Lab 9

Conic Sections

Aim :

- To show conics as the section of a cone as well as the locus of a point

Concepts:

- Cone and its section by a plane
- Locus of a point

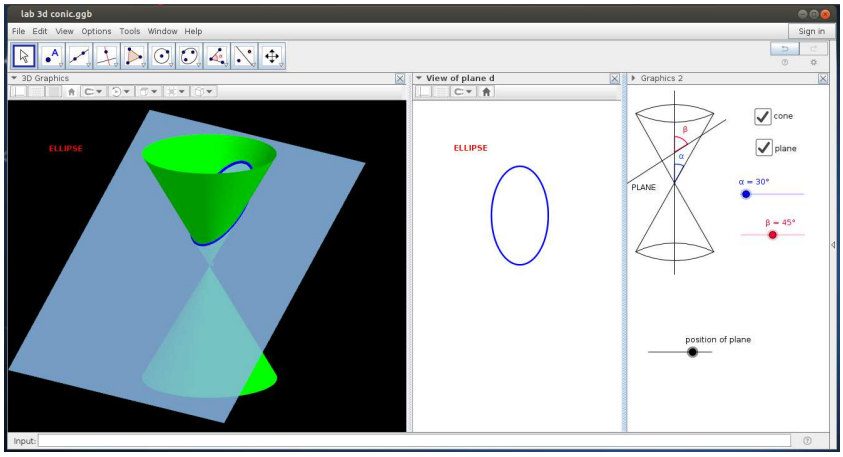
Discussion :

Conic sections are curves obtained by the intersection a double cone by a plane. The angle at which the plane cuts the cone determines the curve. The semi vertical angle of the cone, the position at which the plane cuts the cone etc.. will determine the shape of the curve.
We treat the curves as the locus of a point moving on a plane subjected to certain constraints.

Activity 9.1 Cutting of a Cone by a Plane

Using the applet ML 9.1

About the applet:



In this applet we can see 3 open windows. **Graphics 2** , **3D Graphics** and the third one is **View of plane d**

Graphics 2

- There are three sliders and two check boxes here.
- Using slider α you can change the semi vertical angle of the cone.
- Using slider β you can tilt the plane
- Using slider "Position of the plane" you can change the position of the plane.
- Using the check boxes we can show or hide the cone and the plane.

3D graphics

You can see the 3D view here, using ***Rotate 3D graphics view*** tool, you can rotate the entire view to see it from a convenient angle.

View of plane d

You can see the curve, obtained by intersecting the cone with the plane.

Procedure:

Change the value of β for a fixed α. Observe the curves for different values of β.

α	**Curve**	β
25°	Circle	90°
	Parabola	25°
	Ellipse	$25° < \beta < 90°$
	Hyperbola	
30°	Circle	
	Parabola	
	Ellipse	
	Hyperbola	
45°	Circle	
	Parabola	
	Ellipse	
	Hyperbola	
50°	Circle	
	Parabola	
	Ellipse	
	Hyperbola	

For what values of β do we get the curves - circle, ellipse, parabola and hyperbola?

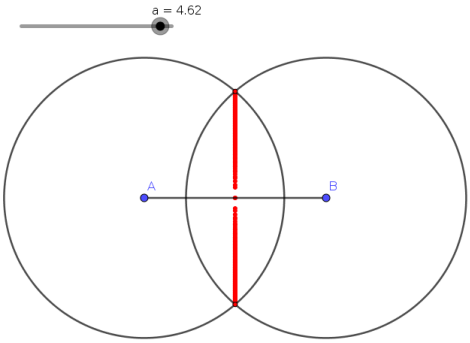
Change the position of the plane and observe the corresponding change in the shape of the curve

Change α and observe corresponding change in the curve.

Activity 9.2 Locus of a point moving equidistant from given points

Procedure:

- Plot two points A and B and join them (Using `line segment` tool)
- Create a number slider **a** with min=0 and max=10
- Draw circles of radius **a** centred at A and B
- Plot the points of intersection of the circles and give their trace.
- Give animation to the slider **a**
- Observe the path of the moving point. Describe the path.
- Save this file as Activity 9.2



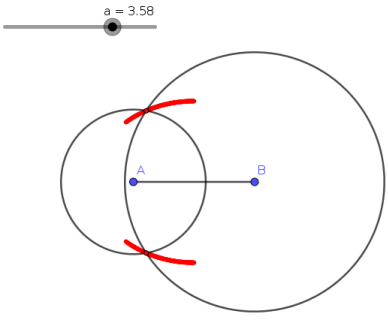
Adjust the increment of the slider as 0.01 or less to get a continuous path. If we use Shift key together with the arrow keys to move the slider, it will decrease the speed of the slider by one tenth. If we use Ctrl key, speed will be increased 10 times

Activity 9.3 Locus of a point the sum of whose distances from two given points is a constant

Procedure:

- As in Activity 9.2, create a slider **a**
- Plot two points A and B and join them.
- Draw a circle of radius **a** centred at A and another circle of radius $10 - \mathbf{a}$ centred at B
- Plot the points of intersection of the circles and give their trace.
- Give animation to the slider **a**
- Observe the path of the moving points. Identify the curve traced.
  (We can show the path of the moving point using `Locus` tool. For this, take the `Locus` tool, click on one of the points of intersection and on the slider. Similarly click on the other point of intersection and on the slider)

We can create this applet by editing the radius of the circle centred at B of the previous Activity 9.2



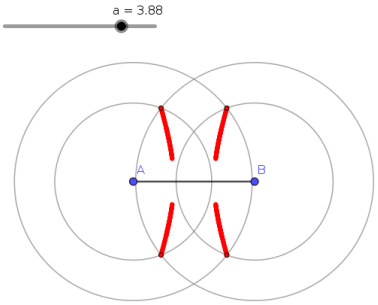
- Change the distance between A and B and observe the change in the shape of the curve
- What happens to the curve when B approaches nearer and nearer to A?

- What is the maximum possible distance between A and B to get a path?

Activity 9.4 Locus of a point the difference of whose distances from two given points is a constant

Procedure:

- As in Activity 9.2, Create a slider **a**
- Plot two points A and B
- Draw a circle of radius **a** centred at A and another circle of radius **a** + 4 centred at B
- Plot the points of intersection of the circles and give their trace.
- Give animation to the slider **a**
- Draw another set of circles of radius **a** centred at B and of radius **a** + 4 centred at A



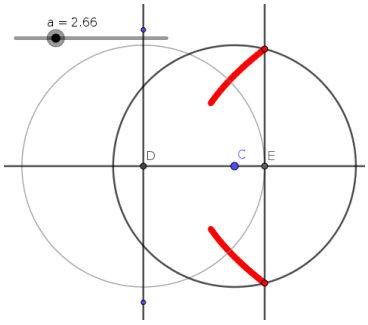
- Plot the points of intersection of these circles and give their trace.
- Give animation to the slider **a**

- Observe the path of the moving points.Identify the curve traced. (Here also we can use the Locus tool as in Activity 9.3)

Activity 9.5 Locus of point equidistant from a point and a fixed line

Procedure:

- Draw a line and plot a point C outside the line.
- Draw perpendicular to the line through C.
- Plot the point of intersection D of the line and its perpendicular.
- Create a slider **a** with min = 0, max = 15 and increment 0.01
- Draw a circle of radius **a** centred at D and plot its point of intersection E with the perpendicular line.
- Draw a line through E and parallel to the first line.



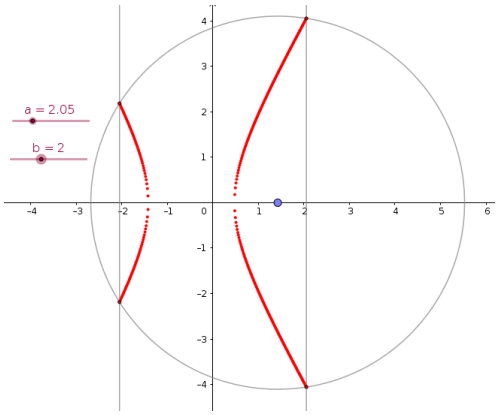
- Draw a circle of radius **a** centred at C and plot its points of intersection with the last line drawn. Trace on these points.
- Give animation to the slider **a**
- Observe the path of the moving points. Identify the curve traced.
- Draw the path using `Locus` tool
- Change the distance between the point and the first line and observe the change in the shape of the curve

Additional Activities

Activity 9.A Focus - Diretrix Definition

Discussion :

We discuss Parabola, Ellipse and Hyperbola as the locus of a point moving on a plane, keeping a specific ratio of distance from a fixed line and a fixed point. **Procedure :**



- Create two sliders **a**, with min = 0 , max = 10 and increment 0.01 and **b** with min = 0 , max = 5 and increment 0.01
- Draw the line $x = 0$ and plot a point A on the positive side of the x axis.
- Draw the lines $x = a$ and $x = -a$
- Draw a circle of radius ab centred at A.
- Plot the point of intersection of the circle with the lines $x = a$ and $x = -a$ (If necessary, you can change the values of the sliders, so that the circle meets the lines.)
- Trace the points of intersection
- Hide the axes, Set **b**=1 and animate slider **a**
- Observe the path of the points. Can you identify the curve ?
- Using `Locus` tool, draw the path of the points.
- Change the value of **b** and observe the path. Can you identify the curves for different values of **b** ?
- Try to define a parabola,an ellipse and a hyperbola in terms of distances from a fixed line and a fixed point.

Activity 9.B Apollonius Circles

Discussion :

We discuss the locus of a point moving on a plane, keeping a specific ratio of distance from two fixed points.

Procedure :

- Create two sliders **a** and **r** with Min = 0 and increment 0.01. Create an input box for **a**.
- Plot two points A and B and join them
- Draw a circle of radius **r** centered at A and another circle of radius **ar** centered at B
- Plot the point of intersections of the circles and trace them. Find the locus of the points using `Locus` tool.
- Observe the path for different values of **a**

-  Can you connect this with internal and external division of a line ?