

MATHEMATICS

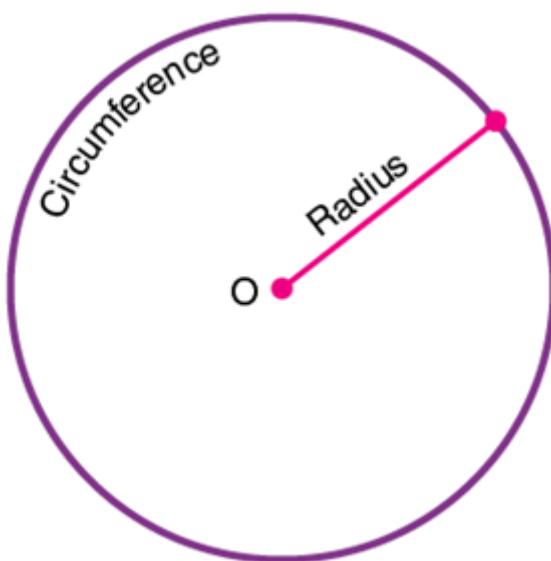
Chapter 10: Circles



Circles

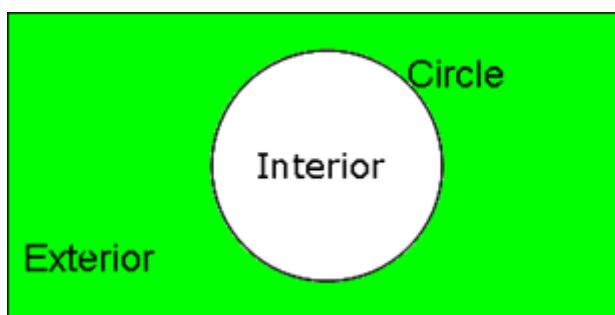
Circle

- The set of all the points in a plane that is at a fixed distance from a fixed point makes a circle.
- A Fixed point from which the set of points are at fixed distance is called the centre of the circle.
- A circle divides the plane into 3 parts: interior (inside the circle), the circle itself and exterior (outside the circle)



Division of a plane using circle

- A circle divides the plane on which it lies into three parts: inside the circle, the circle and outside the circle.
- All the points lying inside a circle are called its **interior points** and all those points which lie outside the circle are called its **exterior points**.
- The collection (set) of all interior points of a circle is called the **interior of the circle** while the collection (set) of all exterior points of a circle is called the **exterior of the circle**.

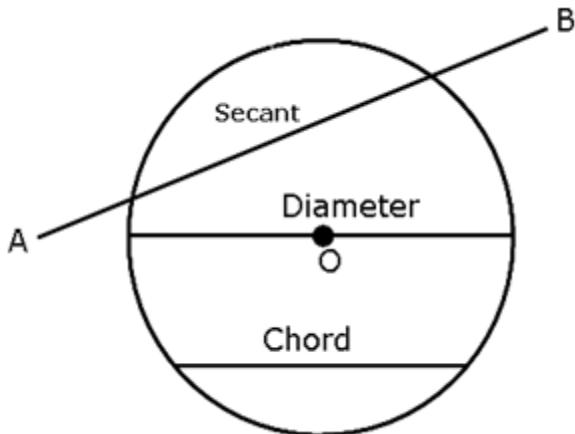


Chord, diameter and secant of a circle

- A line can meet a circle at the most in two points and the line segment joining two

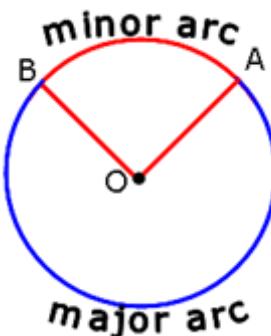
points on a circle is called a **chord** of the circle.

- A chord passing through the center of the circle is called a **diameter** of the circle. A diameter of the circle is its longest chord. It is equal to two times the radius.
- A line which meets a circle in two points is called a **secant** of the circle.



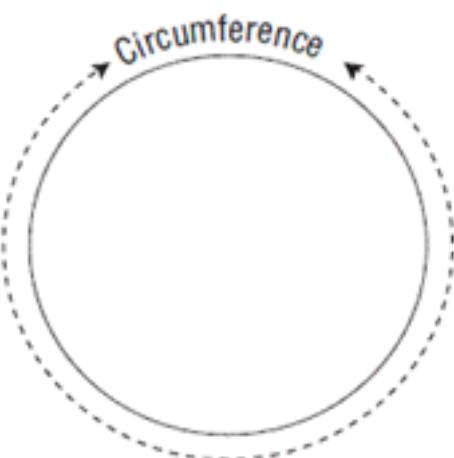
Arc of the circle

- A (continuous) part of a circle is called an **arc** of the circle. The arc of a circle is denoted by the symbol ''.
- When an arc is formed, it divides the circle into two pieces (between the points A and B), the smaller one and the longer one. The smaller one is called the **minor arc** of the circle, and the greater one is called the **major arc** of the circle.

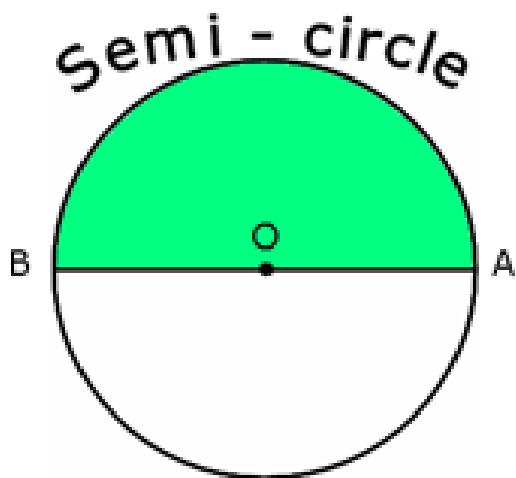


Circumference and Semi-circle

- The length of the complete circle is called the **circumference** of the circle.



- One-half of the whole arc (circumference) of a circle is called a **semi-circle**.



Central angle and Degree measure

- Any angle whose vertex is centre of the circle is called a **central angle**.
- The **degree measure of a minor arc** is the measure of the central angle subtended by an arc.
- The degree measure of a circle is 360° . The degree measure of a semi-circle is 180° (half of the circle).
- The degree measure of a major arc is $(360^\circ - \theta^\circ)$, where θ° is the degree measure of the corresponding minor arc.

Congruent circles and arcs

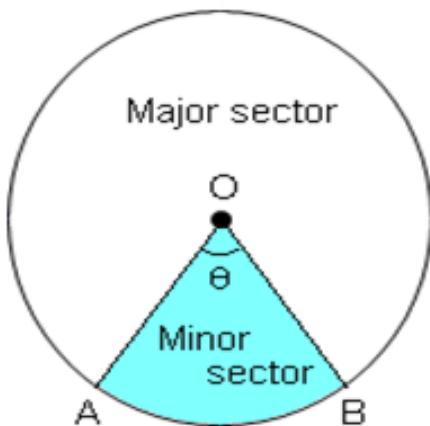
- Two **circles** are said to be **congruent** if and only if either of them can be superposed on the other so as to cover it exactly.
- Accordingly, two **arcs** of a circle (or of congruent circles) are **congruent** if either of them can be superposed on the other so as to cover it exactly.

Sector of a circle

- The part of the plane region enclosed by an arc of a circle and its two bounding radii

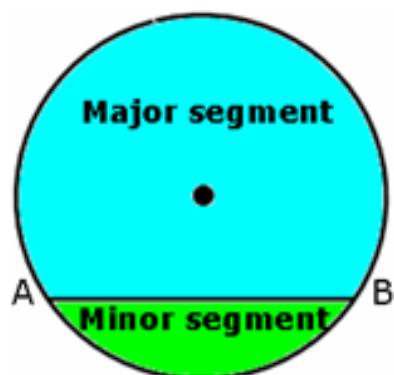
is called **sector** of a circle.

- If the central angle of a sector is more than 180° , then the sector is called a **major sector** and if the central angle is less than 180° , then the sector is called a **minor sector**.



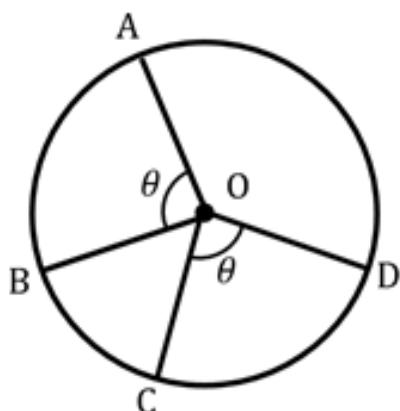
Segment of a circle

- A chord of a circle divides it into two parts. Each part is called a **segment** of the circle.
- The part containing the minor arc is called the **minor segment**, and the part containing the major arc is called the **major segment**.



Angle subtended by a chord and perpendicular drawn to a chord

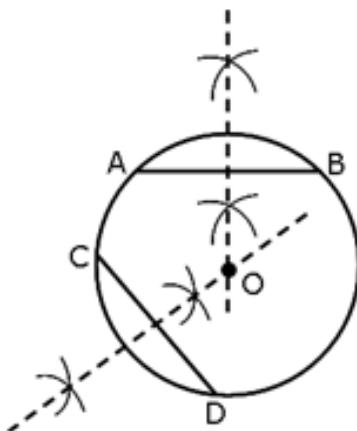
- Equal chords of a circle subtend equal angles at the centre.



- If the angles subtended by the chords of a circle at the centre are equal, then the

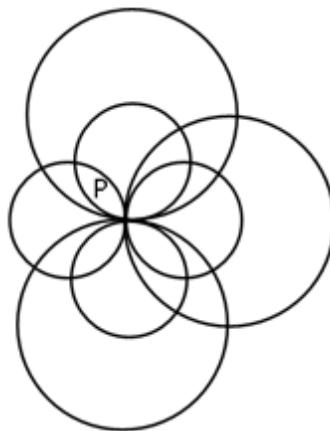
chords are equal.

- In a circle, perpendicular from the center to a chord bisects the chord.
- A line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
- Perpendicular bisectors of two chords of a circle, intersect each other at the centre of the circle.

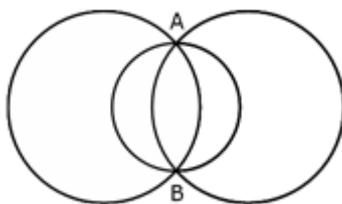


Number of circle through one or more point(s)

- An infinite number of circles can be drawn through a given point, say P.



- An infinite number of circles can be drawn through two given points, say A and B.



- One and only one circle can be drawn through three non-collinear points.

Distance of chord from the centre

- The length of the perpendicular from a point to a line is the distance of the line from the point.

- Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).
- Chords equidistant from the centre of a circle are equal in length.

Angle subtended by an Arc of a circle

- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- If two chords of a circle are equal, then their corresponding arcs are congruent.
- Conversely, if two arcs are congruent, then their corresponding chords are equal.
- Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.

Con-cyclic points

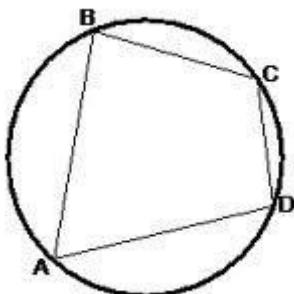
- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points are concyclic, i.e., they lie on the same circle.
- Angles in the same segment of a circle are equal.

Angle in a semi-circle

- An angle in a semi-circle is a right angle.
- The arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

Cyclic quadrilaterals

A quadrilateral, all the four vertices of which lie on a circle is called a **cyclic quadrilateral**. The four vertices A, B, C and D are said to be concyclic points.

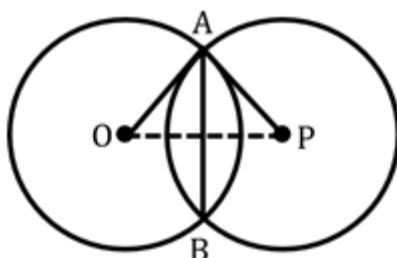


Properties of cyclic quadrilateral

- The opposite angles of a cyclic quadrilateral are supplementary i.e. their sum is 180° .
- If the sum of any pair of opposite angles of a quadrilateral is 180° , then the quadrilateral is cyclic.
- Any exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- The quadrilateral formed (if possible) by the internal angle bisectors of any

quadrilateral is cyclic.

- The line of centres of two intersecting circles subtends equal angles at the two points of intersection.

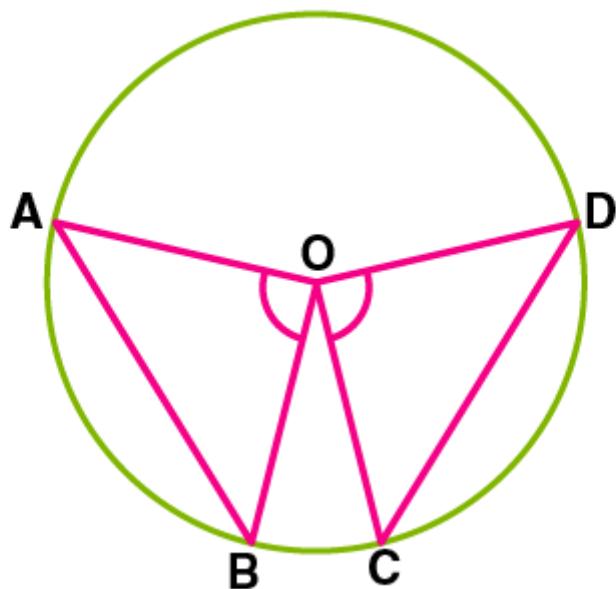


In the figure, $\angle OAM = \angle PAM$.

- If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, then it is a rectangle.
- If the non-parallel sides of a trapezium are equal, then it is cyclic.

Theorem of equal chords subtending angles at the centre.

Equal chords subtend equal angles at the centre.



Proof: AB and CD are the 2 equal chords.

In $\triangle AOB$ and $\triangle COD$

$OB = OC$ [Radii]

$OA = OD$ [Radii]

$AB = CD$ [Given]

$\triangle AOB \cong \triangle COD$ (SSS rule)

Hence, $\angle AOB = \angle COD$ [CPCT]

Theorem of equal angles subtended by different chords.

If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Proof: In ΔAOB and ΔCOD

$$OB = OC \text{ [Radii]} \quad \angle AOB = \angle COD \text{ [Given]}$$

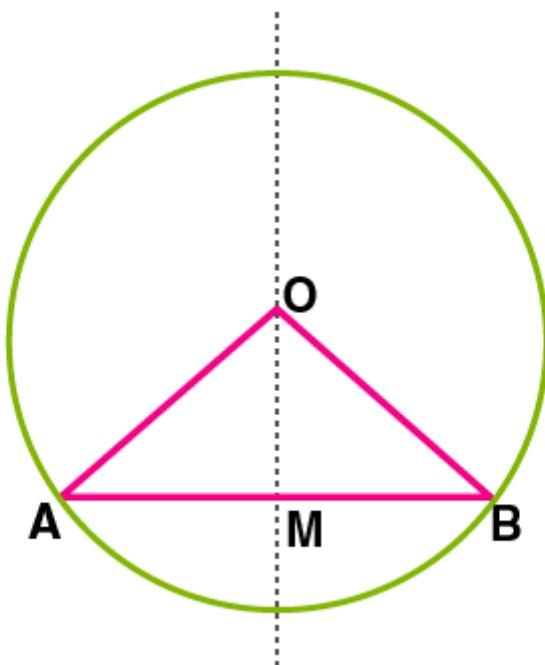
$$OA = OD \text{ [Radii]}$$

$$\Delta AOB \cong \Delta COD \text{ (SAS rule)}$$

$$\text{Hence, } AB = CD \text{ [CPCT]}$$

Perpendicular from the centre to a chord bisects the chord.

Perpendicular from the centre of a circle to a chord bisects the chord.



Proof: AB is a chord and OM is the perpendicular drawn from the centre.

From ΔOMB and ΔOMA ,

$$\angle OMA = \angle OMB = 90^\circ \quad OA = OB \text{ (radii)}$$

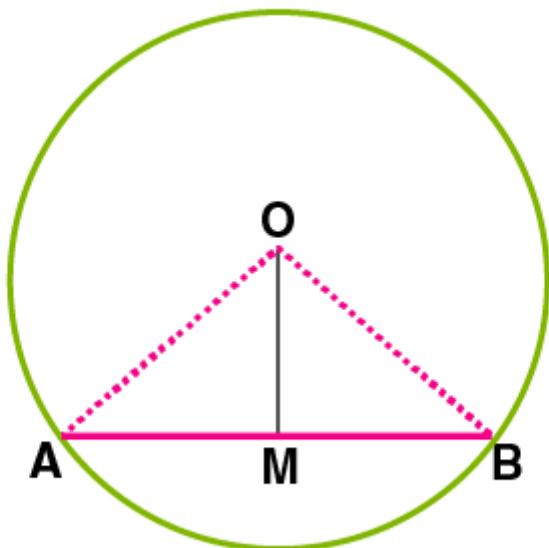
$$OM = OM \text{ (common)}$$

$$\text{Hence, } \Delta OMB \cong \Delta OMA \text{ (RHS rule)}$$

$$\text{Therefore } AM = MB \text{ [CPCT]}$$

A Line through the centre that bisects the chord is perpendicular to the chord.

A line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.



Proof: OM drawn from the center to bisect chord AB.

From ΔOMA and ΔOMB ,

$OA = OB$ (Radii)

$OM = OM$ (common)

$AM = BM$ (Given)

Therefore, $\Delta OMA \cong \Delta OMB$ (SSS rule)

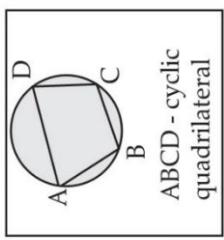
$\Rightarrow \angle OMA = \angle OMB$ (C.P.C.T)

But, $\angle OMA + \angle OMB = 180^\circ$

Hence, $\angle OMA = \angle OMB = 90^\circ \Rightarrow OM \perp AB$

CHAPTER : 10 CIRCLES

Statement	Figure
1. Equal chords of a circle subtend equal angles at the centre.	
2. If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.	
3. The perpendicular from the centre of a circle to a chord bisects the chord.	
4. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.	
5. There is one and only one circle passing through three given non-collinear points.	
6. Equal chords of a circle are equidistant from the centre.	
7. Chords equidistant from the centre of a circle are equal in length.	
8. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.	
9. Angles in the same segment of a circle are equal.	
10. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.	
11. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.	
12. If the sum of a pair of opposite angles of a quadrilateral is 180°, then quadrilateral is cyclic.	



All the four vertices of a quadrilateral ABCD lie on circle

Cyclic quadrilateral

A circle is the locus of a curve equidistant from a point

Fixed point
centre
radius (r)
Fixed distance

Radius
Centre

Related terms
Theorems

Diameter (D)

chord which passes through the centre of a circle
 $D = 2r$
AB = Diameter of a circle

Circle
Interior
Exterior
Major arc
Minor arc
Major segment
Minor segment

Circle
Interior
Exterior
Major arc
Minor arc
Major segment
Minor segment

Important Questions

Multiple Choice questions-

Question 1. If there are two separate circles drawn apart from each other, then the maximum number of common points they have:

- (a) 0
- (b) 1
- (c) 2
- (d) 3

Question 2. D is diameter of a circle and AB is a chord. If AD = 50cm, AB = 48cm, then the distance of AB from the Centre of the circle is

- (a) 6cm
- (b) 8cm
- (c) 5cm
- (d) 7cm

Question 3. In a circle with center O and a chord BC, points D and E lie on the same side of BC. Then, if $\angle BDC = 80^\circ$, then $\angle BEC =$

- (a) 80°
- (b) 20°
- (c) 160°
- (d) 40°

Question 4. The center of the circle lies in _____ of the circle.

- (a) Interior
- (b) Exterior
- (c) Circumference
- (d) None of the above

Question 5. If chords AB and CD of congruent circles subtend equal angles at their centers, then:

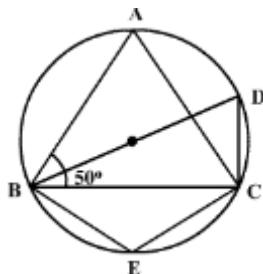
- (a) AB = CD
- (b) AB > CD
- (c) AB < CD
- (d) None of the above

Question 6. Segment of a circle is the region between an arc and of the

circle.

- (a) Perpendicular
- (b) Radius
- (c) Chord
- (d) Secant

Question 7. In the figure, triangle ABC is an isosceles triangle with $AB = AC$ and measure of angle $ABC = 50^\circ$. Then the measure of angle BDC and angle BEC will be



- (a) $60^\circ, 100^\circ$
- (b) $80^\circ, 100^\circ$
- (c) $50^\circ, 100^\circ$
- (d) $40^\circ, 120^\circ$

Question 8. The region between chord and either of the arc is called.

- (a) A sector
- (b) A semicircle
- (c) A segment
- (d) A quarter circles

Question 9. The region between an arc and the two radii joining the Centre of the end points of the arc is called a:

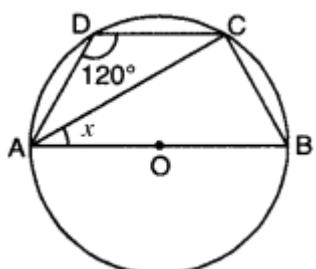
- (a) Segment
- (b) Semi circle
- (c) Minor arc
- (d) Sector

Question 10. If a line intersects two concentric circles with Centre O at A, B, C and D, then:

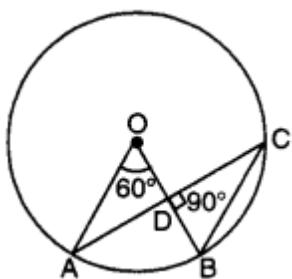
- (a) $AB = CD$
- (b) $AB > CD$
- (c) $AB < CD$
- (d) None of the above

Very Short:

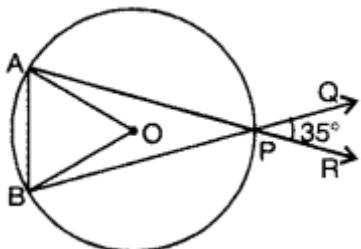
1. In the figure, O is the Centre of a circle passing through points A, B, C and D and $\angle ADC = 120^\circ$. Find the value of x .



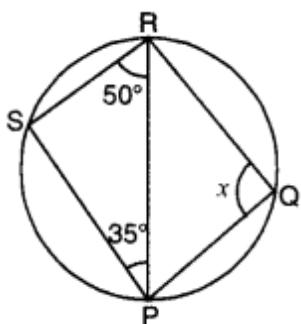
2. In the given figure, O is the Centre of the circle, $\angle AOB = 60^\circ$ and $CDB = 90^\circ$. Find $\angle OBC$.



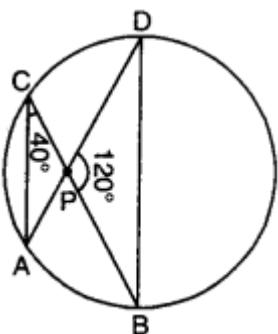
3. In the given figure, O is the Centre of the circle with chords AP and BP being produced to R and Q respectively. If $\angle QPR = 35^\circ$, find the measure of $\angle AOB$.



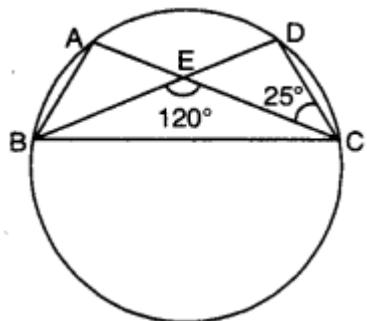
4. In the figure, PQRS is a cyclic quadrilateral. Find the value of x .



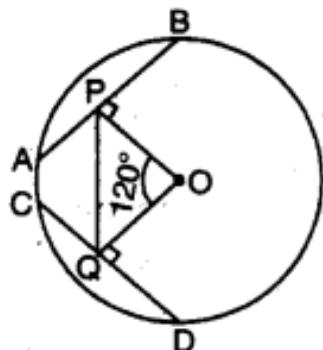
5. In the given figure, $\angle ACP = 40^\circ$ and $BPD = 120^\circ$, then find $\angle CBD$.



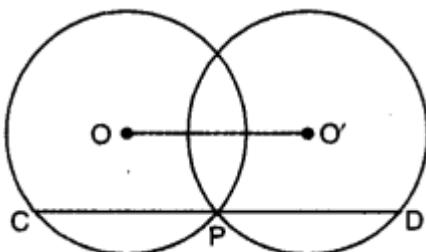
6. In the given figure, if $\angle BEC = 120^\circ$, $\angle DCE = 25^\circ$, then find $\angle BAC$.



7. In the given figure, AB and CD are two equal chords of a circle with Centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If $\angle POQ = 120^\circ$, find $\angle APQ$.

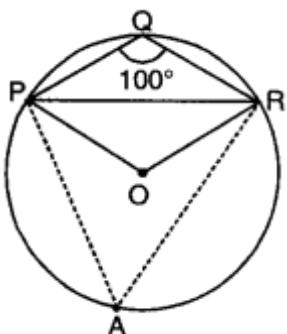


8. Two circles whose centers are O and O' intersect at P. Through P, a line parallel to OO', intersecting the circles at C and D is drawn as shown in the figure. Prove that $CD = 2OO'$

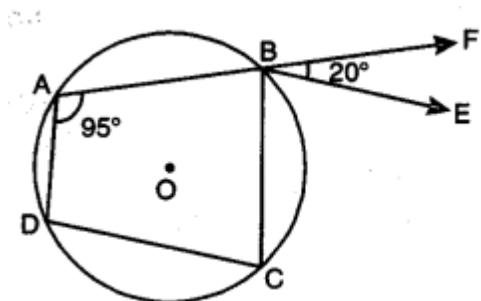


Short Questions:

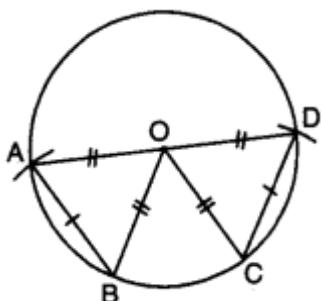
1. In the given figure, $PQR = 100^\circ$, where P, Q and R are points on a circle with Centre O. Find LOPR .



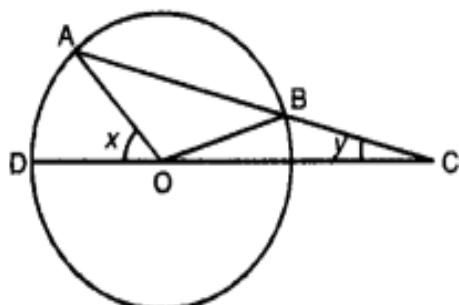
2. In figure, ABCD is a cyclic quadrilateral in which AB is extended to F and BE \parallel DC. If $\angle FBE = 20^\circ$ and $\angle DAB = 95^\circ$, then find $\angle ADC$.



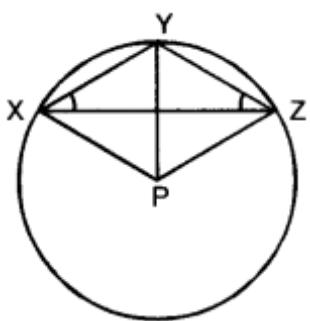
3. If the diagonals of a cyclic quadrilateral are diameters of the circle through the opposite vertices of the quadrilateral. Prove that the quadrilateral is a rectangle.
4. Equal chords of a circle subtend equal angles at the Centre.



5. In the figure, chord AB of circle with Centre O, is produced to C such that BC = OB. CO is joined and produced to meet the circle in D. If $\angle ACD = y$ and $\angle AOD = x$, show that $x = 3y$.

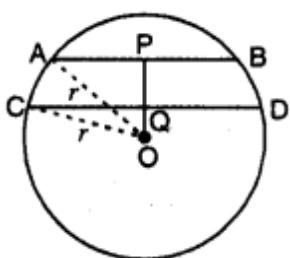


6. In the given figure, P is the Centre of the circle. Prove that: $\angle XPZ = 2(\angle XZY + \angle YXZ)$.

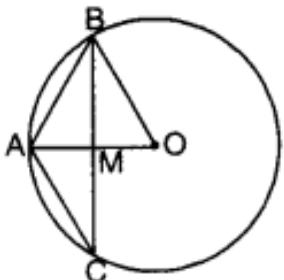


Long Questions:

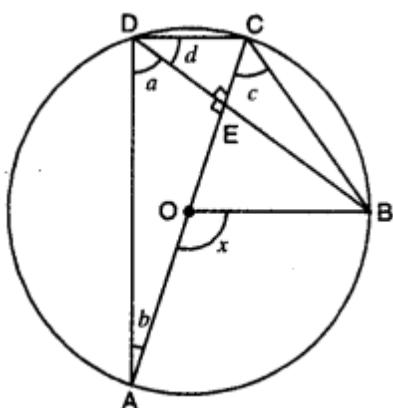
1. In the given figure, O is the Centre of a circle of radius r cm, OP and OQ are perpendiculars to AB and CD respectively and $PQ = 1\text{cm}$. If $AB \parallel CD$, $AB = 6\text{cm}$ and $CD = 8\text{cm}$, determine r



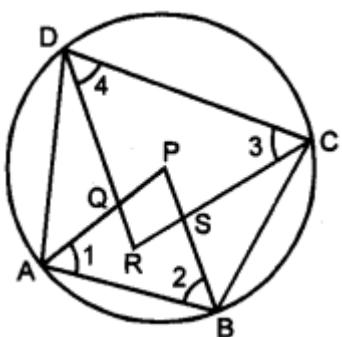
2. In a circle of radius 5cm, AB and AC are two chords such that $AB = AC = 6\text{cm}$, as shown in the figure. Find the length of the chord BC.



3. In the given figure, AC is a diameter of the circle with Centre O. Chord BD is perpendicular to AC. Write down the measures of angles a, b, c and d in terms of x.



4. Show that the quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.



5. PQ and PR are the two chords of a circle of radius r. If the perpendiculars drawn from the Centre of the circle to these chords are of lengths a and b, $PQ = 2PR$, then

$$\text{prove that: } b^2 = \frac{a^2}{4} + \frac{3}{4} r^2$$

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

- a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
- b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement.
- d) Assertion is wrong statement but reason is correct statement.

Assertion: In a circle of radius 6 cm, the angle of a sector 60° . Then the area of the sector is $186/7 \text{ cm}^2$.

Reason: Area of the circle with radius r is πr^2 .

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

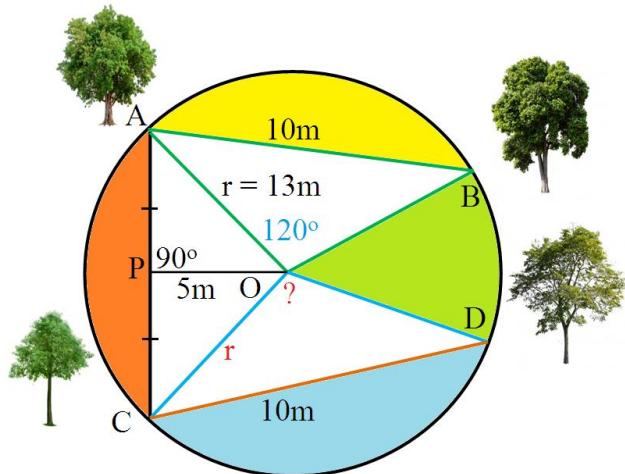
- a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
- b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement.
- d) Assertion is wrong statement but reason is correct statement.

Assertion: The length of the minute hand of a clock is 7cm. Then the area swept by the minute hand in 5 minutes is $12^5/6 \text{ cm}^2$.

Reason: The length of an arc of a sector of angle θ and radius r is given by $l = \theta/360 \times 2\pi r$

Case Study Questions-

1. Read the Source/ Text given below and answer these questions:



A farmer has a circular garden as shown in the picture above. He has a different type of trees, plants and flower plants in his garden. In the garden, there are two mango trees A and B at a distance of $AB = 10\text{m}$. Similarly, he has two Ashoka trees at the same distance of 10m as shown at C and D. AB subtends $\angle AOB=120^\circ$ at the center O, The perpendicular distance of AC from center is 5m . The radius of the circle is 13m .

Now answer the following questions:

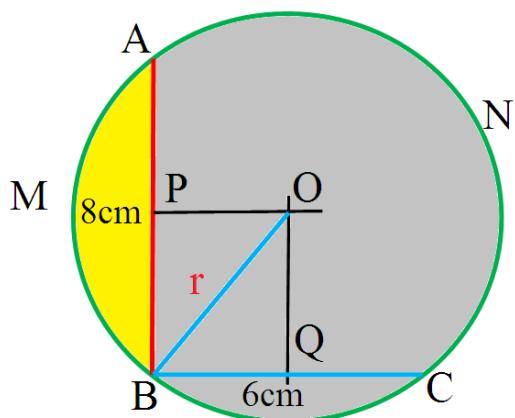
- i. What is the value of $\angle COD$?
 - a. 60°
 - b. 120°
 - c. 100°
 - d. 80°

- ii. What is the distance between mango tree A and Ashok tree C?
 - a. 12m
 - b. 24m
 - c. 13m
 - d. 15m

- iii. What is the value of $\angle OAB$?
 - a. 60°
 - b. 120°
 - c. 30°
 - d. 90°

- iv. What is the value of $\angle OCD$?

- a. 30°
 - b. 120°
 - c. 60°
 - d. 90°
- v. What is the value of $\angle ODC$?
- a. 90°
 - b. 120°
 - c. 60°
 - d. 30°
2. Read the Source/ Text given below and answer these questions:



As Class IX C' s teacher Mrs. Rashmi entered in the class, She told students to do some practice on circle chapter. She Draws two-line AB and BC so that $AB = 8\text{cm}$ and $BC = 6\text{cm}$. She told all students To make this shape in their notebook and draw a circle passing through the three points A, B and C.

- i. Dileep drew AB and BC as per the figure
- ii. He drew perpendicular bisectors OP and OQ of the line AB and BC.
- iii. OP and OQ intersect at O
- iv. Now taking O as centre and OB as radius he drew The circle which passes through A, B and C.
- v. He noticed that A, O and C are collinear.

Answer the following questions:

- i. What you will call the line AOC?
 - a. Arc
 - b. Diameter
 - c. Radius
 - d. Chord

ii. What is the measure of $\angle ABC$?

- a. 60°
- b. 90°
- c. 45°
- d. 75°

iii. What you will call the yellow color shaded area AMB?

- a. Arc.
- b. Sector.
- c. Major segment.
- d. Minor Segment.

iv. What you will call the grey colour shaded area BCNA?

- a. Arc.
- b. Sector.
- c. Major segment.
- d. Minor Segment.

v. What is the radius of the circle?

- a. 4cm
- b. 3cm
- c. 7cm
- d. 5cm

Answer Key:

MCQ:

1. (a) 0
2. (d) 7cm
3. (a) 80°
4. (a) Interior
5. (a) $AB = CD$
6. (c) Chord
7. (b) $80^\circ, 100^\circ$
8. (c) A segment
9. (d) Sector
10. (a) $AB = CD$

Very Short Answer:

1. Since ABCD is a cyclic quadrilateral

$$\angle ADC + \angle ABC = 180^\circ$$

[∴ opp. ∠s of a cyclic quad. are supplementary]

$$120^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 180^\circ - 120^\circ = 60^\circ$$

Now, $\angle ACB = 90^\circ$ [angle in a semicircle]

In rt. ∠ed ΔCB, $\angle ACB = 90^\circ$

$$\angle CAB + \angle ABC = 90^\circ$$

$$x + 60^\circ = 90^\circ$$

$$x = 90^\circ - 60^\circ$$

$$x = 30^\circ$$

2. Since angle subtended at the Centre by an arc is double the angle subtended at the remaining part of the circle.

$$\therefore \angle ACB = \frac{1}{3} \angle AOB = \frac{1}{3} \times 60^\circ = 30^\circ$$

Now, in ACBD, by using angle sum property, we have

$$\angle CBD + \angle BDC + \angle DCB = 180^\circ$$

$$\angle CBO + 90^\circ + \angle ACB = 180^\circ$$

[∴ $\angle CBO = \angle CBD$ and $\angle ACB = \angle DCB$ are the same ∠s]

$$\angle CBO + 90^\circ + 30^\circ = 180^\circ$$

$$\angle CBO = 180^\circ - 90^\circ - 30^\circ = 60^\circ$$

$$\text{or } \angle OBC = 60^\circ$$

3. $\angle APB = \angle RPQ = 35^\circ$ [vert. opp. ∠s]

Now, $\angle AOB$ and $\angle APB$ are angles subtended by an arc AB at Centre and at the remaining part of the circle.

$$\therefore \angle AOB = 2\angle APB = 2 \times 35^\circ = 70^\circ$$

4. In ΔPRS, by using angle sum property, we have

$$\angle PSR + \angle SRP + \angle RPS = 180^\circ$$

$$\angle PSR + 50^\circ + 35^\circ = 180^\circ$$

$$\angle PSR = 180^\circ - 85^\circ = 95^\circ$$

Since PQRS is a cyclic quadrilateral

$$\therefore \angle PSR + \angle PQR = 180^\circ$$

[\because opp. \angle s of a cyclic quad. are supplementary]

$$95^\circ + x = 180^\circ$$

$$x = 180^\circ - 95^\circ$$

$$x = 85^\circ$$

5. $\angle BDP = \angle ACP = 40^\circ$ [angle in same segment]

Now, in $\triangle BPD$, we have

$$\angle PBD + \angle BPD + \angle BDP = 180^\circ$$

$$\Rightarrow \angle PBD + 120^\circ + 40^\circ = 180^\circ$$

$$\Rightarrow \angle PBD = 180^\circ - 160^\circ = 20^\circ$$

$$\text{or } \angle CBD = 20^\circ$$

6. $\angle BEC$ is exterior angle of $\triangle CDE$.

$$\therefore \angle CDE + \angle DCE = \angle BEC$$

$$\Rightarrow \angle CDE + 25^\circ = 120^\circ$$

$$\Rightarrow \angle CDE = 95^\circ$$

7. Arc XY subtends $\angle XPY$ at the Centre P and $\angle XZY$ in the remaining part of the circle.

$$\therefore \angle XPY = 2(\angle XZY)$$

Similarly, arc YZ subtends $\angle YPZ$ at the Centre P and $\angle YXZ$ in the remaining part of the circle.

$$\therefore \angle YPZ = 2(\angle YXZ) \dots \text{(ii)}$$

Adding (i) and (ii), we have

$$\angle XPY + \angle YPZ = 2(\angle XZY + \angle YXZ)$$

$$\angle XPA = 2(\angle XZY + \angle YXZ)$$

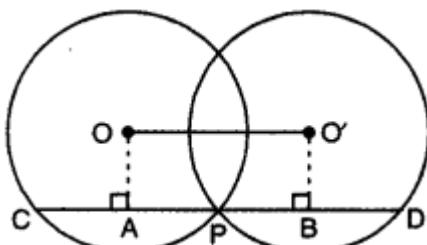
8. Draw $OA \perp CD$ and $O'B \perp CD$

Now, $OA \perp CD$

$OA \perp CP$

$$CA = AP = \frac{1}{2} CP$$

$$CP = 2AP \dots \text{(i)}$$



Similarly, $O'B \perp CD$

$O'B \perp PD$

$$\Rightarrow PB = BD = \frac{1}{2} PD$$

$$\Rightarrow PD = 2PB$$

$$\text{Also, } CD = CP + PD$$

$$= 2AP + 2PB = 2(AP + PB) = 2AB$$

$$CD = 2OO' [\because OABO' \text{ is a rectangle}]$$

Short Answer:

Ans: 1. Take any point A on the circumcircle of the circle.

Join AP and AR.

$\therefore APQR$ is a cyclic quadrilateral.

$\therefore \angle PAR + \angle PQR = 180^\circ$ [sum of opposite angles of a cyclic quad. is 180°]

$$\angle PAR + 100^\circ = 180^\circ$$

\Rightarrow Since $\angle POR$ and $\angle PAR$ are the angles subtended by an arc PR at the Centre of the circle and circumcircle of the circle.

$$\angle POR = 2\angle PAR = 2 \times 80^\circ = 160^\circ$$

\therefore In $APOR$, we have $OP = OR$ [radii of same circle]

$\angle OPR = \angle ORP$ [angles opposite to equal sides]

$$\text{Now, } \angle POR + \angle OPR + \angle ORP = 180^\circ$$

$$\Rightarrow 160^\circ + \angle OPR + \angle ORP = 180^\circ$$

$$\Rightarrow 2\angle OPR = 20^\circ$$

$$\Rightarrow \angle OPR = 10^\circ$$

Ans: 2. Sum of opposite angles of a cyclic quadrilateral is 180°

$$\therefore \angle DAB + \angle BCD = 180^\circ$$

$$\Rightarrow 95^\circ + \angle BCD = 180^\circ$$

$$\Rightarrow \angle BCD = 180^\circ - 95^\circ = 85^\circ$$

$\therefore BE \parallel DC$

$\therefore \angle CBE = \angle BCD = 85^\circ$ [alternate interior angles]

$$\therefore \angle CBF = \angle CBE + \angle FBE = 85^\circ + 20^\circ = 105^\circ$$

$$\text{Now, } \angle ABC + \angle CBF = 180^\circ$$
 [linear pair]

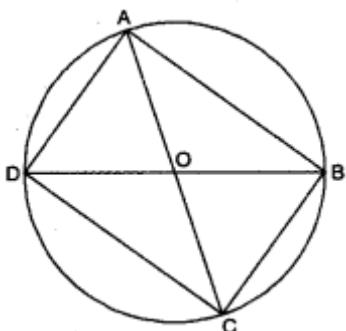
and $\angle ABC + \angle ADC = 180^\circ$ [opposite angles of cyclic quad.]

Thus, $\angle ABC + \angle ADC = \angle ABC + 2CBF$

$$\Rightarrow \angle ADC = \angle CBF$$

$$\Rightarrow \angle ADC = 105^\circ [\because \angle CBF = 105^\circ]$$

Ans: 3. Here, ABCD is a cyclic quadrilateral in which AC and BD are diameters.



Since AC is a diameter.

$$\therefore \angle ABC = \angle ADC = 90^\circ$$

$$[\because \text{angle of a semicircle} = 90^\circ]$$

Also, BD is a diameter

$$\therefore \angle BAD = \angle BCD = 90^\circ [\because \text{angle of a semicircle} = 90^\circ]$$

Now, all the angles of a cyclic quadrilateral ABCD are 90 each.

Hence, ABCD is a rectangle.

Ans: 4. Given: In a circle C(O, r), chord AB = chord CD

To Prove: $\angle AOB = \angle COD$.

Proof: In $\triangle AOB$ and $\triangle COD$

$$AO = CO \text{ (radii of same circle)}$$

$$BO = DO \text{ (radii of same circle)}$$

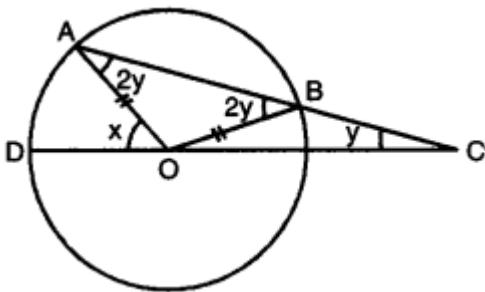
$$\text{Chord } AB = \text{Chord } CD \text{ (given)}$$

$$\Rightarrow \triangle AOB \cong \triangle COD \text{ [by SSS congruence axiom]}$$

$$\Rightarrow \angle AOB = \angle COD \text{ (c.p.c.t.)}$$

Ans: 5. In $\triangle OBC$, OB = BC

$$\Rightarrow \angle BOC = \angle BCO = y \text{ [angles opp. to equal sides are equal]}$$



$\angle OBA$ is the exterior angle of $\triangle BOC$

So, $\angle ABO = 2y$ [ext. angle is equal to the sum of int. opp. angles]

Similarly, $\angle AOD$ is the exterior angle of $\triangle AOC$

$$\therefore x = 2y + y = 3y$$

Ans: 6. Arc XY subtends $\angle XPY$ at the Centre P and $\angle XZY$ in the remaining part of the circle.

$$\therefore \angle XPY = 2(\angle XZY)$$

Similarly, arc YZ subtends $\angle YPZ$ at the Centre P and $\angle YXZ$ in the remaining part of the circle.

$$\therefore \angle YPZ = 2(\angle YXZ) \dots(ii)$$

Adding (i) and (ii), we have

$$\angle XPY + \angle YPZ = 2(\angle XZY + \angle YXZ)$$

$$\angle XPZ = 2(\angle XZY + \angle YXZ)$$

Long Answer:

Ans: 1. Since the perpendicular drawn from the Centre of the circle to a chord bisects the chord. Therefore, P and Q are mid-points of AB and CD respectively.

$$\text{Consequently, } AP = BP = \frac{1}{2} AB = 3\text{cm}$$

$$\text{and } CQ = QD = \frac{1}{2} CD = 4\text{cm}$$

In right-angled $\triangle QAP$, we have

$$OA^2 = OP^2 + AP^2$$

$$r^2 = OP^2 + 3^2$$

$$r^2 = OP^2 + 9$$

In right-angled $\triangle OCQ$, we have

$$OC^2 = OQ^2 + CQ^2$$

$$r^2 = OQ^2 + 4^2$$

$$r^2 = OQ^2 + 16 \dots(ii)$$

From (i) and (ii), we have

$$OP^2 + 9 = OQ^2 + 16$$

$$OP^2 - OQ^2 = 16 - 9$$

$$x^2 - (x - 1)^2 = 16 - 9 \quad [\text{where } OP = x \text{ and } PQ = 1\text{cm given}]$$

$$x^2 - (x^2 - 2x + 1) = 7$$

$$2x = 7 + 1$$

$$x = 4$$

$$\Rightarrow OP = 4\text{cm}$$

From (i), we have

$$r^2 = (4)^2 + 9$$

$$r^2 = 16 + 9 = 25$$

$$r = 5\text{cm}$$

Ans: 2. Here, OA = OB = 5cm [radii]

$$AB = AC = 6\text{cm}$$

\therefore B and C are equidistant from A.

\therefore AO is the perpendicular bisector of chord BC and it intersect BC in M.

Now, in rt. \angle ed ΔAMB , $M = 90^\circ$ (i)

\therefore By using Pythagoras Theorem, we have

$$BM^2 = AB^2 - AM^2$$

$$= 36 - AM^2$$

Also, in rt. \angle ed ΔBMO , $M = 90^\circ$

\therefore By using Pythagoras Theorem, we have

$$BM^2 = BO^2 - MO^2 = 25 - (AO - AM)^2$$

From (i) and (ii), we obtain

$$25 - (AO - AM)^2 = 36 - AM^2$$

$$25 - AOC - AM^2 + 240 \times AM = 36 - AM^2$$

$$25 - 25 + 2 \times 5 \times AM = 36$$

$$10 AM = 36$$

$$AM = 3.6\text{cm}$$

From (i), we have

$$BM^2 = 36 - (3.6)^2 = 36 - 12.96 = 23.04$$

$$BM = \sqrt{23.04} = 4.8\text{cm}$$

$$\text{Thus, } BC = 2 \times BM$$

$$= 2 \times 4.8 = 9.6\text{cm}$$

Hence, the length of the chord BC is 9.6cm.

Ans: 3. Here, AC is a diameter of the circle.

$$\therefore \angle ADC = 90^\circ$$

$$\Rightarrow \angle a + \angle d = 90^\circ$$

In right-angled ΔAED , $\angle E = 90^\circ$

$$\therefore \angle a + 2b = 90^\circ$$

From (i) and (ii), we obtain

$$\angle b = \angle d \dots \text{(iii)}$$

$$\text{Also, } \angle a = \angle c \dots \text{(iv)}$$

[\angle s subtended by the same segment are equal]

Now, $\angle AOB$ and $\angle ADB$ are angles subtended by an arc AB at the Centre and at the remaining part of the circle.

$$\therefore \angle ADB = \frac{1}{2} \angle AOB \Rightarrow \angle a = \frac{x}{2}$$

$$\text{From (iv), we have } \angle a = \angle c = \frac{x}{2}$$

$$\text{Again, } \angle AOB + \angle BOC = 180^\circ$$

$$\Rightarrow \angle BOC = 180^\circ - \angle AOB = 180^\circ - x$$

$\angle BOC$ and $\angle BDC$ are angles subtended by an arc BC at the centre and at the remaining part of the circle.

$$\therefore \angle BDC = \frac{1}{2} \angle BOC$$

$$\Rightarrow \angle d = \frac{1}{2} (180^\circ - x) = 90^\circ - \frac{x}{2}$$

Ans: 4. Given: A cyclic quadrilateral ABCD in which AP, BP, CR and DR are the angle bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively such that a quadrilateral PQRS is formed. To

Prove: PQRS is a cyclic quadrilateral.

Proof: Since ABCD is a cyclic quadrilateral.

$$\therefore \angle A + \angle C = 180^\circ \text{ and } \angle B + \angle D = 180^\circ$$

Also, AP, BP, CR and DR are the angle bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively.

$$\therefore \angle 1 = \frac{1}{2} \angle A, \angle 2 = \frac{1}{2} \angle B, \angle 3 = \frac{1}{2} \angle C \text{ and } \angle 4 = \frac{1}{2} \angle D$$

$$\text{From (i), we have } \frac{1}{2} \angle A + \frac{1}{2} \angle C = \frac{1}{2} (\angle A + \angle C) = \frac{1}{2} \times 180^\circ = 90^\circ$$

$$\text{and } \frac{1}{2} \angle B + \frac{1}{2} \angle D = \frac{1}{2} (\angle B + \angle D) = \frac{1}{2} \times 180^\circ = 90^\circ$$

$$\text{or } \angle 1 + \angle 3 = 90^\circ$$

$$\text{and } \angle 2 + \angle 4 = 90^\circ$$

Now, in $\triangle APB$, by angle sum property of a \triangle

$$\angle 1 + \angle 2 + \angle P = 180^\circ \dots \text{(iii)}$$

Again, in ΔCRD , by angle sum property of a Δ

$$\angle 3 + \angle 4 + \angle R = 180^\circ \dots (\text{iv})$$

Adding (iii) and (iv), we have

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle P + \angle R = 180^\circ + 180^\circ$$

$$90^\circ + 90^\circ + \angle P + \angle R = 360^\circ [\text{using (ii)}]$$

$$\angle P + \angle R = 360^\circ - 180^\circ = 180^\circ$$

i.e., the sum of one pair of the opposite angles of quadrilateral PQRS is 180° .

Hence, the quadrilateral PQRS is a cyclic quadrilateral.

Ans: 5. In circle $C(O, r)$, PQ and PR are two chords, draw $OM \perp PQ$, $OL \perp PR$, such that $OM = a$

and $OL = b$. Join OP. Since the perpendicular from the Centre of the circle to the chord of the circle, bisects the chord.

\therefore We have,

$$PM = MQ = \frac{1}{2} PQ$$

and

$$PL = LR = \frac{1}{2} PR$$

In rt. ΔOMP , $\angle M = 90^\circ$

\therefore By Pythagoras Theorem, we have

$$PM^2 = OP^2 - OM^2$$

$$\left(\frac{1}{2} PQ\right)^2 = r^2 - a^2$$

$$\frac{PQ^2}{4} = r^2 - a^2$$

$$\Rightarrow PQ^2 = 4r^2 - 4a^2 \dots (\text{i})$$

Again, in rt. ΔOLP , $\angle L = 90^\circ$

\therefore By Pythagoras Theorem, we have

$$PL^2 = OP^2 - OL^2$$

$$\left(\frac{1}{2} PR\right)^2 = r^2 - b^2$$

$$\frac{PR^2}{4} = r^2 - b^2$$

$$PR^2 = 4r^2 - 4b^2 \dots (\text{ii})$$

Also,

$$PQ = 2PR \quad [\text{given}]$$

\Rightarrow

$$PQ^2 = 4PR^2 \dots (\text{iii})$$

From (i), (ii) and (iii), we have

$$4r^2 - 4a^2 = 4(4r^2 - 4b^2)$$

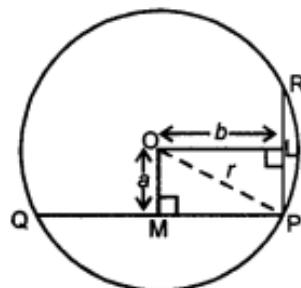
$$\Rightarrow r^2 - a^2 = 4r^2 - 4b^2$$

$$\Rightarrow 4b^2 = 4r^2 - r^2 + a^2$$

$$\Rightarrow 4b^2 = 3r^2 + a^2$$

$$\Rightarrow b^2 = \frac{3}{4}r^2 + \frac{1}{4}a^2$$

$$\text{or } b^2 = \frac{1}{4}a^2 + \frac{3}{4}r^2$$



Assertion and Reason Answers-

1. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

2. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Case Study Answers-

1.

(i)	(b)	120°
(ii)	(b)	24m
(iii)	(c)	30°
(iv)	(a)	30°
(v)	(d)	30°

2.

(i)	(b)	Diameter
(ii)	(b)	90°
(iii)	(d)	Minor Segment.
(iv)	(c)	Major segment.
(v)	(d)	5cm