Introduction to the Theory of Computation Solutions

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4.2

Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable.
Solution: We formulate the problem $EQ_{DFA,REX} = \{\langle \text{A, R} \rangle \mid \text{A is a DFA, R is a regular expression, and } L(A) = L(R)\}$. We will design a TM T that decides $EQ_{DFA,REX}$:
T = "On input ⟨A, R⟩ where A is a DFA, R is a regular expression:

1. Use Theorem 1.54 to convert R into an equivalent DFA B. Therefore, $L(B) = L(R)$.
2. Run EQ_{DFA} on input ⟨A, B⟩. Output what EQ_{DFA} outputs."

Since EQ_{DFA} is decidable, and the conversion from regular expressions to DFAs takes finite time, $EQ_{DFA,REX}$ is decidable.

4.3

Let $ALL_{DFA} = \{\langle \text{A} \rangle \mid \text{A is a DFA and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.
Solution: We will design a TM T that decides ALL_{DFA}:
T = "On input ⟨A⟩ where A is a DFA:

1. Construct a DFA B such that $L(A) = \overline{L(B)}$.
2. Run E_{DFA} on input ⟨B⟩. Output what E_{DFA} outputs."

Since E_{DFA} is decidable, ALL_{DFA} is decidable.

4.13

Let $A = \{\langle \text{R,S} \rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$. Show that A is decidable.
Solution: We will design a TM T that decides A:
T = "On input ⟨R,S⟩ where R and S are regular expressions:

1. Construct a DFA B such that $L(B) = \overline{L(S)} \cap L(R)$.
2. Run E_{DFA} on input ⟨B⟩. Output what E_{DFA} outputs."

Since E_{DFA} is decidable, A is decidable. This construction is correct because $L(R) \subseteq L(S) \Leftrightarrow \overline{L(S)} \cap L(R) = \varnothing$.

4.15

Show that the problem of determining whether a CFG generates all strings in 1* is decidable. In other words, show that $\{\langle \text{G} \rangle \mid \text{G is a CFG over } \{0,1\} \text{ and } 1^* \subset L(G)\}$ is a decidable language.
Solution: Let f be a computable function. Construct a decider D:
D = "On input ⟨G⟩ where G is a CFG:

1. Convert G into an equivalent CFG C in Chomsky Normal Form.
2. Let p be the pumping length of C.
3. Repeat $\forall i \leq f(p)$:
   a. Check whether $1^i \in L(C)$.
   b. If not, *reject* ⟨G⟩.
4. *Accept* ⟨G⟩.

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