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S520: Introduction to Statistics
Problem Set 2

Ans. 1 (a) Tossing 6 fair coins

\# events in sample space or $\#(S) = 2^6$

Now,

\# events where exactly 4 coins show heads or $\#(A) = {}^6C_4$.

Now, probability of $\frac{\#A}{\#S} = \boxed{\frac{{}^6C_4}{2^6}} = \frac{15}{64} = \boxed{0.234}$

(b) # events where there more heads than tails: $(P(X=4) + P(X=5) + P(X=6))$

Let X be a random variable that denotes the # heads

$\#(A) = {}^6C_4 + {}^6C_5 + {}^6C_6$

$\#(S) = 2^6$

Now $\frac{\#(A)}{\#(S)} = \boxed{\frac{{}^6C_4 + {}^6C_5 + {}^6C_6}{2^6}}$

$= \frac{22}{64} = \boxed{0.344}$

(c) $\#(A) = {}^6C_4$ or $X = 4$

$\#(S) = {}^6C_4 + {}^6C_5 + {}^6C_6$ or $X \geq 4$.

Now conditional probability of $\frac{\#A}{\#S} = \boxed{\frac{{}^6C_4}{{}^6C_4 + {}^6C_5 + {}^6C_6}} = \frac{15}{22} = \boxed{0.682}$

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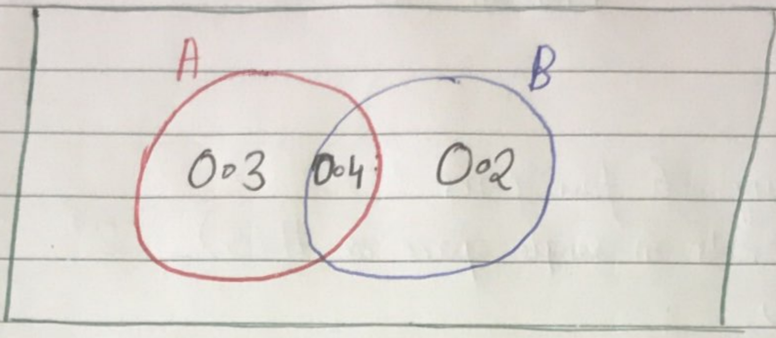
Ans.3(b) Ex 7, Parts (b) to (e)

(b)



$P(A^c \cap B) = 0.2$ [Given]

$P(B) = 0.6$ [Given]

$\therefore P(A \cap B) = 0.6 - 0.2 = 0.4$

Now $P(A) = 0.7$ [Given]

& $P(A \cap B) = 0.4$

$\therefore P(A \cap B^c) = 0.7 - 0.4$

$= 0.3$

NO, it is not possible for A & B to be disjoint events. In Disjoint events $A \cap B = 0$

Here, it is clearly seen through Venn Diagram that $A \cap B \neq 0$

(c) $P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c)$

$= 0.7 + (1 - P(B)) - 0.3$

$= 0.4 + 0.4$

$= \boxed{0.8}$

(d) For A & B to be independent events

$P(AB) = P(A) \times P(B)$

or $0.4 = 0.7 \times 0.6$

$0.40 \neq 0.42$ & hence they are dependent events

(e) $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.4}{0.6} = \frac{2}{3} = \boxed{0.67}$

Ans.4 Exercise 11

(a) A → Person is male Let $P(A) = 0.5$
B → Stuck by lightning

$P(B|A) = 0.85$ [given] $\therefore P(B|A^c) = 0.15$

$$P(A|B) = \frac{P(B|A).P(A)}{\sum_A P(B)} \quad \text{[Bayes' Theorum]}$$

$$= \frac{P(B|A).P(A)}{P(B|A).P(A) + P(B|A^c).P(A^c)}$$

$$= \frac{0.85 \times 0.50}{0.85 \times 0.50 + 0.15 \times 0.5}$$

$$= \frac{0.85 \times \cancel{0.5}}{\cancel{0.5}(0.85 + 0.15)}$$

$$\boxed{P(A|B) = 0.85}$$

$P(A^c|B) = 1 - 0.85$

$$\boxed{P(A^c|B) = 0.15}$$

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(b) $P(A|B) = 0.85$
$P(A) = 0.50$

Now, $P(A|B) \neq P(A)$
That means knowing 'B' affects the probability of A. Hence A & B are dependent events

(c) $P(A|B)$ or Prob. that a person is a man given that he is struck by lightning $= 0.85$

$P(A^c|B)$ or P_r(women / person is struck by lightning) $= 0.15$

$P(A|B) >>> P(A^c|B)$
This means that given a person is struck by lightning, there are 85% chances that the person is male & 15% chances that the person is female. The reasons could be that men, right through the history are more inclined towards working outdoors like farm / fields. Whereas women tend to stay indoors & do household work or work in offices. Hence, men due to more exposure to outdoor activity are more likely to be stuck by lightning.

Ans. 5 Exercise 12, parts (e), (f), (g)

(e) A = Event that the movie was filmed in color
B = " " " " " Western
$P(A|B)$ = Probability that movie was color given it was western
$P(A)$ = " " " " color

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Since P(A|B) is a Subset of P(A) & $P(A|B) \neq P(A)$
Hence A & B are dependent events

(f) A = event, Student will attend College of William & Mary
B = " that " graduated from high school in Virginia

$P_r(B|A)$ is the probability that student will graduate from high school in Virginia given that student attends College of William & Mary

P(B) is simply Student graduates from high school Virginia

There is no mention that Student from high school in Virginia will attend College of William & Mary
That means $P_r(B|A) = P(B)$ or B & A are independent events

(g) A = event that selected person's ~~PHD~~ Ph.D earned before 1950
B = " " " " is female

$P_r(A|B)$ is Probability that Person earned PHD given that she is female
$P_r(A)$ is Prob. that person earned Ph.D before 1950
Since P(A|B) is a Subset of P(A) & $P(A|B) \neq P(A)$
Hence A & B are dependent events

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Ans. 6 C.D.F of Random Variable 'X' $F(y) = \begin{cases} 0 & , \quad y < 0 \\ y/2 & , \quad 0 \le y < 1 \\ (y+1)/4 & , \quad 1 \le y < 3 \\ 1 & \quad y \ge 3 \end{cases}$

(a) $P(X \le 2)$

$F(y) = P(X \le y)$
$F(2) = P(X \le 2)$

$F(2) = \frac{y+1}{4}$

$F(2) = \frac{2+1}{4} = \frac{3}{4} = \boxed{0.75}$

(b) $P(X > 2) = 1 - P(X \le 2)$
$= 1 - \frac{3}{4} = \boxed{0.25}$

(c) $P(0.5 < X \le 2.5)$

$= F(2.5) - F(0.5)$

$= \frac{y+1}{4} - \frac{y}{2}$

$= \frac{2.5+1}{4} - \frac{0.5}{2}$

$= \frac{35}{40} - \frac{05}{20} = \frac{7}{8} - \frac{1}{4} = \boxed{\frac{5}{8}}$

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(d) $P(X=1)$ = Size of jump at $X=1$

$\Rightarrow \frac{y+1}{4} - \frac{y}{2}$

$\Rightarrow \frac{1-y}{\cancel{2}\,4}$ at $P(X=1)$

$\Rightarrow \frac{1-1}{4}$

$\Rightarrow \boxed{P(X=1) = 0}$

(e) $F(q) = 0.6$

Case I

$F(q) = 0.6 = \frac{y}{2}$ or $\frac{q}{2}$

$0.6 = \frac{q}{2}$

$q = 1.2$, since this does not obey the equalities $0 \leq q < 1$ for $F(q) = \frac{q}{2}$, Hence $F(q) \neq \frac{q}{2}$

Case II

$F(q) = \frac{q+1}{4} = 0.6 \quad 1 \leq q < 3$

$\boxed{q = 1.4}$ Here q lies b/w 1 & 3 & hence $F(q) = \frac{q+1}{4}$

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Ans.2

Seattle Seahawks

either

Run

3 possible outcomes

- Touchdown (60%)
- No gain (39%)
- Turnover (1%)

Pass

3 possible outcomes

- Touchdown (65%)
- No gain (31%)
- Turnover (4%)

Winning Probabilities in either cases

P(Win | Touchdown) = 95%
P(Win | NoGain) = 80%
P(Win | Turnover) = 0%

(a) P(Win | Run)

In case, he has decided to run, the ~~P(A)~~ = sample space of events is 'S'

~~P~~(#S) = {{Touch down & Win}, {Touchdown & Not Win}, {Nogain & Win}, {Nogain & lose}, {~~Nogain~~ Turnover & Win}, {Turnover & lose}}

Events that he wins → A

#(A) = {Touchdown & Win}, {Nogain & Win}, {Turnover & Win}

$P(Win \mid Run) = \frac{P(\# A)}{P(\# S)}$

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$$= \frac{0.60 \times 0.95 + 0.39 \times 0.80 + 0.01 \times 0}{(0.60 \times 0.95 + 0.60 \times 0.05) + (0.39 \times 0.80 + 0.39 \times 0.20) + (0.01 \times 0) + 0.01 \times 1}$$

$$= \frac{0.57 + 0.31}{(0.57 + 0.03) + (0.31 \times 0.08) + (0.01)}$$

$$= \frac{0.88}{1}$$

$$\boxed{P(Win|Run) = 88\%}$$

(b) $P(Win|Pass) = \dfrac{0.65 \times 0.95 + 0.31 \times 0.80 + 0.04 \times 0}{0.65(0.95 + 0.05) + 0.31(0.8 + 0.2) + 0.04(0 + 1)}$

$$= \frac{0.6175 + 0.248}{1}$$

$$\boxed{P(Win|Pass) = 0.8655 = 86.55\%}$$

(c) Since $P(Win|Run) > P(Win|Pass)$, Seahawks should run!!