S-520 : Intro to Stats
Problem Set 3

Ans. 1 a) $P(X = x) = \frac{7-x}{20}$ for $x = 1, 2, 3, 4, 5$
& $P(X=6) = 0$

The Probability Mass Fn. $f(x) = P(X=x)$

$$f(x) = \begin{cases} 0.3 \text{ or } 3/10 & x = 1 \\ 0.25 & x = 2 \\ 0.20 & x = 3 \\ 0.15 & x = 4 \\ 0.10 & x = 5 \\ 0 & x = 6 \end{cases}$$

b) To calculate CDF, we know
CDF = $F(y) = P(X \leq y)$
We simply add the probability mass fn. cumulatively

$$F(y) = \begin{cases} 0 & y < 1 \\ 0.3 & 1 \leq y < 2 \\ 0.55 & 2 \leq y < 3 \\ 0.75 & 3 \leq y < 4 \\ 0.90 & 4 \leq y < 5 \\ 1 & 5 \leq y < 6 \\ 1 & y \geq 6 \end{cases}$$

(c) Expected Value of X or $E(x)$

We know,

$$E(x) = \sum_x x \cdot f(x)$$

$$= 1 \times 0.3 + 2 \times 0.25 + 3 \times 0.2 + 4 \times 0.15 + 5 \times 0.1 + 6 \times 0$$

$$\boxed{E(x) = 2.5}$$

(d) $Var(x) = [E(x^2)] - [E(x)]^2$

Now,

$$E(x^2) = \sum_x x^2 \cdot f(x)$$

$$= 1 \times 0.3 + 4 \times 0.25 + 9 \times 0.2 + 16 \times 0.15 + 25 \times 0.1 + 36 \times 0$$

$$= 8$$

$$Var(x) = 8 - (2.5)^2$$

$$\boxed{Var(x) = 1.75}$$

(e) Standard deviation of $x = \sqrt{Var(x)}$

$$= \sqrt{1.75}$$

$$= \boxed{1.3229}$$

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Ans. 2(a) PMF of X for $\{1, 1, 1, 1, 2, 5, 5, 10, 10, 10\}$

X can take 4 values

$$f(x) = P(X = x) = \begin{cases} 4/10 & x = 1 \\ 1/10 & x = 2 \\ 2/10 & x = 5 \\ 3/10 & x = 10 \end{cases}$$

(b) CDF $= F(y) = P(X \leq y)$

$$F(y) = \begin{cases} 0 & y < 1 \\ 4/10 & 1 \leq y < 2 \\ 5/10 & 2 \leq y < 5 \\ 7/10 & 5 \leq y < 10 \\ 1 & y \geq 10 \end{cases}$$

(c) $E(x) = \sum_x x \cdot f(x) = 1 \times \frac{4}{10} + 2 \times \frac{1}{10} + 5 \times \frac{2}{10} \times 10 \times \frac{3}{10}$

$= \boxed{4.6}$

(d) $E(x^2) = \sum_x x^2 f(x) = 1 \times \frac{4}{10} + 4 \times \frac{1}{10} + 25 \times \frac{2}{10} \times 100 \times \frac{3}{10}$

$= 35.8$

$Var(x) = E(x^2) - [E(x)]^2 = 35.8 - (4.6)^2$

$= \boxed{14.64}$

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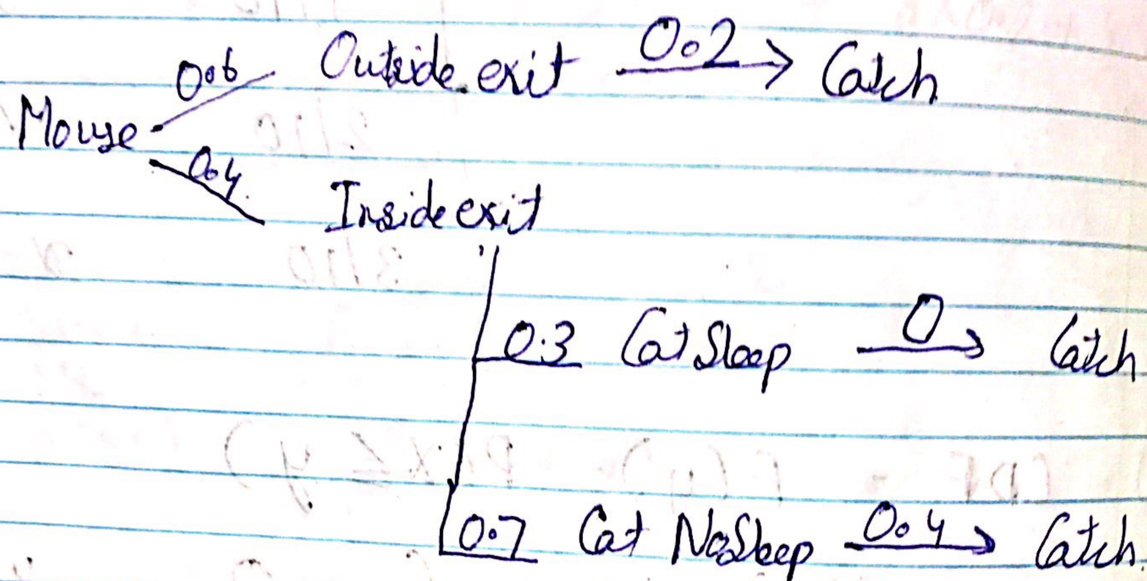
(e) $S.d(x) = \sqrt{Var(x)} = \sqrt{14.64}$

$= \boxed{3.826}$

Ans. 3 a)



Each tree branch is marked with probabilities

Koko can catch the mouse in 2 branches

Outside Catch Prob $= 0.6 \times 0.2 = 0.12$

Inside " " $= 0.4 \times 0.7 \times 0.4 = 0.112$

So, Koko should wait outside for the mouse

(b) let X be the no. of days it takes to catch the mouse if cat waits outside. This follows a binomial distⁿ

Here $p = 0.12$

$1-p = 0.88$ & $n = 7$

$X \sim Binom(7, 0.12)$

Now, $F(X \leq 7) =$ sum(dbinom(1:7, 7, 0.12)) # R code

$= 0.5913244$

Q.4(a) Since it follows a binomial $dist^n$ with $N = 2$ & $p = 0.70$, $1-p = 0.30$
We can Run the following r code to find the answer

⟹ # R code

```
c <- dbinom(0:2, 2, 0.70)
print(c)
```

This will give a vector of length 3

⟹
$P(X=0) = c[1] = 0.09$
$P(X=1) = c[2] = 0.42$
$P(X=2) = c[3] = 0.49$

(b) $F(y) = P(X \leq y)$

$$F(y) = \begin{cases} 0 & y < 0 \\ 0.09 & 0 \leq y < 1 \\ 0.51 & 1 \leq y < 2 \\ 1 & y \geq 2 \end{cases}$$

(c) $E(x) = \sum_x x \cdot f(x)$

$= 0 \times 0.09 + 1 \times 0.42 + 2 \times 0.49$
$= 1.4$

Q.5 a) Let X be a random variable that denotes the no. girls born to these 329 children
$X \sim Binom(329, 0.485)$

$f(X \geq 157) = ?$
We can sum the PMF's from 157 to 329

```
# R code
sum(dbinom(157:329, 329, 0.485))
```

$= 0.6321467$

(b) Here X might not be an independent random variable as X ~~could be influenced by these rich celebrities. With various~~ could be influenced by the ability of these celebrities to conceive a boy or girl.
Out of 329 celebrities selected for the study, maybe for 70% of the celebrities there is a greater than 50% chance to conceive a girl in their family history.
So family history of these celebrities must be taken into consideration.
Also, since these celebrities are rich, they could afford to for expensive medical procedures like test-tube babies & IVF procedure.

Q.6 (i) The problem follows a binomial $dist^n$ with ~~X~~ X being the random variable that denotes # correct identifications

$\therefore X = 0, 1, 2 \text{ --- } 25$

Here $N = 25$ & $p = \frac{1}{5}$ (as there are 5 options to choose from

Now on an average, # symbols that the receiver expects to identify correctly $= E(X)$

$E(X) = n \times p$

$= \cancel{25}^{5} \times \frac{1}{\cancel{5}_1} = \boxed{5}$

(b) $f(X > 7)$ We need to calculate PMFs for 7 to 25 & sum them up

Rcode

```
sum(dbinom(8:25, 25, 0.2))
```

$= \boxed{0.1091228}$

(c) $P(Z \geq 1)$ for $f(X > 7)$

$=$ sum(dbinom(1:20, 20, 0.1091228))

$= \boxed{0.9008353}$

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