Exercise 6.9

1. Solve the following differential equations.

(i) $y'' - 25y = 0$

Solution: Given equation is, $y'' - 25y = 0$

So, its auxiliary equation is,

$$m^2 - 25 = 0 \Rightarrow m^2 = (\pm 5)^2 \Rightarrow m = \pm 5$$

Therefore, the general solution of given equation is,

$$y(x) = c_1 e^{5x} + c_2 e^{-5x}$$

(ii) $y'' - 8y' + 16 = 0$

Solution: Given equation is, $y'' - 8y' + 16 = 0$

So, its auxiliary equation is,

$$m^2 - 8m + 16 = 0 \Rightarrow (m-4)^2 = 0 \Rightarrow m = 4, 4.$$

Therefore, the general solution of given equation is,

$$y(x) = (c_1 + c_2x)\, e^{4x}$$

(iii) **$y'' + y' + 0.25y = 0$**

Solution: Given equation is, $y'' + y' + 0.25y = 0$

So, its auxiliary equation is,

$$m^2 + m + \frac{1}{4} = 0 \Rightarrow 4m^2 + 4m + 1 = 0$$

$$\Rightarrow 4m^2 + 2.2m.1 + (1)^2 = 0$$

$$\Rightarrow (2m+1)^2 = 0 \Rightarrow m = -\frac{1}{2}, -\frac{1}{2}$$

Therefore, the general solution of given equation is,

$$y(x) = (c_1 + c_2x)\, e^{\frac{1}{2x}}$$

(iv) **$8y' - 2y' - y = 0$**

Solution: Given equation is, $8y' - 2y' - y = 0$

So, its auxiliary equation is,

$$8m^2 - 2m - 1 = 0 \Rightarrow 8m^2 - 4m + 2m - 1 = 0$$

$$\Rightarrow 4m(2m-1) + (1(2m-1) = 0$$

$$\Rightarrow (2m-1)(4m+1) = 0$$

$$\Rightarrow m = \frac{1}{2}, -\frac{1}{4}$$

Therefore, the general solution of given equation is,

$$y(x) = c_1e^{\frac{1}{2x}} + c_2e^{\frac{1}{4x}}$$

(v) **$2y'' + 10y' + 25y = 0$**

Solution: Given equation is,

So, its auxiliary equation is,

$$2m^2 + 10m + 25 = 0 \Rightarrow m = \frac{-10 \pm \sqrt{10^2 - 4.2.25}}{2.2}$$

$$= \frac{-10 \pm \sqrt{100}}{4}$$

$$= \frac{-10 \pm \sqrt{100i^2}}{4} = \frac{-10 \pm 10i}{4} = \frac{-5 \pm 5i}{2}$$

Here one real and two imaginary roots of y. Therefore, the general solution of given equation is,

$$y(x) = e^{\frac{-5}{2x}}\left(A\cos\frac{5}{2}x + B\sin\frac{5}{2}x\right)$$

(vii) $y'' - 4y' + 4y = 0$

Solution: Given equation is, $y'' - 4y' + 4y = 0$

So, its auxiliary equation is,

$$m^2 - 4m + 4 = 0 \Rightarrow (m-2)^2 = 0 \Rightarrow m = 2, 2.$$

Thus, y has two equal real roots. Therefore, the general solution of given equation is,

$$y(x) = (c_1 + c_2x)e^{2x}$$

(viii) $4y'' + 4y' - 3y = 0$

Solution: Given equation is, $4y'' + 4y' - 3y = 0$

So, its auxiliary equation is,

$$4m^2 + 4m - 3 = 0 \Rightarrow 4m^2\ 6m - 2m - 3 = 0$$
$$\Rightarrow 2m(2m+3) - 1(2m+3) = 0$$
$$\Rightarrow (2m+3)(2m-3) = 0$$
$$\Rightarrow m = \frac{-3}{2}, \frac{1}{2}$$

Therefore, the general solution of given equation is,

$$y(x) = c_1e^{\frac{-3}{2x}} + c_2e^{\frac{1}{2x}}$$

(ix) $2y'' - 9y' = 0$

Solution: Given equation is, $2y'' - 9y' = 0$

So, its auxiliary equation is,

$$2m^2 - 9m = 0 \Rightarrow m(2m-9) = 0 \Rightarrow m = 0, \frac{9}{2}$$

Therefore, the general solution of given equation is,

$$y(x) = c_1e^0 + c_2e^{\frac{9}{2x}}$$
$$\Rightarrow y(x) = c_1\, c_2e^{\frac{9}{2x}}$$

(x) $y'' + 9y' + 20y = 0$

Solution: Given equation is, $y'' + 9y' + 20y = 0$

So, its auxiliary equation is,

$$m^2 + 9m + 20 = 0 \Rightarrow m^2 + 5m + 4m + 20 = 0$$
$$\Rightarrow m(m+5) + 4(m+5) = 0$$
$$\Rightarrow m = -5, -4.$$

Therefore, the general solution of given equation is,

$$y(x) = c_1e^{-5x} + c_2e^{-4x}$$

(xi) $9y'' - 30y' + 25y = 0$

Solution: Given equation is, $9y'' - 30y' + 25y = 0$

So, its auxiliary equation is,

$$9m^2 - 30m + 25 = 0 \Rightarrow 9m^2 - 2(3m)(5) + 25 = 0$$
$$\Rightarrow (3m-5)^2 = 0 \Rightarrow m = \frac{5}{3}, \frac{5}{3}$$

Therefore, the general solution of given equation is,

$$y(x) = (c_1 + c_2x)e^{\frac{5}{3x}}$$

(xii) $y'' + 2ky' + k^2y = 0$ where k is a constant.

Solution: Given equation is, $y'' + 2ky' + k^2y = 0$

So, its auxiliary equation is,

$$m^2 + 2km + k^2 = 0 \Rightarrow (m+k)^2 = 0$$
$$\Rightarrow m = -k, -k.$$

Therefore, the general solution of given equation is,

$$y(x) = (c_1 + c_2x)e^{-kx}$$

(xiii) $y'' - 3y' - 4y = 0$

Solution: Given equation is, $y'' - 3y' - 4y = 0$

So, its auxiliary equation is,

$$m^2 - 3m - 4 = 0 \Rightarrow m^2 - 4m + m - 4 = 0$$
$$\Rightarrow m(m-4) + 1(m-4) = 0$$
$$\Rightarrow (m-4)(m+1) = 0 \Rightarrow m = 4, -1.$$

Therefore, the general solution of given equation is,

$$y(x) = c_1e^{4x} + c_2e^{-x}$$

(xiv) $y'' - 4y' + y = 0$

Solution: Given equation is, $y'' - 4y' + y = 0$

So, its auxiliary equation is,

$$m^2 - 4m + 1 = 0 \Rightarrow m = \frac{4 \pm \sqrt{16^2 - 4.1.1}}{2.1}$$

$$= \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = (2 \pm \sqrt{3})$$

Therefore, the general solution of given equation is,

$$y(x) = c_1 e^{(2+\sqrt{3})x} + c_2 e^{(2-\sqrt{3})x}$$

(xv) $y'' + 6y' + 9y = 0$

Solution: Given equation is, $y'' + 6y' + 9y = 0$

So, its auxiliary equation is,

$$m^2 + 6m + 9 = 0 \Rightarrow m^2 + 2(3)m + 9 = 0$$

$$\Rightarrow (m+3)^2 = 0 \Rightarrow m = -3, -3$$

Therefore, the general solution of given equation is,

$$y(x) = (c_1 + c_2 x)e^{-3x}$$

(xvi) $16y'' - \pi^2 y = 0$

Solution: Given equation is, $16y'' - \pi^2 y = 0$

So, its auxiliary equation is,

$$16m^2 - \pi^2 = 0 \Rightarrow 16m^2 = \pi^2 \Rightarrow m = \pm\frac{\pi}{4}$$

Therefore, the general solution of given equation is,

$$y(x) = c_1 e^{\frac{\pi}{4}x} + c_2 e^{\frac{-\pi}{4}x}$$

(xvii) $25y'' + 40y' + 16y = 0$

Solution: Given equation is, $25y'' + 40y' + 16y = 0$

So, its auxiliary equation is,

$$16m^2 - 8m + 5 = 0 \Rightarrow m = \frac{8 \pm \sqrt{8^2 - 4.16.5}}{2.16}$$

$$= \frac{8 \pm \sqrt{64 - 320}}{32}$$

$$= \frac{8 \pm \sqrt{-256}}{32}$$

$$= \frac{8 \pm \sqrt{256i^2}}{32} = \frac{8 \pm 16i}{32} = \frac{1 \pm 2i}{4}$$

Therefore, the general solution of given equation is,

$$y(x) = e^{\frac{1}{4x}}\left(A\cos\frac{1}{2^x} + B\sin\frac{x}{2}\right).$$

(xviii) $17y'' - 8y' + 5y = 0$

Solution: Given equation is

$$17y'' - 8y' + 5y = 0 \quad \text{.......(1)}$$

The auxiliary equation of (1) is

$$16m^2 - 8m + 5 = 0$$

$$\Rightarrow \quad m = \frac{8 \pm \sqrt{64 - 320}}{32} = \frac{8 \pm \sqrt{-256}}{32} = \frac{8 \pm i16}{32} = \frac{1}{4} \pm i\frac{1}{2}$$

So, the general solution of (1) is

$$y = e^{x/4}\left(A\cos\frac{x}{2} + B\sin\frac{x}{2}\right)$$

(xix) $y'' - 9\pi^2 y = 0$

Solution: Given equation is, $y'' - 9\pi^2 y = 0$

So, its auxiliary equation is,

$$m^2 - 9\pi^2 = 0 \Rightarrow m^2 - 9\pi^2 = 0 \Rightarrow m = \pm 3\pi$$

Therefore, the general solution of given equation is,

$$y(x) = (c_1 e^{3\pi x} + c_2 e^{-3\pi x}$$

(xx) $y'' - 2\sqrt{2}y' + 2y = 0$

Solution: Given equation is, $y'' - 2\sqrt{2}y' + 2y = 0$

So, its auxiliary equation is,

$$m^2 - 2\sqrt{2}m + 2 = 0 \Rightarrow (m - \sqrt{2})^2 = 0$$

$$\Rightarrow m = \sqrt{2}, \sqrt{2}$$

Therefore, the general solution of given equation is,

$$y(x) = (c_1 + c_2 x)e^{\sqrt{2}x}$$

(2) Solve the following initial value problems

(i) $y'' - 16y = 0, y(0) = 1, y'(0) = 20$

Solution: Given equation is, $y' - 16y = 0$ ...... (i)

$y(0) = 1, y'(0) = 20$ ...... (ii)

It auxiliary equation is, $m^2 - 16 = 0 \Rightarrow m = \pm 4$

Therefore, the general solution of given equation (i) is,

$$y(x) = c_1e^{4x} + c_2e^{-4x} \qquad \text{...... (iii)}$$

By (ii), we have. $1 = c_1e^0 + c_2e^0 \Rightarrow c_1 + c_2 = 1$ ...... (A)

Differential equation (iii) w. r. t. x, then,

$$y'(x) = 4c_1e^{4x} - 4c_2e^{-4x}$$

By (ii), we have, $20 = 4c_1 - 4c_2 \Rightarrow c_1 - c_2 = 5$ ...... (B)

Solving the equations (A) and (B) we get,

$$c_1 = 3 \text{ and } c_2 = -2.$$

Now, equation (iii) becomes,

$$y(x) = 3e^{4x} - 2e^{-4x}$$

(ii) $y'' + 6y' + 9y = 0, y(0) = -4, y'(0) = 14$

Solution: Given equation is, $y'' + 6y' + 9y = 0$ ...... (i)

$y(0) = -4, y'(0) = 14$ ...... (ii)

So, its auxiliary equation is,

$$m^2 + 6m + 9 = 0 \Rightarrow m^2 + 2(3)m + 9 = 0$$

$$\Rightarrow (m+3)^2 = 0 \Rightarrow m = -3, -3.$$

Therefore, the general solution of given equation (i) is,

$$y(x) = (c_1 + c_2x)e^{-3x} \qquad \text{...... (iii)}$$

By (ii), we have, $-4 = (c_1 + 0)e^0 \Rightarrow c_1 = -4$ ...... (A)

Differential equation (iii) w. r. t. x, then,

$$y'(x) = c_2(-3xe^{-3x} + e^{-3x}) - 3c_1e^{-3x}$$

By (ii), we have, $14 = c_2(-3e^0 + e^0) - 3c_1$

$$\Rightarrow 14 = -2c_2 + 12 \Rightarrow c_2 = -1. \qquad \text{[Using (A)]}$$

Now, equation (iii) becomes,

$$y(x) = (-4 + 2x)^{-3x}$$

$$\Rightarrow y(x) = (2x - 4)e^{-3x}$$

(iii) $y'' + y' - 2y = 0, y(0) = 3, y'(0) = 0$

Solution: Given equation is, $y'' + y' - 2y = 0$ ...... (i)

$y(0) = 3, y'(0) = 0$ ...... (ii)

So, its auxiliary equation is,

$$m^2 + m - 2 = 0 \Rightarrow m^2 + 2m - m - 2 = 0$$

$$\Rightarrow m(m+2) - (m+2) = 0$$

$$\Rightarrow (m+2)(m-1) = 0 \Rightarrow m = -2, 1$$

Therefore, the general solution of given equation (i) is,

$$y(x) = c_1e^{-2x} + c_2e^{x} \qquad \text{...... (iii)}$$

By (ii), we have, $c_1 + c_2 = 3$ ...... (A)

Differential equation (iii) w. r. t. x, then,

$$y'(x) = 2c_1e^{-2x} + c_2e^{x}$$

By (ii), we have, $c_2 - 2c_1 = 0$ ...... (B)

Solving (A) and (B) we get,

$$c_1 = 1 \text{ and } c_2 = 2.$$

Now, equation (iii) become,

$$y(x) = e^{-2x} + 2e^{x}$$

(iv) $y'' - 4y' + 5y = 0, y(0) = 1, y'(0) = 2$ **[2011 Fall Q. No. 5(b)]**

Solution: Given equation is, $y'' - 4y' + 5y = 0$ ...... (i)

$y(0) = 1, y'(0) = 2$ ...... (ii)

So, its auxiliary equation is,

$$m^2 - 4m + 5 = 0$$

$$\Rightarrow m = \frac{+4 \pm \sqrt{16 - 4.1.5}}{2.1} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm \sqrt{4}i}{2} = \frac{4 \pm 2i}{2} = (2 \pm i)$$

Therefore, the general solution of given equation (i) is,

$$y(x) = e^{2x}(A\cos x + B\sin x) \qquad \text{...... (iii)}$$

By (ii), we have, $1 = e^0(A\cos 0 + B\sin 0) \Rightarrow A = 1.$

Differential equation (iii) w. r. t. x, then,

$$y'(x) = e^{2x}(-A\sin x + B\cos x) + (A\cos x + B\sin x)e^{2x}$$

By (ii), we have,

$$2 = e^0(A - \sin x + B\cos x) + 2(\cos x + B\sin x)e^{2x}$$

$$\Rightarrow 2 = B + 2A \Rightarrow 2 = B + 2 \qquad [\because A = 1]$$

$$\Rightarrow B = 0$$

Now, equations (iii) becomes,

$$y(x) = e^{2x}\cos x.$$

(v) $y'' - 4y' + 4y = 0, y(0) = 3y'(0) = 1$ **[2009 Spring Q. No. 5(b)]**

Solution: Given equation is, $y'' - 4y' + 4y = 0$ ...... (i)

$y(0) = 3, y'(0) = 1$ ...... (ii)

So, its auxiliary equation is,

$$m^2 - 4m + 4 = 0 \Rightarrow (m-2)^2 = 0 \Rightarrow m = 2, 2.$$

Therefore, the general solution of given equation (i) is,

$$y(x) = (c_1 + c_2x)e^{2x} \qquad \text{...... (iii)}$$

By (ii), we have, $3 = (c_1 + c_2 0)e^0 \Rightarrow c_1 = 3.$

Differential equation (iii) w. r. t. x, then,

$$y'(x) = 2c_1e^{2x} + c_2(x.\ 2e^{2x} + e^{2x})$$

$$\Rightarrow \quad y'(x) = 2c_1e^{2x} + 2c_2xe^{2x} + c_2e^{2x}$$

By (ii), we have, $1 = 2 \times 3e^0 + 0 + c_2e^0 \Rightarrow c_2 = -5$

Now, equations (iii) becomes.

$$y(x) = (3 - 5x)\, e^{2x}$$

(vi) $y'' - y = 0,\ y(0) = 6,\ y'(0) = -4$

Solution: Given equation is, $y'' - y = 0$ ...... (i)

$y(0) = 6,\ y'(0) = -4$ ...... (ii)

So, its auxiliary equation is,

$$m^2 - 1 = 0 \Rightarrow m = \pm 1$$

Therefore, the general solution of given equation (i) is,

$$y(x) = c_1e^x + c_2e^{-x} \quad \text{...... (iii)}$$

By (ii), we have, $6 = c_1 + c_2$ ... (A)

Differential equation (iii) w. r. t. x, then,

$$y'(x) = c_1e_x - c_2e^{-x}$$

By (ii), we have, $-4 = c_1 - c_2$ ... (B)

Solving the equations (A) and (B) we get,

$$c_1 = 1 \text{ and } c_2 = 5$$

Now, equations (iii) becomes,

$$y(x) = e^x + 5e^{-x}$$

(vii) $y'' - 4y' + 3y = 0,\ y(0) = -1,\ y'(0) = -5$

Solution: Given equation is, $y'' - 4y' + 3y = 0$ ...... (i)

$y(0) = -1,\ y'(0) = -5$ ...... (ii)

So, its auxiliary equation is,

$$m^2 - 4m + 3 = 5 \Rightarrow m^2 - 3m - m + 3 = 0$$

$$\Rightarrow m(m-3) - 1(m-1) = 0$$

$$\Rightarrow (m-3)(m-1) = 0 \Rightarrow m = 3, 1.$$

Therefore, the general solution of given equation (i) is,

$$y(x) = c_1e^{3x} + c_2e^x \quad \text{...... (iii)}$$

By (ii), we have, $-1 = c_1 + c_2$ ... (A)

Differential equation (iii) w. r. t. x, then,

$$y'(x) = 3c_1e^{3x} + c_2e^x$$

By (ii), we have, $-5 = 3c_1 + c_2$ ... (B)

Solving the equations (A) and (B) we get,

$$c_1 = -2 \text{ and } c_2 = 1$$

Now, equation (iii) becomes,

$$y(x) = e^x - 2e^{3x}$$

(viii) $y'' + 4y' + 4y = 0\ y(0) = 1,\ y'(0) = 1$

Solution: Given equation is, $y'' + 4y' + 4y = 0$ ...... (i)

$y(0) = 1,\ y'(0) = 1$ ...... (ii)

So, its auxiliary equation is,

$$m^2 + 4m + 4 = 0 \Rightarrow (m+2)^2 = 0 \Rightarrow m = -2, -2.$$

Therefore, the general solution of given equation (i) is,

$$y(x) = (c_1 + c_2x)e^{-2x} \quad \text{...... (iii)}$$

By (ii), we have, $1 = c_1$

Differential equation (iii) w. r. t. x, then,

$$y'(x) = -2c_1e^{-2x} + c_2(-2xe^{-2x} + e^{-2x})$$

By (ii), we have, $1 = -2c_1 + c_2$ .... (A)

Then, $c_2 = 3$ [Using (A)]

Now, equations (iii) becomes,

$$y(x) = (1 + 3x)e^{-2x}$$

(ix) $8y'' - 2y' - y = 0,\ y(0) = -0.2,\ y'(0) = -0.325$

Solution: Given equation is, $8y'' - 2y' - y = 0$ ...... (i)

$y(0) = -0.2,\ y'(0) = -0.325$ ...... (ii)

So, its auxiliary equation is,

$$8m^2 - 2m - 1 = 0 \Rightarrow 8m^2 - 4m + 2m - 1 = 0$$

$$\Rightarrow 4m(2m-1) + 1(2m-1) = 0$$

$$\Rightarrow (2m-1)(4m+1) = 0 \Rightarrow m = \frac{1}{2}, -\frac{1}{4}$$

Therefore, the general solution of given equation (i) is,

$$y(x) = c_1e^{\frac{1}{2}} + c_2e^{\frac{-x}{4}} \quad \text{...... (iii)}$$

By (ii), we have, $-0.2 + c_1 + c_2$ ... (A)

Differential equation (iii) w. r. t. x, then,

$$y'(x) = \frac{1}{2}c_1e^{\frac{x}{2}} - \frac{1}{4}c_2e^{\frac{x}{2}}$$

By (ii), we have, $-0.325 = \frac{1}{2}c_1 - \frac{1}{4}c_2$

$\Rightarrow 2c_1 - c_2 = -1.3$ ... (B)

Solving the equations (A) and (B) we get,

$c_1 = -0.5$ and $c_2 = 0.3$

Now, equation (iii) becomes,

$$y(x) = 0.3e^{\frac{-x}{4}} - 0.5e^{\frac{x}{2}}$$

(x) $y'' + 2.2y' + 1.17y = 0$, $y(0) = 2$, $y'(0) = -2.60$

Solution: Given equation is, $y'' + 2.2y' + 1.17y = 0$ ...... (i)

$y(0) = 2$, $y'(0) = -2.60$ ...... (ii)

So, its auxiliary equation is,

$m^2 + 2.2m + 1.17 = 0$

$\Rightarrow m = \frac{-2.2 \pm \sqrt{2.2^2 - 4.1.1.17}}{2.1} = \frac{-2.2 \pm \sqrt{0.16}}{2} = \frac{-2.2 \pm 0.4}{2}$

$\Rightarrow m = -0.90, -1.30$

Therefore, the general solution of given equation (i) is,

$y(x) = c_1e^{-0.9x} + c_2e^{-1.3x}$ ...... (iii)

By (ii), we have, $2 = c_1 + c_2$ ... (A)

Differential equation (iii) w. r. t. x, then,

$y'(x) = -0.9c_1e^{-0.9x} - 1.3c_2e^{-1.3x}$

By (ii), we have, $-2.60 = 0.9c_1 - 1.3c_2$ ... (B)

Solving the equations (A) and (B) we get,

$c_1 = 0$, $c_2 = 2$

Now, equations (iii) becomes,

$y(x) = 2e^{-1.3x}$

(xi) $4y'' - 4y' - 3y = 0$, $y(-2) = e$, $y'(-2) = -\frac{e}{2}$

Solution: Given equation is, $4y'' - 4y' - 3y = 0$ ...... (i)

$y(-2) = e$, $y'(-2) = -\frac{e}{2}$ .... (ii)

So, its auxiliary equation is,

$4m^2 - 4m - 3 = 0 \Rightarrow 4m^2 - 6m + 2m - 3 = 0$

$\Rightarrow 2m(2m-3) + 1(2m-3) = 0$

$\Rightarrow (2m-3)(2m+1) = 0$

$\Rightarrow m = -\frac{3}{2}, \frac{-1}{2}$

Therefore, the general solution of given equation (i) is,

$y(x) = c_1e^{\frac{3}{2}x} + c_2e^{\frac{-1}{2}x}$ ...... (iii)

By (ii), we have, $e = c_1e^{-3} + c_2e$ ... (A)

Differential equation (iii) w. r. t. x, then,

$y'(x) = \frac{3}{2}c_1e^{\frac{3}{2}x} - \frac{1}{2}c_2e^{\frac{-1}{2}x}$

By (ii), we have, $\frac{-e}{2} = \frac{3}{2}c_1e^{-3} - \frac{1}{2}c_2e$

$\Rightarrow -e = 3c_1e^{-3} - c_2e$ ... (B)

Solving the equations (A) and (B) we get,

$c_1 = 0$, $c_2 = 1$

Now, equations (iii) becomes,

$y(x) = e^{-0.5x}$

(xii) $9y'' + 6y' + y = 0$, $y(0) = 4$, $y'(0) = \frac{-13}{3}$

Solution: Given equation is, $9y'' + 6y' + y = 0$ ...... (i)

$y(0) = 4$, $y'(0) = \frac{-13}{3}$ ...... (ii)

So, its auxiliary equation is,

$9m^2 + 6m + 1 = 0 \Rightarrow (3m+1)^2 = 0 \Rightarrow m = \frac{-1}{3}, \frac{-1}{3}$

Therefore, the general solution of given equation (i) is,

$y(x) = (c_1 + c_2x)e^{\frac{-x}{3}}$ ... (iii)

By (ii), we have, $4 = c_1e^0 \Rightarrow c_1 = 4$

Differential equation (iii) w. r. t. x, then,

$y'(x) = \frac{-1}{3}c_1e^{\frac{-x}{3}} + c_2\left(-\frac{1}{3}xe^{\frac{-x}{3}} + e^{\frac{-x}{3}}\right)$

By (ii), we have, $-\frac{13}{3} = -\frac{1}{3}c_1 + c_2 \Rightarrow -\frac{13}{3} = -\frac{4}{3} + c_2$

$\Rightarrow c_2 = \frac{4}{2} - \frac{13}{3} = \frac{-9}{3} = -3$

Now, equations (iii) becomes,

$y(x) = (4 - 3x)e^{\frac{-x}{3}}$.

(xiii) $y'' - y' - 2y = 0,\ y(0) = -4,\ y'(0) = -17$

Solution: Given equation is, $y'' - y' - 2y = 0$ ...... (i)

$$y(0) = -4,\ y'(0) = -17 \qquad \text{...... (ii)}$$

So, its auxiliary equation is,

$$m^2 - m - 2 = 0 \Rightarrow m^2 - 2m + m - 2 = 0$$

$$\Rightarrow m(m-2) + 1(m-1) = 0$$

$$\Rightarrow (m-2)(m+1) = 0$$

$$\Rightarrow m = 2, -1$$

Therefore, the general solution of given equation (i) is,

$$y(x) = c_1e^{2x} + c_2e^{-x} \qquad \text{... (iii)}$$

By (ii), we have, $-4 = c_1 + c_2$ ... (A)

Differential equation (iii) w. r. t. x, then,

$$y'(x) = 2c_1e^{2x} - c_2e^{-x}$$

By (ii), we have, $-17 = 2c_1 - c_2$ ... (B)

Solving the equations (A) and (B) we get,

$$c_1 = -7,\ c_2 = 3.$$

Now, equation (iii) becomes,

$$y(x) = 3e^{-x} - 7e^{2x}$$