# MATHEMATICS

## Chapter 15: Probability

# Probability

1. **Probability** is a quantitative measure of certainty.
2. Any activity associated to certain outcome is called an **experiment**.

   For example: (i) tossing a coin (ii) throwing a dice (iii) selecting a card.
3. An **outcome** is a result of a single trial of an experiment.

   For example: two possible outcomes of tossing a coin are head and tail.
4. An **event** for an experiment is the collection of some outcomes of the experiment.

   For example: (i) Getting a head on tossing a coin (ii) getting a face card when a card is drawn from a pack of 52 cards.
5. The **empirical (experimental) probability** of an event E denoted as P(E) is given by:

$$P(E) = \frac{\text{Number of trials in which the event happenend}}{\text{Total number of outcomes}}$$

6. Probability of an event lies between 0 and 1. Probability can never be negative.

## Probability

Probability is the measure of the likelihood of an event to occur. Events can't be predicted with certainty but can be expressed as to how likely it can occur using the idea of probability.

Probability can range between 0 and 1, where 0 probability means the event to be an impossible one and probability of 1 indicates a certain event.

Probability means possibility. It is a branch of mathematics that deals with the occurrence of a random event. The value is expressed from zero to one. Probability has been introduced in Maths to predict how likely events are to happen. The meaning of probability is basically the extent to which something is likely to happen. This is the basic probability theory, which is also used in the probability distribution, where you will learn the possibility of outcomes for a random experiment. To find the probability of a single event to occur, first, we should know the total number of possible outcomes.

**Formula for Probability**

The probability formula is defined as the possibility of an event to happen is equal to the ratio of the number of favourable outcomes and the total number of outcomes.

Probability of event to happen P(E) = Number of favourable outcomes/Total Number of outcomes

Sometimes students get mistaken for "favourable outcome" with "desirable outcome". This is the basic formula. But there are some more formulas for different situations or events.

## Solved Examples

1) There are 6 pillows in a bed, 3 are red, 2 are yellow and 1 is blue. What is the probability of picking a yellow pillow?

Ans: The probability is equal to the number of yellow pillows in the bed divided by the total number of pillows, i.e. 2/6 = 1/3.

2) There is a container full of coloured bottles, red, blue, green and orange. Some of the bottles are picked out and displaced. Sumit did this 1000 times and got the following results:

No. of blue bottles picked out: 300

No. of red bottles: 200

No. of green bottles: 450

No. of orange bottles: 50

a) What is the probability that Sumit will pick a green bottle?

Ans: For every 1000 bottles picked out, 450 are green.

Therefore, P(green) = 450/1000 = 0.45

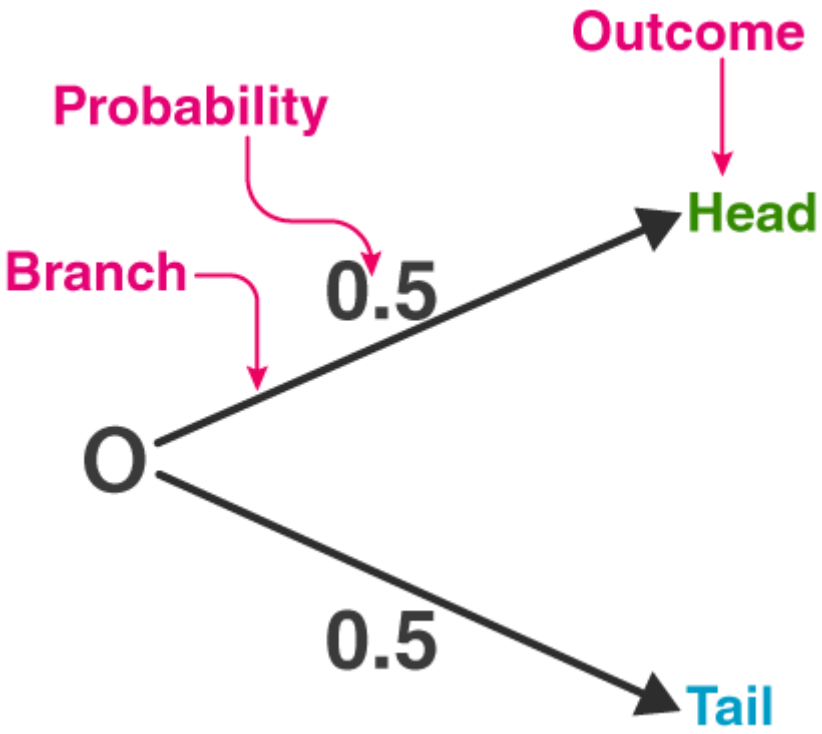
b) If there are 100 bottles in the container, how many of them are likely to be green?

Ans: The experiment implies that 450 out of 1000 bottles are green.

Therefore, out of 100 bottles, 45 are green.

## Probability Tree

The tree diagram helps to organize and visualize the different possible outcomes. Branches and ends of the tree are two main positions. Probability of each branch is written on the branch, whereas the ends are containing the final outcome. Tree diagrams are used to figure out when to multiply and when to add. You can see below a tree diagram for the coin:




## Types of Probability

There are three major types of probabilities:

Theoretical Probability

Experimental Probability

Axiomatic Probability

**Theoretical Probability**

It is based on the possible chances of something to happen. The theoretical probability is mainly based on the reasoning behind probability. For example, if a coin is tossed, the theoretical probability of getting a head will be ½.

**Experimental Probability**

It is based on the basis of the observations of an experiment. The experimental probability can be calculated based on the number of possible outcomes by the total number of trials. For example, if a coin is tossed 10 times and heads is recorded 6 times then, the experimental probability for heads is 6/10 or, 3/5.

**Axiomatic Probability**

In axiomatic probability, a set of rules or axioms are set which applies to all types. These axioms are set by Kolmogorov and are known as Kolmogorov's three axioms. With the axiomatic approach to probability, the chances of occurrence or non-occurrence of the events can be quantified. The axiomatic probability lesson covers this concept in detail with Kolmogorov's three rules (axioms) along with various examples.

Conditional Probability is the likelihood of an event or outcome occurring based on the occurrence of a previous event or outcome.

## Probability of an Event

Assume an event E can occur in r ways out of a sum of n probable or possible equally likely ways. Then the probability of happening of the event or its success is expressed as;

$P(E) = r/n$

The probability that the event will not occur or known as its failure is expressed as:

$P(E') = (n-r)/n = 1-(r/n)$

E' represents that the event will not occur.

Therefore, now we can say;

$P(E) + P(E') = 1$

This means that the total of all the probabilities in any random test or experiment is equal to 1.

## Equally Likely Events

When the events have the same theoretical probability of happening, then they are called equally likely events. The results of a sample space are called equally likely if all of them have the same probability of occurring. For example, if you throw a die, then the probability of getting 1 is 1/6. Similarly, the probability of getting all the numbers from 2,3,4,5 and 6, one at a time is 1/6. Hence, the following are some examples of equally likely events when throwing a die:

- Getting 3 and 5 on throwing a die
- Getting an even number and an odd number on a die
- Getting 1, 2 or 3 on rolling a die

are equally likely events, since the probabilities of each event are equal.

**Complementary Events**

The possibility that there will be only two outcomes which states that an event will occur or not. Like a person will come or not come to your house, getting a job or not getting a job, etc. are examples of complementary events. Basically, the complement of an event occurring in the exact opposite that the probability of it is not occurring. Some more examples are:

It will rain or not rain today

The student will pass the exam or not pass.

You win the lottery or you don't.

**Probability Theory**

Probability theory had its root in the 16th century when J. Cardan, an Italian mathematician and physician, addressed the first work on the topic, The Book on Games of Chance. After its inception, the knowledge of probability has brought to the attention of great mathematicians. Thus, Probability theory is the branch of mathematics that deals with the possibility of the happening of events. Although there are many distinct probability interpretations, probability theory interprets the concept precisely by expressing it through a set of axioms or hypotheses. These hypotheses help form the probability in terms of a possibility space, which allows a measure holding values between 0 and 1. This is known as the probability measure, to a set of possible outcomes of the sample space.

**Probability Density Function**

The Probability Density Function (PDF) is the probability function which is represented for the density of a continuous random variable lying between a certain range of values. Probability Density Function explains the normal distribution and how mean and deviation exists. The standard normal distribution is used to create a database or statistics, which are often used in science to represent the real-valued variables, whose distribution is not known.

## Experiment

**An experiment:**

is any procedure that can be infinitely repeated or any series of actions that have a well-defined set of possible outcomes.

can either have only one or more than one possible outcome. is also called the sample space.

**Trail**

A single event that is performed to determine the outcome is called a trial.

All possible trials that constitute a well-defined set of possible outcomes are collectively called an experiment/ sample space.

## Experimental Probability

**Experimental/Empirical Probability**

The empirical probability of an event that may happen is given by:

Probability of event to happen P(E)=Number of favourable outcomes/Total number of outcomes

You and your 3 friends are playing a board game. It's your turn to roll the die and to win the

game you need a 5 on the dice. Now, is it possible that upon rolling the die you will get an exact 5? No, it is a matter of chance. We face multiple situations in real life where we have to take a chance or risk. Based on certain conditions, the chance of occurrence of a certain event can be easily predicted. In our day to day life, we are more familiar with the word 'chance and probability'. In simple words, the chance of occurrence of a particular event is what we study in probability. In this article, we are going to discuss one of the types of probability called "Experimental Probability" in detail.

**Experimental Probability Vs Theoretical Probability**

There are two approaches to study probability:

- Experimental Probability
- Theoretical Probability

**Experimental Probability**

Experimental probability, also known as Empirical probability, is based on actual experiments and adequate recordings of the happening of events. To determine the occurrence of any event, a series of actual experiments are conducted. Experiments which do not have a fixed result are known as random experiments. The outcome of such experiments is uncertain. Random experiments are repeated multiple times to determine their likelihood. An experiment is repeated a fixed number of times and each repetition is known as a trial. Mathematically, the formula for the experimental probability is defined by;

Probability of an Event P(E) = Number of times an event occurs / Total number of trials.

**Theoretical Probability**

In probability, the theoretical probability is used to find the probability of an event. Theoretical probability does not require any experiments to conduct. Instead of that, we should know about the situation to find the probability of an event occurring. Mathematically, the theoretical probability is described as the number of favourable outcomes divided by the number of possible outcomes.

Probability of Event P(E) = No. of. Favourable outcomes/ No. of. Possible outcomes.

Example: You asked your 3 friends Shakshi, Shreya and Ravi to toss a fair coin 15 times each in a row and the outcome of this experiment is given as below:

| Coin Tossed By: | No. of. Heads | No. of. Tails |
|---|---|---|
| Shakshi | 6 | 9 |
| Shreya | 7 | 8 |

| | | |
|---|---|---|
| Ravi | 8 | 7 |

Calculate the probability of occurrence of heads and tails.

Solution: The experimental probability for the occurrence of heads and tails in this experiment can be calculated as:

Experimental Probability of Occurrence of heads = Number of times head occurs/Number of times coin is tossed.

Experimental Probability of Occurrence of tails = Number of times tails occurs/Number of times coin is tossed.

| Coin Tossed By: | No. of. Heads | No. of. Tails | Experimental Probability for the occurrence of Head | Experimental Probability for the occurrence of Tail |
|---|---|---|---|---|
| Shakshi | 6 | 9 | 6/15 = 0.4 | 9/15 = 0.6 |
| Shreya | 7 | 8 | 7/15 = 0.47 | 8/15 = 0.53 |
| Ravi | 8 | 7 | 8/15 = 0.53 | 7/15 = 0.47 |

We observe that if the number of tosses of the coin increases then the probability of occurrence of heads or tails also approaches to 0.5.

## Coin Tossing Experiment

Consider a fair coin. There are only two possible outcomes that are either getting heads or tails.

Number of possible outcomes = 2

Number of outcomes to get head = 1

The probability of getting head =Number of outcomes to get head/Number of possible outcomes = ½

**Rolling of Dice Experiment**

When a fair dice is rolled, the number that comes up top is a number between one to six. Assuming we roll the dice once, to check the possibility of three coming up.

Number of possible outcomes = 6

Number of outcomes to get three = 1

The probability of getting three = Number of outcomes to get three/Number of possible outcomes=1/6

**Sum of Probabilities of Favorable and Unfavourable events**

- When a trial is done for an expected outcome, there are chances when the expected outcome is achieved. Such a trial/event is called a favourable event.
- When a trial is done for an expected outcome, there are chances when the expected outcome is not achieved. Such a trial/event is called an unfavourable event.
- All favourable and unfavourable event outcomes come from the well-defined set of outcomes.
- Suppose an event of sample space S has n favourable outcomes. Then, there are S-n, unfavourable outcomes.
- The probability of favourable and unfavourable events happening depends upon the number of trials performed. However, the sum of both these probabilities is always equal to one.

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Total number of trials = $6^n$, n is the number of times dice is thrown
$= 6^2 = 36$
Outcomes for getting 11 = (5,6) (6,5)

$$P(E) = \frac{\text{Number of getting 11}}{\text{Total number of trials}} = \frac{2}{36}$$

$$= \frac{1}{18}$$

A dice is thrown 2 times. Find the probability of getting 11

Dice

A coin is tossed 3 times. Find probability of three heads?

Coin

Examples

Total number of trials = $2^n$, $n$ is number of times coin is tossed
$= 2^3$
$= 8$
i.e. if n=3{HHH, HHT, HTT, HTH, TTH, THH, TTT, THT}

$$P(E) = \frac{\text{Number of times three heads occured}}{\text{Total number of trials}}$$

$$= \frac{1}{8}$$

**Probability**

Definition

The extent to which something is likely to happen

Impossible — Even chance — Certain
Less likely — More likely

Related terms

Outcome — Something that follows as a result or consequence

Event — Collection of some outcomes of an experiment

Trial — Action which results in one or several outcomes

Emperical probability

$$P(E) = \frac{\text{Number of trials in which event happened}}{\text{Total number of trials}}$$

Probability value

- Never greater than one
- $0<P(E)<1$
- Never be negative

# Important Questions

## Multiple Choice Questions-

Question 1. Which of the following cannot be the empirical probability of an event?

(a) $\frac{2}{3}$

(b) $\frac{3}{2}$

(c) 0

(d) 1

Question 2. In a survey of 364 children aged 19-36 months, it was found that 91 liked to eat potato chips. If a child is selected at random, the probability that he/she does not like to eat potato chips is:

(a) 0.25

(b) 0.50

(c) 0.75

(d) 0.80

Question 3. In a sample study of 640 people, it was found that 512 people have a high school certificate. If a person is selected at random, the probability that the person has a high school certificate is:

(a) 0.5

(b) 0.6

(c) 0.7

(d) 0.8

Question 4. The probability that a number selected at random from the numbers 1, 2, 3, …, 15 is a multiple of 4 is

(a) $\frac{4}{15}$

(b) $\frac{2}{15}$

(c) $\frac{1}{5}$

(d) $\frac{1}{3}$

Question 5. When a die is thrown, the probability of getting an odd number less than 4 is

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) 0

Question 6. A bag contains 16 cards bearing number 1, 2, 3 …., 16 respectively. One card is drawn at random. What is the probability that a number is divisible by 3?

(a) $\frac{3}{16}$

(b) $\frac{5}{16}$

(c) $\frac{11}{16}$

(d) $\frac{13}{16}$

Question 7. In a cricket match a batsman hits a boundary 4 times out of the 32 balls he plays. In a given ball, what is the probability that he does not hit the ball to the boundary?

(a) $\frac{7}{8}$

(b) $\frac{1}{8}$

(c) $\frac{1}{7}$

(d) $\frac{6}{7}$

Question 8. The sum of the probabilities of all events of a trial is

(a) 1

(b) Greater than 1

(c) Less than 1

(d) Between 0 and 1

Question 9. A die is thrown 300 times and odd numbers are obtained 153 times. Then the probability of getting an even number is

(a) $\frac{153}{300}$

(b) $\frac{147}{300}$

(c) $\frac{174}{300}$

(d) $\frac{147}{153}$

Question 10. Two coins are tossed 1000 times and the outcomes are recorded as below:

| No. of heads | 2 | 1 | 0 |
|---|---|---|---|
| Frequency | 200 | 550 | 250 |

The probability of getting at the most one head is:

(a) $\frac{1}{5}$

(b) $\frac{1}{4}$

(c) $\frac{4}{5}$

(d) $\frac{3}{4}$

## Very Short:

1. The blood groups of some students of Class IX were surveyed and recorded as below:

| *Blood Group* | *A* | *B* | *AB* | *O* |
|---|---|---|---|---|
| *No. of Students* | *19* | *6* | *13* | *12* |

If a student is chosen at random, find the probability that he/she has blood group A or AB

2. A group of 80 students of Class X are selected and asked for their choice of subject to be

taken in Class XI, which is recorded as below:

| *Stream* | *PCM* | *PCB* | *Commerce* | *Humanities* | *Total* |
|---|---|---|---|---|---|
| *Number of Students* | *29* | *18* | *21* | *12* | *80* |

If a student is chosen at random, find the probability that he/she is a student of either commerce or humanities stream.

3. A box contains 50 bolts and 150 nuts. On checking the box, it was found that half of the bolts and half of the nuts are rusted. If one item is chosen at random, find the probability that it is rusted.

4. A dice is rolled number of times and its outcomes are recorded as below:

| *Outcome* | *1* | *2* | *3* | *4* | *5* | *6* |
|---|---|---|---|---|---|---|
| *Frequency* | *35* | *45* | *50* | *38* | *53* | *29* |

Find the probability of getting an odd number.

5. The probability of guessing the correct answer to a certain question is x2 If probability of

not guessing the correct answer is $\frac{2}{3}$, then find x.

6. A bag contains x white, y red and z blue balls. A ball is drawn at the random, then what is the probability of drawing a blue ball.

## Short Questions:

1. 750 families with 3 children were selected randomly and the following data recorded:

| *Number of girls in a family* | *0* | *1* | *2* | *3* |
|---|---|---|---|---|
| *Number of families* | *120* | *220* | *310* | *100* |

If a family member is chosen at random, compute the probability that it has:

(i) no boy child

(ii) no girl child

2. If the probability of winning a race of an athlete is $\frac{1}{6}$ less than the twice the probability of losing the race. Find the probability of winning the race.

3. Three coins are tossed simultaneously 150 times with the following frequencies of different outcomes:

| *Number of tails* | *0* | *1* | *2* | *3* |
|---|---|---|---|---|
| *Frequency* | *25* | *30* | *32* | *63* |

Compute the probability of getting:

(i) At least 2 tails

(ii) Exactly one tail

4. The table shows the marks obtained by a student in unit tests out of 50

| *Unit Test* | *I* | *II* | *III* | *IV* | *V* |
|---|---|---|---|---|---|
| *Marks (Out of 50)* | *34* | *35* | *36* | *34* | *37* |

Find the probability that the student gets 70% or more in the next unit test. Also, the probability that student get less than 70%.

5. Books are packed in piles each containing 20 books. Thirty-five piles were examined for defective books and the results are given in the following table:

## Long Questions:

1. Three coins are tossed simultaneously 250 times. The distribution of

various outcomes is listed below:

(i) Three tails: 30,

(ii) Two tails: 70,

(iii) One tail: 90,

(iv) No tail: 60

Find the respective probability of each event and check that the sum of all probabilities is

2. A travel company has 100 drivers for driving buses to various tourist destination. Given

below is a table showing the resting time of the drivers after covering a certain distance (in km).

| *Distance (in km)* | *After 80 km* | *After 115 km* | *After 155 km* | *After 200 km* |
|---|---|---|---|---|
| *No. of drivers* | *13* | *47* | *30* | *10* |

What is the probability that the driver was chosen at random?

(a) takes a halt after covering 80km.

(b) takes a halt after covering 115km.

(c) takes a halt after covering 155km.

(d) takes a halt after crossing 200km.

3. A company selected 2300 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a home. The information gathered is listed in the table below:

| *Monthly Income* | *Vehicles per Family* | | | |
|---|---|---|---|---|
| *(in ₹)* | *0* | *1* | *2* | *Above 2* |
| *Less than 7000* | *10* | *140* | *25* | *0* |
| *7000 – 10000* | *0* | *295* | *27* | *12* |
| *10000 – 13000* | *1* | *525* | *39* | *11* |
| *13000 – 16000* | *2* | *449* | *29* | *25* |
| *16000 or more* | *1* | *539* | *82* | *88* |

If a family is chosen at random, find the probability that the family is:

(i) earning ₹7000 – ₹13000 per month and owning exactly 1 vehicle.

(ii) owning not more than one vehicle. (iii) earning more than ₹13000 and owning 2 or more than 2 vehicles. (iv) owning no vehicle

4. A survey of 2000 people of different age groups was conducted to find out their preference

in watching different types of movies:

Type I + Family Type II → Comedy and Family

Type III → Romantic, Comedy, and Family 242.

Type IV → Action, Romantic, Comedy and Family

| Age group | Type I | Type II | Type III | Type IV | All |
|---|---|---|---|---|---|
| 18 – 29 | 440 | 160 | 110 | 61 | 35 |
| 30 – 50 | 505 | 125 | 60 | 22 | 18 |
| Above 50 | 360 | 45 | 35 | 15 | 9 |

Find the probability that a person chosen at random is:

(a) in 18-29 years of age and likes type II movies

(b) above 50 years of age and likes all types of movies

(c) in 30-50 years and likes type I movies.:

5. In a kitchen, there are 108 utensils, consisting of bowls, plates, and glasses. The ratio of bowls, plates the glasses is 4:2:3. A utensil is picked at random. Find the probability that:

(i) it is a plate.

(ii) it is not a bowl.

## Assertion and Reason Questions-

**1.** In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

**Assertion:** A die is thrown. Let E be the event that number appears on the upper face is less than 1, then P (E) = $\frac{1}{6}$

**Reason:** Probability of impossible event is 0.

**2.** In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

**Assertion:** A coin is tossed two times. Probability of getting at least two heads is $\frac{1}{4}$.

**Reason:** When a coin is tossed two times, then the sample space is {*HH, HT, TH, TT*}.

## Case Study Questions-

**1.** Read the Source/ Text given below and answer these questions:

Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes given in the table. Read the data given in the table carefully.

| **Outcome** | 3 tails | 2 tails | 1 tail | no tail |
|---|---|---|---|---|
| **Frequency** | 20 | 68 | 82 | 30 |

If the three coins are simultaneously tossed again, compute the probability of:

i. Getting less than 3 tails:

a. 0.9

b. 0.1

c. 0.01

d. 0.02

ii. Exactly 2 Heads:

a. 0.68

b. 0.41

c. 0.34

d. 0.5

iii. Exactly 1 head:

a. 0.68

b. 0.86

c. 0.34

d. 0.11

iv. At least 1 tail:

a. 0.58

b. 0

c. 1

d. 0.85

v. All heads:

a. 0.51

b. 0.55

c. 0.9

d. 0.15

**2.** Read the Source/ Text given below and answer any four questions:

Over the past 200 working days, the number of defective parts produced by a machine in a factory is given in the following table:

| Number of defective parts | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Days | 50 | 32 | 22 | 18 | 12 | 12 | 10 | 10 | 10 | 8 | 6 | 6 | 2 | 2 |

Determine the probability that tomorrow's output will have.

i. No. defective part

a. 0.25

b. 0

c. 0.50

d. 0.025

ii. At least one defective part

a. 0.50

b. 0.75

c. 0.32

d. 0.01

iii. Not more than 5 defective parts

a. 0.12
b. 0.75
c. 0.73
d. 0.60

iv. More than 13 defective parts

a. 0
b. 1
c. -1
d. 0.2

v. At most 3 defective parts

a. -0.12
b. 0.50
c. 0.18
d. 0.61

## Answer Key:

## MCQ:

1. (b) $\frac{3}{2}$
2. (c) 0.75
3. (d) 0.8
4. (c) $\frac{1}{5}$
5. (b) $\frac{1}{3}$
6. (b) $\frac{5}{16}$
7. (a) $\frac{7}{8}$
8. (a) 1
9. (b) $\frac{147}{300}$
10. (c) $\frac{4}{5}$

## Very Short Answer:

1. Here,

total number of students = 19 + 6 + 13 + 12 = 50

Number of students has blood group A or AB = 19 + 13 = 32

Required probability = $\frac{38}{50} = \frac{16}{25}$

2. Here, total number of students = 80

Total number of students of Commerce or Humanities stream = 33

Required probability = $\frac{33}{80}$

3. Total number of nuts and bolts in the box = 150 + 50

= 200

Number of nuts and bolts rusted = $\frac{1}{2} \times 200 = 100$

P(a rusted nut or bolt) = $\frac{100}{200} = \frac{1}{2}$

4. Total number of outcomes = 250

Total number of outcomes of getting odd numbers = 35 + 50 + 53 = 138

∴ P(getting an odd number) = $\frac{138}{250} = \frac{69}{125}$

5. Here, probability of guessing the correct answer = $\frac{x}{2}$

And probability of not guessing the correct answer = $\frac{x}{2}$

Now, $\frac{x}{2} + \frac{2}{3} = 1$

$\Rightarrow 3x + 4 = 6$

$\Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$

6. Number of blue balls = Z

Total balls = x + y + Z

∴ P(a blue ball) = $\frac{z}{x+y+z}$

## Short Answer:

**Ans: 1.** (i) P(no boy child) = $\frac{100}{750} = \frac{2}{15}$

and P (no girl child) = $\frac{120}{750} = \frac{4}{25}$

**Ans: 2**. Let probability of winning the race be p

∴ Probability of losing the race = 1- p

According to the statement of question, we have

$p = 2(1 - p) - \frac{1}{6}$

$\Rightarrow 6p = 12 - 12p - 1$

$\Rightarrow 18p = 11$

$\Rightarrow p = \frac{11}{18}$.

Hence, probability of winning the race is $\frac{11}{18}$.

**Ans: 3**. Here, total number of chances = 150

(i) Total number of chances having at least 2 tails = 32 + 63 = 95

$\therefore$ Required probability $= \frac{95}{150}. = \frac{19}{30}.$

(ii) Total number of chances having exactly one tail = 30

$\therefore$ Required probability $= \frac{30}{150}. = \frac{1}{5}.$

**Ans: 4.** Here, the marks are out of 50, so we first find its percentage (i.e., out of 100)

| Unit Test | I | II | III | IV | V |
|---|---|---|---|---|---|
| Marks (Out of 100) | 68 | 70 | 72 | 68 | 74 |

Total number of outcomes = 5

Probability of getting 70% or more marks = $\frac{3}{5}$.

Probability of getting less than 70% = $\frac{2}{5}$.

**Ans: 5.** Total number of books = 700

(i) P(no defective books) = $\frac{400}{700} = \frac{4}{7}$

(ii) P(more than 0 but less than 4 defective books) = $\frac{269}{700}$

13 (iii) P(more than 4 defective books) = $\frac{13}{700}$

## Long Answer:

**Ans: 1.** Here, the total number of chances = 250

Total number of three tails = 30

$$\therefore \quad \text{P(of three tails)} = \frac{30}{250} = \frac{3}{25}$$

(ii) Total number of two tails = 70

$$\therefore \quad \text{P(of two tails)} = \frac{70}{250} = \frac{7}{25}$$

(iii) Total number of one tail = 90

$$\therefore \quad \text{P(of one tail)} = \frac{90}{250} = \frac{9}{25}$$

(iv) Total number of no tail = 60

$$\therefore \quad \text{P(of no tail)} = \frac{60}{250} = \frac{6}{25}$$

$$\text{Now, sum of all probabilities} = \frac{3}{25} + \frac{7}{25} + \frac{9}{25} + \frac{6}{25} = \frac{25}{25} = 1$$

**Ans: 2.** Total number of drivers = 100

(a) P (takes a halt after covering 80km) = $\frac{13}{100}$

(b) P (takes a halt after covering 115km) = $\frac{60}{100} = \frac{3}{5}$

(c) P (takes a halt after covering 155km) = $\frac{90}{100} = \frac{9}{10}$

(d) P (takes a halt after crossing 200km) = $\frac{10}{100} = \frac{1}{10}$

**Ans: 3.** Here, we have a total number of families = 2300

(i) Number of families earning ₹ 7000 to ₹ 13000 per month and owning exactly 1 vehicle = 295 + 525 = 820

(ii)

$$\therefore \text{ Required probability} = \frac{820}{2300} = \frac{41}{115}$$

(ii) Number of families owning not more than one vehicle = 1962

$$\therefore \text{ Required probability} = \frac{1962}{2300} = \frac{981}{1150}$$

(iii) Number of families earning more than ₹ 13000 and owning 2 or more than 2 vehicles = 224

$$\therefore \text{ Required probability} = \frac{224}{2300} = \frac{56}{575}$$

(iv) Number of families owning no vehicle = 14

$$\therefore \text{ Required probability} = \frac{14}{2300} = \frac{7}{1150}$$

**Ans: 4.** (a) Let $E_1$ be the event, between the age group (18 - 29) years and liking type II movies

Favorable outcomes to event $E_1$ = 160

$\therefore P(E_1) = \frac{160}{2000} = \frac{160}{2000}$

(b) Let $E_2$ be the event, of age group above 50 years and like all types of movies

Favorable outcomes to event $E_2$ = 9

$\therefore P(E_2) = \frac{9}{2000}$

(c) Let $E_3$ be the event, between age group (30 - 50) years and liking type I movies

Favorable outcomes to event $E_3$ = 505

$\therefore P(E_3) = \frac{505}{2000} = \frac{101}{400}$

**Ans: 5.** Total utensils in the kitchen = 108

Let number of bowls be 4x, number of plates be 2x and number of glasses be 3x

$\therefore$ 4x + 2x + 3x = 108

9x = 108

$x = \frac{108}{9} = 12$

Thus, number of bowls = 4 × 12 = 48

Number of plates = 2 × 12 = 24

Number of glasses = 3 × 12 =

(i)P (a plate) = $\frac{24}{108} = \frac{2}{9}$

(ii) P (not a bowl) = $\frac{24+36}{108} = \frac{60}{108} = \frac{5}{9}$

## Assertion and Reason Answers-

**1.** d) Assertion is wrong statement but reason is correct statement.

**Explanation:** When a die is thrown, then number of outcomes are 1, 2, 3, 4, 5, 6

P(number appear on the upper face is less than 1)=0

**2.** a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

**Explanation:** Number of total outcomes when a coin is tossed 2 times i.e., {HH, HT, TH, TT} = 4

P(getting at least two heads) = $\frac{1}{4}$

## Case Study Answers-

**1.**

| | | |
|---|---|---|
| (i) | (a) | 0.9 |
| (ii) | (b) | 0.41 |
| (iii) | (c) | 0.34 |
| (iv) | (d) | 0.85 |
| (v) | (d) | 0.15 |

**2.**

| | | |
|---|---|---|
| i | a | 0.25 |
| ii | b | 0.75 |
| iii | c | 0.73 |
| iv | a | 0 |
| v | d | 0.61 |