**Think and Answer**

**BP#5179**

Please answer all the questions given below. You are allowed to use figures or diagrams to support your answer. Since these questions test your understanding of the whole subject, please refrain from directly asking for answers on Piazza.

**Section 1 - Simple Pendulum**

Q1) Find the eigenvalues of Simple Pendulum at equilibrium point (0, 0). Is the system stable or unstable at this point? (2)

**Solution:**

In a system of simple pendulum, we have the following equations of motion:

(Where x₁ is the angular position of the bob)

₂ = + (Where x₂ is the angular velocity)

(₂ is the angular acceleration)

So, for the given equation, matrix A will be,

A =

A=

So, the B matrix will be,

B=

B=

So, for equilibrium points (0,0):

=

So, the state equation for (0,0) points will be:

u

So, the eigen values will be calculated as:

So,

As the eigen values are purely imaginary, so the system is **marginally stable.**

Q2) Can the Pendulum be balanced at an arbitrary point such as (2π/3, 0) using the Pole Placement or LQR controller? Why? Why Not? Justify your answer. (3)

**Solution:**

Balancing is defined as the system ability to control or hold their equilibrium in relation to gravity only, now in systems like simple pendulum balancing is defined by a system’s controllability which is the ability to drive a state from any initial value to a final value in finite amount of time by providing a suitable input. So, the controllability matrix (R) can be defined as: -

R=

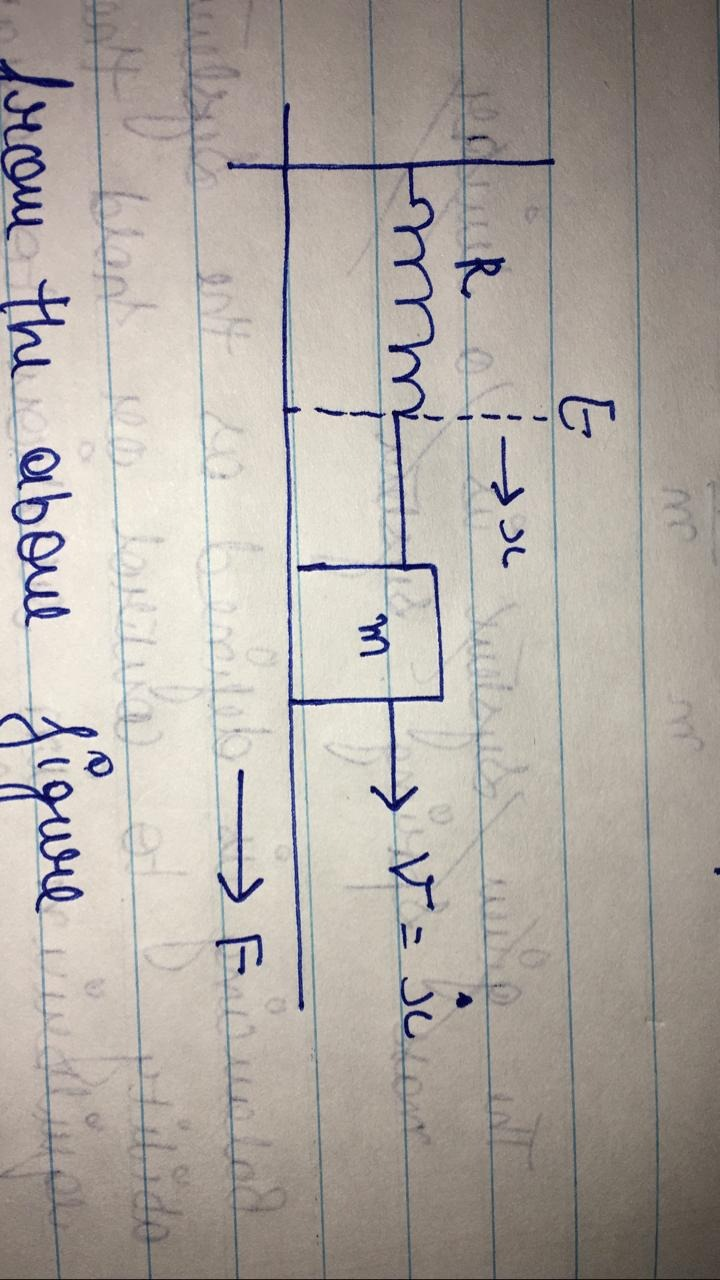
So, the R matrix for the system is: -

The rank of R is 2 which is equal to the number of state variables, thus the system is fully controllable at arbitrary points.

**Section 2 - Mass Spring System**

Q3) Derive the equations of Mass Spring system. (3)

**Solution:**



So from the above figure,

Kinetic Energy = KE =

Potential Energy = PE =

L = KE – PE

L =

So, the equation of mass-spring system is: -

Q4) Is the mass spring system a linear system or non-linear? Justify your answer. (1)

**Solution:**

The given mass-spring system shows non-dampened simple harmonic motion and the system of equations of the arrangement is homogeneous thus the system is considered as a linear system.

Q5) Can the mass spring system be driven to arbitrary state (0.8, 0) using pole placement controller? (Assuming 0.8 is the position and 0 is the velocity). (1)

**Solution:**

If we set the points of the variable y\_setpoint as (0.8, 0) we will see that the block can be driven to the position while running the pole placement function.

**Section 3 - Simple Pulley**

Q6) Under what conditions, will the system remain perfectly at rest? Justify your answer. (1)

**Solution:**

At stable equilibrium points, the system of simple pulley with masses and will be at rest.

Q7) How many equilibrium points does the system have? Are they stable or unstable? Justify your answer. (2)

**Solution:**

Let the position of mass with respect to the pulley be x and velocity be ẋ

So,

The equations are as follows:

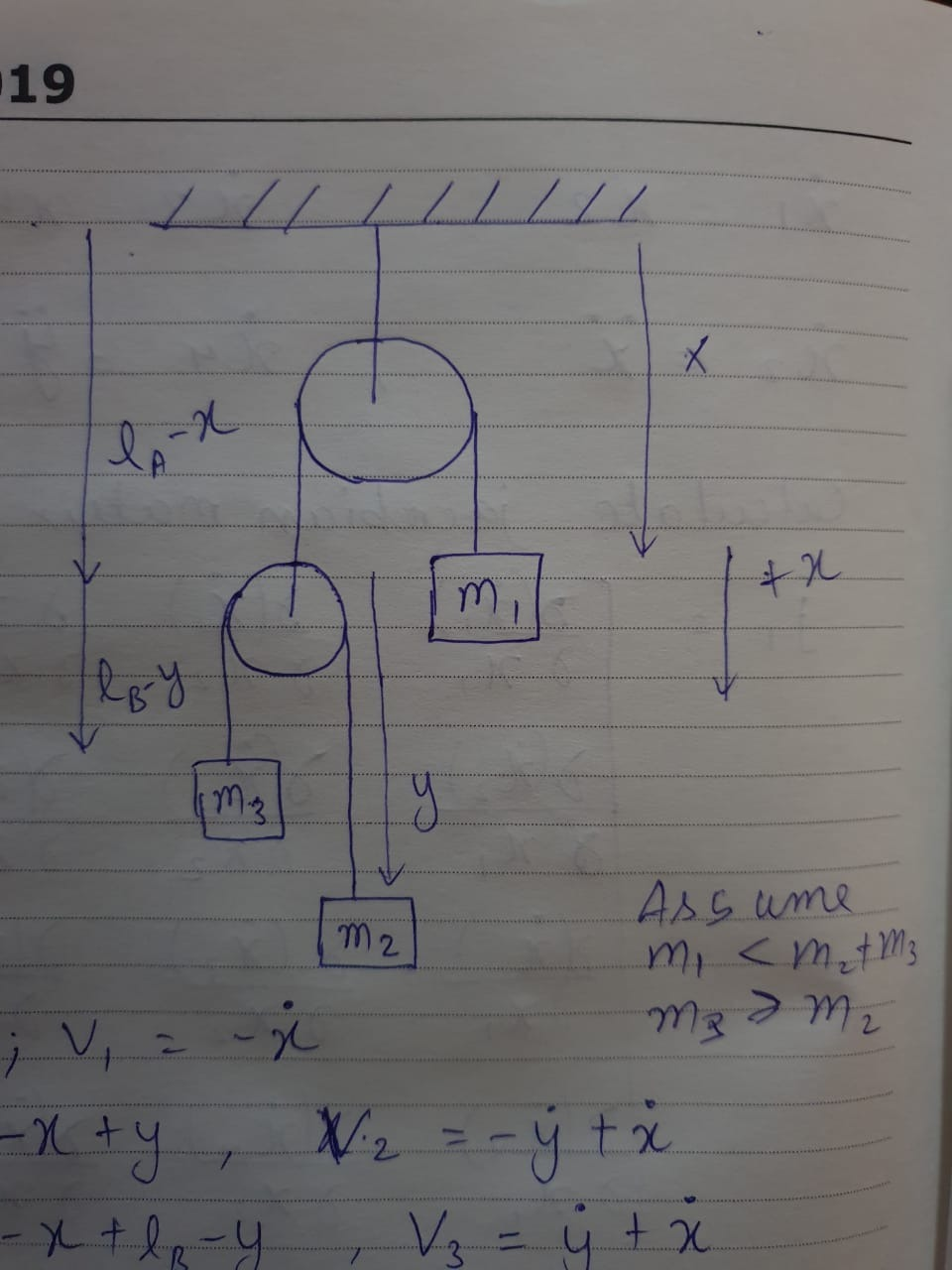
So, for finding equilibrium points, let and . So,

So, for the system, the equilibrium points will have i.e. velocity as 0 whereas can assume any value as it is not being used explicitly in the equation. Thus for these equilibrium points, the system will be at rest and system has infinitely many points. Now all the eigen values of the Jacobian matrix is 0. This shows that the system has **purely imaginary roots** and is **marginally stable**.

**Section 4 - Complex Pulley**

Q8) Derive the equations of motion for the complex pulley system. (5)

**Solution:**



Calculate L,

L= K.E. - P.E.

Find Lagrange equation,

Solving for and ,

Q9) Derive the A and B matrices for the complex pulley system. Is the system linear or nonlinear? (4)

**Solution:**

Jacobian matrix for A matrix of state equation will be:

Since our system has input, we also need to calculate Jacobian for B matrix, will be:

The complex pulley has linear dynamics because this system has 2 control inputs which is same as the degree of freedom i.e. 2.

Q10) Under what conditions, will the system remain perfectly at rest? Justify your answer. (3)

**Solution:**

At stable equilibrium points, the system of complex pulley with masses will be at rest.

The equations are as follows:

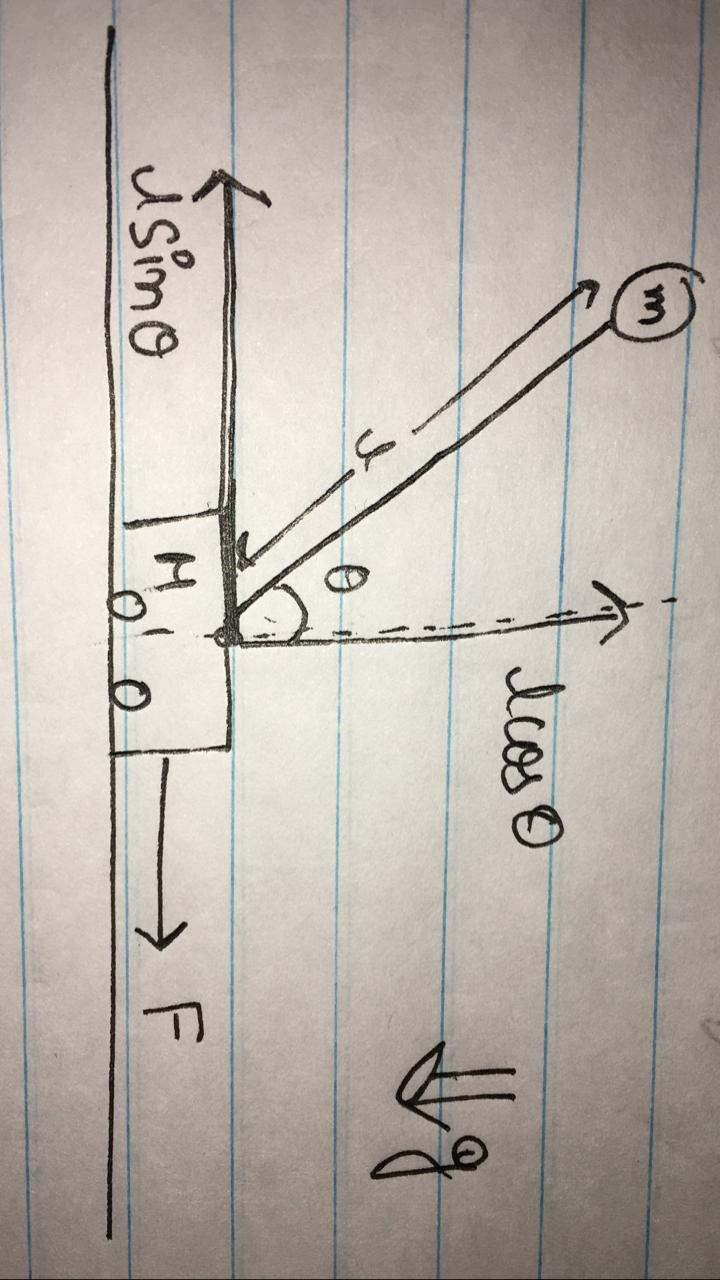
So, for finding equilibrium points, let , , and . So, ,

So, for the system, the equilibrium points will have =0 and =0 whereas and can assume any value as it is not being used explicitly in the equation. Thus for these equilibrium points, the system will be at rest and system has infinitely many points. Now all the eigen values of the Jacobian matrix is 0. This shows that the system has **purely imaginary roots** and is **marginally stable**.

**Section 5 - Inverted Cart Pendulum**

Q11) Derive the equations of motion for the inverted cart pendulum system. Is this system linear or non-linear? Why? (7)

**Solution:**



We know that the Lagrange’s equation is given by: -

() - = Q ….1

where L = T - V ….2

T = Kinetic energy of the system

V = Potential energy of the system

So,

….3

(Where is the velocity of the pendulum and x is the position of cart)

(where is the angular velocity of the pendulum)

……4

Putting 3 and 4 in 2,

……5

Putting 5 in 1,

Where q=x

Clearly,

So, the first equation of motion is: -

……6

…...7

So, the second equation of motion is, 7-6

Put this value in the 1st equation of motion,

The cart pendulum is an inherently unstable system with highly nonlinear dynamics because this system has fewer control inputs than the degree of freedom.

Q12) How many equilibrium points does the inverted cart pendulum system have? Categorize them as stable or unstable? (3)

**Solution:**

Derived equations for the system are:

To find the equilibrium points, we set

, , ,

Equilibrium points will be:

, , ,

, , ,

To determine the stability of system at equilibrium points, we calculate the Jacobian of the system of equations

For each equilibrium point, the Jacobian matrix will be:

Eigen values of 1st Jacobian matrix is

Eigen values of 2nd Jacobian matrix is

The eigen values for (0,0,,0) is purely imaginary. Hence the system will be marginally stable. Marginally stable means that system will be continue to oscillate about the equilibrium point indefinitely.

The eigenvalues for (0,0,0,0) is purely real. One of the eigenvalues have positive real part. Hence the system will be unstable.

Hence we proved that system will be stable for (0,0,,0) and unstable for (0,0,0,0).