Lab 3

Domain and Range

Aim:

- To find the domain and range of functions from their graphs

Concepts:

- Graph of a function, Domain and Range, Shifting of the graph

Discussion :
A graph, being the pictorial representation of a function, gives much information about the properties of that function.
In this lab, we discuss the domain and range of functions with the help of their graphs.We also discuss rational functions, functions with restricted domain and piecewise functions.
In each problem we discuss here, first try to imagine the graph, domain and range of the function and then draw it using GeoGebra

Activity 3.1 Domain and Range of Functions from their Graphs

Procedure:

- Imagine the graphs of the following functions and write their domain and range. (You may use the idea of shifting and reflection of graph as in Lab 2)
- Check your answer by drawing the graphs using GeoGebra

i)x^2+2 ii)x^2-3 iii)$3-|x|$ iv)$(x+2)^2-1$

v)$x^2-6x+12$ vi)$|x-2|$ vii)$|x-2|+3$ viii)$2x^2-8x+5$

ix)$\frac{1}{2}[x]$ x)$[\frac{x}{2}]$ xi) $x-[x]$ xii)$3-x^2$

xiii)$\sqrt{x-2}$ xiv)$\sqrt{4-x}$ xv)$\dfrac{1}{x-2}$ xvi)$\sqrt{x^2-4}$

xvii)$\sqrt{9-x^2}$ xviii)$\dfrac{1}{x^2-9}$ xix)$\dfrac{x^2}{x^2+1}$

Activity 3.2 Rational Functions

Procedure:

- Draw the graph of the function $\frac{x^2-4}{x-2}$. What is the domain of this function?
- On Graphics 2, draw the graph of the function $g(x) = x + 2$

Observe the graphs of f and g. Are they one and the same? Does it mean that $f = g$

- Create a slider **a**
- In Graphics view,plot the point $A(\mathbf{a}, f(\mathbf{a}))$ and in Graphics 2 plot $B(\mathbf{a}, g(\mathbf{a}))$
- Change the value of **a**. We can see that the points move along the graphs.

What happens to the points when **a** reaches 2? What do we infer? comment on it.

To open Graphics2 go to View → Graphics 2

Activity 3.3 Piecewise Functions

Procedure:

- We can draw the graphs of functions with restricted domains using `if` command.
  For example
  `If[-1 < = x < = 2,x^2]` gives the function $f(x) = x^2$ in $[-1, 2]$

  (We can also use the `function` commad.
  `Function[x^2,-1,2]`) gives the same fuction.

  `If[x < = 2,x^2,x > 2,2x]` gives the function $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 2x & \text{if } x > 2 \end{cases}$

The if command in GeoGebra has the following

If[c,f]

gives the function f only for the values of x satisfying the condition c.

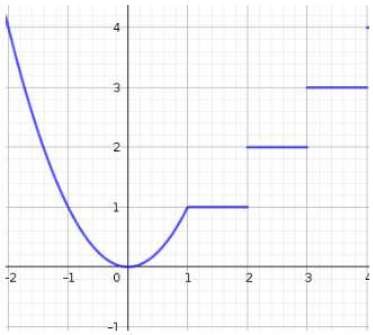
If[c,f,g]

gives the function f for the values of x satisfying the condition c and g for all other values of x.

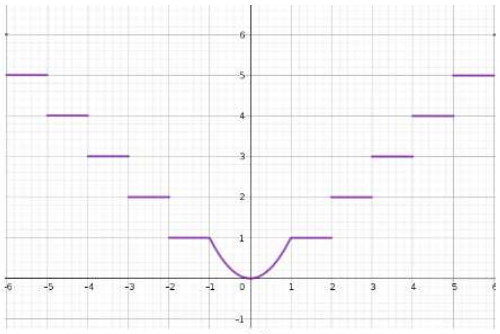
Observe the graphs of the following functions and find their domain and range

1. $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 2x+1 & \text{if } x > 2 \end{cases}$
2. $f(x) = \begin{cases} x^3 & \text{if } x \leq 0 \\ x^2+1 & \text{if } x > 0 \end{cases}$
3. $f(x) = \begin{cases} x^2+2 & \text{if } x < 0 \\ -x^2-2 & \text{if } x > 0 \end{cases}$
4. $f(x) = x^2$ in $[-2, 1]$
5. $f(x) = x^3$ in $[-2, 2]$
6. $f(x) = \frac{1}{x}$ in $[-1, 2]$

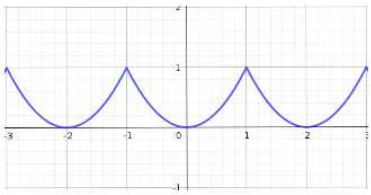
Identify the functions and try to draw the graphs given in the figures.



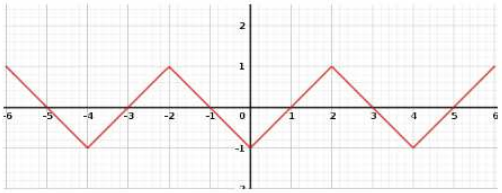
(a)



(b)



(c)



(d)

Additional Activities

Activity 3.A Leaking Tank

A tank holds 50 gallons of water. There is a leak at the bottom of the tank through which water is draining out.

If it takes 20 minutes to completely drain out the water, the volume of water in gallons remaining in the tank at any time t (minutes) is given by Toricelli's law as

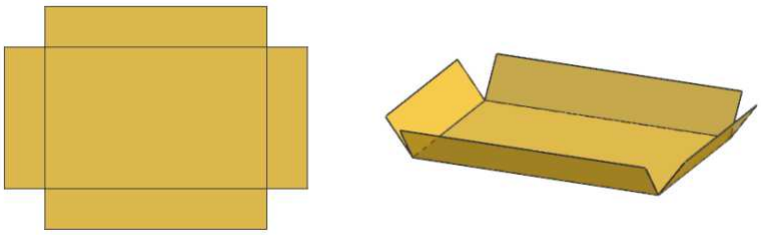
$$V(t) = 50\left(1 - \frac{t}{20}\right)^2 \qquad 0 \le t \le 20$$

Find the volume of water in the tank at times 0,5,10,15 and 20 minutes

What is the domain and range of this function

Activity 3.B The Volume of a Box

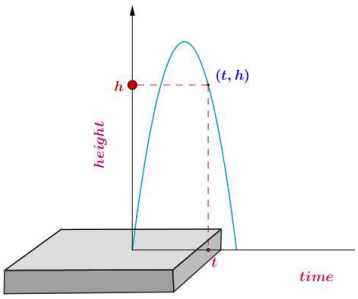
An open box is to be made from a cardboard of size 5 m by 3 m by cutting out squares of equal size from all the four corners of the cardboard and folding up the sides as shown in the figure below.

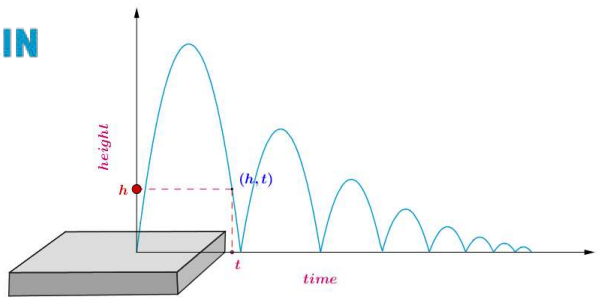


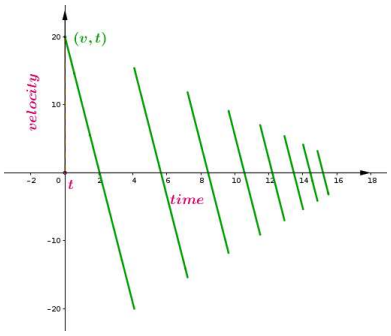
- If x represents the length of the square cut away from the cardboard, write the volume of the box as a function of x
- Plot the graph of this function
- What is the domain of this function ?
- What is the range of this function ?
- Find the volume of the box of maximum volume that can be so made. Also find how big a square may be cut away from the cardboard to get the box of maximum volume.

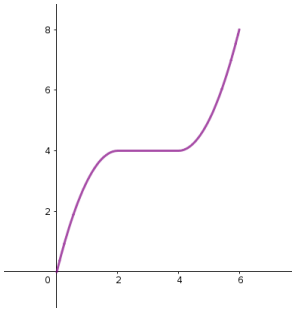
Activity 3.C Some Familiar Graphs from Physics

Observe the following graphs, describe the physical situations involved in the depiction of the graphs. Try to draw the graphs.









Activity 3.D Domain and Range of Relations

Procedure:

Guess the domain and range of the following relations.

1. $R_1 = \{(x, y) : x, y \in R,\ x^2 + y^2 = 4\}$ (Input: `x^2+y^2=4`)
2. $R_2 = \{(x, y) : x, y \in R,\ x^2 + y^2 \leq 4\}$ (Input: `x^2+y^2<=4`)
3. $R_3 = \{(x, y) : x, y \in R,\ x^2 + y^2 \geq 4\}$ (Input: `x^2+y^2>=4`)
4. $R_4 = \{(x, y) : y \leq x^2 + 2\}$ (Input: `y<=x^2+2`)
5. $R_5 = \{(x, y) : y \geq x^2 + 2\}$ (Input: `y>=x^2+2`)

Draw the regions represented by these relations on R. Find their domain and range and verify your answer