Coursework **Fall 2019**
Machine Learning (B555) **Roni Khardon**
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Written Assignment 2

This assignment is due by 10/2 11:59pm. Please submit solutions as a single PDF file through Canvas. Handwritten or typed solutions are fine, so long as the solution is legible and is submitted as a PDF. You might find LaTeX to be a useful tool for typesetting text and math expressions.

1. Solve problem 3.3 (page 174) in the textbook.

2. Consider a multivariate random variable (of dimension 2) $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim \text{uniform}[1,2]^2$ and the random variable y define as $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_2^2 \\ x_1 + 5x_2 \end{pmatrix}$.
   (1) Use the change of variables formulas given in class to calculate the distribution over y.
   (2) What is the range of values of y for which $Pr(y)$ is not zero.
   (3) Verify that $Pr(y)$ calculated in part (1) is normalized; that is, verify that $\int_y Pr(y)dy = 1$.

3. Consider a bi-variate normal variable X distributed $\mathcal{N}(0, I)$ and a univariate Y where $Y|X$ is distributed as $\mathcal{N}(\mu = 3x_1 + 2x_2 + 5, \sigma^2 = 25)$. Calculate an explicit form for $p(X|Y = 4)$ using our template for Bayes theorem for Gaussians. Are x_1, x_2 still independent after Y is observed?

4. Consider a real-valued symmetric matrix S with eigen decomposition $S = V\Lambda V^T$. Now consider the optimization problem:

   $$\text{argmax}_{\{x \mid x^T x \leq 1\}} \; x^T S x$$

   that is, we seek a vector x of norm at most 1 maximizing the quadratic form $x^T Sx$. What is the optimal solution x?

   Hint: can you express x in the basis formed by V?

5. Solve problem 3.7 (page 175) in the textbook.

6. Solve problem 3.11 (page 175) in the textbook.

7. By using Equations (C.22) and (C.26) prove Equation (C.28) in the textbook.