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CS 224

1) ~~When~~ $0 \rightarrow (+A, 0)$
$1 \rightarrow (-A, 0)$

when sending 0, we'll incorrectly detect it (as 1) when $r_x < 0$, i.e, $\alpha s_x + n_x < 0$
$\Rightarrow \boxed{n_x < -\alpha A}$

(since the situation is symmetrical wrt to the y-axis, r_y has no importance ~~for~~ in bit error.)
By the same logic, when '1' is incorrectly detected as a '0'
we have $n_x > \alpha A$

Now, Probability of incorrectly detecting a bit

$$= P = \frac{1}{\sqrt{2\pi}} \left(\int_{\alpha A}^{\infty} e^{-x^2/2} \, dx + \int_{-\infty}^{-\alpha A} e^{-\frac{x^2}{2}} \, dx \right) \times \frac{1}{2}$$

the additional (1/2) at the end is for denoting the probability of getting a 1 (or a 0).

∴ Signal to noise Ratio per symbol

$$= \frac{\alpha^2 A^2}{N_0} = s$$

$\Rightarrow \alpha A = \sqrt{s N_0}$ ... where s is the SNR per symbol

$\Rightarrow$ ~~Prob (incorrect determination~~ of bit)

=

Prob of incorrect determination of bit

$$= \frac{1}{2\sqrt{2\pi}}\left(\int_{\alpha A}^{\infty} e^{-\frac{x^2}{2}}\,dx + \int_{-\infty}^{-\alpha A} e^{-\frac{x^2}{2}}\,dx\right)$$

$$= \frac{1}{2\sqrt{2\pi}}\left(\int_{\sqrt{N_0 S}}^{\infty} e^{-\frac{x^2}{2}}\,dx + \int_{-\infty}^{-\sqrt{N_0 S}} e^{-x^2/2}\,dx\right)$$

2) When working with QPSK, we have to make cases:-

If we send 00, then, if the first bit has an error in detection we have 2 cases

a) $10 \rightarrow \left(\frac{A}{\sqrt{2}}, \frac{-A}{\sqrt{2}}\right)$

b) $11 \rightarrow \left(\frac{-A}{\sqrt{2}}, \frac{-A}{\sqrt{2}}\right)$

$\therefore$ a) $r_x > 0$; $r_y < 0$

$\Rightarrow n_x > \frac{-\alpha A}{\sqrt{2}}$; $n_y < \frac{-\alpha A}{\sqrt{2}}$

b) $r_x < 0$; $r_y < 0$

$\Rightarrow n_x < \frac{-\alpha A}{\sqrt{2}}$; $n_y < \frac{-\alpha A}{\sqrt{2}}$

$$\Rightarrow P = \left(\frac{1}{4}\right) \times \left(\frac{1}{\sqrt{2\pi}}\int_{-\frac{\alpha A}{\sqrt{2}}}^{\infty} e^{-x^2/2} \cdot \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-\frac{\alpha A}{\sqrt{2}}} e^{-x^2/2}\,dx + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-\alpha A/\sqrt{2}} e^{-x^2/2}\,dx \cdot \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-\alpha A/\sqrt{2}} e^{-x^2/2}\,dx\right)$$

$$= \frac{1}{4} \times \frac{1}{2\pi}\left(\int_{-\frac{\alpha A}{\sqrt{2}}}^{\infty} e^{-x^2/2}\,dx \int_{-\infty}^{-\frac{\alpha A}{\sqrt{2}}} e^{-x^2/2}\,dx + \int_{-\infty}^{-\frac{\alpha A}{\sqrt{2}}} e^{-x^2/2}\,dx \cdot \int_{-\infty}^{-\alpha A/\sqrt{2}} e^{-x^2/2}\,dx\right)$$

(1)

basically, we have used the gaussian probability fn to calcaltate the probabibilities for the two cases, and then added them.

The $1/4$ factor comes as we can transmit any symbol from 00, 01, 10, 11.

$\therefore P(00) = 1/4$

Doing the same for 01,
cases :- 11 or 10

we get the $\text{Prob} = \frac{1}{4} \times \frac{1}{2\pi} \left(\int_{-\infty}^{\alpha A/\sqrt{2}} e^{-x^2/2} dx \cdot \int_{-\infty}^{-\alpha A/\sqrt{2}} e^{-x^2/2} dx + \int_{\alpha A/\sqrt{2}}^{} e^{-x^2/2} dx \cdot \int_{-\infty}^{-\alpha A/\sqrt{2}} e^{-x^2/2} dx \right)$ — (1)

Doing the same for 11 and 10,

we get theirs probs $= \frac{1}{4\sqrt{2\pi}} \left(\int_{\alpha A/\sqrt{2}}^{\infty} e^{-x^2/2} dx \right)$

Also, simplifying the expression (1)

we get $\text{prob} = \frac{1}{4\sqrt{2\pi}} \left(\int_{-\infty}^{-\alpha A/\sqrt{2}} e^{-x^2/2} dx \right)$

$\Rightarrow$ Total Prob

$$= \frac{1}{2\sqrt{2\pi}} \left(\int_{-\infty}^{-\alpha A/\sqrt{2}} e^{-x^2/2} dx + \int_{\alpha A/\sqrt{2}}^{\infty} e^{-x^2/2} dx \right)$$

also if s is the SNR per symbol.

$$s = \frac{\alpha^2 A^2}{N_0} \Rightarrow \alpha A = \sqrt{s N_0}$$

$\therefore$ Prob of incorrect bit determination

$$= \frac{1}{2\sqrt{2\pi}} \left(\int_{-\infty}^{-\sqrt{\frac{N_0 k}{2}}} e^{-x^2/2}\,dx + \underbrace{\int_{\sqrt{\frac{N_0 k}{2}}}^{\infty} e^{-x^2/2}\,dx}_{Q\left(\sqrt{\frac{N_0 k}{2}}\right)} \right)$$

Now, since the situation is symmetric wrt to the second bit too,

$\therefore$ ~~secon~~ Prob of second bit incorrectly determined = Prob of first bit incorrectly determined.


Legend
Green: Signal Strength > -85 dBm
Yellow: -105 dBm<Signal Strength<-85 dBm
Red: Signal Strength<-105 dBm
Lab Question
Pipe Line Rd
Hillside Rd
Swimming Pool Rd
CRNTS Rd
Hostel Rd
Indian Institute of Technology Bombay
YP Rd
JYOTIBA PHULE NAGAR
Holy Trinity Church, Powai