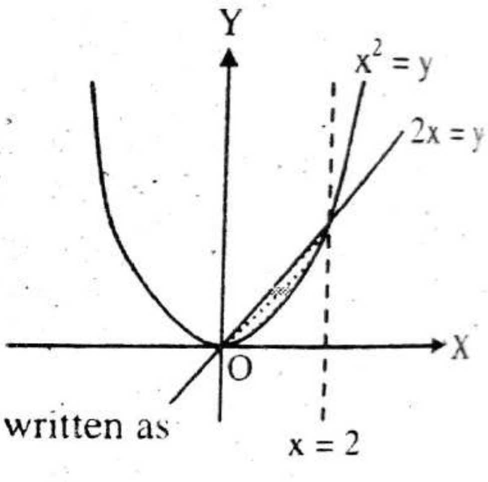
OTHER QUESTIONS FROM SEMESTER END EXAMINATION

Determining the value of Double Integral

2007 Fall Q. No. 3(a)

Let R be the region in the xy plane bounded by the curves $y = x^2$ and $y = 2x$,

evaluate: $\displaystyle\int\int_{R} (x^2 + 4y)dA.$



Solution: Given that,

$$I = \iint_R (x^2 + 4y)\ 2A \qquad \text{............(1)}$$

That is bounded by $y = x^2$ and $y = 2x$.

Solving the curves we get $x = 0, 2$. Then (1) can be written as

$$I = \int_0^2 \int_{2x}^{x^2} (x^2 + 4y)\, dy\, dx \qquad \text{........(2)}$$

Here, region of integration is R: $2x \le y \le x^2, 0 \le x \le 2$.

Clearly, $y = 2x$ is a straight line and $y = x^2$ is a parabola having vertex at (0, 0) and with up open ward.

Clearly, the integral (2) has region of shaded portion in the figure.

Now,

$$|I| = \left| \int_0^2 [x^2y + 2y^2]_{2x}^{x^2} dx \right| = \left| \int_0^2 [(x^2.x^2 + 2x^4) - (x^2.2x + 8x^2)]\, dx \right|$$

$$= \left| \int_0^2 (3x^4 - 2x^3 - 8x^2)\, dx \right|$$

$$= \left| \left[\frac{3x^5}{5} - \frac{2x^4}{4} - \frac{8x^3}{3} \right]_0^2 \right|$$

$$= \left| \frac{3 \times 32}{5} - \frac{2 \times 16}{4} - \frac{8 \times 8}{3} \right|$$

$$= \left| \frac{64}{5} - 8 - \frac{64}{3} \right|$$

$$= \left| 8\left[\frac{8}{5} - 1 - \frac{8}{3}\right] \right| = \left| 8\left(\frac{24 - 15 - 40}{15}\right) \right| = \frac{248}{15}$$

Thus, $I = \frac{248}{15}$

Determining the value of D.I. after changing the Cartesian form to Polar form

2008 Spring Q. No. 3(a)

Evaluate the double integral $\int_0^3 \int_0^{x\sqrt{3}} \frac{y\, dy\, dx}{\sqrt{x^2 + y^2}}$, by changing Cartesian integral to equivalent polar integral.

Solution: Given integral is,

$$I = \int_0^3 \int_0^{x\sqrt{3}} \frac{y\, dy\, dx}{\sqrt{x^2 + y^2}}$$

Here, the variables x varies from $x = 0$ to $x = 3$ and the variable y varies from $y = 0$ to $y = x\sqrt{3}$.

Now, changing the region to polar we substitute $x = r\cos\theta$ and $y = r\sin\theta$.

And, to find r, $x = 0$, $x = 3$

$r\cos\theta = 0$ $\qquad$ $r\cos\theta = 3$

$r = 0$ $\qquad$ $r = 3\sec\theta$

Also, to find θ, $y = 0$ $\qquad$ $y = x\sqrt{3}$

$r\sin\theta = 0$ $\qquad$ $r\sin\theta = \sqrt{3}\, r\cos\theta$

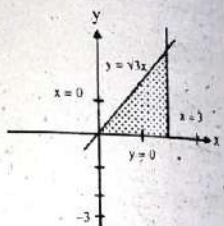
$\theta = 0$ $\qquad$ $\tan\theta = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$.

Now, the above integration change to,

$$I = \int_0^{\pi/3}\int_0^{3\sec\theta} \frac{r\sin\theta\, r\, dr\, d\theta}{\sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}}$$

$$= \int_0^{\pi/3}\int_0^{3\sec\theta} \frac{r^2 \sin\theta\, dr\, d\theta}{r}$$

$$= \int_0^{\pi/3} \sin\theta\, [r]_0^{3\sec\theta}\, d\theta$$



$$= \int_0^{\pi/3} \sin\theta\, 3\sec\theta.\, d\theta = 3\int_0^{\pi/3} \tan\theta.\, d\theta$$

$$= 3\,[\log(\sec\theta)]_0^{\pi/3}$$

$$= 3\left[\log\left(\sec\frac{\pi}{3}\right) - \log(\sec 0)\right] = 3\log(2).$$

Thus, $I = 3\log(2) = \log(8)$.

Determining the value of Double Integral after Reversing the Order of Integration

1999; 2001 Q. No. 3(a)

Change the order of integration and evaluate $\int_0^2\int_0^{4-y^2} y\, dx\, dy$.

Solution: Given integral is

$$I = \int_0^2\int_0^{4-y^2} y\, dx\, dy \quad ....(i)$$

Here, the region of integration is R: $0 < x < 4 - y^2$, $0 < y < 2$.

Since, $x = 4 - y^2 \Rightarrow y^2 = -(x - 4)$ which is a parabola having vertex at (4, 0) and open-left-ward.

Thus, the integral (1) has region of shaded portion as shown in figure (1), that has horizontal strip.

Now, reversing the order of integration, we take the vertical strip as in figure (2) for which y varies from y = 0 to the curve $y = \sqrt{4 - x}$. And, the strip moves from x = 0 to x = 4.

Then,

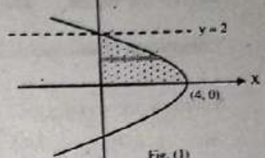


Fig. (1)

$$I = \int_0^4\int_0^{\sqrt{4-x}} y\, dy\, dx$$

$$= \int_0^4 \left[\frac{y^2}{2}\right]_0^{\sqrt{4-x}} dx$$

$$= \int_0^4 \left(\frac{4-x}{2}\right) dx$$

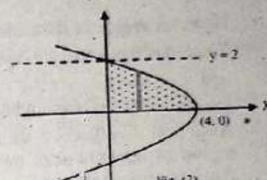


Fig. (2)

$$= \frac{1}{2}\left[4x - \frac{x^2}{2}\right]_0^4$$

$$= \frac{1}{2}\left(16 - \frac{16}{2}\right) = \frac{1}{2}\left(\frac{16}{2}\right) = 4$$

Thus, I = 4 sq. units.

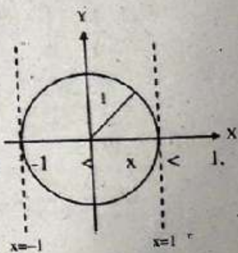
2004 Fall Q. No. 3(a)

Change the following integral into polar form and evaluate:

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2dy\, dx}{(1 + x^2 + y^2)^2}$$

Solution: Given integral is

$$I = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2\, dy\, dx}{(1 + x^2 + y^2)^2}$$



Here, the region of integration be R: $-1 < x < 1$, $-\sqrt{1 - x^2} < y < \sqrt{1 - x^2}$.

Clearly, $(\sqrt{1 - x^2})^2 = y^2 \Rightarrow x^2 + y^2 = 1$.

This shows that the region is a circle with radius r = 1.

So, $0 < r < 1$ and $0 < \theta < 2\pi$.

Set, $x = r\cos\theta$, $y = r\sin\theta$. Then $x^2 + y^2 = r^2$. Also, $dx\, dy = r\,dr\,d\theta$.

Now,

$$I = \int_0^{2\pi}\int_0^1 \frac{dt\,d\theta}{t^2} = \int_0^{2\pi}\left[\frac{t^{-1}}{-1}\right]_1^2 d\theta = \int_0^{2\pi}\left(1-\frac{1}{2}\right)d\theta = \frac{1}{2}\int_0^{2\pi} d\theta = \frac{1}{2}\cdot 2\pi = \pi$$

Thus, $I = \pi$.

2006 Fall Q. No. 3(a)

Sketch the region of integration and evaluate by interchanging the order of integration of the double integral: $\displaystyle\int_{x=0}^{2}\int_{y=0}^{4-x^2} \frac{xe^{2y}}{4-y}\,dy\,dx.$

Solution: Given integral is

$$I = \int_0^2\int_0^{4-x^2} \frac{xe^{2y}}{4-y}\,dy\,dx \qquad \text{.......(1)}$$

Here, the region of integration of (1) is R: $0 \le y \le 4 - x^2, 0 \le x \le 2$.

Since, $y = 0$ is a straight line.

and $y = 4 - x^2 \Rightarrow x^2 = -(y-4)$ is a parabola having vertex at (0, 4) and down-open ward.

Also, both $x = 0$, $x = 2$ are straight line. Thus, the region of integration of (i) is the shaded portion that has vertical strip as shown in figure (1).

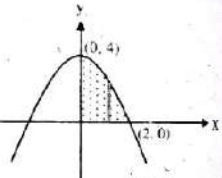


Fig (1)

Now, reversing the order of integration we take the horizontal strip as in figure (2) for which the strip is bounded by $x = 0$ and $x = \sqrt{4-y}$. And, the strip moves from $y = 0$ to $y = 4$.

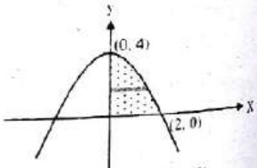


Fig (2)

Therefore, (1) becomes,

$$I = \int_0^4\int_0^{\sqrt{4-y}} \left(\frac{x\,e^{2y}}{4-y}\right)dx\,dy$$

$$= \int_0^4 \frac{e^{2y}}{4-y}\left[\frac{x^2}{2}\right]_0^{\sqrt{4-y}} dy$$

$$= \frac{1}{2}\int_0^4 \frac{e^{2y}}{4-y}(4-y)\,dy = \frac{1}{2}\int_0^4 e^{2y}\,dy = \frac{1}{2}\left[\frac{e^{2y}}{2}\right]_0^4 = \frac{e^8-1}{4}$$

Thus, $\quad I = \dfrac{e^8-1}{4}$

008 Fall Q. No. 3(a)

Sketch the region of integration and evaluate the integral by reversing the order of integration $\displaystyle\int_0^2\int_x^2 y^2 \sin xy\,dy dx.$

Solution: Given that,

$$I = \int_0^2\int_x^2 y^2 \sin xy\,dy\,dx$$

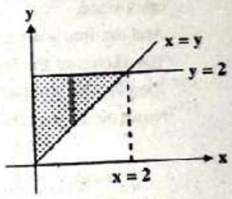
Here the region of integration is bounded by $y = x$, and by $y = 2$.

Since the line $y = x$ passes through the points (0, 0) and (1, 1). And the line $y = 2$ is a straight line that is parallel to x-axis.

Next, the line $x = 0$ is y-axis. And the line $x = 2$ is a straight line that is parallel to y-axis.

On the basis of these boundaries the sketch of figure is shown as in fig-1.

Clearly, the required region generated by the integral (1) is the shaded portion that has vertical strip as shown in figure 1.



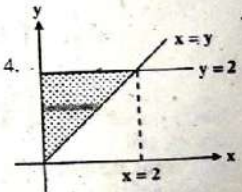
(fig. 1)

Now, reversing the order of integration, region has horizontal strip as in figure 2 in which x varies from $x = 0$ to $x = y$. Also, the strip moves from $y = 0$ to $y = 2$. Then (1) becomes,

$$I = \int_0^2\int_0^y y^2 \sin xy\,dx\,dy$$

$$= \int_0^2 y^2\left[-\frac{\cos xy}{y}\right]_0^y dy$$

$$= -\int_0^2 y\,[\cos(y^2) - 1]\,dy$$



(fig. 2)

Put $y^2 = t$ then $2y\,dy = dt$. Also $y = 0 \Rightarrow t = 0$, $y = 2 \Rightarrow t = 4$.

So,

$$= \frac{1}{2}\int_0^4 [\cos t - 1]\,dt = \frac{1}{2}[\sin t - t]_0^4 = \frac{\sin 4 - 4}{2}$$

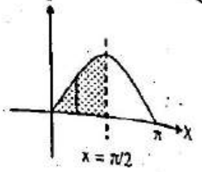
Determining Area by using Double Integral

1999; 2001 Q. No. 3(b)

Using the double integration find the area of the region bounded curves $y = \sin x$ and the line $x = 0$ and $x = \frac{\pi}{2}$.

Solution: Given that the region is bounded by curves $y = \sin x$, $x = 0$ and $x = \frac{\pi}{2}$.

Clearly the region is the shaded portion in the figure that is bounded by $0 \le y \le \sin x$ and $0 \le x \le \frac{\pi}{2}$.



Now,

$$I = \int_0^{\pi/2}\int_0^{\sin x} dy\,dx$$

$$= \int_0^{\pi/2} [y]_0^{\sin x}\,dx = \int_0^{\pi/2} \sin x\,dx = [-\cos x]_0^{\pi/2} = 1 - 0 = 1.$$

Thus, area of the region is 1 sq. unit.

2002 Q. No. 3(b)

Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$ by using double integral.

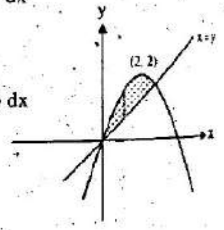
Solution: Given that the region is bounded by $y = 4x - x^2$ and $y = x$. Since, the curve $y = 4x - x^2 \Rightarrow (x-2)^2 = -(y-2)$ which is a parabola having vertex at (2, 2) and equation of line of symmetry be $x - 2 = 0 \Rightarrow x = 2$. So, the parabola is down open ward.

And the line $x = y$ passes through the points (0, 0) and (1, 1).

The sketch of the region is as shown in figure.

Clearly, the point of contact of the curve and the line is (0, 0) and (2, 2).

Now, the area of the region determined by the given curve and the line is



$$\text{Area} = \int_0^2\int_x^{4x-x^2} dy\,dx = \int_0^2 [y]_x^{4x-x^2}\,dx$$

$$\Rightarrow A = \int_0^2 (4x - x^2 - x)\,dx = \int_0^2 (3x - x^2)\,dx$$

$$\Rightarrow A = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^2 = 6 - \frac{8}{3} = \frac{10}{3}$$

Thus, the area of the region is $\frac{10}{3}$ square units.

2002 (II) Q. No. 3(b)

Using the double integrals find the area of the region, $y^2 = 4ax$ and the parabola $x^2 = 4ay$.

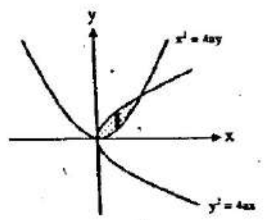
Solution: Given that the required region is bounded the curves $y^2 = 4ax$ and $x^2 = 4ay$. Clearly the curve $y^2 = 4ax$ is a parabola having vertex at (0, 0) and equation of line of symmetry is $y = 0$. So, it has right open ward.

And the curve $x^2 = 4ay$ is a parabola having vertex at (0, 0) and the equation of line of symmetry is $x = 0$. So, it has up open ward.

Moreover, solving these curves, the point of contact between them is (0, 0) and (4a, 4a).

On these bases, the sketch of the region of integration is as in the figure in which the shaded portion is the region of integration.

Now, for the area of the region, taking vertical strip we get,



$$A = \int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$$

$$= \int_0^{4a} [y]_{x^2/4a}^{2\sqrt{ax}}\,dx$$

$$= \int_0^{4a} \left(2\sqrt{ax} - \frac{x^2}{4a}\right) dx$$

$$= \left[2\sqrt{a}\,\frac{x^{3/2}}{3/2} - \frac{1}{4a}\cdot\frac{x^3}{3}\right]_0^{4a} = \frac{4\sqrt{a}}{3}(4a)^{3/2} - \frac{(4a)^4}{12a} = \frac{32a^2}{3} - \frac{64a^3}{12a} = \frac{32a^2}{3} - \frac{16a^2}{3}$$

$$\Rightarrow A = \frac{16a^2}{3}$$

Thus, area of the region bounded by $x^2 = 4ay$ and $y^2 = 4ax$ is $\frac{16a^2}{3}$ sq. units.

Similar Question for Practice;

2006 Fall Q. No. 3(b) OR

Find the area bounded by the parabolas $x = y^2 - 1$ and $x = 2y^2 - 2$ by using double integration.

Determining Volume by using Double Integral

1999(OR); 2001(OR); 2004 Fall Q. No. 3(b)

Find the volume of the solid that is bounded above by the cylinder $z = x^2$ and below by the region enclosed by the parabola $y = 2 - x^2$ and the line $y = x$ is the xy plane.

Solution: Given that the solid is bounded above the cylinder $z = x^2$, below by $y = 2 - x^2$ and is the xy-plane, the solid is bounded by the line $x = y$.

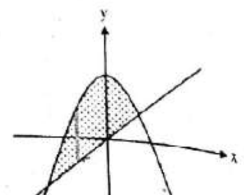
Clearly, the parabola $y = 2 - x^2 \Rightarrow x^2 = -(y - 2)$ has vertex at (0, 2) and line of symmetry is $x = 0$. So, the parabola has down openward. Also, the line $x = y$ passes through the point (0, 0) and (1, 1).

On these bases the sketch base of the solid in xy-plane is as shown in figure.

For the volume of the solid, we integrate z over the region in xy-plane, taking vertical strip. Solving the curves $x = y$ and $y = 2 - x^2$ we get the point of contacts are (-2, -2) and (1, 1).

Now, volume of the solid is

$$V = \int_{-2}^{1}\int_{x}^{2-x^2} x^2\, dy\, dx$$



$$= \int_{-2}^{1} x^2 [y]_x^{2-x^2} dx = \int_{-2}^{1} x^2 (2 - x^2 - x)\, dx$$

$$= \int_{-2}^{1} (2x^2 - x^4 - x^3)\, dx$$

$$= \left[\frac{2x^3}{3} - \frac{x^5}{5} - \frac{x^4}{4}\right]_{-2}^{1}$$

$$= \left(\frac{2}{3} - \frac{1}{5} - \frac{1}{4}\right) - \left(\frac{-16}{3} + \frac{32}{5} - \frac{16}{4}\right)$$

$$= \frac{18}{3} - \frac{33}{5} - \frac{1}{4} + 4$$

$$= 6 - \frac{132 + 5}{20} + 4 = 10 - \frac{137}{20} = \frac{200 - 137}{20} = \frac{63}{20}$$

Thus, volume of the solid is $\frac{63}{20}$ cubic units.

Similar Question for Practice:

2006 Fall Q. No. 3(b)

Find the volume under the parabolic cylinder $z = x^2$ above the region bounded the parabola $y = 6 - x^2$ and the line $y = x$ in xy-plane.

OTHER QUESTIONS FROM FINAL EXAM

2000(OR); 2002 Q. No. 3(b)

Find the volume of the region that lies under the parabolied $z = x^2 + y^2$ and above the triangle enclosed by the lines $y = x$, $x = 0$ and $x + y = 2$.

Solution: Given that the region of integration is enclosed by the lines $x = y$, $x = 0$ and $x + y = 2$.

Clearly, the line $x = y$ passes through (0, 0) and (1, 1).

And $x = 0$ is y-axis and the line $x + y = 2$ passes through the points (2, 0) and (0, 2).

On these bases, the region is sketch as in the figure.

For the volume of the region under the parabolid $z = x^2 + y^2$ and the region shown in figure, we integrate z over the region taking vertical strip.

$$V = \int_{0}^{1}\int_{x}^{2-x} (z)\, dy\, dx = \int_{0}^{1}\int_{x}^{2-x} (x^2 + y^2)\, dy\, dx$$

$$\Rightarrow V = \int_{0}^{1}\left[x^2y + \frac{y^3}{3}\right]_x^{2-x} dx$$

$$\Rightarrow V = \int_{0}^{1}\left[x^2(2 - x) - x^3 + \frac{(2-x)^3}{3} - \frac{x^3}{3}\right]dx$$

$$\Rightarrow V = \int_{0}^{1}\left[2x^2 - x^3 - x^3 - \frac{8 - x^3 - 12x + 6x^2}{3} - \frac{x^3}{3}\right]dx$$

$$\Rightarrow V = \int_{0}^{1}\left[2x^2 - 2x^3 - \frac{8}{3} + 4x - 2x^2\right]dx = \int_{0}^{1}\left(-2x^3 + 4x - \frac{8}{3}\right)dx$$

$$\Rightarrow V = \left[-\frac{2x^4}{4} + 2x^2 - \frac{8x}{3}\right]_0^1 = -\frac{2}{4} + 2 - \frac{8}{3} = -\frac{1}{2} + 2 - \frac{8}{3} = \frac{-3 + 12 - 16}{6} = -\frac{7}{6}$$

Thus, volume of the parabolid under the given boundies is $\frac{7}{6}$ cubic units.

2002 Q. No. 3(b) OR

Find the volume bounded by circle cylinder $x^2 + y^2 = 4$ and the plane $y + z = 4$ and $z = 0$.

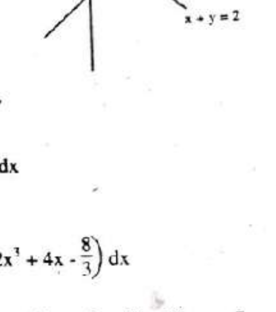
Solution: Given that the solid is bounded by a circle cylinder $x^2 + y^2 = 4$, on the top by the plane $y + z = 4$ and by $z = 0$.

Clearly the plane $y + z = 4$ is parallel to x-axis and makes intercept 4 on y-axis and z-axis.

Then the volume generated by the solid is obtained by integrating z over the circle $x^2 + y^2 = 4$ that has radius 2.

Now, volume of the solid is

$$V = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (z)\, dy\, dx$$



$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4 - y)\, dy\, dx$$

Since $4 - y$ is an odd function. So,

$$V = \int_{0}^{2}\int_{0}^{\sqrt{4-x^2}} (4 - y)\, dy\, dx$$

$$= 16\int_0^2 \int_0^{\sqrt{4-x^2}} dy\,dx - 4\int_0^2 \int_0^{\sqrt{4-x^2}} y\,dy\,dx$$

$$= 16\int_0^2 [y]_0^{\sqrt{4-x^2}} dx - 4\int_0^2 \left[\frac{y^2}{2}\right]_0^{\sqrt{4-x^2}} dx$$

$$= 16\int_0^2 \sqrt{4-x^2}\,dx - 2\int_0^2 (4-x^2)\,dx$$

$$= 16\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - 2\left[4x - \frac{x^3}{3}\right]_0^2$$

$$= 16\,[0 + 2\sin^{-1}(1)] - 2\left(8 - \frac{8}{3}\right) = 16\,.\left(2\,.\frac{\pi}{2}\right) - 2\left(\frac{16}{3}\right) = 16\pi - \frac{32}{\pi}$$

Thus, the volume of the solid is $\left(16\pi - \frac{32}{\pi}\right)$ cubic units.

2003 Fall Q. No. 3(b)

Find the volume of the solid whose base is the region in xy-plane that is bounded by the parabola $y = 3 - x^2$, $y = 2x$ while the top is bounded by the plane $z = x + 1$.

Solution: Given that the base of solid is bounded by the parabola $y = 3 - x^2$ and the line

$y = 2x$. The solid is bounded on the top by the plane $z = x + 1$.

Since, the parabola $y = 3 - x^2 \Rightarrow x^2 = -(y-3)$ has vertex at $(0, 3)$ and equation of line of symmetry is $x = 0$. So, it has down open ward. Also, the line $y = 2x$ passes through the point $(0, 0)$ and $(1, 2)$.

On these bases the sketch of the base of solid is shown in figure.

Now, for the volume of solid, we integrate the plane z over the region of base of solid. So,

$$V = \int_0^1 \int_{2x}^{3-x^2} (z)\,dy\,dx$$

$$= \int_0^1 \int_{2x}^{3-x^2} (x+1)\,dy\,dx$$

$$= \int_0^1 [xy + y]_{2x}^{3-x^2} dx = \int_0^1 [x(3-x^2) + (3-x^2) - (2x^2 + 2x)]\,dx$$

$$= \int_0^1 (3x - x^3 + 3 - x^2 - 2x^2 - 2x)\,dx$$

$$= \int_0^1 (3 + x - 3x^2 - x^3)\,dx$$

$$= \left[3x + \frac{x^2}{2} - x^3 - \frac{x^4}{4}\right]_0^1$$

$$= 3 + \frac{1}{2} - 1 - \frac{1}{4} = \frac{12 + 2 - 4 - 1}{4} = \frac{9}{4}$$

Thus, the volume of the solid is $\frac{9}{4}$ cubic units.

2004 Spring Q. No. 3(b)

Find the volume of the solid in the first octant bounded by the co-ordinate plans, the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 3$.

Solution: Given integral is

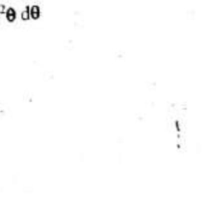
$$I = \int_0^3 \int_0^{x\sqrt{3}} \frac{y\,dy\,x}{\sqrt{x^2+y^2}} \quad \text{........(1)}$$

Clearly, the region is bounded below by $y = 0$, above by $y = x\sqrt{3}$, on the left by $x = 0$ and on the right by $x = 3$.

On these bases, the region of integration is as shown in the figure.

Set $x = r\cos\theta$ and $y = r\sin\theta$. Then $dx\,dy = r\,dr\,d\theta$.

For the radical strip, from the figure, r varies from $r = 0$ to $r = 3\sec\theta$ and θ varies from $\theta = 0$ to $\theta = \frac{\pi}{3}$.



Then (i) becomes,

$$I = \int_0^{\pi/3} \int_0^{3\sec\theta} \frac{r\sin\theta}{\sqrt{r^2}}\,r\,dr\,d\theta$$

$$= \int_0^{\pi/3} \sin\theta \int_0^{3\sec\theta} r\,dr\,d\theta$$

$$= \int_0^{\pi/3} \sin\theta \left[\frac{r^2}{2}\right]_0^{3\sec\theta} d\theta = \frac{1}{2}\int_0^{\pi/3} \sin\theta\,.\,9\sec^2\theta\,d\theta$$

$$= \frac{9}{2}\int_{0}^{\pi/3} \tan\theta \sec\theta \, d\theta$$

$$= \frac{9}{2}[\sec\theta]_{0}^{\pi/3} = \frac{9}{2}\left[\sec\frac{\pi}{3} - \sec 0\right] = \frac{9}{2}(2-1) = \frac{9}{2} = 4.5$$

Thus, the volume of the value of the integral is

SHORT QUESTIONS

2003 Fall: Evaluate: $\int_{0}^{\log 2}\int_{e^y}^{2} dx\, dy.$

Solution: Let,

$$I = \int_{0}^{\log 2}\int_{e^y}^{2} dx\, dy = \int_{0}^{\log 2} [x]_{e^y}^{2} dy = \int_{0}^{\log 2} (2 - e^y)\, dy = [2y - e^y]_{0}^{\log 2}$$

$$\Rightarrow \quad I = [2.\log(2) - e^{\log(2)}] - [0 - e^0] = 2\log(2) - 2 + 1 = 2\log(2) - 1.$$

Thus, $I = 2\log(2) - 1$

2004 Spring: Convert the given integral to polar form $\int_{0}^{3}\int_{0}^{x\sqrt{3}} \frac{y\, dy\, dx}{\sqrt{x^2+y^2}}.$

Solution: See the solution of 2008 Spring.

2009 Spring: Change Cartesian integral $\int_{0}^{2}\int_{0}^{x} y\, dydx$**, to equivalent polar integral.**

Solution: See the required part of Exercise 9.3 Q. No. 3.

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