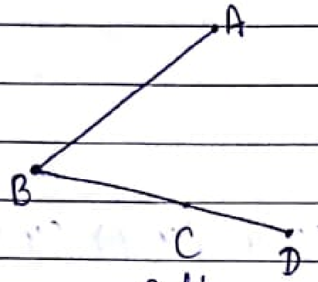
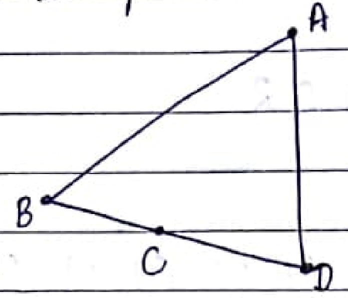
Euler path and circuit

Circuit is a closed path



Path

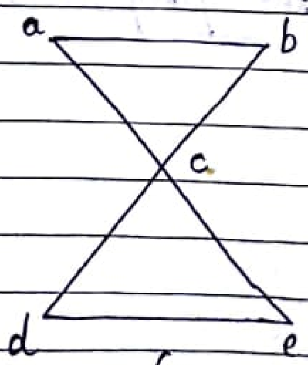


Euler	Hamilton
Path	Path
Ckt	ckt
Euler deals with edges	Hamilton deals with vertices

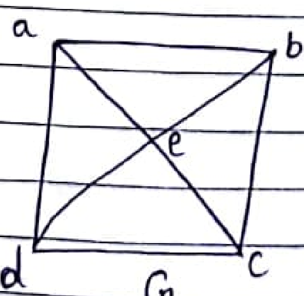
Adjacency matrix → vertices
Incidence matrix → edges

Euler ckt An euler ckt in a graph G is a simple ckt containing every edge of G and Euler path in graph G is a simple path containing every edge of G.

Q.

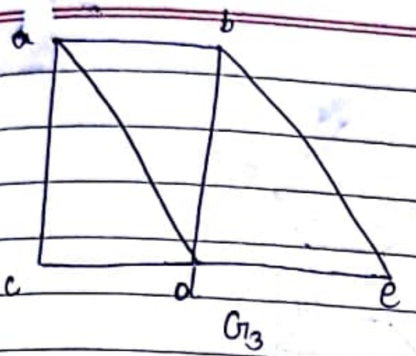


G_1



G_2

Isomerism — No. of vertices
No. of edges
Degree of a vertex



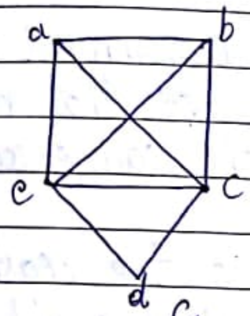
G_3

Which of these undirected graph have an Euler ckt and Euler path?

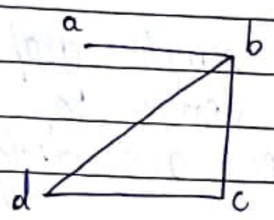
Sol^n^-

	Euler circuit	Euler Path
G_1 -	✓	✓
G_2 -	×	×
G_3 -	×	✓

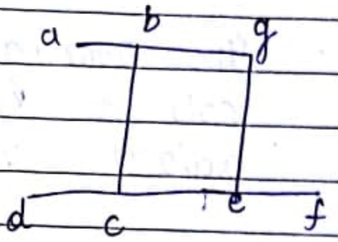
Q.



G_1



G_2

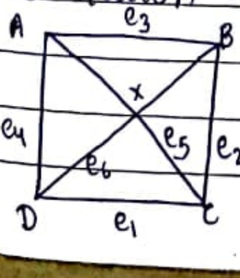


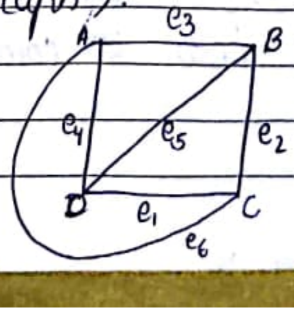
G_3

Which of these simple graphs have Hamilton ckt or if not a Hamilton path?

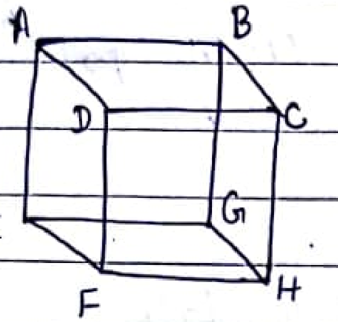
G_1 - a, b, c, d, a Hamilton ckt ✓
G_2 - Hamilton Path ✓
G_3 - Neither Hamilton ckt or Hamilton path

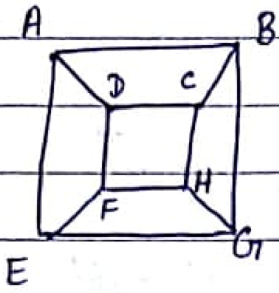
Graph is called planar if it can be drawn in a plane w/o any edges crossing (where a crossing of edges is the intersection of lines or arcs representing them at a pt. other than their common end pt. & such a drawing is called planar representation of a graph).





Planar Graph





Graph Coloring

→ The coloring of simple graph is assignment of color to each vertex of graph so that no two adjacent vertices are assigned the same color.

→ The chromatic no. of a graph is the least no. of color needed for a coloring of this graph.

Chromatic no. of graph → $\chi(G)$

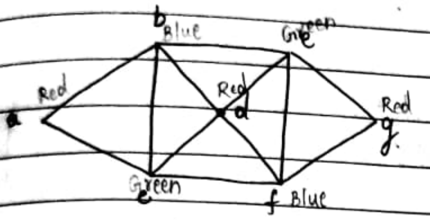
Four Color Theorem

The chromatic no. of a planar graph is no greater than 4. ~~that means~~

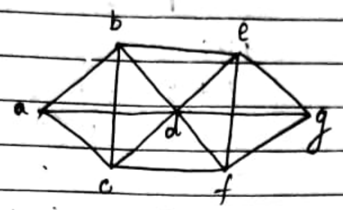
Two things are required to show that the chromatic no. of graph is k:

a) We must show that graph can be colored with k colors. This can be done by constructing such a coloring.

b) We must show that the graph cannot be colored using less than k colors.

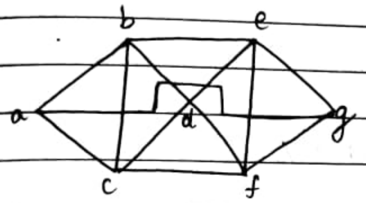


I



II

Not planar



III

13/10/17

Principle of Mathematical Induction

Basis - If $P(n)$ is true for $P(1)$

Inductive - Progression

Q. Show that if n is a +ve integer then

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Sol^n - $P(n) = \frac{n(n+1)}{2}$

For $n = 1$

$$P(1) = \frac{1(1+1)}{2} = 1$$

Let $P(k)$ is true

$$1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}$$

$$P(k+1) = \frac{(k+1)(k+2)}{2}$$

LHS

$1 + 2 + 3 + \cdots k + (k+1)$

$$\Rightarrow \frac{k(k+1)}{2} + (k+1)$$

$$= (k+1)\left(\frac{k}{2} + 1\right)$$

$$= \frac{(k+1)(k+2)}{2}$$

$= RHS$

Hence it is true $\forall n \in N$.

Linear Equations

Degree 1

A linear homogeneous recurrence relation of degree k with constant coefficients is a recurrence relation of the form

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$$

This relation is linear b'coz the right hand side is the sum of previous term ~~previ~~ of the sequence each multiplied by the fn n.

Theorem: Let c_1 & c_2 be the real numbers. Suppose that $r^2 - c_1 r - c_2 = 0$ has two distinct root r_1 & r_2 then the sequence $\{a_n\}$ is the sol^n of recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$ if and only if $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ for $n = 0, 1, 2$ where α_1 & α_2 are constants.

Q What is the solution of recurrence relation

$a_n = a_{n-1} + 2a_{n-2}$

$a_0 = 2$

$a_1 = 7$

Find α_1 & α_2.

Sol^n $r^2 - r - 2 = 0$

$(r-2)(r+1) = 0$

$r_1 = 2$, $r_2 = -1$

$a_n = \alpha_1 (2)^n + \alpha_2 (-1)^n$

$n = 0$

$a_0 = 2 = \alpha_1 + \alpha_2$

$\Rightarrow \alpha_1 + \alpha_2 = 2 \quad -(1)$

$n = 1$

$a_1 = 7 = 2\alpha_1 - \alpha_2$

$\Rightarrow 2\alpha_1 - \alpha_2 = 7 \quad -(2)$

(1) + (2)

$3\alpha_1 = 9$

$\alpha_1 = \frac{9}{3} = 3 \quad \Rightarrow \alpha_2 = -1$

Q.(1) Find the sol^n of

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

with initial conditions

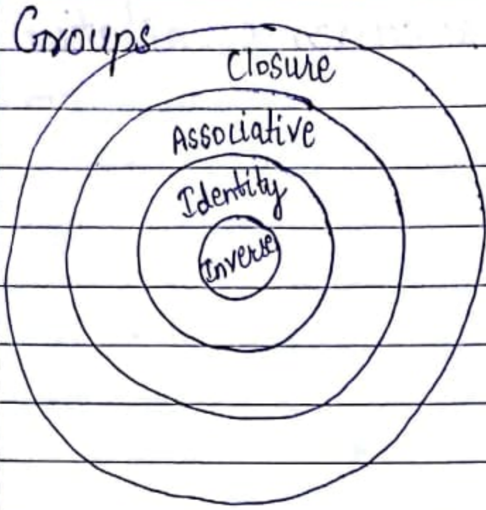
$a_0 = 7$, $a_1 = 6$, $a_2 = 11$

(2) $a_n = -6a_{n-1} - 9a_{n-2}$ for $n \geq 2$

$a_0 = 3 \quad a_1 = -3$

Scanned by CamScanner

Groups



Closure - Algebraic Structure (A
Associative - Semigroup
Identity - Monoid
Inverse - Group

Let A be a set and we are discussing it with any operator,

$$(A, *)$$

$*$ → $+$, $-$, $\times$, $/$

This is called a group

$a, b \in A$

$(A, +)$ $a + b$

Natural No. → $1 - \infty$
Real No. → $-\infty$ - ∞
Integers → +ve numbers 0 -ve numbers
Whole → 0, Natural numbers

1) Closure Property
A set A w.r.t. operation $*$ is said to satisfy closure property if there exist $a, b \in A$ then $a * b \in A$

Algebraic structure (A.S.)
If a set A w.r.t. operation $*$ satisfy closure property then it is called algebraic structure.

Sets	A.S.	Semi gp	Monoid	Group
(N,+)	✓	✓	X	X
(N,−)	X	X	X	X
(N,×)	✓	✓	✓	X
(N,÷)	X	X	X	X
(Z,+)	✓	✓	✓	✓
(Z,−)	✓	X	X	X
(Z,×)	✓	✓	✓	X
(Z,÷)	X	X	X	X
(R,+)	✓	✓	✓	✓
(R,−)	✓	X	X	X
(R,×)	✓	✓	✓	✓
(R,÷)	X	X	X	X
(E,+)	✓	✓	✓	X
(E,×)	✓	✓	✓	X
(O,+)	X	X	X	X
(O,×)	✓	✓	✓	X

* Associative property
A set A w.r.t. operation $*$ is said to satisfy association property if $a, b, c \in A$

$$(a*b)*c = a*(b*c)$$

If A.S. satisfy associative property then it is called semi group.

* Identity
A set A w.r.t. operation $*$ is said to satisfy identity property if ~~A∈~~ $a \in A$ there is an identity element e such that

$$(a*e) = e*a = a$$

	e
$+$	0
$-$	$\cancel{0}\,1$

If semi groups follow identity property then it is called monoid.

Inverse property

A set A w.r.t. operation $*$ is said to satisfy inverse property if $a \in A$ then

$$a * a^{-1} = a^{-1} * a = e$$

If a monoid satisfy inverse property then it is called group.

Langrange's Theorem

$\underbrace{|G|}_{4}$ $\underbrace{|H|}_{2}$

Order of H should divide ~~oder~~ order of ~~3~~ G.

Subset of a group is called subgroup

Homomorphism

Let $(G, 0_1)$ and $(G', 0_2)$ are groups. Then mapping $f: g \to g'$ is said to be homomorphism.

$$f(a\, q\, b) = f(a)\, 0_2\, f(b)$$