List of Table

S. No.	f(t)	$\mathcal{L}\{f(t)\} = F(s)$	S.N.	f(t)	$\mathcal{L}\{f(t)\} = F(s)$
1.	1	$\frac{1}{s}$	6.	cos at	$\frac{s}{s^2+a^2}$
2.	t	$\frac{1}{s^2}$	7.	sin at	$\frac{a}{s^2+a^2}$
3.	t^n	$\frac{n!}{s^{n+1}}$	8.	cosh at	$\frac{s}{s^2-a^2}$
4.	e^{at}	$\frac{1}{s-a}$	9.	sinh at	$\frac{a}{s^2-a^2}$
5.	e^{-at}	$\frac{1}{s+a}$			

List of Formulae

(1) $\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$

(2) $\mathcal{L}\{f''(t)\} = s^2\,\mathcal{L}\{f(t)\} - sf(0) - f'(0)$

(3) $\mathcal{L}\left\{\int_0^t f(u)\,du\right\} = \frac{1}{s}\mathcal{L}\{f(t)\}$

(6) $u_a(t) = u(t-a) = \begin{cases} 0 & \text{for } t < a \\ 1 & \text{for } t \geq a \end{cases}$

(4) $\mathcal{L}\{e^{at} f(t)\} = [F(s)]_{s\to s-a}$

(5) $\mathcal{L}\{e^{-at} f(t)\} = [F(s)]_{s\to s+a}$

(7) $\mathcal{L}\{f(t-a)\,u_a(t)\} = e^{-as}F(s)$

(8) $\mathcal{L}\{t\,f(t)\} = -\frac{d}{ds}(F(s)) = -F'(s)$

(9) $\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}(F(s))$

(11) $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+b^2)^2}\right\} = \frac{t}{2b}\sin \beta t$

(10) $\mathcal{L}^{-1}\left\{\frac{1}{(s^2+b^2)^2}\right\} = \frac{1}{2b^3}(\sin \beta t - \beta t \cos \beta t)$

(12) $\mathcal{L}^{-1}\left\{\frac{1}{(s^2+b^2)^2}\right\} = \frac{1}{2b}(\sin\beta t + \beta t\cos\beta t)$

(13) $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(\bar{s})\,d\bar{s}$

(14) $\mathcal{L}\{f*g\} = F(s)\,G(s)$ where $f*g$ be convolution of f and g.

Exercise 8.1

A. Find the Laplace transform f the following functions where a, b, T, w, θ are constants.

(1) $f(t) = 3t + 4$.

Solution: Let $f(t) = 3t + 4$.

Now, Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{3t+4\} = \int_0^\infty (3t+4)\,e^{-st}\,dt$$

$$= 3\int_0^\infty te^{-st}\,dt + 4\int_0^\infty e^{-st}\,dt$$

$$= 3\left[t\cdot\frac{e^{-st}}{-s} - (1)\cdot\frac{e^{-st}}{(-s)^2}\right]_0^\infty + 4\left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= 3\cdot\frac{1}{s^2} + 4\cdot\frac{1}{s} \qquad [\because e^{-\infty} = 0]$$

$$= \frac{3}{s^2} + \frac{4}{s}$$

Thus, $\mathcal{L}\{3t+4\} = \frac{3}{s^2} + \frac{4}{s}$.

(2) $t^2 + at + b$

Solution: Let, $f(t) = t^2 + at + b$

Then, the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2 + at + b\}$$

$$= \int_0^\infty (t^2 + at + b)\,e^{-st}\,dt$$

$$= \left[(t^2 + at + b)\frac{e^{-st}}{-s} - (2t + a)\frac{e^{-st}}{(-s)^2} + 2\frac{e^{-st}}{(-s)^3}\right]_0^\infty$$

[∵ applying successive integration]

$$= 0 - \left[\frac{b}{-s} - \frac{a}{s^2} + \frac{2}{-s^3}\right] = \frac{2}{s^3} + \frac{a}{s^2} + \frac{b}{s}$$

Thus, $\mathcal{L}\{t^2 + at + b\} = \frac{2}{s^3} + \frac{a}{s^2} + \frac{b}{s}$

3. $\sin\left(\frac{2n\pi t}{T}\right)$

Solution: Let, $f(t) = \sin\left(\frac{2n\pi t}{T}\right)$. where 2, n, π, T are constant.

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \mathcal{L}\left\{\sin\left(\frac{2n\pi t}{T}\right)\right\}$$

$$= \int_0^\infty \sin\left(\frac{2n\pi t}{T}\right)e^{-st}\,dt$$

$$= \left[\frac{e^{-st}}{(-s)^2 + \left(\frac{2n\pi}{T}\right)^2}\left[(-s)\sin\left(\frac{2n\pi t}{T}\right) - \left(\frac{2n\pi}{T}\right)\cos\left(\frac{2n\pi t}{T}\right)\right]\right]_0^\infty$$

$$= 0 - \left[s^2 + \left(\frac{2n\pi}{T}\right)^2\right]^{-1} \left(-\frac{2n\pi}{T}\right) \qquad [\because e^{-\infty} \sim 0 \text{ and } \sin 0 = 0]$$

$$= \left[s^2 + \left(\frac{2n\pi}{T}\right)^2\right]^{-1} \cdot \left(\frac{2n\pi}{T}\right).$$

4. $4e^{5t} + 6t^2 - 4\cos 3t + 3\sin 4t$.

Solution: Let. $f(t) = 4e^{5t} + 6t^2 - 4\cos 3t + 3\sin 4t$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{4e^{5t} + 6t^2 - 4\cos 3t + 3\sin 4t\}$$

$$= 4\,\mathcal{L}\{e^{5t}\} + 6\,\mathcal{L}\{t^2\} - 4\,\mathcal{L}\{\cos 3t\} + 3\,\mathcal{L}\{\sin 4t\} \qquad \text{........ (i)}$$

Since,

$$\mathcal{L}\{e^{5t}\} = \int_0^\infty e^{5t} e^{-st}\, dt = \int_0^\infty e^{-(s-5)t}\, dt$$

$$= \left[\frac{e^{-(s-5)t}}{-(s-5)}\right]_0^\infty = 0 - \frac{1}{-(s-5)} = \frac{1}{s-5}$$

And, $\mathcal{L}\{t^2\} = \int_0^\infty t^2 e^{-st}\, dt$

$$= \left[t^2 \frac{e^{-st}}{-s} - (2t)\frac{e^{-st}}{(-s)^2} + 2\frac{e^{-st}}{(-s)^3}\right]_0^\infty = 0 - \left[0 - 0 + \frac{2}{-s^3}\right] \qquad [\because e^{-\infty} \sim 0]$$

$$= \frac{2}{s^3}$$

Also, $\mathcal{L}\{\cos 3t\} = \int_0^\infty e^{-st} \cos 3t\, dt$

$$= \left[\frac{e^{-st}}{(-s)^2 + 9}[(-s)\cos 3t + 3\sin 3t]\right]_0^\infty$$

$$\left[\because \int e^{at}\cos bt\, dt = \frac{e^{at}}{a^2 + b^2}(a\cos bt + b\sin bt)\right]$$

$$= 0 - \frac{1}{s^2 + 9}[(-s) + 0] \qquad [\because e^{-\infty} \sim 0]$$

$$= \frac{s}{s^2 + 9}$$

Also, $\mathcal{L}\{\sin 4t\} = \int_0^\infty e^{-st} \sin 4t\, dt$

$$= \left[\frac{e^{-st}}{(-s)^2 + 4^2}\{(-s)\sin 4t - 4\cos 4t\}\right]_0^\infty$$

$$\left[\because \int e^{at}\sin bt\, dt = \frac{e^{at}}{a^2 + b^2}(a\sin bt - b\cos bt)\right]$$

$$= 0 - \frac{1}{s^2 + 16}(0 - 4) \qquad [\because e^{-\infty} \sim 0]$$

$$= \frac{4}{s^2 + 16}$$

Therefore (i) becomes,

$$\mathcal{L}\{f(t)\} = 4 \cdot \frac{1}{s-5} + 6 \cdot \frac{2}{s^3} - 4 \cdot \frac{s}{s^2 + 9} + 3 \cdot \frac{4}{s^2 + 16}$$

$$= \frac{4}{s-5} + \frac{12}{s^3} - \frac{4s}{s^2 + 9} + \frac{12}{s^2 + 16}$$

5. $\sin(wt + \theta)$ **[2003 Fall – Short]**

Solution: Let, $f(t) = \sin(wt + \theta)$ where w, θ are constants.

Then Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} \sin(wt + \theta)\, dt$$

$$= \int_0^\infty e^{-st}(\sin wt \cos\theta + \cos wt \sin\theta)\, dt$$

$$= \cos\theta \int_0^\infty e^{-st} \sin wt\, dt + \sin\theta \int_0^\infty e^{-st} \cos wt\, dt$$

$$= \left[\cos\theta\left[\frac{e^{-st}}{(-s)^2 + w^2}\{(-s)\sin wt - w\cos wt\}\right]\right]_0^\infty + \left[\sin\theta\left[\frac{e^{-st}}{(-s)^2 + w^2}\{(-s)\cos wt + w\sin wt\}\right]\right]_0^\infty$$

$$= 0 - \cos\theta \cdot \frac{1}{s2 + w^2}(-w) + 0 - \sin\theta \cdot \frac{1}{s^2 + w^2}(-s) \qquad [\because e^{-\infty} \sim 0]$$

$$= \frac{1}{s^2 + w^2}[w\cos\theta + s\sin\theta]$$

6 $\cos(wt + \theta)$

Solution: Let, $f(t) = \cos(wt + \theta) = \cos wt \cos\theta - \sin wt \sin\theta$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \int_0^\infty (\cos wt \cos\theta - \sin wt \sin\theta)\, e^{-st}\, dt$$

$$= \cos\theta \int_0^\infty e^{-st} \cos wt\, dt - \sin\theta \int_0^\infty e^{-st} \sin wt\, dt$$

$$= \cos\theta\left[\frac{e^{-st}}{(-s)^2 + w^2}\{(-s)\cos wt + w\sin wt\}\right]_0^\infty -$$

$$\sin\theta\left[\frac{e^{-st}}{(-s)^2+w^2}\{(-s)\sin wt - w\cos wt\}\right]_0^\infty$$

$$= 0 - \cos\theta\frac{1}{s^2+w^2}(-s) - 0 + \sin\theta\cdot\frac{1}{s^2+w^2}(-w) \quad [\because e^{-\infty}\sim 0]$$

$$= \frac{s.\cos\theta + w\sin\theta}{s^2+w^2}$$

7. $\cos^2 t$

Solution: Let, $f(t) = \cos^2 t = \dfrac{1+\cos 2t}{2}$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \int_0^\infty\left(\frac{1+\cos 2t}{2}\right)e^{-st}\,dt$$

$$= \frac{1}{2}\int_0^\infty e^{-st}\,dt + \frac{1}{2}\int_0^\infty e^{-st}\cos 2t\,dt$$

$$= \frac{1}{2}\left[\frac{e^{-st}}{-s}\right]_0^\infty + \frac{1}{2}\left[\frac{e^{-st}}{(-s)^2+2^2}\{(-s)\cos 2t + 2\sin 2t\}\right]_0^\infty$$

$$= \frac{1}{2}\left[0 - \frac{1}{-s}\right] + \frac{1}{2}\left[0 - \frac{1}{s^2+4} - (-s)\right] \quad [\because e^{-\infty}\sim 0]$$

$$= \frac{1}{2}\left(\frac{1}{s} + \frac{s}{s^2+4}\right)$$

8. $\cosh^2 3t$ **[2004 Fall – Short]**

Solution: Let, $f(t) = \cosh^2 3t = \left(\dfrac{e^{3t}+e^{-3t}}{2}\right)^2 = \left(\dfrac{e^{6t}+e^{-6t}+2}{4}\right)$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\,e^{-st}\,dt$$

$$= \frac{1}{4}\int_0^\infty (e^{6t}+e^{-6t}+2)\,e^{-st}\,dt$$

$$= \frac{1}{4}\left[\int_0^\infty e^{-(s-6)t}\,dt + \int_0^\infty e^{-(s+6)t}\,dt + 2\int_0^\infty e^{-st}\,dt\right]$$

$$= \frac{1}{4}\left[\frac{e^{-(s-6)t}}{-(s-6)} + \frac{e^{-(s+6)t}}{-(s+6)} + 2\left(\frac{e^{-st}}{-s}\right)\right]_0^\infty$$

$$= \frac{1}{4}\left[0 - \left(\frac{1}{-(s-6)} + \frac{1}{-(s+6)} + 2\cdot\frac{1}{-s}\right)\right] \quad [\because e^{-\infty}\sim 0]$$

$$= \frac{1}{4}\left(\frac{1}{s-6} + \frac{1}{s+6} + \frac{2}{s}\right)$$

9. e^{at+b}

Solution: Let, $f(t) = e^{at+b} = e^{at}\cdot e^{b}$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \int_0^\infty (e^{at}e^{b})\,e^{-st}\,dt$$

$$= e^b\int_0^\infty e^{at}e^{-st}\,dt = e^b\int_0^\infty e^{-(s-a)t}\,dt$$

$$= e^b\left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty$$

$$= e^b\left[0 - \frac{1}{-(s-a)}\right] \quad [\because e^{-\infty}\sim 0]$$

$$= \frac{e^b}{s-a}$$

10. $\sinh^3 2t$

Solution: Let, $f(t) = \sinh^3 2t = \dfrac{\sinh 6t - 3\sinh 2t}{4}$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \frac{1}{4}\int_0^\infty(\sinh 6t - 3\sinh 2t)\,e^{-st}\,dt$$

$$= \frac{1}{4}\int_0^\infty e^{-st}\sinh 6t\,dt - 3\int_0^\infty e^{-st}\sinh 2t\,dt$$

$$= \frac{1}{4}\left[\frac{e^{-st}}{(-s)^2-6^2}(-s\sinh 6t - 6\cosh 6t)\right]_0^\infty - \left[\frac{3e^{-st}}{(-s)^2-2^2}(-s\sinh 2t - 2\cosh 2t)\right]_0^\infty$$

$$= \frac{1}{4}\left\{\left[0 - \frac{1}{s^2-36}(-6)\right] - \left[0 - \frac{1}{s^2-4}(-2)\right]\right\} \quad [\because e^{-\infty}\sim 0]$$

$$= \frac{1}{4}\left\{\frac{6}{s^2-36} - \frac{6}{s^2-4}\right\}$$

$$= \frac{6}{4}\left[\frac{s^2-36-s^2+4}{(s^2-36)(s^2-4)}\right] = -\frac{48}{(s^2-36)(s^2-4)}$$

11. $\cos^2 2t$

Solution: Let, $f(t) = \cos^2 2t = \frac{\cos 6t + 3\cos 2t}{4}$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \mathcal{L}\left\{\frac{\cos 6t + 3\cos 2t}{4}\right\}$$

$$= \frac{1}{4}\int_0^\infty (\cos 6t + 3\cos 2t)\, e^{-st}\, dt$$

$$= \frac{1}{4}\int_0^\infty e^{-st}\cos 6t\, dt + \frac{3}{4}\int_0^\infty e^{-st}\cos 2t\, dt$$

$$= \frac{1}{4}\left[\frac{e^{-st}}{(-s)^2 + 6^2}(-s\cos 6t + 6\sin 6t)\right]_0^\infty + \frac{3}{4}\left[\frac{e^{-st}}{(-s)^2 + s^2}(-s\cos 2t + 2\sin 2t)\right]$$

$$\left[\because \int e^{at}\cos bt\, dt = \frac{e^{at}}{a^2 + b^2}(a\cos bt + b\sin bt)\right]$$

$$= \frac{1}{4}\cdot\frac{(-1)}{s^2 + 36}(-s) + \frac{3}{4}\cdot\frac{(-1)}{s^2 + 4}(-s) \qquad [\because e^{-\infty} \sim 0]$$

$$= \frac{s}{4}\left[\frac{s^2 + 4 + 3s^2 + 108}{(s^2 + 36)(s^2 + 4)}\right]$$

$$= \frac{s(s^2 + 28)}{(s^2 + 36)(s^2 + 4)}$$

12. $\sin \pi t$

Solution: Let, $f(t) = \sin \pi t$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\sin \pi t\}$$

$$= \int_0^\infty e^{-st}\sin \pi t\, dt = \left[\frac{e^{-st}}{(-s)^2 + \pi^2}\{(-s).\sin \pi t - \pi.\cos \pi t\}\right]_0^\infty$$

$$= 0 - \frac{1}{s^2 + \pi^2}(-\pi) \qquad [\because e^{-\infty} \sim 0]$$

$$= \frac{\pi}{s^2 + \pi^2}$$

13. $e^{a - bt}$

Solution: Let, $f(t) = e^{a - bt} = e^a . e^{-bt}$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^a . e^{-bt}\}$$

$$= \int_0^\infty e^a . e^{-bt} e^{-st}\, dt = e^a \int_0^\infty e^{-(s + b)t}\, dt$$

$$= e^a\left[\frac{e^{-(s + b)t}}{-(s + b)}\right]_0^\infty$$

$$= e^a\left[0 - \frac{1}{-(s + b)}\right] \qquad [\because e^{-\infty} \sim 0]$$

$$= \frac{e^a}{s + b}$$

14. Figure see from book.

Solution: Here f(t) is defined as

$$f(t) = \begin{cases} 1 - t & \text{for } 0 \le t \le 1 \\ 0 & \text{for } t > 1 \end{cases}$$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = \int_0^1 e^{-st} f(t)\, dt + \int_1^\infty e^{-st} f(t)\, dt$$

$$= \int_0^1 (1 - t)\, e^{-st}\, dt + 0$$

$$= \int_0^1 (1 - t)\, e^{-st}\, dt$$

$$= \left[(1 - t)\frac{e^{-st}}{-s} - (-1)\frac{e^{-st}}{(-s)^2}\right]_0^1$$

$$= \left(0 + \frac{e^{-s}}{s^2}\right) - \left(\frac{1}{-s} + \frac{1}{s^2}\right) = \frac{1}{s} + \frac{e^{-s} - 1}{s^2}$$

15. Figure see from book.

Solution: Here f(t) is defined as

$$f(t) = \begin{cases} k & \text{for } 0 \le t \le c \\ 0 & \text{for } t > c \end{cases}$$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = \int_0^\infty e^{-st} f(t)\, dt + \int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^c e^{-st} . k\, dt + 0$$

$$= k\int_0^c e^{-st}\, dt = k\left[\frac{e^{-st}}{-s}\right]_0^c = k\left(\frac{e^{-sc} - 1}{-s}\right) = k\left(\frac{1 - e^{-sc}}{s}\right)$$

16. Figure see from book.

Solution: Here f(t) is defined as

$$f(t) = \begin{cases} t & \text{for } 0 \le t \le k \\ 0 & \text{for } t > k \end{cases}$$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = \int_0^k e^{-st} f(t)\, dt + \int_k^\infty e^{-st} f(t)\, dt$$

$$= \int_0^k t\, e^{-st}\, dt + 0$$

$$= \left[t\frac{e^{-st}}{-s} - (1)\frac{e^{-st}}{(-s)^2} \right]_0^k$$

$$= \left(k \cdot \frac{e^{-sk}}{-s} - \frac{e^{-sk}}{s^2} \right) - \left(0 - \frac{1}{s^2} \right) = \frac{1 - e^{-sk}}{s^2} - \frac{k\, e^{-sk}}{s}$$

17. Figure see from book.

Solution: Here, f(t) is defined as

$$f(t) = \begin{cases} 1 - t/2 & \text{for } 0 \le t \le 1 \\ 0 & \text{for } t > 1 \end{cases}$$

Then the Laplace transform of f(t) is

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt = \int_0^1 f(t)\, e^{-st}\, dt + \int_1^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^1 \left(1 - \frac{t}{2}\right) e^{-st}\, dt + 0$$

$$= \left[\left(1 - \frac{t}{2}\right)\frac{e^{-st}}{-s} - \left(\frac{-1}{2}\right)\frac{e^{-st}}{(-s)^2} \right]_0^1$$

$$= \left(\frac{1}{2} \cdot \frac{e^{-s}}{-s} + \frac{1}{2} \cdot \frac{e^{-s}}{s^2} \right) - \left(\frac{1}{-s} + \frac{1}{2} \cdot \frac{1}{s^2} \right)$$

$$= \left(\frac{e^{-s} - 1}{2s^2} \right) - \left(\frac{e^{-s} - 2}{2s} \right)$$

B. Find f(t) if F(s) is as follow:

(i) $\dfrac{7}{s+3}$

Solution: Given that,

$$F(s) = \mathcal{L}\{f(t)\} = \frac{7}{s+3}$$

Now, taking inverse Laplace transform on both sides then

$$\mathcal{L}^{-1}\{\mathcal{L}\{f(t)\}\} = \mathcal{L}^{-1}\left\{\frac{7}{s+3}\right\}$$

$$\Rightarrow f(t) = 7\,\mathcal{L}^{-1}\left\{\frac{1}{s-(-3)}\right\} = 7e^{-3t} \qquad \left[\text{Being, } \mathcal{L}(e^{at}) = \frac{1}{s-a}\right]$$

Thus, $f(t) = 7e^{-3t}$

(ii) $\dfrac{1}{s^2 + 36}$ **[2010 Spring – Short]**

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{1}{s^2 + 36}$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2 + 36}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s^2) + (6)^2}\right\}$$

$$= \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{(s^2) + (6)^2}\right\}$$

$$= \frac{1}{6}\sin 6t \qquad \left[\because L\{\sin at\} = \frac{a}{s^2 + a^2}\right]$$

(iii) $\dfrac{1}{s^4}$

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{1}{s^4}$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \mathcal{L}^{-1}\left(\frac{1}{s^4}\right) = \frac{1}{3!}\mathcal{L}^{-1}\left(\frac{3!}{s^4}\right)$$

$$= \frac{1}{3!}t^3 = \frac{t^3}{6} \qquad \left[\because L\{t^n\} = \frac{n!}{s^{n+1}}\right]$$

(iv) $\dfrac{2s + 6}{s^2 + 4}$

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{2s + 6}{s^2 + 4}$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \mathcal{L}^{-1}\left\{\frac{2s}{s^2 + 4} + \frac{6}{s^2 + 4}\right\} = \mathcal{L}^{-1}\left[2\frac{s}{s^2 + 2^2} + 3\frac{2}{s^2 + 2^2}\right]$$

$$= 2\,\mathcal{L}^{-1}\left[\frac{s}{s^2 + 2^2}\right] + 3\,\mathcal{L}^{-1}\left[\frac{2}{s^2 + 2^2}\right]$$

$$= 2\cos 2t + 3\sin 2t \qquad [\because \text{using table of Laplace transform}]$$

(v) $\dfrac{4}{(s+1)(s+2)}$

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{4}{(s+1)(s+2)} \qquad \text{......(i)}$$

Here,

$$\frac{4}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$\Rightarrow \quad 4 = A(s+2) + B(s+1)$$

$$\Rightarrow \quad 4 = s(A+B) + (2A+B)$$

Equating coefficient of s and the constant term on both sides then we get,

$$A + B = 0 \quad \text{and} \quad 2A + B = 4.$$

Solving we get,

$$A = 4,\ B = -4.$$

Now, equation (i) becomes,

$$\mathcal{L}\{f(t)\} = \left(\frac{4}{s+1} - \frac{4}{s+2}\right)$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \mathcal{L}^{-1}\left\{\frac{4}{(s+1)} - \frac{4}{(s+2)}\right\}$$

$$= 4(e^{-t} - e^{-4t}) \qquad \left[\text{Being, } \mathcal{L}(e^{at}) = \frac{1}{s-a}\right]$$

(vi) $\dfrac{s+1}{s^2+2s}$

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{s+1}{s(s+2)} \qquad \text{........(i)}$$

Here, $\dfrac{s+1}{s(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+2}$ ....(ii)

$$\Rightarrow \quad \frac{s+1}{s(s+2)} = \frac{A(s+2) + Bs}{s(s+2)}$$

This gives, $s + 1 = s(A+B) + 2A$

Equating coefficient of s and the constant term on both sides then we get,

$$A + B = 1 \quad \text{and} \quad 2A = 1$$

Solving, $A = \dfrac{1}{2}$ and $B = \dfrac{1}{2}$

Then, (i) and (ii) becomes,

$$\mathcal{L}\{f(t)\} = \frac{s+1}{s(s+2)} = \left(\frac{1}{2s} + \frac{1}{2(s+2)}\right)$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \mathcal{L}^{-1}\left[\frac{1}{2s} + \frac{1}{2(s+2)}\right] = \frac{1}{2}\mathcal{L}^{-1}\left[\frac{1}{s} + \frac{1}{s+2}\right]$$

$$= \frac{1}{2}(1 + e^{-2t}).$$

(vii) $\dfrac{0.1s + 0.9}{s^2 + 3.24}$

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{0.1s + 0.9}{s^2 + 3.24}$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \mathcal{L}^{-1}\left(\frac{0.1s + 0.9}{s^2 + 3.24}\right)$$

$$= \mathcal{L}^{-1}\left\{\frac{0.1s}{s^2 + 3.24} + \frac{0.9}{s^2 + 3.24}\right\}$$

$$= 0.1\,\mathcal{L}^{-1}\left(\frac{s}{s^2 + 3.24}\right) + \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1.8}{s^2 + 1.8^2}\right)$$

$$= 0.1\cos 1.8t + \frac{1}{2}\sin 1.8t$$

$$\left[\because L\{\sin at\} = \frac{a}{s^2 + a^2} \text{ and } L\{\cos at\} = \frac{s}{s^2 + a^2}\right]$$

(viii) $\dfrac{-s-10}{s^2-s-2}$

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{-s-10}{s^2-s-2} = \frac{-s-10}{s^2-2s+s-2} = \frac{-s-10}{(s-2)(s+1)} \qquad \text{.......(i)}$$

Here,

$$\frac{-s-10}{(s-2)(s+1)} = \frac{A}{(s-2)} + \frac{B}{(s+1)} \qquad \text{....... (ii)}$$

$$\Rightarrow \quad \frac{-s-10}{(s-2)(s+1)} = \frac{A(s+1) + B(s-2)}{(s+1)(s-2)}$$

$$\Rightarrow \quad -s - 10 = s(A+B) + (A - 2B)$$

Equating coefficient of s and the constant term on both sides then we get.

$$A + B = -1 \quad \text{and} \quad A - 2B = -10.$$

Solving we get, $A = -4$ and $B = 3$.

Then (i) and (ii) gives,

$$\mathcal{L}\{f(t)\} = \frac{-4}{s-2} + \frac{3}{s+1}$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \mathcal{L}^{-1}\left[\frac{-4}{s-2} + \frac{3}{s+1}\right]$$

$$= -4e^{2t} + 3e^{-t} = 3e^{-t} - 4e^{2t} \qquad \left[\text{Being, } \mathcal{L}(e^{at}) = \frac{1}{s-a}\right]$$

(ix) $\dfrac{2.4}{s^4} - \dfrac{228}{s^6}$

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{2.4}{s^4} - \frac{228}{s^6}$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \mathcal{L}^{-1}\left(\frac{2.4}{s^4} - \frac{228}{s^6}\right) = \left(\left(\frac{2.4}{3!}\right)L^{-1}\left\{\frac{3!}{s^4}\right\} - \left(\frac{228}{5!}\right)L^{-1}\left\{\frac{5!}{s^6}\right\}\right)$$

$$= \left(\frac{2.4}{6}\right)t^3 - \left(\frac{228}{5\times4\times3\times2\times1}\right)\times t^5 \left[\because L\{t^n\} = \frac{n!}{s^{n+1}}\right]$$

$$= 0.4t^3 - 1.9t^5$$

(x) $\dfrac{s}{L^2s^2 + n^2\pi^2}$

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{s}{L^2s^2 + n^2\pi^2}$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \mathcal{L}^{-1}\left[\frac{s}{L^2\left(s^2 + \frac{n^2\pi 2}{L^2}\right)}\right] = \frac{1}{L^2}\mathcal{L}^{-1}\left[\frac{s}{L^2 + \left(\frac{n\pi}{L}\right)^2}\right]$$

$$= \frac{1}{L^2}\cos\frac{n\pi}{L}t \qquad \left[\because L\{\cos at\} = \frac{s}{s^2 + a^2}\right]$$

(xi) $\displaystyle\sum_{k=1}^{5} \frac{a_k}{s + k^2}$

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \sum_{k=1}^{5} \frac{a_k}{s + k^2}$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \mathcal{L}^{-1}\left\{\sum_{k=1}^{5} \frac{a_k}{s + k^2}\right\} = \sum_{k=1}^{5} a_k\, \mathcal{L}^{-1}\left\{\frac{1}{s - (-k^2)}\right\}$$

$$= \sum_{k=1}^{5} a_k \cdot e^{-k^2 t}$$

(xii) $\dfrac{1}{(s + \sqrt{2})(s - \sqrt{3})}$

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{1}{(s + \sqrt{2})(s - \sqrt{3})} \qquad \text{... (i)}$$

Here, $$\frac{1}{(s + \sqrt{2})(s - \sqrt{3})} = \frac{A}{(s + \sqrt{2})} + \frac{B}{(s - \sqrt{3})} \qquad \text{...... (ii)}$$

$$\Rightarrow \quad \frac{1}{(s + \sqrt{2})(s - \sqrt{3})} = \frac{s(A + B) + (B\sqrt{2} - A\sqrt{3})}{(s + \sqrt{2})(s - \sqrt{3})}$$

$$\Rightarrow \quad 1 = s(A + B) + (B\sqrt{2} - A\sqrt{3})$$

Equating coefficient of s and the constant term on both sides then we get,

$$A + B = 0 \quad \text{and} \quad B\sqrt{2} - A\sqrt{3} = 1$$

Solving we get,

$$A = -\frac{1}{\sqrt{2} + \sqrt{3}} \qquad \text{and } B = \frac{1}{\sqrt{2} + \sqrt{3}}$$

Then (i) and (ii) becomes,

$$\mathcal{L}\{f(t)\} = \frac{1}{(s + \sqrt{2})(s - \sqrt{3})} = -\frac{1}{(s + \sqrt{2})(\sqrt{2} + \sqrt{3})} + \frac{1}{(s - \sqrt{3})(\sqrt{2} + \sqrt{3})}$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \frac{-1}{\sqrt{2} + \sqrt{3}}\left[L^{-1}\left\{\frac{1}{s + \sqrt{2}}\right\}\right] + \frac{1}{(\sqrt{2} + \sqrt{3})}\left[L^{-1}\left\{\frac{1}{3 - \sqrt{3}}\right\}\right]$$

$$= -\frac{-1}{\sqrt{2} + \sqrt{3}}e^{-\sqrt{2}t} + \frac{1}{\sqrt{2} + \sqrt{3}}e^{\sqrt{3}t}$$

$$= \frac{e^{\sqrt{3}t} - e^{-\sqrt{2}t}}{(\sqrt{2} + \sqrt{3})}$$

(xiii) $\dfrac{1}{(s - a)(1 - b)}$ **(a ≠ b).**

Solution: Given that,

$$\mathcal{L}\{f(t)\} = \frac{1}{(s - a)(1 - b)} \qquad \text{... (i)}$$

Here,

$$\frac{1}{(s - a)(1 - b)} = \frac{A}{s - a} + \frac{B}{s - b} = \frac{A(s - b) + B(s - a)}{(s - a)(s - b)} \qquad \text{........ (ii)}$$

$$\Rightarrow \quad 1 = s(A + B) - (Ab + aB)$$

Equating coefficient of s and the constant term on both sides then we get,

$$A + B = 0 \text{ and } -Ab - aB = 1$$

Solving we get,

$$A = -\frac{1}{b - a} \qquad \text{and } B = \frac{1}{b - a}$$

Now (ii) becomes,

$$\frac{1}{(s - a)(1 - b)} = \frac{1}{(a - b)(s - a)} - \frac{1}{(a - b)}\frac{1}{(s - b)}$$

$$= \frac{1}{(a - b)}\left[\frac{1}{s - a} - \frac{1}{(s - b)}\right]$$

Now, taking inverse Laplace transform on both sides then

$$f(t) = \frac{1}{(a - b)}\mathcal{L}^{-1}\left[\frac{1}{s - a} - \frac{1}{(s - b)}\right]$$

$$= \frac{1}{(a - b)}(e^{at} - e^{bt}).$$