Problem set 4

S520

Upload your answers as ONE file (PDF preferred) through the Assignments tab on Canvas by 11:59 pm, Thursday 26th September.

Trosset question numbers refer to the hardcover textbook. Show working (answers only will not get full credit.) You may work with others, but you must write up your homework independently — you should not have whole sentences in common with other students or other sources. You may (and sometimes have to) use R; include your R code where relevant.

1. Let X be a uniform random variable with probability density function (PDF)

$$f(x) = \begin{cases} \frac{1}{20} & 20 \leq x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Verify that f is a PDF. That is, check that $f(x)$ is always non-negative and that the area under $f(x)$ is 1.
   (b) Find the CDF of X, $F(y)$, for all y.
   (c) Find $P(30 \leq X < 50)$.
   (d) Find the expected value of X.
   (e) Find the variance and standard deviation of X. (Hint: You can use calculus if you wish, and it's good practice, but it'll be easier to use the formula for the variance and SD of a uniform random variable.)

2. Let X be a random variable with PDF

$$f(x) = \begin{cases} \frac{1}{30} & 0 \leq x < 20 \\ \frac{1}{60} & 20 \leq x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the CDF of X, $F(y)$, for all y.
   (b) Suppose that buses go past my stop at exactly twenty minutes past the hour and forty minutes past the hour (e.g. 9:20, 9:40) every hour. I arrive at my stop at a completely random time during the day. What is the expected value of the length of time I'll have to wait for a bus?
   (c) Find y such that $F(y) = 0.5$. (That is, set the correct piece of $F(y)$ equal to 0.5, and solve for y.) Is this larger than, smaller than, or the same as $E(X)$?

3. Trosset chapter 5.6 exercise 2, parts (b) and (c).

4. Trosset chapter 5.6 exercise 7. (Use R and give code.)

5. Trosset chapter 5.6 exercise 8. Hints: Remember that Normal$(1, 9)$ denotes a variance of 9, not a SD. Some of the following theorems in the textbook may be relevant: Theorems 4.2, 4.4, 4.7, 4.8, 5.2.

6. (Adapted from the Spring 2017 takehome.) The tempo (speed) of a piece of music is usually measured in beats per minute (BPM.) A study[1] found that the BPM in disco songs was approximately normal with mean 120 and standard deviation 20. (Treat BPM as a continuous random variable for the purpose of this question.) The best tempo for dancing is considered to be 115 to 135 BPM, while anything above 160 BPM is exhausting.

   Suppose a DJ gets lazy and puts her MP3 player containing a large, representative collection of disco songs on shuffle.

   (a) Suppose the MP3 player randomly selects a disco song. What is the probability that the BPM is between 115 and 135?

   (b) Suppose the MP3 player randomly selects ten disco songs. What is the probability at least one of the ten songs has a BPM over 160?

   (c) Suppose the MP3 player randomly selects two disco songs. What is the probability the *average* BPM of these two songs is over 160? (Hint: What's the distribution of the sum of two independent normal random variables? How big does the sum have to be for the average to be over 160?)

Midterm practice questions: Not to be handed in

7. (From Fall 2014 midterm.) Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 0.1 & 0 \le x < 2 \\ 0.2 & 2 \le x < 6 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find F, the cumulative distribution function of X.

   (b) The *median* of X is the value m such that $F(m) = 0.5$. Find the median of X.

   (c) Find the expected value of X.

8. (From Spring 2013 midterm.) Let X be a continuous random variable with probability density function (PDF)

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \le x < 1 \\ \frac{1}{2} & 1 \le x < 2 \\ \frac{1}{2}(3 - x) & 2 \le x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find F, the cumulative distribution function of X.

   (b) Find the expected value of X.

[1] `https://github.com/nikhilunni/BPMFinder/`

(c) Find the standard deviation of X.

9. (From the Spring 2015 takehome.)

   (a) Let X be a normal random variable with mean -5 and standard deviation 10. Find $P(X > 0)$.

   (b) Let Y be a standard normal random variable. Find $P(|Y| > 1.5)$.

   (c) Let $Z_1, Z_2, \ldots, Z_{10}$ be ten independent standard normal random variables. Find the probability that at least six of them are positive.