Lab 11

Ellipse and Hyperbola

Aim :

- To explore different methods of drawing ellipse and hyperbola using GeoGebra tools, commands and equations

Concepts:

- Definitions of ellipse and hyperbola
- Equations of ellipse and hyperbola

Discussion :

As in Lab 10 we use different tools and commands to draw Ellipse and Hyperbola.We need a thorough knowledge about the curve and its equation for drawing them with a specific tool or command. Sometimes it may need some calculations also.

Activity 11.1 Ellipse 1

Procedure:

If the foci and a point on the ellipse are known, we can draw it in the following way.

- Using `Ellipse` tool, select the foci one by one and then a point on the ellipse (or give input `Ellipse[focus,focus,point]`)
- Using `Ellipse` tool draw the following ellipses

  1. Foci $(\pm 3, 0)$, passing through the point $(5, 2)$
  2. Foci $(0, \pm 4)$ and length of major axis 10
  3. Foci $(\pm 2, 0)$ and length of minor axis 5
  4. $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$
  5. Using `Ellipse` tool draw the following ellipses and find the length of the latus rectum geometrically

     (a) Foci $(\pm 4, 0)$, passing through the point $(5, 2)$

     (b) $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

Activity 11.2 Ellipse 2

Procedure:

We can draw an ellipse if we know its foci and length of semi major axis using `Ellipse` command. For example, the command `Ellipse[(3,0),(-3,0),5]` gives the ellipse with foci $(\pm 3, 0)$ and length of semi major axis 5

- Using the above command draw the following ellipse

  1. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$
  2. Foci $(0, \pm 5)$, passing through the point $(2, 6)$

  Create a slider **a**. Draw the ellipse using the command, `Ellipse[(-a,0),(a,0),5]`. Change the value of **a** and observe the corresponding change in the shape of the curve.

Activity 11.3 Hyperbola

Procedure:

If the foci and a point on the hyperbola are known, we can draw it in the following way.

- Using `Hyperbola` tool, select the foci one by one and then a point on the hyperbola (or give input `Hyperbola[focus, focus,point]`) to get the hyperbola with first the two points as foci and passing through the third point

- Using `Hyperbola` tool draw the following hyperbola

  1. Foci $(\pm 3, 0)$, passing through the point $(5, 2)$
  2. Foci $(0, \pm 4)$ and length of transverse axis 6
  3. Foci $(\pm 3, 0)$ and length of conjugate axis 5
  4. $\dfrac{x^2}{16} - \dfrac{y^2}{25} = 1$
  5. Using `Hyperbola` tool draw the following hyperbola and find the length of the latus rectum geometrically
     (a) Foci $(\pm 4, 0)$, passing through the point $(5, 2)$
     (b) $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

Activity 11.4 Hyperbola 2

Procedure:

- We can draw a hyperbola, if we know its foci and length of transverse axis using `Hyperbola` command. For example,the command `Hyperbola[(3,0),(-3,0),2]` gives the hyperbola with foci $(\pm 3, 0)$ and length of transverse axis 4

- Using the above command draw the following hyperbola

  1. $\dfrac{x^2}{25} - \dfrac{y^2}{9} = 1$
  2. Foci $(0, \pm 5)$, passing through the point $(2, 6)$

Additional Activities

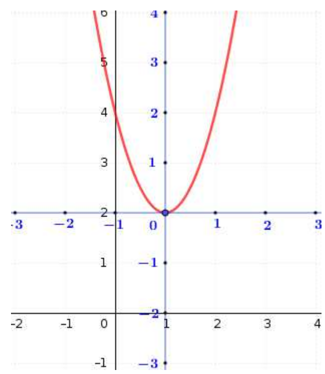
Activity 11.A Parabola in General

Aim:

To find the focus, directrix and length of the latus rectum of the parabola $y = 4x^2 - 2x + 5$

Procedure:

- Open ML 8.4 applet and draw the parabola $y = 4x^2 - 2x + 5$.
- Shift the origin to the vertex of the parabola.
- Find the transformed equation of the parabola
- Using this, find its focus, directrix, and length of the latus rectum with respect to the new origin
- Find the coordinates of the focus and the equation of the directrix with respect to the original system of axis
- Find the equation of the ellipse with foci (-2,3) and (6,3) and passing through (5,5)
- Find the equation of the hyperbola with foci (3,6) and (3,0) and passing through the origin



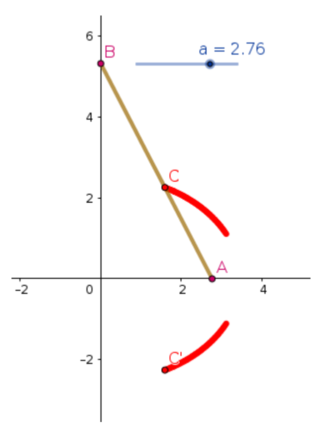
Activity 11.B Locus of a Point on a sliding Rod

Aim:

To find the path of a point on a rod of fixed length (say, 6 units) sliding between two coordinate axes

Procedure:

- Create a slider **a** with min = -6, max = 6 and increment = .01
- Plot the point A(a,0)
- Draw the circle with centre at A and radius 6
- Mark the point of intersection B of this circle with the y axis.
- Draw the line segment AB
- Hide the circle
- Plot a point C on the line segment and trace on it.
- Animate slider **a**, and observe the path of C
- To get the complete curve, reflect C on the x axis and trace on it



- Using `Locus` tool, draw the locus of the point C and its reflection

  What happens to the path if C is

  - Near to A
  - Near to B
  - At the mid point of AB