Lab 10

Circle and Parabola

Aim :

- To explore different methods of drawing Circles and Parabolas using GeoGebra tools, commands and equations

Concepts:

- Definitions of Circle and Parabola
- Equations of Circle and Parabola

Discussion :
Different tools and commands are available with GeoGebra for drawing Conic Sections according to the given data. To draw a given curve using a specific tool or command, the curve may be interpreted in a different form other than the given one. This needs a thorough knowledge about the curve.
We visualise some problems in Circles and Parabolas.We also discuss the change in the curve according to the constant in the standard equation of the Parabola.

Activity 10.1 Circle

We can draw a circle in different ways

- Centre and a point on the circle are given
  - Using `Circle with Centre through Point` tool, click on the centre and then on the point
  - Give input in the following manner. `Circle(centre point,point)`
- Centre and radius are given
  - Using `Circle with centre and radius` tool, click on the centre and enter radius.
  - Give input in the following manner. `Circle(Point,Radius)`
- Three points on the circle are given
  - Using `Circle through 3 Points` tool,click on the points
  - Give input in the following manner. `Circle(Point,Point,Point)`
- Input the equation of the circle
  For eg: `(x-1)^2 + (y-2)^2 = 4`
  `x^2 + y^2 + 2x = 5`

- Find the centre and radius of the following circles. Draw the circle and verify your answer.You can do it in any of the following ways.

  ◦ Draw the circle by direct input of the equation, find its centre and radius. Compare with your answer.

  ◦ Draw the circle using `Circle with Centre and Radius` tool, using the center and radius that you found. Compare its equation with the given equation.

  > We can find the centre of a circle **c** using the input command **Center(c)** and its radius by the input command **Radius(c)**

  1. $(x+5)^2+(y-3)^2=36$
  2. $x^2+y^2-4x-8y-45=0$
  3. $2x^2+2y^2-8=0$

- Find the equations of the following circles. Input the equations obtained,draw the circles and verify your answer

  1. Centre (-2,3) and radius 4
  2. Centre (2,2) and passing through the point(4,5)

- Construct the following circles without using `Circle through 3 Points` tool or input commands

  1. Passing through the points (2,3) and (-1,1) and with centre on the line $x-3y-11=0$
  2. Passing through the points (1,2) (5,4) and (3,6).
  3. If three points are given, how can we find the equation of the circle passing through them (without using GeoGebra. Hint: above problem)

Activity 10.2 Parabola 1

Procedure:

- Using the tool `Parabola` select a line and a point to get a parabola with the line as directrix and the point as focus
- We can also draw a parabola using input command,for example,the input command `Parabola[(2,0),x+2=0]` gives the parabola with focus $(2,0)$ and directrix $x+2=0$
- If A represents a point and f represents a line then the command `Parabola[A,f]` gives the parabola with focus A and directrix f

Draw a line and plot a point. Draw the corresponding parabola. Change the distance between the line and the point,observe the corresponding change in the shape of the parabola

Find the focus and directrix of the following parabolas. Using `Parabola` tool, draw them. Check whether the equation of the parabola that you have drawn is same as the given equation.

i) $y^2=8x$ ii) $x^2=4y$

iii) $x^2=-4y$ iv) $y^2=-10x$

Activity 10.3 Parabola 2

Procedure:

- By giving the equation of the parabola directly in the input bar, we can draw the parabola
- Create a slider **a** and give the input $y^2 = 4ax$ and $x^2 = 4ay$
- Change the value of **a** and observe the shape of the parabolas
- Find the focus and length of latus rectum of the following parabolas. Verify your answer geometrically as follows;
  Input the equation and draw the parabola. Using focus command (`Focus`[name of parabola]), we can find its focus. Draw the line through the focus and perpendicular to the axis of the parabola. Mark the points of intersection of this line with the parabola and join them with a line segment. Hide the line and measure the length of the latus rectum.

  i) $y^2 = 6x$ ii) $x^2 = -8y$

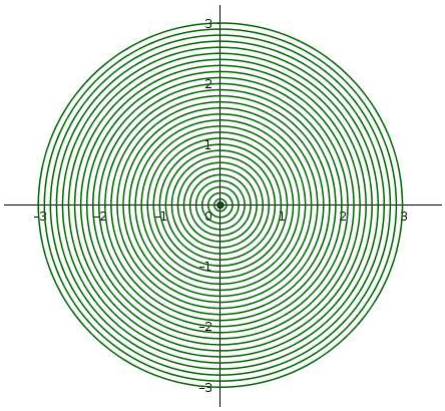
  iii) $x^2 = 10y$ iv) $y^2 = -4x$

Additional Activities

Activity 10.A Family of Circles

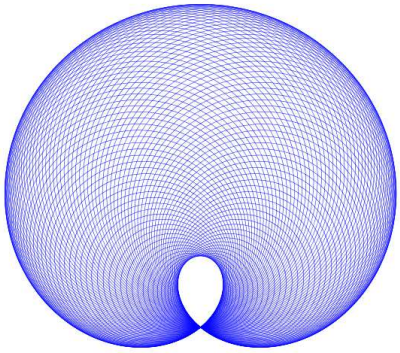
Procedure:

- Input the following sequence command and draw the pattern
  `Sequence[x^2+y^2=r^2,r,0,3,.1]` which gives a family of circles with centre at the origin and radius varying from 0 to 3 by an increment 0.1



- Imagine the pattern obtained by the following commands and then draw them
  1. `Sequence[(x-r)^2 +y^2=r^2,r,0,3,.1]`
  2. `Sequence[(x^2 +(y+r)^2 = r^2,r,0,3,.1]`
- Draw the following family of circles using sequence command

1. Centres lie on the line $y = x$, and pass through the origin
2. Family of 100 circles of radius 3, whose centres lie at equal distance on the circle of radius 3 centred at the origin.(Hint: Use the concept of trigonometry-coordinates of points on a circle)
3. Family of 100 circles, whose centers lie at equidistant points on the circle of radius 3 centred at the origin and passing through the point $(3, 0)$
4. Do the above activity with a slider n to change the number of circles and another slider a so that all the circles pass through $(a, 0)$ or $(0, a)$ instead of $(3, 0)$. Change the value of a and observe the change in the pattern.



Activity 10.B Parabola with Given Focus and Directrix

Aim:

To create a parabola whose focus and directrix are given.

Procedure:

- Plot a point A and draw a line BC using `line through 2 points` tool.
- Take a point D on the line using `point on Object` tool
- Draw the line segment AD and its perpendicular bisector
- Draw the line perpendicular to the first line and passing through D
- Take the intersection of the above line with the perpendicular bisector and trace on this point
- Animate the point D and observe the path of the above point