Lab 7

Shifting and scaling of graphs of trigonometric functions

Aim :

- To study the effect of the constants **a**, **b** and **c** of the trigonometric function $asin(bx + c)$ on the graph of the function.

- To study the periodicity of trigonometric functions.

Concepts:

- Graphs of trigonometric functions

- Periodicity of trigonometric functions

Discussion :

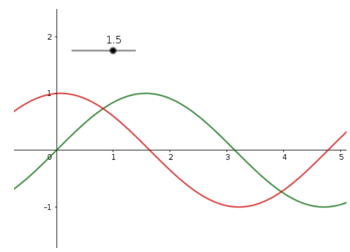
We have already seen the translation and reflection of the graph of the function $f(x)$ according to some changes in the definition of the function. Here we discuss the scaling of the graph of trigonometric functions along with translation and reflection. These concepts will be useful in the study of waves in Physics.

Activity 7.1 Shifting

Procedure:

> A function f for which there exist a real number a such that $f(x + a) = f(x)$, for all x is called a periodic function. The smallest positive number satisfying this identity is called the period of f.

- Change the distance on the x axis in terms of π.
- Draw the graph of $f(x) = \sin x$.
- Create a number slider **a** with min=-10 and max=10, with increment 0.01
- Draw the graph of $\sin(x+a)$ (By giving input $f(x+a)$)
- Find the minimum positive value of **a**, for which $\sin(x + a) = \sin x$
(This value of **a** is called period of $\sin x$)
- Observe the change in the graph according to **a**. (Refer Activity 2.2)
- Set the value of slider **a** at 0



- Draw the graph of $\cos x$
- Compare the graphs of $\sin(x + a)$ and $\cos x$.
- Can you predict the value of **a** for which the graph of $\sin(x + a)$ coincides with that of $\cos x$?
- Draw the graphs of the following functions (Set-1)
1) $\sin(\frac{\pi}{2} - x)$ 2) $\sin(\frac{\pi}{2} + x)$ 3) $\sin(\pi - x)$ 4) $\sin(\pi + x)$
5) $\sin(\frac{3\pi}{2} - x)$ 6) $\sin(\frac{3\pi}{2} + x)$ 7) $\sin(2\pi - x)$ 8) $\sin(2\pi + x)$
- Also draw the graphs of $\sin x$, $-\sin x$, $\cos x$, and $-\cos x$ (Set-2)
- Compare the graphs of functions in Set-1 with the graphs of functions in Set-2 and note down the observations in the following table

Sl. No	Trig.Function	Reduced form
1	$\sin(\frac{\pi}{2} - x)$	
2	$\sin(\frac{\pi}{2} + x)$	
3	$\sin(\pi - x)$	
4	$\sin(\pi + x)$	
5	$\sin(\frac{3\pi}{2} - x)$	
6	$\sin(\frac{3\pi}{2} + x)$	
7	$\sin(2\pi - x)$	
8	$\sin(2\pi + x)$	

> You can hide or show graphs (or any object) by clicking on the bullets in the Algebra window

- Draw the graphs of the following functions (Set-3)
1) $\cos(\frac{\pi}{2} - x)$ 2) $\cos(\frac{\pi}{2} + x)$ 3) $\cos(\pi - x)$ 4) $\cos(\pi + x)$
5) $\cos(\frac{3\pi}{2} - x)$ 6) $\cos(\frac{3\pi}{2} + x)$ 7) $\cos(2\pi - x)$ 8) $\cos(2\pi + x)$

- Compare the graphs of functions in Set-3 with the graphs of functions in Set-2 and note down the observations in the following table

Sl. No	Trig.Function	Reduced form
1	$\cos(\frac{\pi}{2} - x)$	
2	$\cos(\frac{\pi}{2} + x)$	
3	$\cos(\pi - x)$	
4	$\cos(\pi + x)$	
5	$\cos(\frac{3\pi}{2} - x)$	
6	$\cos(\frac{3\pi}{2} + x)$	
7	$\cos(2\pi - x)$	
8	$\cos(2\pi + x)$	

Activity 7.2 Scaling

Procedure:

- Draw the graph of $f(x) = \sin x$
- Create a slider **a**

- Draw the graph of $a \sin x$ (By giving input $a * f$)
- Observe the change in the graph according to **a**
- How does the value of **a** affect the domain and range of $a * f$
- Change the function f to $\cos x$
- Repeat the above observations and make notes

Activity 7.3 Periods of Trigonometric Functions

Procedure:

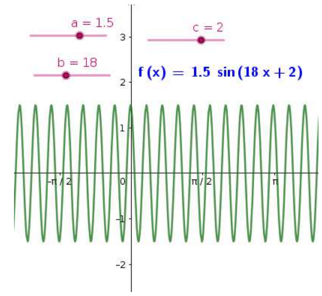
- Draw the graph of $f(x) = \sin x$
- Create an integer slider **n**
- Draw the graph of $\sin(nx)$ (By giving input $f(nx)$)
- Observe the change in the graph according to **n**
- Write the periods of the following functions
  1) $\sin 2x$ 2) $\sin 3x$ 3) $\sin 5x$
- If n is a positive integer, what is the period of $\sin nx$
- Draw the graph of $\sin(\frac{x}{n})$ (Hide all other graphs and input `f(x/n)`)
- Identify the periods of $\sin(\frac{x}{2})$, $\sin(\frac{x}{3})$, $\sin(\frac{x}{4})$ *etc.*
- Change the function f to $\cos x$
- Repeat the above observations and make notes

From the graph of $\sin x$, we can see that its period is 2π. Observe the graph of $\sin x$ in the intervals $[0, 2\pi]$, $[2\pi, 4\pi]$, $[4\pi, 6\pi]$ etc. We realise that the portions of the graph are identical in these intervals. Note that these intervals are of length 2π. We can also observe that the portions of the graph are not identical in any interval with length less than 2π

Activity 7.4 Shifting and Scaling

Procedure:

- Draw the graph of $f(x) = \sin x$
- Create three sliders **a**,**b** and **c**
- Draw the graph of $a\sin(bx + c)$ (By giving input `a*sin(b*x+c)`)
- Observe the changes in the graph according to the changes in the values of the sliders
- Change the function f to $\cos x$



- Repeat the above observations and make notes

Additional Activities

Activity 7.A Waves

Discussion :
This activity is related to the topic "Waves" in Physics. A wave, when viewed mathematically is a function of the displacement x from the origin and time t and is expressed as

$$f(x,t) = a\sin(kx \pm \omega t)$$

where a is the amplitude of the wave, ω is the frequency of the wave and k is a scaling factor. In this activity we explore the geometrical nature of a wave by making use of the wave equation.

Procedure:

- Create four sliders **a**,**k**,ω,**t** all with min=0 and max=10, 500, 10 and 100 respectively.
- Draw the graph of $f(x) = a\sin(kx - \omega t)$
- Animate **t**, to get a propagating wave with amplitude **a**
- Draw the graph of $g(x) = a\sin(kx + \omega t)$
- Compare the waves f and g. What do you observe?
- Input $f + g$ which gives the resultant of the above waves,which is a standing wave.(You may learn more about waves in Physics class.)
- Change the values of **k**, ω and **t** and observe the changes and make notes.

Activity 7.B Music and Math

Discussion :

Being a wave, sound can be represented in terms of trigonometric functions. Using GeoGebra we can produce sound of required frequency and amplitude.

Procedure:

- Create a slider **a** with minimum value 0 and maximum value 5
- Create an integer slider **n** with minimum value 0 and maximum value 1500
- Draw the graph of $f(x) = a\sin(n.2\pi x)$. We get a sine wave of amplitude **a** and frequency **n**. (You may learn more about waves in Physics class.)
- You can play the corresponding sound using `PlaySound` command. For this, create a button with caption Play (Take the Button tool and click on the Graphics View)
  Write the command as `PlaySound[f,0,100]` in the Scripting tab.
  (Which means, on clicking the button, it will play the pure sine tone of frequency **n** corresponding to the function f between 0 to 100).
  To stop the sound, take another button with caption Stop and write the script as `PlaySound[False]`.
  Clicking on the button will stop playing the sound
- Change the value of **a**, and observe the difference in the sound
- Change the value of **n** and observe the change in frequency of the sound. (You can verify the frequency of sound using a pitch analyser which is available in smart phones as mobile app.)

Activity 7.C Harmonic Sound

Discussion :

In this activity we explore the superposition property of waves. This is achieved by adding two or more functions representing waves. We also discuss beats, which is observed when two sound waves of close frequency are superimposed. Similarly we discuss harmonic sounds, sounds whose frequency is an integer multiple of a fundamental frequency.

Procedure:

- Create two sliders **a**,**b** with minimum value 0 and maximum value 5
- Create two integer sliders **m** and **n** with minimum value 1 and maximum value 1500
- Input two functions $f(x) = a\sin(n.2\pi x)$ and $g(x) = b\sin(m.2\pi x)$
- Create the function $h = f + g$
- Create 4 buttons with captions Sound 1, Sound 2, Resultant and Stop with scripts as follows
  Sound 1 $\rightarrow$ `PlaySound[f,0,100]`
  Sound 2 $\rightarrow$ `PlaySound[g,0,100]`
  Resultant $\rightarrow$ `PlaySound[h,0,100]`
  Stop $\rightarrow$ `PlaySound[False]`

- Set **m** = **n** = 250. Then Sound 1, Sound 2 and Resultant will play sounds of the same frequency
- Set **m** = 250, **n** = 251. While playing Resultant, we experience a beat sound of frequency 1 (We can experience the beat sound of frequency 1 with any two frequencies of numerical difference 1)
- Set **m** = 250, **n** = 252. While playing Resultant, we experience a beat sound of frequency 2
- Set **m** = 250, **n** = 500 (or any integer multiple of 250) and play the resultant sound. Pitch analyser will show the frequency as 250. These two sounds are said to be harmonic.

You can repeat this exercise for any number of sounds.

Activity 7.D Blood Pressure

Each time your heart beats, your blood pressure first increases and then reduces as the heart rests between beats. The maximum and minimum blood pressures are called systolic and diastolic pressures respectively. Your blood pressure reading is written as systolic/diastolic. A reading of 120/80 is considered as normal.
A certain person's blood pressure was written as

$$p(t) = 115 + 25\sin(160\pi t)$$

where $p(t)$ is the pressure in mm of mercury at time t measured in minutes.

- Draw the graph of this function
- Find the period of this function
- How many times does his heart beat per minute?
- Is this person's blood pressure normal?