* Successive differentiation

1. $y = \dfrac{1}{ax+b} \Rightarrow y_n = \dfrac{(-1)^n a^n n!}{(ax+b)^{n+1}}$

2. $y = e^{ax} \Rightarrow y_n = a^n e^{ax}$

3. $y = \sin(ax+b) \Rightarrow y_n = a^n \sin\left(ax+b+\dfrac{n\pi}{2}\right)$

4. $y = \cos(ax+b) \Rightarrow y_n = a^n \cos\left(ax+b+\dfrac{n\pi}{2}\right)$

5. $y = e^{ax}\sin(bx+c) \Rightarrow y_n = (a^2+b^2)^{n/2} e^{ax} \left\{\sin\left(bx+c+n\tan^{-1}\dfrac{b}{a}\right)\right\}$

6. $y = e^{ax}\cos(bx+c) \Rightarrow y_n = (a^2+b^2)^{n/2} e^{ax} \cos\left(bx+c+n\tan^{-1}\dfrac{b}{a}\right)$

7. $y = \log(ax+b) \Rightarrow y_n = \dfrac{(-1)^{n-1}(n-1)!\, a^n}{(ax+b)^n}$

8. $y = a^{bx} \Rightarrow y_n = b^n (\log a)^n \cdot a^{bx}$

9. $y = x^n \Rightarrow y_n = n!$

10. Leibnitz theorem

$$(UV)_n = U_n V + {}^nC_1 U_{n-1} V_1 + {}^nC_2 U_{n-2} V_2 + \dots + {}^nC_n U V_n$$

11. Taylor's theorem

$$f(a+h) = f(a) + h f'(a) + \frac{h^2}{2!} f''(a) + \dots + \frac{h^{n-1}}{(n-1)!} f^{n}(a)$$

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11. Taylor's Theorem
$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \dots + \frac{h^n}{n!}f^n(a+\theta h)$$

12. Maclaurin's Theorem
$$f(a) = f'(a) + af'(0) + \frac{a^2}{2!}f''(0) + \dots + \frac{a^n}{n!}f^n(\theta h)$$

* Infinite Series

1. $\lim_{n\to\infty} n^{1/n} = 1$
2. $\lim_{n\to\infty} \left(1+\frac{\lambda}{n}\right)^n = e^{\lambda}$
3. $\lim_{n\to\infty} \log n = 0$
4. $\lim_{n\to 0} \frac{\sin n}{n} = 1$

* Tests for convergence & divergence

1. Sequence of partial sums (SOPS)
If S_n convgs, then U_n also convgs.

2. Geometric series
$\sum ar^{n-1}$; $r<1 \rightarrow$ convg
$r \geq 1 \rightarrow$ divg.

3. Limit Comparison Test (LCT)
$\lim_{n\to\infty} \frac{U_n}{V_n}$ is finite
if $V_n \rightarrow$ convgs, then $U_n \rightarrow$ convgs
$V_n \rightarrow$ divgs, then $U_n \rightarrow$ divgs.

4. p-series test
$\sum \frac{1}{n^p}$ if $p>1 \rightarrow$ convg
if $p \leq 1 \rightarrow$ divg.

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5. Cauchy n^{th} Root test

$\lim_{n\to\infty} (U_n)^{1/n} = l$; $l < 1 \to$ convg
$l > 1 \to$ divg
$l = 1 \to$ test fails

6. Ratio Test

$\lim_{n\to\infty} \frac{U_n}{U_{n+1}} = l$; $l > 1 \to$ convg
$l < 1 \to$ divg
$l = 1 \to$ test fails

7. Raabe's Test

$\lim_{n\to\infty} n\left(\frac{U_n}{U_{n+1}} - 1\right) = l$; $l > 1 \to$ Convg
$l < 1 \to$ divg
$l = 1 \to$ test fails

8. Logarithmic Test

$\lim_{n\to\infty} n\left(\log \frac{U_n}{U_{n+1}}\right) = l$; $l > 1 \to$ convg
$l < 1 \to$ divg
$l = 1 \to$ fails

9. Gauss Test

$\frac{U_n}{U_{n+1}} = 1 + \frac{\lambda}{n} + O\left(\frac{1}{n^2}\right)$; $\lambda > 1 \to$ convg
$\lambda \leq 1 \to$ divg

* Expansions

1. $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots$

2. $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots$

3. $\tan\theta = \theta + \frac{\theta^3}{3!} + \frac{2}{15}\theta^5 + \ldots$

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4. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$

5. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

6. $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

7. $\log(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots\right)$

* ~~Radius of curvature~~

* Absolute convergence

~~Leibnitz test~~

Let a series be $U_1 + U_2 + U_3 + \dots + U_n$

→ If $|U_1| + |U_2| + |U_3| + \dots + |U_n|$ is convg then $\sum U_n$ ~~be~~ is absolutely convgt.

→ If $\sum |U_n|$ is divg. but $\sum U_n$ is convg, then $\sum U_n$ is conditionally convgt.

* Radius of curvature

1. Cartesian curve $y = f(x)$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

2. Parametric curve $y = f(t)$ $x = g(t)$

$$\rho = \frac{(x'^2 + y'^2)^{3/2}}{x'y'' - y'x''}$$

3. Polar curve $r = f(\theta)$

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$$

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4. Newton's method

$\rho = \lim_{\substack{x \to 0 \\ y \to 0}} \frac{x^2}{2y}$ [when x-axis is tangent]

$\rho = \lim_{\substack{x \to 0 \\ y \to 0}} \frac{y^2}{2x}$ [when y-axis is tangent]

5. Expansion method $y = f(x)$ ($y = x$ is tangent)

Let $y = px + q\frac{x^2}{2!} + \dots$

where $p = f'(0)$ $q = f''(0)$

$\therefore \rho = \frac{(1+p^2)^{3/2}}{q}$

* Area under curve

1. $y = f(x)$, $x = a$, $x = b$

$A = \int_a^b y \, dx$

2. $x = f(y)$, $y = a$, $y = b$

$A = \int_a^b x \, dy$

3. $x = \phi(t)$ $y = f(t)$ x-axis

$A = \int_{t_1}^{t_2} y \cdot \frac{dx}{dt} \cdot dt$

4. $x = \phi(t)$ $y = f(t)$ y-axis

$A = \int_{t_1}^{t_2} x \cdot \frac{dy}{dt} \cdot dt$

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5. $r = f(\theta)$

$$A = \frac{1}{2}\int_a^b r^2 \, d\theta$$

* Length of curve

1. $y = f(x)$ $\quad x = a \quad x = b$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

2. $x = f(y)$ $\quad y = a \quad y = b$

$$L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$$

3. $x = f(t)$ $\quad y = g(t)$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

4. $r = f(\theta)$

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$$

* Volume of revolution

1. Revolution about x-axis $\Rightarrow \int_a^b \pi y^2 \, dx$

2. Revolution about y-axis $\Rightarrow \int_a^b \pi x^2 \, dy$

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3. Revolution about initial line $(\theta = 0) \Rightarrow \int_a^b \frac{2\pi}{3} r^3 \sin\theta \, d\theta$

4. Revolution about $\left(\theta = \frac{\pi}{2}\right) \Rightarrow \int_a^b \frac{2\pi}{3} r^3 \cos\theta \, d\theta$

* Surface area of Revolution

1. Revolution about x-axis $\Rightarrow \int_a^b 2\pi y \frac{dl}{dx} \cdot dx$

2. Revolution about y-axis $\Rightarrow \int_a^b 2\pi x \frac{dl}{dy} \cdot dy$

* Differential Eqⁿs.

1. Linear eqⁿs.

$$\frac{dy}{dx} + Py = Q$$

$\Rightarrow \quad IF = e^{\int P dx}$

Solⁿ $\Rightarrow \quad y(IF) = \int Q \times IF \, dx$

2. Exact diff. eqⁿ $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad (Mdx + Ndy = 0)$

Solⁿ $\Rightarrow \int_{y\,const} M dx + \int N^* dy = C$

If $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, then we will reduce it:

$\rightarrow$ Homo. eqⁿ $\Rightarrow \quad IF = \frac{1}{Mx + Ny}$

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→ $f_1(x,y)\,y\,dx + f_2(x,y)\,x\,dx = 0$; $IF = \dfrac{1}{Mx - Ny}$

→ for $M dx + N dy = 0$

if $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$, then $e^{\int f(x) dx}$ is IF

if $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = f(y)$, then $e^{\int f(y) dy}$ is IF

* Method of Variation Parameters

→ ~~Proo~~ For find particular integral (PI)

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$PI = -y_1 \int \frac{y_2 X}{W} dx + y_2 \int \frac{y_1 X}{W} dx$$

13.7 WORKING PROCEDURE TO SOLVE THE EQUATION

$$\frac{d^n y}{dx^n} + k_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + k_{n-1} \frac{dy}{dx} + k_n y = X$$

of which the *symbolic form* is

$$(D^n + k_1 D^{n-1} + \dots + k_{n-1} D + k_n) y = X .$$

Step I. To find the complementary function

(*i*) *Write the A.E.*

i.e., $D^n + k_1 D^{n-1} + \dots + k_{n-1} D + k_n = 0$ and solve it for D.

(*ii*) Write the *C.F.* as follows :

Roots of A.E.	*C.F.*
1. $m_1, m_2, m_3 \dots$ (real and different roots)	$c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + \dots$
2. $m_1, m_1, m_3 \dots$ (two real and equal roots)	$(c_1 + c_2 x) e^{m_1 x} + c_3 e^{m_3 x} + \dots$
3. $m_1, m_1, m_1, m_4 \dots$ (three real and equal roots)	$(c_1 + c_2 x + c_3 x^2) e^{m_1 x} + c_4 e^{m_4 x} + \dots$
4. $\alpha + i\beta, \alpha - i\beta, m_3 \dots$ (a pair of imaginary roots)	$e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x) + c_3 e^{m_3 x} + \dots$
5. $\alpha \pm i\beta, \alpha \pm i\beta, m_5 \dots$ (2 pairs of equal imaginary roots)	$e^{\alpha x}[(c_1 + c_2 x) \cos \beta x + (c_3 + c_4 x) \sin \beta x] + c_5 e^{m_5 x} + \dots$