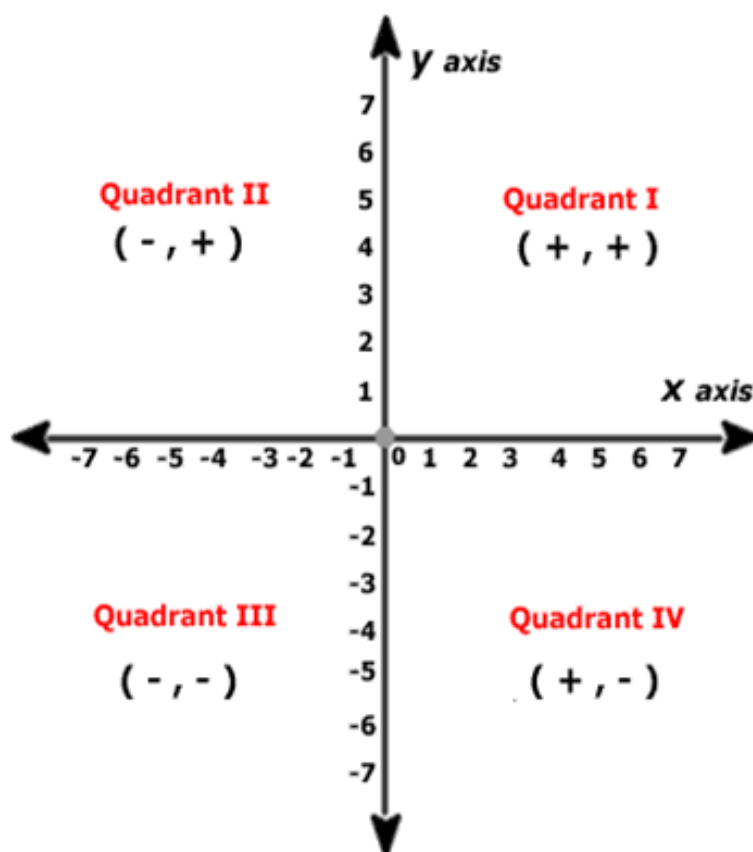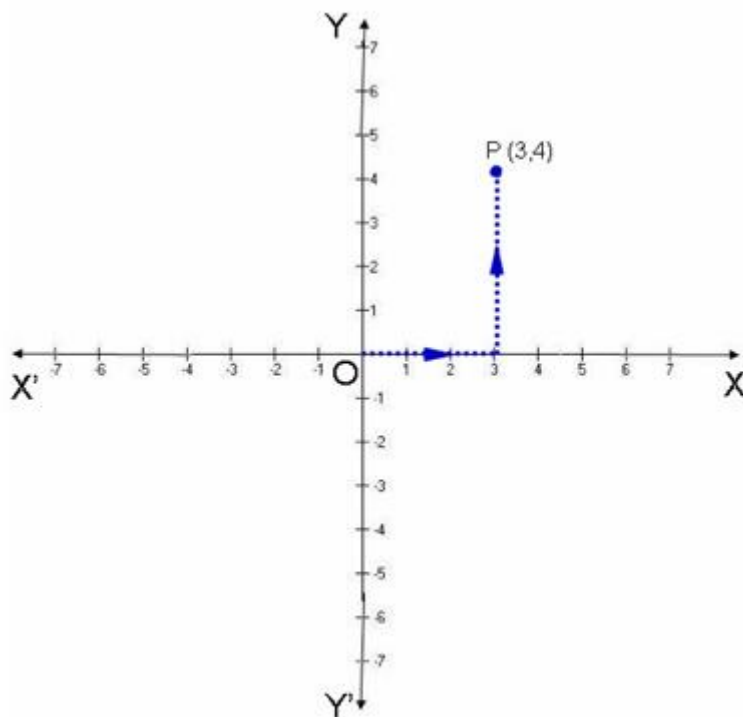# MATHEMATICS

Chapter 3: Coordinate Geometry

Coordinate Geometry

1. Two perpendicular number lines intersecting at point zero are called **coordinate axes**. The horizontal number line is the **x-axis** (denoted by X'OX) and the vertical one is the **y-axis** (denoted by Y'OY). The point of intersection of x-axis and y-axis is called **origin** and denoted by 'O'.

2. **Cartesian plane** is a plane obtained by putting the coordinate axes perpendicular to each other in the plane. It is also called coordinate plane or xy plane.

3. The **x-coordinate** of a point is its perpendicular distance from y-axis.

4. The **y-coordinate** of a point is its perpendicular distance from x-axis.

5. The point where the x axis and the y axis intersect is represented by coordinate points (0, 0) and is called the **origin**.

6. The **abscissa** of a point is the x-coordinate of the point. The **ordinate** of a point is the y-coordinate of the point.

7. If the abscissa of a point is x and the ordinate of the point is y, then (x, y) are called the **coordinates** of the point.

8. The axes divide the Cartesian plane into four parts called the **quadrants** (one fourth part), numbered I, II, III and IV anticlockwise from OX.

9. Sign of coordinates depicts the quadrant in which it lies. The coordinates of a point are of the form (+, +) in the first quadrant, (-, +) in the second quadrant, (-, -) in the third quadrant and (+, -) in the fourth quadrant.

10. The coordinates of a point on the x-axis are of the form (x, 0) and that of the point on y-axis are (0, y).

11. To plot a point P (3, 4) in the Cartesian plane, start from origin and count 3 units on the positive x axis then move 4 units towards positive y axis. The point at which we will arrive will be the point P (3, 4).

12. If x ≠ y, then (x, y) ≠ (y, x) and if (x, y) = (y, x), then x = y.

Cartesian System

Cartesian plane & Coordinate Axes

Cartesian Plane: A cartesian plane is defined by two perpendicular number lines, A horizontal line(x–axis) and a vertical line (y–axis).
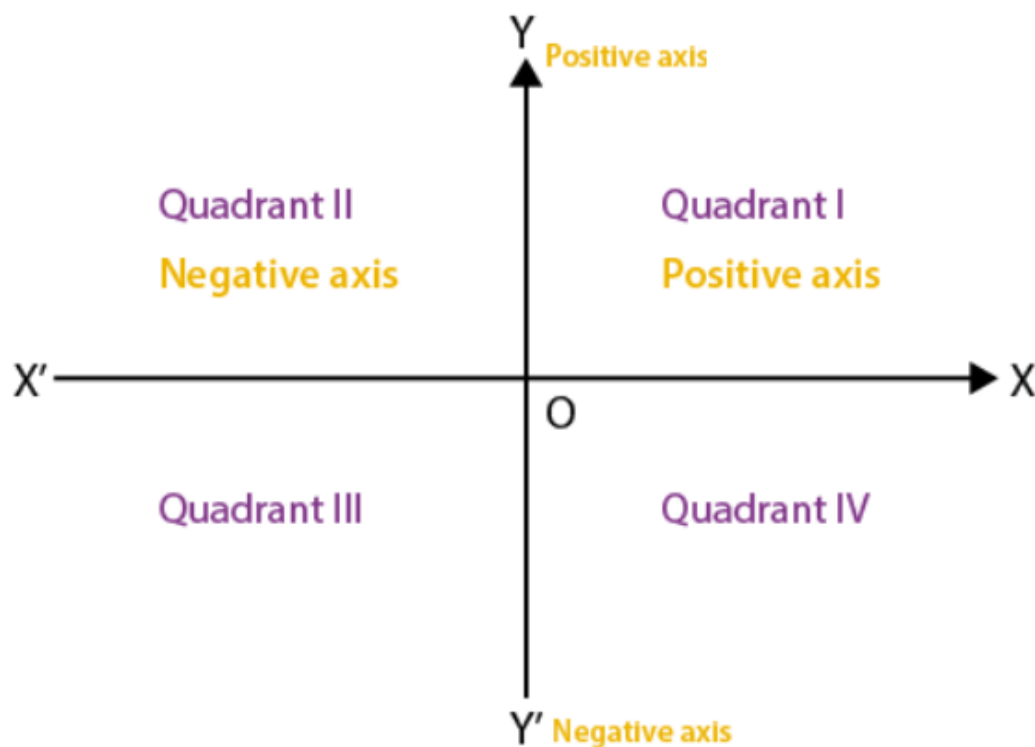
These lines are called coordinate axes. The Cartesian plane extends infinitely in all directions.

Origin: The coordinate axes intersect each other at right angles, The point of intersection of these two axes is called Origin.

Co-ordinate system is used to locate the position of a point in a plane using two perpendicular lines. Points are represented in the form of coordinates (x, y) in two-dimension with respect to x- and y- axes. In this article, we will learn about Cartesian Coordinate system.

To understand the need of coordinate system, let us consider an example, suppose Rina is a girl in your class and she sits on the 3rd column and 5th row. Then, this position can be represented as (3, 5).

Two axes – vertical axis and perpendicular axis are reference lines of a rectangular system from which distances are measured. They are obtained as follows:

Explanation:

Take two number lines XX' and YY'. Place XX' in horizontal and write the numbers on it as we write in the number line. Similarly, place YY' in vertical and proceed writing numbers on it as we write in a number line. Combine both the lines in such a way that the two lines cross each other at their zeroes or origins. The horizontal line XX' is called the x-axis and the vertical line YY' is called the y-axis. The point where XX' and YY' cross is called the origin, and is denoted by O. Since the positive numbers lie on the directions OX and OY, OX and OY are called the positive directions of the x-axis and the y-axis respectively. Similarly, OX' and OY' are called the negative directions of the x- and y-axes respectively.

Important Terms:

Quadrants:

Moreover, the axes divide the plane into four parts and these four parts are called quadrants (one-fourth part). Thus, we have four quadrants numbered I, II, III and IV anticlockwise from OX.

Points in different Quadrants.

Signs of coordinates of points in different quadrants:

I Quadrant: '+' x – coordinate and '+' y – coordinate. E.g. (2, 3)

II Quadrant: '-' x – coordinate and '+' y – coordinate. E.g. (-1, 4)

III Quadrant: '-' x – coordinate and '-' y – coordinate. E.g. (-3, -5)

IV Quadrant: '+' x – coordinate and '-' y – coordinate. E.g. (6, -1)

Cartesian Plane:

A plane consists of axes and quadrants. Thus, we call the plane the Cartesian Plane, or the Coordinate Plane, or the x-y plane. The axes are called the coordinate axes.

Cartesian coordinate system for one dimensional:

The Cartesian coordinate system for one dimensional space consists of a line. We choose a point O, origin on the line, a unit of length and orientation for the line. The orientation chooses which of the two half lines determined by O is the positive, and which is negative. Each point P of the line can be specified by its distance from O, taken with a negative or positive sign.

Number line:

A line with a chosen Cartesian system is called a number line. Every real number has a unique location on the line. Every point on the number line can be interpreted as a number.

Important Note:

The above depicts a two-dimensional system. In case of a three-dimensional system, we have three mutually perpendicular axes, namely x, y and z. It can be generalized to create n coordinates for any point in n-dimensional Euclidean space.

Abscissa and Ordinate

The x-coordinate of a point is its perpendicular distance from the y-axis measured along the x-axis and it is known as Abscissa.

The y-coordinate of a point is its perpendicular distance from the x-axis measured along the y-axis and it is known as Ordinate.

In writing the coordinates of a point in the coordinate plane, the x-coordinate comes first and then the y-coordinate. We place the coordinates in brackets as (x, y). The coordinates describe a point in the plane uniquely. It implies (3,1) ≠ (1,3) or in general (x, y) ≠ (y, x).

Consider an example point (5,6). Here abscissa = 5 and ordinate = 6.

Different Types of Coordinate Systems

We have mainly two types of coordinate systems as listed below:

Cartesian coordinate system

As stated above, it uses the concept of mutually perpendicular lines to denote the coordinate of a point. To locate the position of a point in a plane using two perpendicular lines, we use the cartesian coordinate system. Points are represented in the form of coordinates (x, y) in two-dimension with respect to x- and y- axes.

The x-coordinate of a point is its perpendicular distance from the y-axis measured along the x-axis and it is known as Abscissa. The y-coordinate of a point is its perpendicular distance from the x-axis measured along the y-axis and it is known as Ordinate.

Polar Coordinate System

Here, a point is chosen as the pole and a ray from this point is taken as the polar axis. Basically, we have two parameters namely angle and radius. The angle Θ with the polar axis has a single line through the pole measured anti-clockwise from the axis to the line.

The point will have a unique distance from the origin (r). Thus, a point in Polar coordinate

system is represented as a pair of coordinates (r, Ө). The pole is represented by (0, Ө) for any value of Ө, where r = 0.

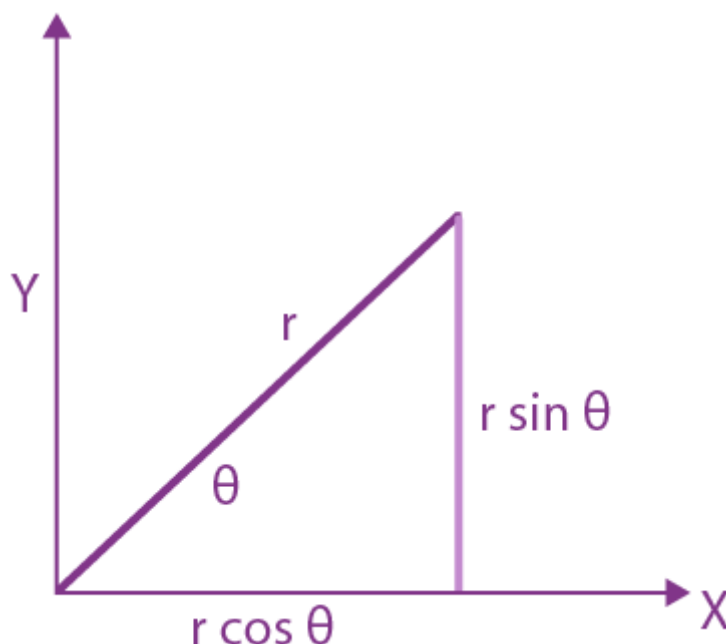(r, Ө), (r, Ө + 2π) and (-r, Ө + π) are all polar coordinates for the same point.

The distance from the pole is called the radial coordinate, radial distance or simply radius and the angular coordinate, polar angle or azimuth.

Consider the figure below that depicts the relationship between polar and cartesian coordinates.

$X = r \cos Ө$ and $y = r \sin Ө$

$r = (x^2 + y^2)^{1/2}$ and $\tan Ө = (y/x)$

Polar equation of a curve consists of points of the form (r, Ө).

In case of circle, the general equation for a circle with centre at (R, β) and

radius a is $r^2 - 2rR \cos (Ө - β) + R^2 = a^2$.

Radial lines (those running through the pole) are represented by the equation: Ө = β.

Cartesian Formulae for the Plane

Distance between two points

The distance between two points of the plane (x_1, y_1) and (x_2, y_2) is given by

$$d = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}$$

In case of three-dimensional system, the distance formula between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$d = [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^{1/2}$$

Representation of a vector

In two-dimensions, the vector from the origin to the point with the cartesian coordinates (x, y) can be written as r = xi + yj where i = (1,0) and j = (0,1) are unit vectors in the direction of the x-axis and y-axis respectively.

In case of three-dimensions, we will have r = xi + yj + zk, where k = (0,0,1) is the unit vector in the direction of z-axis.
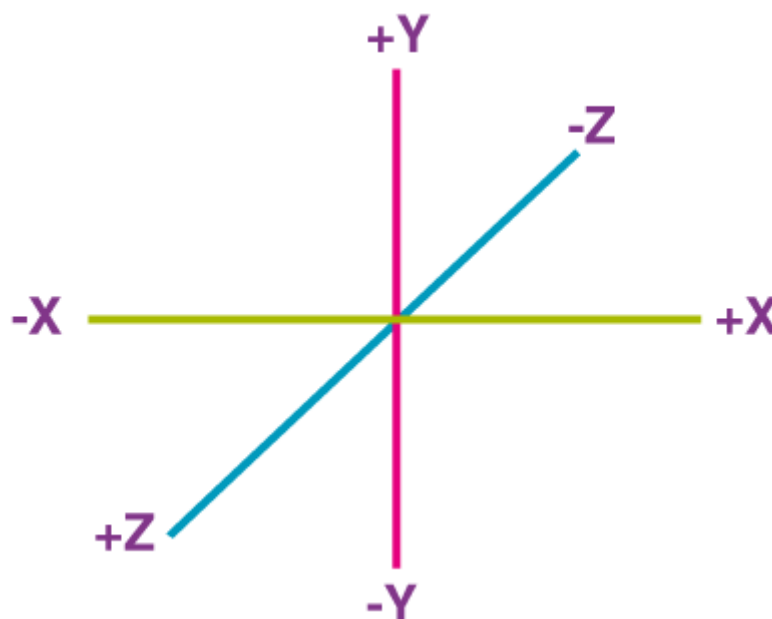
Three Dimensional Geometry

3D geometry involves the mathematics of shapes in 3D space and involving 3 coordinates which are x-coordinate, y-coordinate and z-coordinate. In a 3d space, three parameters are required to find the exact location of a point. For JEE, three-dimensional geometry plays a major role as a lot of questions are included in the exam. Here, the basic concepts of geometry involving 3-dimensional coordinates are covered which will help to understand different operations on a point in 3d plane.

Coordinate System in 3D Geometry

In 3 dimensional geometry, a coordinate system refers to the process of identifying the position or location of a point in the coordinate plane. To understand more about coordinate planes and system, refer to the coordinate geometry lesson which covers all the basic concepts, theorems, and formulas related to coordinate or analytic geometry.
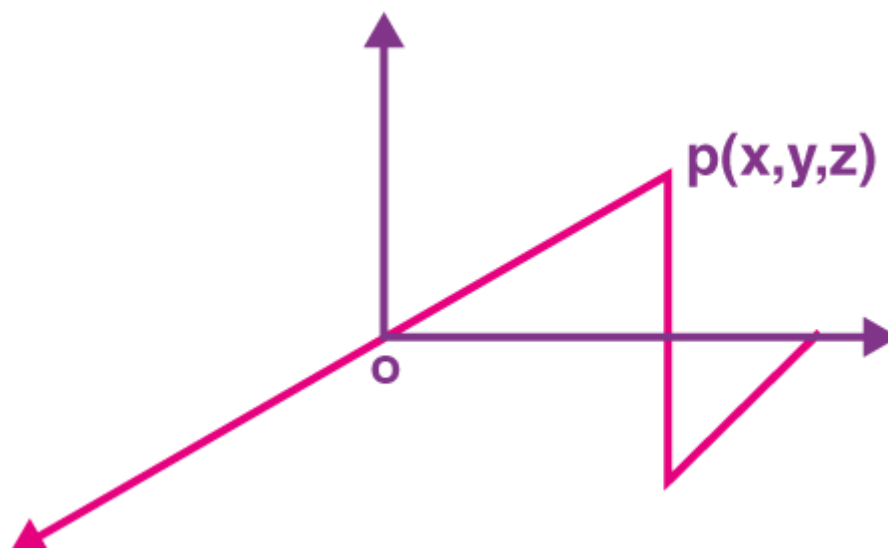
Rectangular coordinate system

Three lines perpendicular to each other pass through a common point. That common point is called the origin, the 3 lines the axes. They are x-axis, y-axis, z-axis respectively. O is the observer with respect to his position of any other point is measured. The position or coordinates of any point in 3D space is measured by how much he has moved along x, y and z-axis respectively. So if a point has a position (3, -4, 5) means he has moved 3 unit along positive x-axis, 4 unit along negative y-axis, 5 unit along positive z-axis.

Rectangular coordinate system – 3D Geometry
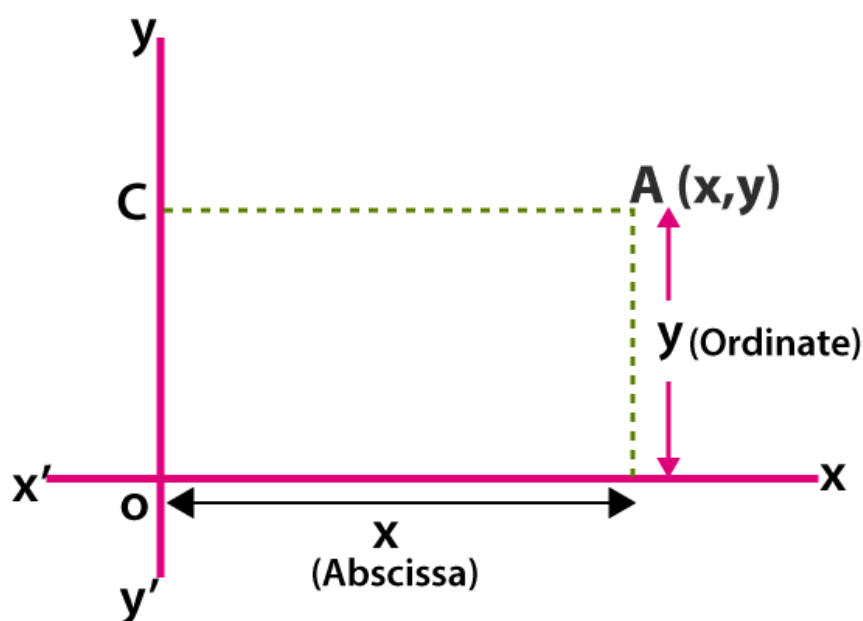
Distance from the Origin

Distance from the Origin in 3D Space – 3D Geometry

Plotting on a Graph

Representation of a point on the Cartesian plane

Using the co-ordinate axes, we can describe any point in the plane using an ordered pair of numbers. A point A is represented by an ordered pair (x, y) where x is the abscissa and y is the ordinate of the point.
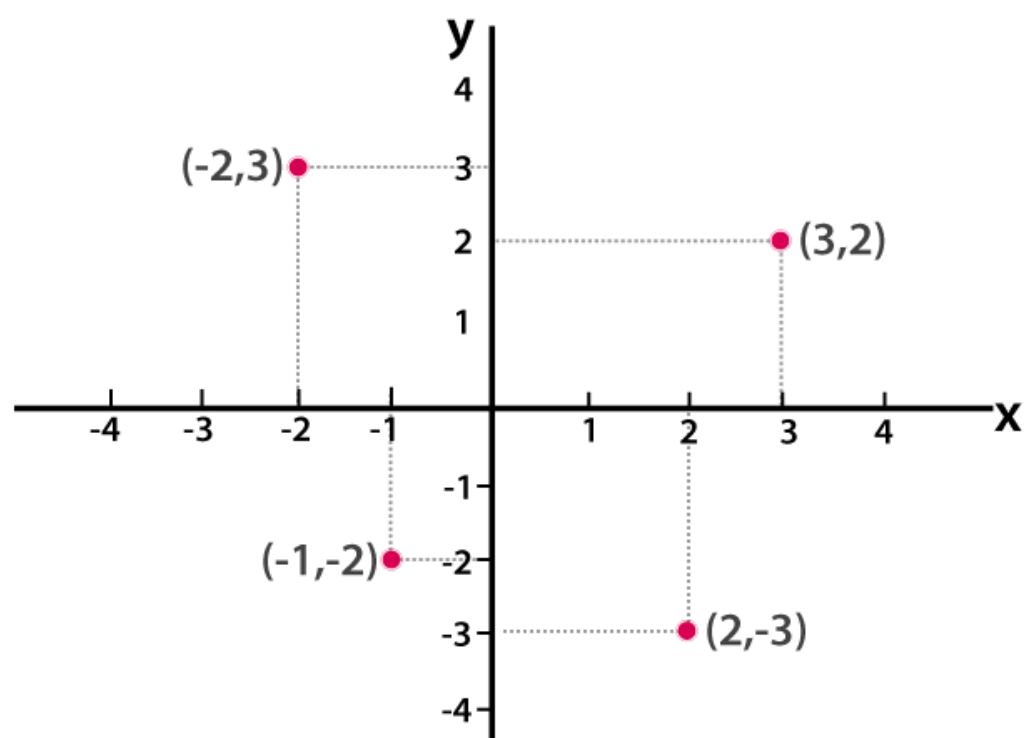

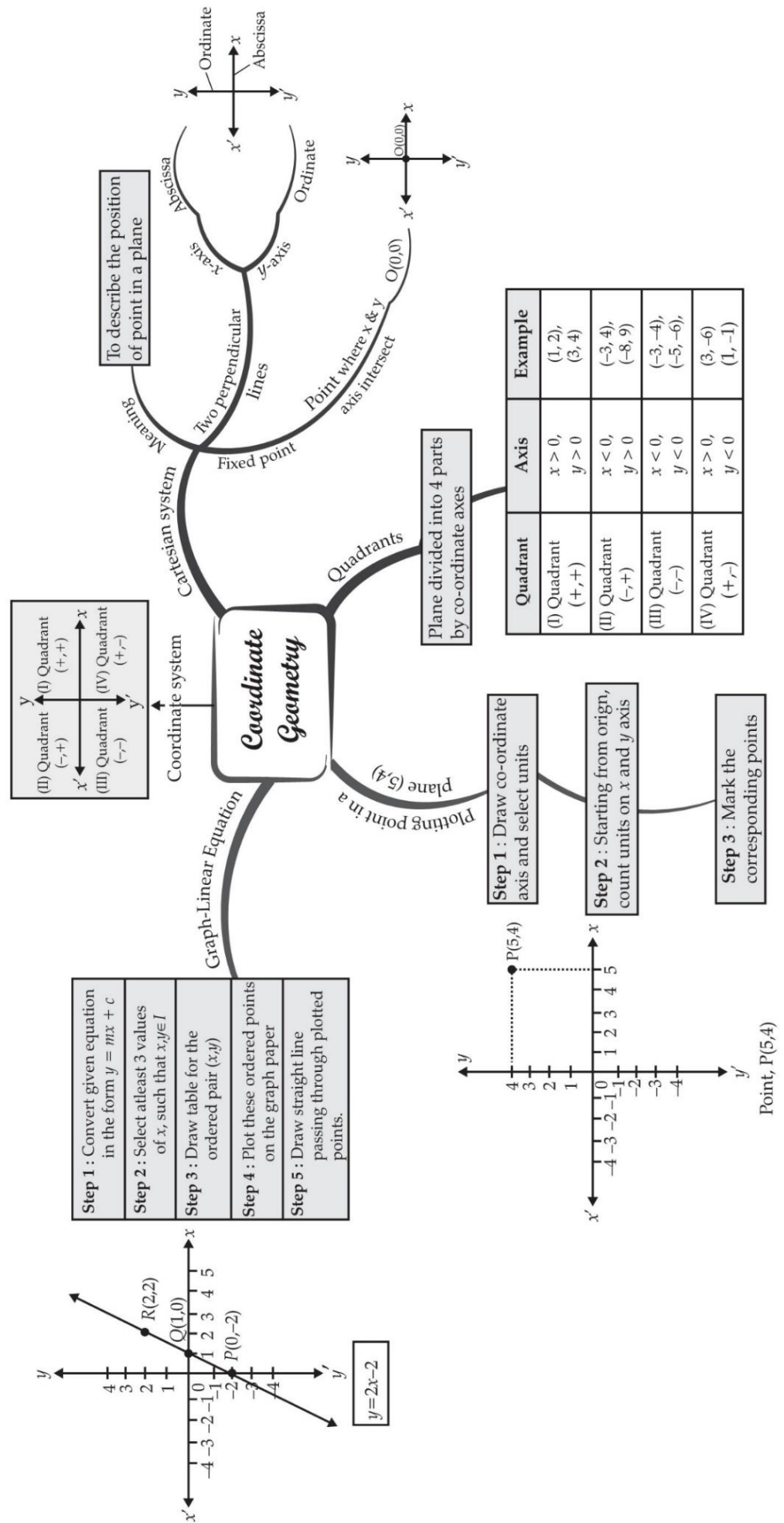Position of a point in a plane

Plotting a point

The coordinate points will define the location in the cartesian plane. The first point (x) in the coordinates represents the horizontal axis, and the second point in the coordinates (y) represents the vertical axis.

Consider an example, Point (3, 2) is 3 units away from the positive y-axis and 2 units away

from the positive x-axis. Therefore, point (3, 2) can be plotted, as shown below. Similarly, (-2, 3), (-1, -2) and (2, -3) are plotted.

CHAPTER : 3 COORDINATE GEOMETRY

Coordinate Geometry

Cartesian system

- Meaning: Two perpendicular lines
- To describe the position of point in a plane

Ordinate — Abscissa

Abscissa — x-axis
Ordinate — y-axis

- Fixed point
- Point where x & y axis intersect — O(0,0)

Coordinate system

- (II) Quadrant (−,+) | (I) Quadrant (+,+)
- (III) Quadrant (−,−) | (IV) Quadrant (+,−)

Quadrants

Plane divided into 4 parts by co-ordinate axes

Quadrant	Axis	Example
(I) Quadrant (+,+)	$x > 0$, $y > 0$	(1, 2), (3, 4)
(II) Quadrant (−,+)	$x < 0$, $y > 0$	(−3, 4), (−8, 9)
(III) Quadrant (−,−)	$x < 0$, $y < 0$	(−3, −4), (−5, −6),
(IV) Quadrant (+,−)	$x > 0$, $y < 0$	(3, −6) (1, −1)

Plotting point in a plane (5,4)

Step 1 : Draw co-ordinate axis and select units

Step 2 : Starting from orign, count units on x and y axis

Step 3 : Mark the corresponding points

Point, P(5,4)

Graph-Linear Equation

Step 1 : Convert given equation in the form $y = mx + c$

Step 2 : Select atleast 3 values of x, such that $x, y \in I$

Step 3 : Draw table for the ordered pair (x, y)

Step 4 : Plot these ordered points on the graph paper

Step 5 : Draw straight line passing through plotted points.

$y = 2x - 2$

P(0,−2), Q(1,0), R(2,2)

Important Questions

Multiple Choice Questions-s

Question 1. If the coordinates of a point are (0, -4), then it lies in:

a) X-axis

b) Y-axis

c) At origin

d) Between x-axis and y-axis

Question 2. If the coordinates of a point are (3, 0), then it lies in:

a) X-axis

b) Y-axis

c) At origin

d) Between x-axis and y-axis

Question 3. If the coordinates of a point are (-3,4), then it lies in:

a) First quadrant

b) Second quadrant

c) Third quadrant

d) Fourth quadrant

Question 4. If the coordinates of a point are (-3, -4), then it lies in:

a) First quadrant

b) Second quadrant

c) Third quadrant

d) Fourth quadrant

Question 5. The name of horizontal line in the cartesian plane which determines the position of a point is called:

a) Origin

b) X-axis

c) Y-axis

d) Quadrants

Question 6. The name of vertical line in the cartesian plane which determines the position of a point is called:

a) Origin

b) X-axis

c) Y-axis

d) Quadrants

Question 7. The section formed by horizontal and vertical lines determining the position of point in a cartesian plane is called:

a) Origin

b) X-axis

c) Y-axis

d) Quadrants

Question 8. The point of intersection of horizontal and vertical lines determining the position of point in a cartesian plane is called:

a) Origin

b) X-axis

c) Y-axis

d) Quadrants

Question 9. Points (1,2), (-2,-3), (2,-3);

a) First quadrant

b) Do not lie in the same quadrant

c) Third quadrant

d) Fourth quadrant

Question 10. If x coordinate of a point is zero, then the point lies on:

a) First quadrant

b) Second quadrant

c) X-axis

d) Y-axis

Very Short:

1. Write the signs convention of the coordinates of a point in the second quadrant.

2. Write the value of ordinate of all the points lie on x-axis.

3. Write the value of abscissa of all the points lie on y-axis.

4. If in coordinates of a point B(3, -2), signs of both coordinates are interchanged, then it will lie in which quadrant?

5. Find distances of points C(-3, -2) and D(5, 2) from x-axis and y-axis.

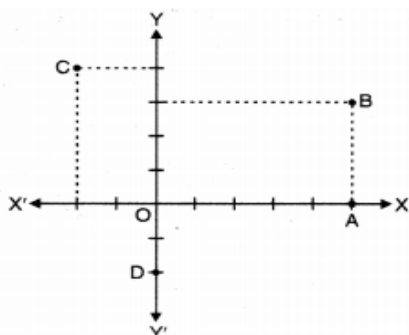6. Find the values of x and y, if two ordered pairs $(x - 3, - 6)$ and $(4, x + y)$ are equal.

7. In which quadrant does the point (-1, 2) lie?

8. Find the distance of the point (0, -5) from the origin.

9. Write the shape of the quadrilateral formed by joining (1, 1), (6, 1), (4, 5) and (3, 5) on graph paper.

Short Questions:

1. In the given figure, ABCD is a rectangle with length 6 cm and breadth 3 cm. O is the mid-
   point of AB. Find the coordinates of A, B, C and D.

2. Write the coordinates of A, B, C and D from the figure given alongside.

3. A point lies on x-axis at a distance of 9 units from y-axis. What are its coordinates? What will be the coordinates of a point, if it lies on y-axis at a distance of -9 units from x-axis?

4. Plot the point P(2, -6) on a graph paper and from it draw PM and PN perpendiculars to x-axis and y-axis respectively. Write the coordinates of the points M and N.

5. Without plotting the points indicate the quadrant in which they lie, if :

   (i) ordinate is 5 and abscissa is – 3

   (ii) abscissa is -5 and ordinate is – 3

   (iii) abscissa is – 5 and ordinate is 3

   (iv) ordinate is 5 and abscissa is 3

6. Plot the points A(1, 4), B(-2, 1) and C(4, 1). Name the figure so obtained on joining them in order and also, find its area.

7. Plot the following points, join them in order and identify the figure thus formed: A(1, 3), B(1, -1), C(7, -1) and D(7, 3)

Long Questions:

1. Plot the points A(3, 2), B(-2, 2), C(-2, -2) and D(3, -2) in the cartesian plane. Join these points and name the figure so formed.

2. Write the coordinates of two points on X-axis and two points on Y-axis which are at equal distances from the origin. Connect all these points and make them as vertices of quadrilateral. Name the quadrilateral thus formed.

3. On environment day, class-9 students got five plants of mango, silver oak, orange, banyan and amla from soil department. Students planted the plants and noted their locations as (x, y).

	Mango	Silver Oak	Orange	Banyan	Amla
x	2	3	0	–3	–2
y	0	4	7	4	0

Plot the points (x, y) in the graph and join them in the given order. Name the figure you get. Which social act is being done by students of class-9 ?
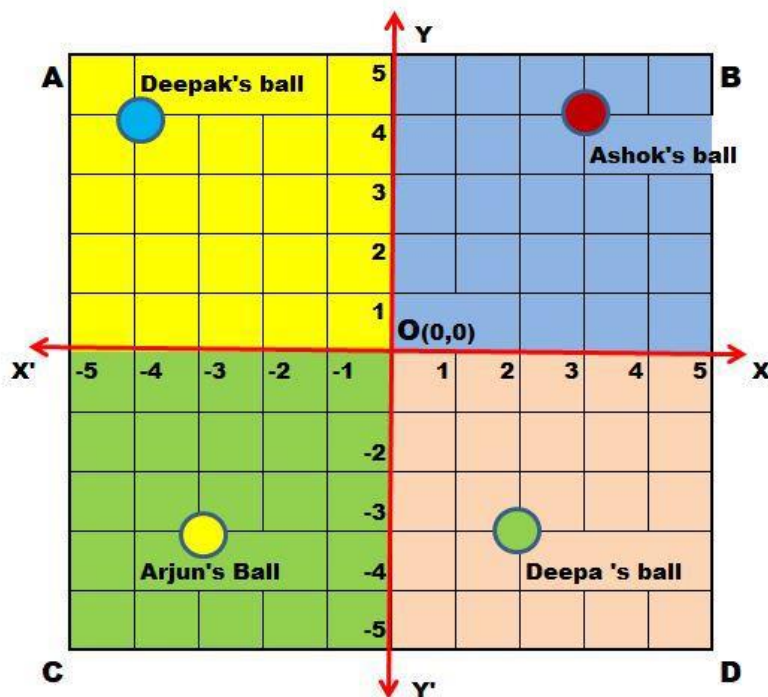
Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: The points (- 3,5) and (5, - 3)are at different positions in the coordinate plane.

Reason: If $x \neq y$, then $(x, y) \neq (y,x)$

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: The point (- 5 , 0) lies on y – axis and (0, - 4) on x -axis.

Reason: Every point on the x -axis has zero distance from x -axis and every point on the y - axis has zero distance from y -axis.

Case Study Questions-

1. Read the Source/ Text given below and answer these questions:

There is a square park ABCD in the middle of Saket colony in Delhi. Four children Deepak, Ashok, Arjun and Deepa went to play with their balls. The colour of the ball of Ashok, Deepak, Arjun and Deepa are red, blue, yellow and green respectively. All four children roll their ball from centre point O in the direction of XOY, X'OY, X'OY' and XOY'. Their balls stopped as shown in the above image.

Answer the following questions:

i. What are the coordinates of the ball of Ashok?

    a. (4, 3)

    b. (3, 4)

    c. (4, 4)

    d. (3, 3)

ii. What are the coordinates of the ball of Deepa?

    a. (2, -3)

    b. (3, 2)

    c. (2, 3)

    d. (2, 2)

iii. What the line XOX' is called?

    a. y-axis.

    b. ordinate.

    c. x-axis.

    d. origin.
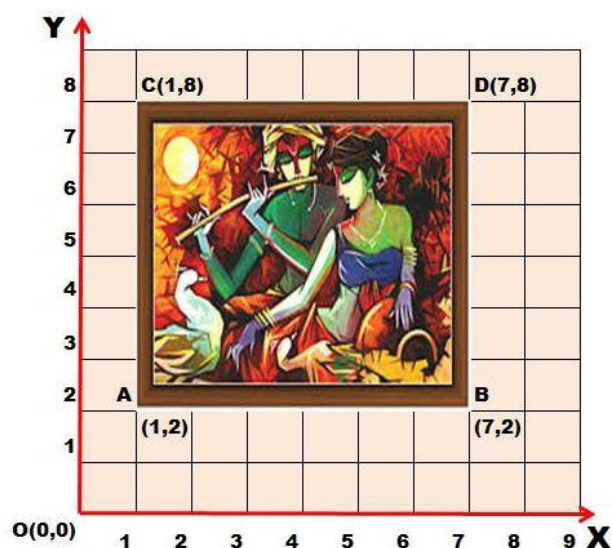
iv. What the point O(0, 0) is called?

    a. y-axis.

    b. ordinate.

    c. x-axis.

    d. origin.

v. What is the ordinate of the ball of Arjun?

    a. -3

    b. 3

    c. 4

    d. 2

2. Read the Source/ Text given below and answer any four questions:

Rohit was putting up one of his paintings in his living room. Before this Rohit had put a grid on the wall where each unit measured equal to a foot. The upper-left corner of the frame is at point C(1, 8) and the upper-right corner at D(7, 8). The bottom-left corner is at A(1, 2) and the bottom-right corner at B(7, 2).

Please answer the following questions:

i. What is the width of the painting plus frame?

    a. 5 feet

    b. 8 feet

    c. 9 feet

    d. 6 feet

ii. What is the length of the painting plus frame?

    a. 9 feet

    b. 8 feet

    c. 6 feet

    d. 5 feet

iii. Which sides of the painting are parallel to x-axis?

    a. AB and CD

    b. AC and BD

    c. Diagonals AD and BC

    d. No one

iv. Which sides of the painting are parallel to y-axis?

    a. AB and CD

    b. AC and BD

    c. Diagonals AC and BD

    d. No one

v. Point A, B, C and D lie in which quadrant?

    a. I

    b. II

    c. III

    d. IV

Answer Key:

MCQ:

1. (b) Y-axis

2. (a) Y-axis

3. (b) Second quadrant

4. (c) Third quadrant

5. (b) X-axis

6. (c) Y-axis

7. (d) Quadrants

8. (a) Origin

9. (b) Do not lie in the same quadrant

10.(d) Y-axis

Very Short Answer:

1. (-ve, +ve)

2. 0

3. 0

4. When signs of both coordinates of B(3, -2) are interchanged, then coordinates of new point are B'(-3, 2) and it will lie in second quadrant.

5. Distances of point C(-3, -2) from x-axis is 2 units in the negative direction and from y-axis is 3 units in the negative direction. Distances of point D(5, 2) from x-axis is 2 units and from y-axis is 5 units.

6. Here, two ordered pairs are equal.

⇒ Their first components are equal, and their second components are separately equal.

⇒ x – 3 = 4 and x + y = -6

⇒ x = 7 and 7 + y = -6 ⇒ y = − 13

Hence, x = 7 and y = − 13.

7. (-1, 2) lie in second quadrant.

8. 5 units.

9. Trapezium.

Short Answer:

Ans: 1.

We have taken 1cm = 1 unit and origin O is the mid-point of AB

∴ OA = OB = 3cm

and BC = AD = 3cm

Thus, the coordinates of A are (-3, 0)

the coordinates of B are (3, 0)

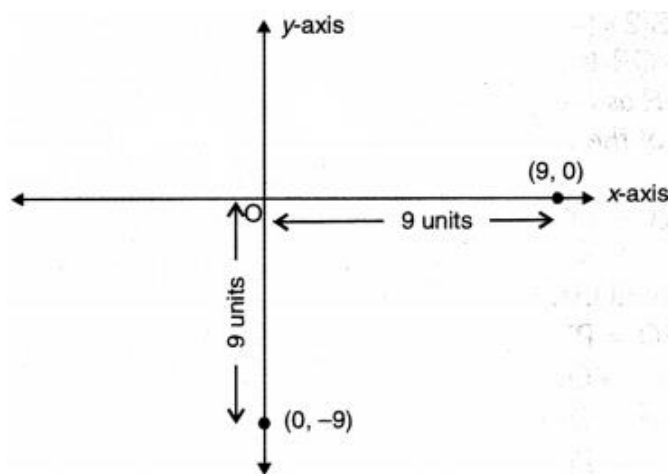the coordinates of C are (3, 3)

the coordinates of D are (-3, 3)

Ans: 2. Coordinates of the point A are (5, 0)
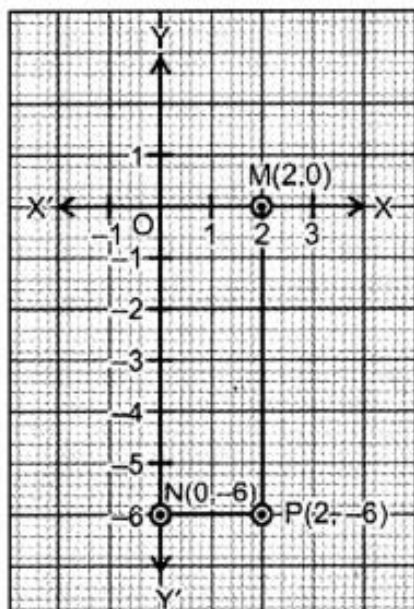
Coordinates of the point B are (5, 3)

Coordinates of the point C are (-2, 4)

Coordinates of the point D are (0, -2)

Ans: 3. As shown in graph, the coordinates of a point which lies on x-axis at a distance of 9 units from y-axis are (9, 0) and the coordinates of a point which lies at a distance of -9 units from x-axis are
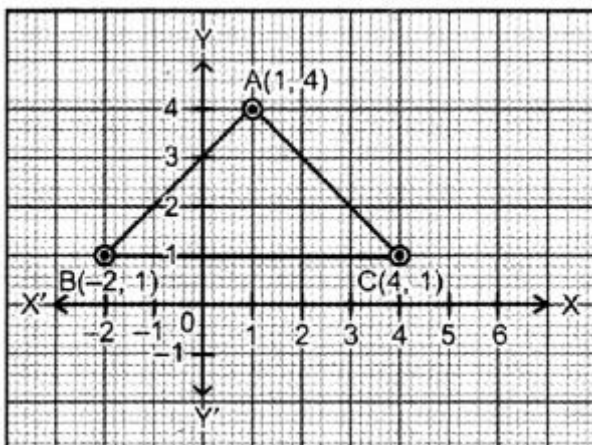
(0, -9).

Ans: 4.

Ans: 5. (i) Clearly, point (-3, 5) lies in 2nd quadrant.

(ii) Clearly, point (-5, − 3) lies in 3rd quadrant.

(ii) Clearly, point (-5, 3) lies in 2nd quadrant.
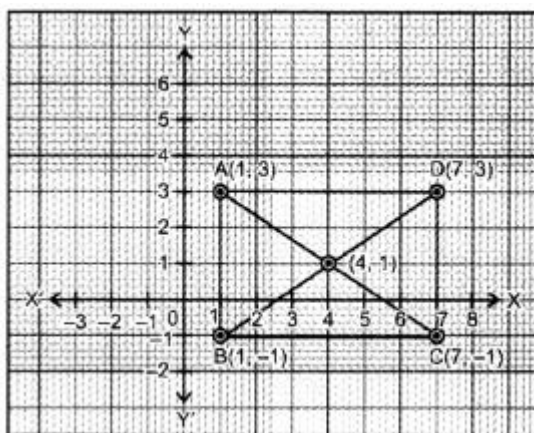
(iv) Clearly, point (3, 5) lies in 1st quadrant.

Ans: 6.

Triangle.

Area of $\triangle ABC = \frac{1}{2} \times BC \times \text{Height}$

$= \frac{1}{2} \times 6 \times 3$

$= 9$ sq. units

Ans: 7.

ABCD is a rectangle.

Point of intersection of the diagonals AC and BD is (4, 1)
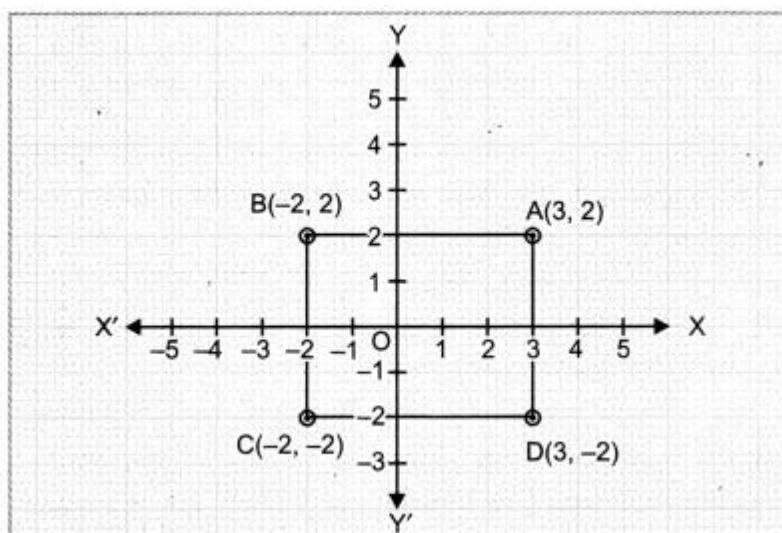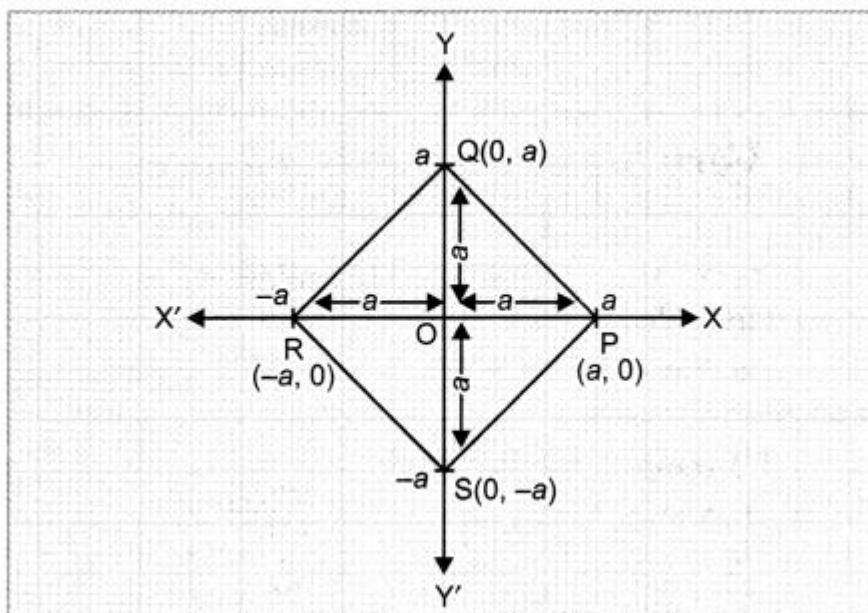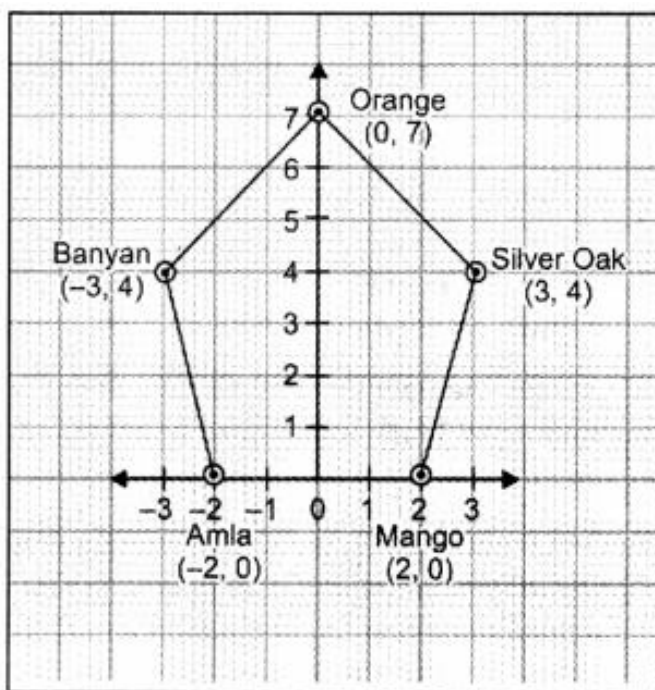
Long Answer:

Ans: 1.

Figure so formed is ABCD a rectangle.

Ans: 2. Let a be the equal distance from origin on both axes. Now, the coordinates of two points on equal distance 'a 'on x-axis are Pla, 0) and R(-a, 0). Also, the coordinates of two points on equal distance 'a' on Y-axis are Q(0, a) and S(0, -a). Join all the four points on the graph. Now, PQRS, thus formed is a square.

Ans: 3.

The given trees (points) are Mango (2, 0), Silver Oak (3, 4), Orange (0,7), Banyan (-3, 4) and Amla (-2, 0). The location of these trees are Orange (0,7) shown in the graph.

On joining the points of mango, silver oak, orange, banyan and amla in order, the figure so formed is a regular pentagon.

Planting more trees helpful in reducing pollution and make the environment clean and green for the coming generations.

Assertion and Reason Answers-

1. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation:

Assertion (A) :

The points(-3,5) and (5,-3) are at different positions in the coordinate plane.

For the point (-3,5)

Abscissa = - 3 & ordinate = 5

The point lies is 2nd quadrant

For the point (5,-3)

Abscissa = 5 & ordinate = - 3

The point lies is 4th quadrant

Since 5 ≠ - 3

So the points(-3,5) and (5,-3) are at different positions in the coordinate plane.

So Assertion (A) is correct

Reason(R) :

If x is not equal to y then the position of (x,y) in the cartesian plane is different from the position of (y,x)

We know that two point (a, b) & (c, d) are the same point iff a = c , b = d

So if x is not equal to y then the position of (x,y) in the cartesian plane is different from the position of (y,x)

Therefore Reason(R) is correct

Also reason (R) is the correct explanation of assertion

Hence the correct option is:

(a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. d)   Assertion is wrong statement but reason is correct statement.

Explanation: (-5,0) lies on x-axis because the first part of co-ordinate shows X-axis and second part show y-axis and also the point (0,-4) lies on y axis not x axis

Hence the assertion is false but the reason is 100% true statement.

Case Study Answers-

1.

(i)	(b)	(3, 4)
(ii)	(a)	(2, -3)
(iii)	(c)	x-axis.
(iv)	(d)	origin.
(v)	(a)	-3

2.

i	d	6 feet
ii	c	6 feet
iii	a	AB and CD
iv	b	AC and BD
v	a	I