 6x^3 - x^4$$

$$\Rightarrow -c_3x^4 + x^3(6c_3 - c_4) + x^2\left(12c_3 + \frac{9}{2}c_4 - c_5\right) + x(6c_4 + 3c_5 - c_6) + \left(2c_5 + \frac{3}{2}c_6 - c_7\right) = 12x^2 + 5x^3 - c^4$$

Comparing coefficient on both side then,

$$-c_3 = -1, \qquad 6c_3 - c_4 = 6, \qquad 12c_3 + \frac{9}{2}c_4 - c_5 = 12,$$

$$6c_4 + 3c_5 - c_6 = 0, \qquad 2c_3 + \frac{3}{2}c_6 - c_7 = 0$$

Solving we get, $c_3 = 1$, $c_4 = 0$, $c_5 = 0$, $c_6 = 0$, $c_7 = 0$.

Then (iii) becomes, $y_p = x^4$

Now, general equation of (i) is,

$$y_p = y_h(x) + y_p$$

$$\Rightarrow y(x) = c_1e^{-2x} + c_2e^{-0.5x} + x^4 \qquad \text{....... (iv)}$$

Since, by (ii), $4 = c_1 + c_2$ — (A)

And differentiating (iv) w. r. t. x. we get,

$$y'(x) = -2c_1e^{-2x} + 0.5\, c_2e^{0.5x} + 4x^3$$

Since, by (ii), $-8 = -2c_1 + \frac{1}{2}c_2$ — (B)

Solving (A) and (B) we get, $c_2 = 0$ and $c_1 = 4$.

Now (iv) becomes,

$$y(x) = 4e^{-2x} + x^4$$

(v) $y'' - 4y = e^{-2x} - 2x$, $y(0) = 0$, $y'(0) = 0$

Solution: Given that, $y'' - 4y = e^{-2x} - 2x$ .... (i)

$y(0) = 0$, $y'(0) = 0$ .... (ii)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 - 4 = 0 \Rightarrow m = \pm 2.$$

So, its solution is, $y_h(x) = c_1e^{-2x} + c_2e^{2x}$

And for the particular solution of (i), here $R = e^{-2x} - 2x$, so let,

$$y_p = c_3xe^{-2x} + c_4x \qquad \text{...... (iii)}$$

+ c5

Then, $y'_p = -2c_3xe^{-2x} + c_3e^{-2x} + c_4$

and $y''_p = 4c_3xe^{-2x} - 2c_3e^{-2x} - 2c_3e^{-2x}$

So (i) becomes,

$$4c_3xe^{-2x} - 4c_3e^{-2x} - 4c_3xe^{-2x} - 4c_4x = e^{-2x} - 2x$$

$$\Rightarrow -4c_3e^{-2x} - 4c_4x = e^{-2x} - 2x$$

Comparing coefficient on both side then,

$$-4c_3 = 1, \quad -4c_4 = -2$$

Solving we get, $c_3 = \frac{1}{2}$, $c_4 = \frac{1}{2}$

Then (iii) becomes, $y_p = \frac{-1}{4}xe^{-2x} + \frac{x}{2}$

Now, general equation of (i) is,

$$y_p = y_h(x) + y_p$$

$$\Rightarrow y(x) = c_1e^{-2x} + c_2e^{2x} - \frac{x}{4}e^{-2x} + \frac{x}{2} \qquad \text{...... (iv)}$$

Since, by (ii), $0 = c_1 + c_2$ — (A)

And differentiating (iv) w. r. t. x. we get,

$$y'(x) = -2c_1e^{-2x} + 2c_2e^{2x} - \frac{1}{4}(-2xe^{-2x} + e^{-2x}) + \frac{1}{2}$$

$$\Rightarrow y'(x) = -2c_1e^{-2x} + 2c_2e^{2x} + \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + \frac{1}{2}$$

Since, by (ii), $0 = -2c_1 + 2c_2 - \frac{1}{4} + \frac{1}{2}$

$$\Rightarrow -2c_1 + 2c_2 + \frac{1}{4} = 0 \qquad \text{— (B)}$$

Solving (A) and (B) we get,

$$c_1 = \frac{1}{16}, \qquad c_2 = -\frac{1}{16}$$

Now (iv) becomes

$$y(x) = \frac{1}{16}e^{-2x} - \frac{1}{16}e^{2x} - \frac{xe^{-2x}}{4} + \frac{x}{2}$$

$$= -\frac{1}{8}\left(\frac{e^{2x} - e^{-2x}}{2}\right) - \frac{x}{4}e^{-2x} + \frac{x}{2}$$

$$= -\frac{1}{8}\sinh 2x + \frac{x}{2} - \frac{x}{4}e^{-2x}$$

(vi) $y'' + 1.2y' + 0.36y = 4e^{-0.6x}$, $y(0) = 0$, $y'(0) = 1$.

Solution: Given that, $y'' + 1.2y' + 0.36y = 4e^{-0.6x}$ .... (i)

$y(0) = 0$, $y'(0) = 1$ .... (ii)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 1.2m + 0.36 = 0$$

$$m = \frac{-1.2 \pm \sqrt{(1.2)^2 - 4.1.036}}{2} = \frac{-1.2 \pm \sqrt{1.44 - 1.44}}{2} = -0.6, -0.6$$

So, its solution is, $y_h(x) = (c_1 + c_2x)e^{-0.6x}$

And for the particular solution of (i), here $R = 4e^{-0.6x}$, so let,

$y_p = c_2x^2 e^{-0.6x}$ .... (iii)

Then, $y'_p = -0.6c_2x^2e^{-0.6x} + 2c_2xe^{-0.x}$

And $y''_p = -0.6c_2.(2xe^{-0.6x} - x^2 0.6e^{-0.6x}) + 2c_2(-0.6xe^{-0.6x} + e^{-0.6x})$

$= -1.2c_2xe^{-0.6} + 0.36c_2x^2e^{-0.6x} - 1.2c_2xe^{-0.6x} + 2c_2e^{-0.6x}$

$= 0.36c_2x^2e^{-0.6x} - 2.4c_2xe^{-0.6x} + 2c_2e^{-0.6x}$

So (i) becomes,

$0.36c_2x^2e^{-0.x} - 2.4xc_2xe^{-0.6x} + 2c_2e^{-0.6x} - 0.72c_2x^2e^{-0.6x} + 2.4c_2xe^{-0.6x} + 0.36c_2x^2e^{-0.6x} = 4e^{-0.6x}$

$\Rightarrow 2c_2e^{-0.6x} = 4e^{-0.6x}$

$\Rightarrow 2c_2 = 4$

$\Rightarrow c_2 = 2.$

Then (iii) becomes, $y_p = 2x^2e^{-0.6x}$

Now, general equation of (i) is,

$y(x) = y_h(x) + y_p$

$= (c_1 + c_2x)e^{-0.6x} + 2x^2e^{-0.6x}$ .... (iv)

Since, by (ii), $0 = c_1$

And differentiating (iv) w. r. t. x. we get,

$y'(x) = 0.6c_1e^{-0.6x} + c_2e^{-0.6x} - 0.6c_2xe^{-0.6x} + 4xe^{-0.6x} - 1.2x^2e^{-0.6x}$

Since, by (ii), $1 = 0.6c_1 + c_2$

$\Rightarrow c_2 = 1$ [Being $c_1 = 0$]

Now (iv) becomes

$y(x) = xe^{-0.6x} + 2x^2e^{-0.6x}$

$= (x + 2x^2)e^{-0.6x}$

(vii) $y'' + y' = 2 + 2x + x^2$, $y(0) = 8$, $y'(0) = -1$

Solution: Given that, $y'' + y' = 2 + 2x + x^2$ .... (i)

$y(0) = 8$, $y'(0) = -1$ .... (ii)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + m = 0 \Rightarrow m(m+1) = 0 \Rightarrow m = 0, -1$$

So, its solution is, $y_h(x) = c_1 + c_2e^{-x}$

And for the particular solution of (i), here $R = x^2$ so let,

$y_p = (c_3x^2 + c_4x + c_5)x = c_3x^3 + c_4x^2 + c_5x$ ....(iii)

Then, $y_p' = 3c_3x^2 + 2c_4x + c_5$ and $y''_p = 6c_3x + 2c_4$

So (iii) becomes,

$6c_3 + 2c_4 + 3c_3x^2 + 3c_4x + c_5 = 2 + 2x + x^2$

$\Rightarrow 3c_3x^2 + x(6c_3 + 2c_4) + (2c_4 + c_5) = 2 + 2x + x^2$

Comparing coefficient on both side then,

$3c_3 = 1$, $6c_3 + 2c_4 = 2$, $2c_4 + c_5 = 2$.

Solving we get. $c_3 = \frac{1}{3}$, $c_5 = 2$ and $c_4 = 0$

Then (iii) becomes, $y_p = \frac{1}{3}x^3 + 2x$

Now, general equation of (i) is,

$y(x) = y_h(x) + y_p$

$\Rightarrow y(x) = c_1 + c_2e^{-x} + \frac{x^3}{3} + 2x$ .... (iv)

Since, by (ii), $8 = c_1 + c_2$ .... (A)

And differentiating (iv) w. r. t. x. we get,

$y'(x) = -c_2e^{-2x} + x^2 + 2$

Since, by (ii), $-1 = c_2 + 2 \Rightarrow c_2 = 3.$

Then (A) gives, $c_1 = 5.$

Now (iv) becomes

$$y(x) = 5 + 3e^{-x} + \frac{x^3}{3} + 2x.$$

(viii) $y'' + 2y' + y = e^{-x}$, $y(0) = -1$, $y'(0) = 1$ **[2008 Fall Q. No. 5(b)]**

Solution: Given that, $y'' + 2y' = e^{-x}$ .... (i)

$y(0) = -1$, $y'(0) = 1$ .... (ii)

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 2m + 1 = 0 \Rightarrow (m+1)^2 = 0 \Rightarrow m = -1, -1$$

So, its solution is, $y_h(x) = (c_1 + c_2x)e^{-x}$

And for the particular solution of (i), here $R = e^{-x}$ so let,

$y_p = c_3x^2e^{-x}$ .... (iii)

Then, $y'_p = 2c_3xe^{-x} - c_3x^2e^{-x}$

And, $y''_p = 2c_3(-e^{-x}x + e^{-x}) - c_3(2xe^{-x} - x^2e^{-x})$

$= -2c_3xe^{-x} + 2c_3e^{-x} - 2c_3xe^{-x} + c_3x^2e^{-x}$

$= -2c_3xe^{-x} + 2c_3e^{-x} + c_3x^2e^{-x}$

So (i) becomes,

$$-4c_3xe^{-x} + 2c_3e^{-x} + c_3x^2e^{-x} + 4c_3xe^{-x} - 2c_3x^2e^{-x} + c_3x^2e^{-x} = e^{-x}$$

$$\Rightarrow \quad 2c_3e^{-x} = e^{-x}$$

Comparing coefficient on both side then,

$$2c_3 = 1 \Rightarrow c_3 = \frac{1}{2}$$

Then (iii) becomes, $\quad y_p = \frac{1}{2}x^2e^{-x}$

Now, general equation of (i) is,

$$y_p = y_h(x) + y_p$$

$$\Rightarrow y(x) = (c_1 + c_2x)\,e^{-x} + \frac{1}{2}x^2e^{-x} \qquad \text{........(iv)}$$

Since, by (ii), $-1 = c_1$

And differentiating (iv) w. r. t. x. we get,

$$y'(x) = -c_1e^{-x} + c_2e^{-x} - c_2xe^{-x} + xe^{-x} - \frac{1}{2}x^2e^{-x}$$

Since, by (ii), $1 = -c_1 + c_2 \Rightarrow c_2 = 0.$

Now (iv) becomes

$$y(x) = -e^{-x} + \frac{1}{2}x^2e^{-x}$$

$$\Rightarrow y(x) = \left(\frac{x^2}{2} - 1\right)e^{-x}$$

(ix) $y'' + 2y' + 5y = 1.25e^{0.5x} + 40\cos 4x - 55\sin 4x,\ y(0) = 0.2,\ y'(0) = 60.1$

Solution: Given that, $\quad y'' + 2y' + 5y = 1.25e^{0.5x} + 40\cos4x - 55\sin4x \qquad \text{.......(i)}$

$y(0) = 0.2,\ y'(0) = 60.1 \qquad \text{........ (ii)}$

The auxiliary equation of homogeneous part of (i) is,

$$m^2 + 2m + 5 = 0$$

$$\Rightarrow m = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4i}{2} = (-1 \pm 2i)$$

So, its solution is

$$y_h(x) = e^{-x}(A\cos2x + B\sin2x)$$

And for the particular solution of (i), here $R = x^2$ so let,

$$y_p = c_1e^{0.5x} + c_2\cos4x + c_3\sin4x \qquad \text{........ (iii)}$$

Then, $y_p' = 0.5c_1e^{0.5x} - 4c_2\sin4x + 4c_3\cos4$

And, $y_p'' = 0.25c_1e^{0.5x} - 16c_2\cos4x - 16c_3\sin4x$

So (i) becomes,

$$0.25c_1e^{0.5x} - 16c_2\cos4x - 16c_3\sin4x + c_1e^{0.5x} - 8c_2\sin4x + 8c_3\cos4x + 5c_1e^{0.5x} + 5c_2\cos4x + 5c_3\sin4x = 1.25e^{0.5x} + 40\cos4x - 55\sin4x$$

$$\Rightarrow 6.25c_1e^{0.5x} + (-11c_2 + 8c_3)\cos4x - (11c_3 + 8c_2)\sin4x = 1.25e^{0.5x} + 40\cos4x - 55\sin4x$$

Comparing coefficient on both side then,

$$6.25c_1 = 1.25, \qquad -11c_2 + 8c_3 = 40, \qquad -11c_3 - 8c_2 = -55$$

Solving we get, $c_1 = \frac{1}{5}$, $c_2 = 0$ and $c_3 = 5$.

Then (iii) becomes, $y_p = \frac{1}{5}e^{0.5x} + 5\sin 4x$

Now, general equation of (i) is,

$$y(x) = y_h(x) + y_p$$
$$= e^{-x}(A\cos 2x + B\sin 2x) + 0.2e^{0.5x} + 5\sin 4x) \qquad \text{......... (iv)}$$

Since, by (ii), $0.2 = A + 2 \Rightarrow A = -1.8$

And differentiating (iv) w. r. t. x., we get,

$$y'(x) = -2A\sin 2xe^{-x} - Aco2xe^{-x} - e^{-x}B\sin 2x + 2B\cos 2xe^{-x} + 20\cos 4x$$

Since, by (ii),

$$60.1 = -A + 2B + 1 + 20$$
$$\Rightarrow 60.1 = 1.8 + 2B + 21$$
$$\Rightarrow 2B = 60.1 - 22.8$$
$$\Rightarrow B = \frac{37.3}{2} = 18.65$$

Now (iv) becomes

$$y(x) = e^{-x}(18.65\sin 2x - 1.8\cos 2x) + 0.2e^{0.5x} + 5\sin 4x$$