Problem set 3

S520

Upload your answers as ONE file (PDF preferred) through the Assignments tab on Canvas by 11:59 pm, Thursday 19th September.

Trosset question numbers refer to the hardcover textbook. Show working. You may work with others, but you must write up your homework independently — you should not have whole sentences in common with other students or other sources. Some questions are best solved with R; if you use R, include your code.

1. Trosset chapter 4.5 exercise 2.
2. Trosset chapter 4.5 exercise 3.
3. Trosset chapter 4.5 exercise 7. Note: Submitting the tree diagram is optional.
4. According to Pew Research, 70% of Generation Z Americans (those aged 13 to 21 in 2018) believe "government should do more to solve problems." I select two Generation Z Americans at random. Let X be a discrete random variable representing the number of the people I selected who believe "government should do more to solve problems." Since the number of Generation Z Americans is very large, X will have very close to a binomial distribution.
   (a) Find $P(X = 0)$, $P(X = 1)$, and $P(X = 2)$.
   (b) Write down an expression for $F(y)$, the cumulative distribution function (CDF) of X, for all y-values from $-\infty$ to ∞.
   (c) Find the expected value of X.
5. (From the Fall 2015 takehome.) A controversial issue in evolutionary psychology (and statistics) is whether and to what extent characteristics of parents, such as physical beauty, affect the probability of giving birth to a girl. Gelman and Weakliem (2009) counted all children born to the members of People Magazine's 50 Most Beautiful People list from 1995–2000 (up to August 2007,) so, for example, Julia Roberts contributes two boys and a girl to the data set. They found that of the 329 children born to these celebrities, 157 were girls. In the general population, 48.5% of births are girls.
   (a) Suppose the sexes of all children are independent, and each child has a 48.5% probability of being a girl. Use the binomial distribution to find the probability that at least 157 out of 329 children are girls.
   (b) Explain why the sexes of the children in the study might not all be independent.
6. Trosset chapter 4.5 exercise 14.

Extra practice questions: Not to be handed in

7. (From the Spring 2014 midterm.) Let X be a discrete random variable with probability mass function

$$P(X = x) = \begin{cases} 0.1 & x = 1 \\ 0.2 & x = 2 \\ 0.2 & x = 3 \\ 0.3 & x = 4 \\ 0.2 & x = 5 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find $F(y)$, the cumulative distribution function of X, for all $y \in (-\infty, \infty)$.

   (b) Find the expected value and the variance of X.

   (c) Let X_1 and X_2 be independent random variables with the same distribution as X. What is the probability that $X_1 + X_2 = 4$?

8. (From the Spring 2013 midterm.) Let X be a discrete random variable with probability mass function

$$P(X = x) = \begin{cases} kx & x \in \{1, 2, 3, 4, 5\} \\ 0 & \text{otherwise.} \end{cases}$$

   where k is a constant.

   (a) Find k.

   (b) Find $F(y)$, the cumulative distribution function of X, for all $y \in (-\infty, \infty)$.

   (c) Find the expected value and variance of X.

9. (From the Summer 2016 takehome.) Suppose that among likely voters in an upcoming election, 46% support Billary, 39% support Ronald, while the remaining 15% support Other. (You may assume that the population of likely voters is very large.)

   (a) I randomly select two likely voters (in no particular order.) What is the probability that both support Ronald?

   (b) I randomly select two likely voters (in no particular order.) What is the probability that one supports Billary and the other supports Ronald?

   (c) I randomly select 1000 likely voters (in no particular order.) What is the probability that at least 480 of the sample support Billary?

10. (From the Fall 2017 takehome.) In English Premier League soccer, a team gets 3 points for a win, 1 point for a draw, and 0 points for a loss. In an upcoming game, Newcastle (an average team) will play Liverpool (a good team) at home. Suppose Newcastle has a choice between two strategies:

    - Attacking: 25% chance of win, 25% chance of draw, 50% chance of loss.
    - Defensive: 15% chance of win, 50% chance of draw, 35% chance of loss.

    Which strategy gives the higher number of expected points? Show your working.