Why Numerical Methods ?

→ To avoid rigorous mathematics (Basically)

By now, we know :

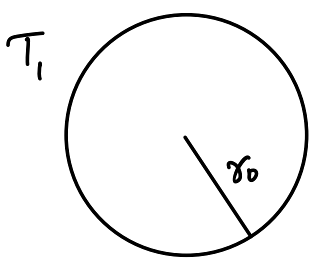
① To make ODE

② Expressing B. C.

③ Solving diff eqn & applying B.C.

Ex :- for 1 D steady Condn :-

Sphere $$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{e}}{k}=0$$



B.C $$\frac{d(T(0))}{dr}=0 \quad \& \quad T(r_0)=T_1$$

⇒ Using this if we solve,

⇒ Analytical Soln,

$$\Rightarrow T(r) = T_1 + \frac{\dot{e}}{6k}\left(r_0^2 - r^2\right)$$

By this we can also find, rate of Heat transfer ($\dot{Q}$) = ? now.

⇒ Fourier's law,

$$\dot{Q}(r) = -kA\frac{dT}{dr} = \frac{4\pi r^3 \dot{e}}{3}$$ Simple!

* Complexities comes with complex geometries where getting analytical solⁿ is very difficult

* Even simple geometries can be complex if thermal condⁿ are not sufficiently simple.

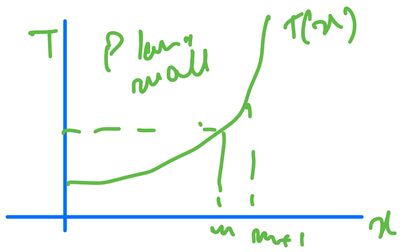
ESO-208 ⇒ finite Difference Method

forward | Centeral | Backward

Proof

$$\frac{dT}{dx} \simeq \frac{T_m - T_{m-1}}{\Delta x}$$

$$\frac{d^2T}{dx^2} = \frac{\left.\frac{dT}{dx}\right|_m - \left.\frac{dT}{dx}\right|_{m-1}}{\Delta x}$$



* for 1D heat transfer plane

finite difference form.

$$\Rightarrow \left\{ \underbrace{\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2}}_{\frac{d^2T}{dx^2}} + \frac{\dot{e}_m}{k} = 0 \right\}$$

need?

Heat Conduction eq^n involves this term.

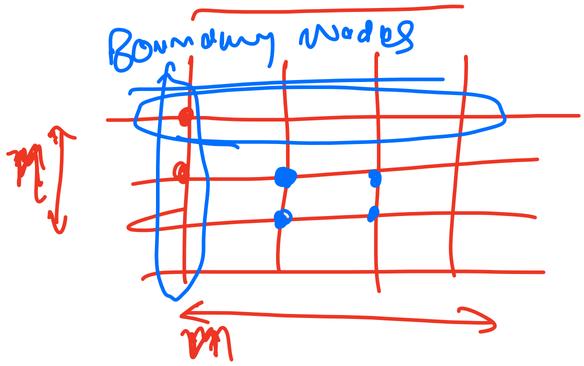
Diff eq^n is $\frac{d^2T}{dx^2} + \frac{\dot{e}}{k} = 0$

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) \quad \text{— } y, z$$

Extending to 2D, 3D ...

$$\frac{T_{m+1,n} - 2T_{m,n} + T_{m-1,n}}{\Delta x^2} + \frac{(\qquad)}{\Delta y^2} + \frac{\dot{e}_{m,n}}{k} = 0$$

* Nodes Used ?



x divided in m region

y divided in n region

Total nodes ⇒ $(m+1)(n+1)$

Finite diff applied to $(m-1)(n-1)$ nodes

To overcome these B.C., we will be using energy conservation method.

* Rate of $\left(\text{Cond} + \text{Gen} = \text{Change in Energy.} \right)$

Example

Large Uranium plate of thickness $L = 4\ cm$

$k = 28 \frac{W}{m.K}$

$\dot{e} = 5 \times 10^{6} \frac{W}{m.K}$

$h = 45\ W/m^2.K$

3 Nodes (equal spaced)

estimate exposed surface temp.?
@ s. state

$\Delta x = ? \qquad \frac{L}{M-1} = 0.02\ m$

$T_0 = 0°C$ (B.C)

$T_1, T_2 = ?$ ⇒ unknown nodal Temp

⇒ apply finite diff method.

$m = 1 \Rightarrow \frac{T_0 - 2T_1 + T_2}{\Delta x^2} + \frac{\dot{e}_1}{k} = 0 \rightarrow 1$

NODE - 2 B.C $\Rightarrow$ Energy conservation

$$h \cancel{A} (T_\infty - T_2) + k \cancel{A} \frac{(T_1 - T_2)}{\Delta x} + \dot{e}_2 \left(\cancel{A} \frac{\Delta x}{2} \right) = 0 \longrightarrow (2)$$

solve for T_1 & T_2