Lab 5

Trigonometric Identities

Aim:

- To construct applets to establish the relation among various trigonometric functions like $\sin\left(\frac{n\pi}{2} \pm x\right)$ with $\sin x$, $\cos x$ *etc.*
- To confirm the findings geometrically

Concepts:

- Trigonometric functions are defined by means of coordinates of a point on the unit circle centred at origin
- Concept of congruent triangles

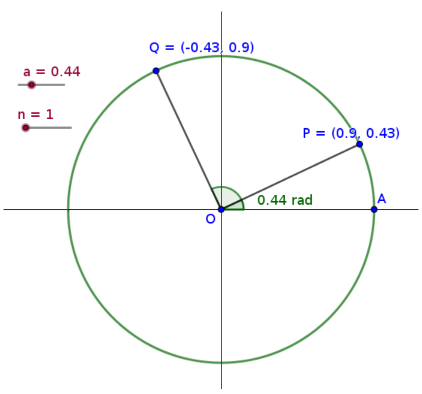
Discussion :

Similar to the applet constructed in Lab 4 we construct two points on the unit circle centred at the origin. One point P has a rotation of x radians from $(1, 0)$ and the second point Q has a rotation of $\frac{n\pi}{2} + x$. Comparing the coordinates of P and Q we establish the relation between $\sin(\frac{n\pi}{2} + x)$, $\cos(\frac{n\pi}{2} + x)$, $\tan(\frac{n\pi}{2} + x)$ with $\sin x$, $\cos x$, $\tan x$ *etc.* for different integral values of **n**. We repeat the activity by changing the rotation of Q as $\frac{n\pi}{2} - x$.

Activity 5.1 $\sin(\frac{n\pi}{2} + x)$

Procedure:

- As in activity Activity 4.1, draw a unit circle centred at the origin.(With the same initial settings as in Activity 4.1)
- Take a point A$(1, 0)$. Create a slider **a** with min=0 and plot the point P on the circle such that $\angle$ AOP $=$ a
- Create an integer slider n with minimum 1 and maximum 8
- Plot another point Q on the circle such that $\angle$ AOQ $= \frac{n\pi}{2} + a$ (Using *angle with given size* tool; click on A and O in that order and give `n*pi/2+a` in the box provided for entering the angle)



For different values of **a**, observe the coordinates of P and Q and complete the following table.

Sl.No	a	$\sin a$	$\cos a$	$\sin(\frac{\pi}{2}+a)$	$\cos(\frac{\pi}{2}+a)$	$\sin(\pi+a)$	$\cos(\pi+a)$	$\sin(\frac{3\pi}{2}+a)$	$\cos(\frac{3\pi}{2}+a)$

- Establish the relation between $\sin(\frac{n\pi}{2}+x)$, $\cos(\frac{n\pi}{2}+x)$, $\sin x$ and $\cos x$ for different values of n

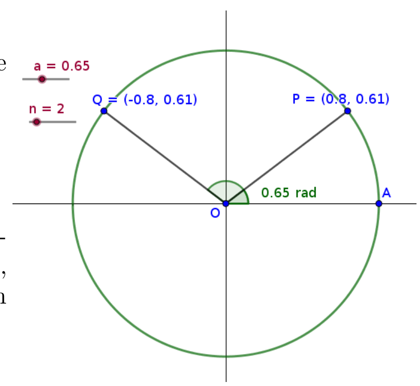
- Do you observe any peculiarity for even and odd values of n ?
- What is the relation between $\tan(\frac{n\pi}{2}+x)$ and $\tan x$?
- Save this file as Activity 5.1

Activity 5.2 $\sin(\frac{n\pi}{2}-x)$

Procedure:

- As in Activity 5.1, draw a unit circle centred at the origin. Plot points A and P.
- While plotting Q, give angle as `n*pi/2-a`
- Show the coordinates of P and Q

  (Or you can edit the applet of Activity 5.1 as follows- Open the applet, using *save as* option from *file* menu, save the applet as Activity 5.2. Then double click on Q and change the angle as `n*pi/2-a`)



For different values of a, observe the coordinates of P and Q and complete the following table.

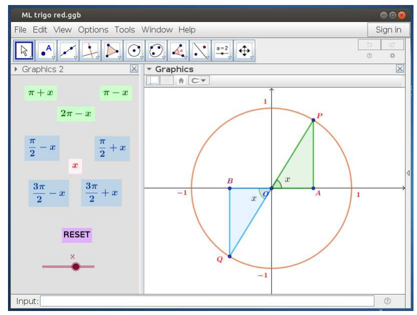
Sl.No	a	$\sin a$	$\cos a$	$\sin(\frac{\pi}{2}-a)$	$\cos(\frac{\pi}{2}-a)$	$\sin(\pi-a)$	$\cos(\pi-a)$	$\sin(\frac{3\pi}{2}-a)$	$\cos(\frac{3\pi}{2}-a)$

- Establish the relation between $\sin(\frac{n\pi}{2} - x)$, $\cos(\frac{n\pi}{2} - x)$, $\sin x$ and $\cos x$ for different values of n
- Do you observe any peculiarity for even and odd values of n?
- What is the relation between $\tan(\frac{n\pi}{2} - x)$ and $\tan x$
- Save this file as Activity 5.2

Activity 5.3 Geometrical Proof

Procedure:

Use applet ML 5.3



About the applet

Using the slider **x** we can rotate the point P by an angle x along the unit circle.
Using the buttons, we can select the rotation of the point Q as $\pi + x$, $\pi - x$, $\frac{\pi}{2} + x$ etc

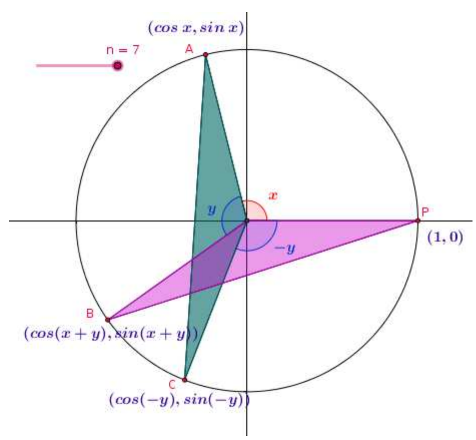
With the help of this applet, try to give a geometrical proof of the result that we found on activities Activity 5.1 to Activity 5.2 (Hint:Use properties of congruent trianges)

Additional Activities

Activity 5.A $\cos(x + y)$

Discussion :

Using an applet we prove the result
$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$



Procedure :

Use Applet ML 5.A

About the applet

By moving the slider **n** we can plot 3 points whose coordinates are $(\cos(x), \sin(x))$, $(\cos(x + y), \sin(x + y))$ and $(\cos(-y), \sin(-y))$.

- Using congruence of triangles prove the result

$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

Activity 5.B Simple Harmonic Motion

Discussion :

Simple Harmonic Motion (SHM) is a periodic function for which the displacement is a sinusoidal function of time (displacement can be expressed as a function of *sine* or *cosine*).The equation

$$x(t) = A\cos(\omega t + \phi)$$

represents an SHM with amplitude A , angular frequency ω and initial phase ϕ. We can identify an SHM as a projection of a uniforn circular motion on a straight line.

Procedure :

- Make the initial settings as in Activity 4.1
- Create sliders **A**, ω with Min = 0 and Max = 10, ϕ and t with Min = 0, Max = 50, Increment 0.01.
- Draw a circle of radius A centred at the origin and plot the point O(0,0).
- Plot the point $B(A, 0)$
- Plot the point B' such that $\angle\, BOB' = \phi$ radian
- Plot the point B'' such that $\angle\, B'OB'' = \omega\mathbf{t}$ radian. (Now the rotation of B'' from B is $\omega t + \phi$.
- Draw OB''
- Draw a perpendicular from B'' to x axis and plot the point of intersection C. Hide the perpendicular line and draw $B''C$ with a line segment.
- Write x coordinates C in terms of A, ω, ϕ and t.
- What is the x coordinate of C when $t = 0$?
- If x represents the displacement of the point C from the origin at time t, write its equation of motion.