Problem set 2

S520

Upload your answers as ONE file (PDF preferred) through the Assignments tab on Canvas by 11:59 pm, Thursday 12th September.

Trosset question numbers refer to the hardcover textbook. Show working. You may work with others, but you must write up your homework independently — you should not have whole sentences in common with other students or other sources.

1. (From the Fall 2015 midterm.)

    (a) I toss six fair coins. What is the probability exactly four of the coins show heads?

    (b) I toss six fair coins. What is the probability that there are more heads than tails?

    (c) I toss six fair coins. Given that there are more heads than tails, what is the conditional probability exactly four of the coins show heads?

2. (From the Spring 2015 takehome.) In the last minute of Super Bowl XLIX, the Seattle Seahawks, who are behind by four points, must decide whether to run or pass. To simplify, suppose that in each case, there are three mutually exclusive outcomes: a touchdown, no gain, and turnover.

    If the Seahawks run, the probabilities of each outcome are:

    - Touchdown: 60%
    - No gain: 39%
    - Turnover: 1%

    If the Seahawks pass, the probabilities are:

    - Touchdown: 65%
    - No gain: 31%
    - Turnover: 4%

    No matter whether they decide to run or pass, the probability of winning depends only on whether there is a touchdown, no gain, or a turnover. The probabilities are:

    - After a touchdown, the Seahawks have a 95% chance of winning.
    - After no gain, the Seahawks have an 80% chance of winning.
    - After a turnover, the Seahawks have no chance of winning.

    (a) Suppose the Seahawks decide to run. What is the probability they win?

(b) Suppose the Seahawks decide to pass. What is the probability they win?

(c) Based on your answers to parts (a) and (b), should the Seahawks run or pass?

3. Trosset chapter 3.7 exercise 7, parts (b) to (e) (probabilities and conditional probabilities.) (You should draw the Venn diagram, but you don't have to submit it.)

4. Trosset chapter 3.7 exercise 11.

5. Trosset chapter 3.7 exercise 12, parts (e), (f), and (g) (independent or dependent.) Verbal explanations are sufficient, though feel free to Google data if it helps you.

6. Let X be a random variable with the following cumulative distribution function (CDF):

$$F(y) = \begin{cases} 0 & y < 0 \\ y/2 & 0 \le y < 1 \\ (y+1)/4 & 1 \le y < 3 \\ 1 & y \ge 3 \end{cases}.$$

(a) What's $P(X \le 2)$?

(b) What's $P(X > 2)$?

(c) What's $P(0.5 < X \le 2.5)$?

(d) What's $P(X = 1)$?

(e) Let q be a number such that $F(q) = 0.6$. What's q?