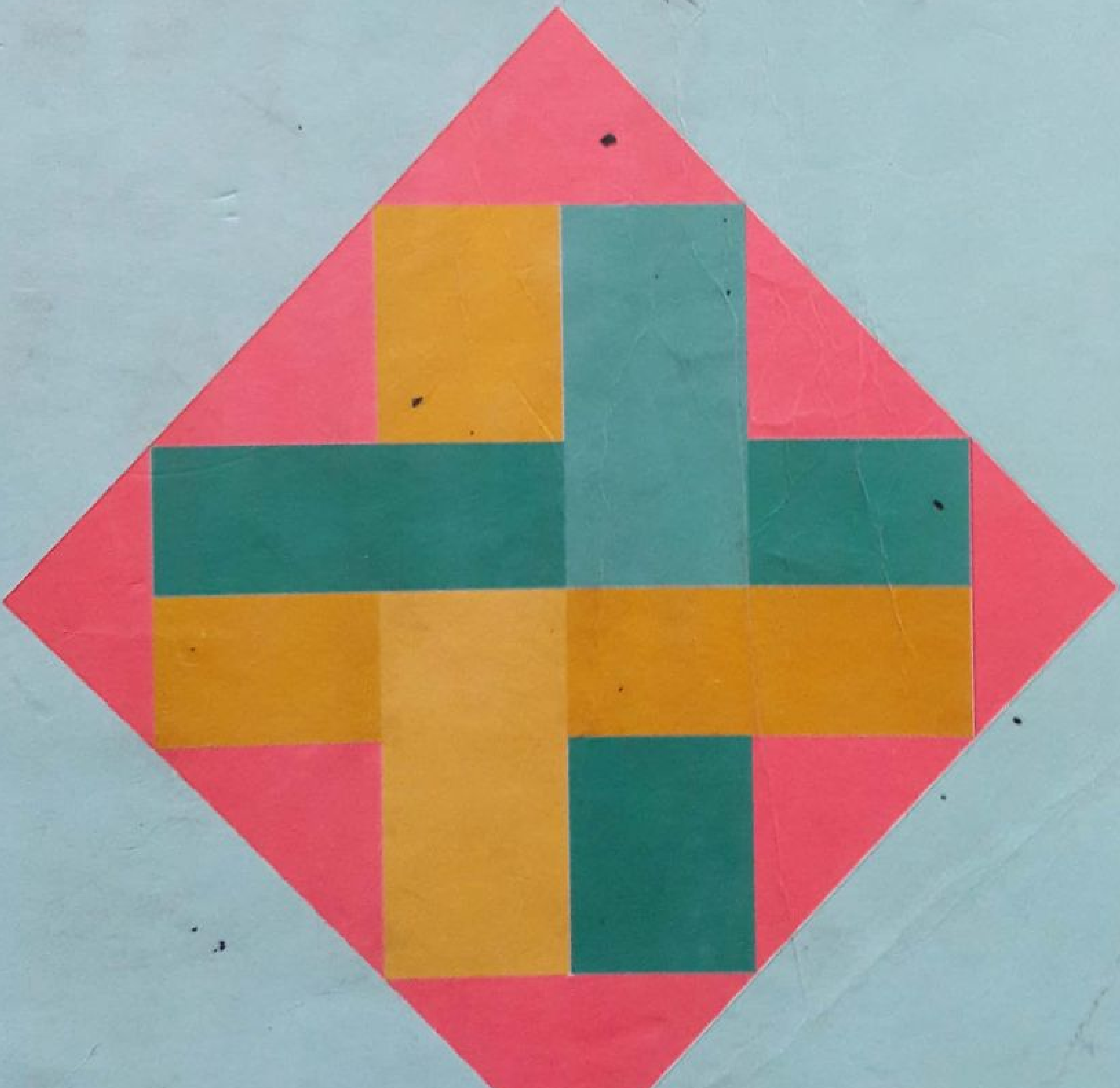
Universities Press

Mathematics

Techniques of Problem Solving



Steven G. Krantz

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CMP 102.3 Problem Solving Techniques (3-2-0)

Evaluation:

	Theory	Practical	Total
Sessional	50	-	50
Final	50	-	50
Total	100	-	100

Course Objectives:

A large part of everyday activity involves problem solving in some form. On order to solve problem one must think analytically to find a solution to a problem. The main aim of this course is:

1. To improve and impart conceptual clarity in thinking analytically and logically.
2. To provide fundamental means of approach how to translate verbal discussion onto analytical data and then how to solve it by computer.

Course Contents:

1. **Basic Concepts of Problem Solving** **(10 hrs)**
   Introduction to Problem Solving Approach. How to count. Use of induction principle. Problems of Logic and Issues of Parity.

2. **Application of Geometry** **(10 hrs)**
   Classical Planar Geometry. Analytic Geometry. Solid Geometry and miscellaneous problems.

3. **Miscellaneous Problem Solving Techniques** **(15 hrs)**
   Probabilistic approach to solving Counting Problems. Logic Problems (Simple logic, theory of games. Tracing routes. Learning from Parity. Mysterious arithmetic problems and surprise). Problems from Recreational math. (Magic square and Weighing problems). Problems of Algebra and Analysis (Inequality, Trignometry and related ideas).

4. **Solving Miscellaneous Real Life Problems** **(10 hrs)** ২০
   Miscellaneous problems, impossible problems, Problems from everyday life and Statistics.

Laboratory Work:

Realization and Implementation of the numerous problems and various problem-solving techniques learned is to be implemented in C Programming Language. However, the practical implementation is also considered as an assignment for the "Programming in C" course module.

Textbooks:

1. Krantz, Steven G., *Techniques of Problem Solving*, University Press, 1998, ISBN:81-737-116-X

Reference Books:

1. Etter, D. M., *Engineering Problem Solving with ANSI C*, Prentice Hall, NJ, 1995,
2. Lakatos, *Proofs and Refutation*, Cambridge University Press, 1976.
3. Polya, G., *How to Solve It*, Princeton University Press, Princeton, 1998.

Subject	Daily Leson Plan
1.2.1 ltes 1	How many zeros are there at the end of no. 100! → 100 X 99 X 98 x . x 2 x 1 when we multiply by 10 we get 0 at end. the prime factorization of 10 is 10 = 5 * 2. so, for this prablem we solue by counting the factor of 5 in 100!
	1 .... 10 → 5, 10 → 2 zeros 11 .... 20 → 15, 20 → 2 " 21 − 30 → 25, 30 → 3 " (5×5, 10×3) 31 − 40 → 35, 40 → 2 41 − 50 → 45, 50 → 3 (9×5, 10×15) 51 − 60 → 55, 60 → 2 " 61 − 70 → 65, 70 → 2 $\frac{100}{5} = 20$ $\frac{20}{5} = 4$ 24
	71 − 80 → 75, 80 → 3 81 − 90 → 85, 90 → 2 91 − 100 → 95, 100 → 3 24 zeros

1.2.2	6 student each students shakes hands with each other. How many hand shake in tatal 2 student = 1 hand shake 3 " = 2 4 " = 3 5 " = 4 6 " = 5
	Tatal = 15 hand shake in tatal. Q for 12 students → $1+2+\cdots+11=66$ no. hand shake.

Subject	Daily Leson Plan
1.2.3 pter 1	Find sum of first k positive integers.

$1+2+3+\cdots+(k-1)+k = Sk.$

$S(k+1) = 1+2+3+\cdots+(k-1)+k+(k+1).$

use lineal equ^n $(n+1)^2-n^2=2n+1.$

$2^2-1^2=4-1=3 = 2\cdot1+1$

$3^2-2^2=9-4=5=2\cdot2+1$

$4^2-3^2=16-9=7=2\cdot3+1$

$(k+1)^2-k^2=k^2+2k+1-k^2=2\cdot k+1$

$(\cancel{2^2}-1^2)+(\cancel{3^2}-\cancel{2^2})+(\cancel{4^2}-\cancel{3^2})+\cdots+(k+1)^2-\cancel{k^2}=$

$= \cancel{k}\ \underline{2\cdot1+1}+\underline{2\cdot2+1}+\underline{2\cdot3+1}+\underline{2k+1}$

$(k+1)^2-1^2 = 2(1+2+3+\cdots+k)+(1+1+1+\cdots+1).$

$(k+1)^2-1^2 = 2\cdot Sk\cdot+k$

$\cdot k^2+\cancel{2k}+\cancel{1}-\cancel{1}=2Sk\cdot\cancel{+k}$

$k^2+k=2Sk$

$Sk=\dfrac{k(k+1)}{2}$ $\left[S_n=\dfrac{n(n+1)}{2}\right]$

1.2.4 Q Find the sum of first 'K' sq. numbers.

$\Rightarrow$ $1^2+2^2+3^2+4^2+\cdots\cdots+K^2+(K+1)^2$

using linear equation: $(l+1)^3-l^3=3l^2+3\cdot l+1$

$2^3-1^3=8-1=7=3\cdot 1^2+3\cdot 1+1$

$3^3-2^3=27-8=19=3\cdot 2^2+3\cdot 2+1$

$4^3-3^3=64-27=37=3\cdot 3^2+3\cdot 3+1$

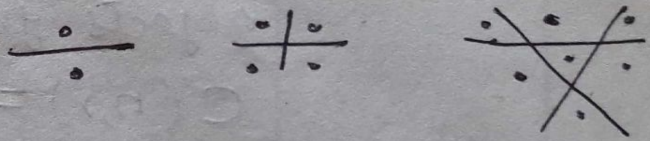
$\vdots$

$(K+1)^3-K^3=K^3+3K^2+3K+1-K^3=3K^2+3K+1$

SO, $2^3-1^3+3^3-2^3+4^3-3^3+\cdots\cdots+(K+1)^3-K^3=3(1^2+2^2+3^2+\cdots\cdots+K^2)+(1+1+1+\cdots\cdots+1).$

$(K+1)^3-1^3=3\cdot S_n+3\left(\frac{K^2+K}{2}\right)+K$

$S_n=\frac{K(K+1)(2K+1)}{6}$ //.

Subject	Daily Leson Plan
1.2.6	with 3 st. line what is the max^m region to divide a 2D plane? → 1 line : 2 Region 2 line : 4 Region 3 line : 7 Region. 

1.3.1 From K-objects given, how many ordered pair are there?

1^{st} object $\{a_0, a_1, a_2, \cdots a_k\}$ k is possible choice.

2^{nd} object $\{ - \cdot \cdot \quad \cdot \quad - \}$ $(k-1)$ possible choice

$\therefore$ for possible pair $k(k-1)$.

e.g a, b, c, d

(a,b) (a,c), (a,d) $4 \times 3 = 12$ pairs

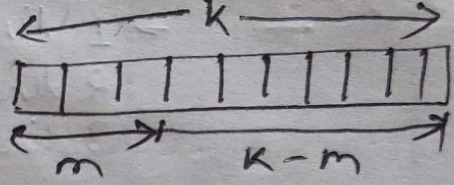
(ba) (b c) (bd)

(c a) (cb) (cd)

(d a) (db) (dc)

$\therefore$ $K(K-1)$ = possible pairs.

1.3.2	From k-objects how many permutation (Ordering) are there? for 1, 2, 3, how many permutation are there? 123 132 $\therefore$ $3*2*1=6$ 213 231 for 4 objects $4*3*2*1=24$. 321 312 $k!$ for k objects
	combination (random) $=\binom{k}{m}=\frac{k!}{m!\,(k-m)!}$ permutation : $\binom{k}{m}=\frac{k!}{(k-m)!}$ k = total m = taken at a time $P(n,r)=$ $=\frac{n!}{(n-r)!}$

bject	Daily Leson Plan
	How many different ways are there to pick up m objects from a table of k-objects? $C(n,r) = \frac{n!}{r!(n-r)!}$  $\therefore \frac{k!}{m!(k-m)!}$

1.3.4	How many hands are passible in 5 card poker! $\binom{n}{r}$ or $\binom{k}{m}$ $\binom{52}{5}$ $C = \frac{k!}{m!(k-m)!} = \frac{52!}{5!(52-5)!}$ $= \frac{52!}{5!47!} = \frac{52 \times 51 \times 50 \times 49 \times 48 \times \cancel{47!}}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 * \cancel{47!}} = 2598960.$

1.3.5	Bridge hand → find pair of Bridge hand from 52 cards. $C1 = \binom{52}{13} = \frac{52!}{13!\,(52-13!)} = \frac{52!}{13!\,39!} = 6.5 * 10^{20}$ $C2 = \binom{39}{13} = \frac{39!}{13!\,(39-13)!} = \frac{39!}{13!\,26!} = 5.2 * 10^{21}$ Total = C1 * C2;

Q 1.5.5

There are adults > boys > girls > families,
but a family must have 2 adults and at least 3 or more children (family 1 or 2 adults and $\geq$ 3 children)

$\therefore$ condition 1: 1 family $\rightarrow$ 2 girls $\rightarrow$ 3 boys $\rightarrow$ 4 adults
here, 4 adults means 2 families but we have 1 family so not possible

condition 2: 2 families $\rightarrow$ 3 girls $\rightarrow$ 4 boys $\rightarrow$ 5 adults
5 adults so 2 families ~~no~~ & children are also 7 so not possible

condition 3: 3 families $\rightarrow$ 4 girls $\rightarrow$ 5 boys $\rightarrow$ 6 adults
here 6 adults means 3 families. and $5+4=9$ children can be divided 3 in each family. So possible.

Subject	Daily Leson Plan
1.6.2	We have container 6l & 4l use only two jug & make just 3l water in one of the jug.
parity	6+4=10 l 6-4=2 l. Not possible

1-6-3Q u have 9l & 4l containee put 6l in 9l containee?

	9 lit	4 lit
① Fill 9l jug	9	0
② Fill 4l from 9l jug	5	4
③ empty 4l jug	5	0
④ again fill 4l jug from 9l jug	1	4
⑤ empty 4l jug	1	0
⑥ ~~fill~~ poure 1 lit into 4l	0	1
⑦ Fill 9l jug	9	1
⑧ Fill 4l with 9l jug	6	4

Now 9l jug contains 6l water.

		water.
page=39	1.6.5	cows + people. & man A heard of cows & runs randomly. There are 120 heads & 300 feets in total. Find no. of cows & men. people 'p' + cows 'C' = 120 ... (1) Legs ⇒ 2p + 4C = 300 feets -- (11)
		multiplg eq^n (1) by 2 = 2p+2C = 240 .. (111) ∴ Now $\cancel{2p} + 4C = 300$ ii − iii $-\cancel{2p} + _{-}2C = _{-}240$ $2C = 60$ $C = \frac{60}{2} = 30.$ $P = 90.$

Class	Subject	Daily Leson Plan
41		1.6.9: ⇒ A sheep takes 1 day to clear a field of grass. A cow ,, $\frac{1}{2}$,, ,, ,, ,, ,, ,, ,, . How many days would it take both together? ⇒ 1 cow = 2 sheep. 1 cow + 1 sheep = 2 sheep + 1 sheep = 3 sheep. 1 sheep takes 1 day to finish grass.
		3 ,, ,, $\frac{1}{3}$,, ,, ,, ,, .

1.6.11	What is the last digit of 3^{4798}? $3^1 = 3$ $3^5 = 243$ $4x = 1$ $3^2 = 9$ $3^6 = 729$ $4x+1 = 3$ 0, 1, 2, 3 are remainders. $3^3 = 27$ $3^7 = 2187$ $4x+2 = 9$ $3^4 = 81$ $3^8 = 6561$ $4x+3 = 7$
	when we div 4798 by 4 & if its remainder comes 2 then its last digit is 9.

Subject	Daily Leson Plan
1.6.12 Page=42	what is the last digit of 7^{65432}? $7^0=1$ $7^1=7$ $\quad$ $7^5=16807$ $7^2=49$ $\quad$ $7^6=117649$ $7^3=343$ $\quad$ $7^7=823543$ $7^4=2401$ $\quad$ $7^8=5764801$
	divide 65432 by 4 (because pattern repeats after 4 number) if rem=0 last digit =1 ,, rem = 1 ,, ,, = 7 rem = 2 ,, ,, = 9 rem = 3 ,, ,, = 3 Ans = rem=0 ∴ 1