STUDENT LEARNING OBJECTIVE

From studying this chapter you will learn

- Why Engineer needs economics concepts?
- The terminologies used in the engineering economic analysis
- Principle of Engineering Economics.
- To develop the cash flow diagram
- How econonic system is carried out?

1.1 Definition of Economics

Definition 1

Economics is the social science that examines how people choose to use limited or scarce resources in attempting to satisfy their unlimited wants. (N. Gregory Mankiw)

Definition 2

Economics is a science which studies human behavior as a relationship between ends and scarce means which have alternative uses. (Lionell Robbins)

Why Engineering Economics?

The field of engineering economy is concerned with the systematic evaluation of the benefits and costs of the projects involving engineering design and analysis. Engineering economy quantifies the benefits and costs associated with engineering projects to determines if they make (or save) enough money to warrant their capital investment. In manufacturing or construction, engineering is involved in every detail of a product's production (about 85%) from conceptual design to distribution. Engineers must decide if the benefits of a project exceed its costs and must make this comparison in a unified framework. The framework within which to make this comparison is the field of engineering economics.

In the development of any product, a company's engineers are called upon to translate an idea into reality. A firm's growth and development largely depends upon a constant flow of

ideas for new products, and for the firm to remain competitive, it has to make existing products better or produce them at a lower cost. Traditionally, a marketing department would propose a product and pass the recommendation to the engineering department. The engineering department would work up a design and pass it on a manufacturing, which would make the product. With this type of product development cycle, a new product normally takes several months (or even years) to reach a market. Decisions made by the engineers are commonly the result of choosing one alternative over another. Decisions often reflect a person's educated choice of how to best invest funds, also called capital. The amount of capital is usually restricted, just as the cash available to an individual is usually limited. The decision of how to invest the capital will invariably change the future, hopefully for the better: that is, it will be value adding. Engineers play a major role in capital investment decisions based on their analysis, synthesis, and design efforts. The factors considered in making the decision are a combination of economic and noneconomic factors.

Hence we can define Engineering Economics in a following different way:

Definition 1"Engineering economics is the application of economic techniques to the evaluation of design and engineering alternatives. The role of engineering economics is to assess the appropriateness of a given project, estimate its value, and justify it from an engineering standpoint". (Dr. John M.Watts)

Definition 2"Engineering economics deals with the methods that enable one to take economics decision towards minimizing the cost or maximizing benefits to business organization".

Definition 3"Engineering economics deals with the concepts and techniques of analysis useful in evaluating the worth of systems, products, and services in relation to their costs".

Definition 4 "Engineering economy involves formulating, estimating, and evaluating the economic outcomes when alternatives to accomplish a defined purpose are available. Another way to define engineering economy is as a collection of mathematical techniques that simplify economic comparison".

Knowing how to correctly apply these techniques is especially important to engineers, since virtually any project will affect costs and/or revenues.

1.2 Origin of Engineering Economy

Cost considerations and comparisons are fundamental aspects of engineering practice. However, the development of engineering economy methodology, which is now used in all engineering work, is relatively recent. This does not mean that, historically, costs were usually overlooked in engineering decisions. However, the perspective that ultimate economy is a primary concern to the engineer and the availability of sound techniques to address this concern differentiates this aspect of modern engineering practices from that of the past.

A pioneer in the field was Arthur M. Wellington, a civil engineer, who in the latter part of the nineteenth century specifically addressed the role of economic analysis in engineering projects. His interest was railroad building in USA. His work was followed by Eugene Grant who published the first edition of his textbook which was the milestone in the development of engineering economy as we know it today. He placed emphasis on developing an economic point of view in engineering. In 1942 Woods and DeGarmo wrote the first edition of this book, later entitled Engineering Economy.

1.3 Role of Engineering Economy

People make decisions: computers, mathematics and other tools do not make. The techniques and models of engineering

economy assist people in making decisions. Since decision affect what will be done, the time frame of engineering economy is primarily the future. Therefore, numbers used in an engineering economic analysis are best estimates of what is expected to occur. These estimates often involve the three essential elements: cash flows, time of occurrence and interest rate. These estimates are about future, and will be somewhat different than what actually occurs, primarily because of changing circumstances and unplanned for events. In other words stochastic nature of estimate will likely make the observed value in the future differ from the estimate made now. Especially, sensitivity analysis is performed during the engineering economic study to determine how the decision might change based on varying estimate (discussed in chapter 8).

Engineering economy can be used equally to analyze outcomes have met or not met a specified criterion, such as rate of return requirement (discussed in chapter 4). There is an important procedure used to address the development and selection of alternatives. Commonly referred as the problem-solving approach or the decision making process, the steps in the approach follow:

1. Understand the problem and define the objective.
2. Collect relevant information.
3. Define the feasible alternative solutions and make realistic estimates.
4. Identify the criteria for decision making using one or more attributes.
5. Evaluate each alternative, using sensitivity analysis to enhance the evaluation.
6. Select the best alternative.
7. Implement the solution and monitor the results.

Strategic Economic Decisions

Once project ideas are identified, they are typically classified as:

1. Equipment and process selection
2. Equipment Replacement
3. New product development and product expansion
4. Cost reduction, and
5. Service improvement

1.4 Principles of Engineering Economics

The principle of engineering economics can be highlighted in the seven points as below:

Principle 1

Develop the Alternatives: The choice is among alternatives. The alternatives need to be identified and then defined for subsequent analysis.

Principle 2

Focus on the Differences: Only the differences in expected future outcomes among the alternatives are relevant to their comparison and should be considered in the decision.

Principle 3

Use a Consistent Viewpoint: The prospective outcomes of the alternatives, economic and other, should be consistently developed from a defined viewpoint (perspective).

Principle 4

Use a Common Unit of Measure: Using a common unit of measurement to enumerate as many of the prospective outcomes as possible will make easier the analysis and comparison of the alternatives.

Principle 5

Consider All Relevant Criteria: Selection of preferred alternative requires the use of criteria or several criteria. The decision process should consider both the outcomes enumerated in the monetary unit and those expressed in some other unit of measurement or made explicit in the descriptive manner.

Principle 6

Make Uncertainty Explicit: Uncertainty is inherent in projecting the future outcomes of the alternatives and should be recognized in their analysis and comparison.

Principle 7

Revisit the Decision: Improved decision making results from an adaptive process to the extent practicable, the initial projected outcomes of the selected alternative should be subsequently compared with actual results achieved.

1.5 Essential Economics Terminology

1. **Annuity:**
   - An amount of money payable to a beneficiary at regular intervals for a prescribed period of time out of a fund reserved for that purpose.
   - A series of equal payments occurring at equal periods of time
     - Amount paid annually/monthly/weekly etc., including reimbursement of borrowed capital and payment of interest.
2. **Assets:** An economic resource of entity (including money resources, physical resources, and intangible resources).
3. **Breakeven point:**
   - A graphic representation of the relation between total income and total costs for various levels of production and sales indicating areas of profit and loss.
   - A point where the organization is in no gain and no loss state.

4. **Capital:**
- The financial resources involved in establishing and sustaining an enterprise or project.
- A term describing wealth which may be utilized to economic advantage. The form that this wealth takes may be as cash, land, equipment, patents, raw materials, finished products, etc.

5. **Cash flow:** The statement showing actual amount coming into the firm and/or going out of the firm.
6. **Capital recovery:** It is the annual equivalent cost of capital cost.
7. **Discount rate:** The interest rate used to calculate the present value of the future cash flows.
8. **Decision making:** A program of action undertaken as a result of established policy to influence the final decision.
9. **Decision making under certainty:** Simple decisions that assume complete information and no uncertainty connected with the analysis of the decisions.
10. **Decision making under uncertainty:** Decision for which the analyst elects to consider several possible futures, the probabilities of which cannot be estimated.
11. **Decisions under risk:** A decision problem in which the analyst elects to consider several possible futures, the probabilities of which can be estimated.
12. **Depreciation:**
- Decline in value of a capitalized asset.
- A form of capital recovery applicable to a property with two or more years' life span, in which an appropriate portion of the asset's value is periodically charged to current operations.

13. **Economic life:** The timeframe an asset will be economically useful.
14. **Economic efficiency:** It is the ratio of output to input of a business system

Economic efficiency (%) = Output/Input*100

= Worth/ Cost *100

15. **Interest:** Interest is the fee that is charged for use of someone else's money. The size of the fee will depend upon the total amount of money borrowed and the length of time over which it is borrowed.
<u>Simple interest</u>: It is defined as the fixed percentage of the principal (the amount of money borrowed) multiplied by the life of the loan.
<u>Compound interest</u>: The type of interest that is periodically added to the amount investment (or loan) so that subsequent interest is based on the cumulative amount.
16. **Inflation:** An increase in the average price paid for goods and services bringing about reduction in the purchasing power. The inverse of inflation is *deflation*.
17. **Intangibles:**
- In economic studies, conditions or economy factors that cannot be readily evaluated in quantitative terms as in money.
- In accounting, the assets that cannot be reliably evaluated (e.g., goodwill, social values).

18. **Labor:** The capacity of human effort (both mind and muscles) available for use in producing goods and services.
19. **Opportunity cost:**
- The value of benefits sacrificed in selecting a course of action among alternatives.
- The value of the next best opportunity foregone by deciding to do one thing rather than another.

20. **Salvage Value:** Receipt at project termination for sale or transfer of the equipment (can be a salvage cost).
21. **Time value of money:** Since money has the ability to earn interest, its value increases with time. Hence it is the relationship between interest and time.

22. **Utility:** Satisfaction that a consumer obtains from goods and services that are consumed. It is a measure of satisfaction

1.6 Definition of cash flow

Cash flow is the stream of monetary (Rupees) values—costs (inputs) and benefits (outputs)—resulting from a project investment. The analysis of events and transactions that affects the cash position of company is termed as cash flow. A cash flow is the difference between total receipts (inflows) and total cash disbursement (outflows) for a given period of time. It is the statement that shows the actual amount coming into firm or going out of the firm.

Cash Inflows: Actual rupees coming into a firm.

Cash outflows: Actual rupees going out from the firm.

Cash Flow diagrams (CFD)

The costs and benefits of engineering projects over time are summarized on a cash flow diagram (CFD). Specifically, CFD illustrates the size, sign, and timing of individual cash flows, and forms the basis for engineering economic analysis. It is difficult to solve a problem if you cannot see it. The easiest way to approach problems in economic analysis is to draw a picture. The picture should show three things:

1. A time interval divided into an appropriate number of equal periods
2. All cash outflows (deposits, expenditures, etc.) in each period.
3. All cash inflows (withdrawals, income, etc.) for each period.

Unless otherwise indicated, all such cash flows are considered to occur at the end of their respective periods.

Drawing a Cash Flow Diagram

- In a cash flow diagram (CFD) the end of period t is the same as the beginning of period (t+1).

- Beginning of period cash flows are: rent, lease, and insurance payments
- End-of-period cash flows are: O&M, salvages, revenues, overhauls
- The choice of time zero is arbitrary. It can be when a project is analyzed, when funding is approved, or when construction begins
- One person's cash outflow (represented as a negative value) is another person's inflow (represented as a positive value)
- It is better to show two or more cash flows occurring in the same year individually so that there is a clear connection from the problem statement to each cash flow in the diagram
- Arrow lengths are approximately proportional to the magnitude of cash flow.

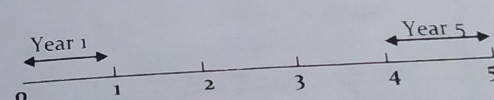
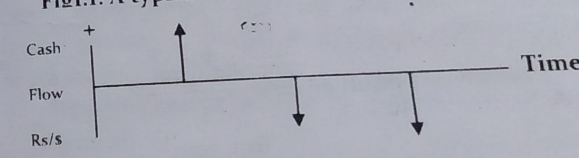
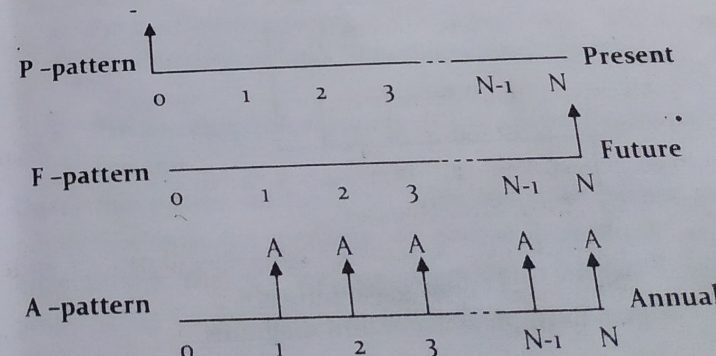


Fig1.1: A typical cash flow time scale for 5 years





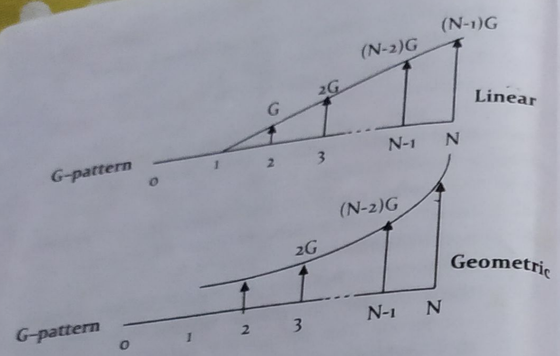


Fig 1.3: Pattern of Cash Flow

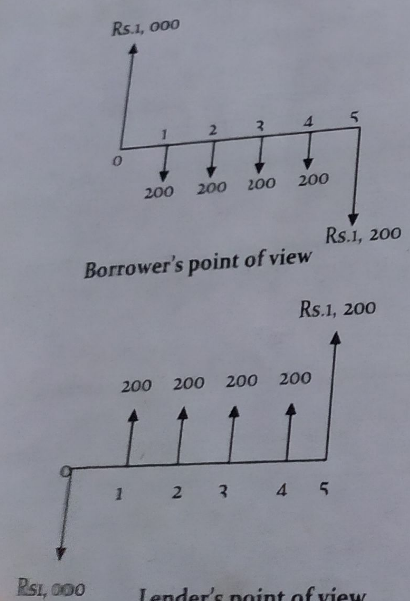


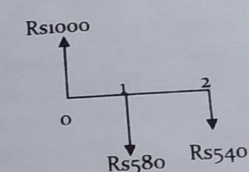
Fig1.4: Example of Cash flow diagrams

Example 1.1

A man borrowed Rs. 1,000 from a bank at 8% interest. Two end-of-year payments: at the end of the first year, he will repay half of the Rs.1000 principal plus the interest that is due. At the end of the second year, he will repay the remaining half plus the interest for the second year.

Cash flow for this problem is:

End of year	Cash flow
0	+Rs 1, 000
1	-Rs 5, 80 (-Rs500 - Rs80)
2	-Rs 5, 40 (-Rs500 - Rs40)



1.7 Economic System

Economic system is the institutional framework within which a society or country carries on its economic activities. Mainly, there are three types of systems:

1. **Private enterprise system (Capitalistic Economic System)**
2. **Pure socialistic system**
3. **Combination of both (Mixed Economic System)**

1. **Private enterprise system (Capitalistic Economic System)**

Under this system, all firms, factories and other means of productions are the property of private individuals and organizations. They are free to use them with a view to making profit. What to produce how to produce and for whom to produce, all these central problems of economic are settled by the free working of the forces of demand and supply.

Features

- *Right of private property*
- *Freedom of enterprise*
- *Freedom of choice by consumers*
- *Profit motive*
- *Class conflict*

Merits

- *Individual initiative*
- *Perfect competition*
- *Dynamic economy*

Demerits

- *Inequality of incomes*
- *Inefficient production*
- *Monopoly and exploitation*
- *Unemployment*

2. *Pure socialistic system*

In pure socialistic system there is no private property. Resources, goods and services are owned and controlled by the government. Production takes place in government enterprises and the government specifies the conditions under which exchange can occur.

Features

- Social ownership of means of production
- No private enterprise
- Economic equality
- Equality of opportunity
- Economic planning
- Social welfare and social security.

Merits

- In the absence of private profit, production will be shifted from more profitable goods to more useful goods.
- Many things, consumption of which is considered essential for health and efficiency, may be supplied free or below cost.
- Socialist economy will prevent cyclical fluctuations in business activity and will bring about smooth working of the economy.

Demerits

- Bureaucratic running of the system
- It will lead to concentration of both political and economic power in the hands of government.
- There is no proper basis of cost calculation and in the absence of such a basis, the economy cannot function in an efficient manner or allocate the resources in the best possible way.

3. Combination of both (Mixed Economic System)

The private enterprise system is decentralized where as socialistic system is highly centralized. In present days, economy is the mixture of socialism and private enterprises. According to mixed economy, some part of an economy's output will be produced by the profit oriented private sectors and another part will be produced in a socialistic manner by the public sector. There are also nonprofit sectors like hospitals, schools etc.

Features

- Co-existence of the public and private sectors
- Role of government directions
- Government protection of labor
- Control of monopoly

Review Questions

1. Define Engineering economics.
2. Why Engineering Economics is necessary for the Engineers? Discuss.
3. Discuss on Principle of Engineering Economics.
4. Discuss briefly the terminologies used in the engineering economics.
5. Suppose that today you borrowed Rs 10,000 from a friend and you asked a friend to repay the loan within 5 years beginning with Rs 2000 at the end of first year, Rs 1,500 at the end of 2^{nd} year, Rs 1,000 at the end of 3^{rd} year, Rs 500 at the end 4^{th} year and 5^{th} year. Draw the cash flow diagram of this transaction.
6. Define economic system. Discuss on its types.

Chapter 2

Cost Classification and Analysis

CHAPTER OUTLINE

- *Elements of Cost*
- *Classification of Cost*
- *Cost Variance Analysis*

Condition of Adverse and favorable variance

Material Variance

Wage Variance

Variable overhead Variance

Fixed overhead Variance

Job Costing and Process Costing

7. The actual data and the data from the standard cost card are taken from a manufacturing company. Calculate all the variances.

Particulars	Standard	Actual
Production (units)	1,800	1,600
Direct materials (kg)	3,600	3,600
Direct material cost (Rs)	720,000	6,00,000
Direct labor days	9,000	8,500
Direct labor cost (Rs)	1,800,000	1,700,000
Fixed overhead (Rs)	100,000	95,000
Variable overhead (Rs)	1,00,00	1,25,000

8. Complete the three process when the output is 100 kg and overhead cost is Rs 1000 (200%of direct wages)

Items	Total Rs	Process		
		I	II	III
Direct material	2,200	1,800	300	100
Direct wages	400	100	200	100
Direct expenses	500	300	-	200

Chapter 3

Interest and Time Value of Money

CHAPTER OUTLINE

- *Interest (Simple interest, compound interest)*
- *Economic Equivalence*
- *Nominal and Effective Interest Rate*
- *Discrete Cash Flow and Discrete Compounding Formulas*
- *Continuous Compounding Formulas*
- *Interest Calculation for Gradient Series*
- *Introduction to Life Cycle Costing*
- *Introduction to Financial and Economic Analysis*

STUDENT LEARNING OBJECTIVE

From studying this chapter you will learn

- To understand the concept of simple interest and compound interest
- To understand the concept of economic equivalence
- To derive the formula for compound interest
- To derive the formula for the discrete cash flow and discrete compounding
- To derive the formula for continuous compounding and continuous cash flow
- To derive the formula for gradient series (linear and geometric)
- To solve the numerical problems of the discrete and continuous compounding and gradient series

3.1 Time value of Money

Time value of money is defined as the time-dependent value of money stemming both from changes in purchasing power of money (inflation or deflation) and from the real earning potential of alternative investments over time. Since money has the ability to earn interest, its value increases with time. Hence it is the relationship between interest and time.

Interest

Most of us are familiar in a general way with the concept of interest. We know the money left in the savings account earns interest so that the balance over time is greater than the sum of the deposits. Whenever we go for any investment, we will have to consider the following three factors:

(a) Liquidity

Once it is invested, it is not so easy to convert it to cash and when needed immediately, we will not be able to spend on another project or on other financial expenses. In other words, it is the reward for not being able to use your money while you are holding the stock or mortgage or promise.

(b) Risk premium

There is always a certain degree of risk associated with any financial investment. For example, if you lend someone Rs 1,000, it is not sure that you may get it back either because of his nature or market scenario. The situation is worse in case when you are making investment on businesses, shares etc. where you might also loose your principal amount. It is common that most of the people fear for investing, knowingly or unknowingly they are conscious about risks associated with it. In other words, it is the reward for any chance that you would not get your money back or that it will have declined in value while invested.

(c) Inflation factor

Purchasing power of money goes down at a constant rate annually and we call it inflation. The money we invested should at least earn to cover the loss in its value due to inflation. In other words, it is the compensation for decrease in purchasing power between the time you invest it and time it is returned to you.

Every investor, because of these factors, looks for some return on their investment and charges a cost of investment known as *interest rate*. It is a percentage that is periodically applied and added to an amount (or varying amounts) of money over a specified length of time. When money is borrowed, the interest paid is the charge to the borrower for the use of the lender's property; when money is loaned or invested, the interest earned is the lender's gain from providing a good to another. Interest, then may be defined as the cost of having money available for use.

Elements of transaction involving interest

1. An initial amount of money that, in transaction involving debt or investment, is called the **Principal.**

2. The **interest rate** that measures the cost or price of money and that is expressed as a percentage per period of time.
3. A period of time called **interest period** that determines how frequently interest is calculated.
4. A specified length of time that marks the duration of the transaction and thereby establishes a certain **number of interest periods**.
5. A future amount of money that results from the cumulative effects of the interest rate over a number of interest periods.

Simple Interest: It uses fixed percentage of the principal (the amount of money borrowed) i.e. if the total amount of interest earned is directly proportional to the initial principal amount, then the interest is said to be simple.

For a deposit of P dollars at a simple interest rate of i for N periods, the total earned interest I would be $\mathbf{I = (i{*}P)\,N}$

The total amount available at the end of N periods, F, thus would be

$$F = P + I = P(1 + iN)$$

Compound interest: When the total time period is subdivided into several interest periods (one year, half yearly, quarterly, monthly, weekly); interest is credited at the end of each interest period, and is allowed to accumulate from one interest period to next, then the interest is said to compounded.

3.2 Derivation of compound interest formula (single cash flow)

(To find the single future sum (F) of the initial payment P)

During a given interest period, the current interest is

determined as a percentage of the total amount owed (i.e. principal plus the previous accumulated interest)

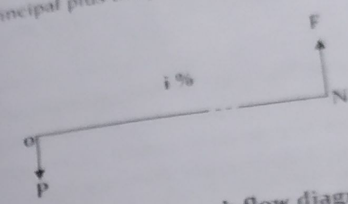


Fig: 3.1Single cash flow diagram

For the 1st interest period

Interest $I_1 = i{*}P$

Total accumulated amount at the end of 1st year

$F_1 = P + I_1 = P + i{*}P = P(1+i)$

For the 2nd interest period

Interest $I_2 = i{*}F_1 = i{*}(1+i)$ Total accumulated amount at the end of 2nd year

$F_2 = F_1 + I_2 = P(1+i) + i{*}(1+i)P = P(1+i)^2$

For the 3rd interest period

$I_3 = F_2{*}i = P(1+i)^2{*}i$

Total accumulated amount at the end of 2nd year

$F_3 = F_2 + I_3 = P(1+i)^2 + P(1+i)^2{*}i = P(1+i)^3$

Continuing,

If there is 'N' interest period

$$F = P(1+i)^N$$

The factor in the bracket is called the **Single Payment Compound Factor**

Functionally, $F = P(F/P, i\%, N)$

(To find the initial payment (P) of the single future sum (F))

$$P = F(1+i)^{-N}$$

The factor in the bracket is called the ***Single Payment Present worth factor***

Functionally, $P = F\,(P/F, i\%, N)$

Example3.1 -

Suppose you deposit Rs 1,000 in savings account that pays interest at a rate of 8%, compounded annually. Assume that you don't withdraw the interest earned at the end of each period (year), but let it accumulate. How much would you have at the end of year 3?

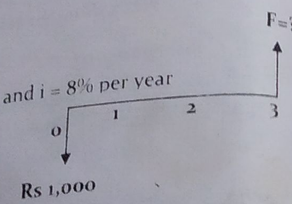
Solution

Given: P = Rs 1,000, N=3 years, and i = 8% per year

$F = P\,(F/P, i\%, N)$

$F = \text{Rs } 1{,}000\,(1+0.08)^3$

F = **Rs 1,259.71 (Ans)**



Example3.2

You have just purchased shares @Rs 100 per share. If you expect the stock price to increase 20% per year, how long do you expect to double its market price?

Solution

$F = 2P$

$P\,(1+0.2)^N = 2P$

$1.2^N = 2$

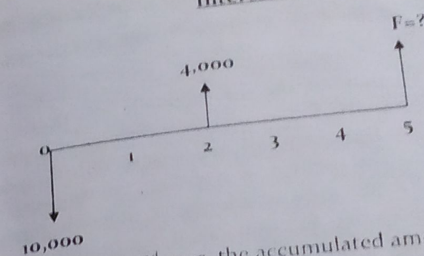
Taking log on both sides

$N \log 1.2 = \log 2$

$N = \log 2 / \log 1.2 = 3.80 \approx 4$ **years (Ans)**

Example3.3

Mr. X deposits Rs 10,000 now in a bank which gives 8% interest per year. He draws Rs 4,000 at the end of 2nd year. What will be the remaining amount at the end of 5th year?



At the end of the 2nd year, the accumulated amount will be

$10{,}000\,(F/P, 8\%, 2) = 10{,}000\,(1+0.08)^2 = \text{Rs } 11{,}664$

After drawing 4000, the remaining deposit amount at the end of 2nd year will be,

$11{,}664 - 4{,}000 = \text{Rs } 7{,}664$

At the end of the fifth year the total accumulated amount will be

$7{,}664\,(F/P, 8\%, 3) = 7{,}664\,(1+0.08)^3 =$ **Rs 9,654.5 (Ans)**

Alternatively

$F = \{10{,}000\,(1.08)^2 - 4{,}000\}\,(1.08)^3$

= **Rs 9,654.5 (Ans)**

RULE OF 72

Rule of 72 can determine approximately how long it will take for a sum of money to 'double'. The rule states that "to find the time it takes for the present sum of money to grow by a factor of 2, we divide 72 by the interest rate"

As in the previous example 3.2, 72/20 = 3.60 or roughly we can say 4 years

3.3 Economic Equivalence

Which option would you prefer?

- Receiving Rs. 20,000 today and Rs. 50,000 ten years from now.

- Receiving Rs. 8,000 each year for the next ten years.

Calculations for determining the economic effects of one or more cash flows are based on the concept of economic equivalence. The time value of money and the interest rate helps to develop the concept of economic equivalence which means that different sums of money at different times are equal in economic value. For example, if the interest rate is 6% per year, Rs 100 today (present time) is equivalent to Rs 106 one year from today. If someone offered you a gift of Rs 100 today or Rs 106 one year from today, it would make no difference which offer you accept. In either case you have Rs 106 one year from today. However, the two sums of money are equivalent to each other only when the interest rate is 6% per year. At a higher or lower interest rate, Rs 100 today is not equivalent to Rs 106 one year from today.

Calculations for determining the economic effects of one or more cash flows are based on the concept of economic equivalence. ***Economic Equivalence*** refers to the fact that a cash flow- whether single payment or a series of payment- can be converted to an equivalent cash flow at any point in time. Economic Equivalence exists between cash flows that have the same economic effect.

Example3.4

Suppose we invest Rs. 1000 at 12% annual interest for 5 years.

From Compound interest formula,

$F=P(1+i)^N$ expresses the equivalence between present amount, P, and future amount, F, for given interest, i, and number of interest periods N.

$F = Rs.\ 1000(1+0.12)^5 = Rs.\ 1762.34$

We can say that at 12% interest, Rs. 1000 received now is equivalent to Rs. 1762.34 received in 5 years, and that we can

trade Rs. 1000 now for the promise of receiving Rs. 1762.34 in 5 years.

Example3.5

Suppose you are offered the alternative of receiving either Rs. 3000 at the end of 5 years or P Rs today. There is no question that the Rs. 3000 will be paid in full (no risk). Because you have no current need for the money, you would deposit the P Rs in an account that pays 8% interest. What value of P would make you indifferent to your choice between P Rs today and the promise of Rs. 3000 at the end of 5 years?

Solution

Our job is to determine the present amount that is economically equivalent to $3000 in 5 years given the investment potential of 8% per year.

Given: F = Rs. 3000, N = 5 years, i=8% per year.

P = F (1+i)-N = 3000 (1+0.08)-5 = Rs. 2042

- If P is anything less than Rs. 2042, you would prefer the promise of Rs. 3000 in 5years to P Rs today.
- If P were greater than Rs. 2042, you would prefer P.

Equivalence calculation: General Principles

Principle 1

Equivalence calculations made to compare alternatives requires a common time basis.

- In example 3.5, if we had been given magnitude of each cash flow and had been asked to determine their equivalency, we should choose the reference point and find the value of each cash flow at that point.
- For selecting the reference point, commonly present time (present worth) or some point in future (future worth) is used.
- The choice of point is chosen as per convenience.

Example3.6

In example 3.5, we determined that, given an interest rate of 8% per year, receiving Rs. 2042 today is equivalent to receiving Rs. 3000 in 5 years. Are these cash flows also equivalent at the end of year 3?

Solution

Here base period is 3 years.

In fig (a)

P = Rs. 2042, i=8% per year, N = 3 years

In fig (b)

F = Rs. 3000, i=8% per year, N = 2 years

Figure (a)

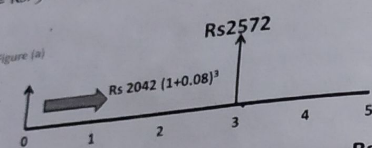
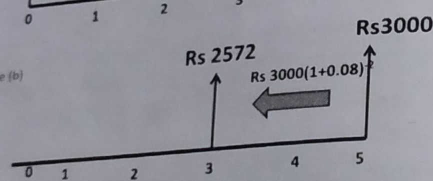


Figure (b)



Principle 2

Equivalence depends on interest rate

- The equivalence between two cash flows is a function of both the cash flow pattern and the interest rate that operates on those cash flows.
- Change in the interest rate will destroy the equivalence between these two sums.

Example3.6

In example 3.5, we determined, at an interest rate of 8% per year, receiving Rs. 2042 today is equivalent to receiving Rs. 3000 in 5 years. Are these cash flows equivalent at an interest rate of 10%?

Solution

Lets select base period N= 5 years.

F = Rs. 2042 (1+0.1)5 = Rs. 3829

Since the amount is > Rs. 3000, change in interest rate destroys the equivalence between two cash flows.

Principle 3

Equivalence Calculations may require the conversion of multiple payment cash flows to a single cash flow.

Example3.7

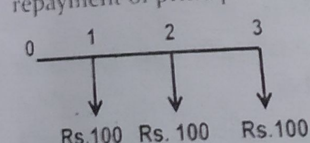
Suppose that you borrow Rs. 1000 from a bank for 3 years at 10% annual interest. The bank offers two options: (1) Repaying the interest charges for each year at the end of that year and repaying the principal at the end of 3 year. (2) Repaying the loan all at once (including both interest and principal) at the end of year 3.

Option	Year 1	Year 2	Year 3
End of year repayment of interest and principal repayment at end of loan.	Rs. 100	Rs.100	Rs.1100
One end of loan repayment of both principal and interest	0	0	Rs. 1331

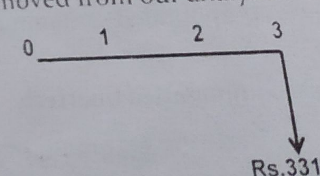
Determine whether these options are equivalent, assuming the interest rate for comparison is 10%?

Solution

Since we pay the principal after 3 years in either plan, the repayment of principal can be removed from our analysis.



Option 1



Option 2

Since option 2 is already a single payment at n=3 years, it is simplest to convert option 1 cash flow pattern to a single value at n=3

We must convert the three disbursement of option 1 to their respective values at n=3.

F_3 for Rs. 100 at n=1: Rs. 100 $(1+0.10)^{3-1}$ = Rs. 121

F_2 for Rs. 100 at n=2: Rs. 100 $(1+0.10)^{3-2}$ = Rs. 110

F_1 for Rs. 100 at n=3: Rs. 100 $(1+0.10)^{3-3}$ = Rs. 100

Total = Rs. 331

3.4 Nominal and Effective interest rate

If a financial institution uses a unit of time other than a year – a month or a quarter (e.g. when calculating interest payments), the institution usually quotes the interest rate on an annual basis. Commonly this rate is stated as

r% Compounded M-ly

Where, r = the nominal interest rate per year

M = the compounding frequency or the number of interest periods per year

r/M = the interest rate per compounding period.

Suppose that if a bank express the interest rate as "18% compounded monthly", we say that 18% is the *nominal interest rate or annual percentage rate (APR)* and the compounding frequency is monthly i.e. number of interest period per year is 12.

A nominal interest rate r may be stated for any time period – 1 year, 6 months, quarter, month, week day etc. Let us see the following examples for expressing the nominal interest rate.

r = 12% compounded semiannually, M=2,
i.e.12%/2 (6% per 6 months)

r = 12% compounded quarterly, M=4,
i.e. 12%/4 (3% per 3 months)

r = 18% compounded monthly, M=12,
i.e. 18%/12 (1.5% per month)

r = 15%compunded weekly, M= 52
i.e.15%/52 (0.29% per week)

When M $\rightarrow \infty$, compounded continuously

Suppose that Rs 1,000 to be invested at a nominal rate of 12% compounded semiannually.

The interest earned during first six months is 1,000* 0.12/2
= Rs 60

Total principal at the end of the first six months = Rs (1,000+60)
= Rs 1,060

Interest earned during the second six months is Rs 1,060*0.12/2
= Rs 63.60

Total interest at the end of 1 year = Rs 60+ Rs 63.60
= Rs 123.60

The effective annual interest rate for the entire year = 123.60/1,000 * 100 = 12.36%

The *exact or the actual rate* of interest earned on the principal during one year is known as the ***effective interest (i)***. The effective interest rates are always expressed on an annual basis unless specifically stated otherwise.

Relation between effective (i) and nominal (r) interest rate

$i = (1 + r/M)^M - 1$, M is the compounding period per year.

As from the above example, the effective interest rate for 12% compounded semi annually,

$i = (1 + r/M)^M - 1 = (1+0.12/2)^2 - 1 = 12.36\%$

Alternatively

$i_N = (1+i)^M - 1$,

Where, N = number of compounding periods.

i = interest rate per compounding period

M = compounding periods per year

Example 3.8

What is the effective interest rate of the nominal interest rate 9% per year if the compounding is a) yearly b) quarterly c) monthly (d) daily (e) continuously ($N \longrightarrow \infty$)

Solution

For compounding yearly,

$i = (1+0.09/1)^{1} - 1 = 0.09 =$ 9%

For compounding quarterly,

$i = (1+0.09/4)^{4} - 1 = 0.09308 =$ 9.308%

For compounding monthly,

$i = (1+0.09/12)^{12} - 1 = 0.09380 =$ 9.380%

For compounding daily,

$i = (1+0.09/365)^{365} - 1 = 0.0941 = 9.41\%$

For compounding continuously,

$i = (1+0.09/\infty)^{\infty} - 1 = e^{r} - 1 = e^{0.09} - 1 = 9.417\%$

Example 3.9

A person deposits a sum of Rs 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity of the deposit after 10 years.

Solution

P = Rs 5,000, N = 10 years, i=12%compounded quarterly

Interest period in a year = 4

Total interest period for 10 years = 40

i= 12%/4 = 3%

Using single payment compound amount factor

$F = P(F/P, 3\%, 40) = 5{,}000(1+0.03)^{40} =$ **Rs 16,310 (Ans).**

Alternatively

iyear =(1+0.12/4)4 -1 = 12.5508% per year

N =10 years

F = 5,000 (F/P, 12.55%, 10) = 5,000 (1+0.125508)10

= **Rs 16,310 (Ans)**

3.5 Development of Interest formulas

As we begin to compare series of cash flows instead of single payments, the required analysis becomes more complicated. However, when patterns in cash flow transactions can be identified, we can take advantage of these patterns by developing concise expression for computing either present or future worth of the series. We will classify the five major categories of cash flow transactions; develop interest formulas for them and present several working examples for them.

Types of Cash Flow

1. ***Single cash flow:*** the simplest case involves the equivalence of a single present amount and its future worth. The single cash flow formulas deal with the only two amounts; a single present amount P and its future worth F.

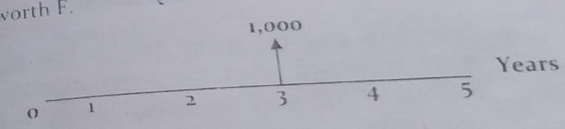


Fig: Single cash flow

2. **Equal (uniform) series:** In this type, transactions are arranged as a series of equal cash flows at regular intervals, known as an equal payment series (uniform series) (fig: (b). This describes the cash flows of the common installment loan contract, which arranges repayment of the loan in equal periodic installments.

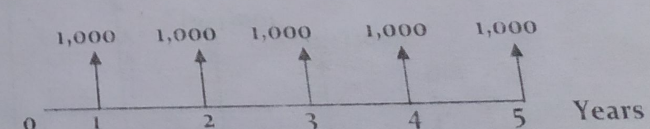


Fig: Equal (uniform) payment series

3. ***Linear gradient series:*** While many transactions involve series of cash flows, the amounts are not always uniform; they may vary in some regular way. One common pattern of variation occurs when each cash flow in a series increases (or decreases) by a fixed amount. For example, A 10 year loan repayment plan might specify a series of annual payments that increase by Rs 1000 each year. This type of cash flow pattern is called linear gradient series.

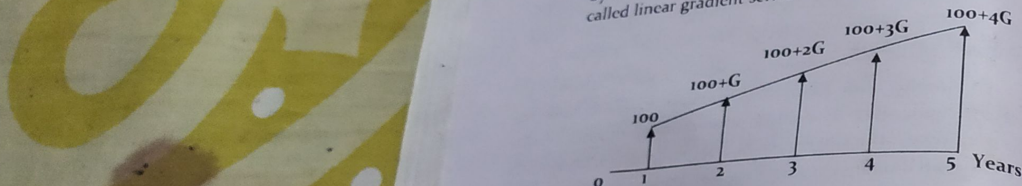


Fig: Linear Gradient series

4. **Geometric gradient series:** Another kind of gradient series is formed when the series in cash flow is determined, not by a fixed amount like Rs 1,000, but by some fixed rate, expressed as a percentage. For example, in a 10 year financial plan for a project, the cost of particular raw material might be budgeted to increase at a rate of 4% per year. The curving gradient in the diagram of such a series suggests its name which is geometric gradient series. However, we don't deal with the formulas for geometric gradient series.

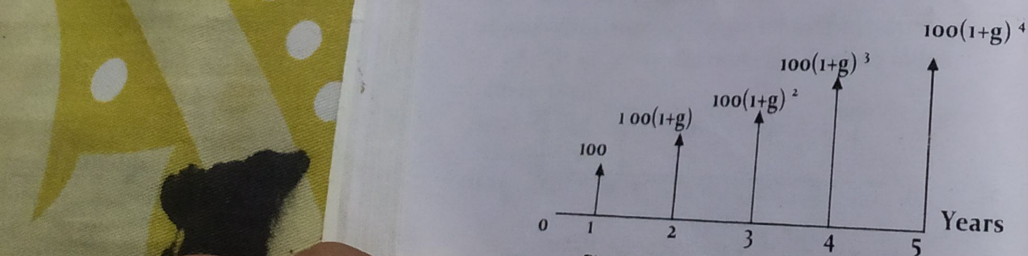


Fig: Geometric Gradient series

5. ***Irregular Series:*** A series of cash flow may be irregular. It doesn't exhibit an overall regular pattern.

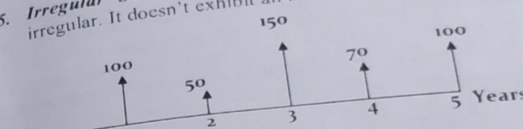


Fig: Irregular series

Factor Notation

We will express the resulting compound interest factors in a conventional notation that can be substituted in a formula to indicate precisely which factor to use in solving an equation. In the preceding examples the formula derived for the single cash flow as $F = P(1+i)^N$. In ordinary language, this tells us that to determine what future amount F is equivalent to a present amount P, we need to multiply P by a factor expressed as 1 plus the interest rate, raised to the power given by the number of interest periods. The factor in the functional form is expressed as: (F/P, i%, N) which is read as "Find F, when P, i, and N given". The factor notation is included for each of the formulas derived in the following sections.

3.6 Discrete compounding and discrete cash flow

Interest formula relating a uniform (equal) series

(1) To Find F when A is given

Let's consider the following cash flow as shown at the end of each period.

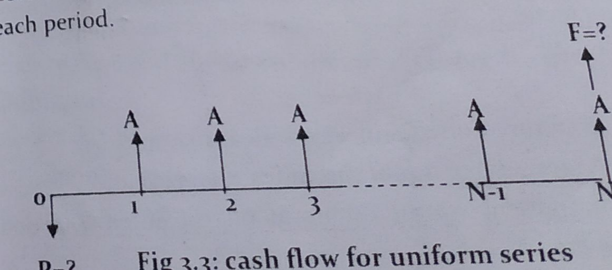


Fig 3.3: cash flow for uniform series

If the amount 'A' occurs at the end of each period for N periods, and i% is the interest rate per period, the Future worth (F) at the end of the last (Nth) period is:

$F = A \{(1+i)^{N-1} + (1+i)^{N-2} + (1+i)^{N-3} + \ldots\ldots + (1+i)^{1} + 1\}$

The bracketed term comprise a geometric progression with common ratio (1+i)

$S_N = \frac{a_1 - b a_N}{1-b}$ $(b \neq 1)$

Where a_1 = first term, b= common ratio, a_N = last term

If we let $b = (1+i)^{-1}$, $a_1 = (1+i)^{N-1}$ and $a_N =$ 1, then

$$F = A \left\{(1+i)^{N-1} - (1+i)^{-1} * 1\right\} / \left\{1 - (1+i)^{-1}\right\}$$

$$F = A \{((1+i)^N - 1)/i\}$$

Alternatively,

$F = A (1+i)^{N-1} + A (1+i)^{N-2} + \ldots\ldots + A(1+i) + A \ldots\ldots\ldots\ldots (1)$

Multiplying both sides by (1+i)

$F (1+i) = A (1+i)^N + A (1+i)^{N-1} + \ldots\ldots\ldots + A (1+i)^2 + A (1+i) \ldots\ldots\ldots (2)$

Subtracting equation (1) from equation (2), we get

$F (1+i) - F = A (1+i)^N - A$

$F^* i = A \{(1+i)^N - 1\}$

$$F = A \{((1+i)^N - 1)/i\}$$

The quantity in a bracket is called the *uniform series compound amount factor.*

Functionally, F = A (F/A, i%, N)

Note: F (future worth) coincides with last annuity 'A'.

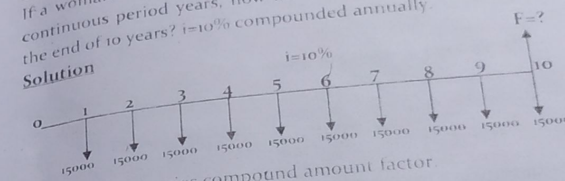
A (annuity) occurs at the end of each period

P (present worth) occurs at on the interest period before the first 'A'

Example 3.10

If a woman deposits Rs 15,000 at the end of each year for 10 continuous period years, how much money is accumulated at the end of 10 years? i=10% compounded annually.

Solution



Using uniform series compound amount factor.

F = A (F/A, 10%, 10)

$F = 15,000 \{((1+0.1)^{10} - 1)/0.1\}$

F = **Rs 2, 39,061.36 (Ans)**

(1) To Find P when A is given

Consider the cash flow diagram in fig: 3.3

We know, $F = A [(1+i)^N - 1] / i$

Also, $F = P (1+i)^N$

$P (1+i)^N = A [(1+i)^N - 1] / i$

$$P = A [(1+i)^N - 1] / [i * (1+i)^N]$$

The quantity in a bracket is called the *uniform series present worth factor*

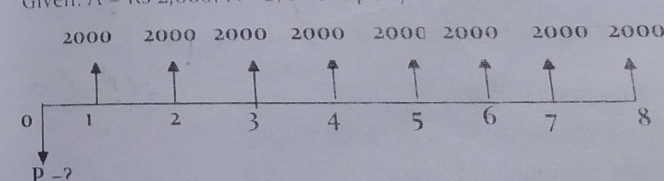
Functionally, P = A (P/A, i%, N)

Example 3.11

How much money should you deposit now in a saving account earning 10% interest rate compounded annually so that you may make 8 end of year withdrawal of Rs 2000 each?

Solution

Given: A = Rs 2,000, N =8, i=10% per year



Using uniform series present worth factor

$P = A\,(P/A, 10\%, 8)$

$P = 2{,}000\ [(1+0.1)^{8} - 1] / [i * (1+0.1)^{8}]$

$P =$ **Rs. 10, 669.85 (Ans)**

(2) To Find A when F is given

We know, $F = A\,\{[(1+i)^{N} - 1]/i\}$

$$A = F\,[i / ((1+i)^{N} - 1)]$$

The quantity in a bracket is called the *sinking fund factor*

Functionally, $A = F\,(A/F, i\%, N)$

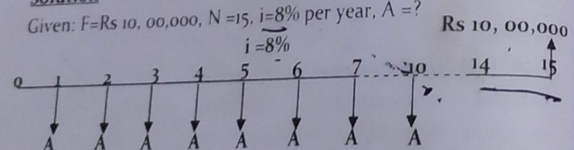
Sinking fund is an interest bearing account into which a fixed sum is deposited each interest period: it is commonly established for the purpose of replacing fixed assets. A/F, 8%, 10)

Example 3.12

Mr. Ramesh wants to have Rs 10, 00,000 for the studies of his daughter after the period of 15 years. How much rupees does he has to deposit each year for 10 continuous years in a saving account that earns 8% interest annually. (TU, IOE-2061)

Solution

Given: F=Rs 10, 00,000, N =15, i=8% per year, A =?



Discounting Rs. 10, 00,000 to the year 10 $(1+i)^{N}$ e^{rN}

$P = 10, 00,000\ (P/F, 8\%, 5)$

$= 10, 00,000\ (1+0.08)^{-5} =$ Rs. 6, 80,583.197

Using the sinking fund factor

$A = F\,(A/F, 8\%, 10)$

$A = 6, 80{,}583.197\ [i / ((1+i)^{N} - 1)]$

$A = 6, 80{,}583.197\ [0.08 / ((1.08)^{10} - 1)]$

$A =$ **Rs 46,980.31 (Ans)**

e^{rN} $e^{rN} - 1$ $\frac{e^{rN} - 1}{e^{r} - 1}$

Example 3.13

How many deposits of Rs 25,000 each should Dr. Thakur make each month so that the final accumulated amount will be Rs 10, 00,000 if the bank interest rate is 12% per year? (TU, IOE 2063)

Solution

Given: A = 25,000 per month, F = 10, 00,000, i =12% per year

Monthly interest rate, $i_m = (1 + i_{yr})^{1/12} - 1$

$i_m = (1+i_{year})^{1/12} - 1$

$i_m = (1+0.12)^{1/12} - 1$

$i_m = 0.0094 = 0.94\%$ i_m

Using the uniform series compound amount factor

$F = A\,(F/A, 0.94\%, N)$ N $F = A\,(F/A, i\%, N)$

$10, 00{,}000 = 25{,}000\ \{((1+0.0094)^{N} - 1)/ 0.0094\}$

$0.376 = (1.0094)^{N} - 1$

$1.376 = (1.0094)^{N}$

Taking log on both sides

Log 1.376 = N Log 1.0094

N = 34, Dr. Thakur should make **34 deposits (Ans)**

(1) To Find A when P is given $P = A\,\frac{(1+i)^{N} - 1}{i(1+i)^{N}}$

We know, $P = A\,[(1+i)^{N} - 1] / [i * (1+i)^{N}]$

$$A = P\,[i^{*}(1+i)^{N}] / [(1+i)^{N} - 1]$$

The quantity in a bracket is called the *capital recovery factor*

Functionally, **A= P (A/P, i%, N)**

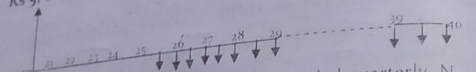
Capital recovery is the annual equivalent of capital cost

Example 3.14

A man aged 40 years now had borrowed Rs. 5, 00,000 from a bank for his further studies at the age of 20 years. Interest was charged at 11% per year compounded quarterly. He wished to pay loan in semiannual equal installments with the first installment beginning 5 years after receiving the loan. He has just cleared the loan now. What amount did he pay in each installment? (TU, IOE-2062)

Solution

Rs 5, 00,000



Given: P = Rs 5, 00,000, i=11%per compounded quarterly, N=20 years, A =?

Quarterly interest rate,

$i_q = 11\%/4 = 2.75\%$

Semiannual interest rate

$i_{semi} = (1+i_q)^2 - 1$

$i_{semi} = (1+0.0275)^2 - 1$

$i_{semi} = 5.57\%$

Using the single payment compound amount factor

F =5, 00,000 (F/P, 5.57%, 40) .... (1)

Using the uniform series compound amount factor

F = A (F/A, 5.57%, 30) .... (2)

Equating equation (1) and (2)

$5,00,000\,(1+0.057)^{40} = A\,\{((1+0.057)^{30} - 1)/0.057\}$

A = Rs 61,217.3 is the semiannual payment (Ans)

Interest factor and symbols

To find	Given	Factor	Factor name	Functional symbol
SINGLE CASHFLOW				
F	P	$(1+i)^N$	Single payment compound amount	{F/P, i%, N}
P	F	$1/(1+i)^N$	Single payment present equivalent	{P/F, i%, N}
UNIFORM SERIES				
F	A	$(1+i)^N -1/ i$	Uniform series compound amount	{F/A, i%, N}
P	A	$\frac{(1+i)^N-1}{i*(1+i)^N}$	Uniform Series present worth	{P/A, i%, N}
A	F	$\frac{i}{(1+i)^N-1}$	Sinking fund	{A/F, i%, N}
A	P	$\frac{i*(1+i)^N}{(1+i)^N-1}$	Capital recovery	{A/P, r%, N}

3.7 Continuous compounding and continuous compounding formulas

(a) Continuous compounding and discrete cash flow

The interest formula (for this) assumes, cash whenever it is available, can usually be used profitably. Here cash flows occurs at discrete interval (e.g. once per year) but it is compounded continuously throughout the interval.

Let the nominal rate of interest per year be r, if the interest is compounded continuously for M times and if the principal amount is Rs 1 then the amount at the end of year will be

$1\,(1+r/M)^M$

$= (1+r/M)^M - 1$ .... (1)

Let $M/r = p$, $M = pr$

Equation one becomes $(1+1/p)^{pr} = \{(1+1/p)^p\}^r$

Here as $M \to \infty$, $p \to \infty$

As $\lim_{P \to \infty} (1+1/p)^p = e$

$\lim_{M \to \infty} (1+(1/p))^{pr} = e^r - 1$