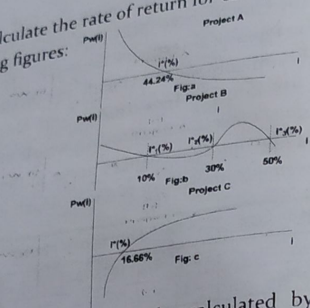
Project B is the non –Simple investment
Project C is the simple borrowing

If we calculate the rate of return for each project, we obtain the following figures:




The IRR of a project can be calculated by the following methods.

1. **Direct Solution Method**
2. **Trial and error method**
3. **Computer Solution method**

1. **Direct Solution Method**

For the very special case of a project with only a two-flow transaction (an investment followed by a single future payment) or service life of 2 years of return, we can apply direct mathematical solution for determining the rate of return.

**Example 4.15**

| Period (n) | Project 1 | Project 2 |
|---|---|---|
| 0 | - Rs1,000 | -Rs2,000 |
| 1 | Rs0 | Rs1,300 |
| 2 | Rs0 | Rs 1,500 |
| 3 | Rs0 | |
| 4 | Rs1500 | |





Consider two investment projects with the cash flow transactions as above. Compute the rate of return for each project.

**Project A**

$$Rs1,000 = Rs1,500(P/F,i,4)$$
$$Rs1,000 = Rs1,500(1+i)^{-4}$$
$$0.6667 = (1+i)^{-4}$$
$$\frac{\ln 0.6667}{-4} = \ln(1+i)$$
$$0.101365 = \ln(1+i)$$
$$e^{0.101365} = 1+i$$
$$i = e^{0.101365} - 1$$
$$= 10.67\%$$

**Project B**

$$PW(i) = -Rs2,000 + \frac{Rs1,300}{(1+i)} + \frac{Rs1,500}{(1+i)^2} = 0$$

Let $x = \frac{1}{1+i}$, then

$$PW(i) = -2,000 + 1,300x + 1,500x^2$$

Solve for $x$:

$x = 0.8$ or $-1.667$

Solving for $i$ yields

$$0.8 = \frac{1}{1+i} \rightarrow i = 25\%, \quad -1.667 = \frac{1}{1+i} \rightarrow i = -160\%$$

Since $-100\% < i < \infty$, the project's $i^* = 25\%$.

1. **Trial and error method**

Step 1: Guess an interest rate, say $i$ =15%

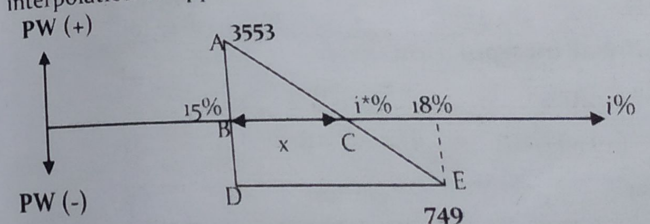
Step 2: Compute PW ($i$) at the guessed $i$ value. (Consider the project C of example 4.8)

PW (15%) = Rs 3,553

Step 3: If PW ($i$) > 0, then increase $i$. If PW ($i$) < 0, then decrease $i$.

PW (18%) = - Rs 749

Step 4: If you bracket the solution, you use a linear interpolation to approximate the solution.




Here, i*% = 15% + BC

Using the equilateral triangle,

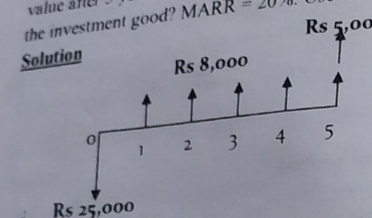
BC/DE = AB/AD

x/(18-15)% = 3553/ (3553 +749)
x= 3% *[3553/ (3553+ 749)] = 2.45%
i*% = 15% + 2.45% = 17.45%
If i*> MARR, select the project.
If i*< MARR, reject the project
*This rule does not work for a situation where an investment has multiple rates of return*

**Example 4.16**

Consider the project with the initial investment = Rs 25,000, salvage value after 5 years = Rs 5,000, Net revenue = Rs.8, 000 / year. Is the investment good? MARR = 20%. Use IRR method.

**Solution**




**Using PW formulation**

PW ($i^*$) = 0

PW inflow - PW outflow = 0

5,000 (P/F, $i^*$15) + 8000 (P/A, $i^*$, 5) = 25,000

$$5{,}000(1/(1+i^*)^5)+8{,}000\frac{(1+i^*)^5-1}{i^*(1+i^*)^5}-25{,}000=0 \quad \text{.................} \quad (1)$$

By trial and error

| | | |
|---|---|---|
| At | $i^*$= 20%, | PW = 934.30 |
| | $i^*$= 25% | PW = -1847.10 |

**Using linear interpolation**

$$\frac{25\%-20\%}{934.30-(-1847.10)} = \frac{i^*\%-20\%}{934.30-0}$$

$i^*$ = 21.679%

Putting $i^*$ = 21.679% in eq.1, PW (21.679%) = -58.20

IRR lies between 20% and 21.679%

**Again using linear interpolation**





$$\frac{21.679\%-20\%}{934.30-(-58.20)} = \frac{i^*-20\%}{934.30-0}$$

$i^*$ = 21.58%

Putting $i^*$ = 21.58% in eq.1

PW (21.58%) = -1.40

IRR lies between 20% and 21.58%

**Again, interpolating**

IRR, $i^*$ = 21.577%, where PW (21.577%) = 0

**Here, IRR > MARR, investment is acceptable**

**Unrecovered Project Balance Calculation**

| Year | Unpaid balance at beginning year (Rs) | Return on Unpaid Balance (Rs) | Payment received | Unpaid balance at end of year (Rs) |
|---|---|---|---|---|
| 0 | -25,000 | 0 | 0 | -25,000 |
| 1 | -25,000 | -5,394.25 | 8,000 | -22,394.25 |
| 2 | -22,394.25 | -4,832 | 8,000 | -19,226.25 |
| 3 | -19,226.25 | -4,148.45 | 8,000 | -15,374.70 |
| 4 | -15,374.70 | -3317.40 | 8,000 | -10,692 |
| 5 | -10,692 | -2307 | 13,000 | 0 |

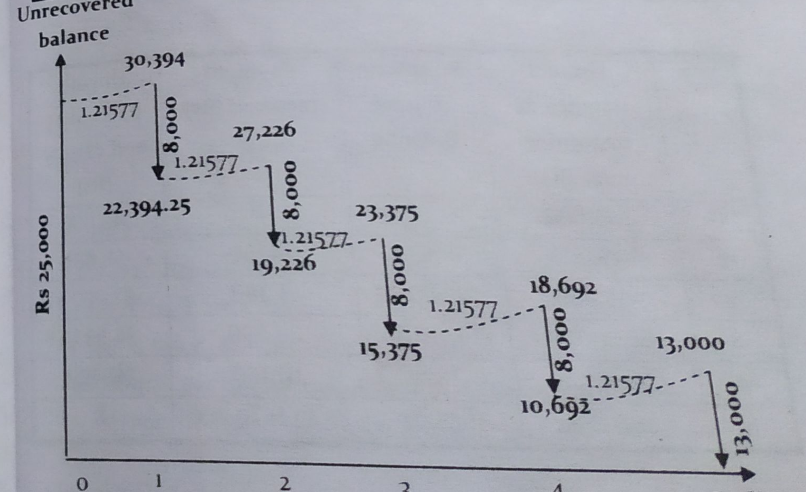



Fig: Investment Balance diagram of example 4.10

**Example 4.17**
Consider the investment project which has a project cash flow as follows. Is the project feasible if MARR = 10%. Use IRR method and draw investment balance diagram.

| End of year | Net Cash flow |
|---|---|
| 0 | - Rs 45,000 |
| 1 | -Rs 4250 |
| 2 | Rs 9280 |
| 3 | Rs 38600 |
| 4 | Rs 61460 |
| 5 | -Rs 20220 |

**Solution**

**Using PW formulation**

PW inflow – PW outflow = 0

Or, 4250 (P/F, i*, 1) + 92800 (P/F, i*, 2) + 38600 (P/F, i*, 3) +61460 (P/F, i*, 1) - 20220 (P/F, i*, 5) – 45,000 =0

By solving we get i*% = 21.40%

Unrecovered Project Balance Calculation

| Year | Unpaid balance at beginning year (Rs) | Return on Unpaid Balance | Payment received (Rs) | Unpaid balance at end of year (Rs) |
|---|---|---|---|---|
| 0 | -45,000 | 0 | 0 | -45,000 |
| 1 | -45,000 | -9,630 | -4250 | -58,880 |
| 2 | -58,880 | -12,600.32 | 9,280 | -62,200.32 |
| 3 | -62,200.32 | -13310.86 | 38,600 | -36,911.18 |
| 4 | -36,911.18 | -7899 | 61460 | 16649.8 |
| 5 | 16649.8 | 3563.06 | 20,212≈20,200 | 0 |





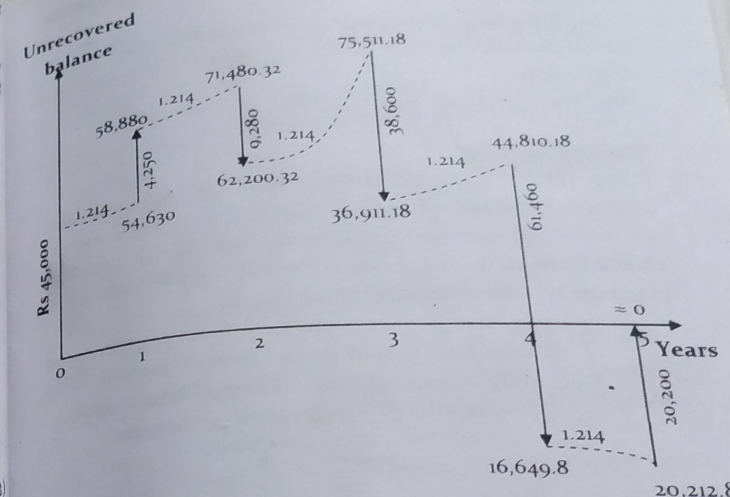



**Fig: Investment balance diagram of example 4.11**

2. **Computer Solution Method**
   Consider the example 4.12,
   PW (i* %) = 0

8,000 (P/A, i*%, N) + 5,000 (P/F, i*%, N) -25,000 = 0

$8{,}000\ ((1+i^*)^5 - 1)/((1+i^*)^{5} * i) + 5{,}000\ (1+i^*)^{-5} - 25{,}000 = 0$

$8\ ((1+i^*)^5 - 1)/((1+i^*)^{5} * i^*) + 5\ (1+i^*)^{-5} - 25 = 0$

**Write the equation in calculator as**

**$8\ ((1+A)^5 - 1)/((1+A)^{5} * A) + 5\ (1+A)^{-5} - 25 = 0$**

Press SHIFT → CALC

**It may take some time to calculate. The value in the calculator gives the value of A and this is the IRR.**

**In Microsoft Excel 2007and 2010,**

Click Formula

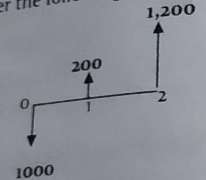
Click Auto Sum

Click More Function
Type IRR in Search for a Function, Click Ok
Insert the value of Cash Flow
Click Enter.
The result gives the value of IRR.

**Drawbacks of IRR**

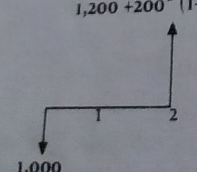
Using IRR is common. Unfortunately, its misuse is also common. Although net present value is a more stable metric the use of IRR is common, despite its shortcomings. Investors should know all the drawbacks of the IRR and when using it is inappropriate. The drawbacks of the IRR are:

1. It assumes all inter temporal cash flows are reinvested at its own rate, usually higher than is possible. This leads to the concept of External Rate of Return (ERR). Let us consider the following cash flow diagram




For the above cash flow pattern, we find IRR =20%, but at the end of 1st year, recovered fund is Rs 200 which may be reinvested in other than 20%. If it is invested in say 30%. The new cash flow diagram will be

$1{,}200 + 200^{*}(1+0.3) = 1{,}460$




IRR will be

$1000\,(1+i)^{2} = 1{,}460$

$i = 20.83\%$

2. IRR method involves linear interpolation of non linear function and when solved manually by trial and error method, may not give accurate result and it is more time consuming.
3. There are situations in which its iterative calculation process fails to produce a solution.
4. When the algebraic sign of the cash flow changes in the middle of the series it is possible to obtain two "right" answers.

Let us consider the following cash flow

| EOY | Net Cash Flow |
|---|---|
| 0 | -1,000 |
| 1 | +2,300 |
| 2 | -1,320 |

From the above cash flow pattern, we find IRR =10% and 20%, but both of them are incorrect. So we may abandon the IRR method for practical purpose and use the NPW criterion to make the decision.

5. When mutually exclusive projects are considered it can recommend the wrong investment and does not consider the scale of the investment.

Let us consider the two mutually exclusive projects.

| EOY | Mutually Exclusive Alternatives | |
|---|---|---|
| | A | B |
| 0 | -Rs. 100 | -Rs 500 |
| 1 | +Rs. 150 | +Rs 650 |
| IRR | 50% | 30% |
| PW (10%) | 36 | 90.90 |

Here both projects are acceptable at MARR =10%, but project B with higher PW is more worthful, whereas from IRR point of view, Project A seems better. Since IRR is a relative measure of investment worth, this inconsistency in ranking occurs. Under this circumstance, incremental analysis is necessary.

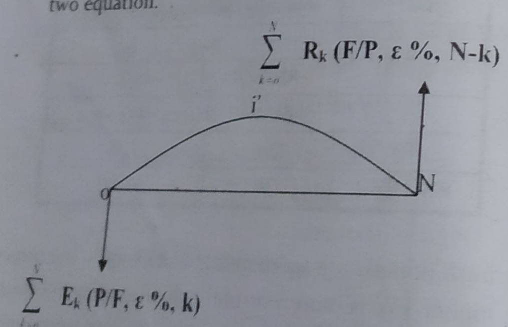
### 4.6 External Rate of Return (ERR)/ Modified IRR

The drawback of the IRR method (reinvestment assump[tion]) may not be valid in the engineering economy. For example, if firm's MARR is 20% per year and the IRR for a project is 42.4%, it may not be possible for the firm to reinvest net cash proceeds from the project at much more than 20%. This situation, coupled with the computational demands and possible multiple interest rates associated with the IRR method, has given rise to other rate of return methods that can remedy this weakness which is referred as *External rate of Return or Modified IRR.*

The external Rate of Return (i') is the unique rate of return for a project that assumes that net positive cash flows, which represent money not immediately needed by the project, are reinvested at the reinvestment rate ε %. The reinvestment rate depends upon the market rate available for investments.

**Steps of ERR calculation**

- All cash outflows are discounted to period zero (present) at ε % per compounding period
- All cash inflows are Compounded to period N at ε %
- ERR is the interest rate that equivalence between the two equation.




$$\sum_{k=0}^{N} E_k (P/F, \varepsilon\%, k)(F/P, i', N) = \sum_{k=0}^{N} R_k (F/P, \varepsilon\%, N-k)$$





Where,

$R_k$ = receipts in period k

$E_k$ = expenditures in period k

N = project life or number of study period

ε % = external reinvestment rate per period.

**Accept /Reject Decision Rule**

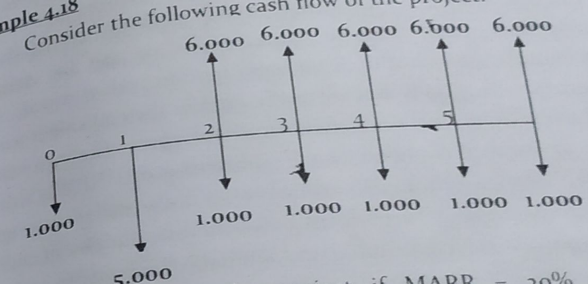
If *ERR > MARR,* — *accept the project*

*ERR = MARR,* — *remain indifferent*

*ERR < MARR,* — *reject the project*

**Example 4.18**

Consider the following cash flow of the project.




Calculate the ERR of the project if MARR = 20% and reinvestment rate ε % =15%. Is the project accepted?

**Solution**

1. Discounting all the cash outflows to the time zero at 15%.

$1,000 + 5,000 (P/F, 15\%, 1) = 1,000 + 5,000 (1+0.15)^{-1} = 5,347.82$

2. Compounding all the cash inflows to the year 6 at 15%

$5,000 (F/A, 15\%, 5) = 5,000 \{(1.15)^5 - 1 / 0.15\} = 33,711.90$

3. Establishing the equivalence between the two equation

$5,347.82 (F/P, i', 6) = 33,711.90$

$5,347.82 (1+i')^6 = 33,711.90$

$(1+i')^6 = 6.303$

$\therefore i' = 1.359 - 1 = \mathbf{35.91\%}$ is the ERR of the project.

Here ERR (35.91%) > MARR (20%), the project is accepted.

**Advantages of ERR over IRR**

- It does not need trial and error approach determination of i*%
- There is no possibility of multiple rate of return.

**4.7 Benefit Cost Analysis (B/C ratio)**

Benefit-Cost-Analysis is a decision making tool used systematically develop useful information about the desirab and undesirable effects of public project. Benefit Cost Analy estimates and totals up the equivalent money value of th benefits and costs to the community of projects to establi whether they are worthwhile. These projects may be dam irrigation, water supply highways etc. or can be trainin programs and health care systems. In other words th benefit/cost ratio is defined as the ratio of the equivalent wort of benefits to the equivalent worth of costs. The equivalen worth measure applied can be present worth (PW), future worth"(FW), an annual worth (AW). It is also called "savings-investment ratio". To perform an economic analysis of public alternatives, the cost (initial and annual), the benefits and the dis-benefits, if considered, must be estimated as accurately as possible in monetary units. It is difficult to estimate and agree upon the economic impact of benefits and dis-benefits for a public sector alternative. Dis-benefit may not be known at the time of economic analysis.

*Two types of B/C ratio*

1. *Conventional B/C ratio*
2. *Modified B/C ratio*

__Benefits__ - advantages to be experienced by the owners, the public. (Favorable outcomes)

__Dis-benefits__ - expected undesirable or negative consequences to the public, if the alternative is implemented. (Unfavorable outcomes)

***User's benefit = Benefits - Dis-benefits***





__Cost:__ estimated expenditures to the government entity for construction, operation and maintenance of the project, less any expected salvage value. Include capital investment and annual operating costs.

***Sponsor's Cost = Capital cost + Operating and Maintenance costs - Revenues***

**__Public Sector Project__**

Public sector project are owned, used and financed by the citizenry of any government level. Public sector projects have a primary purpose to provide services to the citizenry for the public good at no profit. Areas such as health, safety, economic welfare, and utilities comprise a majority of alternatives that require engineering economic analysis. Usually public sector investment decisions involve a great deal of expenditure, and their benefits are expected to occur over an extended period of time. One of the important issues that the toll authority must address is how it can determine whether its decisions, which affect the use of public funds, are, in fact, in the best public interest. In identifying the benefits of a project of this nature, we need to consider both primary benefits –the ones directly attributable to the project- and the secondary benefits – the ones indirectly attributable to the project.

**__Accept /Reject Decision Rule__**

For the project to be feasible

Benefit (B) > Cost (C)

Or B/C > 1

| | | |
|---|---|---|
| If | $B/C > 1$, | ***accept the project*** |
| | $B/C = 1$, | ***remain indifferent*** |
| | $B/C < 1$, | ***reject the project*** |

**1. PW method**

$$\text{Conventional B/C ratio} = \frac{\text{PW (benefits of proposed project)}}{\text{PW (total cost of proposed project)}}$$

$$= \frac{\mathbf{PW\ (B)}}{\mathbf{(I) - PW\ (S) + PW\ (O\&M)}}$$

Modified B/C ratio $= \dfrac{PW\,(B) - PW\,(O\&M)}{(I) - PW\,(S)}$

**Where**, PW (B) = Present worth of benefits of project.
PW (O&M) = Present worth of Operation and maintenan
project.
PW (S) = Salvage value of investment.
I = Initial investment of the project.

**2. FW method**

Conventional B/C ratio $= \dfrac{FW\,(B)}{FW\,(I) - (S) + FW\,(O\&M)}$

Modified B/C ratio $= \dfrac{FW\,(B) - FW\,(O\&M)}{FW\,(I) - (S)}$

**Where,**
FW (B) = Future worth of benefits of project.
FW (O&M) = Future worth of Operation and maintenance
project.
PW (S) = Salvage value of investment.
I = Initial investment of the project.

**3. AW method**

Conventional B/C ratio $= \dfrac{AW\,(\text{benefits of proposed project})}{AW\,(\text{total cost of proposed project})}$

$= \dfrac{AW\,(B)}{CR + AW\,(O\&M)}$

Modified B/C ratio $= \dfrac{AW\,(B) - AW\,(O\&M)}{(CR)}$

**Where,**
AW (B) = Annual worth of benefits of project.
CR = Capital Recovery.
AW (O&M) = Annual worth of operation and maintenance cost
I = Initial investment of the project.





*Note: Whichever method we use, value remains same, similarly*
$B/C_{modified} > B/C_{conventional}$, if $B/C > 1$
$B/C_{modified} < B/C_{conventional}$, if $B/C < 1$
$B/C_{modified} = B/C_{conventional}$, if $B/C = 1$

**Example 4.19**
Find both types of B/C ratio using PW, FW and AW method.

| | | |
|---|---|---|
| Initial investment | = | Rs 20,000 |
| Revenue / year | = | Rs 10,000 |
| Expenses/ year | = | Rs 4,400 |
| Salvage value | = | Rs 4,000 |
| Useful life | = | 5 years |
| MARR | = | 8% |

**Solution**
Using PW method

PW (B) $= 10{,}000\ (P/A,\ 8\%,\ 5) = 10{,}000\ \left[\dfrac{(1.08)^5 - 1}{(1.08)^5 * 0.08}\right]$

= Rs 39,927

PW (S) $= 4{,}000\ (P/F,\ 8\%,\ 5)$

$= 4{,}000\ (1/\ (1.08)^5)$

= Rs 2,722.33

PW (O&M) $= 4{,}400\ (P/A,\ 8\%,\ 5)$

$= 4{,}400\left[\dfrac{(1.08)^5 - 1}{(1.08)^5 * 0.08}\right]$

= Rs 17,567.90

Conventional B/C ratio $= \dfrac{PW\,(B)}{(I) - PW\,(S) + PW\,(O\&M)}$

$= \dfrac{39{,}927}{20{,}000 - 2{,}722.33 + 17{,}567.90}$

= **1.146 > 1 (justified)**

Modified B/C ratio $= \dfrac{PW(B) - PW(O\&M)}{(I) - PW(S)}$

$= \dfrac{39{,}927 - 17567.90}{20{,}000 - 2722.33}$

$= 1.294 > 1$ **(justified)**

Using AW method

Conventional B/C ratio $= \dfrac{AW(B)}{CR + AW(O\&M)}$

CR $= 20{,}000\,[A/P, 8\%, 5] - 4{,}000\,(A/F, 8\%, 5)$

$= 20{,}000 \left[\dfrac{(1.08)^5 * 0.08}{(1.08)^5 - 1}\right] - 4{,}000\,(0.08/(1.08)^5 - 1)$

$=$ Rs 4,327

Conventional B/C ratio $= 10{,}000/4{,}327 + 4{,}400$

$= 1.146 > 1$ **(justified)**

Modified B/C ratio $= \dfrac{AW(B) - AW(O\&M)}{(CR)}$

$= 10{,}000 - 4{,}400 / 4{,}327$

$= 1.294 > 1$ **(justified)**

**4.8 Payback Period Method (PB)**

Payback considers the initial investment costs and the resulting annual cash flow. The payback time (period) is the length of time needed before an investment makes enough to recoup the initial investment. The payback method screens the projects on the basis of how long it takes for net receipts to equal investment. But the payback method doesn't account for savings after the initial investment is paid back from the profits (cash flow) generated by the investment (project).





**Simple payback period** -

Simple payback period doesn't considers the time value of money (i=0). Simple Payback period is calculated using the following equation if the annual savings are equal:

$$\textit{Simple Payback Period (in years)} = \frac{\textit{Initial Investment}}{\textit{Annual Savings (Cashinflow)}}$$

*Where:*

*Initial Investment = Initial investment for a project*

*Annual Savings (Cash Flow) = Annual savings derived from the investment*

In other way, the simple payback period for a project having one time investment at time zero can be computed as follows:

$$\text{Simple payback period: } (\theta) = \sum_{k=1}^{\theta} (R_k - E_k) - I \geq 0$$

**Advantages of simple payback period**

- Easy to calculate
- It is interpreted in tangible terms (in years)
- It doesn't require any assumptions about the project in terms of timing, life time or interest rates.

**Disadvantages of simple payback period**

- It takes no account of any savings after the payback period.
- It takes no account of the residual value in the capital asset.
- It takes no account of the time value of money.

**Example 4.20**

Consider an example of evaluating the purchase of pollution prevention equipment for a cost of Rs.8, 000, but provide a net annual operational saving of Rs.3, 500. When the net annual savings is divided into the initial investment, the simple payback period is calculated as follows:

Payback Period (in years) = $\frac{\text{Initial Investment}}{\text{Annual Savings (Cash Flow)}}$ =

= 2.3 (years)

**Example 4.16**

Calculate the simple payback period for the given cash flow [of] the project

| Period | Net Cash flow (Rs) |
|---|---|
| 0 | -25,000 |
| 1 | +8,000 |
| 2 | +8,000 |
| 3 | +8,000 |
| 4 | +8,000 |
| 5 | +13,000 |

**Solution**

| Period | Net Cash flow (Rs) | Cumulative Cash Flow (Rs) |
|---|---|---|
| 0 | -25,000 | -25,000 |
| 1 | +8,000 | -17,000 |
| 2 | +8,000 | -9,000 |
| 3 | +8,000 | -1,000 |
| 4 | +8,000 | +7,000 |
| 5 | +13,000 | +20,000 |

Here the cumulative Cash flow turns to positive in year 4. Therefore Payback period lie between year 3 and 4. By interpolating, we get the payback period = 3+1000/8000 = 3.125 years (Ans)

**Discounted payback period**

The problem with the Simple payback period is that it ignores the time value of money. In order to correct this, we can use





discounted cash flows in calculating the payback period. Discounted Payback Period is one of several methods to determine if an investment is a good one or not. In this method, you discount each of the future cash flows and then count the number of years necessary to recoup your investment.

In other way, the discounted payback period for a project having one time investment at time zero can be computed as follows:

Discounted payback period (θ')

$$\sum_{k=1}^{\theta'} (R_k - E_k)\ (P/F, i\%, k) - I \geq 0$$

**Advantages of discounted payback period**

- Considers the time value of money
- Considers the riskiness of the project's cash flow (through the cost of capital)

**Disadvantages of discounted payback period**

- No concrete decision criteria that indicate whether the investment increases the firm's value.
- Requires an estimate of the cost of capital in order to calculate the payback.
- Ignores cash flows beyond the discounted payback period

**Example 4.21**

Consider the example 4.12 and Calculate the discounted payback period at i=20%

**Solution**

| Discounted Period | Net cash flow (Rs) | PW of net cash flow | Cumulative Cash Flow |
|---|---|---|---|

| (n) | | (i=20%) | (Rs) |
|---|---|---|---|
| 0 | -25,000 | -25,000 | -25,000 |
| 1 | +8,000 | 6,667 | -18,333 |
| 2 | +8,000 | 5,556 | -12,777 |
| 3 | +8,000 | 4,630 | -8,147 |
| 4 | +8,000 | 3,858 | -4289 |
| 5 | +13,000 | 5,223 | +934 |

Here the cumulative Cash flow turns to positive in year 5. Therefore Payback period lie between year 4 and 5. By interpolating, we get the payback period = 4+ 4,289/5,223 = 4.82 years (Ans)

## Some solved examples

1. Find IRR of the following project with the initial investment of 350,000 and the cash flow as shown. Also draw the investment balance diagram.

| End of year | Cash Outflow | Cash Inflow |
|---|---|---|
| 1 | 15,000 | 50,000 |
| 2 | 15,000 | 75,000 |
| 3 | 15,000 | 1,00,000 |
| 4 | 15,000 | 1,25,000 |
| 5 | 15,000 | 1,50,000 |

**Solution**

Net cash flow for each year

| End of year | Net cash flow | A | G |
|---|---|---|---|
| 1 | 35,000 | 35,000 | 0 |
| 2 | 60,000 | 35,000 | 25,000 |
| 3 | 85,000 | 35,000 | 50,000 |





| | | | |
|---|---|---|---|
| 4 | 110,000 | 35,000 | 75,000 |
| 5 | 135,000 | 35,000 | 100,000 |

Here the cash flow is in gradient series
Using gradient to present worth series factor

PW ($i^*$ %) = 0

$-3,50,000 + 35,000\,(P/A, i^*, 5) + 25000\,(P/G, i^*, 5) = 0$

$-3,50,000+35,000[((1+i^*)^5-1)]/[i^*(1+i^*)^5] +25,000 \{((1+i^*)^5 -5^*i^*-1/ i^{*2} (1+i^*)^5\} = 0$

Solving by hit and trial, we get **IRR ($i^*$ %) = 5.634%**

**Unrecovered project balance**

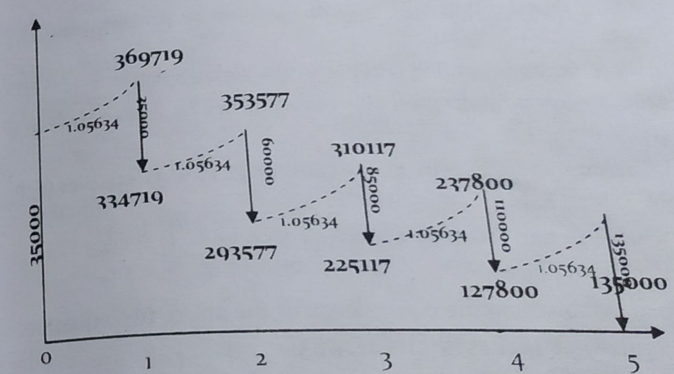


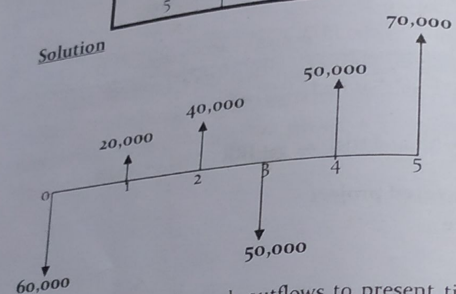

**Fig: Investment balance Diagram**

2. Find the ERR (external rate of return) when ε %=15%. (TU, IOE, 2064)

| End of year | Annual Cash Flow |
|---|---|
| 0 | Rs - 60000 |

| | |
|---|---|
| 1 | Rs +20000 |
| 2 | Rs +40000 |
| 3 | Rs - 50000 |
| 4 | Rs +50000 |
| 5 | Rs +70,000 |

**Solution**




Step 1: Discounting all the cash outflows to present time at ε =15%.

$60{,}000 + 50{,}000\ (P/F, 15\%, 3) = \text{Rs } 92{,}875.8$

Step 2: Compounding all the cash inflows to future time at ε =15%.

$20{,}000\ (F/P, 15\%, 4) + 40{,}000\ (F/P, 15\%, 3) + 50{,}000\ (F/P, 15\%, 1) + 70{,}000$

$= 34{,}980 + 60{,}835 + 57{,}500 + 70{,}000$

$= \text{Rs } 223{,}315$

Step 3: Establishing the equivalence of the above two equation

$92875.8\ (F/P, i'\%, 5) = 22,\ 3315$

$(1+i')^5 = 2.40$

$i' = 19.135\%$

The external rate of return (ERR) is **19.315% (Ans)**

3. The annual income of the project starts from Rs 5, 00,000 at the end of first year and decreases at the rate of Rs 4000/yr for 15 years. What is the equivalent present worth when the MARR is 12%. **(TU,IOE, 2064)**

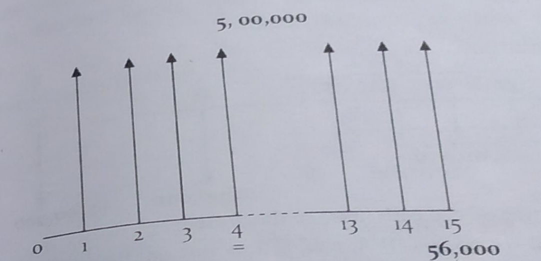


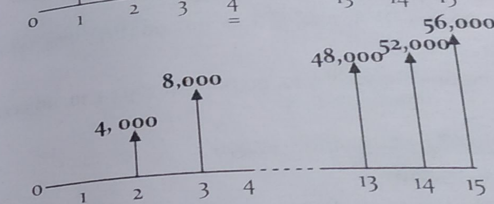


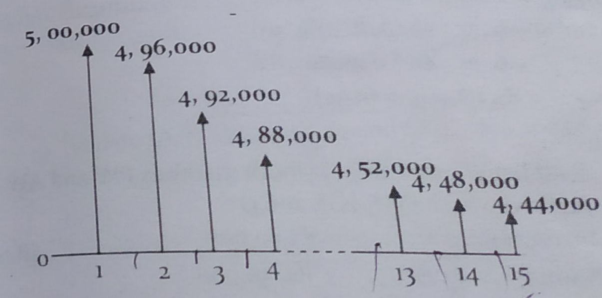
**Solution:**

Here, the cash flow is in decreasing gradient series.

$N=15$, $G= 4000$, $i=12\%$,




Applying gradient to present worth series factor we get

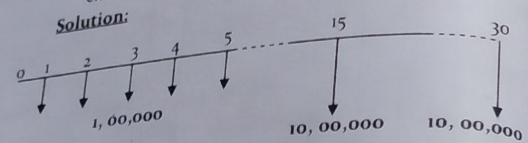
$PW\ (12\%) = 5,\ 00{,}000\ (P/A, 12\%, 15) - 4000\ (P/G, 12\%, 15)$

$= 5,\ 00{,}000 * 6.8109 - 4{,}000 * 33.9202$

$=$ **Rs 32, 69,769.2 (Ans)**

4. Maintenance cost for a new communication tower with an expected 50 years life are estimated to be Rs 100,000 each years for the first 5 years, followed by a Rs 10,00,000 expenditure in the $15^{th}$ year and another Rs 10,00,000 expenditure in the $30^{th}$ year. If MARR = 10%, what is the equivalent uniform annual cost over the entire 50 period? **(PU 2006)**

**Solution:**




Discounting all the cash flows to the present

PW (10%) = 1, 00,000 (P/A, 10%, 5) + 10, 00,000 (P/F, 10%, 15) + 10, 00,000 (P/F, 10%, 30)

= 1, 00,000 * 3.7908 + 10, 00,000 * 0.2394 + 10, 00,000 * 0.0573

= 3, 79,080 + 2, 39,400 + 57,300

= **Rs 6, 75,780**

Applying the capital recovery factor

AW (10%) = 6, 75,780 (A/P, 10%, 50)

= 6, 75,780 * 0.10086

= **Rs 68,159.17 (Ans)**

5. Find benefit cost ratio by both methods PW and AW method where **(TU, IOE 2064)**

| | |
|---|---|
| Investment | = Rs 90,000 |
| Annual revenue | = Rs, 50,000 |
| Annual Cost | = Rs 2000 |
| Salvage value | = Rs 20,000 |
| MARR | = 12% |
| N | = 10 years |

Solution





**Using PW formulation**

PW (12%) net benefits = 50,000 (P/A, 12%, 10) = 50,000 *5.6502

= Rs 2, 82,510

PW (12%) annual cost = 2,000 (P/A, 12%, 10) = 2,000 *5.6502

= Rs 11,300.4

PW (12%) salvage value = 20,000 (P/F, 12%, 10) = 20,000* 0.3220

= Rs 6,440

Conventional B/C ratio $= \dfrac{\text{PW (B)}}{\text{(I)} - \text{PW (S)} + \text{PW (O\&M)}}$

$= \dfrac{282510}{90{,}000 - 6{,}440 + 11{,}300.4}$

= **2.98 > 1 (Justified)**

Modified B/C ratio $= \dfrac{\text{PW (B)} - \text{PW (O\&M)}}{\text{(I)} - \text{PW (S)}}$

$= \dfrac{2,82{,}510 - 11{,}300.4}{90{,}000 - 6{,}440}$

= **3.25>1 (justified)**

**Using AW formulation**

Conventional B/C ratio $= \dfrac{\text{AW (B)}}{\text{CR + AW (O\&M)}}$

$= \dfrac{50{,}000}{\{90{,}000\ (A/P,\ 12\%,\ 10) - 20{,}000\ (A/F,\ 12\%,\ 10)\} + 2{,}000}$

= **2.97 > 1 (Justified)**

Modified B/C ratio $= \dfrac{\text{AW (B)} - \text{AW (O\&M)}}{\text{(CR)}}$

$= \dfrac{50{,}000 - 2{,}000}{14{,}788.6}$

= **3.25 >1 (justified)**

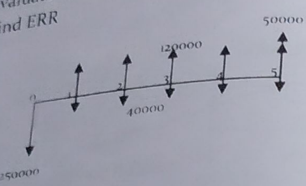
6. Equipment costs 2, 50,000 and has salvage value of 50,0 at the end of its expected life 5 years. Annual expenses will 40,000. It will produce revenue of 120,000 per year. MARR 20%=ε

(i) Evaluate IRR using AW formulation

(ii) Evaluate both types of B/C ratio with FW formulati

(iii)Find ERR

**Solution**




(i) Calculation of IRR

$AW(i^*\%) = 0$

$AW_{inflow} - AW_{outflow} = 0$

$120000 + 50000\,(A/F,\, i^*\%,\, 5) - 40000 - 250000\,(A/P, i^*\%, 5) = 0$

$120000 + 500000\,\{ i^* / (1+i^*)^5 - 1\} - 40000 - 250000\,\{ ((1+i^*)^5 * i^*) / ((1+i^*)^5 - 1)\}$

Solving by hit and trial (calculator)

We get, $i^*\% = 0.21577 = 21.577\%$

IRR = 21.577% (Ans)

(ii) Finding both B/C ratio using FW formulation

$$\text{Conventional B/C ratio} = \frac{FW(B)}{FW(I) - (S) + FW(O\&M)}$$

$$\text{Modified B/C ratio} = \frac{FW(B) - FW(O\&M)}{FW(I) - (S)}$$

FW (B) = 120,000 (F/A, 20%, 5) = 120,000* 7.4416 = 892992

FW (I) = 250,000 (F/P, 20%, 5) = 250,000 * 2.4883 = 622075

FW (O&M) = 40,000 (F/A, 20%, 5) = 40,000 * 7.4416 =297664





$$\text{Conventional B/C} = \frac{892992}{622075 - 50000 + 297664}$$

$= 1.026 > 1$ (justified)

$$\text{Modified B/C} = \frac{892992 - 297664}{622075 - 50000}$$

$= 1.04 > 1$ (justified)

(iii) Calculating ERR

Step 1: Discount all the cash outflow to year zero at ε%

= 250000 + 40000 (P/A, 20%, 5)

= 250000 + 40000 * 2.9906 = 369624

Step II: Compound all the cash inflow to period N at ε%

= 120000 (F/A, 20%, 5) + 50,000

= 120000* 7.4416 + 50,000 = 942992

Step III: Establishing the equivalence between above two equations

369624 (F/P,i', 5) = 942992

Or, $369624\,(1+i')^5 = 942992$

Or, $(1+i')^5 = 2.5512$

Or, $(1+i') = 1.206$

Or, $i' = 1.206 - 1 = 0.206$

Or, $i' = 20.6\%$

External rate of Return (ERR) = **20.6%** (Ans)

6. For the cash flow given below, compute Payback Period, Net annual worth, IRR and BCR if MARR is 12%

**Solution**

**Using payback Period.**

| Period | Cash | PW at 12% | Cumulative |
|---|---|---|---|

| | Flow | | Cash flow |
|---|---|---|---|
| 0 | -25,000 | -25,000 | -25,000 |
| 1 | 10,000 | $10{,}000\,(1+0.12)^{-1} = 8928.57$ | -16,071.4 |
| 2 | -5,000 | $-5{,}000\,(1+0.12)^{-2} = -3985.97$ | -20,057.4 |
| 3 | 25,000 | $25{,}000\,(1+0.12)^{-3} = 17794.51$ | -2,262.89 |
| 4 | 30,000 | $30{,}000\,(1+0.12)^{-4} = 19065.54$ | 16802.65 |

Here the Cumulative cash flow is negative on year 3 and positive on year 4. Hence the payback period lies between year 3 and 4. By Interpolating we get,

3+ 2262.89/19065.54 = 3.118 years (ans)

**Using Net annual worth**

We know that, AW (i %) = R – E – CR

Since salvage value is not given, we don't calculate CR

Annual Revenues (R) = {10,000 (P/F, 12%, 1) + 25,000 (P/F, 12%, 3) + 30,000 (P/F, 12%, 4)} (A/P, 12%,4)

= {(10,000 *0.8929) + (25,000 *0.7118) + (30,000 * 0.6355)} * 0.3292

= (8929 + 17795 +19065) * 0.3292

= 45789 *0.3292 = 15073.74

Annual Expenses (E) = 5000 (P/F, 12%,2) (A/P,12%,4)

= 3985.97 * 0.3292 = 1312.18

AW (12%) = 15073.74 - 1312.18 = **13761.56**

**Using IRR method**

Using PW formulation

PW (i* %) = 0

$PW_{inflow} - PW_{outflow} = 0$

10,000 (P/F, 12%, 1) + 25,000 (P/F, 12%, 3) + 30,000 (P/F, 12%, 4) = 0

$10{,}000\,(1+i^*)^{-1} + 25{,}000\,(1+i^*)^{-3} + 30{,}000\,(1+i^*)^{-4} - 5{,}000\,(1+0.12)^{-2} - 25{,}000 = 0$

Solving by trial and error,

We get i*% = 32.827%, hence IRR = 32.827 %(Ans)

**Using BCR method**





Using PW formulation

BCR = PW (B) / PW (C)

= {10,000 (P/F, 12%, 1) + 25,000 (P/F, 12%, 3) + 30,000 (P/F, 12%, 4) } / 5000 (P/F, 12%,2) + 25,000

= (8929 + 17795 + 19065) / 1312.18 +25,000

= 45789 / 26312.18

= 1.740 >1 (Justified)

**Review Questions**

1. What are the methodologies for evaluating the project? Explain briefly
2. Explain present worth, future worth and annual worth?
3. What do you understand by capital recovery? Explain.
4. Define IRR and explain its drawbacks.
5. What is External rate of return (ERR) and what are its advantages over IRR.
6. What are simple and non simple investments? Explain.
7. Explain simple and discounted payback period method. List down its advantages and disadvantages.
8. What do you understand by benefit/cost analysis? Explain its types.
9. What do you understand by public sector project?

**Exercises**

**Equivalent worth Method**

10. Consider the following set of investment projects. All projects have 3 year investment life. Compute the Present worth and Future Worth of each project. MARR =10%.

| Project's cash flow (Rs) | | | | |
|---|---|---|---|---|
| N | A | B | C | D |
| 0 | -1000 | -1000 | -1000 | -1000 |
| 1 | 0 | 600 | 1200 | 900 |
| 2 | 0 | 800 | 800 | 900 |
| 3 | 3000 | 1500 | 1500 | 1800 |

(PW: 1253.94, 1333.58, 1879.038, 1914 FW: 1669, 1775, 2501, 2548)

11. A company wishes to set aside money now to inv over the next 4 years in automating its custome service department. The company can earn 10% on lump sum deposited now and it wishes to withdraw th money in the following increment.
Year 1: Rs 25,000 to purchase a computer and databa software.
Year 2: Rs 3,000 to purchase additional hardware.
Year 3: No expenses. Year 4: Rs. 5,000 to purcha software upgrades.
How much money must be deposited now to cove anticipated payments over the next 4 years? (28621.67)

12. Large food processing corporations is considering using laser technology to seed up and eliminates waste in the potato-peeling process. To implement the system, the company anticipates needing Rs.3 million to purchase the industrial strengths lasers. The system will save Rs.1, 2, 00,000 per year in labor and materials. However it will require an additional operating and maintenance cost of Rs. 2, 50,000. Annual income taxes will also increase by Rs. 1, 50,000. The system is expected to have a 10 year service life and will have a salvage value of about Rs. 2, 00, 000. If the company's MARR is 18% justify the economics of the project based on the NPW method. (633481.929)

13. Consider the following set of investment projects. All projects have 3 year investment life. Compute the Future worth of each project. Which project will you select?

| Project's cash flow (Rs) | | | | |
|---|---|---|---|---|
| N | A | B | C | D |
| 0 | -2,500 | -1,000 | -2,500 | -3,000 |





| | | | | |
|---|---|---|---|---|
| 1 | 5,400 | 3,000 | 7,000 | 1,500 |
| 2 | 14,400 | 1,000 | 2,000 | 5,500 |
| 3 | 7,200 | 3,000 | 4,000 | 6,500 |

(26246.5, 6399, 11342.5, 10372)

14. The owner of the business is considering investing Rs.55, 000 in new equipment. He estimates that the net cash flows during the first year will be Rs 5,000 but these will increase by Rs 2,500 per year the next year and each year thereafter. The equipment is estimated to have a 10 year service life and a net salvage at this time of Rs 6,000. The firm interest rate is 12%.
(a) Determine the annual capital cost (ownership cost) for the equipment. (9392.224)
(b) Determine the equivalent annual savings (revenues) (9303.75)
(c) Determine if this is a wise investment? (-499)

15. An industrial firm is can purchase a special machine for Rs 40, 000. A down payment of Rs.4, 000 is required, and the balance can be paid in five equal year-end installments at 7% interest on the unpaid balance. As an alternative, the machine can be purchased for Rs 36,000 in cash. If the MARR is 10%, determine which alternative should be accepted, based on the annual equivalent method?

16. A construction firm is considering establishing an engineering computing center. This center will be equipped with three engineering workstations that would cost Rs.25, 000 each, and each has a service life of 5 years. The expected salvage value of each workstation is Rs.2, 000. The annual operating and maintenance cost would be Rs.15, 000 for each work station. At a MARR of 15%, determine the equivalent annual costs for operating the engineering center? (66483.77)

[illegible]

## Rate of Return Method

18. The board of directors is considering a proposal to establish a facility to manufacture an electronically controlled "intelligent" crop sprayer. This crop sprayer project would require an investment of Rs [illegible] million in assets and would produce an annual after tax benefit of Rs [illegible] million over a service life of 8 years. When the project terminates, the corporation would earn Rs [illegible] million from the assets. Compute the IRR generated by this project, if the corporation MARR is [illegible]%. Should the corporation accept this project? Show the investment balance diagram also. ([illegible]%)

19. Calculate the IRR of the project having the initial investment of Rs [illegible], salvage value after 15 years is Rs [illegible], net revenue /year of Rs [illegible], MARR = [illegible]%. Show also the investment balance diagram. ([illegible]%)

20. Consider three investments with the following sequences of cash flow:

| Year | Net Cash Flow (Rs) | | |
|---|---|---|---|
| | Project A | Project B | Project C |
| 0 | -18,000 | -20,000 | -56,500 |
| 1 | 10,000 | 32,000 | +25,00 |
| 2 | 20,000 | 32,000 | -64,59 |
| 3 | 35,000 | -22,000 | -78,345 |





Identify simple and non-simple investment and compute the IRR of each project.

(74.231%, 111.1%, -195.57%)

21. Calculate the ERR for the following project if MARR = $\epsilon$=15%

(a)

| EOY | Net Cash Flow |
|---|---|
| 0 | -25,000 |
| 1 | 8,000 |
| 2 | 8,000 |
| 3 | 8,000 |
| 4 | 8,000 |
| 5 | 13,000 |

(b)

| EOY | Net Cash Flow |
|---|---|
| 0 | -1,50,000 |
| 1 | 30,000 |
| 2 | 50,000 |
| 3 | 60,000 |
| 4 | 80,000 |
| 5 | -35,000 |
| 6 | 45,000 |

(c)

| EOY | Net Cash Flow |
|---|---|
| 0 | -4,50,000 |
| 1 | -42,000 |
| 2 | 92,800 |
| 3 | 386000 |
| 4 | 614600 |
| 5 | -202700 |

(18.71%, 15.13%, 18.259%)

## Benefit Cost Analysis

22. Find by using both types of B/C ratio using AW and FW method.