

# Introduction to the Theory of Computation Solutions

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## Contents

1	Chapter 4	5
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# Chapter 1

## Chapter 2

## Chapter 3

# 1 Chapter 4

## 4.2

Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable.

**Solution:** We formulate the problem  $EQ_{DFA,REX} = \{\langle A, R \rangle \mid A \text{ is a DFA, } R \text{ is a regular expression, and } L(A) = L(R)\}$ . We will design a TM  $T$  that decides  $EQ_{DFA,REX}$ :

$T$  = “On input  $\langle A, R \rangle$  where  $A$  is a DFA,  $R$  is a regular expression:

1. Use Theorem 1.54 to convert  $R$  into an equivalent DFA  $B$ . Therefore,  $L(B) = L(R)$ .
2. Run  $EQ_{DFA}$  on input  $\langle A, B \rangle$ . Output what  $EQ_{DFA}$  outputs.”

Since  $EQ_{DFA}$  is decidable, and the conversion from regular expressions to DFAs takes finite time,  $EQ_{DFA,REX}$  is decidable.

## 4.3

Let  $ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$ . Show that  $ALL_{DFA}$  is decidable.

**Solution:** We will design a TM  $T$  that decides  $ALL_{DFA}$ :

$T$  = “On input  $\langle A \rangle$  where  $A$  is a DFA:

1. Construct a DFA  $B$  such that  $L(A) = \overline{L(B)}$ .
2. Run  $E_{DFA}$  on input  $\langle B \rangle$ . Output what  $E_{DFA}$  outputs.”

Since  $E_{DFA}$  is decidable,  $ALL_{DFA}$  is decidable.

## 4.4

Let  $A_{\epsilon CFG} = \{\langle G \rangle \mid G \text{ is a CFG that generates } \epsilon.\}$ . Show that  $A_{\epsilon CFG}$  is decidable.

**Solution:** We will design a TM  $T$  that decides  $A_{\epsilon CFG}$ :

$T$  = “On input  $\langle G \rangle$  where  $G$  is a CFG:

1. Convert  $G$  into an equivalent CFG  $C = (V, \Sigma, R, S)$  in Chomsky Normal Form.
2. *Accept* if  $C$  includes the rule  $S \rightarrow \epsilon$ , reject otherwise.”

Since converting a CFG into CNF is decidable,  $A_{\epsilon CFG}$  is decidable.

## 4.13

Let  $A = \{\langle R, S \rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$ . Show that  $A$  is decidable.

**Solution:** We will design a TM  $T$  that decides  $A$ :

$T$  = “On input  $\langle R, S \rangle$  where  $R$  and  $S$  are regular expressions:

1. Construct a DFA  $B$  such that  $L(B) = \overline{L(S)} \cap L(R)$ .
2. Run  $E_{DFA}$  on input  $\langle B \rangle$ . Output what  $E_{DFA}$  outputs.”

Since  $E_{DFA}$  is decidable,  $A$  is decidable. This construction is correct because  $L(R) \subseteq L(S) \Leftrightarrow \overline{L(S)} \cap L(R) = \emptyset$ .

## 4.15

Show that the problem of determining whether a CFG generates all strings in  $1^*$  is decidable. In other words, show that  $\{\langle G \rangle \mid G \text{ is a CFG over } \{0, 1\} \text{ and } 1^* \subset L(G)\}$  is a decidable language.

**Solution:** Let  $f$  be a computable function. Construct a decider  $D$ :

$D$  = “On input  $\langle G \rangle$  where  $G$  is a CFG:

1. Convert  $G$  into an equivalent CFG  $C$  in Chomsky Normal Form.
2. Let  $p$  be the pumping length of  $C$ .
3. Repeat  $\forall i \leq f(p)$ :
  - a. Check whether  $1^i \in L(C)$ .
  - b. If not, *reject*  $\langle G \rangle$ .
4. *Accept*  $\langle G \rangle$ .

## Chapter 5

## Chapter 6



## Chapter 7

## Chapter 8

## Chapter 9