Introduction to the Theory of Computation Solutions

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4.2

Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable.
Solution: We formulate the problem $EQ_{DFA,REX} = \{\langle A, R\rangle \mid A \text{ is a DFA, R is a regular expression, and } L(A) = L(R)\}$. We will design a TM T that decides $EQ_{DFA,REX}$:
T = "On input $\langle A, R\rangle$ where A is a DFA, R is a regular expression:

1. Use Theorem 1.54 to convert R into an equivalent DFA B. Therefore, $L(B) = L(R)$.
2. Run EQ_{DFA} on input $\langle A, B\rangle$. Output what EQ_{DFA} outputs."

Since EQ_{DFA} is decidable, and the conversion from regular expressions to DFAs takes finite time, $EQ_{DFA,REX}$ is decidable.

4.3

Let $ALL_{DFA} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.
Solution: We will design a TM T that decides ALL_{DFA}:
T = "On input $\langle A\rangle$ where A is a DFA:

1. Construct a DFA B such that $L(A) = \overline{L(B)}$.
2. Run E_{DFA} on input $\langle B\rangle$. Output what E_{DFA} outputs."

Since E_{DFA} is decidable, ALL_{DFA} is decidable.

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