1. Find the second order partial derivative of

$$u = e^{x^2+xy+y^2}$$

2. If $u = log\left(x^3 + y^3 + z^3 - 3xyz\right)$**, then show that**

(i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

(ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$

3. If $u = f(xyz)$**show that** $x^2u_{xx} = y^2u_{yy} = z^2u_{zz}$

4. If $x = r\cos\theta, y = r\sin\theta$ **Prove that**

$$\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right]$$

5. If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ **prove that** $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

6. If $u = f(r)$**,then prove that**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r).$$

7. If $u = x^2 + y^2 + z^2$ **then show that** $xu_x + yu_y + zu_z = 2u$

8. If $u = log\sqrt{x^2 + y^2 + z^2}$**, then show that**

$$(x^2 + y^2 + z^2)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1.$$

9. If $u = e^{xyz}$,then prove that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = \left(1 + 3xyz + x^2y^2z^2\right)e^{xyz}$$

10. If $z = \phi(x + ay) + \Psi(x - ay)$,then show that

$$a^2 \frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}.$$

11. State and prove Euler's theorem on homogeneous function of two independent variables

12. Verify the Euler's theorem of the function

i. $u = \frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}}$

ii. $u = x^n \sin\left(\frac{y}{x}\right)$

iii. $u = \frac{x^2y^2}{x^3+y^3}$

iv. $u = \frac{x^3y}{x^2+y^2}$

v. $u = x^n \tan^{-1}\left(\frac{y}{x}\right)$

vi. $u = xf\left(\frac{y}{x}\right)$

vii. $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$

13. If $u = \sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$

14. If $u = \tan^{-1}\left\{\frac{x^3+y^3}{x-y}\right\}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

15. . If $u = log\frac{x^2+y^2}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

16. . if $= logx$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

17. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$$

18.

If $u = \text{cosec}^{-1}\left(\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}\right)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{6}\tan u$$

19.

If $\sin u = \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

20.

Given that $u = \sin\left(\frac{x}{y}\right)$,$x = e^t, y = t^2$, find $\frac{du}{dt}$

Ans. $\frac{(t-2)}{t^3}e^t\cos\left(\frac{e^t}{t^2}\right)$

21. Examine and find the maximum and minimum value of

$$8 - 4x + 4y - x^2 - y^2$$

Ans Mxx value=16

22. Examine and find the maximum and minimum values of

$$\boldsymbol{20 - x^2 - y^2 - z^2}$$

Ans Max. value=20

23. Obtain the maximum value of xyz such that $x + y + z = 24$.

Ans Max.value=512

24. Find the extreme value of $x^2 + y^2 + z^2$ connected by the relation
$x + z = 1$ and $2y + z = 2$

Ans Min.value=1

25. Find the minimum value of
$\boldsymbol{x^2 + xy + y^2 + 3z^2}$ **under the condition**
$\boldsymbol{x + 2y + 4z = 60.}$

Ans Min.value= $\frac{2700}{7}$

26. Find the minimum value of $\boldsymbol{x^2 + y^2 + z^2}$ **connected by the relation** $\boldsymbol{ax + by + cz = p.}$

Ans Min.value= $\frac{p^2}{(a^2+b^2+c^2)^2}$

27. Find the maximum value of xyz under the condition

$x + y + z = 8.$

Ans Max. value$= \frac{512}{27}$

28. Obtain the minimum value of $x^2 + y^2 + z^2$ subject to the condition

$$x + y + z - 1 = 0 \; and \; xyz + 1 = 0.$$

Ans Min. value$= 3$

Hint : Using $y^2 + z^2 = (y + z)^2 - 2yz$

29. Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 1.$

Ans Min . value$= \frac{1}{3}$

30. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1.$

Ans Min.value$= 27$