

# Basic Data Structures: Stacks and Queues

Neil Rhodes

Department of Computer Science and Engineering  
University of California, San Diego

Data Structures  
Data Structures and Algorithms

# Outline

1 Stacks

2 Queues

## Definition

**Stack:** Abstract data type with the following operations:

## Definition

**Stack:** Abstract data type with the following operations:

- **Push(Key):** adds key to collection

## Definition

**Stack:** Abstract data type with the following operations:

- `Push(Key)`: adds key to collection
- `Key Top()`: returns most recently-added key

## Definition

**Stack:** Abstract data type with the following operations:

- `Push(Key)`: adds key to collection
- `Key Top()`: returns most recently-added key
- `Key Pop()`: removes and returns most recently-added key

## Definition

**Stack:** Abstract data type with the following operations:

- `Push(Key)`: adds key to collection
- `Key Top()`: returns most recently-added key
- `Key Pop()`: removes and returns most recently-added key
- `Boolean Empty()`: are there any elements?

## Balanced Brackets

**Input:** A string *str* consisting of '(', ')', '[', ']' characters.

**Output:** Return whether or not the string's parentheses and square brackets are balanced.



## Balanced Brackets

Balanced:

- “( [ ] ) [ ] ( )”,
- “((( [ ( [ ] ) ] ) ) ( ) )”

Unbalanced:

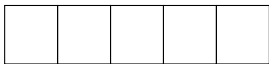
- “( [ ] ] ( )”
- “] [”

## IsBalanced(*str*)

```
Stack stack
for char in str:
    if char in ['(', '[']:
        stack.Push(char)
    else:
        if stack.Empty(): return False
        top ← stack.Pop()
        if (top = '[' and char != ']') or
           (top = '(' and char != ')'):
            return False
return stack.Empty()
```

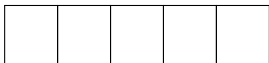
# Stack Implementation with Array

numElements: 0



# Stack Implementation with Array

numElements: 0



Push(a)

# Stack Implementation with Array

numElements: 1

a				
---	--	--	--	--

Push(a)

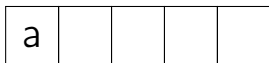
# Stack Implementation with Array

numElements: 1

a				
---	--	--	--	--

# Stack Implementation with Array

numElements: 1



Push(b)

# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

Push(b)



# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

Top()

# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

Top()  $\rightarrow$  b

# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

Push(c)

# Stack Implementation with Array

numElements: 3

a	b	c		
---	---	---	--	--

Push(c)

# Stack Implementation with Array

numElements: 3

a	b	c		
---	---	---	--	--

# Stack Implementation with Array

numElements: 3

a	b	c		
---	---	---	--	--

Pop()



# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

Pop()  $\rightarrow$  c

# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

Push(d)

# Stack Implementation with Array

numElements: 3

a	b	d		
---	---	---	--	--

Push(d)

# Stack Implementation with Array

numElements: 3

a	b	d		
---	---	---	--	--

# Stack Implementation with Array

numElements: 3

a	b	d		
---	---	---	--	--

Push(e)

# Stack Implementation with Array

numElements: 4

a	b	d	e	
---	---	---	---	--

Push(e)

# Stack Implementation with Array

numElements: 4

a	b	d	e	
---	---	---	---	--



# Stack Implementation with Array

numElements: 4

a	b	d	e	
---	---	---	---	--

Push(f)

# Stack Implementation with Array

numElements: 5

a	b	d	e	f
---	---	---	---	---

Push(f)

# Stack Implementation with Array

numElements: 5

a	b	d	e	f
---	---	---	---	---

# Stack Implementation with Array

numElements: 5

a	b	d	e	f
---	---	---	---	---

Push(g)

# Stack Implementation with Array

numElements: 5

a	b	d	e	f
---	---	---	---	---

Push(g) → ERROR

# Stack Implementation with Array

numElements: 5

a	b	d	e	f
---	---	---	---	---

# Stack Implementation with Array

numElements: 5

a	b	d	e	f
---	---	---	---	---

Empty()

# Stack Implementation with Array

numElements: 5

a	b	d	e	f
---	---	---	---	---

Empty()  $\rightarrow$  False



# Stack Implementation with Array

numElements: 5

a	b	d	e	f
---	---	---	---	---

# Stack Implementation with Array

numElements: 5

a	b	d	e	f
---	---	---	---	---

Pop()

# Stack Implementation with Array

numElements: 4

a	b	d	e	
---	---	---	---	--

Pop()  $\rightarrow$  f

# Stack Implementation with Array

numElements: 4

a	b	d	e	
---	---	---	---	--

# Stack Implementation with Array

numElements: 4

a	b	d	e	
---	---	---	---	--

Pop()

# Stack Implementation with Array

numElements: 3

a	b	d		
---	---	---	--	--

Pop()  $\rightarrow$  e

# Stack Implementation with Array

numElements: 3

a	b	d		
---	---	---	--	--

# Stack Implementation with Array

numElements: 3

a	b	d		
---	---	---	--	--

Pop()



# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

Pop()  $\rightarrow$  d

# Stack Implementation with Array

numElements: 2

a	b			
---	---	--	--	--

# Stack Implementation with Array

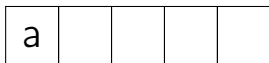
numElements: 2

a	b			
---	---	--	--	--

Pop()

# Stack Implementation with Array

numElements: 1



Pop()  $\rightarrow$  b

# Stack Implementation with Array

numElements: 1

a				
---	--	--	--	--

# Stack Implementation with Array

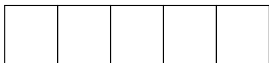
numElements: 1

a				
---	--	--	--	--

Pop()

# Stack Implementation with Array

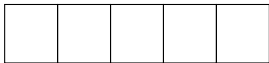
numElements: 0



Pop()  $\rightarrow$  a

# Stack Implementation with Array

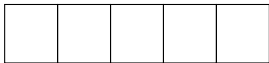
numElements: 0





# Stack Implementation with Array

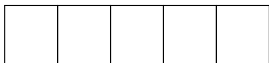
numElements: 0



Empty()

# Stack Implementation with Array

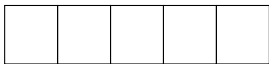
numElements: 0



Empty()  $\rightarrow$  True

# Stack Implementation with Array

numElements: 0



# Stack Implementation with Linked List



head

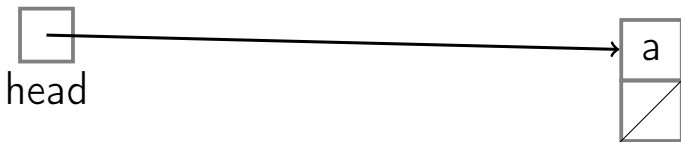
# Stack Implementation with Linked List



head

Push(a)

# Stack Implementation with Linked List



Push(a)

# Stack Implementation with Linked List



# Stack Implementation with Linked List



Push(b)



# Stack Implementation with Linked List



Push(b)

# Stack Implementation with Linked List



# Stack Implementation with Linked List



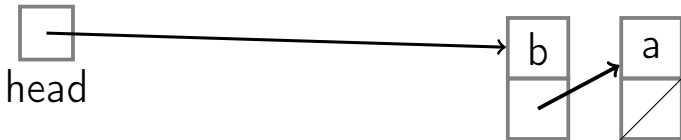
Top()

# Stack Implementation with Linked List

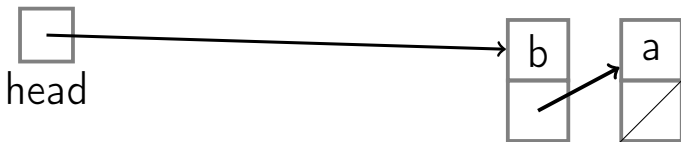


$\text{Top}() \rightarrow b$

# Stack Implementation with Linked List

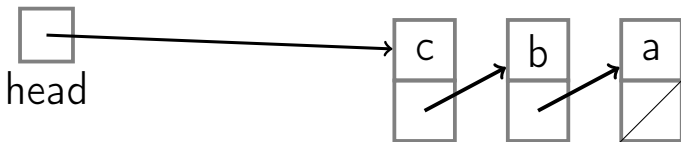


# Stack Implementation with Linked List



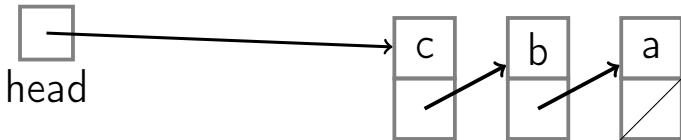
Push(c)

# Stack Implementation with Linked List



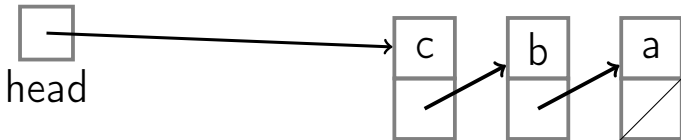
Push(c)

# Stack Implementation with Linked List





# Stack Implementation with Linked List



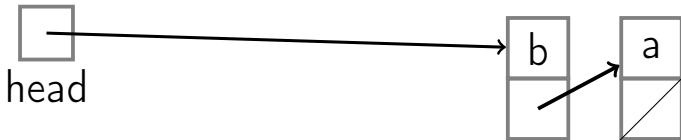
Pop()

# Stack Implementation with Linked List



$\text{Pop()} \rightarrow c$

# Stack Implementation with Linked List

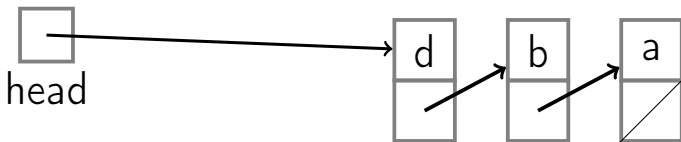


# Stack Implementation with Linked List



Push(d)

# Stack Implementation with Linked List

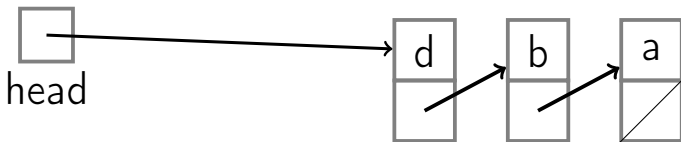


Push(d)

# Stack Implementation with Linked List

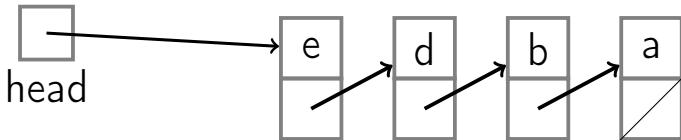


# Stack Implementation with Linked List



Push(e)

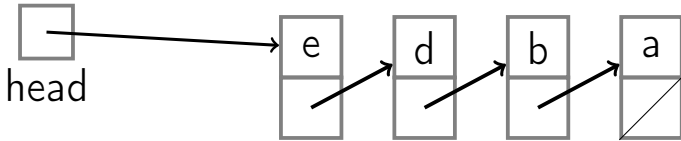
# Stack Implementation with Linked List



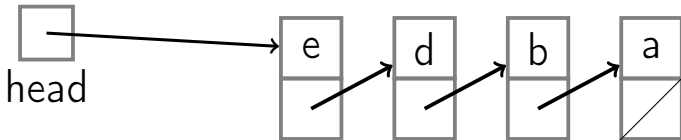
Push(e)



# Stack Implementation with Linked List

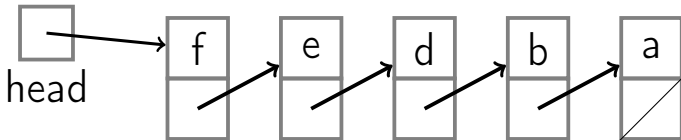


# Stack Implementation with Linked List



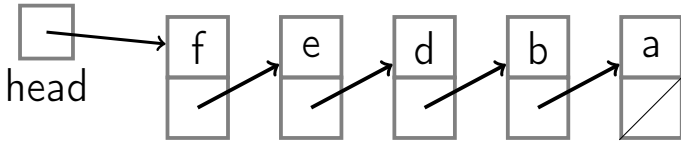
Push(f)

# Stack Implementation with Linked List

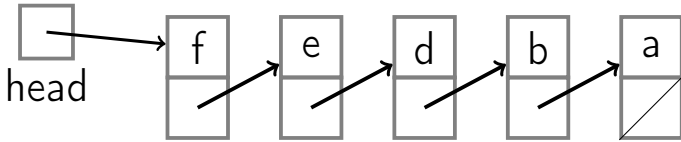


Push(f)

# Stack Implementation with Linked List

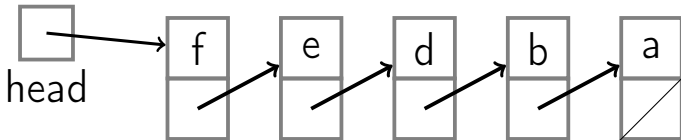


# Stack Implementation with Linked List



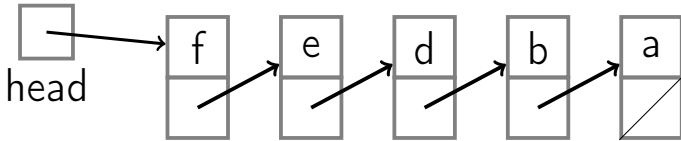
Empty()

# Stack Implementation with Linked List

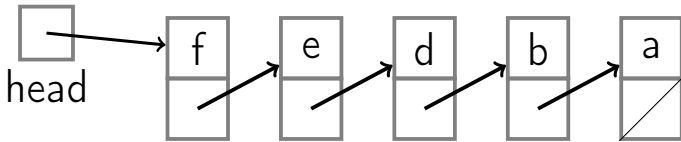


`Empty()`  $\rightarrow$  `False`

# Stack Implementation with Linked List



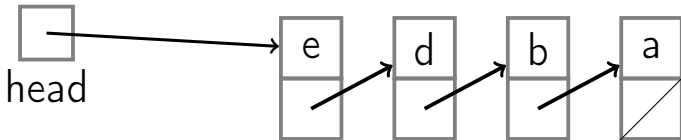
# Stack Implementation with Linked List



Pop()

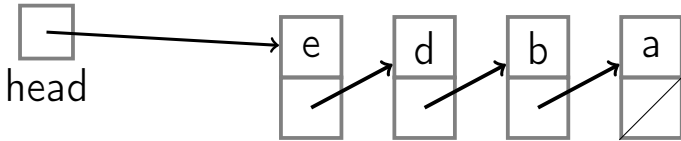


# Stack Implementation with Linked List

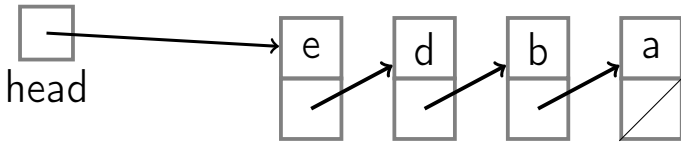


Pop() → f

# Stack Implementation with Linked List

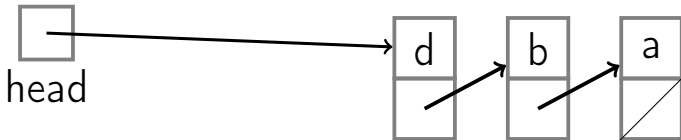


# Stack Implementation with Linked List



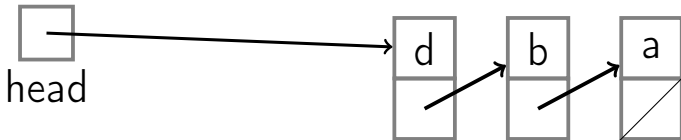
Pop()

# Stack Implementation with Linked List



Pop() → e

# Stack Implementation with Linked List

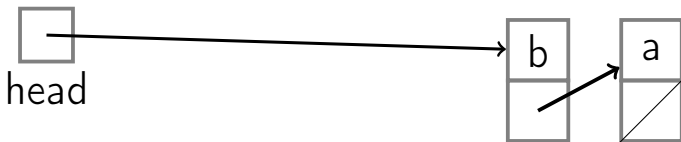


# Stack Implementation with Linked List



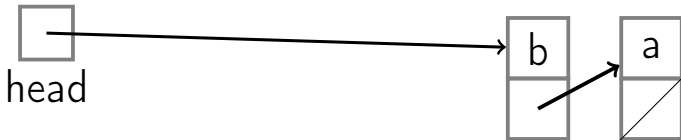
Pop()

# Stack Implementation with Linked List



Pop()  $\rightarrow$  d

# Stack Implementation with Linked List





# Stack Implementation with Linked List



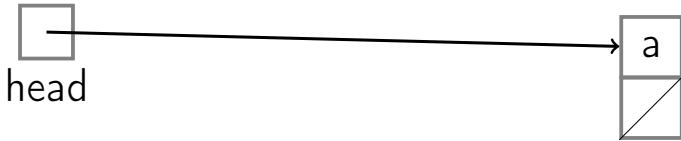
Pop()

# Stack Implementation with Linked List



Pop() → b

# Stack Implementation with Linked List



# Stack Implementation with Linked List



Pop()

# Stack Implementation with Linked List



Pop()  $\rightarrow$  a

# Stack Implementation with Linked List



head

# Stack Implementation with Linked List



head

Empty()

# Stack Implementation with Linked List



head

`Empty() → True`



# Stack Implementation with Linked List



head

# Stack Implementation with Linked List

# Stack Implementation with Linked List

# Stack Implementation with Linked List

# Summary

- Stacks can be implemented with either an array or a linked list.

# Summary

- Stacks can be implemented with either an array or a linked list.
- Each stack operation is  $O(1)$ : Push, Pop, Top, Empty.

# Summary

- Stacks can be implemented with either an array or a linked list.
- Each stack operation is  $O(1)$ : Push, Pop, Top, Empty.
- Stacks are occasionally known as LIFO queues.

# Outline

1 Stacks

2 Queues



# Definition

**Queue:** Abstract data type with the following operations:

# Definition

**Queue:** Abstract data type with the following operations:

- **Enqueue(Key):** adds key to collection

# Definition

**Queue:** Abstract data type with the following operations:

- `Enqueue(Key)`: adds key to collection
- `Key Dequeue()`: removes and returns least recently-added key

# Definition

**Queue:** Abstract data type with the following operations:

- `Enqueue(Key)`: adds key to collection
- `Key Dequeue()`: removes and returns least recently-added key
- `Boolean Empty()`: are there any elements?

# Definition

**Queue:** Abstract data type with the following operations:

- `Enqueue(Key)`: adds key to collection
- `Key Dequeue()`: removes and returns least recently-added key
- `Boolean Empty()`: are there any elements?

FIFO: First-In, First-Out

# Queue Implementation with Linked List



head



tail

# Queue Implementation with Linked List



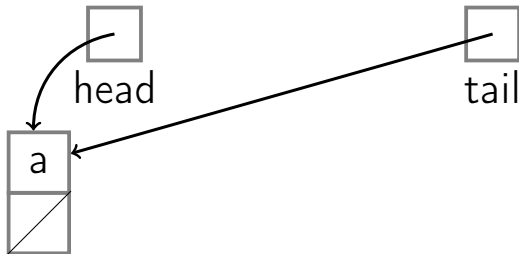
head



tail

Enqueue(a)

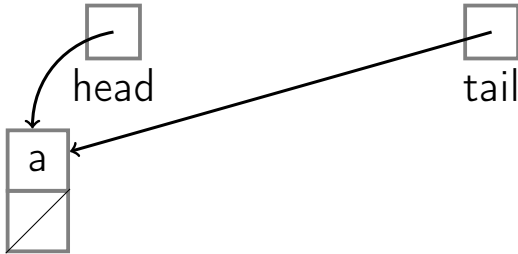
# Queue Implementation with Linked List



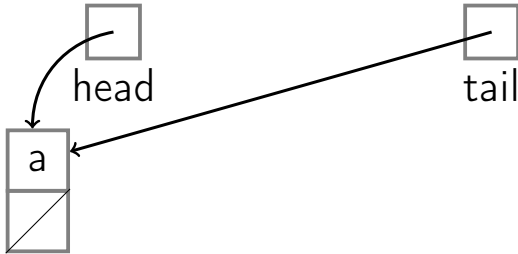
Enqueue(a)



# Queue Implementation with Linked List

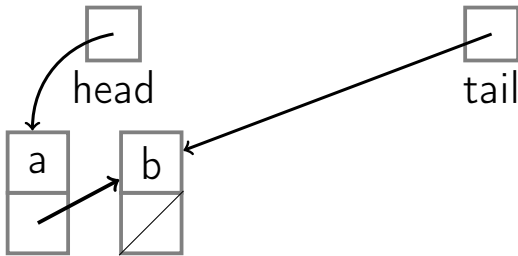


# Queue Implementation with Linked List



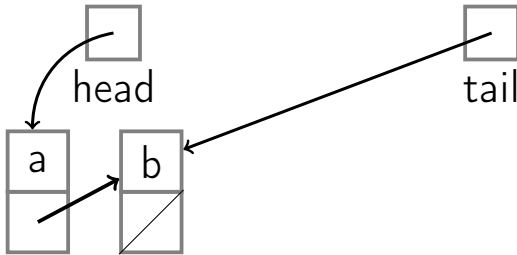
Enqueue(b)

# Queue Implementation with Linked List

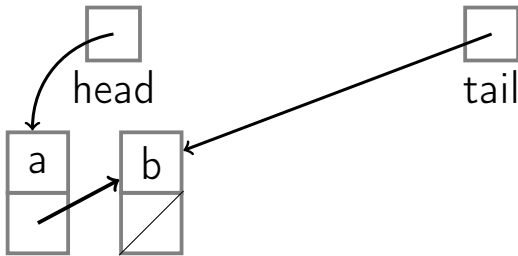


Enqueue(b)

# Queue Implementation with Linked List

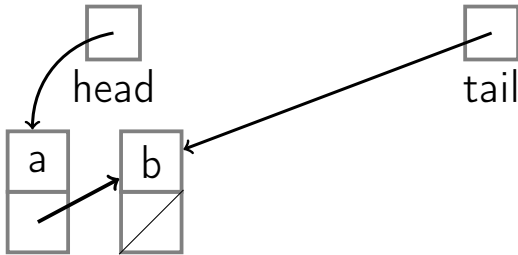


# Queue Implementation with Linked List



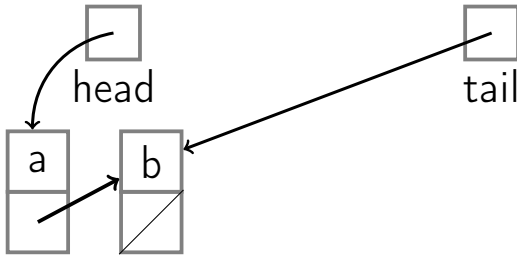
Empty()

# Queue Implementation with Linked List

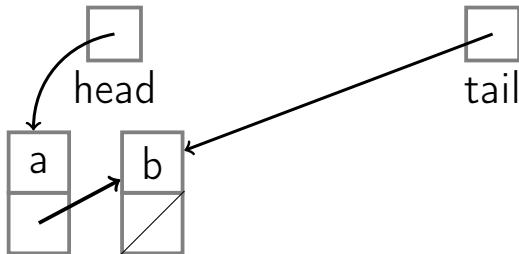


`Empty() → False`

# Queue Implementation with Linked List



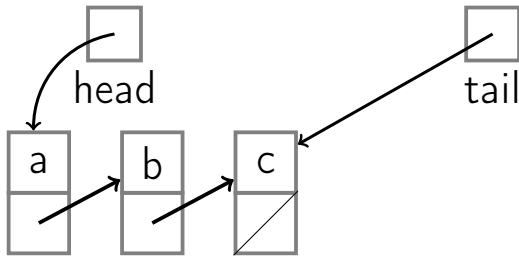
# Queue Implementation with Linked List



Enqueue(c)

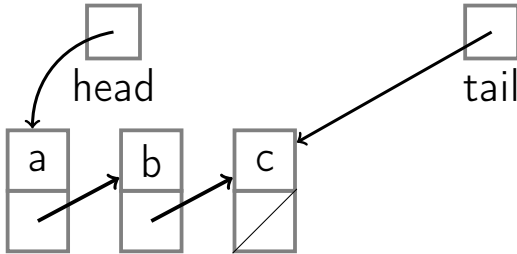


# Queue Implementation with Linked List

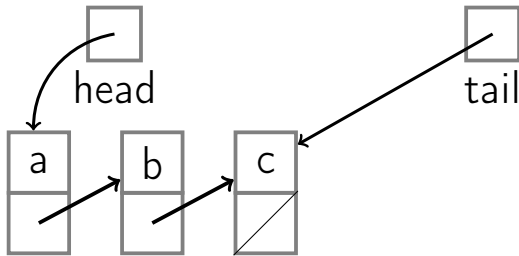


Enqueue(c)

# Queue Implementation with Linked List

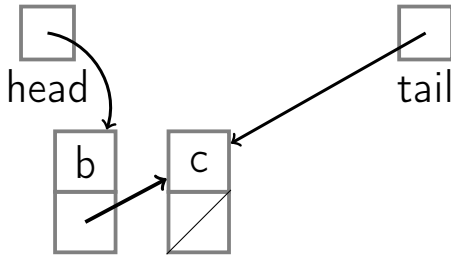


# Queue Implementation with Linked List



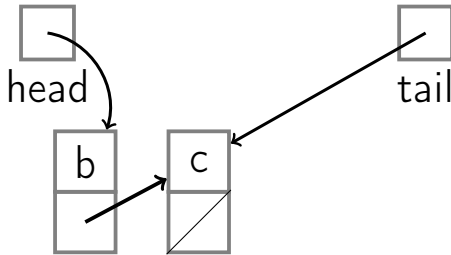
Dequeue()

# Queue Implementation with Linked List

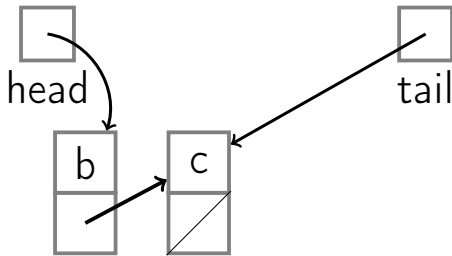


Dequeue()  $\rightarrow$  a

# Queue Implementation with Linked List

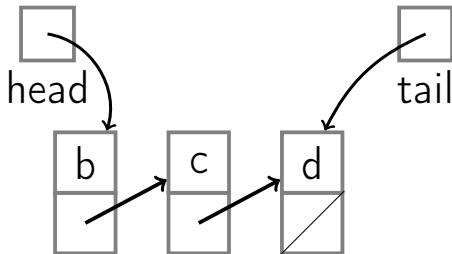


# Queue Implementation with Linked List



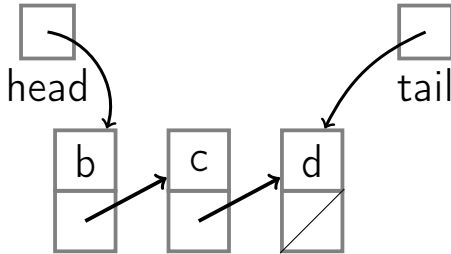
Enqueue(d)

# Queue Implementation with Linked List



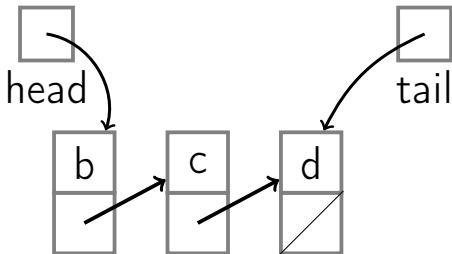
Enqueue(d)

# Queue Implementation with Linked List



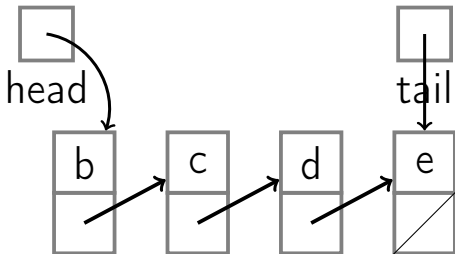


# Queue Implementation with Linked List



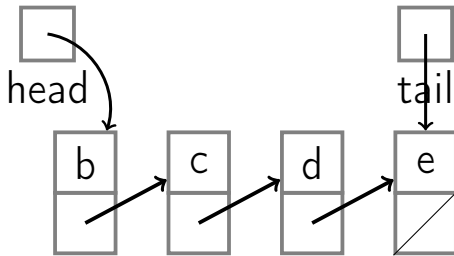
Enqueue(e)

# Queue Implementation with Linked List

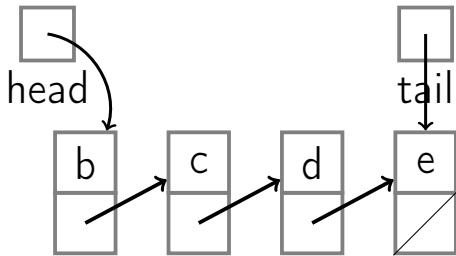


Enqueue(e)

# Queue Implementation with Linked List

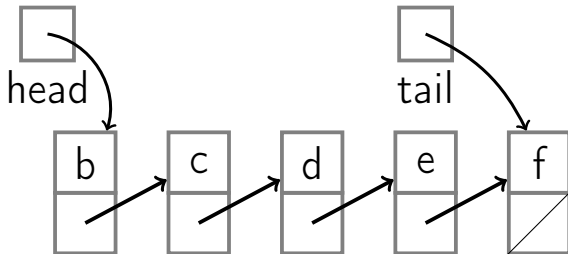


# Queue Implementation with Linked List



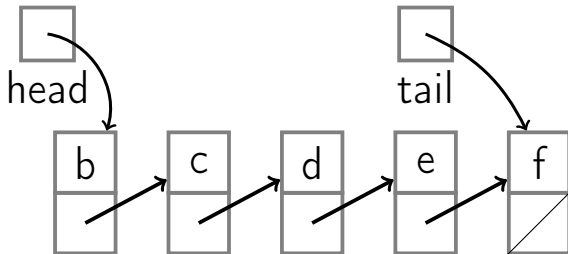
Enqueue(f)

# Queue Implementation with Linked List

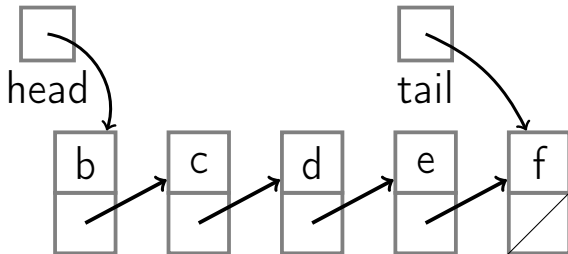


Enqueue(f)

# Queue Implementation with Linked List

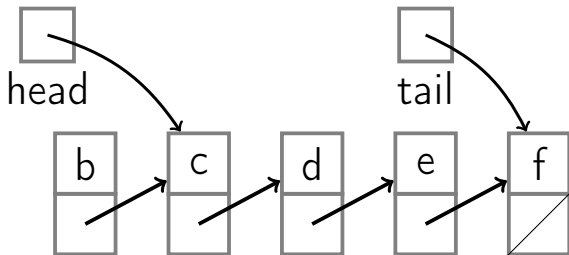


# Queue Implementation with Linked List



Dequeue()

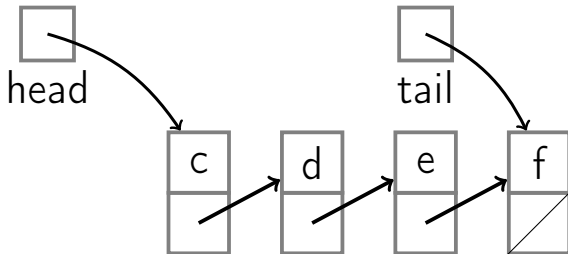
# Queue Implementation with Linked List



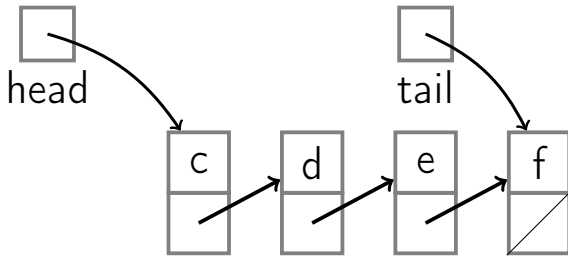
Dequeue() → b



# Queue Implementation with Linked List

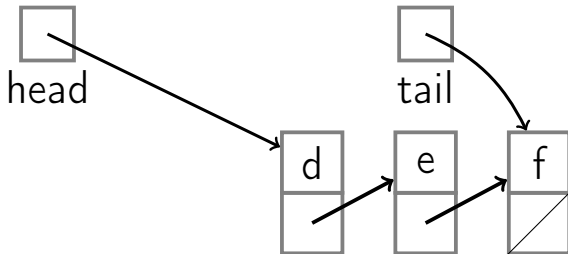


# Queue Implementation with Linked List



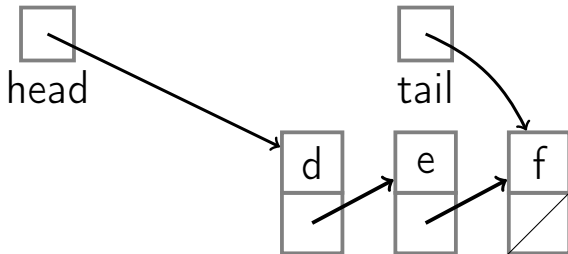
Dequeue()

# Queue Implementation with Linked List

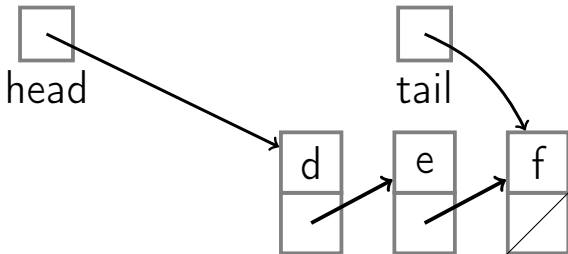


Dequeue()  $\rightarrow$  c

# Queue Implementation with Linked List

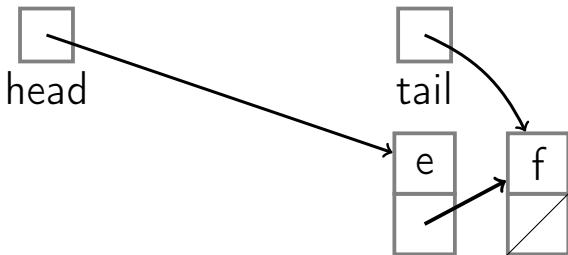


# Queue Implementation with Linked List



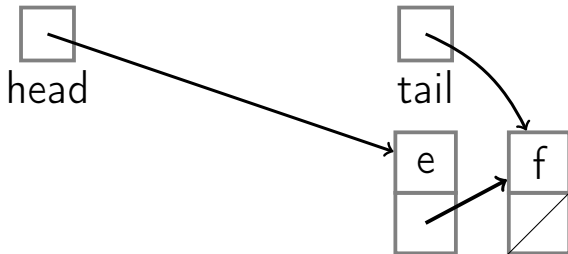
Dequeue()

# Queue Implementation with Linked List

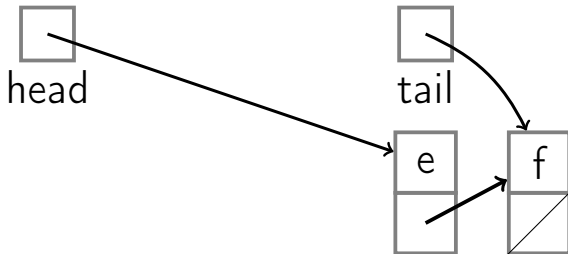


Dequeue()  $\rightarrow$  d

# Queue Implementation with Linked List



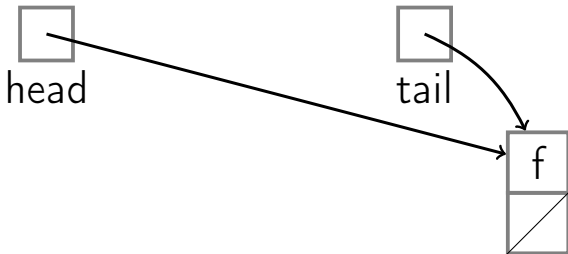
# Queue Implementation with Linked List



`Dequeue()`

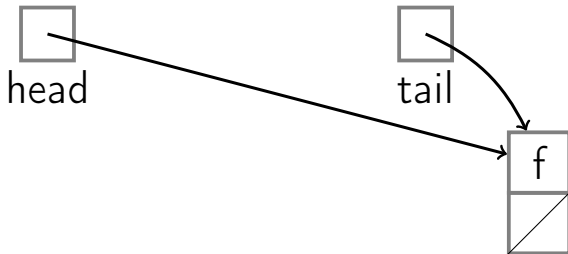


# Queue Implementation with Linked List

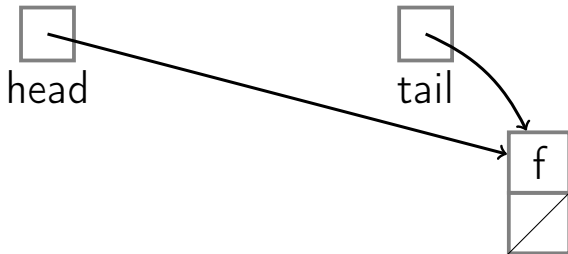


Dequeue()  $\rightarrow$  e

# Queue Implementation with Linked List



# Queue Implementation with Linked List



Dequeue()

# Queue Implementation with Linked List



head



tail

Dequeue()  $\rightarrow$  f

# Queue Implementation with Linked List



head



tail

# Queue Implementation with Linked List



head



tail

Empty()

# Queue Implementation with Linked List



head



tail

`Empty()`  $\rightarrow$  `True`

# Queue Implementation with Linked List



head



tail



# Queue Implementation with Linked List

- Enqueue: use `List.PushBack`

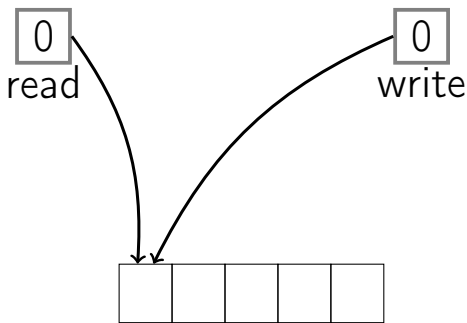
# Queue Implementation with Linked List

- Enqueue: use `List.PushBack`
- Dequeue: use `List.TopFront` and `List.PopFront`

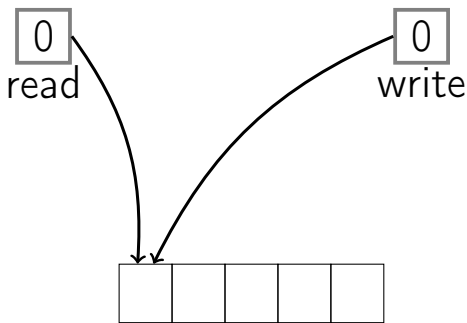
# Queue Implementation with Linked List

- Enqueue: use `List.PushBack`
- Dequeue: use `List.TopFront` and `List.PopFront`
- Empty: use `List.Empty`

# Queue Implementation with Array

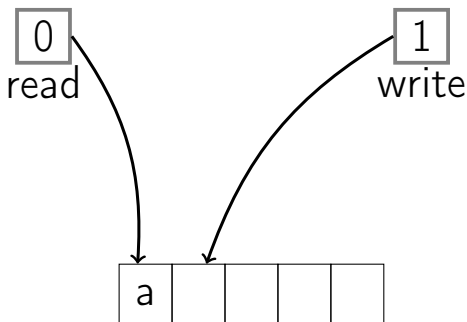


# Queue Implementation with Array



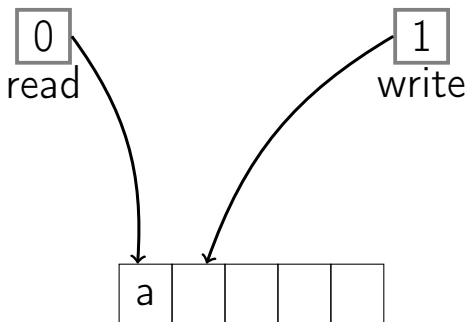
Enqueue(a)

# Queue Implementation with Array

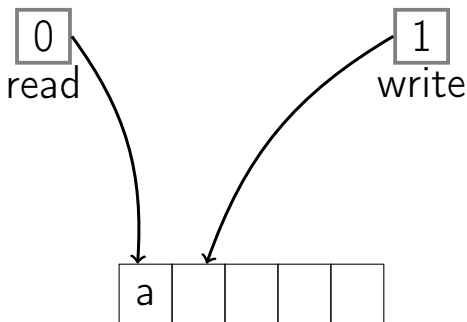


Enqueue(a)

# Queue Implementation with Array



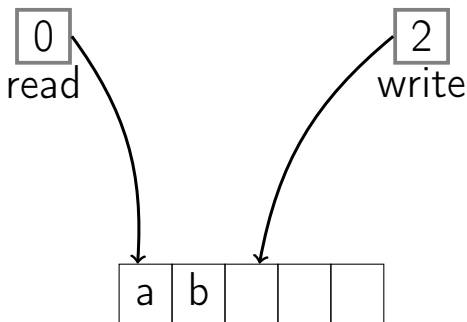
# Queue Implementation with Array



Enqueue(b)

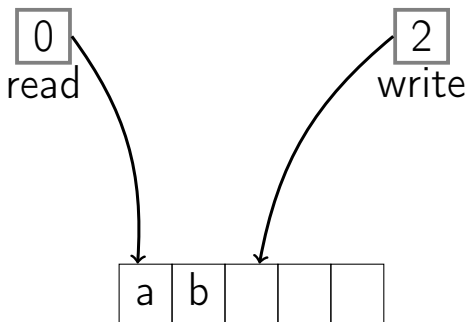


# Queue Implementation with Array

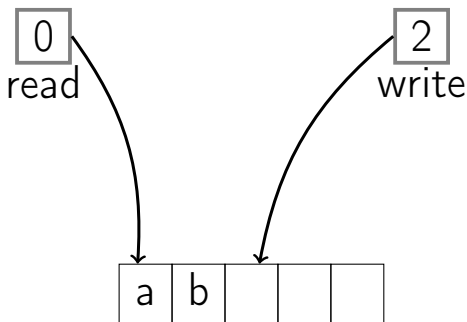


Enqueue(b)

# Queue Implementation with Array

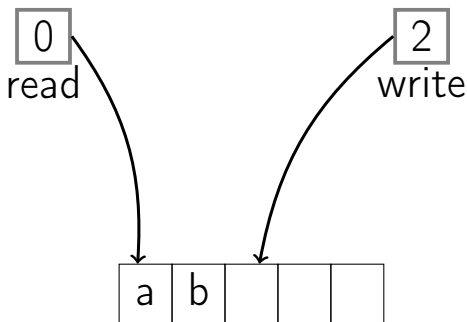


# Queue Implementation with Array



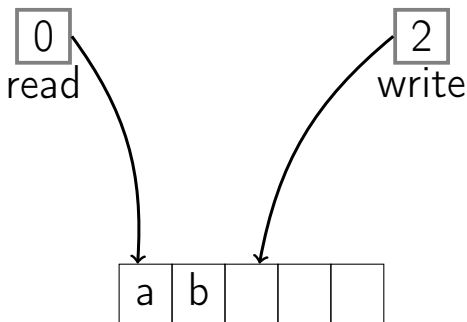
Empty()

# Queue Implementation with Array

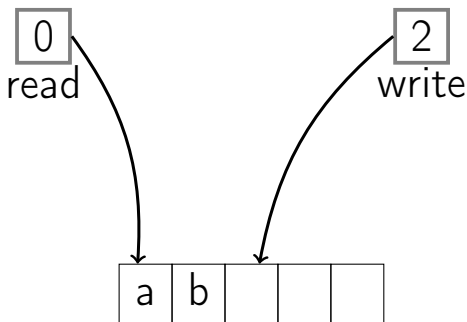


`Empty()`  $\rightarrow$  `False`

# Queue Implementation with Array

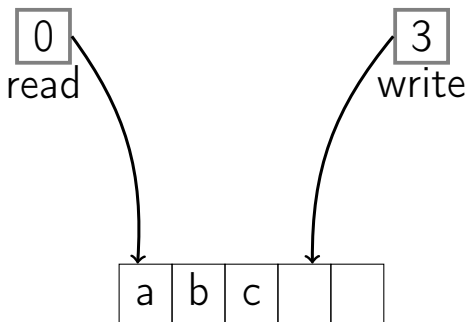


# Queue Implementation with Array



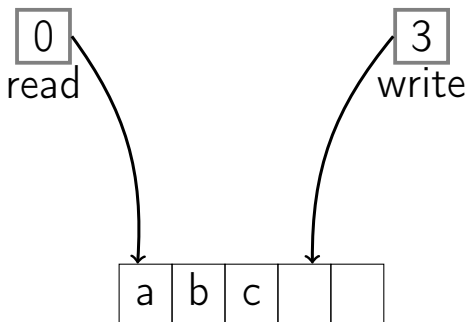
Enqueue(c)

# Queue Implementation with Array



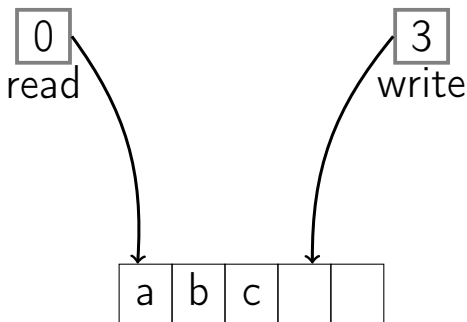
Enqueue(c)

# Queue Implementation with Array



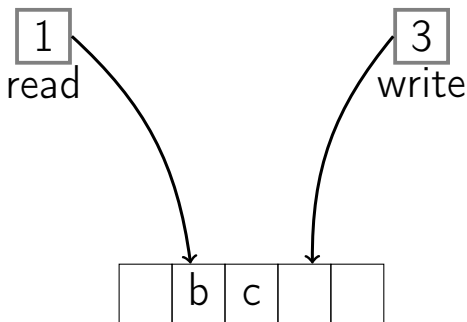


# Queue Implementation with Array



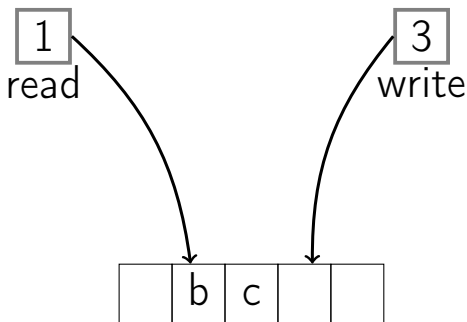
Dequeue()

# Queue Implementation with Array

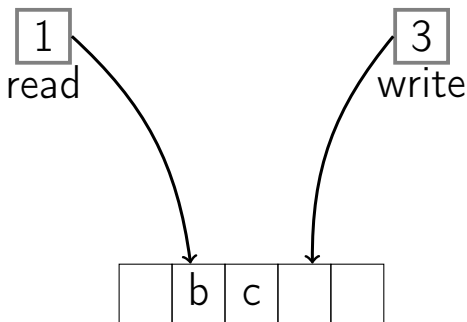


Dequeue()  $\rightarrow$  a

# Queue Implementation with Array

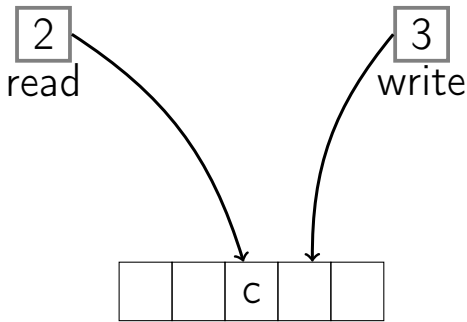


# Queue Implementation with Array



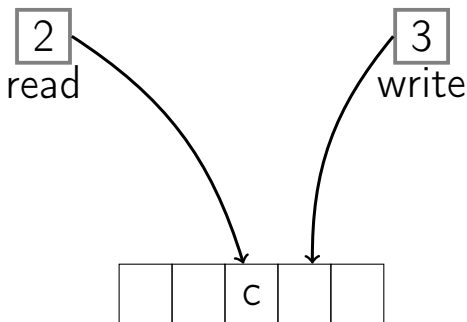
Dequeue()

# Queue Implementation with Array

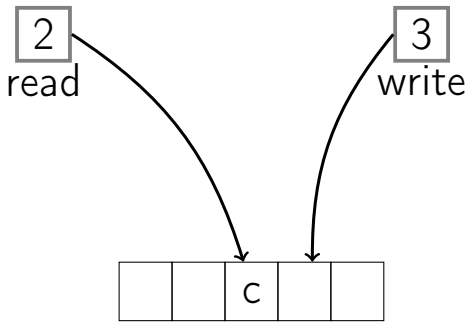


Dequeue()  $\rightarrow$  b

# Queue Implementation with Array

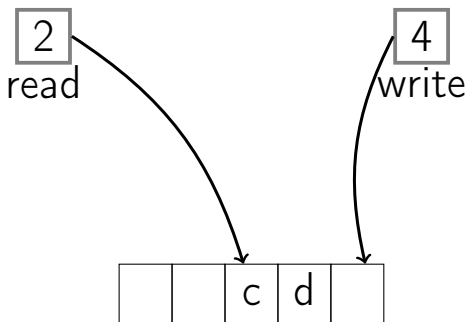


# Queue Implementation with Array



Enqueue(d)

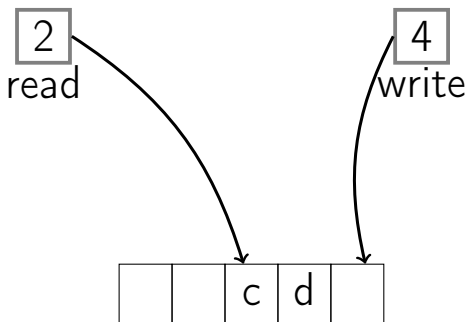
# Queue Implementation with Array



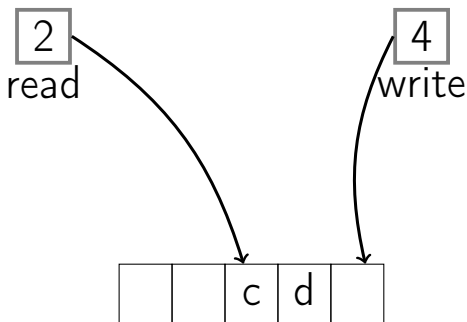
Enqueue(d)



# Queue Implementation with Array

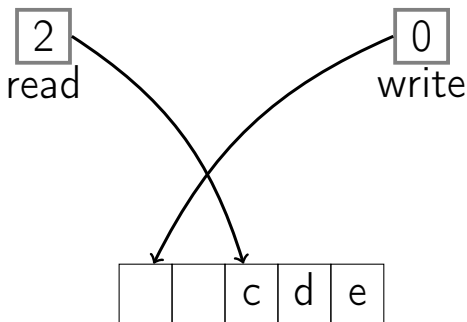


# Queue Implementation with Array



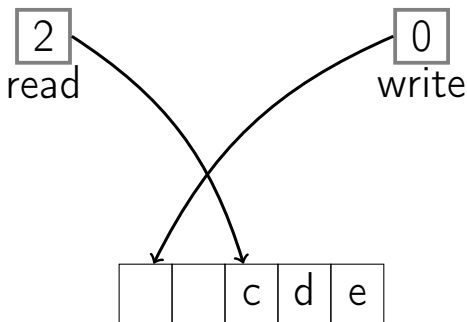
Enqueue(e)

# Queue Implementation with Array

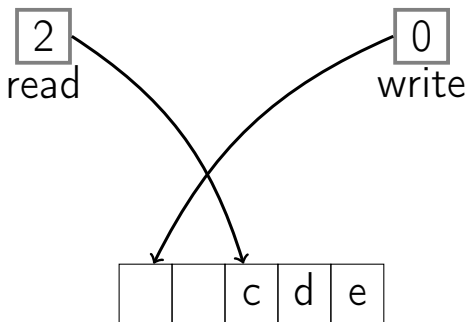


Enqueue(e)

# Queue Implementation with Array

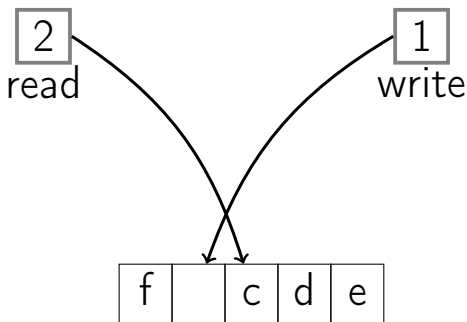


# Queue Implementation with Array



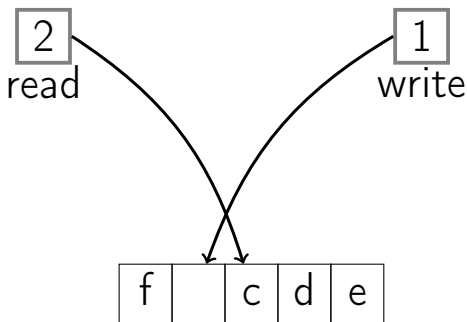
Enqueue(f)

# Queue Implementation with Array

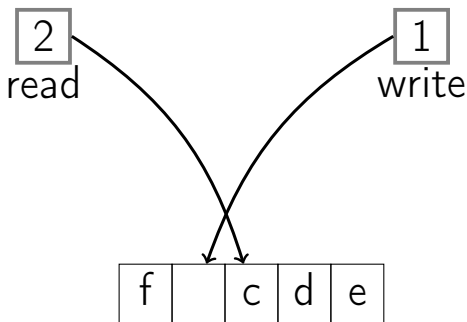


Enqueue(f)

# Queue Implementation with Array



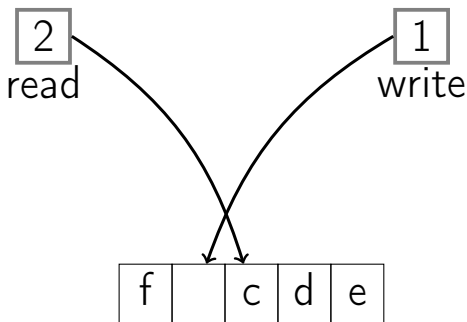
# Queue Implementation with Array



Enqueue(g)

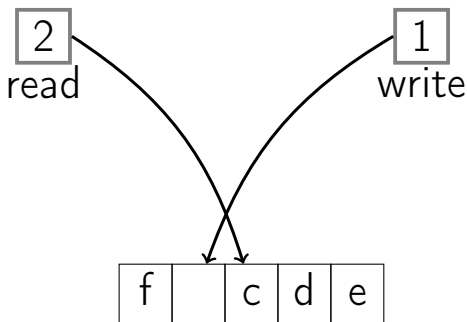


# Queue Implementation with Array

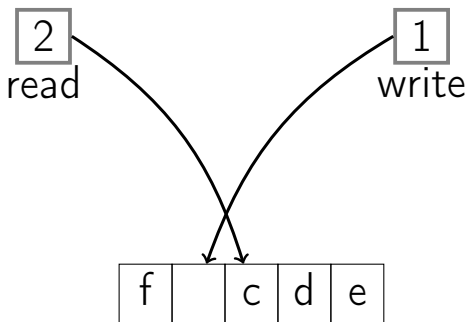


Enqueue(g)  $\rightarrow$  ERROR

# Queue Implementation with Array

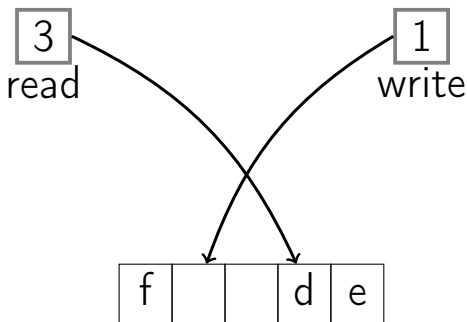


# Queue Implementation with Array



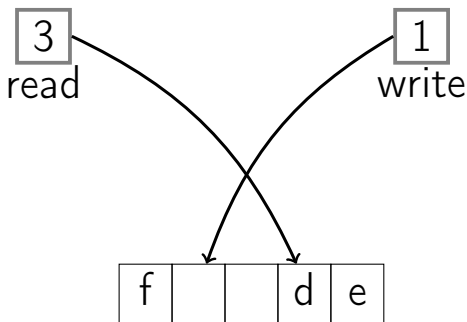
Dequeue()

# Queue Implementation with Array

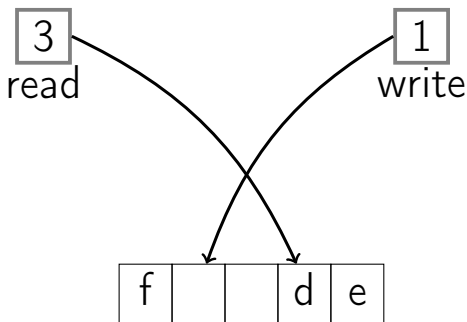


Dequeue()  $\rightarrow$  c

# Queue Implementation with Array

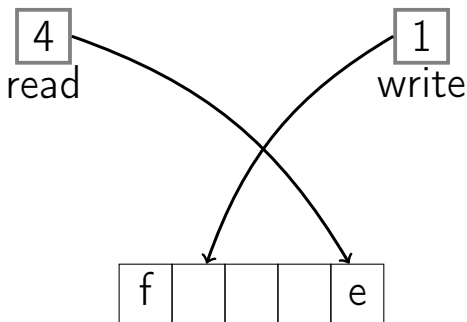


# Queue Implementation with Array



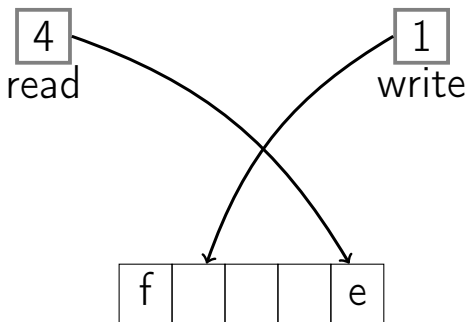
Dequeue()

# Queue Implementation with Array



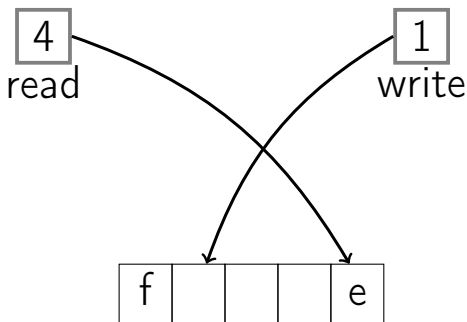
Dequeue()  $\rightarrow$  d

# Queue Implementation with Array



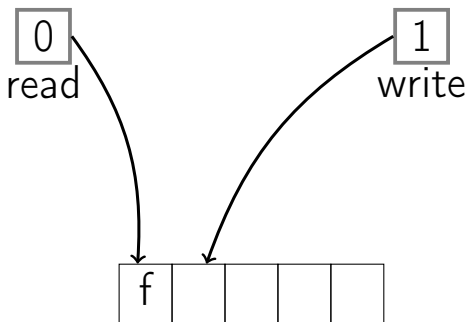


# Queue Implementation with Array



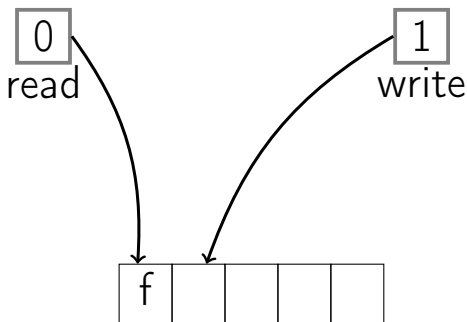
Dequeue()

# Queue Implementation with Array

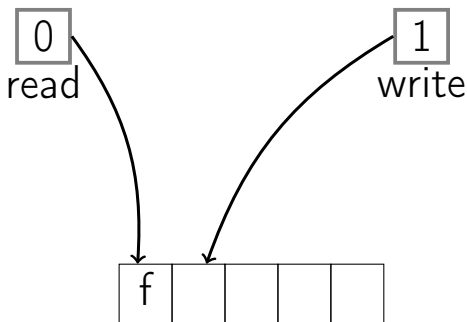


Dequeue()  $\rightarrow$  e

# Queue Implementation with Array

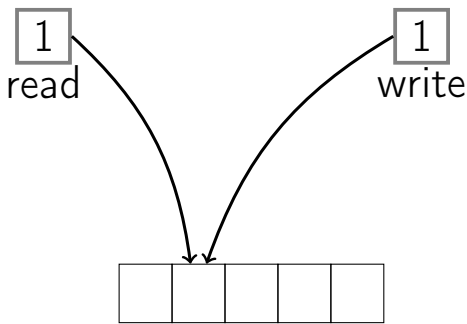


# Queue Implementation with Array



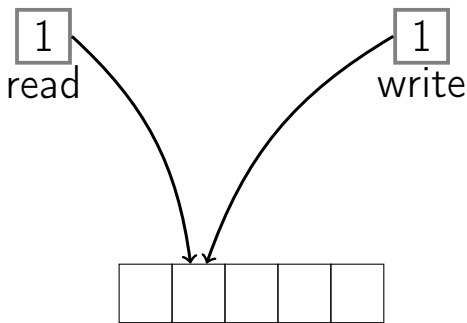
Dequeue()

# Queue Implementation with Array

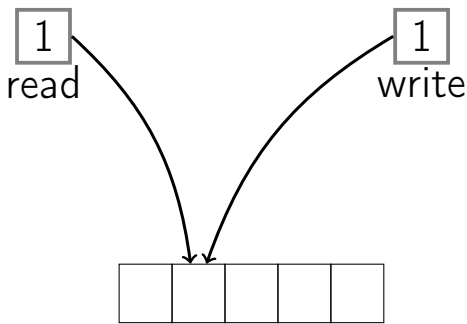


Dequeue()  $\rightarrow$  f

# Queue Implementation with Array

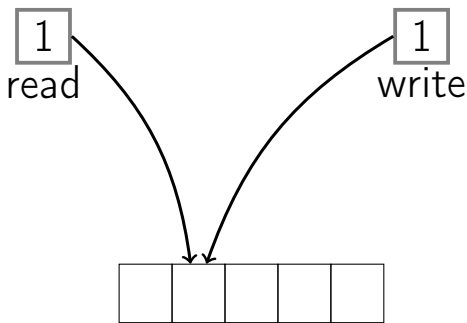


# Queue Implementation with Array



Empty()

# Queue Implementation with Array



`Empty() → True`



# Summary

# Summary

- Queues can be implemented with either a linked list (with tail pointer) or an array.

# Summary

- Queues can be implemented with either a linked list (with tail pointer) or an array.
- Each queue operation is  $O(1)$ : Enqueue, Dequeue, Empty.