Assignment 4

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October 27, 2017

Part I
Single Pendulum

1 Math representations

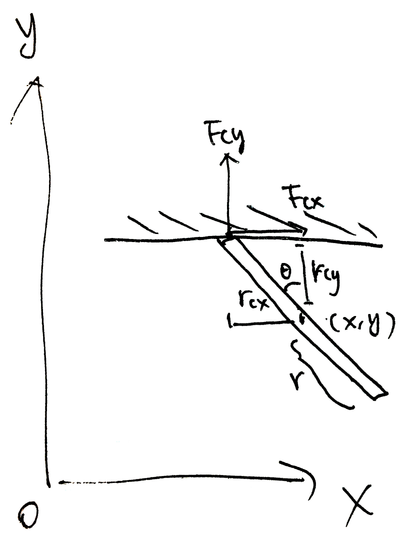

Figure 1: Illustration of the single pendulum scenario

Refer to Figure 1, we have $A = \begin{bmatrix} m & 0 & 0 & -1 & 0 \\ 0 & m & 0 & 0 & -1 \\ 0 & 0 & I & r_{cy} & -r_{cx} \\ -1 & 0 & r_{cy} & 0 & 0 \\ 0 & -1 & -r_{cx} & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ mg \\ 0 \\ -\dot{\theta}^2 r_{cx} \\ \dot{\theta}^2 r_{cy} \end{bmatrix}$ and $x = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ F_{cx} \\ F_{cy} \end{bmatrix}$

where m is the mass of the pendulum, I is the moment of inertia and $I = mr^2$, F_{c*} is the x or y component of the contact force by the wall on the pendulum and r_{c*} denotes the x or y component of the coordinate of the contact point. We have

$$Ax = \begin{bmatrix} m & 0 & 0 & -1 & 0 \\ 0 & m & 0 & 0 & -1 \\ 0 & 0 & I & r_{cy} & -r_{cx} \\ -1 & 0 & r_{cy} & 0 & 0 \\ 0 & -1 & -r_{cx} & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ F_{cx} \\ F_{cy} \end{bmatrix} = \begin{bmatrix} 0 \\ mg \\ 0 \\ -\dot{\theta}^2 r_{cx} \\ \dot{\theta}^2 r_{cy} \end{bmatrix} = B$$

2 Simulation

Here I tracked the kinetic energy and potential energy during the simulation under four scenarios: (1) simple simulation, (2) simulation with energy preservation, (3) simulation with simple damping and (4) simulation with both simple damping and energy preservation shown in Figure 2 and Figure 3.

Energy Preservation An experimental process to correct kinetic energy towards the ideal value. The ideal kinetic energy $KE_i = KE_0 + PE_0 - PE - DE$, where KE_0 denotes the initial total kinetic energy, PE_0 denotes the initial total potential energy and DE denotes the energy loss from damping. Refer to the source codes for detailed algorithm.

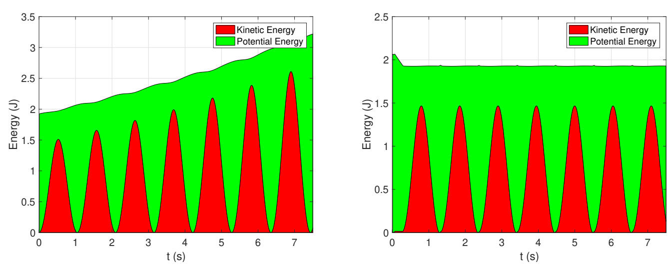

Figure 2: left, simple simulation; right, simple simulation with energy preservation

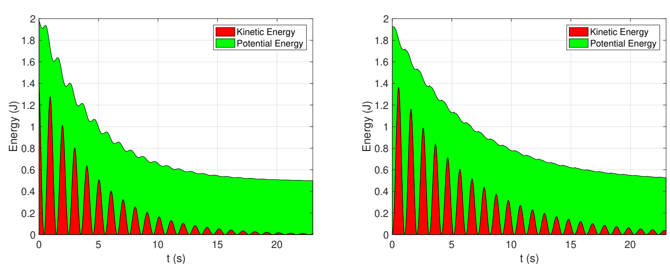

Figure 3: left, simple simulation with damping; right, simple simulation with damping and energy preservation

Part II
Four Pendulums

For details source code, visit project page https://github.com/Aahung/cpsc526-pendulum-simulation. Math representation is omitted here.

3 Simulation

Similarly, I tracked the energy change in Figure 4 and Figure 5. In Figure 5, the damping without energy preservation does not guarantee the total energy decreases over time, however, with the energy preservation, it is more real.

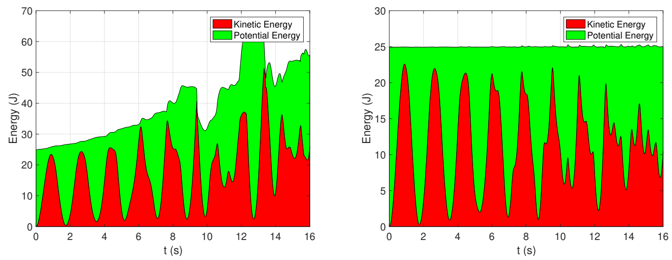

Figure 4: left, 4-pendulum simulation; right, 4-pendulum simulation with energy preservation

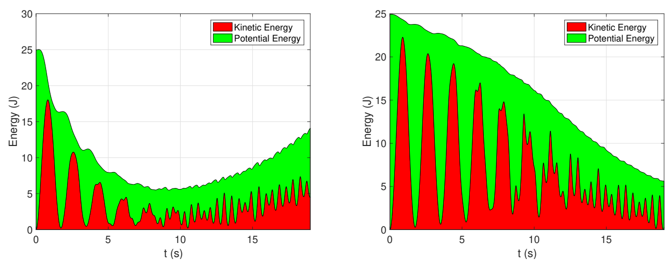

Figure 5: left, 4-pendulum simulation with damping; right, 4-pendulum simulation with damping and energy preservation

Part III
Ground Plane and Improvement

4 Adding Ground Plane

Adding a ground plane will leads to 2 extra rows (cols) in matrix A and 2 extra rows in B and x. The spring force over mass F_g/m should be added (after negation) to $B[last]$. Detail algorithm see codes.

5 Improvements

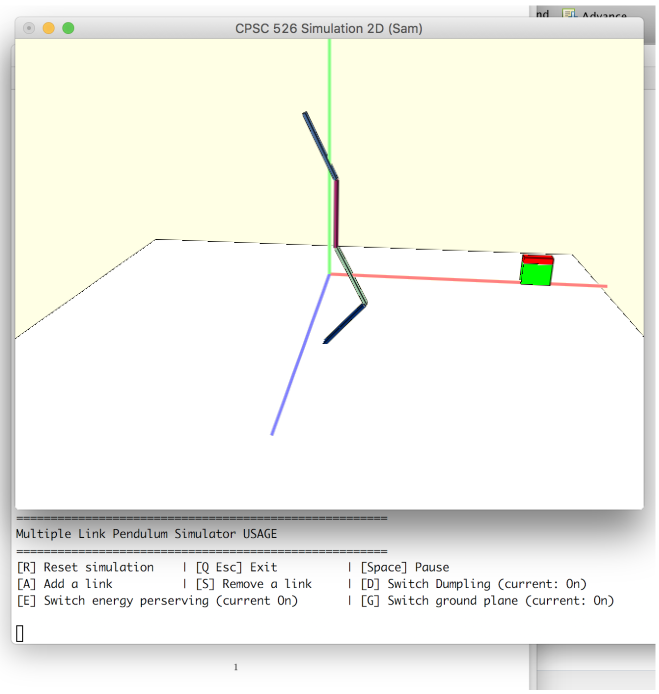

Figure 6: Screenshot of simulation interface

Refer to Figure 6, there are a couple of improvements made:

1. Display kinetic energy (red) and potential energy in real-time
2. Provides fruitful options to configure the simulation in runtime
   (a) Add/remove pendulum
   (b) Toggle energy perseverance
   (c) Toggle damping
   (d) Toggle ground plane
3. Colorful pendulums, color keeps changing over time