Standard Representation

- Any logical expression can be expressed in the following two forms:
  - ✓ Sum of Product (SOP) Form
  - ✓ Product of Sum (POS) Form

SOP Form

For Example, logical expression given is;

Sum

$$Y = A.B + B.C + A.C$$

Product

POS Form

For Example, logical expression given is;

Product

$$Y = (A + B).(B + C).(A + C)$$

Sum

Standard or Canonical SOP & POS Forms

- ✓ We can say that a logic expression is said to be in the standard (or canonical) SOP or POS form if each product term (for SOP) and sum term (for POS) consists of all the literals in their complemented or uncomplemented form.

Standard SOP

$$Y = ABC + A\overline{B}\,\overline{C} + \overline{A}BC$$

Each product term consists all the literals

Standard POS

$$Y = (A + B + C).(A + \overline{B} + \overline{C}).(\overline{A} + B + C)$$

Each sum term consists all the literals

Examples

Sr. No.	Expression	Type
1	$Y = AB + AB\overline{C} + \overline{A}BC$	Non Standard SOP
2	$Y = AB + A\overline{B} + \overline{A}\overline{B}$	Standard SOP
3	$Y = (\overline{A} + B).(A + \overline{B}).(\overline{A} + \overline{B})$	Standard POS
4	$Y = (\overline{A} + B).(A + \overline{B} + C)$	Non Standard POS

Conversion of SOP form to Standard SOP

Procedure:

1. Write down all the terms.
2. If one or more variables are missing in any product term, expand the term by multiplying it with the sum of each one of the missing variable and its complement .
3. Drop out the redundant terms.

Example 1

Convert given expression into its standard SOP form $Y = AB + A\overline{C} + BC$

$$Y = AB + A\overline{C} + BC$$

Missing literal is A

Missing literal is B

Missing literal is C

$$Y = AB.(C + \overline{C}) + A\overline{C}.(B + \overline{B}) + BC.(A + \overline{A})$$

Term formed by ORing of missing
literal & its complement

Example 1

Continue ....

$$Y = AB.(C + \overline{C}) + A\overline{C}.(B + \overline{B}) + BC.(A + \overline{A})$$

$$Y = ABC + AB\overline{C} + AB\overline{C} + A\overline{B}\,\overline{C} + ABC + \overline{A}BC$$

$$Y = ABC + AB\overline{C} + AB\overline{C} + A\overline{B}\,\overline{C} + ABC + \overline{A}BC$$

$$Y = ABC + AB\overline{C} + A\overline{B}\,\overline{C} + \overline{A}BC$$

Standard SOP form
Each product term consists all the literals

Conversion of POS form to Standard POS

Procedure:

1. Write down all the terms.
2. If one or more variables are missing in any sum term, expand the term by adding the products of each one of the missing variable and its complement.
3. Drop out the redundant terms.

Example 2

Convert given expression into its standard SOP form $Y = (A+B).(A+C).(B+\overline{C})$

$$Y = (A+B).(A+C).(B+\overline{C})$$

Missing literal is A

Missing literal is B

Missing literal is C

$$Y = (A + B + C\overline{C}).(A + C + B\overline{B}).(B + \overline{C} + A\overline{A})$$

Term formed by ANDing of missing literal & its complement

Example 2

Continue ....

$$Y = (A + B + C\overline{C}).(A + C + B\overline{B}).(B + \overline{C} + A\overline{A})$$

$$Y = (A + \underline{B + C})(A + \underline{B + \overline{C}}).(A + \underline{B + C})(A + \overline{B} + C).(A + \underline{B + \overline{C}})(\overline{A} + B + \overline{C})$$

$$Y = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + \overline{C})$$

$$Y = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + \overline{C})$$

Standard POS form
Each sum term consists all the literals

Concept of Minterm and Maxterm

- ✓ **Minterm:** Each individual term in the standard SOP form is called as “Minterm”.
- ✓ **Maxterm:** Each individual term in the standard POS form is called as “Maxterm”.

- ✓ The concept of minterm and max term allows us to introduce a very convenient shorthand notation to express logic functions

Minterms & Maxterms for 3 variable/literal logic function

Variables			**Minterms**	**Maxterms**
A	B	C	mi	Mi
0	0	0	$\overline{A}\,\overline{B}\,\overline{C} = m_0$	$A + B + C = M_0$
0	0	1	$\overline{A}\,\overline{B}C = m_1$	$A + B + \overline{C} = M_1$
0	1	0	$\overline{A}B\overline{C} = m_2$	$A + \overline{B} + C = M_2$
0	1	1	$\overline{A}BC = m_3$	$A + \overline{B} + \overline{C} = M_3$
1	0	0	$A\overline{B}\,\overline{C} = m_4$	$\overline{A} + B + C = M_4$
1	0	1	$A\overline{B}C = m_5$	$\overline{A} + B + \overline{C} = M_5$
1	1	0	$AB\overline{C} = m_6$	$\overline{A} + \overline{B} + C = M_6$
1	1	1	$ABC = m_7$	$\overline{A} + \overline{B} + \overline{C} = M_7$

Representation of Logical expression using minterm

$$Y = \underbrace{ABC}_{m_7} + \underbrace{\overline{A}BC}_{m_3} + \underbrace{A\overline{B}\,\overline{C}}_{m_4} + \underbrace{A\overline{B}C}_{m_5}$$

Logical Expression

Corresponding minterms

$$Y = m_7 + m_3 + m_4 + m_5$$

$$Y = \Sigma m(3,4,5,7)$$

OR

$$Y = f(A,B,C) = \Sigma m(3,4,5,7)$$

where Σ denotes sum of products

Representation of Logical expression using maxterm

$$Y = \underbrace{(A + \overline{B} + C)}_{M_2} . \underbrace{(A + B + C)}_{M_0} . \underbrace{(\overline{A} + \overline{B} + C)}_{M_6}$$

Logical Expression

Corresponding maxterms

$$Y = M_2 . M_0 . M_6$$

$$Y = \Pi M(0, 2, 6)$$

OR

$$Y = f(A, B, C) = \Pi M(0, 2, 6)$$

where $\prod$ denotes product of sum

Conversion from SOP to POS & Vice versa

- ✓ The relationship between the expressions using minters and maxterms is complementary.

- ✓ We can exploit this complementary relationship to write the expressions in terms of maxterms if the expression in terms of minterms is known and vice versa

Conversion from SOP to POS & Vice versa

- ✓ For example, if a SOP expression for 4 variable is given by,

$$Y = \Sigma m(0,1,3,5,6,7,11,12,15)$$

- ✓ Then we canget the equivalent POS expression using complementary relationship as follows:

$$Y = \Pi M\ (2,4,8,9,10,13,14)$$

Examples

1. Convert the given expression into standard form

$$Y = A + BC + ABC$$

2. Convert the given expression into standard form

$$Y = (A + B).(A + \overline{C})$$

Karnaugh Map (K-map)

- ✓ In the algebraic method of simplification, we need to write lengthy equations, find the common terms, manipulate the expressions etc., so it is time consuming work.

- ✓ Thus "K-map" is another simplification technique to reduce the Boolean equation.

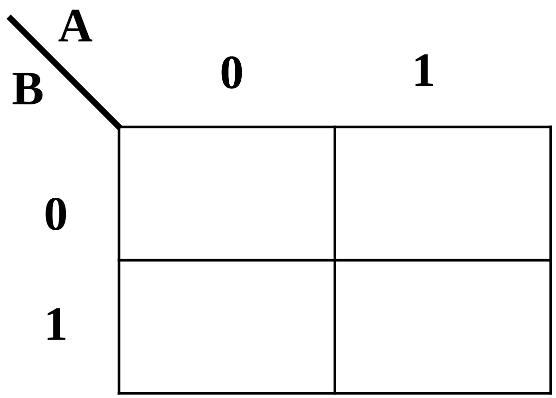
Karnaugh Map (K-map)

- ✓ It overcomes all the disadvantages of algebraic simplification techniques.

- ✓ The information contained in a truth table or available in the SOP or POS form is represented on K-map.

Karnaugh Map (K-map)

- K-map Structure - 2 Variable
  - ✓ A & B are variables or inputs
  - ✓ 0 & 1 are values of A & B
  - ✓ 2 variable k-map consists of 4 boxes i.e. $2^2=4$



Karnaugh Map (K-map)

➢ K-map Structure - 2 Variable

✓ Inside 4 boxes we have enter values of Y i.e. output

B \ A		$\overline{A}$ 0	A 1
$\overline{B}$	0	$\overline{A}\,\overline{B}$	$\overline{A}B$
B	1	$A\overline{B}$	AB

B \ A		$\overline{A}$ 0	A 1
$\overline{B}$	0	m_0	m_1
B	1	m_2	m_3

K-map & its associated minterms

Karnaugh Map (K-map)

✓Relationship between Truth Table & K-map

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

B \ A		$\overline{A}$ **0**	A **1**
$\overline{B}$	**0**	0	0
B	**1**	1	1

A \ B		$\overline{B}$ **0**	B **1**
$\overline{A}$	**0**	0	1
A	**1**	0	1

Karnaugh Map (K-map)

- K-map Structure - 3 Variable
  - ✓ A, B & C are variables or inputs
  - ✓ 3 variable k-map consists of 8 boxes i.e. $2^3=8$

A \ BC	00	01	11	10
0				
1				

Karnaugh Map (K-map)

✓ 3 Variable K-map & its associated product terms

A \ BC	00	01	11	10
0	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}\,\overline{B}C$	$\overline{A}BC$	$\overline{A}B\overline{C}$
1	$A\overline{B}\,\overline{C}$	$A\overline{B}C$	ABC	$AB\overline{C}$

Karnaugh Map (K-map)

✓ 3 Variable K-map & its associated minterms

C \ AB	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

BC \ A	0	1
00	m_0	m_4
01	m_1	m_5
11	m_3	m_7
10	m_2	m_6

Karnaugh Map (K-map)

- K-map Structure - 4 Variable
  - ✓ A, B, C & D are variables or inputs
  - ✓ 4 variable k-map consists of 16 boxes i.e. $2^4=16$

AB \ CD	00	01	11	10
00				
01				
11				
10				

Karnaugh Map (K-map)

✓ 4 Variable K-map and its associated product terms

CD \ AB	00	01	11	10
00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}\,\overline{D}$	$AB\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}\,\overline{D}$
01	$\overline{A}\,\overline{B}\,\overline{C}D$	$A\overline{B}\,\overline{C}D$	$AB\overline{C}D$	$A\overline{B}\,\overline{C}D$
11	$\overline{A}\,\overline{B}CD$	$A\overline{B}CD$	$ABCD$	$A\overline{B}CD$
10	$\overline{A}\,\overline{B}C\overline{D}$	$A\overline{B}C\overline{D}$	$ABC\overline{D}$	$A\overline{B}C\overline{D}$

AB \ CD	00	01	11	10
00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$\overline{A}\,\overline{B}\,\overline{C}D$	$\overline{A}\,\overline{B}CD$	$\overline{A}\,\overline{B}C\overline{D}$
01	$A\overline{B}\,\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$
11	$AB\overline{C}\,\overline{D}$	$AB\overline{C}D$	$ABCD$	$ABC\overline{D}$
10	$A\overline{B}\,\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$

Karnaugh Map (K-map)

✓ 4 Variable K-map and its associated minterms

CD \ AB	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{11}	m_{10}

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Representation of Standard SOP form expression on K-map

For example, SOP equation is given as

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

- ✓ The given expression is in the standard SOP form.
- ✓ Each term represents a minterm.
- ✓ We have to enter '1' in the boxes corresponding to each minterm as below

$\overline{A}\,\overline{B}\,\overline{C}$ $\overline{A}\,\overline{B}C$

A \ BC	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	1	1	0	0
A 1	1	0	1	1

$AB\overline{C}$

$A\overline{B}\,\overline{C}$ ABC

Simplification of K-map

- ✓ Once we plot the logic function or truth table on K-map, we have to use the grouping technique for simplifying the logic function.
- ✓ Grouping means the combining the terms in adjacent cells.
- ✓ The grouping of either 1's or 0's results in the simplification of Boolean expression.

Simplification of K-map

- ✓ If we group the adjacent 1's then the result of simplification is SOP form

- ✓ If we group the adjacent 0's then the result of simplification is POS form

Grouping:

- ✓ While grouping, we should group most number of 1's.
- ✓ The grouping follows the binary rule i.e we can group 1,2,4,8,16,32,……..…number of 1's.
- ✓ We cannot group 3,5,7,………number of 1's
- ✓ ***Pair***: A group of two adjacent 1's is called as Pair
- ✓ ***Quad***: A group of four adjacent 1's is called as Quad
- ✓ ***Octet***: A group of eight adjacent 1's is called as Octet

Grouping of Two Adjacent 1's : Pair

✓ ***A pair eliminates 1 variable***

$\bar{A}BC$ $\quad$ $\bar{A}B\bar{C}$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ **0**	**0**	**0**	**1**	**1**
A **1**	**0**	**0**	**0**	**0**

$$Y = \bar{A}BC + \bar{A}B\bar{C}$$

$$Y = \bar{A}B(C + \bar{C})$$

$$Y = \bar{A}B \qquad (\because C + \bar{C} = 1)$$

Grouping of Two Adjacent 1's : Pair

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	0	0
A 1	1	0	0	1

A \ BC	$\overline{BC}$ 00	BC 01	BC 11	BC 10
$\bar{A}$ 0	0	1	1	1
A 1	0	0	1	0

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	0	0
A 1	0	1	0	0

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	1	1
A 1	1	0

Possible Grouping of Four Adjacent 1's : Quad

✓ ***A Quad eliminates 2 variable***

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0	0	0	0
$\overline{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\overline{B}$ 10	1	1	1	1

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0	1	0	0
$\overline{A}B$ 01	0	1	0	0
AB 11	0	1	0	0
$A\overline{B}$ 10	0	1	0	0

Possible Grouping of Four Adjacent 1's : Quad

- ✓ ***A Quad eliminates 2 variable***

AB \ CD		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	0	0	0	0
$\bar{A}B$	01	1	1	0	0
AB	11	1	1	0	0
$A\bar{B}$	10	0	0	0	0

AB \ CD		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	0	1	1	0
$\bar{A}B$	01	0	0	0	0
AB	11	0	0	0	0
$A\bar{B}$	10	0	1	1	0

Possible Grouping of Four Adjacent 1's : Quad

✓ ***A Quad eliminates 2 variable***

AB \ CD		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	1	0	0	1
$\bar{A}B$	01	0	0	0	0
AB	11	0	0	0	0
$A\bar{B}$	10	1	0	0	1

AB \ CD		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	0	0	0	0
$\bar{A}B$	01	1	0	0	1
AB	11	1	0	0	1
$A\bar{B}$	10	0	0	0	0

Possible Grouping of Four Adjacent 1's : Quad

✓ ***A Quad eliminates 2 variable***

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	1	1	1
AB 11	0	1	1	1
$A\bar{B}$ 10	0	0	0	0

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	1	1	0

Possible Grouping of Eight Adjacent 1's : Octet

✓ ***A Octet eliminates 3 variable***

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	0	0	0
AB 11	1	1	1	1
$A\bar{B}$ 10	1	1	1	1

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	1	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	1	1	0

Possible Grouping of Eight Adjacent 1's : Octet

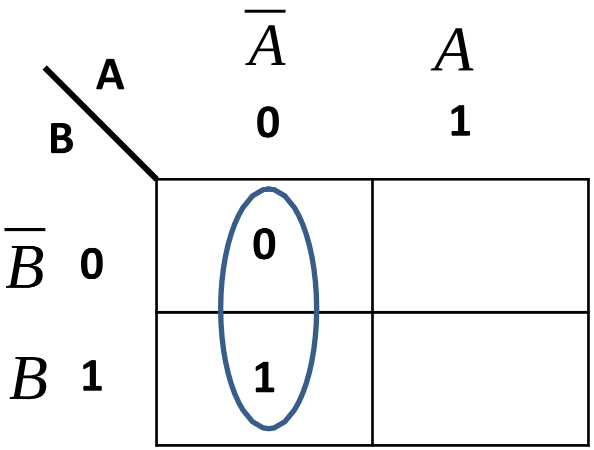
✓ ***A Octet eliminates 3 variable***

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	1	1	1	1
$\overline{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\overline{B}$ 10	1	1	1	1

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	1	0	0	1
$\overline{A}B$ 01	1	0	0	1
AB 11	1	0	0	1
$A\overline{B}$ 10	1	0	0	1

Rules for K-map simplification

1. Groups may not include any cell containing a zero.



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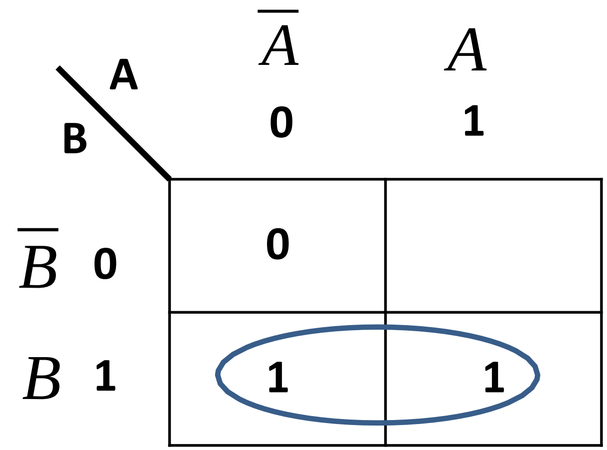

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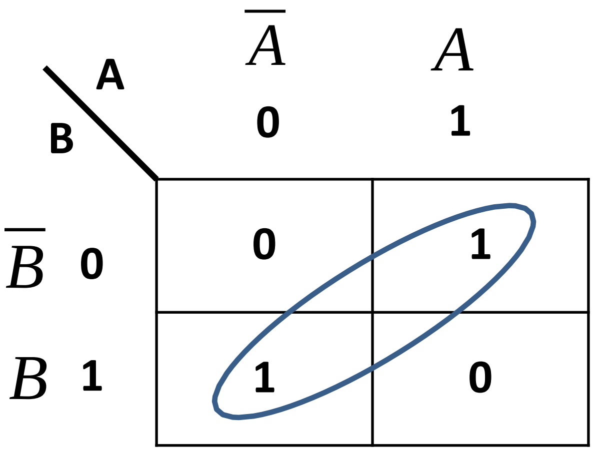
Rules for K-map simplification

2. Groups may be horizontal or vertical, but may not be diagonal



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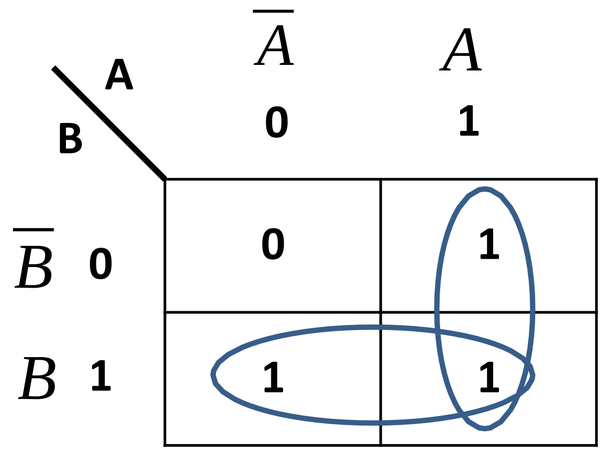


Accepted

5. Each cell containing a one must be in at least one group

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	0	1
A 1	0	0	1	0

Rules for K-map simplification

6. Groups may be overlap

A \ BC		$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$	0	1	1	1	1
A	1	0	0	1	1

Rules for K-map simplification

7. Groups may wrap around the table. The leftmost cell in a row may be grouped with rightmost cell and the top cell in a column may be grouped with bottom cell

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1	1	1
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	1	1	1

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	0	0	1
A 1	1	0	0	1

Rules for K-map simplification

9. A pair eliminates one variable.

10. A Quad eliminates two variables.

11. A octet eliminates three variables

Example 1

For the given K-map write simplified Boolean expression

C \ AB		$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$	0	0	1	1	1
C	1	0	0	1	0

Example 1

continue.....

C \ AB		$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}$	**0**	0	1	1	1
C	**1**	0	0	1	0

$A\bar{C}$

$B\bar{C}$

AB

Simplified Boolean expression

$$Y = B\overline{C} + AB + A\overline{C}$$

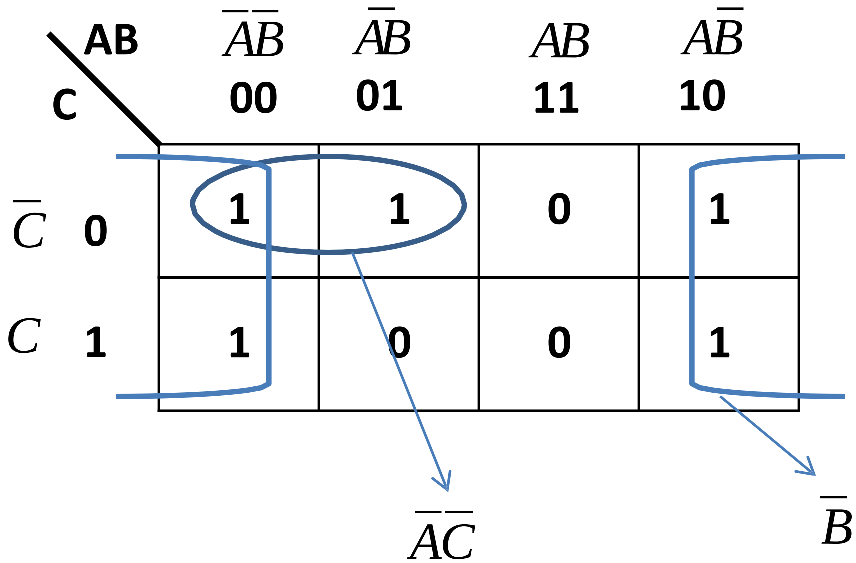
Example 2

For the given K-map write simplified Boolean expression

C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	1	1	0	1
C 1	1	0	0	1

Example 2

continue.....



Simplified Boolean expression

$$Y = \bar{B} + \bar{A}\bar{C}$$

Example 3

A logical expression in the standard SOP form is as follows;

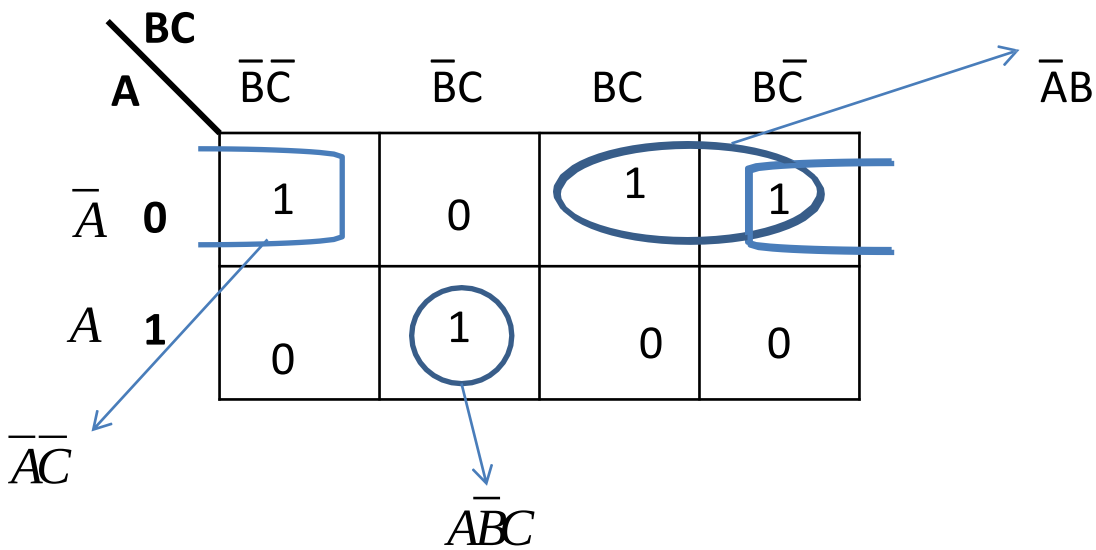
$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,C$$

Minimize it with using the K-map technique

Example 3

continue……

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,C$$



Simplified Boolean expression

$$Y = \overline{A}\,\overline{C} + \overline{A}B + A\overline{B}C$$

Example 4

A logical expression representing a logic circuit is;

$$Y = \Sigma m \ (0, 1, 2, 5, 13, 15)$$

Draw the K-map and find the minimized logical expression

Example 4

continue…..

$$Y = \Sigma m(0, 1, 2, 5, 13, 15)$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0: 1	1: 1	3: 0	2: 1
$\bar{A}B$ 01	4: 0	5: 1	7: 0	6: 0
AB 11	12: 0	13: 1	15: 1	14: 0
$A\bar{B}$ 10	8: 0	9: 0	11: 0	10: 0

$\bar{A}\bar{B}\bar{D}$

$\bar{A}\bar{C}D$

ABD

Simplified Boolean expression

$$Y = \bar{A}\bar{B}\bar{D} + \bar{A}\bar{C}D + ABD$$

Example 5

Minimize the following Boolean expression using K-map ;

$$f(A,B,C,D)=\Sigma m(1,3,5,9,11,13)$$

Example 5 continue.....

$$f(A, B, C, D) = \Sigma m(1, 3, 5, 9, 11, 13)$$

AB \ CD	$\bar C \bar D$ 00	$\bar C D$ 01	$C D$ 11	$C \bar D$ 10
$\bar A \bar B$ 00	0 (0)	1 (1)	1 (3)	0 (2)
$\bar A B$ 01	0 (4)	1 (5)	0 (7)	0 (6)
$A B$ 11	0 (12)	1 (13)	0 (15)	0 (14)
$A \bar B$ 10	0 (8)	1 (9)	1 (11)	0 (10)

$\bar C D$

$\bar B D$

Simplified Boolean expression

$$f = \bar B D + \bar C D$$

$$f = D(\bar B + \bar C)$$

Example 6

Minimize the following Boolean expression using K-map ;

$$f(A, B, C, D) = \Sigma m(4,5,8,9,11,12,13,15)$$

Example 6

continue…..

$$f(A,B,C,D) = \Sigma m(4,5,8,9,11,12,13,15)$$

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0 **0**	1 **0**	3 **0**	2 **0**
$\overline{A}B$ 01	4 **1**	5 **1**	7 **0**	6 **0**
AB 11	12 **1**	13 **1**	15 **1**	14 **0**
$A\overline{B}$ 10	8 **1**	9 **1**	11 **1**	10 **0**

$B\overline{C}$

$A\overline{C}$

AD

Simplified Boolean expression

$$f = B\overline{C} + A\overline{C} + AD$$

Example 7

Minimize the following Boolean expression using K-map ;

$$f_2(A, B, C, D) = \Sigma m(0,1,2,3,11,12,14,15)$$

Example 7

continue.....

$$f_2(A,B,C,D) = \Sigma m(0,1,2,3,11,12,14,15)$$

AB \ CD		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	1 (0)	1 (1)	1 (3)	1 (2)
$\bar{A}B$	01	0 (4)	0 (5)	0 (7)	0 (6)
AB	11	1 (12)	0 (13)	1 (15)	1 (14)
$A\bar{B}$	10	0 (8)	0 (9)	1 (11)	0 (10)

$\bar{A}\bar{B}$

$AB\bar{D}$

ACD

Simplified Boolean expression

$$f_2 = \bar{A}\bar{B} + AB\bar{D} + ACD$$

Example 8

Solve the following expression with K-maps;

1 $f_1(A,B,C) = \Sigma m(0,1,3,4,5)$

2 $f_2(A,B,C) = \Sigma m(0,1,2,3,6,7)$

Example 8

continue……

$$f_1(A,B,C)=\Sigma m(0,1,3,4,5)$$

$\overline{A}C$

A \ BC	$\overline{B}\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	1 (0)	1 (1)	1 (3)	0 (2)
A 1	1 (4)	1 (5)	0 (7)	0 (6)

$\overline{B}$

Simplified Boolean expression

$$f_1=\overline{A}C+\overline{B}$$

$$f_2(A,B,C)=\Sigma m(0,1,2,3,6,7)$$

A \ BC	$\overline{B}\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	1 (0)	1 (1)	1 (3)	1 (2)
A 1	0 (4)	0 (5)	1 (7)	1 (6)

$\overline{A}$ B

Simplified Boolean expression

$$f_2=\overline{A}+B$$

Example 9

Simplify ;

$$f(A,B,C,D)=\Sigma m(0,1,4,5,7,8,9,12,13,15)$$

Example 9 continue.....

$$f(A,B,C,D) = \Sigma m(0,1,4,5,7,8,9,12,13,15)$$

CD / AB	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$ 00	0: **1**	1: **1**	3: **0**	2: **0**
$\overline{A}B$ 01	4: **1**	5: **1**	7: **1**	6: **0**
AB 11	12: **1**	13: **1**	15: **1**	14: **0**
$A\overline{B}$ 10	8: **1**	9: **1**	11: **0**	10: **0**

$\overline{C}$

BD

Simplified Boolean expression

$$f = \overline{C} + BD$$

Example 10

Solve the following expression with K-maps;

1 $$f_1(A, B, C, D) = \Sigma m(0,1,3,4,5,7)$$

2 $$f_2(A, B, C) = \Sigma m(0,1,3,4,5,7)$$

Example 10

continue......

$$f_1(A,B,C,D) = \Sigma m(0,1,3,4,5,7)$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0: **1**	1: **1**	3: **1**	2: **0**
$\bar{A}B$ 01	4: **1**	5: **1**	7: **1**	6: **0**
AB 11	12: **0**	13: **0**	15: **0**	14: **0**
$A\bar{B}$ 10	8: **0**	9: **0**	11: **0**	10: **0**

$\bar{A}\bar{C}$ $\bar{A}D$

Simplified Boolean expression

$$f_1 = \bar{A}\bar{C} + \bar{A}D$$

$$f_2(A,B,C) = \Sigma m(0,1,3,4,5,7)$$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0: **1**	1: **1**	3: **1**	2: **0**
A 1	4: **1**	5: **1**	7: **1**	6: **0**

$\bar{B}$ C

Simplified Boolean expression

$$f_2 = \bar{B} + C$$

K-map for Product of Sum Form (POS Expressions)

- ✓ Karnaugh map can also be used for Boolean expression in the Product of sum form (POS).
- ✓ The procedure for simplification of expression by grouping of cells is also similar

K-map for Product of Sum Form (POS Expressions)

- ✓ The letters with bars (NOT) represent 1 and unbarred letters represent 0 of Binary.
- ✓ A zero is put in the cell for which there is a term in the Boolean expression
- ✓ Grouping is done for adjacent cells containing zeros.

Example 11

Simplify ;

$$f(A,B,C,D)=\Pi M(0,1,3,5,6,7,10,14,15)$$

Example 11 **continue.....**

$$f(A,B,C,D) = \Pi M(0,1,3,5,6,7,10,14,15)$$

AB \ CD	CD 00	$C\overline{D}$ 01	$\overline{C}\overline{D}$ 11	$\overline{C}D$ 10
AB 00	0 (0)	0 (1)	0 (3)	1 (2)
$A\overline{B}$ 01	1 (4)	0 (5)	0 (7)	0 (6)
$\overline{A}\overline{B}$ 11	1 (12)	1 (13)	0 (15)	0 (14)
$\overline{A}B$ 10	1 (8)	1 (9)	1 (11)	0 (10)

$A+\overline{D}$

$\overline{B}+\overline{C}$

$A+B+C$

$\overline{A}+\overline{C}+D$

Simplified Boolean expression

$$f = (A+\overline{D})(\overline{B}+\overline{C})(\overline{A}+\overline{C}+D)(A+B+C)$$

Example 12

Simplify ;

$$f(A,B,C,D) = \Pi M(4,6,10,12,13,15)$$

Example 12

$$f(A,B,C,D) = \Pi M(4,6,10,12,13,15)$$

AB \ CD		CD 00	$C\bar{D}$ 01	$\bar{C}\bar{D}$ 11	$\bar{C}D$ 10
AB	00	1 (0)	1 (1)	1 (3)	1 (2)
$A\bar{B}$	01	0 (4)	1 (5)	1 (7)	0 (6)
$\bar{A}\bar{B}$	11	0 (12)	0 (13)	0 (15)	1 (14)
$\bar{A}B$	10	1 (8)	1 (9)	1 (11)	0 (10)

$A+\bar{B}+D$

$\bar{A}+\bar{B}+C$

$\bar{A}+B+\bar{C}+D$

$\bar{A}+\bar{B}+\bar{D}$

Simplified Boolean expression

$$f = (\bar{A}+B+\bar{C}+D)(A+\bar{B}+D)(\bar{A}+\bar{B}+\bar{D})(\bar{A}+\bar{B}+C)$$

K-map and don’t care conditions

- ✓ For SOP form we enter 1’s corresponding to the combinations of input variables which produce a high output and we enter 0’s in the remaining cells of the K-map.
- ✓ For POS form we enter 0’s corresponding to the combinations of input variables which produce a high output and we enter 1’s in the remaining cells of the K-map.

K-map and don’t care conditions

- ✓ But it is not always true that the cells not containing 1’s (in SOP) will contain 0’s, because some combinations of input variable do not occur.
- ✓ Also for some functions the outputs corresponding to certain combinations of input variables do not matter.

K-map and don’t care conditions

- ✓ In such situations we have a freedom to assume a 0 or 1 as output for each of these combinations.
- ✓ These conditions are known as the “Don’t Care Conditions” and in the K-map it is represented as ‘X’, in the corresponding cell.
- ✓ The don’t care conditions may be assumed to be 0 or 1 as per the need for simplification

K-map and don't care conditions - Example

Simplify ;

$$f(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,5)$$

K-map and don’t care conditions - Example

$$f(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,5)$$

CD / AB	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$ **00**	x	1	1	x
$\overline{A}B$ **01**	0	x	1	0
AB **11**	0	0	1	0
$A\overline{B}$ **10**	0	0	1	0

$$f = CD + \overline{A}\,\overline{B} + \overline{A}D$$

EXAMPLE:

- Simplify the following Boolean function using don’t care conditions:

$F(W,X,Y,Z)= W(\bar{X}Y+\bar{X}\bar{Y}+XYZ)+X\bar{Z}(Y+W)$

$D(W,X,Y,Z)=\bar{W}\bar{X}(\bar{Y}Z+Y\bar{Z})$

And implement using an universal gate.

Five variable k- map:

AB \ CDE	000	001	011	010	110	111	101	100
00	**0**	**1**	**3**	**2**	**6**	**7**	**5**	**4**
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

Five-variable k-map

- Simplify the following Boolean function:

$$f(A, B, C, D, E) = \Sigma m(0,2,4,6,9,11,13,15,17,21,25,27,29,31)$$

AB \ CDE	000	001	011	010	110	111	101	100
00	1	0	0	1	1	0	0	1
01	0	1	1	0	0	1	1	0
11	0	1	1	0	0	1	1	0
10	0	1	0	0	0	0	1	0

$$F=\bar{A}\bar{B}\bar{E}+A\bar{D}E+BE$$