

Ans 02

$$\mu = 5 \text{ hrs}$$

$$\sigma^2 = 0.5 \text{ hrs}$$

let X be the no. of hrs that a laser printer can work with a pair of AAAA batteries

$$X \sim N(5, 0.5^2)$$

let Y be the duration that a laser printer can work with 20 pairs of AAAA batteries

$$Y \sim N(100, 20 \times 0.5^2)$$

$$P(Y \geq 105)$$

$$= 1 - P(Y < 105)$$

$$= 1 - \text{pnorm}(105, 100, \sqrt{20 \times 0.5})$$

$$= \boxed{0.01267366}$$

Ans 04

X is a D.R.V with PMF

$$P(X=x) = \begin{cases} 0.3 & x = -2 \\ 0.6 & x = -1 \\ 0.1 & x = 12 \\ 0 & \text{otherwise} \end{cases}$$

let X_1, \dots, X_n be IID with same distⁿ as X & \bar{X} be sample mean of (X_1, \dots, X_n)

$$a) E(X) = \sum x f(x)$$

$$= -2 \times 0.3 - 1 \times 0.6 + 12 \times 0.1$$

$$= -0.6 - 0.6 + 1.2$$

$$= \boxed{0}$$

$$(b) \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= \left[(-2)^2 \times 0.3 + (-1)^2 \times 0.6 + (12)^2 \times 0.1 \right] - 0^2$$

$$= 1.2 + 0.6 + 14.4 - 0$$

$$= \boxed{16.2}$$

$$(c) E(\bar{X}) = \mu = \boxed{0}$$

$$(d) \text{Variance}(\bar{X}) = \frac{\sigma^2}{n} = \frac{16.2}{n}$$

$$(e) \text{For } n = 100$$

$$\bar{X} \sim N\left(0, \frac{16.2}{100}\right)$$

$$P(\bar{X} > 0.5)$$

$$1 - P(\bar{X} \leq 0.5)$$

#rcond

$$s.d. = \frac{\sqrt{16.2}}{10} = 0.40249$$

$$1 - \text{pnorm}(0.5, 0, 0.40249)$$

$$= \boxed{0.107073}$$

Ans 3 (a) Fluctuations of Value Stock are X_1, X_2, \dots, X_{400}
 " " " " Growth " " " " Y_1, Y_2, \dots, Y_{400}

$$E[X_i] = 0.01; \text{Var}[X_i] = 0.01; E[Y_j] = 0; \text{Var}[Y_j] = 0.25$$

let combined fluctuation be for Value Stock $X = X_1 + X_2 + \dots + X_{400}$
 " " " " " " Growth stock $Y = Y_1 + Y_2 + \dots + Y_{400}$

$$E[X] = 400 \times 0.01 = 4 \quad \text{Var}[X] = 400 \times 0.01 = 4$$

$$E[Y] = 400 \times 0 = 0 \quad \text{Var}[Y] = 400 \times 0.25 = 100$$

$$(a) \quad P(X > 0) = 1 - P(X \leq 0)$$

Rcode

$$= 1 - \text{pnorm}(0, 4, 2)$$

$$= \boxed{0.9772499}$$

$$(b) \quad P(Y > 0) = 1 - P(Y \leq 0)$$

Rcode

$$= 1 - \text{pnorm}(0, 0, 10)$$

$$= \boxed{0.5}$$

$$(c) \quad P(X \geq 20) = 1 - P(X \leq 20)$$

$$= 1 - \text{pnorm}(20, 4, 2)$$

$$= 6.661338e^{-16}$$

$$\approx 0$$

$$(d) P(Y \geq 20)$$

$$= 1 - P(Y \leq 20)$$

r code

$$= 1 - \text{pnorm}(20, 0, 10)$$

$$= \boxed{0.02275013}$$

$$(e) P(Y > X)$$

$$P(Y - X > 0)$$

$$= 1 - P(Y - X \leq 0)$$

For $Y - X$

$$E(Y - X) = 0 - 4 = -4$$

$$\text{Var}(Y - X) = 100 + (-1)^2 \cdot 4$$

$$\text{Var}(Y - X) = 104$$

$$= 1 - \text{pnorm}(0, -4, \sqrt{104})$$

$$= \boxed{0.03474433}$$