Unit-II Number System and Codes(6hrs)

Topics covered:

- Introduction to Different types of number system
- Binary to decimal and decimal to binary conversions, Octal, hexadecimal number system and conversions,
- Binary arithmetic, 1's complement and 9's complements,
- gray code, Excess 3 Code.
- instruction codes,
- alphanumeric characters,
- Modulo 2 system and 2's complement,
- Binary coded decimal (BCD) and hexadecimal codes,
- Parity method for error detection

Number System

✓ A number system defines a set of values used to represent quantity.

- Positional number system.
- Non Positional number system

Different Number Systems

- ✓ Decimal Number System :Base/Radix 10
- ✓ Binary Number System

:Base/Radix 2

✓ Octal Number System

:Base/Radix 8

✓ Hexadecimal Number System

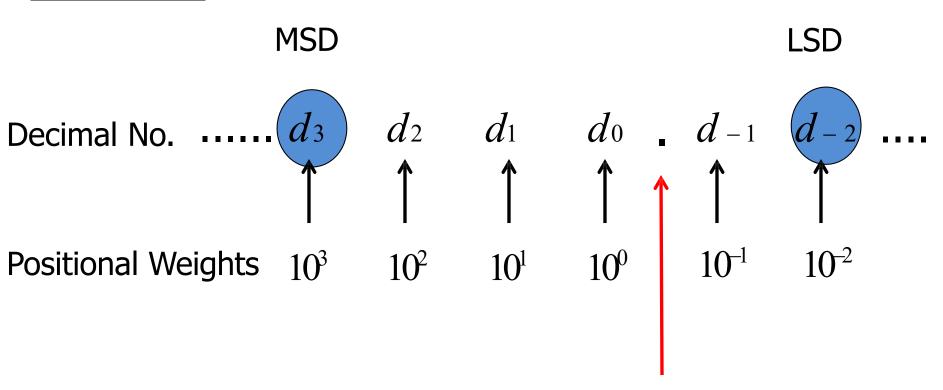
:Base/Radix 16

- ✓ Decimal number system contains ten unique symbols 0,1,2,3,4,5,6,7,8 and 9
- ✓ Since counting in decimal involves ten symbols, we can say that its base or radix is ten.

✓ It is a positional weighted system

- ✓ In this system, any number (integer, fraction or mixed) of any magnitude can be represented by the use of these ten symbols only.
 - ✓ Each symbols in the number is called a "Digit"

Structure:



Decimal Point

✓ MSD: The leftmost digit in any number representation, which has the greatest positional weight out of all the digits present in that number is called the "Most Significant Digit" (MSD)

✓ LSD: The rightmost digit in any number representation, which has the least positional weight out of all the digits present in that number is called the "Least Significant Digit" (LSD)

> Examples

1214

1897

9875.54

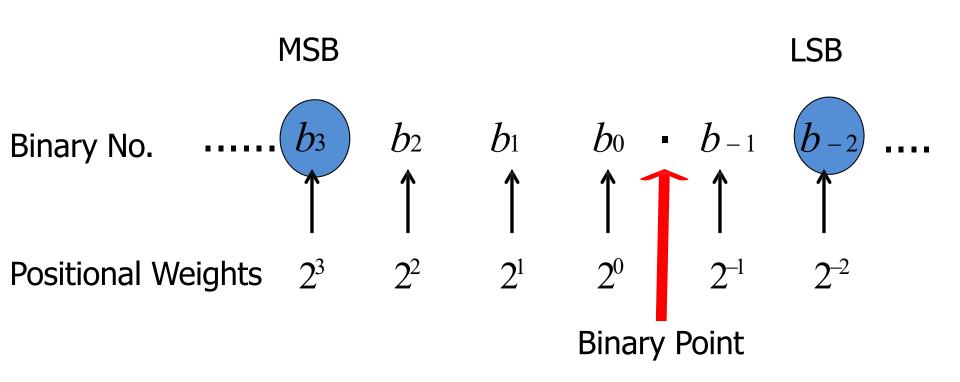
✓ Binary number system is a positional weighted system

- ✓ It contains two unique symbols 0 and 1
- ✓ Since counting in binary involves two symbols, we can say that its base or radix is two.

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- ✓ A binary digit is called a "Bit"
- ✓ A binary number consists of a sequence of bits, each of which is either a 0 or a 1.
- ✓ The binary point separates the integer and fraction parts

Structure:



✓ MSB: The leftmost bit in a given binary number with the highest positional weight is called the "Most Significant Bit" (MSB)

✓ LSB: The rightmost bit in a given binary number with the lowest positional weight is called the "Least Significant Bit" (LSB)

Decimal No.	Binary No.
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111

Decimal No.	Binary No.
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

✓ **BIT:** The binary digits (0 and 1) are called bits.

- Single unit in binary digit is called "Bit"

- Example: 1

✓ NIBBLE: A nibble is a combination of 4 binary bits.

Example: 1110

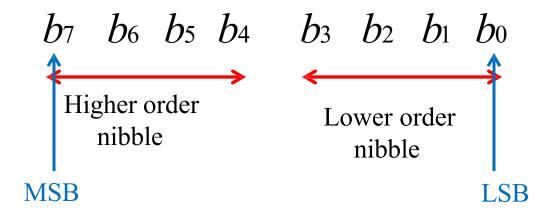
0000

1001

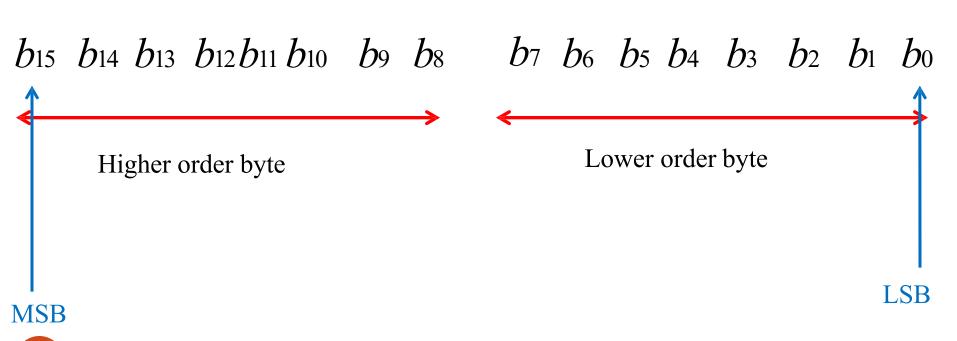
0101

✓ **BYTE:** A byte is a combination of 8 binary bits.

✓ The number of distinct values represented by a byte is 256 ranging from 0000 0000 to 1111 1111.



✓ WORD: A word is a combination of 16 binary bits. Hence it consists of two bytes.



✓ **DOUBLE WORD:** A double word is exactly what its name implies, two words.

-It is a combination of 32 binary bits.

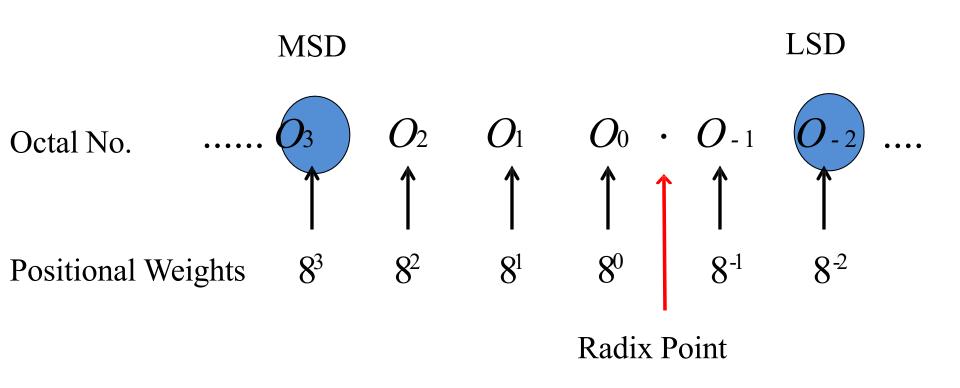
- ✓ Octal number system is a positional weighted system.
- ✓ It contains eight unique symbols 0,1,2,3,4,5,6 and 7
- ✓ Since counting in octal involves eight symbols, we can say that its base or radix is eight (8).

✓ The largest value of a digit in the octal system will be 7.

✓ That means the octal number higher than 7 will not be 8, instead of that it will be 10.

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Structure:



✓ Since its base $8 = 2^3$, every 3 bit group of binary can be represented by an octal digit.

✓ An octal number is thus 1/3rd the length of the corresponding binary number

Decimal No.	Binary No.	Octal No.
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7

✓ Binary numbers are long. These numbers are fine for machines but are too lengthy to be handled by human beings. So there is a need to represent the binary numbers concisely.

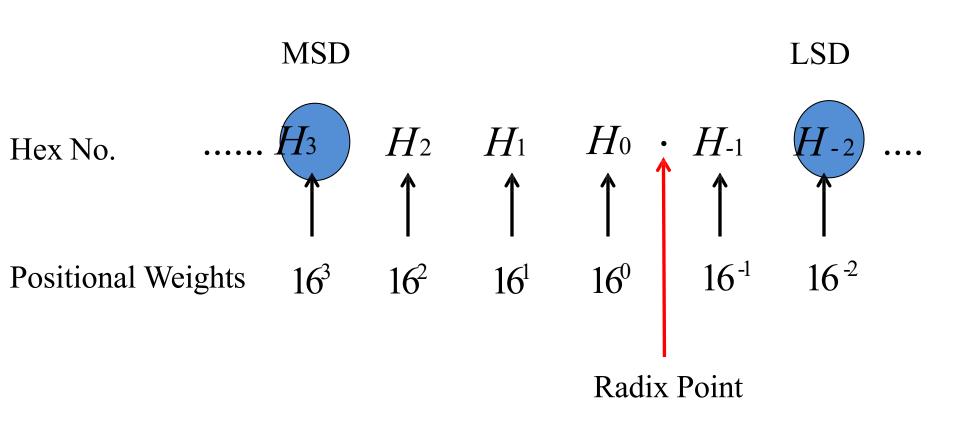
✓ One number system developed with this objective is the hexadecimal number system (or Hex)

✓ Hex number system is a positional weighted system

✓ It contains sixteen unique symbols 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E and F.

✓ Since counting in hex involves sixteen symbols, we can say that its base or radix is sixteen.

Structure:



- ✓ Since its base $16 = 2^4$, every 4 bit group of binary can be represented by an hex digit.
- ✓ An hex number is thus 1/4th the length of the corresponding binary number
- ✓ The hex system is particularly useful for human communications with computer

Decimal No.	Binary No.	Hex No.
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7

Decimal No.	Binary No.	Hex No.
8	1000	8
9	1001	9
10	1010	А
11	1011	В
12	1100	С
13	1101	D
14	1110	E
15	1111	F

Conversion of Decimal number into Binary number (Integer Number)

Procedure:

- 1. Divide the decimal no by the base 2, noting the remainder.
- 2. Continue to divide the quotient by 2 until there is nothing left, keeping the track of the remainders from each step.
- 3. List the remainder values in reverse order to find the number's binary equivalent

Example: Convert 105 decimal number in to it's equivalent binary number.

2	105		
2	52	1	LSB
2	26	0	
2	13	0	
2	6	1	
2	3	0	
2	1	1	
	0	1	MSB

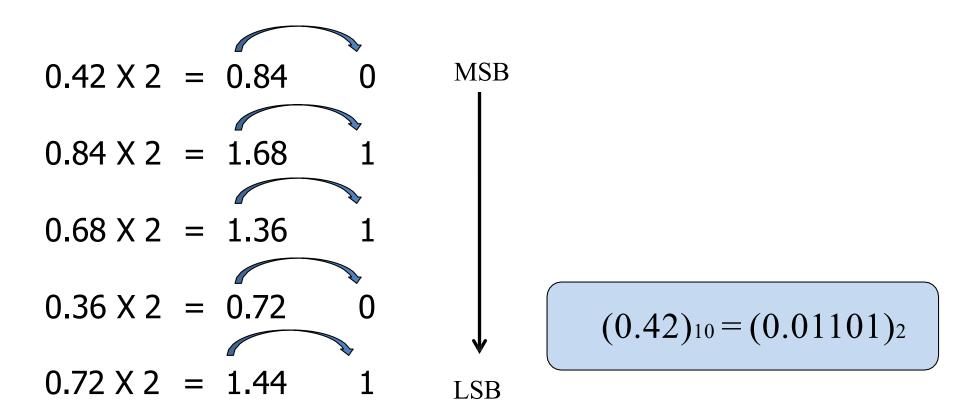
 $(105)_{10} = (1101001)_2$

Conversion of Decimal number into Binary number (Fractional Number)

Procedure:

- 1. Multiply the given fractional number by base 2.
- 2. Record the carry generated in this multiplication as MSB.
- 3. Multiply only the fractional number of the product in step 2 by 2 and record the carry as the next bit to MSB.
- 4. Repeat the steps 2 and 3 up to 5 bits. The last carry will represent the LSB of equivalent binary number

Example: Convert 0.42 decimal number in to it's equivalent binary number.



Exercise:

• Convert following Decimal Numbers in to its equivalent Binary Number:

$$(8476.47)_{10} = (?)_2$$

Exercise:

• Convert following Decimal Numbers into its equivalent Binary Number:

$$(8476.47)_{10} = (?)_2$$

Conversion of Decimal Number into Octal Number (Integer Number)

Procedure:

- 1. Divide the decimal no by the base 8, noting the remainder.
- 2. Continue to divide the quotient by 8 until there is nothing left, keeping the track of the remainders from each step.
- 3. List the remainder values in reverse order to find the number's octal equivalent

Example: Convert 204 decimal number in to it's equivalent octal number.

	8	204	Remainder	Lab
	8	25	4	LSD ↑
•	8	3	1	
•		0	3	MSD

$$(204)_{10} = (314)_8$$

Conversion of Decimal Numberinto Octal Number (Fractional Number)

Procedure:

- 1. Multiply the given fractional number by base 8.
- 2. Record the carry generated in this multiplication as MSD.
- 3. Multiply only the fractional number of the product in step 2 by 8 and record the carry as the next bit to MSD.
- 4. Repeat the steps 2 and 3 up to 5 bits. The last carry will represent the LSD of equivalent octal number

Example: Convert 0.6234 decimal number in to it's equivalent Octal number.

$$0.6234 \times 8 = 4.9872$$
 4 MSD
 $0.9872 \times 8 = 7.8976$ 7
 $0.8976 \times 8 = 7.1808$ 7
 $0.1808 \times 8 = 1.4464$ 1
 $0.4464 \times 8 = 3.5712$ 3 LSD (0.6234)10=(0.47713)8

Exercise:

• Convert following Decimal Numbers in to its equivalent Octal Number:

$$(420.6)_{10} = (?)_8$$

Conversion of Decimal Number into Hexadecimal Number (Integer Number)

Procedure:

- 1. Divide the decimal no by the base 16, noting the remainder.
- 2. Continue to divide the quotient by 16 until there is nothing left, keeping the track of the remainders from each step.
- 3. List the remainder values in reverse order to find the number's hex equivalent

Example: Convert 2003 decimal number in to it's equivalent Hex number.

16	2003 <u>R</u>			
16	125	3	3	LSD ↑
16	7	13	D	
	0	7	7	MSD

$$(2003)_{10} = (7D3)_{16}$$

Conversion of Decimal Number into Hexadecimal Number (Fractional Number)

Procedure:

- 1. Multiply the given fractional number by base 16.
- 2. Record the carry generated in this multiplication as MSD.
- 3. Multiply only the fractional number of the product in step 2 by 16 and record the carry as the next bit to MSD.
- 4. Repeat the steps 2 and 3 up to 5 bits. The last carry will represent the LSD of equivalent hex number

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Example: Convert 0.122 decimal number in to it's equivalent Hex number.

 $(0.122)_{10} = 0.1F3B6)_{16}$

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Exercise:

• Convert following Decimal Numbers in to its equivalent Hex Number:

$$(420.6)_{10} = (?)_{16}$$

Conversion of Binary Number into Decimal Number

Procedure:

- 1. Write down the binary number.
- 2. Write down the weights for different positions.
- 3. Multiply each bit in the binary number with the corresponding weight to obtain product numbers to get the decimal numbers.
- 4. Add all the product numbers to get the decimal equivalent

Example: Convert $(1001.01)_2$ binary number in to it's equivalent decimal number.

$$= (1 \times 2^{3}) + (0 \times 2^{2}) + (0 \times 2^{1}) + (1 \times 2^{0}) \cdot (1 \times 2^{-1}) + (1 \times 2^{-2})$$

$$= 8 + 0 + 0 + 1 \cdot 0.5 + 0.25$$

$$= 9.75$$

$$(1001.01)_2 = (9.75)_{10}$$

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Conversion of Octal Number into Decimal Number

Procedure:

- 1. Write down the octal number.
- 2. Write down the weights for different positions.
- 3. Multiply each bit in the binary number with the corresponding weight to obtain product numbers to get the decimal numbers.
- 4. Add all the product numbers to get the decimal equivalent

Example: Convert 365.24 octal number in to it's equivalent decimal number.

Octal No.

Positional Weights

$$=(3\times8^2)+(6\times8^1)+(5\times8^0).(2\times8^{-1})+(4\times8^{-2})$$

$$= 192 + 48 + 5 \cdot 0.25 + 0.0625$$

$$(365.24)_8 = (245.3125)_{10}$$

Exercise

 Convert following Octal Numbers into its equivalent Decimal Number:

$$(273.56)_8 = (?)_{10}$$

Example: Convert 273.56 octal number in to it's equivalent decimal number.

Octal No.

Positional Weights

$$=(2\times8^2)+(7\times8^1)+(3\times8^0).(5\times8^{-1})+(6\times8^{-2})$$

$$= 128 + 56 + 3 \cdot 0.625 + 0.09375$$

Conversion of Hexadecimal Number into Decimal Number

Procedure:

- 1. Write down the hex number.
- 2. Write down the weights for different positions.
- 3. Multiply each bit in the binary number with the corresponding weight to obtain product numbers to get the decimal numbers.
- 4. Add all the product numbers to get the decimal equivalent

Example: Convert 5826 hex number in to it's equivalent decimal number.

Hex No.

Positional Weights

$$=(5\times16^3)+(8\times16^2)+(2\times16^1)+(6\times16^0)$$

$$=20480 + 2048 + 32 + 6$$

$$= 22566$$

$$(5826)_{16} = (22566)_{10}$$

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Exercise

• Convert following Hexadecimal Numbers in to its equivalent Decimal Number:

$$(6B7)_{16} = (?)_{10}$$

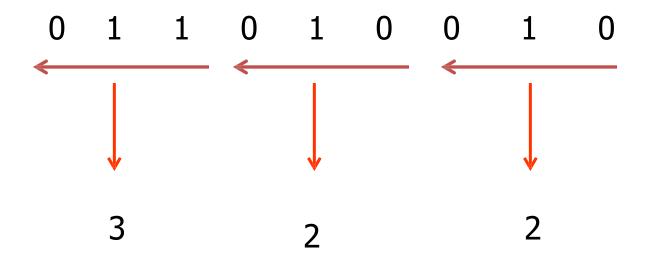
Conversion of Binary Number into Octal Number

Procedure:

- 1. Group the binary bits into groups of 3 starting from LSB.
- 2. Convert each group into its equivalent decimal.

 As the number of bits in each group is restricted to 3, the decimal number will be same as octal number

Example: Convert 11010010 binary number in to it's equivalent octal number.



$$(11010010)_2 = (322)_8$$

Exercise:

 Convert following Binary number into its equivalent Octal Number:

$$(1101.11)_2 = (?)_8$$

Ans:

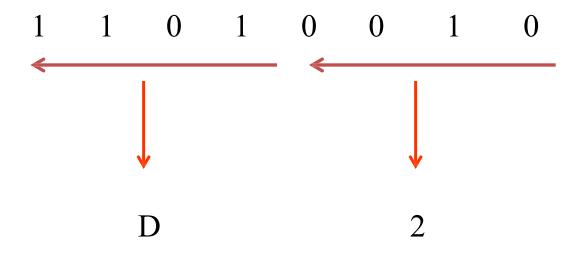
Conversion of Binary Number to Hexadecimal Number

Procedure:

- 1. Group the binary bits into groups of 4 starting from LSB.
- 2. Convert each group into its equivalent decimal.

As the number of bits in each group is restricted to 4, the decimal number will be same as hex number

Example: Convert 11010010 binary number in to it's equivalent hex number.



$$(11010010)_2 = (D2)_{16}$$

Exercise

• Convert following Binary Numbers into its equivalent Hexadecimal Number:

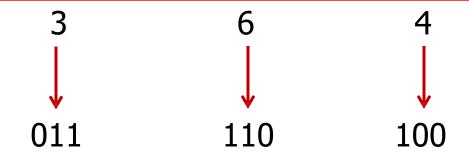
$$(10001.01)_2 = (?)_{16}$$

Ans:

Conversion of Octal Number into Binary Number

✓ To get the binary equivalent of the given octal number we have to convert each octal digit into its equivalent 3 bit binary number

Example: Convert 364 octal number in to it's equivalent binary number.



$$(364)_8 = (0111110100)_2$$

OR

$$(364)_8 = (111110100)_2$$

Exercise

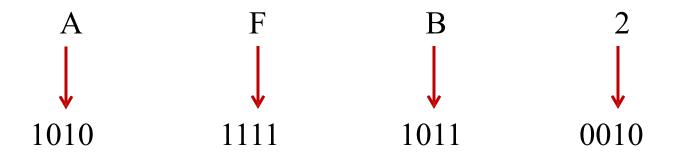
 Convert following Octal Numbers in to its equivalent Binary Number:

$$(6534.04)_8 = (?)_2$$

Conversion of Hexadecimal Number into Binary Number

✓ To get the binary equivalent of the given hex number we have to convert each hex digit into its equivalent 4 bit binary number

Example: Convert AFB2 hex number in to it's equivalent binary number.



 $(AFB2)_{16} = (10101111110110010)_2$

Exercise

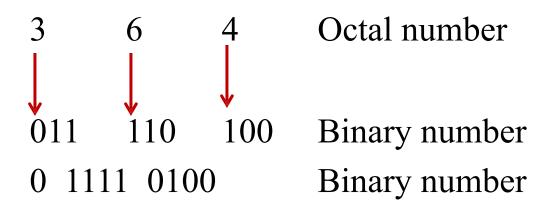
• Convert following Hexadecimal Numbers in to its equivalent Binary Number:

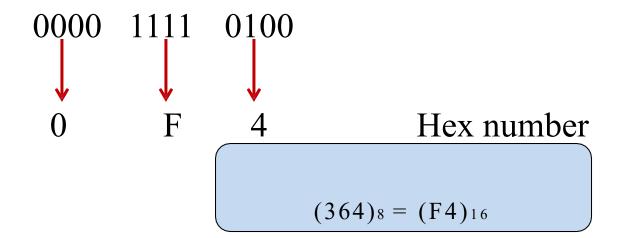
$$(8E47.AB)_{16} = (?)_2$$

Conversion of Octal Number into Hexadecimal Number

✓ To get hex equivalent number of given octal number, first we have to convert octal number into its 3 bit binary equivalent and then convert binary number into its hex equivalent.

Example: Convert 364 octal number in to it's equivalent hex number.





Exercise

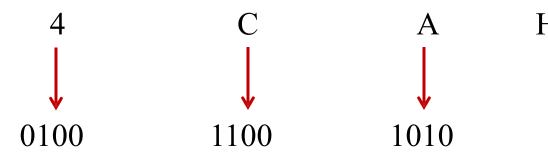
• Convert following Octal Numbers in to its equivalent Hex Number:

$$(6534.04)_8 = (?)_{16}$$

Conversion of Hexadecimal Number into Octal Number

✓ To get octal equivalent number of given hex number, first we have to convert hex number into its 4 bit binary equivalent and then convert binary number into its octal equivalent.

Example: Convert 4CA hex number in to it's equivalent octal number.



Hex Number

Binary Number

Binary Number

Octal Number

$$(4CA)_{16} = (2312)_8$$

Binary Addition

Example: Perform $(1101.101)_2 + (111.011)_2$

$$(1101.101)_2 + (111.011)_2 = (10101.000)_2$$

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Exercise

• Perform Binary Addition of following:

$$(1011)_2 + (1101)_2 + (1001)_2 + (1111)_2$$

Binary Subtraction

➤ Following are the four most basic cases for binary subtraction

			Su	btraction	Borrow
0	-	0	=	0	0
0	-	1	=	1	1
1	-	0	=	1	0
1	-	1	=	0	0

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Binary Subtraction

Example: Perform (1010.010)₂-(111.111)₂

	1	10	1	1	10		
0	10	0	10	10	0	10	
1	0	1	0.	0	1	0	
-	1	1	1.	1	1	1	
0	0	1	0.	0	1	1	

$$(1010.010)_2 - (111.111)_2 = (0010.011)_2$$

Exercise

• Perform Binary Subtraction of following:

$$(10001.01)_2$$
- $(1111.11)_2$

Binary Multiplication

Following are the four most basic cases for binary multiplication

```
0 \quad X \quad 0 \quad = \quad 0
```

$$0 X 1 = 0$$

$$1 X 0 = 0$$

$$1 X 1 = 1$$

Binary Multiplication

Example: Perform (1001)₂*(1000)₂

Binary Multiplication

```
Example: Perform (1001)_2*(1000)_2
                             X X
                             X
 (1001)_2 * (1000)_2 = (1001000)_2
```

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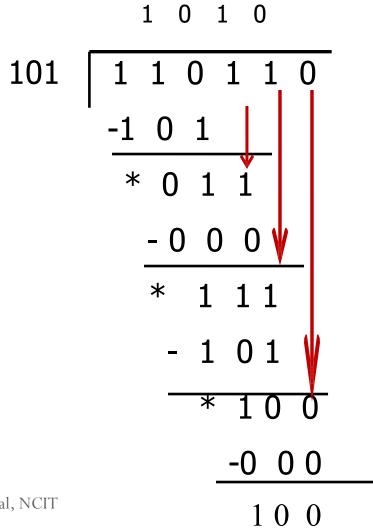
Exercise

• Perform Binary Multiplication of following:

 $(1101.11)_2 X (101.1)_2$

Binary Division

Example: Perform $(110110)_2/(101)_2$



Remainder: 100 Quotient: 1010

Exercise

• Perform Binary Division of following:

$$(1010)_2$$
 by $(11)_2$

- The 1's complement of a number is obtained by simply complementing each bit of the number that is by changing all 0's to 1's and all 1's to 0's.
- This system is called as 1's complement because the number can be subtracted from 1 to obtain result

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Example: Obtain 1's complement of the 1010

1's complement of the 1010 is 0101

Sr. No.	Binary Number	1's Complement
1	1101 0101	0010 1010
2	1001	0110
3	1011 1111	0100 0000
4	1101 1010 0001	0010 0101 1110
5	1110 0111 0101	0001 1000 1010
6	1011 0100 1001	0100 1011 0110
7	1100 0011 0010	0011 1100 1101
8	0001 0010 1000	1110 1101 0111

Subtraction Using 1's Complement

- ➤ In 1's complement subtraction, add the 1's complement of subtrahend to the minuend.
- ➤ If there is carry out, bring the carry around and add it to LSB.
- ➤ If carry is present, the answer is positive and it is true binary form
- ➤ If there is no carry, the answer is negative and it is in 1's complement.

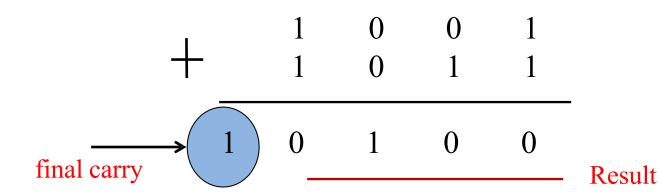
Subtraction using 1's Complement

$$(9)_{10}$$
 - $(4)_{10}$

$$(4)_{10} = (0100)_2$$

=1011

Step 2: Add 9 with 1' complement of 4



Step 3:If carry is generated add final carry to the result

Subtraction using 1's Complement

Example: Perform using 1' complement

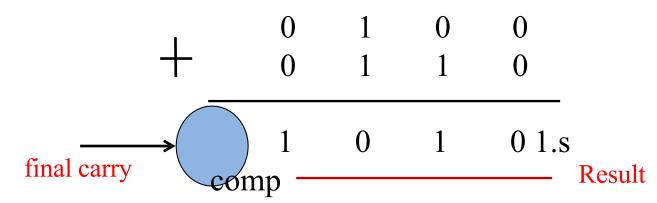
$$(4)_{10}$$
 - $(9)_{10}$

Step 1: Take 1' complement of

$$(9)_{10} = (1001)_2$$

=0110

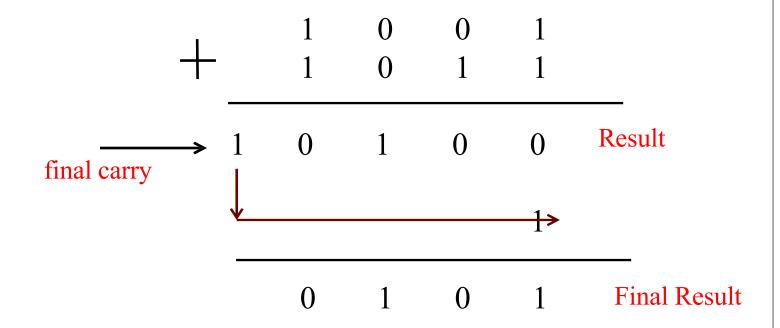
Step 2: Add 4 with 1' complement of 9



Step 3:If carry is generated add final carry to the result

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Example Continue



When the final carry is produced the answer is positive and is in its true binary form

✓ The 2's complement of a number is obtained by adding 1 to the 1's complement of that number

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Sr. No.	Binary Number	1's Complement	2's Complement
1	1101 0101	0010 1010	0011 1011
2	1001	0110	0111
3	1011 1111	0100 0000	0100 0001
4	1101 1010 0001	0010 0101 1110	0010 0101 1111
5	1110 0111 0101	0001 1000 1010	0001 1000 1011

Subtraction Using 2's Complement

- ✓ In 2's complement subtraction, add the 2's complement of subtrahend to the minuend.
- ✓ If carry is generated then the result is positive and in its true binary form.
- ✓ If the carry is not produced, then the result is negative and in its 2's complement form.

*Carry is always to be discarded

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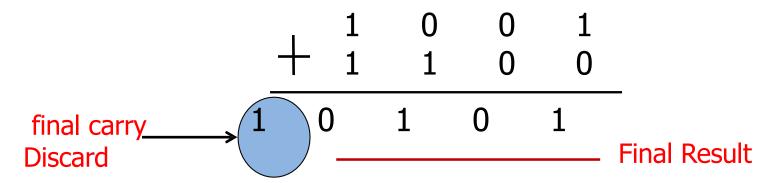
Subtraction Using 2's Complement

Example: Perform using 2' complement $(9)_{10}-(4)_{10}$

Step 1: Take 2' complement of
$$(4)_{10} = (0100)_2$$

= $1011 + 1 = 1100$

Step 2: Add 9 with 2' complement of 4



If Carry is generated, discard carry. The result is positive and its true binary form.

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BCD or 8421 Code

- ✓ The smallest BCDnumber is (0000) and the largest is (1001). The next number to 9 will be 10 which is expressed as (0001 0000) in BCD.
- ✓ There are six illegal combinations 1010, 1011, 1100, 1101, 1110 and 1111 in this code i.e. they are not part of the 8421 BCD code

Decimal to BCD Conversion

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Sr. No.	Decimal Number	BCD Code
1	8	1000
2	47	0100 0111
3	345	0011 0100 0101
4	99	1001 1001
5	10	0001 0000

Gray Code

- ✓ The gray code is non-weighted code.
- ✓ It is not suitable for arithmetic operations.
- ✓ It is a cyclic code because successive code words in this code differ in one bit position only i.e. unit distance code

Binary	Gray code
000	001
001	001
010	011
011	010

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Binary to Gray Code Conversion

- >Keep MSB of gray code same as MSB of binary code.
- Going from left to right, add adjacent pairs of binary code to get next gray code bit.
- ➤ Discard any carries

Example 1: Convert 1011 Binary Number into Gray Code

Binary Number

Gray Code

Binary and Corresponding Gray Codes

Decimal No.	Binary No.	Gray Code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101

Gray Code to Binary Conversion

>Keep MSB of binary code same as MSB of gray code.

Add each binary code generated to the gray code bit in the next adjacent.

Discard carries.

Example 1:: Convert 1110 Gray code into Binary Number.

Gray Code 1 1 1 0

Binary Number 1 0 1 1

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Excess-3 Code (XS-3)

Example 1: Obtain Xs-3 Code for 428 Decimal

	4	2	8	
	0100	0010	1000	
+	0011	0011	0011	
	0111	0101	1011	

Excess-3 Code (XS-3)

Decimal No.	BCD Code	Excess-3 Code= BCD + Excess-3
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

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Excess 3 code is a self complementary code

Decimal Digit	BCD code	Excess-3 code
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

Alphanumeric code

- Alphanumeric codes are also called character codes, are binary codes used to represent alphanumeric data. The codes write alphanumeric data, including letters of the alphabet, numbers, mathematical symbols and punctuation marks, in a form that is understandable and process able by a computer.
- Using these code we can interface, input output devices such as keyboards, monitors, printers etc. With computer.

ASCII Codes

- ✓ The American Standard Code for Information Interchange is a character-encoding scheme originally based on the English alphabet.
- ✓ ASCII codes represent text in computers, communications equipment, and other devices that use text.
- ✓ Most modern character-encoding schemes are based on ASCII, though they support many additional characters.

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ASCII Codes

✓ ASCII includes definitions for 128 characters: 33 are non-printing control characters (many now obsolete) that affect how text and space is processed and 95 printable characters, including the space (which is considered an invisible graphic)

✓ Example: a=97

A = 65

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- ✓ The BCD addition is performed by individually adding the corresponding digits of the decimal number expressed in 4 bit binary groups starting from LSD.
- ✓ If there is no carry & the sum term is not an illegal code, no correction is needed.

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✓ If there is a carry out of one group to the next group or if the sum term is an illegal code then 6 i.e. 0110 is added to the sum term of that group and resulting carry is added to the next group.

✓ This is done to skip the six illegal states.

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Sum≤9, Carry=0

Sum≤9, Carry=1

Sum>9, Carry=0

Answer is correct.

No correction required.

Add 6 to the sum term to get the correct answer

Add 6 to the sum term to get the correct answer

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Example: Perform in BCD (57)10+(26)10

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Thus we have to add 0110 in illegal BCD code

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Example



0 0 0 1

$$(57)_{10} + (26)_{10} = (83)_{10}$$

Add 0110 in only invalid code

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Exercise

• Perform BCD Addition

$$(88.7)_{10} + (265.8)_{10}$$

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Parity method for error detection

• Parity bit: an extra bit included with message to make total number of 1's either odd or even.

message	P(odd)	P(even)
000	1	0
001	0	1
010	0	1
011	1	0
100	0	1
101	1	0
110	1	0
111	0	1

Subtraction of decimal using 9's complement

• 9"s complement of number is obtained by subtracting each decimal digit from 9.

• Example: 9's complement of 523 is: 476.

Subtraction of decimal using 10's complement

• 10"s complement of number is obtained by adding 1 to the 9's complement

• Example: 10's complement of 523 is: 476+1=477.

Thank you!