Problem set 5

S520

Upload your answers as ONE file (PDF preferred) through the Assignments tab on Canvas by 11:59 pm, Thursday 3rd October.

Trosset question numbers refer to the hardcover textbook. Show working (answers only will not get full credit.) You may work with others, but you must write up your homework independently — you should not have whole sentences in common with other students or other sources. You may (and sometimes have to) use R; include your R code where relevant.

1. Trosset chapter 6.4 exercise 1.

2. Trosset chapter 6.4 exercise 3.

3. (From the Spring 2017 midterm.) Let X be a continuous random variable with probability density function (PDF)

$$f(x) = \begin{cases} 0.1 & 0 \leq x < 1 \\ 0.2 & 1 \leq x < 2 \\ 0.4 & 2 \leq x < 3 \\ 0.3 & 3 \leq x < 4 \\ 0 & \text{otherwise.} \end{cases}$$

   To answer the following questions, it may help to draw a graph of this PDF.

   (a) Complete $F(y)$, the cumulative distribution function of X, by complete the expression below:

$$F(y) = \begin{cases} \\ \\ \\ \\ \\ \end{cases} \begin{matrix} y < 0 \\ 0 \leq y < 1 \\ 1 \leq y < 2 \\ 2 \leq y < 3 \\ 3 \leq y < 4 \\ y \geq 4 \end{matrix}$$

   (b) Is the median of X equal to 2, less than 2, or greater than 2? Explain why or calculate the median.

   (c) Is the expected value of X equal to 2, less than 2, or greater than 2? Explain why or calculate the expected value.

4. The heights of adult women follow an approximately normal distribution with mean 63.8 inches and SD 2.9 inches.

   (a) What percentage of adult women are taller than 65.5 inches?

(b) What is the interquartile range of adult women's heights?

(c) Complete the following, rounding to the nearest 0.1 of an inch: "The shortest 2.5% of women are shorter than —, while the tallest 2.5% of women are taller than —."

5. Let X be a continuous random variable with PDF

$$f(x) = \begin{cases} 0.3 & 0 \leq x < 1 \\ 0.7 & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Find the constant value a that minimizes $E|X - a|$.

(b) Find the constant value b that minimizes $E[(X - b)^2]$.
Hint: See Trosset section 6.3.

6. The file `IU_Salary_List-2014-2015.xls` contains the salaries of IU employees for the 2014–15 academic year (this is publicly available information.) Read this data into R as a data frame called `IUsalaries` — you'll need to either convert the spreadsheet to a .csv file, or install package (e.g. `rio`) that allows R to read in Excel spreadsheets directly.

What percentage of IU academic employees earn more than Dr. Luen? (Take "academic employees" to mean those for whom "Plan" is "AC1.".) Include your R code.

Practice questions: Not to be handed in

7. (From the Fall 2014 takehome. Not to be handed in.) Let X be a standard normal random variable. Let $Y = X^2$.

(a) Find $P(Y > 1)$.

(b) Find the 0.9-quantile of Y.

8. (From the Spring 2014 takehome. Not to be handed in.) Let A, B, C, D, and E be independent standard normal random variables. What is the probability that at least two of the five variables are greater than 1?