Lab 30

Maxima and Minima

Aim

- To explore the concept of local maxima and local minima
- To find points of local maxima, local minima, absolute maximum and absolute minimum geometrically

Concepts

- Local maxima and local minima
- Absolute maximum and absolute minimum

Discussion

We discuss the method of finding points of local maxima, local minima, local maximum value, local minimum value, absolute maximum and absolute minimum with the help of graph of a function and graph of its derivative.

Activity 30.1 Local Maxima and Local Minima

If a function has a local maximum or a local minimum at a point and if the function is differentiable at that point, then the derivative at that point is zero. So it is enough to search for points of local maxima or local minima at the points where either the derivative equal to zero or the function is not differentiable.

Find the points of local maxima and local minima of the function $f(x) = 8x^3 - 33x^2 + 36x - 8$ Also find local maximum and local minimum values.

Procedure

First Derivative test

- Draw the graph of $f(x)$ and $f'(x)$.
- Find the points A and B at which the graph of $f'(x)$ cuts the x axis. Show the coordinates of A and B.
- From the graph of $f(x)$ it is easy to identify whether the x coordinates of A and B are points of local maxima or local minima.
- The input commands `f(x(A))` and `f(x(B))` gives the corresponding values.

Second Derivative test

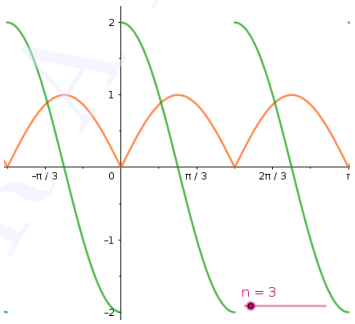
- Draw the graph of $f(x)$ and $f'(x)$ and $f''(x)$

- Find the points A and B at which the graph of $f'(x)$ cuts the x axis. Show the coordinates of A and B.
- Find the sign of $f''(x(A))$ and $f''(x(B))$. [This is possible by observing the graph of $f''(x)$ or by the input commands `f''(x(A))` and `f''(x(B))`].

Find the points of local maxima and local minima of the function $|\sin(2x)|$
Also find the local maximum and local minimum values

Procedure

- Create an integer slider **n** and mark the distance on x axis as $\frac{\pi}{n}$. (Right click on the Graphics View → Graphics → x Axis → Distance → type `pi/n`).
- Draw the graphs of the functions $f(x) = |\sin(2x)|$ and $f'(x)$.
- Identify the points at which either $f(x)$ is not differentiable or $f'(x) = 0$.
- Find points of local maxima and local minima.



Find the points of local maxima and local minima of the following functions
Also find the local maximum and local minimum values

1. $9x^2 + 12x + 2$
2. $x^3 + 1$
3. $\sin x - \cos x, 0 < x < \pi$
4. $\dfrac{1}{x^2 + 2}$
5. $\sin|x|$

Activity 30.2 Absolute Maximum and Absolute Minimum

Find absolute maximum value and absolute minimum value of the following functions in the given intervals.

1. $f(x) = 2x^3 - 15x^2 + 36x + 1$ in [1,5]

   Procedure

   - Using the input command `Derivative(2x^3-15x^2+36x+1)` draw the graph of $f'(x)$.
   - Find the points A and B at which above graph meets x axis.
   - Find $f(1), f(5), f(x(A))$ and $f(x(B))$. Highest among them is the absolute maximum value and least is the absolute minimum value.
2. $3x^4 - 8x^3 + 12x^2 - 48x + 25$ in [0,3]
3. $\sin x + \cos x,\ x \in [0, \pi]$