Discrete Signal Processing on Graphs: Sampling Theory

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Abstract/Aim of your project

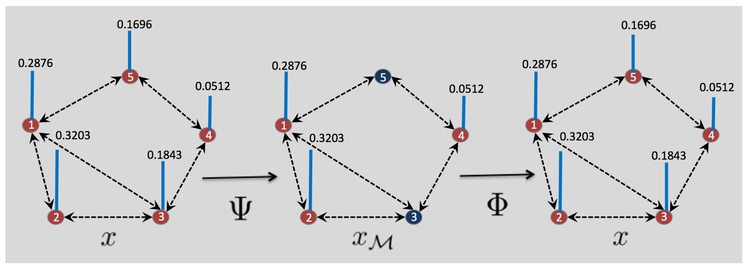
- Optimal Sampling of a graph signal
- Perfect Reconstruction of the graph signal

Applications or importance of the task

1) We only require a part of the initial data in order to reconstruct the data.
2) We can sample a fraction of users in a network along with their information and then recover all of the users' information.
3) We can analyse road system densities.

Challenges or motivation of work

1) Sampling on graphs is difficult because graphs lie on irregular structures
2) Previous sampling methods generally work on undirected graphs



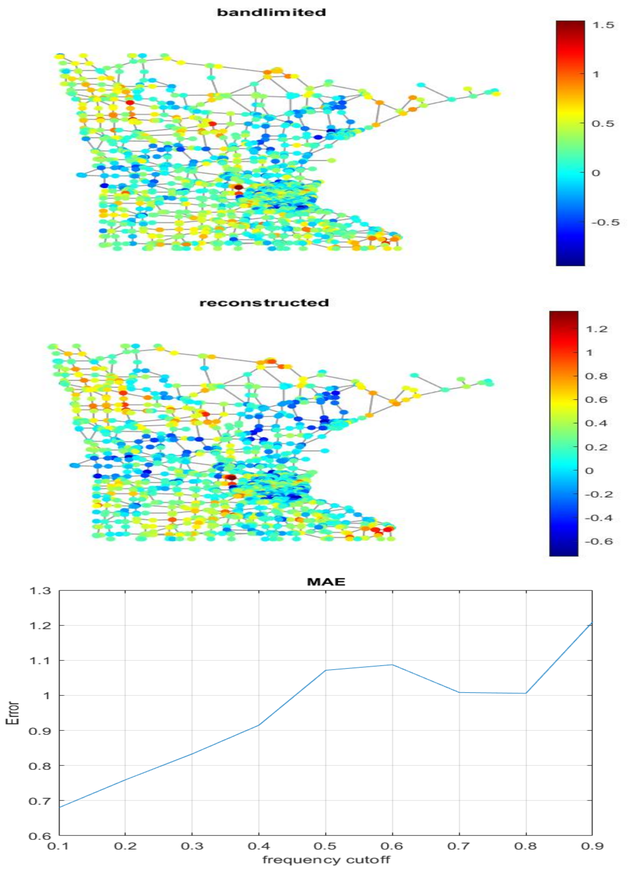
Method /Algorithms/ important concepts

$$Adj = \mathrm{V}\Delta V^{-1}$$
$$x' = V^{-1}x$$
$$V = inverse\ Fourier\ transform$$
$$k = number\ of\ non-zero\ elements\ of\ x'$$
$$\Psi = sampling\ operator(samples\ any\ k\ rows)$$
$$\Phi = interpolation\ operator(V_k(\Psi * V_k)^{-1})$$
$$x_m = \Psi * \mathrm{x}$$
$$x = \Phi * x_m$$

Any relevant discussion

This paper takes on a different type of sampling based off the bandlimited signal. Here we take into account the number of non-zero frequencies to sample the signal. This model does not consider noise.
In the case of noise being present we find the optimal sampling operator. The optimal sampling operator can be found when the least singular value of $\Psi * V_k$ is the maximum. Previous papers have shown a greedy algorithm gives a good approximation to global optimum.

Results/ MATLAB plots



Conclusion

Any graph can be perfectly reconstructed after sampling if the graph is bandlimited. We can also alter the sampling operator in order to make the method robust to noise.

References

[1] S. K. Narang, A. Gadde and A. Ortega, "Signal processing techniques for interpolation ingraph structured data," 2013 IEEE International Conference on Acoustics, Speech and Signal Processing, 2013, pp. 5445-5449, doi: 10.1109/ICASSP.2013.6638704
[2] A. Anis, A. Gadde and A. Ortega, "Towards a sampling theorem for signals on arbitrary graphs," 2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2014, pp. 3864-3868, doi: 10.1109/ICASSP.2014.6854325