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College. Roll.no: 191725.
Level : Bachelors.
Programme : Software.
Semester : 2^{nd}
Subject : Mathematical Foundation for computer science.
Signature of Examinee/student : Date : 18/03/2078.

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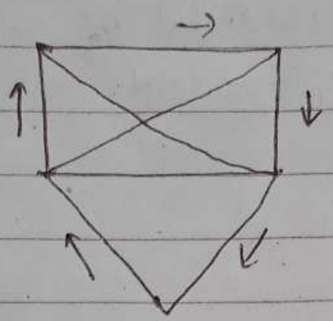
Q.N.1.

A. Ans.

Hamilton Graph:

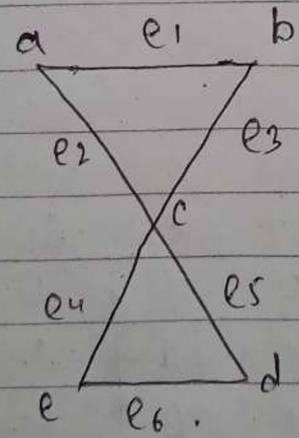
It is the graph in which the ckt passes through every vertex of Graph.



Euler Graph:

Euler Graph is a connected graph whose all vertices are of even degrees.

Example:



Euler Graph	Hamilton Graph.
• The graph must inclide all edge vertexes.	• The graph shouldn't include all edge vertices.
• Edge is not repeated & vertex may be repeated in euler Graph.	• Edge isn't repeated & vertex mayn't be repeated in hamilton graph.

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B. Ans.

Adjacent matrix:

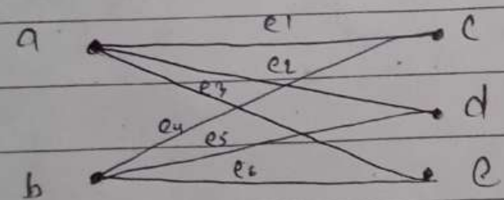
The matrix A is called adjacent matrix if $a_{ij} = a_{ji}$.

Incidence matrix:

The matrix which is obtained by plotting the relationship of edge being incident to vertex is called incidence matrix.

Second part:

Complete bipartiate graph $K_{2,3}$



It's incidence matrix is:

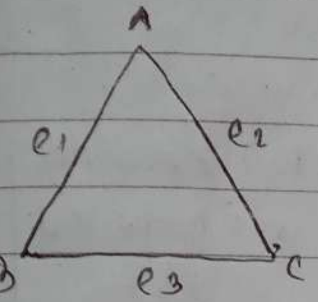
$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

C. Ans.

- A graph that can be drawn on the plane without crossing edges are called planar graph.

To prove euler's formula let us take some planar graphs.

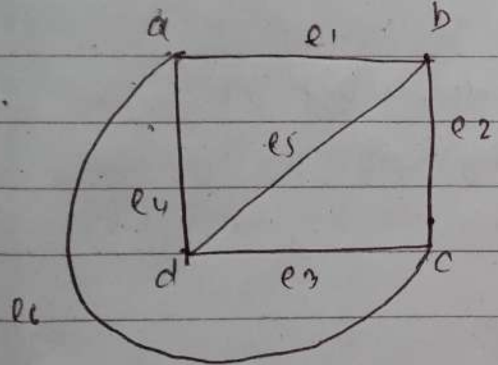
1.



Planner graph 1.

Here,
No. of Edges (E) = $\{e_1, e_2, e_3\}$ = 3.
No. of Vertex (V) = 3.
No. of Faces (F) = 2.

2.



Here,
No. of Edges (E) = $\{e_1, e_2, e_3, e_4, e_5, e_6\}$
= 6.
No. of Vertex (V) = 4.
No. of Faces (F) = 4.

As we know that,

$V - E + F = 2$ is Euler's formula.

where,

V = No. of Vertex.
E = No. of Edges.
F = No. of Faces.

In first case, $3 - 3 + 2 = 2 \Rightarrow 2 = 2$, Euler's formula is verified.
In second graph, $4 - 6 + 4 = 2 \Rightarrow 2 = 2$, Euler's formula is verified.
Hence, planner graph satisfies Euler formula.

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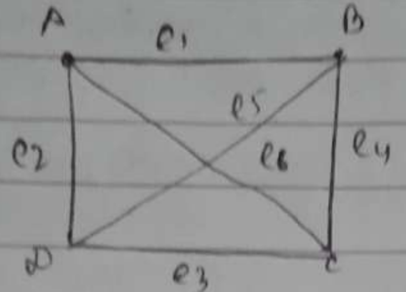
Q.N.2.

A. Ans.



Let us consider a graph with 'n' vertices. Since the graph is complete graph each vertex are connected to each other by distinct edges.

So, the total number of degree for each vertex is $(n-1)$.

The sum of all degree of graph is given by,

$$deg(V_1) + deg(V_2) + \dots + deg(V_n)$$

or, $(n-1) + (n-1) + \dots + (n-1)$.

or, $n(n-1)$

Again,

We know that the sum of all degree of vertices is equal to the twice number of edges.

$\therefore n(n-1) = 2e$.

$\Rightarrow e = \frac{n(n-1)}{2}$.

Hence, In complete graph. for n no. of vertices, the number of edges is given by $\frac{n(n-1)}{2}$.

B. Ans.

Dijkstra's algorithm:

This algorithm helps in determining the shortest path. The technique to implement this algorithm is given below:

Step 1 : Label the initial vertex of graph with weight zero.

Step 2 : Calculate the weight of all vertices adjacent to the intial vertex corresponding to the weight of the edges incident on the initial vertex.

Step 3 : Label these vertices with smallest possible value of their weights.

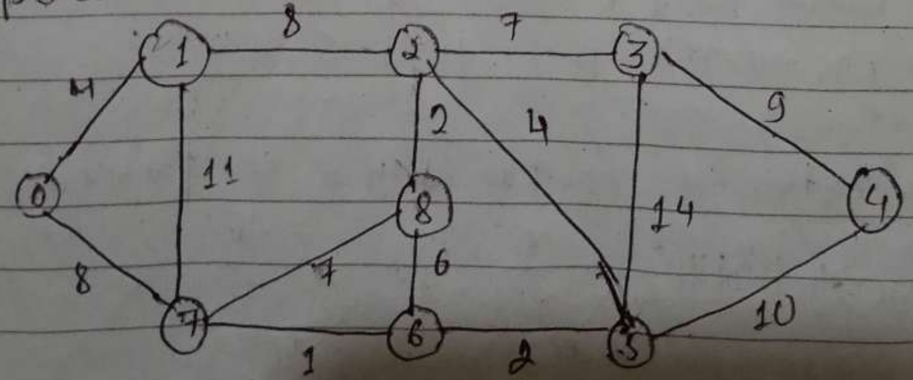
Step 4 : Calculate the weights of all vertices which are adjacent to the vertices with minimum weight determined in step 3.

Step 5 : Label these steps with minimum weight.

Step 6 : Continue this process until all the vertices of weighted graphs are labelled.

Step 7 : Trace the path of cumulative min^{m} weight from intial to final vertex.

Second part:



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Step 1 :

Here, the starting vertex is 0. The adjacent vertex to starting vertex are 1 and 7. The shortest path is to 1. So we take path from 0 to 1.

Step 2 :

There are two adjacent vertex for vertex 1. The shortest path betn two adjacent vertexs 2 and 7 is 8. So, we take path from 1 to 2.

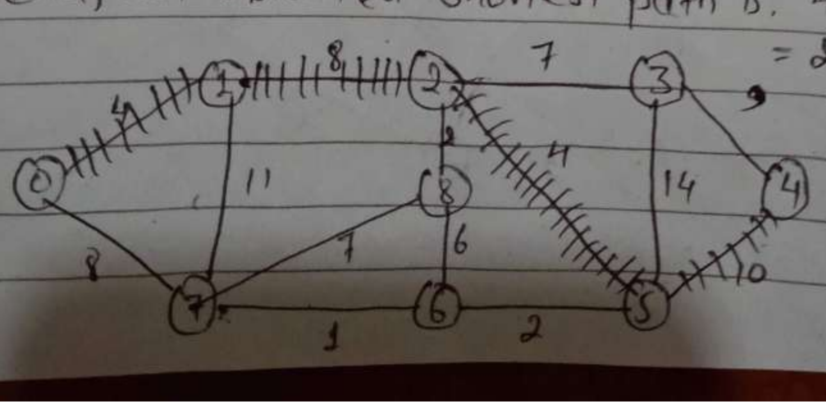
Step 3 :

There are three adjecent vertexs (8, 5, 3) for vertex 2. If we take shortest path of weight 2 then it takes 2+6+2 = 10 distance to reach vertex 5, but if we take vertex 5 itself then the distance will be 4. So we take path from 2 to 5.

Step 4 :

There are two adjacent vertexs (3, 4) to reach vertex 4 (the final node). Here, if we take vertex 3 then it will take 14+9 = 25 whereas if we select the final node itself (Vertex 4) the distance will be short. So, we take path from 5 to 4.

Hence, our obtained shortest path is. 4 + 8 + 4 + 10 = 26.



C. Ans.

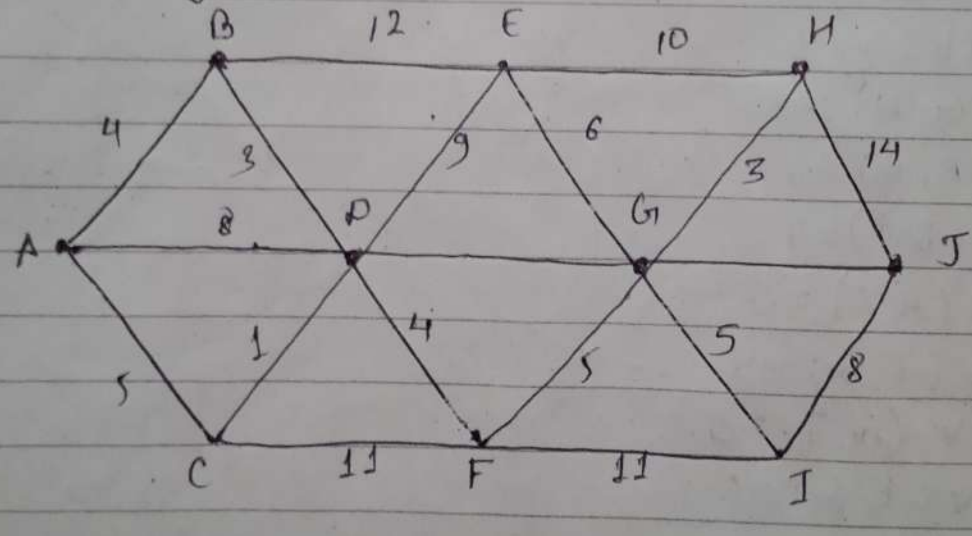
Spanning tree of a graph is a sub-graph of 'G' that contains all the vertices of 'G' and doesn't contain a cycle.

Whereas, the minimum spanning tree is the tree which is constructed with minimum cost.

There are two algorithm for constructing minimum spanning tree and they are:

i. Prism's Algorithm.

ii. Kruskal's algorithm.



Let us solve by using kruskal's algorithm:

Kruskal's algorithm:

- In this algorithm we list all the pair of vertices of given graph in ascending order of their weight.

- Then we choose the vertex pair with least weight and add it to the tree being formed, after that the vertex pair with next minimum weight from the list is selected and added to the tree, and soon.

- During the process of adding vertex pair if any vertex pair with minmum weight forms a cycle, we discard that vertex pair.

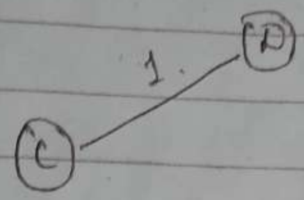
- This process is continued until the list becomes empty and Hence the tree obtained is minimum spanning tree.

Solution,

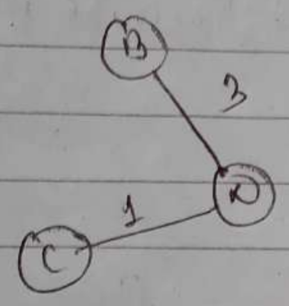
Arranging the vertex pair according to their weight in ascending order.

V (C, D) = 1
V (B, D) = 3
V (G, H) = 3
V (A, B) = 4
V (D, F) = 4
V (A, C) = 5
V (F, G) = 5
V (G, I) = 5.
V (E, G) = 6
V (A, D) = 8
V (I, J) = 8
V (D, F) = 9.
V (E, H) = 10 V (D, G) = 10
V (C, F) = 11
V (F, I) = 11
V (B, E) = 12
V (H, J) = 14.
V (G, J) = 15.

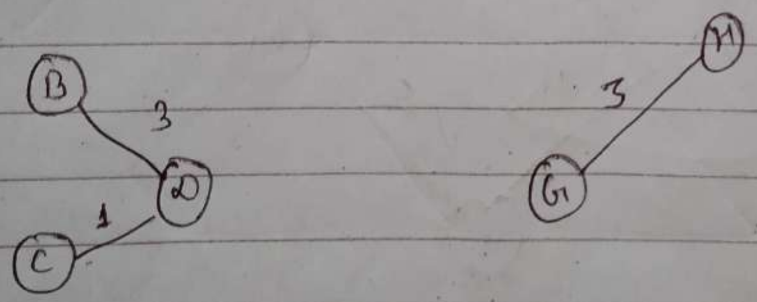
Here, V(C,D) = 1 is the least weight vertex pair so we add it to our tree being constructed.



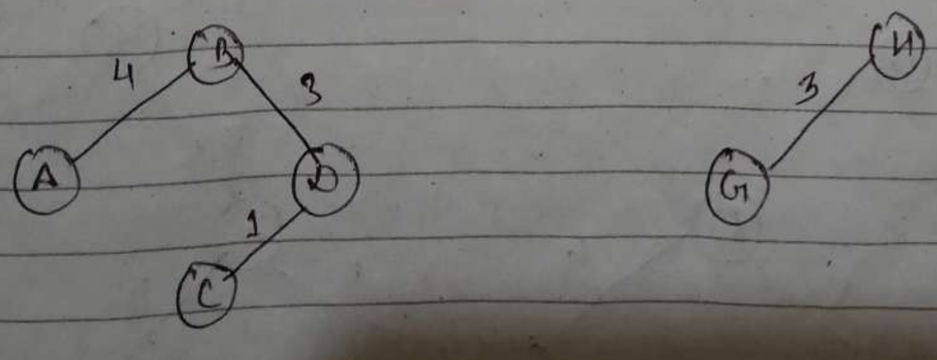
Add (B,D) = 3 on our tree.



(G,H) is the next minimum weight so we add it to the tree.



(A, B) is the next candidate to be added.



(D, F) is the next one.

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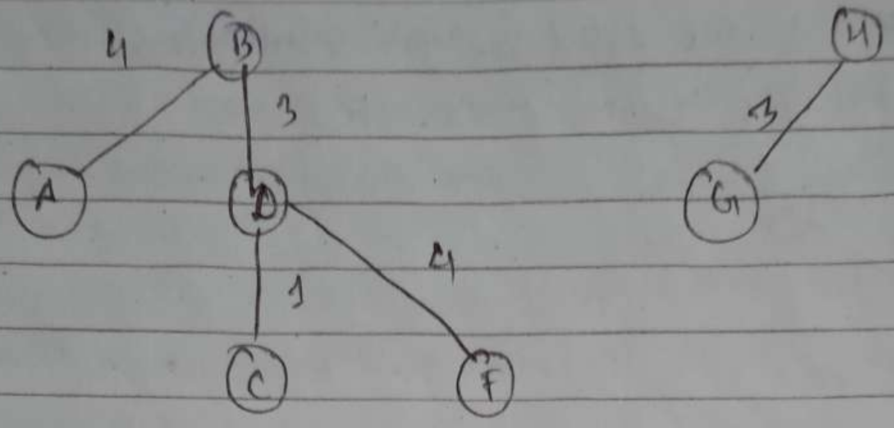
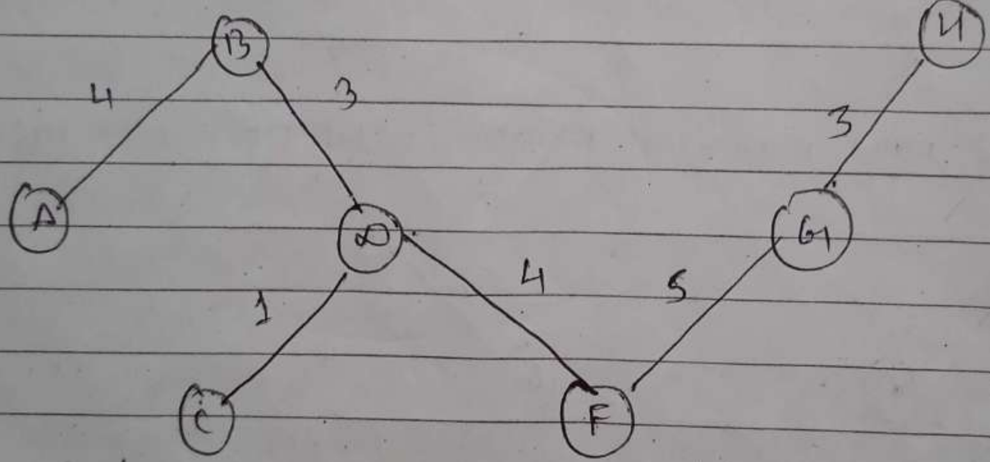
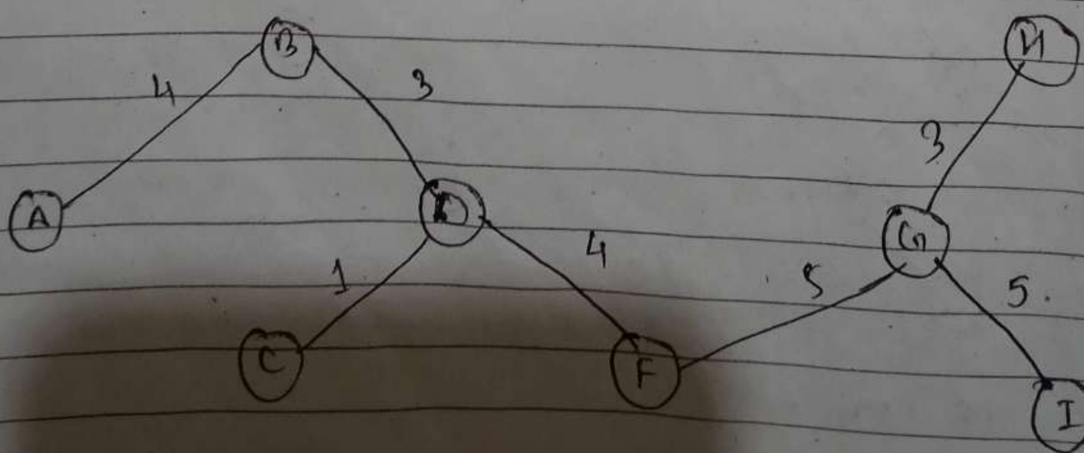


The vertex pair (A, C) is discarded because it forms cycle.

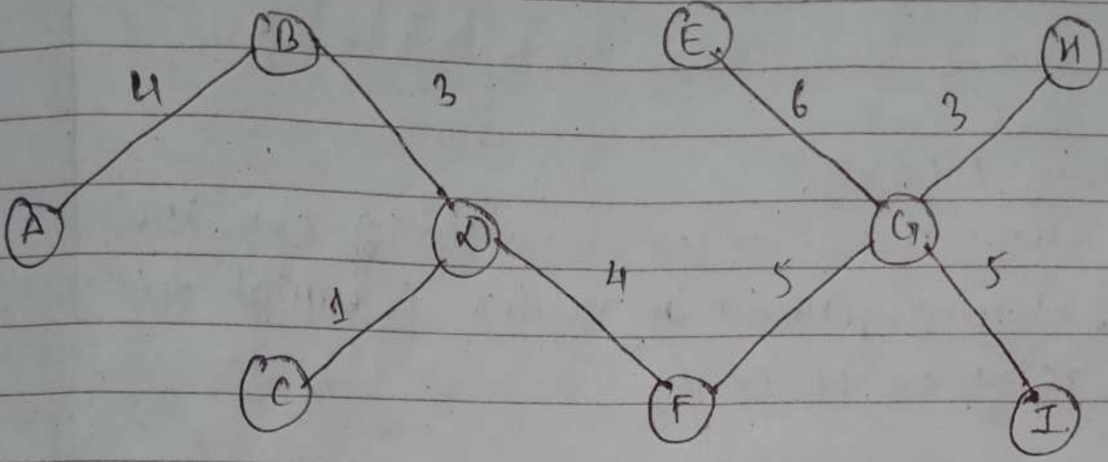
Add (F, G).



Add vertex pair (G, I) on the tree

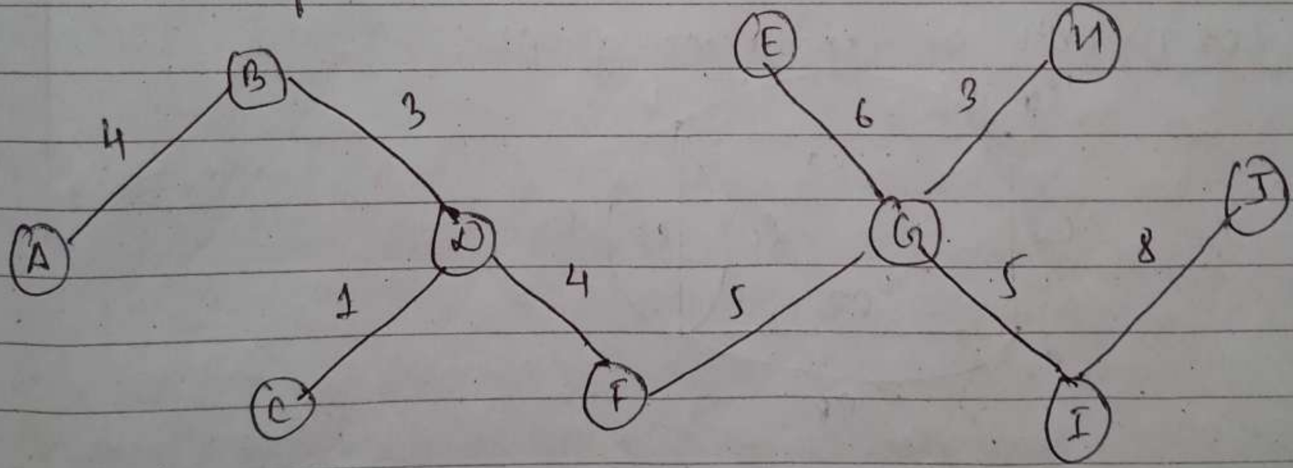


Add vertex pair (E, G) on the tree.



Discard vertex pair (A, D) because it forms cycle.

Add vertex pair (I, J) on the tree.



Total Cost is $4+3+1+4+5+6+3+5+8$
$= 39$.

Q. N. 3.

A. Ans.

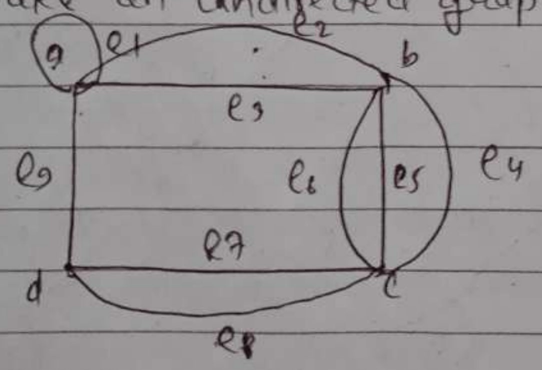
Handshaking theorem:

It states that, "In an undirected graph, the total no. of degree formed at vertex is equal to twice the total no. of edges".

i.e. Total no. of degree = 2.e..

Verification:

Let us take an undirected graph:



Here, No. of edges = {$e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9$}

(e) = 9.

And,

Degree of vertex a = deg(a) = 5.

lly, deg(b) = 5, deg(c) = 5, deg(d) = 3.

∴ Total no. of degree = deg(a) + deg(b) + deg(c) + deg(d)

= 5+5+5+3

= 18.

Hence, Handshaking theorem = 2e. is proved.

B. Ans.

i. To show: $(p \rightarrow q) \equiv p \vee \sim q$.

p	q	$p \rightarrow q$	$\sim q$	$p \vee \sim q$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(Question is wrong)

ii. To show: $\sim p(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$.

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$p \vee (\sim p \wedge q)$	$\sim(p \vee (\sim p \wedge q))$	$\sim p \wedge \sim q$
T	T	F	F	F	T	F	F
T	F	F	T	F	T	F	F
F	T	T	F	T	T	F	F
F	F	T	T	F	F	T	T

from above table,

$$\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q.$$

C. Ans.

Given proposition: "The home team wins whenever it is raining".

Let p = the home team wins.
q = It is raining.

So,
Converse $= q \rightarrow p$.
= If it is raining then home team wins.

Inverse = $\sim p \to \sim q$.
= The hometeam doesn't win whenever it is not raining.

Contrapositive = $\sim q \to \sim p$.
= It is not raining whenever hometeam is not wining.

Q.N.4.

A. Ans.

Universal quantification:

Let $p(x)$ be the proportional function with domain of discourses D, the statement
for every/all x, $P(x)$ of D.
ie. $\forall x, P(x)$ is called universal quantified statement. And the symbol $\forall$ is called universal quantifier.

Example:

Let $P(x)$ be the statement "$x+1>x$" for domain of all real numbers.
then, $\forall P(x)$ is true, which is universal quantifier.

Existencial Quantification:

let $p(x)$ be the proportional function with domain of Discourses D, the statement

· "If there exists some element in x belonging to $P(x)$" then such existance is called existancial quantified statement. And the symbol $\exists$ is called existancial quantifier.

example:

Let $P(x)$ be the statement: "$x > 3$" where domain of discourses is all real number. Here universal quantifier is false as there exists values greater than 3 which make statemont false. Hence, existancial quantifier is true i.e. $\exists x\, P(x)$ is true.

Quantifier :

statemont.	when true	when false.
$\forall x, P(x)$ universal.	$P(x)$ is true for every x.	there is an x for which $P(x)$ is false.
$\exists x, P(x)$ existancial	There is an x for which $P(x)$ is true.	There is an x for which $P(x)$ is false.

B. Ans.

Here, $\overline{\forall x P(x)}$ implies the negation of universal quantifier. The negation of universal quantifier is existancial quantifier. ($\overline{\forall} = \exists$)

· The negation doesn't apply to x. as it is value of domain discourses ($\overline{x} = x$).

$\therefore \overline{\forall x\, P(x)} = \exists x\, \overline{P(x)}$

C. Ans.

Identifying the individual sentences.

a. Dilendra work in bank.
b. He is bank manager.
c. He use tally.
d. Dilendra uses Guncase.
e. Dilendra is Bank Manager.

Writing the given statement into propositional logic statements.

Hypothesis

i. $\sim a \rightarrow \sim b$
ii. $\sim c$
iii. $a \rightarrow d$
iv. $e \vee c$

Conclusion : d

Q.N.5.

A. Ans.
Direct proof:
If $p \rightarrow q$ be an implication, in direct proof we assume that hypothesis is true i.e. 'p' is true

then by using different theorem and already proven facts we conclude that conclusion is also true. (i.e. q is true).

Indirect proof:

Sometimes, in certain cases the direct proof is not appropriate for verifying the statement, such as in case of "there exists infinite prime numbers." So, In such type of cases, we verify the statement by using different technique where we sometime assume or sometime solve from opposite hand. So, this types of proof are indirect proofs.

Second part:

let us suppose that difference of rational number and irrational number is rational.

As we know that rational numbers are those numbers which is in the form of $\frac{p}{q}$; where p and q are positive integers and $q \neq 0$.
whereas, irrational numbers are not in form of p/q.
let it be a.

So, A/C to question,

$$\frac{p}{q} - a = \frac{p'}{q'}$$ (Another rational number as per our assumption)

or, $\frac{p - qa}{q}$

$= \frac{\cancel{q}(p/q - a)}{\cancel{q}} = \frac{p}{q} - a$ is not rational number. our supposition was wrong. the difference gives rational number.

B. Ans.

Given statement : $8^n - 3^n$ is divisible by 5 for $n \geq 1$.

For $n=1$, $8^1 - 3^1 = 5$ is divisible by 5, which is true.

let us suppose the given statement is true for $n=k$. i.e. $8^k - 3^k$ is divisible by 5. —(i)

Now, Let us prove for $n=k+1$.
i.e. $8^{k+1} - 3^{k+1}$ is divisible by 5.

Here,

$$
\begin{aligned}
8^{k+1} - 3^{k+1} &= 8^k \cdot 8^1 - 3^k \cdot 3^1 \\
&= 8^k \cdot 8 - 3^k \cdot (8-5) \\
&= (8^k) \cdot 8 - 3^k \cdot 8 + 3^k \cdot 5 \\
&= 8(8^k - 3^k) + 5 \cdot 3^k.
\end{aligned}
$$

Since, $8^k - 3^k$ is divisible by 5. from statement (i). Also, $5 \cdot 3^k$ is divisible by 5.

Hence. the given statement $8^n - 3^n$ is divisible by 5 for $n \geq 1$.

C. Ans.

Q.N.6.

A. Ans.

Given,

$$a_n = 5a_{n-1} - 6a_{n-2} + 7^n \text{ ; where } a_0 = 1 \text{ and } a_1 = 2. \quad \text{(i)}$$

It is a non-homogenous recurrance relation.

The associated part is,

$$a_n = 5a_{n-1} - 6a_{n-2}.$$

The characteristics eqn is,

$$r^2 = 5r - 6$$

or, $r^2 - 5r + 6 = 0$

or, $r^2 - 3r - 2r + 6 = 0$

or, $r(r-3) - 2(r-3) = 0$

$\therefore r = 3, 2.$

The general solution for distinct roots is,

$$a_n = k_1 2^n + k_2 3^n \quad \ldots\ldots \text{(ii)}$$

The non-homogenous part is,

$$a_n = c.7^n \quad \ldots \text{(iii)}$$

So, particular integral is given by, [from (i) and (iii)

$$c.7^n = 5c.7^{n-1} - 6c.7^{n-2} + 7^n$$

or, $c.7^2 = 5c\,7 - 6c + 7^2$

or, $49c = 35c - 6c + 49$ $\quad \therefore a_n(P) = \left(\frac{49}{20}\right)7^n$

$\therefore$ Solution is, $a_n = k_1 2^n + k_2 3^n + \frac{49}{20}.7^n.$

B. Ans.

Finite state machine :

A finite state machine (FCM) is defined mathematically by 5 tuples.

i.e. $M = (Q, I, O, F, G)$.

where,

M = FCM.
Q = Finite set of state.
I = Finite set of Inputs.
O = Finite set of output.
F = transition function
G = Output relation.

Eg: Fan as a FSM.

Q = { on, off }.
I = { pressButton }
O = { Fan on, Fan off }.

F consits of,

(on, pressButton) → Fan off
(off, pressButton) → Fan on.

G consits of,

(on, pressButton) → (off, fan off)
(off, pressButton) → (on, fan off)

The representation of FSA can be done using transition diagram and transition table.

Transition Diagram Construction:

- It is represented using the weighted directed graph where states are represented by vertices.
- Transition from one state to another is represented directed graph.
- Value given to each edge is its input
- Starting state are represented by single circle by pointing an arrow head and final state is represented by double circle.

C. Ans.

DFA	NDFA.
• In DFA, for each input symbol, one can determine the state to which the machine will move.	• In NDFA, for particular input symbol the machine to move in different state.
• Similar to FSA. $M = (Q, \Sigma, \delta, q_0, F)$.	• $M = (Q, \Sigma, \delta, q_0, F)$. Transition function as $Q \times \Sigma \rightarrow 2^Q$.

Q.N. 7.

B. Ans.

Language: The collection of all possible string over some given alphabet. It is denoted by L.
E.g. $L = \{0, 1, 11, 001 \ldots\}$

Types of Grammer:

1. Unrestricted Grammer: If no restriction is applied to the production rule of the grammer then it is called unrestricted grammer.

2. Context sensitive grammer: A grammer is said to be context sensitive grammer if its production rule is of the form

$$w_1 \alpha w_2 \rightarrow w_1 \beta w_2$$

3. Context free grammer: A grammer is said to be context free grammer if its production rule is of the form.

$$\alpha \rightarrow \beta.$$

4. Regular grammer: A grammer is said to be regular grammer if its production rule is of the form.

non-terminal → exactly one terminal.