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SE
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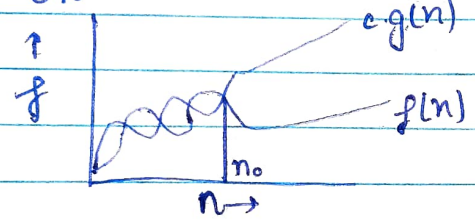
Assignment - 1.

DESIGN AND ANALYSIS OF ALGORITHM

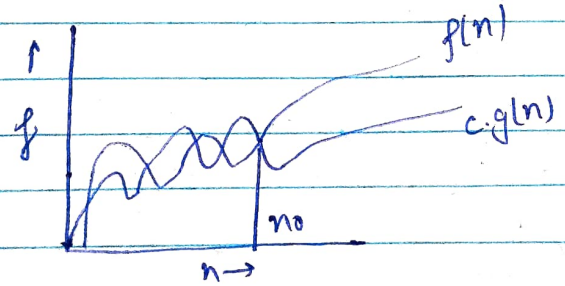
Ans 1. Asymptotic notations to analyse an algorithm running time identifying it's behaviour as the input size for the algorithm increases. These notation are used to tell the complexity of an algorithm when input is very large.
type of asymptotic notations

1. Big - Oh



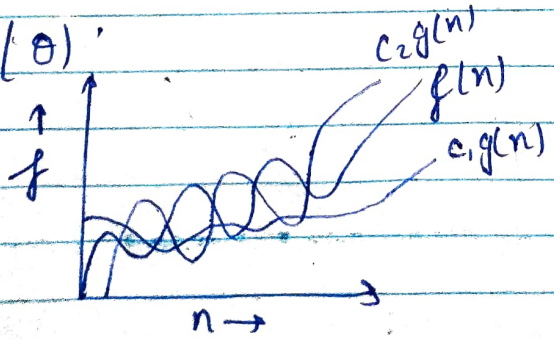
$f(n) = O(g(n))$
if and only if
$f(n) \leq c \cdot g(n)$
$\forall n \geq n_0$

2. Big Omega (Ω)



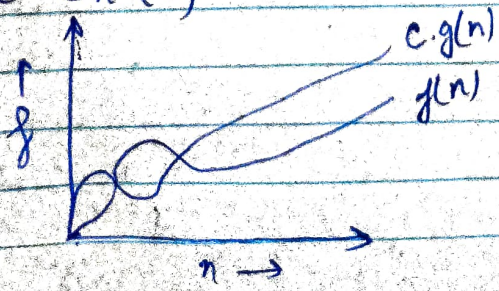
$f(n) = \Omega g(n)$
if and only if
$f(n) \geq c \cdot g(n)$
$\forall n > n_0$

3. Theta (θ)



$f(n) = \theta g(n)$
if and only if
$c_1 g(n) \leq f(n) \leq c_2 g(n)$
$\forall n \geq \max(n_1, n_2)$

4. Small - Oh (o)



$f(n) = o(g(n))$
$f(n) < c \cdot g(n) \; \forall \; n > n_0$

Ans 2. $i = 1, 2, 4, 8, \ldots n$

$2^0, 2^1, 2^2, 2^3, \ldots 2^k \longrightarrow$ GP

$a = 1, \quad r = 2$

$t_k = ar^{k-1}$

$= 1 \times 2^{k-1}$

$n = \frac{2^k}{2}$

$2^k = 2n \qquad k = \log_2(2n) \; ; \; k = \log_2(n) + \log_2(2)$

$= \log_2(n) + 1$

$T.C = O(\log_2(n)+1) = \boxed{O(\log n)}$

Ans 3. $T(n) = 3T(n-1) \; -①\; n > 0$

$T(1) = 1$

put $n = n-1$ in eq ①

$T(n-1) = 3T(n-2) \; -②$

put $T(n-1)$ in eq ①

$T(n) = 3[3T(n-2)]$

$T(n) = 9T(n-2) \; -③$

put $n = n-2$ in eq ①

$T(n-2) = 3T(n-3) \; -④$

put $T(n-2)$ in eq ③

$T(n) = 9(3T(n-3))$

$T(n) = 27T(n-3)$

$\vdots$

$T(n) = 3^k T(n-k) \; -⑤$

$T(1) = 1$

$n-k = 1$

$k = n-1. \; -⑥$

from ⑤ & ⑥

$T(n) = 3^{n-1} T(1)$

$T(n) = \frac{3^n}{3} \times 1 =$ $\boxed{T.C = O(3^n)}$

Ans 4. $T(n) = 2T(n-1) - 1$ — ①

$T(1) = 1$

put $n = n-1$ in eq ①

$T(n-1) = 2T(n-2) - 1$ ..

put $T(n-1)$ in eq ①

$T(n) = 2(2T(n-2) - 1) - 1$

$T(n) = 4T(n-2) - 2 - 1$ — ②

put $n = n-2$ in eq ①

$T(n-2) = 2T(n-3) - 1$

put $T(n-2)$ in eq ②

$T(n) = 4(2T(n-3) - 1) - 2 - 1$

$T(n) = 8T(n-3) - 4 - 2 - 1$ — ③

⋮

$T(n) = 2^k [T(n-k)] - 2^{k-1} - 2^{k-2} - \cdots 2^1 - 2^0$ — ④

$T(1) = 1$

$n - k = 1$

$k = n - 1$ — ⑤

from ④ & ⑤

$T(n) = 2^{n-1}[T(n-(n-1))] - 2^{n-2} - 2^{n-3} \cdots 2^0$

$= 2^{n-2} - 2^{n-1} - 2^{n-3} - \cdots 1$

$= \frac{1}{2}[2^n - (2^n - 1)]$

$= \frac{1}{2} \times 1 = \frac{1}{2}$ $\boxed{T.C = O(1)}$

Ans 5. $n = 1, 3, 6 \ldots K \rightarrow AP$

$T.C = \frac{k(k+1)}{2}$

$O\left(\frac{k^2+k}{2}\right)$

$O(k^2)$ $\boxed{T.C = O(n^2)}$

Ans 6. Void function (int n) {
int i, count = 0; → 1
for (i = 1, i * i <= n; i ++)
1 $(n+1)^2$ n
count ++; }
n

$1 + 1 + (n+1)^2 + n + n$

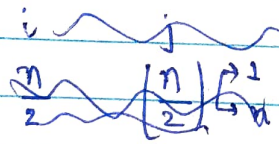
$2 + n^2 + 2n + 1 + 2n$

$n^2 + 4n + 3$

$O(n^2 + 4n + 3)$

$O(n^2)$ $\boxed{T.C = O(n^2)}$

Ans 7.



Void function (int n) {
int i, j, k, count = 0;
for (i = n/2, i <= n; i ++) — O(n)
for (j = 1, j <= n; j = j * 2) — log(n)
for (k = 1; k <= n; k = k * 2)
count ++; } — log(n)

$T.C = \log(n) * \log(n)$

$= \log^2(n)$

$= O[\log^2(n)]$

Ans 8. function (int n) {
if (n == 1) return; → 1
for (i = 1 to n)
for (j = 1 to n) → n * n
printf ("*"); → 1
}
}
function (n - 3) → $n * n^2$
}

$1 + n^2 + 1 + n^3$

$n^3 + n^2 + 2$ $O(n^3)$

Ans 9.

```
for (i=1 to n)
    for (j=1; j<=n; j=j+1)
        printf("*");
}
```

i	j	times
1	1 → n	$\frac{n+1}{2}$
2	1 → n	$n+1/2$
⋮		
n	1 → n	$n+1/2$

$$T.C = \log n \cdot \left(\frac{n+1}{2}\right)$$

$$= O\left(\frac{n+1}{2} \log n\right)$$

$$= O(n \log n)$$

Ans 10.

$$n^k \leq c a^n$$

$$a^n + n^k \leq c a^n \rightarrow a^n$$

$$a^n + n^k \leq a^n (c-1)$$

$$\frac{a^n + n^k}{a^n} \leq (c-1)$$

$$c \geq 1 + \frac{n_0^k}{a^{n_0}} + 1$$

$$c \geq 2 + \frac{n_0^k}{a^n}$$

$$c \geq 2 + \frac{n_0'}{1.5^n}$$

$$n_0 = 1$$

$$c \geq 2 + \frac{1}{1.5}$$

$$c \geq 3.0 + 1$$

$$c \geq 4$$

Ans 11. Time complexity = $O(n)$

The execution of different code lines here are :-

1) while $\Rightarrow (n-1)$
2) $i = i+j \Rightarrow (n)$
3) $j++ \Rightarrow (n)$

$$T.C = n + n + n - 1$$

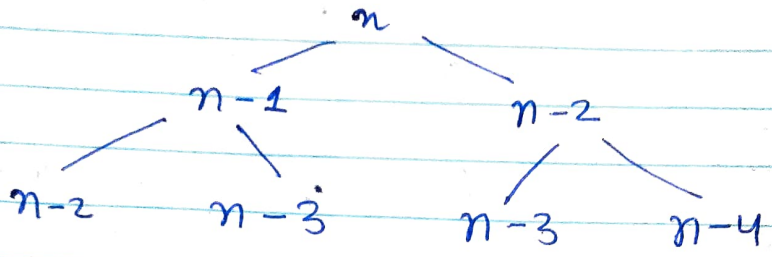
$$= 3n - 1$$

$$T.C = O(3n-1)$$

$$T.C = O(n)$$

Ans 12. the main working of fibonacci series is

$$f(n) = f(n-1) + f(n-2)$$



$$T(n) = 1 + 2 + 4 + \dots + 2^n$$

$a = 1, \quad r = 2$

$$\frac{a(r^n - 1)}{r-1} = \frac{1(2^{n+1} - 1)}{2-1} = 2^{n+1}$$

$$T(n) = O(2^{n+1}) = O(2^n * 2^1) = O(2^n)$$

Ans 13. $O(n(\log n))$

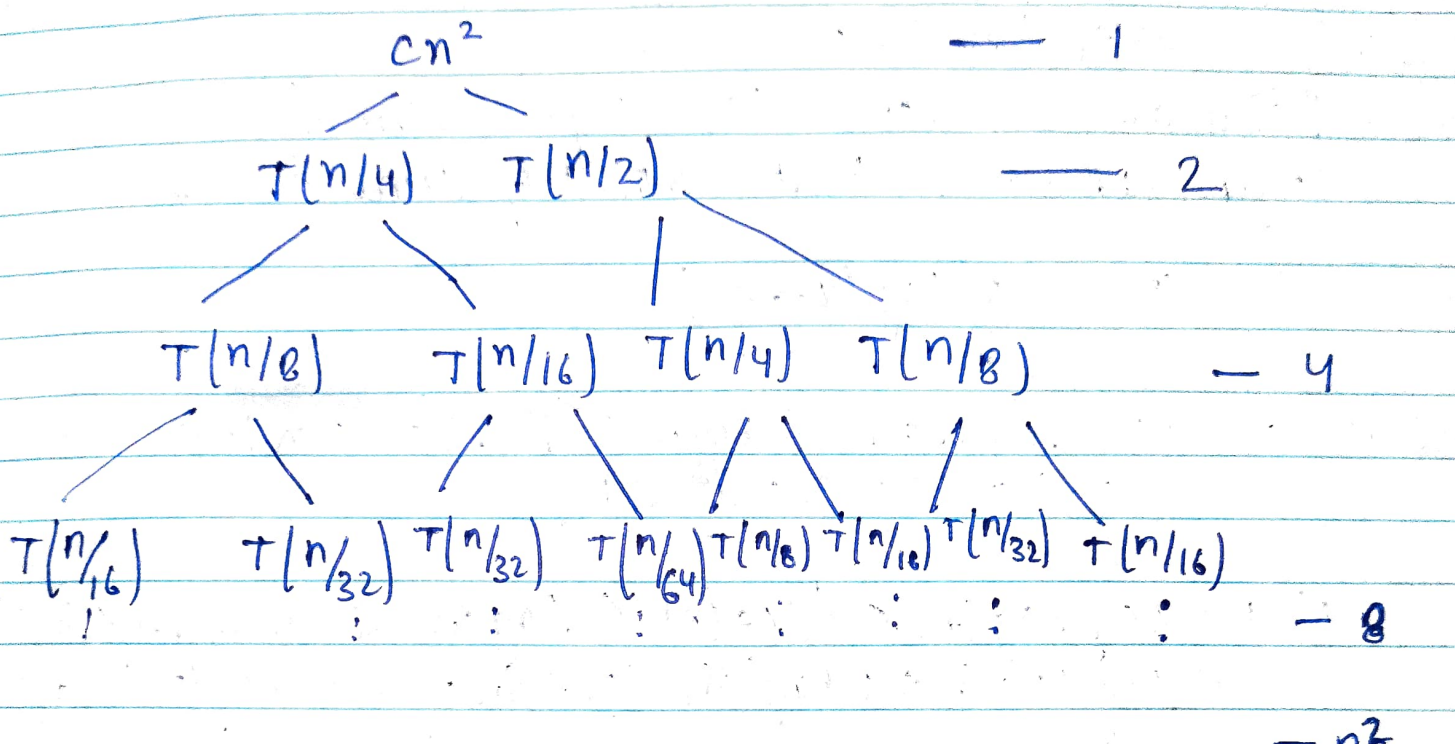
```
int n;
for (int i = 0; i<n ; i++) {
    for (int j = n; j > 0; j /= 2) {
        printf ("*");
    }
}
```

Ans 13 $O(n^3)$

```
int i, j, k;
for (i = 1, i<=n; i++) {
    for (j = 1; j <= n; j++) {
        for (k = 1; k <= n; k++) {
            printf ("*");
        }
    }
}
```

$n^3 + n^2 + 2$

Ans 14. $T(n) = T(n/4) + T(n/2) + cn^2$



$$T(n) = c\left[n^2 + 5\left(\frac{n^2}{16}\right) + 25\left(\frac{n^2}{256}\right) + \cdots\right]$$

ratio $= 5/16$

$$= \frac{n^2}{1 - 5/16} = O(n^2)$$

Ans 15.

```
int fun(int n) {
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j += i) {
            O(1); }
    }
```

i	j	times
1	1 → n	n
2	1 → n	n/2
⋮		
n		n/n

$$T(n) = n + \frac{n}{2} + \frac{n}{3} + \cdots \frac{n}{n}$$

$$= n\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n}\right)$$

$$T(n) = n \log(n)$$

Ans 16. $i = 2, 2^c, 2^{c^2}, 2^{c^3} \ldots\ldots 2^{c^{\log_c \log(n)}}$

The last term has to be $<= n$.

$2^{c \log_c(\log(n))} = 2^{\log n} = n$

There are in total $\log_c(\log(n))$ iterations each take a constant amount of time to run

$T.C = \theta(\log(\log n))$

Ans 18. a) $100 < \log\log n < \log n < \sqrt{n} < n < n\log n = \log(n!)$
$< n^2 < 2^n < 2^2 < 4^n < n!$

b) $1 < \log\log(n) < \sqrt{\log(n)} < \log n < 2n < 4n < 2(2^n)$
$< \log(2N) < 2\log(n) < n < n\log n = \log(n!) < n < n$

c) $96 < \log_2(n) = \log_8(n) < n\log_6(n) = n\log_2(n) = \log(n!)$
$< 5n < 8n^2 < 7n^3 < 8^{2n}$

Ans 19.
```
Int fun (int arr[N], Key) {
    for (i=0 to n-1) {
        if (ARRay[i] = key) {
            return i; }
    }
    return -1;
}
```

Ans 20. a Iterative Insertion Sort.

```
Void InsertionSort (int arr[], int n) {
    int i, temp, j;
    for (int i=1; i<=n-1; i++)
        temp = arr[i];
        j = i-1;
        while (j>=0 && arr[j] > temp) {
            arr[j+1] = arr[j];
            j = j-1; }
        arr[j+1] = temp;
```

Recursive Insertion sort

```
Void InsertionSort (int arr[], int n) {
    if (n<2)
        return;
    InsertionSort (arr, n-1);
    last = arr[n-1], j = n-2;
    while [j >=0 && arr (j] > temp) {
            arr [j+1] = arr [j];
            j = j-1; }
    arr [j+1] = last;
}
```

Ans 21.

Algorithm	Best case	Average case	Worst case
Bubble	$O(n^2)$	$O(n^2)$	$O(n^2)$
Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion	$O(n)$	$O(n^2)$	$O(n^2)$
Merge	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quick	$O(n \log n)$	$O(n \log n)$	$O(n^2)$
Heap	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$

Ans 22.

Algorithm	In-place	Stable	Online
Bubble	✓	✓	X
Selection	✓	X	X
Insertion	✓	✓	✓
Merge	X	✓	X
Quick	X	X	X
Heap	✓	X	X

ITERATIVE BINARY SEARCH

Ans 23.

```
int BinarySearch(int arr[], int l, int r, int x)
    while (l <= r) {
        int m = (l + r) / 2;
        if (arr[m] == x)
            return m;
        else if (arr[m] < x)
            l = m + 1;
        else r = m - 1; }
    return -1;
}
```

RECURSIVE BINARY SEARCH

```
int BinarySearch (int arr[], int l, int r, int x)
    if (l > r)
        return -1;
    int m = (l + r) / 2;
    if (arr[m] == x)
        return m;
    else if (arr[m] < x)
    return BinarySearch (arr, m+1, r, x);
    else
    return BinarySearch (arr, l, m-1, x);
}
```

Best

Time complexity,		Space complexity
Linear (Recursive) →	O(n)	O(1)
Binary (Recursive) →	O(log n)	O(log n)
Linear (Iterative) →	O(1)	O(1)
Binary (Iterative) →	O(1)	O(1)

Ans 24. Recurrence relation for binary search

$$T(n) = T(n/2) + 1.$$