

Logic Assignment.

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.

Ans. It is proposition.

It's Truth value is T.

b) Miami is the capital of Florida.

Ans. It is proposition.

It's Truth value is F.

c) $2+3=5$

Ans. It is proposition.

It's Truth value is T.

d) $5+7=10$.

Ans. It is proposition.

It's Truth value is F.

e) $x+2=11$.

Ans. It is functional proposition.

It's truth value can't be said unless domain discloses or value of x is provided.

f) Answer this question.

Ans. It is not proposition.

2. What is the negation of each of these propositions?

a) Jennifer and Teja are friends.

Ans. Jennifer and Teja are not friends.

b) There are 13 items in a baker's dozen.

Ans. There are not 13 items in a baker's dozen.

c) Abby sent more than 100 text messages every day.

Ans. Abby didn't send more than 100 text messages every day.

d) 121 is a perfect square.

Ans. 121 is not a perfect square.

3. Let p and q be the propositions p : I bought a lottery ticket this week. q : I won the million dollar jackpot.

Express each of these propositions as an English sentence.

a. $\neg p$: I didn't buy a lottery ticket this week.

b. $\neg q$:

c. $p \rightarrow q$: If I buy a lottery ticket this week then I will win the million dollar jackpot.

d. $p \wedge q$: I ~~will~~ bought a lottery ticket this week and won the million dollar jackpot.

e. $p \leftrightarrow q$: Buying a lottery ticket is necessary and sufficient for winning million dollar jackpot.

- f. $\neg p \rightarrow \neg q$: If I don't buy a lottery ticket this week then I won't win the million dollar's jackpot.
- g. $\neg p \wedge \neg q$: I didn't buy a lottery ticket this week and didn't win the million dollar jackpot.
- h. $\neg p \vee (p \wedge q)$: I don't buy a lottery ticket this week or I buy a lottery ticket this week and win the million dollar jackpot.

4. Let p and q be the propositions p : It is below freezing.
 q : It is snowing. Write these propositions using p and q and logical connectives (including negations).

- a. It is below freezing and snowing: $p \wedge q$
- b. It is below freezing but not snowing: $p \wedge \neg q$
- c. It is not below freezing and it is not snowing: $\neg(p \wedge q)$
- d. It is either snowing or below freezing (or both): $q \vee p$.
- e. If it is below freezing, it is also snowing: $p \rightarrow q$.
- f. Either it is below freezing or it is snowing, but it is not snowing if it is below freezing:
- g. That it is below freezing is necessary and sufficient for it to be snowing: $p \leftrightarrow q$

5. Construct a truth table for each of the following statements.

a. $p \rightarrow \sim q$.

p	q	$\sim q$	$p \rightarrow \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

b. $\sim p \leftrightarrow q$

p	q	$\sim p$	$\sim p \leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

c. $(p \rightarrow q) \vee (\sim p \rightarrow q)$

p	q	$p \rightarrow q$	$\sim p$	$\sim p \rightarrow q$	$(p \rightarrow q) \vee (\sim p \rightarrow q)$
T	T	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	F	T

d. $(P \rightarrow Q) \wedge (\sim P \rightarrow Q)$

P	Q	$P \rightarrow Q$	$\sim P$	$\sim P \rightarrow Q$	$(P \rightarrow Q) \wedge (\sim P \rightarrow Q)$
T	T	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

e. $(P \leftrightarrow Q) \vee (\sim P \leftrightarrow Q)$

P	Q	$P \leftrightarrow Q$	$\sim P$	$\sim P \leftrightarrow Q$	$(P \leftrightarrow Q) \vee (\sim P \leftrightarrow Q)$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	F	T	T	T
F	F	T	T	F	T

f. $(\sim P \leftrightarrow \sim Q) \leftrightarrow (P \leftrightarrow Q)$

P	Q	$\sim P$	$\sim Q$	$\sim P \leftrightarrow \sim Q$	$P \leftrightarrow Q$	$(\sim P \leftrightarrow \sim Q) \leftrightarrow (P \leftrightarrow Q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

6. Show that $\sim(\sim P)$ and P are logically equivalent.

Ans

P	$\sim P$	$\sim(\sim P)$
T	F	T
F	T	F

From above truth table $\sim(\sim P) \equiv P$

7. Use a truth table to verify the first De Morgan law
 $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Here, $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$.

8. Determine whether $(\sim P \wedge (P \rightarrow Q)) \rightarrow \sim Q$ is tautology.

P	Q	$\sim P$	$P \rightarrow Q$	$(\sim P \wedge (P \rightarrow Q))$	$\sim Q$	$(\sim P \wedge (P \rightarrow Q)) \rightarrow \sim Q$
T	T	F	T	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Above truth table doesn't show tautology

9. Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

p	q	$\neg q$	$p \rightarrow q$	$(\neg q \wedge (p \rightarrow q))$	$\neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

From above truth table, $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

10. Let $Q(x)$ be the statement " $x+1 > 2x$." If the domain consists of all integers, what are these truth values?

a) $Q(0) \Rightarrow 1 > 0$ is true.

b) $Q(-1) \Rightarrow -1+1 > 2(-1)$
 $\Rightarrow 0 > -2$ is true.

c) $Q(1) \Rightarrow 1+1 > 2 \times 1$
 $\Rightarrow 2 > 2$ is false.

d) $\exists x. Q(x)$: Because $Q(x)$ is true for some values such as for $x=1$, $x=-1$, so, $\exists x. Q(x)$ is true.

e) $\forall x. Q(x)$: Since, there are some values like, $x=1$ in which statement is false. So, $\forall x. Q(x)$ is false.



11. Express each of these statements using quantifiers.

a) All dogs have fleas.

b) There is a horse that can add.

c) Every koala can climb.

d) No monkey can speak French.

e) There exists a pig that can swim and catch fish.