Lab 13

Limits

Aim :

- To geometrically explore the concept of the limit of a function at a point.

Concepts:

- Value of a function at a point
- Graph of a function

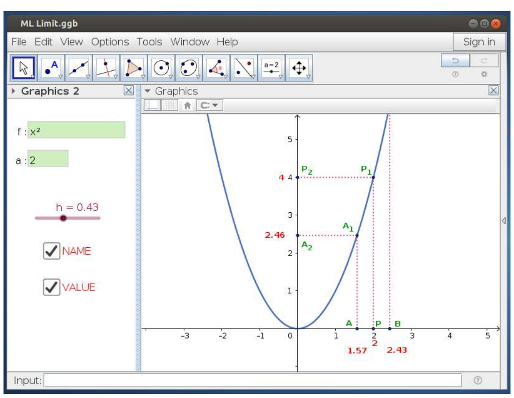
Discussion:

We geometrically explore the concept of limit at a point. We discuss the existence and different cases of non existence of limit, the nature of the graph at a point where limit exists/does not exist, the concept of left limit and right limit etc..
We geometrically interpret some standard limits also.

Activity 13.1 Geometrical Interpretation of Limits

Procedure:

Use the applet 13.1-Limits



About the applet :

- You can see the graph of a function $f(x)$, 3 points A, B, P on the x axis, corresponding points A_1, B_1, P_1 on the graph and A_2, B_2, P_2 on the y axis
- 'NAME' Check Box: By clicking on it you can show/hide the names of the points
- VALUE Check Box: By clicking on it you can show/hide the x coordinates of the points A, B and P and the y coordinates of the points A_2, B_2 and P_2
- Slider **h**: Using this we can bring the points A and B towards P
- Input box **a**: To change the position of P

- Input box **f**: To change the function

<u>Initial settings</u>

- $f(x) = x^2$
- $\mathbf{a} = 2$
- $\mathbf{h} = 1$
- Show the names of the points

Gradually change the value of **h** from 1 to 0. Observe the movements of the points.What happens to A_2 and B_2 as A and B approaches P ?

Show the values of the points. Set **h**=1 and gradually bring it to 0. Observe the values.What happens to the x coordinates of the points A and B? What happens to the y coordinates of A_2 and B_2?

We can record the value of points to spreadsheet as follows. Open spreadsheet view $\longrightarrow$ Spreadsheet.
Right click on A_1 $\longrightarrow$ record to spreadsheet $\longrightarrow$ tick Row limit(10) $\longrightarrow$ Close. Similarly record the point B_1 to spreadsheet.

We can observe that as the x coordinates of A and B approach to 2, the y coordinates of A_2 and B_2 approach 4.
If we call the x coordinates of A and B as x, then the y coordinates of A_2 and B_2 are $f(x)$
So we observe that as $x \to 2$, $f(x) \to 4$
ie, the limit of $f(x)$ at $x = 2$ is 4

What happens to the points A, B, A_2 and B_2 when $\mathbf{h} = 0$

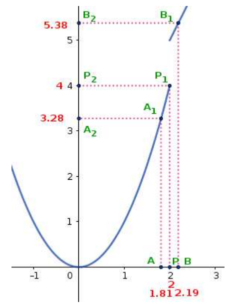
Activity 13.2 Limit of Rational Functions

Procedure:

- In the above applet, change the function to $f(x) = \dfrac{x^2 - 4}{x - 2}$
- Move the slider **h** from 1 to 0
- What is the limit of this function at $x = 2$
- What happens to the points A_2 and B_2 when $\mathbf{h} = 0$ (Refer Activity 3.2)

Activity 13.3 Limit of Piecewise Functions

Procedure:



Using the above applet, discuss the limit of the following functions

1. $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ 2x+1 & \text{if } x > 2 \end{cases}$ at $x = 2$

   (Input `If[x<=2,x^2,2x+1]`)

   What happens to $f(x)$ as x approaches to 2 from left and right?

2. Change $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ 2x & \text{if } x > 2 \end{cases}$ and discuss the limit at $x = 2$

Discuss the existence of limit for the following functions

1. $f(x) = \dfrac{1}{x}$, at $x = 0$

2. $f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$

3. $f(x) = \begin{cases} x-2 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x+2 & \text{if } x > 0 \end{cases}$ (Input `If[x<0,x-2,x>0,x+2,0]` Or `If[x<0,x-2,x==0,0,x+2]`)

4. $f(x) = \begin{cases} x^2-1 & \text{if } x \le 1 \\ -x^2-1 & \text{if } x > 1 \end{cases}$

Activity 13.4 Limit of Trigonometric Functions

Procedure:

- Plot the graphs of $\sin x$ and x in the same graphics view. Zoom it at the origin. What do you see? What inference do you get from this?

- Using the applet used in the previous activity, discuss the limit of $\dfrac{\sin x}{x}$ at $x = 0$

Plot the graphs of x^2, $\sin(x^2)$, $\sin^2 x$, $\tan(x^2)$ and $\tan^2 x$ on the same graphics view, Zoom it at the origin and what do you see? What inference you get from this?

Activity 13.5 Limit of Exponential and Logarithmic Functions

Procedure:

- Input **a**=0.We get a slider in the Algebra view. Show it in the graphics view by clicking on it
- Draw the graph of the function $f(x) = e^x - a$
- Input `y = x` to get the line
- Using `Reflect about line` tool, click on the graph and on the line, we get the reflection of the graph of e^x on the line $y = x$, which represents the graph of $log_e(x)$

- Using the slider **a**, move the graph of f downwards until the line becomes tangent to the curve
- What happens to the reflection ?
- What are the definitions of the functions represented by the curves?
- Zoom it at the origin until the three curves seem to be one

- What do you infer from this?
- Write down some limits using this inference

Additional Activities

Activity 13.A Some more problems

Procedure:

With the help of the applet, discuss the limit of the following functions

1. $f(x) = \sin\left(\frac{1}{x}\right)$ at $x = 0$
2. $f(x) = x\sin\left(\frac{1}{x}\right)$ at $x = 0$
   Draw the lines $y = x$ and $y = -x$. Why does the graph of $x\sin\left(\frac{1}{x}\right)$ lie between these lines?
3. $f(x) = x^2\sin\left(\frac{1}{x}\right)$ at $x = 0$
   Draw the curves $y = x^2$ and $y = -x^2$. Discuss the existence of the limit of $x^2\sin\left(\frac{1}{x}\right)$ at 0 with the help of these graphs
4. $f(x) = \sqrt{x}\sin\left(\frac{1}{x}\right)$ at $x = 0$
   Draw the parabola $y^2 = x$. Discuss the existence of limit of the above function with the help of this curve