

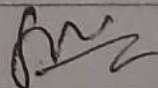
College Roll.No: 191725.

Level: Bachelors

Programme: Software

Semester: 2nd

Subject: Engineering Mathematics II.

Signature of the examinee/ student:  Date: 14/03/2078

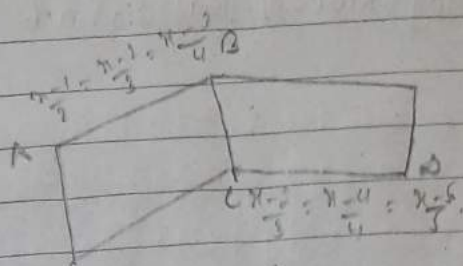
Q.N.1.

A. Ans.

The given eqⁿ of lines are:

$$\frac{x-1}{2} = \frac{x-2}{3} = \frac{x-3}{4} \dots (i)$$

$$\frac{x-2}{3} = \frac{x-4}{4} = \frac{x-5}{5} \dots (ii)$$

Let l, m, n be direction cosine of BC .Since, $BC \perp AB$ then, $2l + 3m + 4n = 0 \dots (iii)$ Also, $BC \perp CD$ then, $3l + 4m + 5n = 0 \dots (iv)$

Solving (iii) and (iv) by cross method.

$$\frac{l}{15-16} = \frac{m}{-(10-12)} = \frac{n}{8-9}$$

$$\text{or, } \frac{l}{-1} = \frac{m}{2} = \frac{n}{-1}$$

$$\text{or, } \frac{l}{1} = \frac{m}{-2} = \frac{n}{1} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{(1)^2+(-2)^2+(1)^2}}$$

Taking 1st and last ratio.

$$\frac{l}{1} = \frac{\sqrt{1}}{\sqrt{6}}$$

$$\therefore l = \frac{1}{\sqrt{6}}$$

and,

$$\frac{m}{-2} = \frac{1}{\sqrt{6}}$$

$$\therefore m = \frac{-2}{\sqrt{6}}$$

Also,

$$\frac{n}{1} = \frac{1}{\sqrt{6}}$$

$$\therefore n = \frac{1}{\sqrt{6}}$$

Now,

$$\text{Magnitude of short distance (SD)} = \left| \frac{1(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)}{\sqrt{1^2 + m^2 + n^2}} \right|$$

$$= \left| \frac{1(2-1) + \left(\frac{-2}{\sqrt{6}}\right) \cdot (4-2) + \frac{1}{\sqrt{6}}(5-3)}{\sqrt{6}} \right|$$

$$= \left| \frac{1}{\sqrt{6}} - \frac{2 \times 2}{\sqrt{6}} + \frac{2}{\sqrt{6}} \right|$$

$$= \left| \frac{3}{\sqrt{6}} - \frac{4}{\sqrt{6}} \right|$$

$$= \left| \frac{-1}{\sqrt{6}} \right|$$

$$\therefore \text{S.D.} = \frac{1}{\sqrt{6}}$$

And, the eqⁿ of shortest distance is,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l & m & n \end{vmatrix} = 0 = \begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_2 & m_2 & n_2 \\ l & m & n \end{vmatrix}$$

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{vmatrix} = 0 = \begin{vmatrix} x-2 & y-4 & z-5 \\ 3 & 4 & 5 \\ \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{vmatrix}$$

B. Ans.

The equation of sphere through the given circle, $x^2 + y^2 + z^2 = 1$ and $2x + 4y + 5z = 6$ be,

$$(x^2 + y^2 + z^2 - 1) + k(2x + 4y + 5z - 6) = 0.$$

$$\text{or, } x^2 + y^2 + z^2 + 2kx + 4ky + 5kz - (1 + 6k) = 0 \dots (i)$$

Comparing eqⁿ (i) with general equation of sphere, i.e. w^t

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ we get,

$$u = k, \quad v = 2k \quad \text{and} \quad w = \frac{5k}{2}, \quad d = -(1 + 6k).$$

\therefore Center of eqⁿ (i) is, $(-u, -v, -w) = (-k, -2k, -5k/2)$.
and,

Radius of (i) is given by:

$$r = \sqrt{u^2 + v^2 + w^2 - d}.$$

$$= \sqrt{k^2 + 4k^2 + \frac{25k^2}{4} + (1 + 6k)}$$

$$= \sqrt{\frac{4k^2 + 16k^2 + 25k^2 + 4 + 24k}{4}}$$

$$= \frac{1}{2} \sqrt{45k^2 + 24k + 4}$$

As the sphere (i) touches the plane $z = 0$. So, the length of radius of sphere is equal to z-coordinate value.

$$\text{i.e. } \frac{1}{2} \sqrt{45k^2 + 24k + 4} = \frac{5k}{2}.$$

$$\text{or, } 45k^2 + 24k + 4 = 25k^2.$$

$$\text{or, } 20k^2 + 24k + 4 = 0$$

$$\text{or, } 5k^2 + 6k + 1 = 0$$

$$a_1, (5k+1)(k+1)=0$$

$$\therefore k = -1, -1/5.$$

Hence, the eqⁿ (1) becomes;

$$x^2 + y^2 + z^2 - 2x - 4y - 5z + 5 = 0 \quad \text{and,}$$

$$5(x^2 + y^2 + z^2) - 2x - 4y - 5z + 1 = 0.$$

which are required eqⁿ of sphere.

$$Q \cdot N \cdot 2.$$

B. Ans.

Given function:

$$f(x, y) = x^3 + y^3 - 3axy \dots (i)$$

Now, Diff. eqⁿ (i) partially we get,

$$f_x = 3x^2 - 3ay.$$

$$f_y = 3y^2 - 3ax$$

$$f_{xx} = 6x.$$

$$f_{yy} = 6y.$$

$$f_{xy} = -3a.$$

For stationary point,

$$f_x = 0$$

$$f_y = 0$$

i.e. $3x^2 - 3ay = 0$

i.e. $3y^2 - 3ax = 0$

or, $x^2 - ay = 0 \dots (ii)$

or, $y^2 - ax = 0 \dots (iii)$

Solving (ii) and (iii) we get,

$$x = 0, a \quad \text{and} \quad y = 0, a.$$

Also, $f_{xx} \cdot f_{yy} - (f_{xy})^2$

$$\Rightarrow 0 \times 0 - (-3a)^2 = 0$$

or, $9a^2 = 0$

$$\therefore a = 0$$

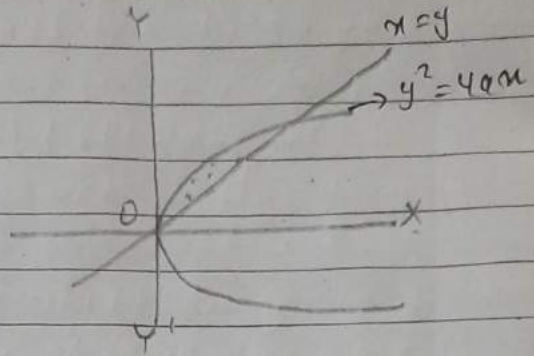
At point (0,0) or at $x=0$ and $y=0$.

The function gives no information.

Q.N.3.

A. Ans.

$$\int_0^{4a} \int_{\frac{y^2}{4a}}^y \frac{x^2 - y^2}{x^2 + y^2} \cdot dx \cdot dy.$$



$$\text{let } I = \int_0^{4a} \int_{\frac{y^2}{4a}}^y \frac{x^2 - y^2}{x^2 + y^2} \cdot dx \cdot dy.$$

Here, the integrand is first integrated w.r.t. x along the horizontal strip which varies from $x = \frac{y^2}{4a}$ to $x = y$. and w.r.t. y in vertical strip.

To change it into polar coordinates,

let $x = r \cos \theta$ and $y = r \sin \theta$.

$\therefore x^2 + y^2 = r^2$ and $dy \cdot dx = r \cdot dr \cdot d\theta$ in given integral.

We get, $\theta = \frac{\pi}{4}$ to $\theta = \frac{\pi}{2}$ and $r = 0$ to $r = 4a \cot \theta \cdot \operatorname{cosec} \theta$.

Now, our integral becomes:

$$I = \int_{\pi/4}^{\pi/2} \int_0^{4a \cot \theta \cdot \operatorname{cosec} \theta} \frac{r^2 \cos^2 \theta - r^2 \sin^2 \theta}{r^2 \cos^2 \theta + r^2 \sin^2 \theta} \cdot r \cdot dr \cdot d\theta$$

$$\text{in } \because 4ax = y^2$$

$$\text{or } 4a r \cos \theta = r^2 \sin^2 \theta$$

$$\text{or } 4 \sin^2 \theta - 4a r \cos \theta = 0$$

$$\text{or } r [r \sin^2 \theta - 4a \cos \theta] = 0$$

$$\therefore r = 0$$

$$+ r \sin^2 \theta = 4a \cos \theta$$

$$\therefore r = 4a \cot \theta \cdot \operatorname{cosec} \theta$$

$$= \int_{\pi/4}^{\pi/2} \int_0^{4a \cot \theta \cdot \operatorname{cosec} \theta} \frac{r^3 (\cos^2 \theta - \sin^2 \theta)}{r^2} \cdot dr \cdot d\theta$$

$$= \int_{\pi/4}^{\pi/2} \int_0^{4a \cot \theta \cdot \operatorname{cosec} \theta} r \cdot (\cos^2 \theta - \sin^2 \theta) \cdot dr \cdot d\theta$$

$$= \int_{\pi/4}^{\pi/2} \cos 2\theta \left[\frac{r^4}{4} \right]_0^{4a \cot \theta \cdot \operatorname{cosec} \theta} d\theta$$

$$= \int_{\pi/4}^{\pi/2} \frac{\cos 2\theta}{2} [16a^2 \cot^2 \theta \cdot \operatorname{cosec}^2 \theta] d\theta$$

$$= 8a^2 \int_{\pi/4}^{\pi/2} (\cos^2 \theta - \sin^2 \theta) (\cot^2 \theta \cdot \operatorname{cosec}^2 \theta) d\theta$$

$$= 8a^2 \int_{\pi/4}^{\pi/2} (\cos^2 \theta - \sin^2 \theta) \cdot \left(\frac{\cos^2 \theta}{\sin^4 \theta} \right) d\theta$$

$$= 8a^2 \int_{\pi/4}^{\pi/2} \left(\frac{\cos^4 \theta}{\sin^4 \theta} - \frac{\cos^2 \theta \cdot \sin^2 \theta}{\sin^4 \theta} \right) d\theta$$

$$= 8a^2 \int_{\pi/4}^{\pi/2} (\cot^4 \theta - \cot^2 \theta) d\theta$$

$$= 8a^2 \int_{\pi/4}^{\pi/2} [\cot^2 \theta (\operatorname{cosec}^2 \theta - 1) - \cot^2 \theta] d\theta$$

$$= 8a^2 \int_{\pi/4}^{\pi/2} [\cot^2 \theta \cdot \operatorname{cosec}^2 \theta - 2\cot^2 \theta] d\theta$$

$$= 8a^2 \left\{ \left[-\frac{\cot^3 \theta}{3} \right]_{\pi/4}^{\pi/2} - \int_{\pi/4}^{\pi/2} 2\cot^2 \theta d\theta \right\}$$

$$= 8a^2 \left\{ -\frac{1}{3} [\cot^3 \theta]_{\pi/4}^{\pi/2} - 2 \int_{\pi/4}^{\pi/2} (\operatorname{cosec}^2 \theta - 1) d\theta \right\}$$

$$= 8a^2 \left\{ -\frac{1}{3} [0-1] - 2 \left[-(\cot \theta - 0) \right]^{\pi/12} \right\}$$

$$= 8a^2 \left\{ \frac{1}{3} + 2 \left[\cot \theta + 0 \right]^{\pi/12} \right\}$$

$$= 8a^2 \cdot \left\{ \frac{1}{3} + 2 \left[\frac{\pi}{2} - 1 - \frac{\pi}{4} \right] \right\}$$

$$= 8a^2 \left\{ \frac{1}{3} + 2 \times \left(\frac{\pi-4}{4} \right) \right\}$$

$$= 8a^2 \cdot \left\{ \frac{1}{3} + \frac{\pi-4}{2} \right\}$$

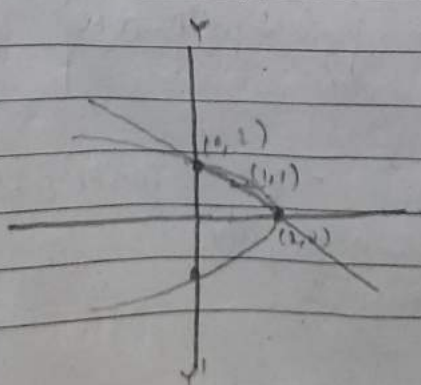
$$= \frac{8a^2}{6} \left(\frac{2+\pi-4}{1} \right)$$

B. Ans.

Given eqⁿ of curves:

$$y = 2-x \text{ and } y^2 = 2(2-x).$$

Here, $y = 2-x$ is the eqⁿ of straight line whereas, $y^2 = 4-2x$ is eqⁿ of parabola, having vertex at $(2, 0)$.



$$\text{for } y = 2-x$$

x	0	2	1
y	2	0	1

$$\text{for } y^2 = 4-2x$$

x	2	0
y	0	± 2

For point of intersection.

$$(2-x)^2 = 4-2x$$

$$\text{or, } 4 - 4x + x^2 = 4 - 2x$$

$$\text{or, } x^2 = 2x$$

$$\text{or, } x^2 - 2x = 0$$

$$\text{or, } x(x-2) = 0$$

$$\therefore x = 0 \text{ or, } x = 2.$$

$$\text{At, } x = 0, y = 2$$

$$\text{At } x = 2, y = 0.$$

Taking horizontal strip.

$$\text{So, the required Area} = \int_0^2 \int_{2-y}^{4-\frac{y^2}{2}} dx \cdot dy.$$

$$= \int_0^2 \left[\frac{4-y^2}{2} - (2-y) \right] \cdot dy.$$

$$= \int_0^2 \left[\frac{4-y^2}{2} - 2 + y \right] \cdot dy.$$

$$= \int_0^2 \left[\cancel{2} - \frac{y^2}{2} - \cancel{2} + y \right] \cdot dy$$

$$= \int_0^2 \left[y - \frac{y^2}{2} \right] \cdot dy$$

$$= \left[\frac{y^2}{2} - \frac{y^3}{6} \right]_0^2$$

$$= \left[\frac{(2)^2}{2} - \frac{(2)^3}{6} \right] - 0$$

$$= \frac{4}{2} - \frac{8}{6}$$

$$= 2 - \frac{4}{3}$$

$$= \frac{2}{3} \text{ sq. units.}$$

Hence, the area bounded by curve is $\frac{2}{3}$ sq. units.

Q.N.4.

A. ans.

Linear differentiated equation of first order is the differential equation which only involves the differential that has order power raised to one. It is usually in form of $\frac{dy}{dx} + Py = Q$.

Example: $\frac{dy}{dx} = \sin x$.

where, P & Q are function of x or constant.

second part:

$$x \log x \cdot \frac{dy}{dx} + y = 2 \log x.$$

Dividing both sides by $x \cdot \log x$ we get,

$$\frac{dy}{dx} + \frac{1}{x \cdot \log x} \cdot y = \frac{2 \log x}{x \cdot \log x}$$

$$\text{or, } \frac{dy}{dx} + \frac{1}{x \cdot \log x} \cdot y = \frac{2}{x} \quad \dots (i)$$

Here, eqⁿ (i) is in the form of, linear differential eqⁿ.

$$\therefore P = \frac{1}{x \cdot \log x} \quad \text{and, } Q = \frac{2}{x}$$

$$\therefore \text{I.F.} = e^{\int P \cdot dx}$$

$$= e^{\int \frac{1}{x \log x} \cdot dx}$$

$$= e^{\int \frac{dz}{z}}$$

$$= e^{\log z}$$

$$= e^{\log(\log x)}$$

$$= \log x$$

$$\begin{aligned} \text{Let } \log x &= z \\ \therefore \frac{1}{x} \cdot \frac{dx}{dz} &= 1 \\ \therefore \frac{dx}{x} &= dz \end{aligned}$$

Multiplying eqⁿ (i) by I.F. we get,

$$y \times e^{\int P \cdot dx} = \int Q \cdot e^{\int P \cdot dx} \cdot dx + c$$

$$\text{or, } y \times \log x = \int \frac{2}{x} \times \log x \cdot dx + c$$

$$\text{or, } y \log x = 2 \cdot \int \frac{\log x}{x} \cdot dx + c$$

$$\text{or, } y \log x = 2 \cdot \frac{(\log x)^2}{2} + c$$

$$\text{or, } y \log x = 2 \log x + c$$

$$\text{or, } y^{\log x} = 2 + c' \quad \text{where, } c' = \frac{c}{\log x}$$

$\therefore y^{\log x} = 2 + c'$ is required solution.

Q.N. 5.

Ans.

Here, $y'' - 4y = e^{-2x} - 2x \dots (i)$

Taking Laplace transform on both sides,

$$\mathcal{L}y'' - 4\mathcal{L}y = \mathcal{L}e^{-2x} - \mathcal{L}2x$$

Let the solution of eqⁿ (i) be,

$$y = y_h + y_p$$

where y_h is the solution of homogeneous part

The homogeneous eqⁿ of (i) is,

$$y'' - 4y = 0 \dots (ii)$$

and, the auxiliary eqⁿ of (ii) is,

$$m^2 - 4 = 0 \Rightarrow m = \pm 2$$

Here, m has real and distinct value, so solution of (ii) is,

$$y_h(x) = C_1 e^{-2x} + C_2 e^{2x} \dots (iii)$$

Comparing (i) with $y'' + Py' + Qy = R$ we get,

$$R = e^{-2x} - 2x$$

Clearly R has repeated value of e^{-2x} . so let's choose particular solⁿ of (i).

$$y'_p = -2C_3 x e^{-2x} + C_3 e^{-2x} + C_4$$

$$y''_p = 4C_3 x e^{-2x} - 2C_3 e^{-2x} - 2C_3 e^{-2x}$$

B. Ans.

Given differential eqⁿ is,

$$y'' = 9y$$

$$\text{or, } y'' - 9y = 0 \dots (i)$$

Assume the general solution of (i) as,

$$y = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + \dots (ii)$$

Diff. (ii) w.r.t. x we get,

$$y' = C_1 + 2C_2x + 3C_3x^2 + 4C_4x^3 + 5C_5x^4 + \dots$$

Again, diff. w.r.t. x we get,

$$y'' = 2C_2 + 6C_3x + 12C_4x^2 + 20C_5x^3 + \dots$$

Now,

Putting value of y'' and y in given eqⁿ

$$(2C_2 + 6C_3x + 12C_4x^2 + 20C_5x^3 + \dots) = 9(C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + \dots) = 0$$

$$\text{or, } (2C_2 - 9C_0) + (6C_3 - 9C_1)x + (12C_4 - 9C_2)x^2 + (20C_5 - 9C_3)x^3 + \dots = 0$$

Equating to zero the coefficient of various powers of x we get,

$$2C_2 - 9C_0 = 0 \Rightarrow C_2 = \frac{9}{2}C_0$$

$$6C_3 - 9C_1 = 0 \Rightarrow C_3 = \frac{9C_1}{6} = \frac{3C_1}{2}$$

$$12C_4 - 9C_2 = 0 \Rightarrow C_4 = \frac{9C_2}{12} = \frac{3C_2}{4} = \frac{3}{4} \times \frac{9}{2}C_0 = \frac{27}{8}C_0$$

$$20C_5 - 9C_3 = 0 \Rightarrow C_5 = \frac{9C_3}{20} = \frac{9}{20} \times \frac{3}{2}C_1 = \frac{27}{40}C_1$$

Putting value in eqⁿ (ii) we get,

$$y = C_1 + C_1 x + \frac{9}{2} C_0 x^3 + \frac{3}{2} C_1 x^3 + \frac{27}{8} C_0 x^4 + \frac{27}{40} C_1 x^5 + \dots$$

$$\therefore y = C_1 \left(1 + x + \frac{3}{2} x^3 + \frac{25}{40} x^5 + \dots \right) + C_0 \left(\frac{9}{2} x^3 + \frac{27}{8} x^4 + \dots \right)$$

Q.N.6.

A. Ans.

First shifting theorem:

$$\text{If } \mathcal{L}(f(t)) = F(s) \text{ then, } \mathcal{L}(e^{at} f(t)) = F(s-a).$$

Proof:

$$\begin{aligned} \text{we have: } \mathcal{L}(f(t)) &= \int_0^{\infty} e^{-st} f(t) \cdot dt \\ &= F(s) \dots (i) \end{aligned}$$

$$\begin{aligned} \text{Now, } \mathcal{L}(e^{at} \cdot f(t)) &= \int_0^{\infty} e^{-st} e^{at} \cdot f(t) \cdot dt \\ &= \int_0^{\infty} e^{-(s-a)t} \cdot f(t) \cdot dt \\ &= F(s-a) \text{ for } (s-a) > 0. \end{aligned}$$

$$\therefore \mathcal{L}(e^{at} \cdot f(t)) = F(s-a) \text{ which proves first shifting theorem.}$$

Second shifting theorem:

$$\text{If } \mathcal{L}(f(t)) = F(s) \text{ then, } \mathcal{L}[u(t-a) \cdot f(t-a)] = e^{-as} \cdot F(s)$$

Proof:

By definition of Laplace transform, $\mathcal{L}(f(t)) = \int_0^{\infty} e^{-st} \cdot f(t) \cdot dt$
 $= F(s).$

Now,

$$\begin{aligned} \mathcal{L}[u(t-a) \cdot f(t-a)] &= \int_0^{\infty} e^{-st} \cdot u(t-a) \cdot f(t-a) \cdot dt \\ &= \int_0^a e^{-st} \cdot 0 \cdot f(t-a) \cdot dt + \int_a^{\infty} e^{-st} \cdot 1 \cdot f(t-a) \cdot dt \\ &= \int_a^{\infty} e^{-st} \cdot f(t-a) \cdot dt. \end{aligned}$$

$$\text{Put } (t-a) = z.$$

$$\therefore 1-0 = \frac{dz}{dt},$$

$$\therefore dz = dt.$$

$$\text{When } t=0 \text{ then } z=0$$

$$\text{When } t=\infty \text{ then } z=\infty.$$

$$= \int_0^{\infty} e^{-s(z+a)} \cdot f(z) \cdot dz.$$

$$= \int_0^{\infty} e^{-sz-a} \cdot f(z) \cdot dz.$$

$$= e^{-a} \int_0^{\infty} e^{-sz} \cdot f(z) \cdot dz.$$

$$= e^{-a} \cdot F(s).$$

which proves the second shifting theorem.

B. Ans.

Here, $y'' + 4y' + 4y = e^{-t}$.

Taking Laplace transform on both sides,

$$\mathcal{L}(y'' + 4y' + 4y) = \mathcal{L}(e^{-t})$$

$$\text{or, } \mathcal{L}y'' + 4\mathcal{L}y' + 4\mathcal{L}y = \frac{1}{s+1}$$

$$\text{or, } [s^2\mathcal{L}y - s \cdot y(0) - y'(0)] + 4[s \cdot \mathcal{L}(y) - y(0)] + 4\mathcal{L}y = \frac{1}{s+1}$$

$$\text{or, } [s^2\mathcal{L}(y) - s \times 0 - 0] + 4[s \cdot \mathcal{L}(y) - 0] + 4 \cdot \mathcal{L}(y) = \frac{1}{s+1}$$

$$\text{or, } s^2\mathcal{L}(y) + 4s\mathcal{L}(y) + 4 \cdot \mathcal{L}(y) = \frac{1}{s+1}$$

$$\text{or, } \mathcal{L}(y) [s^2 + 4s + 4] = \frac{1}{s+1}$$

$$\text{or, } \mathcal{L}(y) = \frac{1}{(s+1)(s^2+4s+4)}$$

$$= \frac{1}{(s+1)(s+2)^2}$$

$$\text{Let } \mathcal{L}(y) = \frac{1}{(s+1)(s+2)^2} = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2} \dots \dots (i)$$

$$\therefore 1 = A(s+2)^2 + (s+1)(s+2)B + (s+1) \cdot C$$

Put $s = -2$, $1 = -C \therefore C = -1$.

Put $s = -1$, $1 = A \therefore A = 1$

On Equating coefficients we get,

$$1 = 4A + 2B + C$$

$$\text{or, } 1 = 4 + 2B - 1$$

$$\text{or, } 2 = 4 + 2B$$

$$\text{or, } -2 = 2B$$

$$\therefore B = -1.$$

Now, substituting value of A, B and C in eqⁿ (i) we get.

$$L(y) = \frac{1}{s+1} - \frac{1}{s+2} - \frac{1}{(s+2)^2}$$

$$\text{or, } y = L^{-1}\left(\frac{1}{s+1}\right) - L^{-1}\left(\frac{1}{s+2}\right) - L^{-1}\left(\frac{1}{(s+2)^2}\right)$$

$$= e^{-t} - e^{-2t} - t \cdot e^{-2t}$$

$$\therefore y = e^{-t} - e^{-2t}(1+t)$$

Q.N.7.

A. Ans.

$$\text{Given: } (1+x)y \cdot dx + (1+y)x \cdot dy = 0.$$

7. B. Ans.

To find $\mathcal{L}(t \cdot e^t)$.

$$\text{since, } \mathcal{L}(t \cdot f(t)) = (-1) \cdot \frac{d}{ds} \cdot F(s)$$

$$= (-1) \cdot \frac{d}{ds} \cdot \mathcal{L}(f(t))$$

$$= (-1) \cdot \frac{d}{ds} \cdot \mathcal{L}(e^t)$$

$$= (-1) \cdot \frac{d}{ds} \cdot \left(\frac{1}{s-a} \right)$$

$$= (-1) \cdot \log(s-a)$$

$$\therefore \mathcal{L}(t \cdot e^t) = -\log(s-a)$$

7. C. Ans.

$$\text{we have: } f = ax^2 + 2hxy + by^2 \dots (i) \quad \text{--- u.}$$

According to Euler theorem,

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = nu, \text{ where } n \text{ is degree of homogeneous eq.}$$

then,

Diff. (i) partially, w.r.t. x

$$\frac{\partial u}{\partial x} = 2ax + 2hy$$

lly, w.r.t. y .

$$\frac{\partial u}{\partial y} = 2hx + 2by$$

Now,

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y}$$

$$= x(2ax + 2hy) + y(2hx + 2by)$$

$$= 2ax^2 + 2hxy + 2hxy + 2by^2$$

$$= 2(ax^2 + 2hxy + by^2)$$

$$= 2u \quad \text{A.}$$

Hence, Euler's formula is verified.

7. D. Ans.

Since the required plane is parallel to $3x - 4y + 5z = 0$,

so our required eqⁿ is,

$$3x - 4y + 5z + K = 0 \dots (i)$$

Also, eqⁿ (i) passes through $(1, 1, 1)$ so,

$$3 \times 1 - 4 \times 1 + 5 \times 1 + K = 0$$

$$\text{or, } 4 + K = 0$$

$$\therefore K = -4$$

Hence, the eqⁿ of required plane is, $3x - 4y + 5z - 4 = 0$.