Problem Set 5
S520

Ans 2

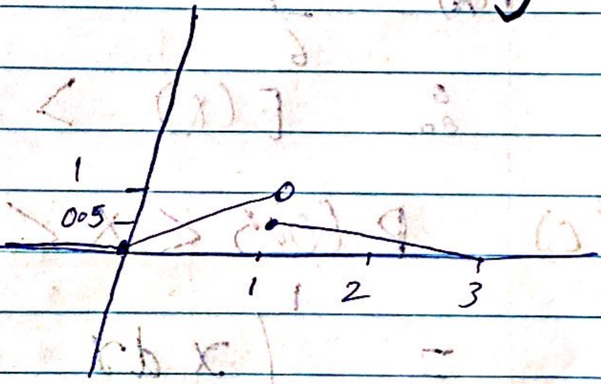
X is a continuous random variable

PDF of x:

$$f(x) = \begin{cases} 0 & x < 0 \\ x & x \in (0,1) \\ (3-x)/4 & x \in (1,3) \\ 0 & x > 3 \end{cases}$$

(a) $q_2(x)$

$P(X < q_2) = 0.5$



$\Rightarrow P[X < 1] = \int_0^1 x \, dx = \left[\frac{x^2}{2}\right]_0^1 = 0.5$

$\therefore \boxed{q_2 = 1}$

(b) $E(x) = \int_x x \cdot f(x) \, dx$

$= \int_0^1 x^2 \, dx + \int_1^3 \frac{3x - x^2}{4} \, dx$

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$$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{3x^2}{8}\right]_1^3 - \left[\frac{x^3}{12}\right]_1^3$$

$$= \frac{1}{3} + \frac{3}{\cancel{8}}\left[\cancel{8}\right] - \frac{1}{\cancel{12}_6}\left[\cancel{26}^{13}\right]$$

$$= \frac{1}{3} + \frac{3}{1} - \frac{13}{6}$$

$$= \frac{2+18-13}{6}$$

$$E(X) = \frac{7}{6}$$

$\therefore \quad E(X) > q_2(x)$

(c) $P(0.5 < X < 1.5)$

$$= \int_{0.5}^{1} x\,dx + \int_{1}^{1.5} \frac{3-x}{4}\,dx$$

$$= \left[\frac{x^2}{2}\right]_{0.5}^{1} + \frac{1}{4}\left(3x - \frac{x^2}{2}\right)_1^{1.5}$$

$$= \frac{1}{2}\left[1 - 0.25\right] + \frac{1}{4}\left[4.5 - \frac{2.25}{2} - \left(3 - \frac{1}{2}\right)\right]$$

$$= 0.375 + \frac{1}{4}\left[\frac{1.75}{2}\right]$$

$$= 0.375 + \frac{1.75}{8}$$

$$= \boxed{0.59375}$$

(d) $q_3(x) - q_1(x) = iqr(x)$

$P(X < q_1) = 0.25$

$\Rightarrow \int_0^{q_1} x\,dx = 0.25$

$\Rightarrow \left[\frac{x^2}{2}\right]_0^{q_1} = 0.25$

$\Rightarrow q_1^2 = 0.50$

$\Rightarrow \boxed{q_1 = 0.7071}$

$P(X < q_3) = 0.75$

$\Rightarrow 0.5 + \int_1^{q_3} \left(\frac{3-x}{4}\right) dx = 0.75$

$\Rightarrow \frac{1}{4}\left(\frac{3x}{1} - \frac{x^2}{2}\right)_1^{q_3} = 0.25$

$\Rightarrow (6x - x^2)_1^{q_3} = 2$

$\Rightarrow 6q_3 - q_3^2 - 5 = 2$

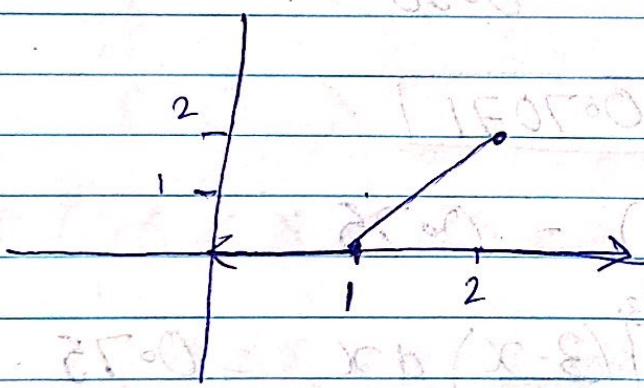
$\Rightarrow q_3^2 - 6q_3 + 7 = 0$

$\Rightarrow q_3 = \frac{6 \pm \sqrt{8}}{2} = 4.41, 1.59$

$q_3 \in (1,3) \therefore q_3 = 1.59$

$$IQR(X) = q_3 - q_1$$
$$= 1.59 - 0.7071$$
$$= \boxed{0.8829}$$

Ans.1 $f(x) = \begin{cases} 0 & \text{if } x < 1 \\ 2(x-1) & \text{if } 1 \le x \le 2 \\ 0 & \text{if } x > 2 \end{cases}$



$q_2(x)$: $P(X < q_2) = 0.5$

$$\Rightarrow \int_1^{q_2} 2(x-1)\,dx = 0.5$$

$$\Rightarrow 2\left[\frac{x^2}{2} - x\right]_1^{q_2} = 0.5$$

$$\Rightarrow 2\left[\frac{q_2^2}{2} - q_2 - \left(\frac{1}{2} - 1\right)\right] = 0.5$$

$\Rightarrow q_2^2 - 2q_2 + 1 = 0.5$

$\Rightarrow 2q_2^2 - 4q_2 + 1 = 0$

$\Rightarrow q_2 = \dfrac{4 \pm \sqrt{16-8}}{4}$

$= 1 \pm \dfrac{2\sqrt{2}}{4}$

$q_2 = 1 \pm \dfrac{1}{\sqrt{2}}$

$q_2 \in [1, 2]$

$\therefore q_2 = 1 + \dfrac{1}{\sqrt{2}} = \boxed{1.70716}$

$IQR(x) = q_3(x) - q_1(x)$

$q_1(x) = P(X < q_1) = 0.25$

$\Rightarrow \int_1^{q_1} 2(x-1)\,dx = 0.25$

Following from previous calculations

$\Rightarrow q_1^2 - 2q_1 + 1 = 0.25$

$\Rightarrow \cancel{q_1^2 - 2q_1 + 0.75 = 0}$ $(q_1 - 1)^2 = \dfrac{1}{4}$

$\Rightarrow q_1 - 1 = \pm \dfrac{1}{2}$

$\Rightarrow \boxed{q_1 = \dfrac{3}{2}}$ $\qquad q_1 \in [1, 2]$

$$q_3(x) = P(X < q_3) = \int_1^{q_3} 2(x-1)\,dx = 0.75$$

Following from previous calculations

$$(q_3-1)^2 = 0.75$$

$$q_3 - 1 = \pm\frac{\sqrt{3}}{2}$$

$$q_3 = 1 \pm 0.866$$

$$q_3 = 1.866$$

$$I.Q.R(X) = q_3 - q_1$$

$$= 1.866 - 1.500$$

$$\boxed{IQR(x) = 0.366}$$

Ans.4 $\textbf{Height} \sim N(63.8, (2.9)^2)$

(a) $P(\text{Height} > 65.5)$

$$= 1 - P(\text{Height} < 65.5)$$

let Height = X

$$= 1 - P\left(\frac{X-\mu}{\sigma} < \frac{65.5-63.8}{2.9}\right)$$

$$= 1 - P\left(Z < \frac{1.7}{2.9}\right)$$

$$= 1 - P(Z < 0.586206897)$$

Rcode

1- pnorm(0.586206897, 0, 0)

[1] $\boxed{0.2788682}$ = 27.88682 %

(b) $IQR(X)$

$Q_3(X) - Q_1(X)$

R code

$Q_3(X)$ = qnorm(0.75, 63.8, 2.9)

$Q_1(X)$ = qnorm(0.25, 63.8, 2.9)

$IQR(X) = Q_3(X) - Q_1(X)$

[1] $\boxed{3.912041}$

(c) The shortest 2.5 % of women are shorter than: #Rcode

qnorm(0.025, 63.8, 2.9)

= 58.1161 inches ≈ $\boxed{58.1 \text{ inches}}$

The tallest 2.5 % of women are taller than: #Rcode

qnorm(0.975, 63.8, 2.9)

= 69.4839 ≈ $\boxed{69.5 \text{ inches}}$

For R.V X, PDF of X:

Ans.5 $f(x) = \begin{cases} 0.3 & 0 \le x < 1 \\ 0.7 & 1 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$

(a) $E|X-a|$

We know, the value of const. a that minimizes $E|X-a|$ is $q_2(X)$

$F(y)$: for $0 \le y < 1$ $= 0.3y$

$F(y)$: $1 \le y < 2$: $\int_1^y 0.7\,dy = 0.7(y-1) + (0.3y)_{y=1}$

$= 0.7y - 0.7 + 0.3$

$= \underline{0.7y - 0.4}$

$\therefore F(y) = \begin{cases} 0 & y < 0 \\ 0.3y & 0 \le y < 1 \\ 0.7y - 0.4 & 1 \le y < 2 \\ 1 & y > 2 \end{cases}$

$q_2(X) = F(q_2) = 0.5$

$\Rightarrow 0.7q_2 - 0.4 = 0.5 \Rightarrow q_2 = \frac{9}{7} = 1.286$

$\therefore \boxed{a = 1.286}$

(b) We know from Tarset Theorem 6.1 that value of c that minimizes $E(X-c)^2$ is $c = \mu$

$E(X) = \int_x x f(x)\,dx$

$= \int_0^1 0.3x\,dx + \int_1^2 x \times 0.7$

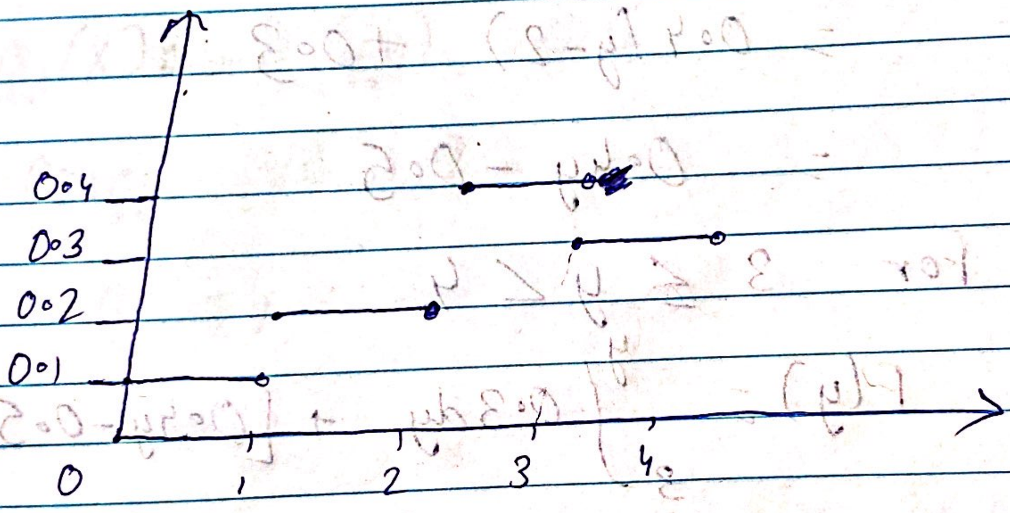
$= \frac{0.3}{2}[x^2]_0^1 + \frac{0.7}{2}[x^2]_1^2$

$= 0.15 \times 1^2 + 0.35[3]$

$E(x) = \boxed{1.20 = b}$

Ans. 3

$$f(x) = \begin{cases} 0.1 & 0 \le x < 1 \\ 0.2 & 1 \le x < 2 \\ 0.4 & 2 \le x < 3 \\ 0.3 & 3 \le x < 4 \\ 0 & \text{otherwise} \end{cases}$$



~~For y < 0~~

$F(y) = P(X < x)$

For $y < 0$

$F(y) = 0$

For $0 \le y < 1$

$F(y) = \int_0^y 0.1\, dy$

$= 0.1y$

For $1 \le y < 2$

$F(y) = \int_1^y 0.2\, dy + [0.1y]_{\text{at } y=1}$

$= [0.2y]_1^y + 0.1$

$= 0.2(y-1) + 0.1$

$= 0.2y - 0.1$

For $2 \le y < 3$

$$F(y) = \int_2^y 0.4\,dy + [0.2y - 0.1]_{at\ y=2}$$

$$= [0.4y]_2^y + 0.3$$

$$= 0.4(y-2) + 0.3$$

$$= 0.4y - 0.5$$

For $3 \le y < 4$

$$F(y) = \int_3^y 0.3\,dy + [0.4y - 0.5]_{at\ y=3}$$

$$= [0.3y]_3^y + 0.7$$

$$= 0.3(y-3) + 0.7$$

$$= 0.3y - 0.9 + 0.7$$

$$= 0.3y - 0.2$$

For $y \ge 4$

$$F(y) = 1$$

$$F(y) = \begin{cases} 0 & y < 0 \\ 0.1y & 0 \le y < 1 \\ 0.2y - 0.1 & 1 \le y < 2 \\ 0.4y - 0.5 & 2 \le y < 3 \\ 0.3y - 0.2 & 3 \le y < 4 \\ 1 & y \ge 4 \end{cases}$$

(b) $q_2(x)$ means $F(q_2) = 0.5$

$$0.4q_2 - 0.5 = 0.5$$

$$0.4q_2 = 1$$

$$q_2 = \frac{10}{0.4}$$

$$\boxed{q_2 = 2.5}$$ lies b/w 2 & 3

Yes It's greater than 2

(c) $E(x) = \int_x x\, f(x)\, dx$

$$= \int_0^1 x \times 0.1 dx + \int_1^2 x \times 0.2 dx + \int_2^3 x \times 0.4\, dx + \int_3^4 0.x \times 0.3\, dx$$

$$= \frac{0.1}{2} \times [x^2]_0^1 + \frac{0.2}{2}[x^2]_1^2 + \frac{0.4}{2}[x^2]_2^3 + \frac{0.3}{2}[x^2]_3^4$$

$$= 0.05 + 0.1[3] + 0.2[5] + 0.15[7]$$

$$= 0.05 + 0.30 + 1. + 1.05$$

$$= 0.35 + 2.05$$

$$= \boxed{2.40}$$ It is greater than 2

Ans 6 53.53613 %
R code handed over for this calculation in separate file