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RAJ RAJESH INSTITUTE

(STUDY MATERIAL)

ENGLISH MEDIUM

GEOMETRY & MENSURATION

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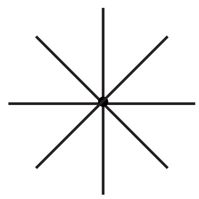
GEOMETRY

ZERO DIMENSION / NO DIMENSION

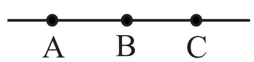
1) **POINT (बिन्दु) :** A point is a mark of position which has no dimension, i.e. no shape or size .

Note-1 : *Infinite lines can be drawn through a points.*

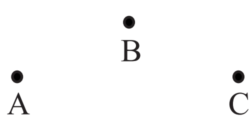


Note -2 : *One and only one line can be drawn through two distinct point.*

Note – 3 : *Three or more than three points said to be collinear (सरेख) if a line segment passing through them, Otherwise they are called Non-collinear. (असरेख)*



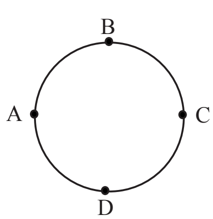
A, B & C collinear points



A, B & C are non-collinear points.

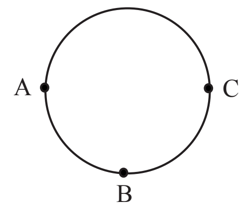
=> Two points are always collinear

Note- 4 : *Four or more than four points said to be concyclic (एकवृत्तीय) if a circle passes through them.*



A, B, C & D are concylic points.

Note-5 : *Any three non-collinear points are always concyclic.*



Note-6 : *A circle always passes through three non-collinear points.*

ONE DIMENSION

1) **LINE (रेखा) :** A line is a straight path that extends indefinitely in both the directions. It has no end points.

Line PQ ($\overrightarrow{PQ}$)

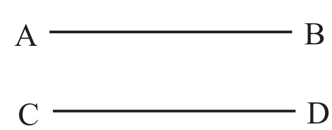
2) **LINE SEGMENT (रेखाखण्ड) :** A line segment is the portion of a line with two fixed end points.

Line Segment ($\overleftarrow{PQ}$ or PQ)

3) **RAY (किरण) :** A line segment extended endlessly in one direction is called ray.

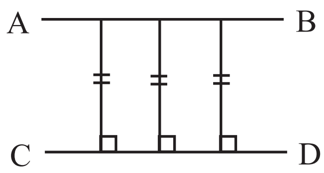
Ray PQ ($\overrightarrow{PQ}$)

4) **PARALLEL LINES (समान्तर / समानांतर या अप्रतिच्छेदी रेखाएँ) :** Two lines in a plane are called parallel if they do not meet when produced indefinitely on either side



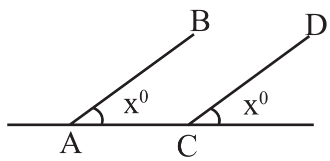
AB | | CD

Note-1: *Perpendicular distance between two parallel lines are always equal.*



AB | | CD

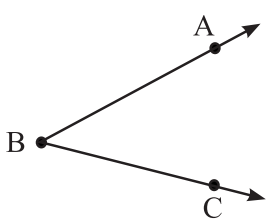
Note-2 : *If two straight lines make equal angle with same plane then they are parallel and vice-versa.*



AB | | CD

TWO DIMENSION

1) **ANGLE** (कोण) **:** An angle is formed when two line segments or two rays have a common end-point. The two line segments forming an angle are called arms of the angle, whereas their common end-point is called the vertex (शीर्ष) of the angle.

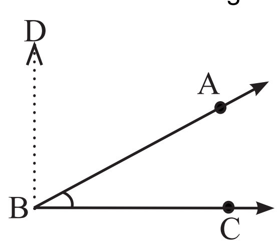


Angle ABC ($\angle$ABC)

TYPES OF ANGLES

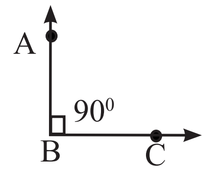
(According to measurement of angle)

1) **Acute Angle** (न्यूनकोण) : An angle measuring less than 90^0 is called an Acute Angle.



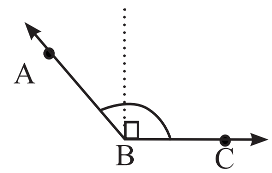
Acute Angle ABC ($\angle$ABC $<$ 90^0)

2) **Right Angle** (समकोण) **:** An angle whose measure is 90^0, is called a right angle. The arms of a right angle are perpendicular (लम्ब) to each others.



Right Angle ABC ($\angle$ABC = 90^0)

3) **Obtuse Angle** (अधिक कोण) **:** An angle greater than a right angle, but less than 180^0, is called an obtuse angle.

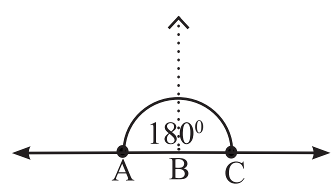


Obtuse angle ABC (90^0 < $\angle$ABC < 180^0)

4) **Straight Angle** (ऋजुकोण) **:** An angle equal to two right angles.

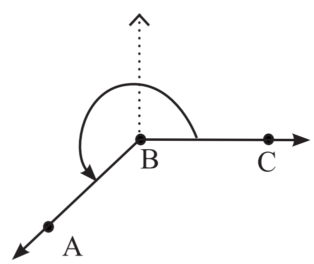
or

An Angle whose measure is 180^0 is called a straight angle.



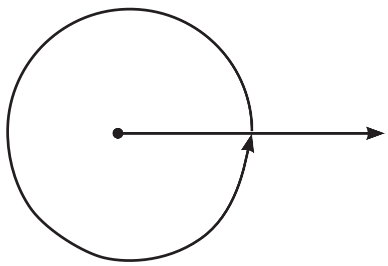
Straight angle ABC ($\angle$ABC = 180^0)

5) **Reflex Angle** (पुनर्युक्त कोण या प्रतिवर्त्ती कोण)**:** An angle whose measure is more than 180^0 and less than 360^0 is called a reflex angle.



Reflex angle ABC (180^0 < $\angle$ABC $\angle$$360^0$)

6) **Complete Angle** (सम्पूर्ण कोण) **:** The measure of a complete angle is 360^0 .



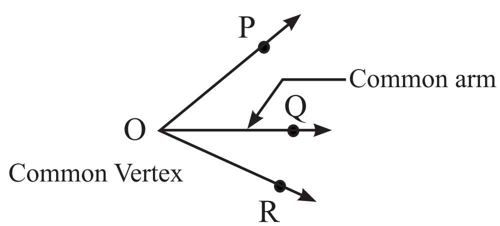
<u>**TERMS RELATED TO ANGLE**</u>

(कोण से संबंधित शब्द)

1) **Complementary Angles** (पूरक कोण)**:** Two angles are said to be complementary if the sum of their degree measure is 90^0 .

2) **Supplementary Angles** (सम्पूरक कोण) **:** Two angels are said to be supplementary if the sum of their degree measures is 180^0.

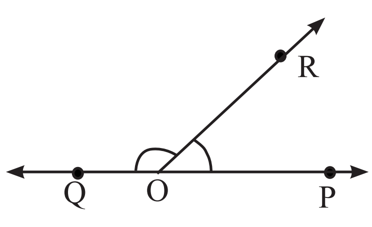
3) **Adjacent Angles** (आसन्न कोण) **:** Two angles are said to be adjacent if they have a common vertex and a common arm between two other arms.



$\angle POQ$ and $\angle QOR$ are adjacent angles.

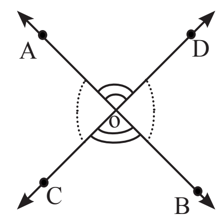
4) **Linear Pair Angles** (रैखिक युग्म कोण) **:** A pair of adjacent angles is said to form a linear pair if the outer arms of the angles lie on one line.

A linear pair (consisting of two angles) is measured to be 180^0 .



$\angle POR + \angle QOR = 180^0$

5) **Vertically Opposite Angles** (शीर्षाभिमुख कोण) **:** The angles opposite to the common vertex formed by the intersection of two lines having no common arm are known as vertically opposite angles.

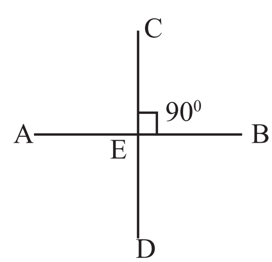


($\angle AOD$ and $\angle BOC$) form one pair of vertically opposite angles, and ($\angle AOC$ and $\angle BOD$) form another pair of vertically opposite angles.

Note : *When two lines intersect, vertically opposite angles are always equal.*

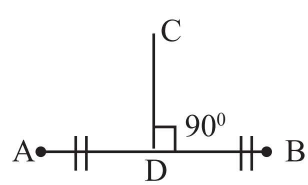
$\angle AOC = \angle BOD$ and $\angle AOD = \angle BOC$

6) **Perpendicular** (लम्ब) **:** The two lines are said to be perpendicular to each other, if they contain an angle of 90^0 or one right angle between them.



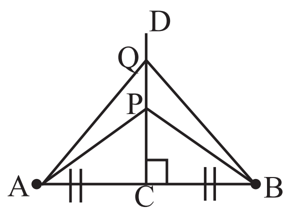
AB and CD are perpendicular to each other. $CD \perp AB$

7) **Perpendicular Bisector** (लम्ब समद्विभाजक) **:** If a line passes through the mid-point of a line segment and perpendicular to it, then the line is called the perpendicular bisector of the line segment.



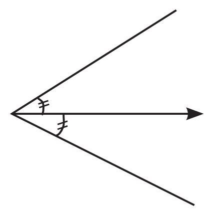
CD is perpendicular bisector of AB.

Note : *Every point on perpendicular bisector is equidistant* (समान दूरी पर) *from both ends.*

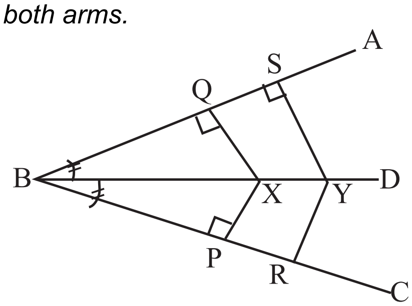


AP = PB, AQ = QB

8) Angle Bisector (अर्द्धक) **:** If a line bisects (समद्विभाजित) an angle, then the line is called the bisector of the angle.



Note : *Every point on angle bisector is equidistant from both arms.*

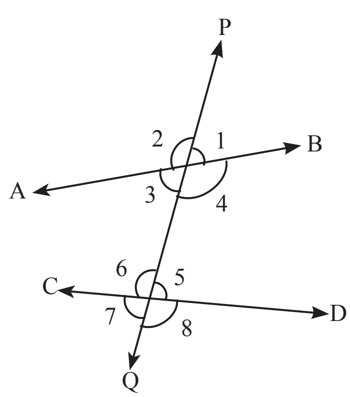


PX = QX
RY = SY

TRANSVERSAL (तिर्यक)

Transversal (तिर्यक)**:** A line which cuts two or more given lines at different points is called a transversal.

Angle formed by transversal



PQ is transversal.

** ***Transversal makes Eight Angles***

1) **Exterior Angles** (बाह्य कोण) **:** 1, 2, 7, & 8

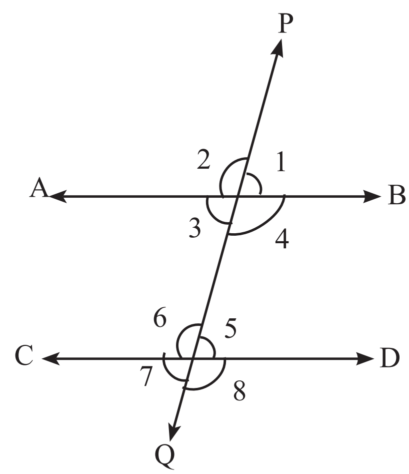
2) **Interior Angles** (अंत: कोण) **:** 3, 4, 5 & 6

3) **Four pairs of corresponding angles** (संगत कोण)**:** (2, 6), (1,5), (3,7) & (4,8)

4) **Two pairs of Alternate Interior Angles** (एकांतर अंत: कोण) **:** (3, 5) & (4, 6)

5) **Two pairs of Alternate Exterior Angles** (एकांतर बाह्य कोण) **:** (2, 8) & (1, 7)

PROPERTIES OF PARALLEL LINES



AB | | CD & PQ is transversal

1) Pairs of corresponding angles are equal.

$\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$

& $\angle 4 = \angle 8$

2) Pairs of alternate (interior or exterior) angles are equal.

$\angle 3 = \angle 5$, $\angle 4 = \angle 6$, $\angle 2 = \angle 8$

& $\angle 1 = \angle 7$

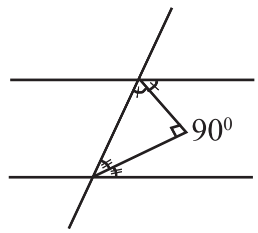
3) Sum of interior angles or exterior angles on the same side of the transversal is equal to 180^0 .

$\angle 3 + \angle 6 = \angle 4 + \angle 5 = \angle 2 + \angle 7$

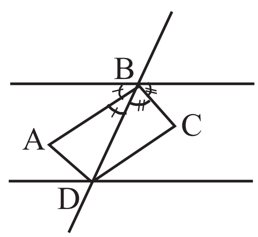
$= \angle 1 + \angle 8 = 180^0$

Note : *If two lines are intersected by transversal and if any one of the above mentioned condition is true, then the two lines are parallel.*

4) Bisector of interior angles intersect at 90^0.

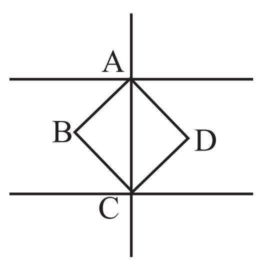


5) Bisectors of interior angles encloses a rectangle.



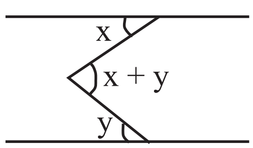
ABCD is a rectangle.

6) If Transversal is perpendicular then it form a square.

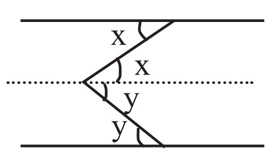


ABCD is a square.

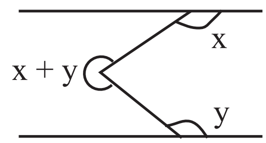
7)



Proof :



8)



<u>MEASUREMENT OF ANGLE</u>

(कोण की माप)

There are three system used for the measurement of angles.

1) Sexagesimal System or English System (Degree)
2) Centesimal System or French System (Grade)
3) Circular measurement (radian)

1) Sexagesimal or English System (Degree) : Here a right angle is divided into 90^0 equal parts known as degrees. Each degree is divided into 60 equal parts called minutes and each minutes is further divided into 60 equal parts called seconds.

60 seconds (60") = 1 minute (1')

60 minutes (60') = 1 degree (1^0)

90 degree (90^0) = 1 right angle.

2) Centesimal system or French System (Grade) : Here a right angle is divided into 100 equal parts called grades and each grade is divided into 100 equal parts, called minute and each minute is further divided into 100 seconds.

100 seconds	(100") = 1 minute (1')
100 minutes	(100') = 1 grade (1^g)
100 grade	(100^0) = 1 right angle

3) Circular measurement or Radian measure : The number of radians in an angle subtended by an arc of a circle at the centre is equal to arc.

The angle subtended by arc of length radius at the centre is equal to one radian.

$$\theta = \frac{l}{r} \; \frac{\Rightarrow \text{length of arc}}{\Rightarrow \text{radius}}$$

$$\pi^C = 180^\circ$$

$$1^C = \left(\frac{180}{\pi}\right)^\circ$$

$$x^C = \left(\frac{180}{\pi} \times x\right)^\circ$$

$$x^\circ = \left(\frac{\pi}{180} \times x\right)^C$$

$$x^\circ y' z'' = \left[x \times \frac{\pi}{180} + y \times \frac{\pi}{180 \times 60} + z \times \frac{\pi}{180 \times 60 \times 60}\right]^C$$

Angle made by Needle of a Clock

Hours Needle -

1 dial	= 360^0
12 hours	= 360^0
1 hours	**= 30^0**
60 minute	= 30^0
1 minute	= $\frac{1^\circ}{2}$

Minutes Needle

1 dial	= 360^0
60 minute	= 360^0
1 minute	**= 6^0**

Ex – **9 : 32**

Angle $= \left[9 \times 30 + 32 \times \frac{1}{2}\right] - [32 \times 6]$

$= [270 + 16] - [192]$

$= 286 - 192 = 94^0$

POLYGON (बहुभुज)

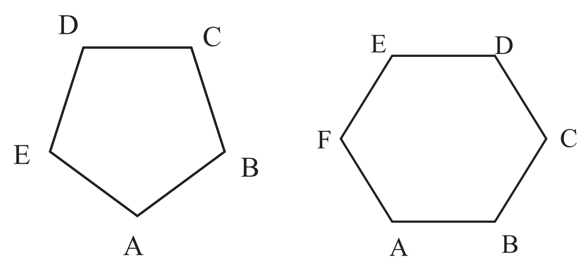
Polygon : A plane geometrical figure, bounded by atleast three line segments, is called a polygon.

Name of Polygons

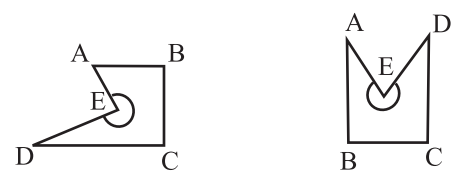
No. of Sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Types of Polygons

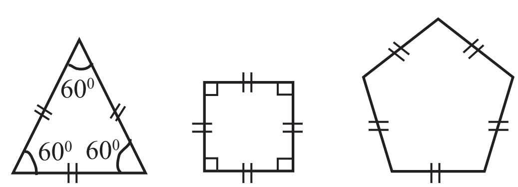
i) **Convex Polygon** (उत्तल बहुभुज) **:** If each angle of a polygon is less than 180^0, it is called a convex polygon.



ii) **Concave Polygon** (अवतल बहुभुज) **:** If at least one angle of a polygon is more than 180^0, it is called a concave polygon.

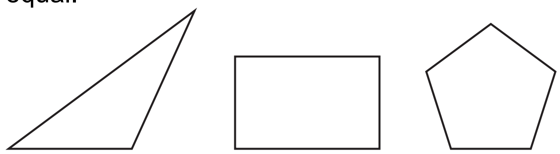


iii) **Regular Polygon** (सम बहुभुज) **:** A regular polygon is a polygon with all its sides and all its angles equal.



Equilateral Triangle Square Regular Pentagon

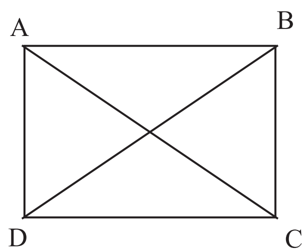
iv) **Non-Regular Polygon** (विषम बहुभुज) **:** A polygon is called a non-regular polygon, if all the sides are not equal.



Scalene triangle   Rectangle   Pentagon

<u>Terms related to Polygon</u>

Diagonal (विकर्ण) **:** Line segment joining any two non-consecutive vertices.



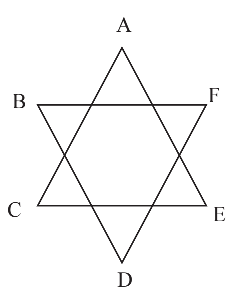
AC & BD are diagonals.

<u>FORMULA related to Polygon</u>

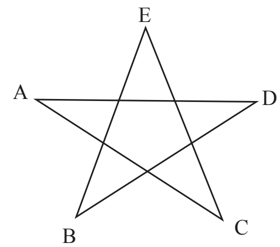
1) Sum of interior angles of a polygon
$= (n-2) \times 180^0$
2) Sum of exterior angles
$= 360^0$
3) Number of diagonals

$$= \frac{n(n-3)}{2} = {}^nC_2 - n$$

4) Sum of vertices angles of n sided star shaped polygon = $(n-4) \times 180^0$



$\angle A + \angle B + \angle C + \angle D + \angle E$
$= (6-4) \times 180^0 = 360^0$



$\angle A + \angle B + \angle C + \angle D + \angle E$
$= (5-4) \times 180^0 = 180^0$

5) Interior angle + Exterior angle = 180^0
6) Exterior angle = 180^0 – Interior angle

*****For a regular polygon of n sided***

1) Each interior angle = $\dfrac{(n-2)\times 180^\circ}{n}$

2) Each exterior angle = $\dfrac{360^\circ}{n}$

3) Number of Sides = $\dfrac{360^\circ}{\text{each exterior angle}}$

4) Area = $\dfrac{n}{4}a^2 \cot\left(\dfrac{180}{n}\right)$

Where a = length of sides

5) Area of equilateral triangle = $\dfrac{\sqrt{3}}{4}a^2$

6) Area of square = a^2

7) Area of hexagon = $\dfrac{3\sqrt{3}}{2}a^2$

<u>Properties related to Polygon</u>

1. *In any polygon (except triangle and quadrilateral) sum of interior angles is greater than sum of exterior angles.*
2. *Triangle is only one polygon in which sum of interior angles is half of sum of exterior angles.*
3. *Quadrilateral is only one polygon in which sum of interior angles is equal to sum of exterior angles.*

TRIANGLE (त्रिभुज)

Triangle - A triangle is a plane and closed geometrical figure, bounded by three line segments. A triangle has three sides (भुजा), three angles (कोण) and three vertices (शीर्ष).

Types of triangle (According to side)

1) **Equilateral Triangle** (समबाहु त्रिभुज) **:** A triangle in which all the three sides are equal.

Note (1) : In equilateral triangle all angles are equal.

Note (2) : In equilateral triangle each angle is equal to 60^0 .

2) **Isosceles Triangle (**समद्विबाहु त्रिभुज) **:** A triangle in which any of two sides are equal.

Note (3) : In Isosceles triangle two angles are equal.

Note (4) : If two sides of a triangle are equal than angle opposite to them are equal.

Note (5) : If two angles of a triangle are equal than sides opposite to them are equal.

3) **Scalene Triangle (**विषमबाहु त्रिभुज**) :** A triangle in which all the sides are unequal.

Note (6) : In scalene triangle all the three angles are unequal.

Note (7) : If two sides of a triangles are unequal than greater side has greater angle opposite to it.

Note (8) : If two angles of a triangle are unequal than greater angle has greater side opposite to it.

Types of Triangle (According to angle)

1) **Acute-angled Triangle** (न्यूनकोण त्रिभुज) **:** If all the three angles of a triangle are acute angles it is called an acute-angled triangle.

Note (9) : In acute angle triangle sum of any two angles is greater than 90^0.

Note (10) : In acute angle triangle $c^2 < a^2 + b^2$ (where a, b & c are length of sides and C is greatest side.

2) **Right-angled Triangle** (समकोण त्रिभुज) **:** If one of the angles of a triangle is a right angle, it is called right-angled triangle.

Note (11) : In right angle triangle sum of other two angle is equal to 90^0 .

Note (12) : If sum of two angles is equal to third angle than triangle is right angled triangle.

Note (13) : In right angled triangle $c^2=a^2 + b^2$ where a, b & c are length of sides and c is greatest side.

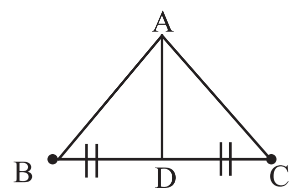
3) **Obtuse-angled Triangle** (अधिक कोण त्रिभुज)**:** If one of the angles of a triangle is an obtuse angle, it is called an obtuse-angled triangle.

Note (14) : If sum of two angle is less than 90^0, than triangle is obtuse angled triangle.

Note (15) : In Obtuse angled triangle $c^2 > a^2 + b^2$ where a, b & c are length of sides and c is greatest side.

Terms related to Triangle

1) **Median** (माध्यिका) **:** The straight line joining a vertex of a triangle to the mid-point of the opposite side is called a median. A triangle has three medians.

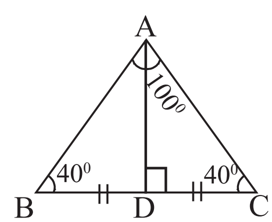


If BD = DC than AD is median

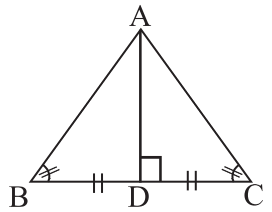
Note (16) : In equilateral triangle all the three medians are equal in length.

Note (17) : In Isosceles triangle medians drawn from vertex of each equal angles are equal. That means in Isosceles triangle two medians are equal.

Note (18) : In isosceles triangle median drawn from vertex of unequal angle is perpendicular to side.



Note (19) : In Isosceles triangle median drawn from vertex unequal angle is bisect the vertex angle.



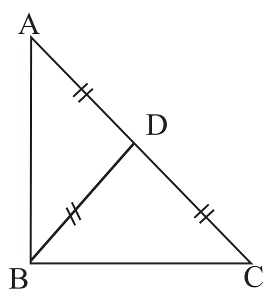
$\angle BAD = \angle DAC$

Note (20) : In scalene triangle all the three medians are unequal.

Note (21) : In any triangle median lie always inside of triangle.

Note (22) : The three medians of a triangle are concurrent (एक बिन्दुगामी) . That means they have a common point of intersection.

Note (23) : In right-angle triangle median drawn from vertex of right angle to hypotenuse is equal to half of the hypotenuse.

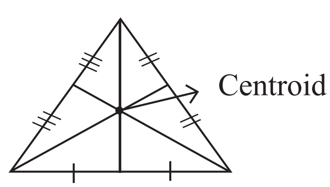


Or,

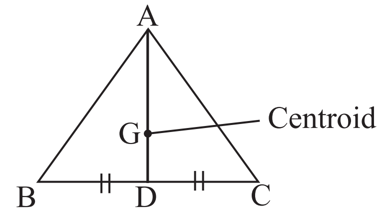
$$BD = \frac{1}{2} AC$$

If median is equal to half of its corresponding side than triangle must be right-angled triangle.

2) Centroid (केन्द्रक या गुरूत्व केन्द्र) **:** The three medians of a triangle always intersect each-other at the same point. This point of intersection of the medians is called centroid of the triangle.

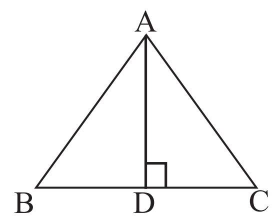


Note (24) : Centroid divides the median into the ratio of 2 : 1.



AG : GD = 2 : 1

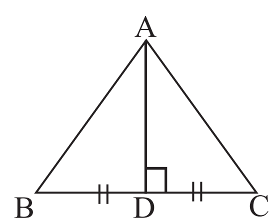
3) Altitude / Perpendicular / Height (ऊँचाई / लम्ब) : An altitude of a triangle, with respect to a side, is the perpendicular line segment drawn to the side from the opposite vertex.



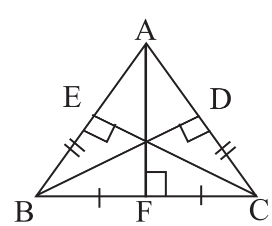
AD is altitude with respect to the side BC.

Note (25) : In equilateral triangle all the three perpendicular are equal in length.

Note (26) : In equilateral triangle perpendicular and median are same line segment.



Note (27) : In Isosceles triangle two perpendicular drawn from equal angles to equal sides are equal and perpendicular drawn from unequal angle to unequal side is also median and also angle bisector.



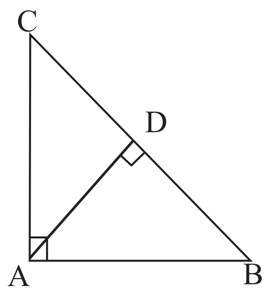
If AB = AC than BD = CE

AF is bisector of $\angle$A and also median.

Note (28) : In scalene triangle all the three perpendicular are unequal.

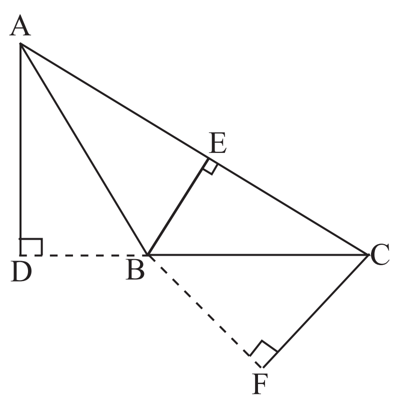
Note (29) : In acute angle triangle all the three perpendicular lie inside of triangle.

Note (30) : In right angle triangle two sides containing right angles are also altitude and one altitude from vertex of right angle is inside of triangle.



AC, AB & AD are altitudes.

Note (31) : In obtuse angle triangle two altitudes from acute angles are outside of triangle and one altitude is inside of triangle.



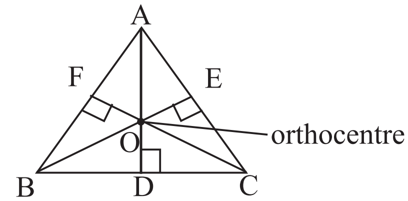
AD, BE and CF are altitudes on the sides BC, AC & AB respectively.

Note (32) : Greatest side has least altitude and least side has greatest altitude.

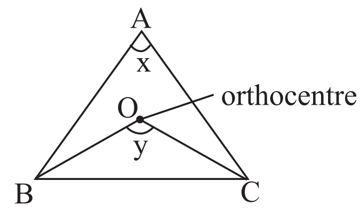
Note (33) : In any line segment joining vertex to opposite side perpendicular is shortest.

Note (34) : The three perpendiculars are concurrent.

4) Orthocentre (लम्बकेन्द्र) : The three altitudes of a triangle always intersect each-other at the same point. This point of intersection of the altitudes is called orthocenter of the triangle



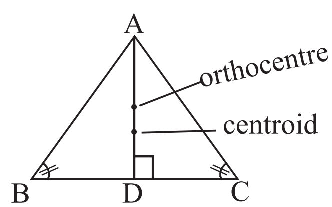
Note (35) : Angle made by any side on ortho-centre is supplementary of opposite angle.



$x + y = 180^0$

Note (36) : In equilateral triangle centroid and orthocentre are same point.

Note (37) : In isosceles triangle centroid and orthocenter are two different points lie on the perpendicular or median drawn from unequal angle to unequal side.

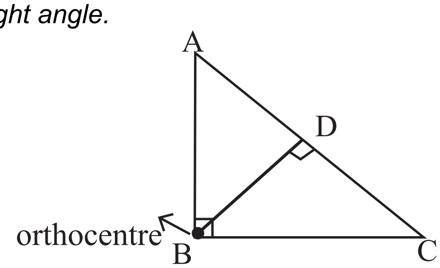


Note (38) : In isosceles triangle vertex centroid and orthocenter are collinear points.
Note (39) : A line segment joining centroid and orthocenter makes 90^0 with side or bisect the side then triangle is isosceles triangle.

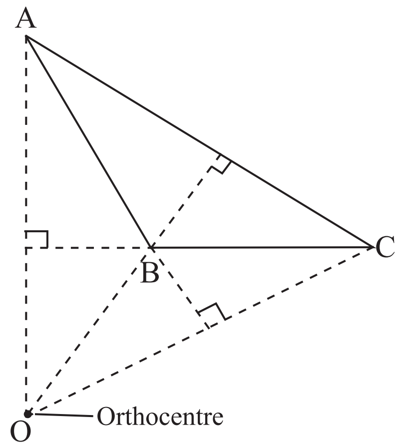
Note (40) : In scalene triangle vertex, centroid and orthocenter are three non-collinear points.

Note (41) : In acute angle triangle orthocenter lie inside of triangle.

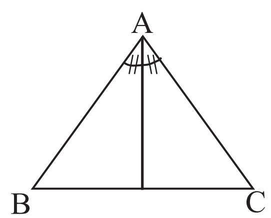
Note (42) : In right angle triangle orthocenter is vertex of right angle.



Note (43) : In obtuse angle triangle orthocentre lie outside of triangle.



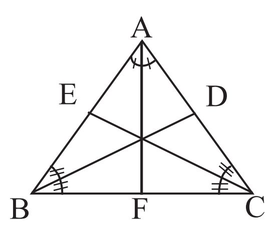
5) **Angle Bisector** (अर्द्धक) **:** A line segment joining vertex to opposite side such that it bisect the vertex angle.



Note (44) : In equilateral triangle all the three angle bisectors are equal in length.

Note (45) : In equilateral triangle angle bisector, perpendicular and median are same line segment.

Note (46) : In isosceles triangle angle bisectors drawn from equal angles are equal in length and angle bisector drawn from unequal angle is also perpendicular and median.



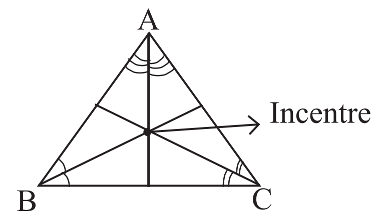
In Δ ABC, $\angle$B = $\angle$C & BD, CE & AF are angle bisectors than BD = CE

Note (47) : In scalene triangle all the three angle bisectors are unequal in length.

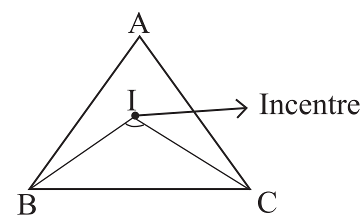
Note (48) : Angle bisector lie always inside of Δ.

Note (49) : All the three angle-bisectors are concurrent.

6) **Incentre** (अंतः केन्द्र) **:** The point of intersection of the internal bisectors of the angles of a triangle is called its incentre.

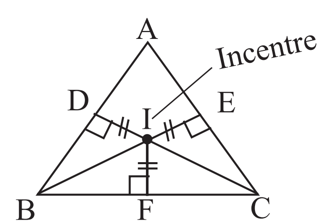


Note (50) : Angle made by any side on incentre is equal to 90^0 + half of opposite angle.



$$\angle BIC = 90^\circ + \frac{1}{2}\angle A$$

Note (51) : In-centre is equidistant from all the three sides.



ID = IE = IF

Note (52) : In equilateral, triangle centroid, orthocenter, and incentre are same point.

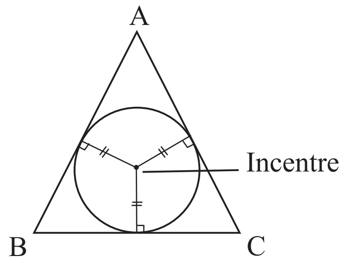
Note (53) : In Isosceles triangle centroid, orthocenter and incentre are three different points situated on median/ perpendicular/ angle-bisector drawn from unequal angle to unequal side.

Note (54) : In isosceles triangle centroid, orthocentre and incentre are three different collinear points.

Note (55) : In scalene triangle centroid, orthocenter and incentre are three different non-collinear points.

Note (56) : In any triangle incentre lie inside of the triangle.

7) **Incircle (अंतःवृत्त) :** A circle inside of triangle touches all the three sides of triangle and its centre is incentre of triangle.



Note (57) :

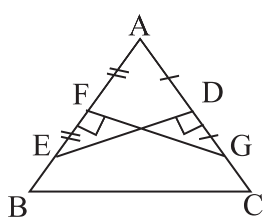
$$\text{Inradius} = \frac{\text{Area of triangle}}{\text{semi perimetre of triangle}}$$

8) **Perpendicular Bisector (लम्ब समद्विभाजक) :** If a line passes through the mid-point of a side of a triangle and perpendicular to it, then the line is called the perpendicular bisector of the line segment.

Note (58) : In equilateral triangle all the three perpendicular bisector are equal in length.

Note (59) : In equilateral triangle perpendicular bisector, median, perpendicular and angle bisector are same line segment.

Note (60) : In isosceles triangle perpendicular bisectors drawn on equal sides are equal in length.



DE = FG

Note (61) : In isosceles triangle perpendicular bisectors drawn on equal sides not going through vertices.

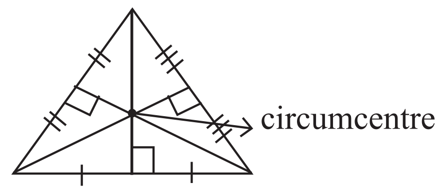
Note (62) : In isosceles triangle perpendicular bisector drawn on unequal side is also median, perpendicular and angle bisector.

Note (63) : In scalene triangle all the three perpendicular bisectors are unequal.

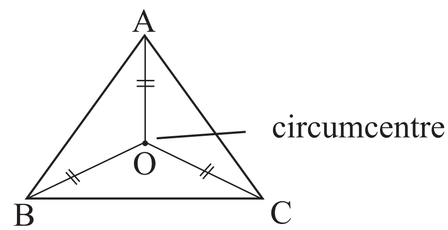
Note (64) : In scalene triangle all the three perpendicular bisector not going through vertices.

Note (65) : All the three perpendicular bisectors are concurrent.

9) **Circumcentre (परिकेन्द्र) :** The point of intersection of the perpendicular bisectors of the sides of a triangle is called circumcentre.

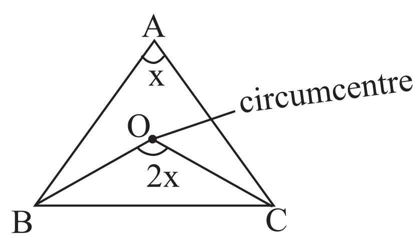


Note (66) : Circumcentre is equidistant from all the three vertices.



OA = OB = OC

Note (67) : Angle made by any side on circumcentre is twice of opposite angle.



Note (68) : In quilateral triangle centroid, orthocenter, incentre and circumcentre are same point.

Note (69) : In Isosceles triangle centroid, orthocenter, incentre and circumcentre are four different points situated on one line segment joining vertex and mid-point of unequal side.

Note (70) : In Isosceles triangle centroid orthocenter, incentre and circumcentre are four different collinear points.

Note (71) : In scalene triangle centroid, orthocenter, incentre and circumcentre are four different non-collinear points.

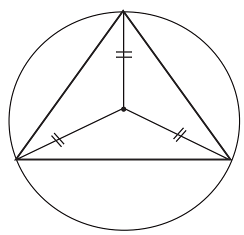
Note (72) : In acute triangle circumcentre lie inside of triangle.

Note(73): In right angle triangle circumcentre is mid-point of hypotenuse.

Note (74) : In obtuse angle triangle circumcentre lies outside of the triangle.

Note(75): Line segment joining circumcentre and mid-point of side is perpendicular on side or vice-versa.

10) **Circumcircle (परिवृत्त) :** Circumcircle is a circle passing through all the three vertices of triangle and its centre is circumcentre of triangle .



Note (76) : Length of circum radius =

$$\frac{\text{product of sides}}{4 \times \text{Area of triangle}}$$

Note (77) : In right angle triangle circum radius is equal to half of its hypotenuse.

Note (78) : In right angle triangle hypotenuse is the diameter of circumcircle.

Note (79) : In equilateral triangle

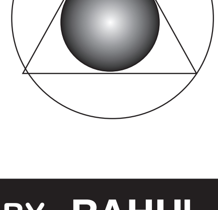
(i) Inradius : circumradius = 1 : 2

(ii) Area of incircle : Area of circumcircle= 1 : 4

(iii) In radius = $\frac{a}{2\sqrt{3}}$

(iv) Circum radius = $\frac{a}{\sqrt{3}}$

(v) Area of shaded region : Area of unshaded region = 1 : 3



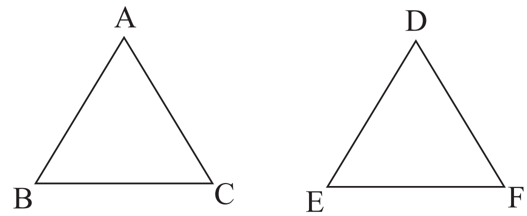
Congruence of Triangle

(त्रिभुजों की सर्वांगसमता)

Two triangle are said to be congruent if they are equal in shape and size both.

Or,

Two triangles are congruent if and only if one of them can be made to superpose (एक दूसरे पर रेख देना) on the other, so as to cover it exactly.



If ΔABC superposes on ΔDEF exactly such that the vertices of ΔABC fall on the vertices of ΔDEF in the following order.

A $\leftrightarrow$ D, B $\leftrightarrow$ E, C $\leftrightarrow$ F

Then we have following six equalities –

AB = DE, BC = EF, CA = FD

(i.e., corresponding sides are congruent)

$\angle$A = $\angle$D, $\angle$B = $\angle$E, $\angle$C = $\angle$F

(i.e., corresponding angles are congruent)

(i) Corresponding sides (संगत भुजा) = Sides opposite to equal angle.

(ii) Corresponding angles (संगत कोण) = Angles opposite to equal sides.

(iii) If ΔABC $\cong$ ΔDEF that means

$\angle$A = $\angle$D, $\angle$B = $\angle$E, $\angle$C = $\angle$F and

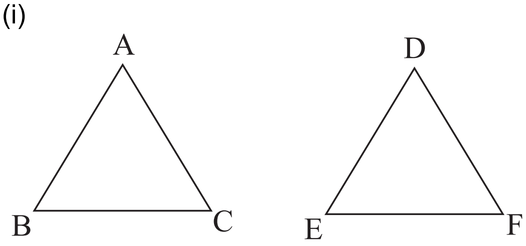
AB = DE, BC = EF, Ac = DF

(iv) If ΔDEF $\cong$ ΔABC and ΔDEF $\cong$ ΔPQR then ΔABC $\cong$ ΔPQR

SUFFICIENT CONDITIONS (CRITERIA) FOR CONGRUENCE OF TRIANGLES

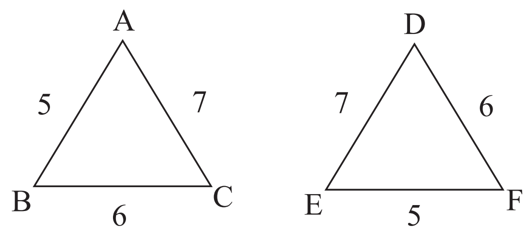
1) **SIDE – SIDE – SIDE (S-S-S) Congruence Criterion :** Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.

(i)



If AB = DE, BC = EF & AC = DF, then Δ ABC $\cong$ Δ DEF.

(ii)



(a) Δ ABC $\cong$ Δ EFD (RIGHT)

(b) Δ ABC $\cong$ Δ DFE (WRONG)

(c) Δ ABC $\cong$ Δ FED (WRONG)

(d) Δ CAB $\cong$ Δ DEF (RIGHT)

(e) Δ BAC $\cong$ Δ FED (RIGHT)

(iii) PQ = LM, QR = MN & PR = LN

(a) Δ PQR $\cong$ Δ LMN (RIGHT)

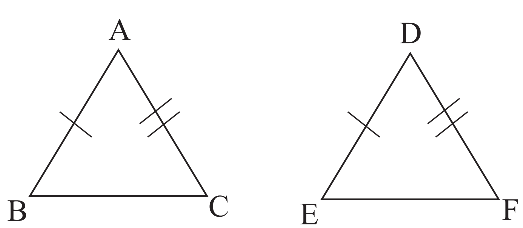
(b) Δ PRQ $\cong$ Δ LNM (RIGHT)

(c) Δ QRP $\cong$ Δ MNL (RIGHT)

(d) Δ QPR $\cong$ Δ LMN (WRONG)

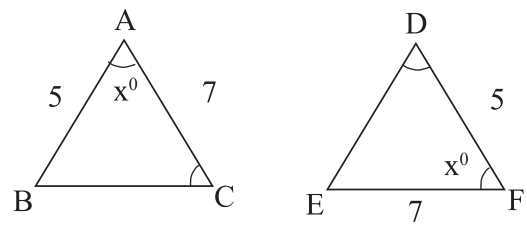
2) **SIDE-ANGLE-SIDE (S-A-S) Congruence Criterion :** Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle.

(i)



If AB = DE, AC = DF & $\angle A = \angle D$

then Δ ABC $\cong$ Δ DEF



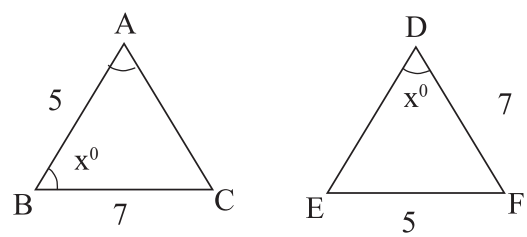
(a) Δ ACB $\cong$ Δ FED (RIGHT)

(b) Δ BAC $\cong$ Δ DFE (RIGHT)

(c) Δ BCA $\cong$ Δ FED (WRONG)

(d) Δ CBA $\cong$ Δ DEF (WRONG)

(iii)

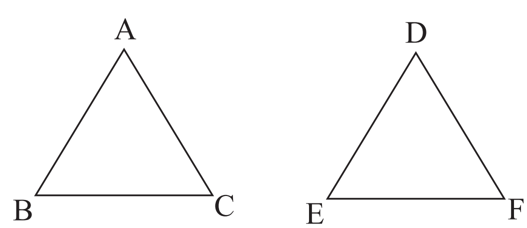


Two triangles are not congruent because the equal angle should be the angle included between the sides.

(iv) PQ = ST, QR = TM & $\angle Q = \angle T$

then, Δ PQR $\cong$ Δ STM

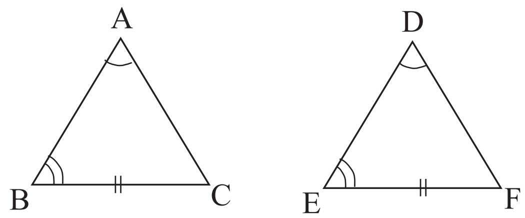
3) **ANGLE-SIDE-ANGLE (A-S-A) Congruence Criterion :** Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.



If $\angle B = \angle E$, $\angle C = \angle F$ & BC = EF then,

Δ ABC $\cong$ Δ DEF

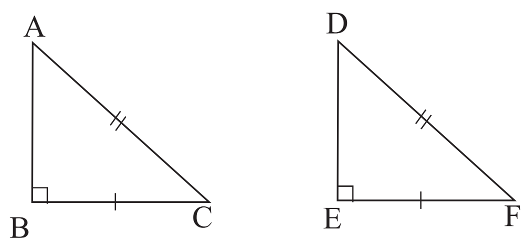
4) **ANGLE-ANGLE-SIDE (A-A-S) Criterion of Congruence :** If any two angles and a non-included side of one triangle are equal to the corresponding angles and side of another triangle, then two triangles are congruent.



If $\angle A = \angle D$, $\angle B = \angle E$ & $BC = EF$

then, $\Delta ABC \cong \Delta DEF$

5) **RIGHT ANGLE-HYPOTENUSE-SIDE (R-H-S) Congruence Criterion :** Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle.



$AC = DF$, $BC = EF$ & $\angle B = \angle E = 90^0$
then,

$\Delta ABC \cong \Delta DEF$

<u>PROPERTIES RELATED TO CONGRUENCE OF TRIANGLES</u>

Note-1 : *If two triangles are congruent then their corresponding sides are equal.*

Note-2 : *If two triangles are congruent then their corresponding angles are equal.*

Note-3 : *If two triangles are congruent then they must be equiangular but if two triangles are equiangular then they need not be congruent.*

Note-4 : *If two triangles are congruent then they are equal in area and perimeter.*

Note-5 : *If two triangles are congruent then their all corresponding parts are equal.*

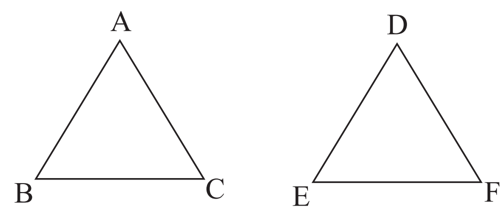
SIMILARITY OF TRIANGLES

(त्रिभुजों की समरूपता)

Two triangle are said to be similar if they are equal in shape (आकार) but need not to be equal in size.

Or

Two triangles said to be similar if their corresponding angles are equal and their corresponding sides are proportional

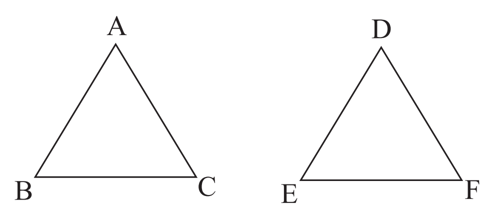


If $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and

$\frac{AD}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ then ΔABC $\Delta ABC \sim \Delta DEF$

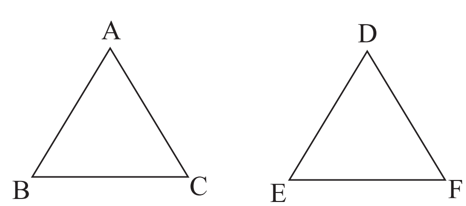
<u>CRITERIA OF SIMILARITY</u>

1) **A – A / A – A – A :** If two triangles are equiangular, then they are similar.



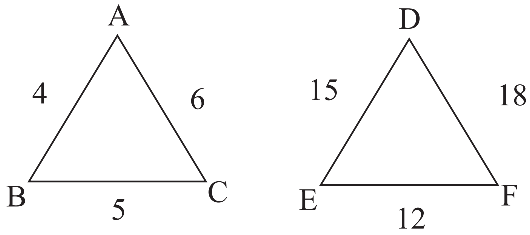
$\angle A = \angle D$, $\angle B = \angle E$ & $\angle C = \angle F$ then $\Delta ABC \sim \Delta DEF$.

2) **S – S – S :** If the corresponding sides of two triangles are proportional, then they are similar.
(i)



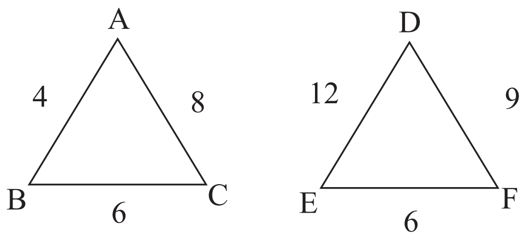
If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\Delta ABC \sim \Delta DEF$

(ii)



$$\frac{AB}{EF}=\frac{BC}{DE}=\frac{AC}{DF}$$, then $\Delta ABC \sim \Delta FED$

(iii)

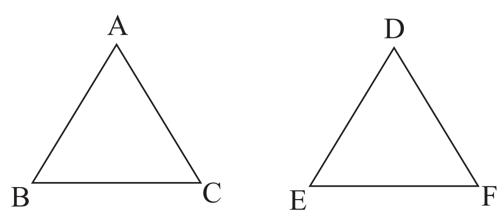


$$\begin{array}{ccc}4 & 6 & 8\\ \downarrow & \downarrow & \downarrow\\ 6 & 9 & 12\end{array} \qquad \frac{4}{6}=\frac{6}{9}=\frac{8}{12}$$

$$\frac{AB}{EF}=\frac{BC}{FD}=\frac{CA}{DE}$$, then $\Delta ABC \sim \Delta EFD$

3) S – A – S : If in two triangles, one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar.

(i)



$\angle A = \angle D$ and $\dfrac{AB}{DE}=\dfrac{AC}{DF}$

then $\Delta ABC \sim \Delta DEF$

(ii) $\dfrac{AB}{EF}=\dfrac{AC}{DF}$ & $\angle A = \angle F$

then $\Delta ABC \sim \Delta FED$

<u>Properties related to Similarity</u>

Note-1 : *If two triangles are similar than their corresponding sides are proportional.*

If $\Delta ABC \sim \Delta DEF$ then $\dfrac{AB}{DE}=\dfrac{AC}{DF}=\dfrac{BC}{EF}$

Note-1 : *If two triangles are similar than their all corresponding parts (except angles) are proportional. That means ratio of corresponding sides = ratio of corresponding median = ratio of corresponding height = ratio of corresponding angle bisector = ratio of corresponding perpendicular bisector.*

Note- 3 : *If two triangles are similar than they are equiangular and if two triangles are equiangular then they are similar.*

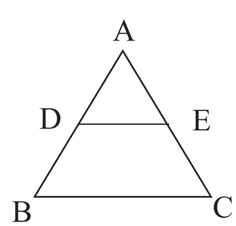
If $\Delta ABC \sim \Delta DEF$ then

$\angle A = \angle D$, $\angle B = \angle E$ & $\angle C = \angle F$

Note- 4 : *If two triangles are similar than ratio between their perimeter is equal to ratio between their corresponding sides.*

Note-5 : *If two triangles are similar than ratio between their area is equal to ratio between square of their corresponding sides.*

Note-6 : *Line segment joining two sides parallel to third side is divide triangle into two parts and forms a new triangle similar to original triangle.*



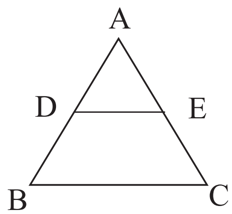
If DE | | BC then there are two triangles ΔADE & ΔABC

$\angle A = \angle A$
$\angle D = \angle B$
$\angle E = \angle C$

then $\Delta ADE \sim \Delta ABC$

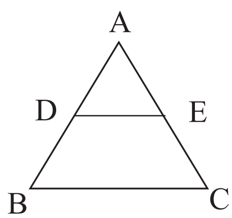
$\because \quad \dfrac{AD}{AB} = \dfrac{AE}{AC} = \dfrac{DE}{BC}$

Note -7 : *(Basic proportionality theorem or Thales theorem)- If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.*



If DE | | BC then $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Note – 8 :



If DE | | BC then,

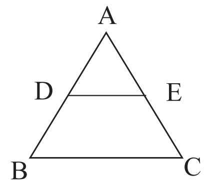
(i) $\Delta ADE \sim \Delta ABC$

(ii) $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ (iii) $\dfrac{DB}{AD} = \dfrac{EC}{AE}$

(iv) $\dfrac{AB}{AD} = \dfrac{AC}{AE}$ (iv) $\dfrac{AD}{AB} = \dfrac{AE}{AC}$

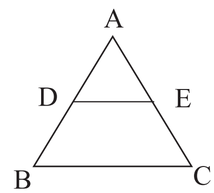
(vi) $\dfrac{AB}{DB} = \dfrac{AC}{EC}$ (vii) $\dfrac{DB}{AB} = \dfrac{EC}{AC}$

Note-9 : *(Converse of the basic proportionally theorem) – If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.*



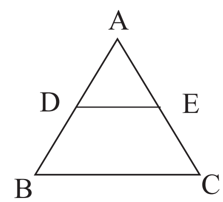
If $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ then DE | | BC

Note-10 : *The line drawn from the mid-point of one side of a triangle parallel to another side bisect the third side.*



If DE | | BC & D is midpoint of AB then AE = EC

Note-11 : *The line joining the midpoint of two sides of a triangle is parallel and equal to half of third side.*



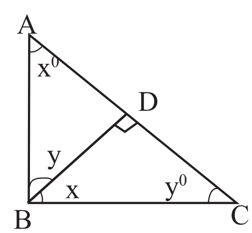
If D & E are midpoints of AB & AC then,
(i) DE | | BC

(ii) DE = $\dfrac{1}{2}$ BC

(iii) Area of Δ ADE : Area of Δ ABC = 1 : 4

(iv) Area of ΔADE : Area of $\square$ DBCE = 1 : 3
(v) Area of Δ ABC : Area of $\square$ DBCE = 4 : 3

Note-12 : *In right angle triangle perpendicular drawn from the vertex of right angle to hypotenuse is divide the triangle into two parts and forms two small triangle similar to each other and also similar to original triangle.*



$\Delta ABD \sim \Delta BCD \sim \Delta ACB$

(a) (i) $AB^2 = AC \times AD$

(ii) $BC^2 = AC \times CD$

(iii) $BD^2 = AD \times CD$

(b) (i) $\frac{AB^2}{BD^2} = \frac{AC}{CD}$

(ii) $\frac{BC^2}{BD^2} = \frac{AC}{AD}$

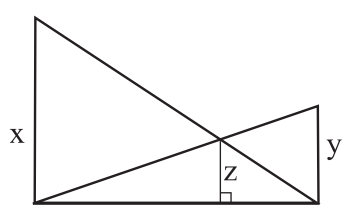
(iii) $\frac{AB^2}{BC^2} = \frac{AD}{CD}$

(c) (i) $\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$

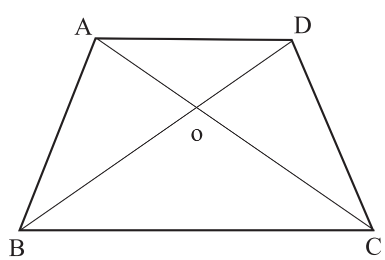
(ii) $BD = \frac{AB \times BC}{AC}$

Note-13 : *Two poles of height x and y metres are 'p' meters apart (x > y). The height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is z metre than relation between x, y & z is –*

(i) $\frac{1}{z} = \frac{1}{x} + \frac{1}{y}$ (ii) $z = \frac{xy}{x+y}$



Note-14 : *Diagonal of trapezium divide it into four triangles. Two triangles along with parallel side are similar and two triangles along with non-parallel sides are equal in area.*

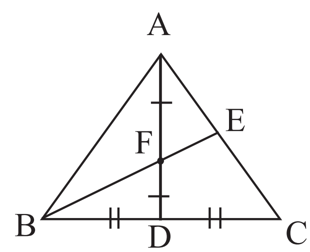


If AD | | BC then

(i) $\Delta AOD \sim \Delta COB$ &

(ii) Area of Δ AOB = Area of ΔCOD

Note-15 : *Line segment joining vertex and midpoint of a median divide the third side into the ratio of 1 : 2.*

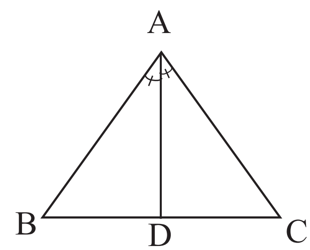


If AD is median and F is midpoint of AD then

(i) AE : EC = 1 : 2

(ii) AE = $\frac{1}{3}AC$

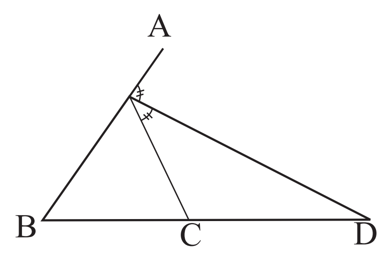
Note-16 : *The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.*



$$\frac{BD}{DC} = \frac{AB}{AC}$$

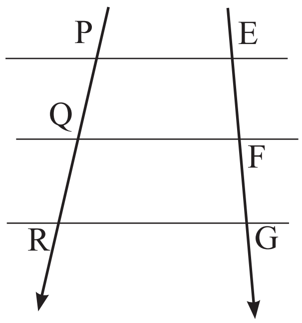
Note-17 : *If a line through one vertex of a triangle divides the opposite sides in the ratio of other two sides, then, the line bisects the angle at the vertex.*

Note-18 : *The external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle.*



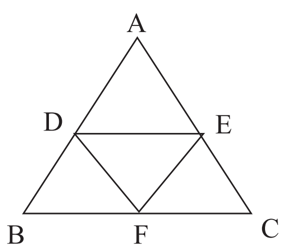
$$\frac{BD}{CD} = \frac{AB}{AC}$$

Note -19 : *If three or more parallel lines are intersected by two transversals, than the intercepts made by them on the transversals are proportional.*



$$\frac{PQ}{QR}=\frac{EF}{FG}$$

Note -20 : *The line segment joining the midpoints of the sides of a triangle form four congruent triangles, each of which is similar to the original triangle. That means all the four triangles are equal in area.*



PROPERTIES RELATED TO TRIANGLE

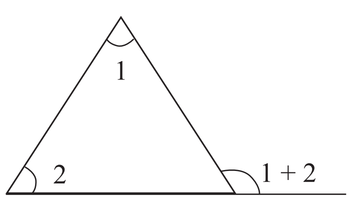
(त्रिभुज के गुण)

1) The sum of the three angles of a triangle is 180^0 .

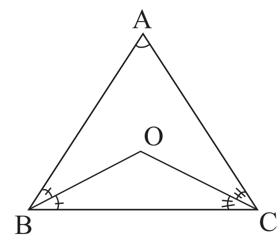
2) Sum of any two sides of a triangle is greater than the third side.

3) Difference between any two sides of a triangle is less than the third side.

4) If a side of triangle is produced the exterior angle so formed is equal to the sum of the two interior opposite angles.

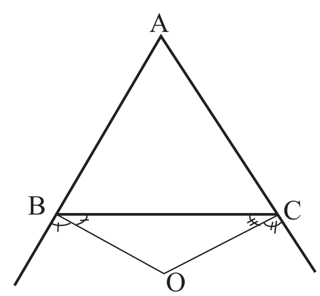


5) Angle made by bisector of any two angles is equal to $90^\circ+\frac{1}{2}$ of third angle.



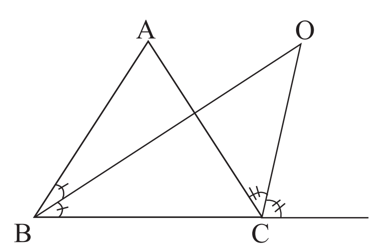
$$\angle BOC = 90^\circ + \frac{1}{2}\angle A$$

6) Angle made by bisectors of any two exterior angles is equal to $90^\circ-\frac{1}{2}$ of third angle.



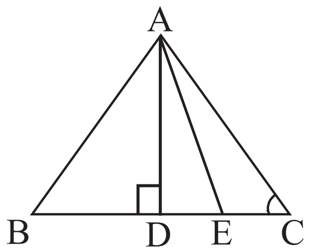
$$\angle BOC = 90^\circ - \frac{1}{2}\angle A$$

7) Angle made by bisectors of one interior and one exterior angle is equal to half of third angle.



$$\angle BOC = \frac{1}{2}\angle A$$

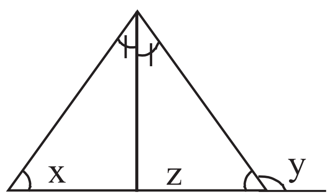
8) Angle made by perpendicular and angle bisector on vertex is equal to half of difference of other two angles.



If AD ⊥ BC and AE is bisector of ∠A then

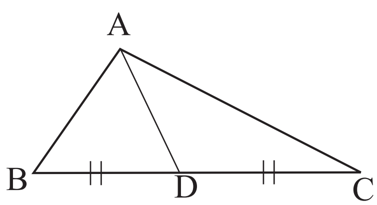
$$\angle DAE = \frac{1}{2}(\angle B - \angle C)$$

9) Sum of exterior and interior opposite angle is equal to twice of the angle made by angle bisector on the same side.



x + y = 2z

10) Sum of any two sides greater than twice the median drawn to the third side.

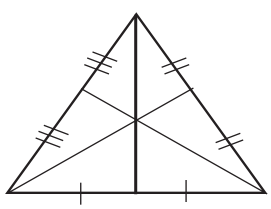


AB + AC > 2 AD

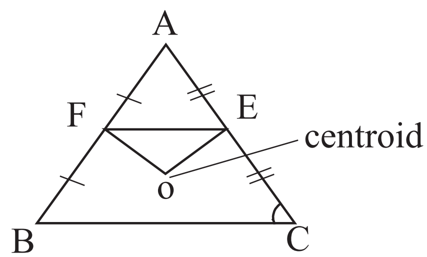
11) Perimetre of a triangle is greater than the sum of its three medians.

12) The sum of three altitudes of a triangle is less than the sum of three sides of the triangle.

13) Median divide the triangle into six small triangles equal in area.

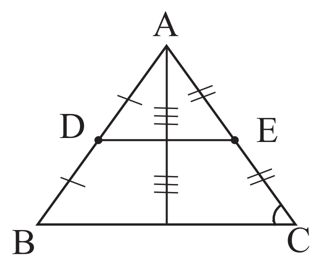


14) Area of triangle form by centroid and midpoint of any two sides is equal to $\frac{1}{12}$ of original triangle.

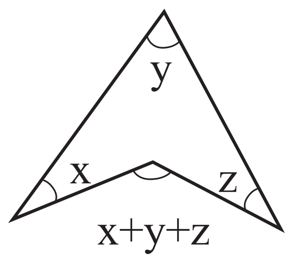


Area of ΔOEF = $\frac{1}{12}$ Area of ΔABC

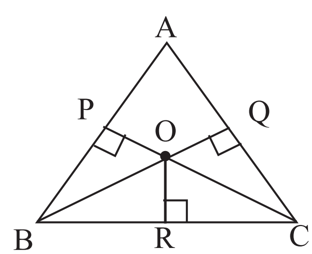
15) Any line segment joining vertex to opposite side is bisected by the line segment joining midpoints of others two sides.



16)



17) In equilateral triangle, sum of perpendicular distances of all the three sides from any point inside of triangle is equal to height of triangle.



Height of equilateral triangle ABC = OP + OQ + OR

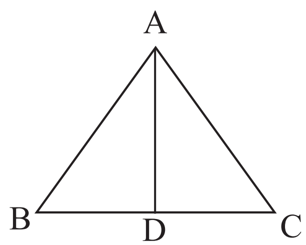
18) In equilateral triangle –

(i) Side : Height = $2 : \sqrt{3}$

(ii) $(\text{Side})^2 : (\text{Height})^2$ = 4 : 3

(iii) $3 \times \text{side}^2$ = $4 \times \text{Height}^2$

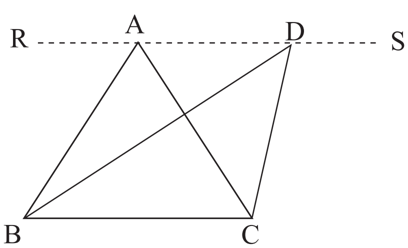
19) Line segment joining vertex to opposite side, divides triangle into two parts and ratio between area of these two triangles is equal to ratio between their bases.



$$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ADC} = \frac{BD}{DC}$$

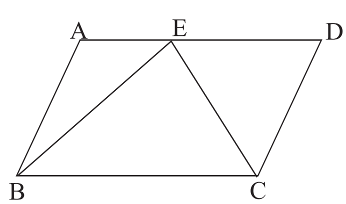
20) Median divides the triangle into two parts equal in area.

21) Two triangle having same base and between same parallel equal in area.



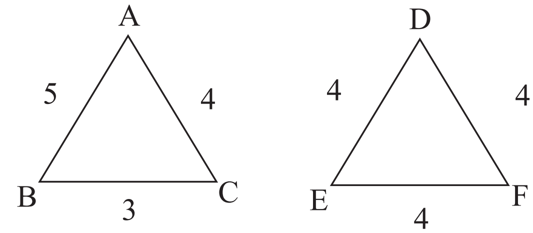
If RS | | BC then
Area of ΔABC = Area of ΔBDC

22) Area of triangle is equal to half of area of a parallelogram having same base and between same parallel.



Area of ΔBEC = $\frac{1}{2}$ Area of parallelogram ABCD

23) Two triangle having equal perimeter, equilateral triangle is maximum in area.

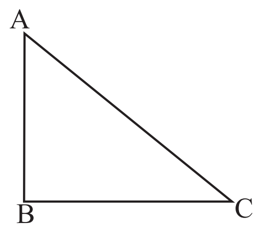


Area of ΔDEF > Area of ΔABC

24) Two triangle inscribed in circle equilateral triangle is maximum in area.

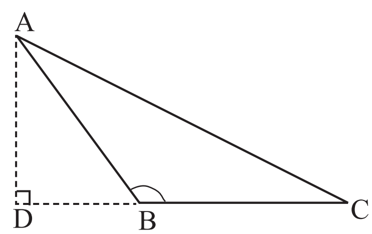
25) PYTHAGORAS THEOREM

(i) In Right Angle Triangle –



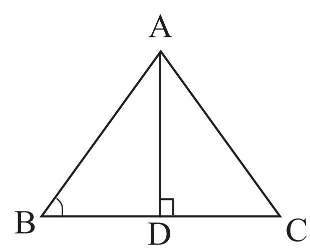
$AC^2 = AB^2 + BC^2$

(ii) In Obtuse Angle Triangle –



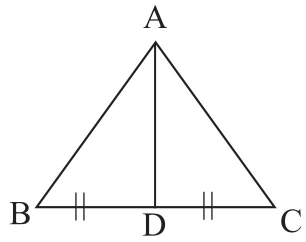
$AC^2 = AB^2 + BC^2 + 2BC \times BD$

(iii) In Acute Angle Triangle –



$AC^2 = AB^2 + BC^2 - 2BC \times BD$

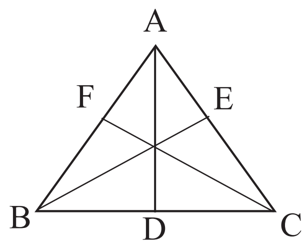
(iv)



AD is median

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

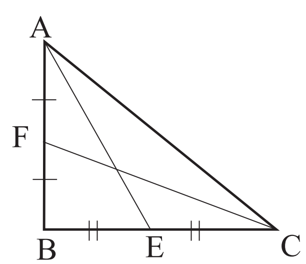
(v)



AD, BE & CF are medians

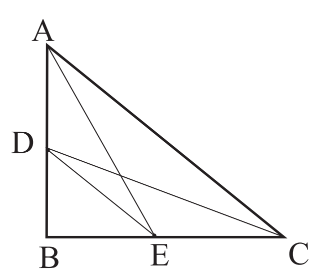
$3 (AB^2 + BC^2 + CA^2) = 4 (AD^2 + BE^2 + CF^2)$

(vi) In a right angle triangle, four times the sum of the square of the medians drawn from the acute angles is equal to five times the square of the hypotenuse.



$4 (AE^2 + CF^2) = 5AC^2$

(vii) ABC is right triangle right-angled at B. D and E be any points on AB and BC respectively. Then $AE^2 + CD^2 = AC^2 + DE^2$



(viii) Basic Pythagorean Triplets –

(3, 4, 5),	(5, 12, 13),	(7, 24, 25),
(8, 15, 17),	(9, 40, 41),	(11, 60, 61)

$$n + \frac{n}{2n+1}$$

If n = 1 =>

$$1 + \frac{1}{2\times 1+1} = 1 + \frac{1}{3} = \frac{(4)_{\pm 1}}{(3)}(5)$$

(3, 4, 5)

If n = 2 =>

$$2 + \frac{2}{2\times 2+1} = 2 + \frac{2}{5} = \frac{(12)_{\pm 1}}{(5)}(13)$$

(5, 12, 13)

<u>FORMULA RELATED TO TRIANGLE</u>

1) Area of triangle $= \frac{1}{2}$ x base x height

2) Area of triangle $= \sqrt{s(s-a)(s-b)(s-c}$

Where, s $= \frac{a+b+c}{2}$ and a, b & c are length of sides.

3) Area of triangle $= \frac{4}{3}\sqrt{s(s-a)(s-b)(s-c}$

Where, s $= \frac{a+b+c}{2}$ and a, b & c are length of medians.

4) Height of equilateral triangle $= \frac{\sqrt{3}}{2}\times \text{side}$

5) Length of side of equilateral triangle

$= \frac{2}{\sqrt{3}}\times \text{height}$

6) Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times \text{side}^2$

7) Area of equilateral triangle = $\frac{(\text{height})^2}{\sqrt{3}}$

8) Height of isosceles triangle = $\frac{1}{2}\sqrt{4b^2 - a^2}$
Where b is equal side

9) Area of isosceles triangle = $\frac{a}{4}\sqrt{4b^2 - a^2}$

QUADRILATERAL (चतुर्भुज)

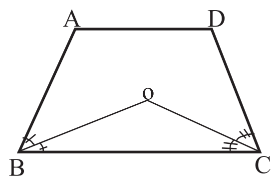
A geometrical figure bounded by four line segment is called quadrilateral.

Properties related to quadrilateral

1) Sum of interior angles is equal to 360^0

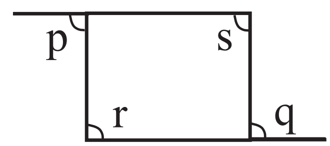
2) Sum of exterior angles is equal to 360^0

3) Angle made by bisectors of any two consecutive angles is equal to half of sum of others to angles.



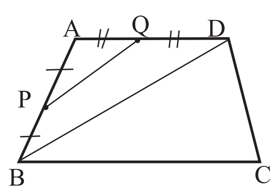
$$\angle BOC = \frac{1}{2}(\angle A + \angle D)$$

4) Sum of pair of interior opposite angle is equal to sum of pair of other two exterior opposite angles.



r + s = p + q

5) Line segment joining midpoints of any two adjacent sides is parallel and equal to half of corresponding diagonal.



PQ || BD & PQ = $\frac{1}{2}BD$

6) Quadrilateral formed by line segment joining midpoints of sides of original quadrilateral is a parallelogram.

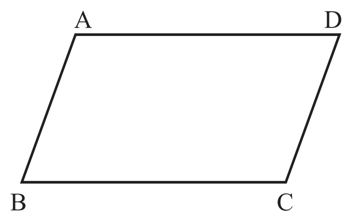
7) Area of quadrilateral joining midpoints of sides is equal to half of original quadrilateral.

TYPES OF QUADRILATERAL

1) Parallelogram (समांतर चतुर्भुज)
2) Rectangle (आयत)
3) Square (वर्ग)
4) Rhombus (समचतुर्भुज)
5) Trapezium (समलम्ब चतुर्भज)

Parallelogram (समांतर या समानांतर चतुर्भुज)

A quadrilateral whose both pair of opposite sides are parallel, is called a parallelogram.



AD | | BC & AB | | CD

A quadrilateral is a parallelogram if any one of the following holds :

1) Each pair of opposite sides are parallel.

Or

2) Each pair of opposite sides are equal.

Or

3) Each pair of opposite angles are equal.

Or

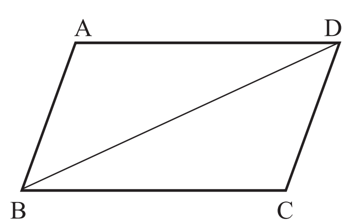
4) One pair of opposite sides are parallel and equal.

Or

5) Diagonal bisect each other.

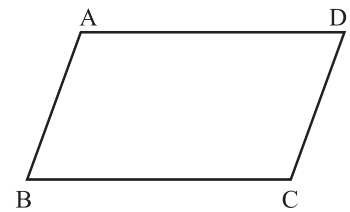
Properties related to parallelogram

1) In parallelogram, diagonal bisect each other and each diagonal bisect the parallelogram into two congruent triangles.



$\Delta ABD \cong \Delta CDB$

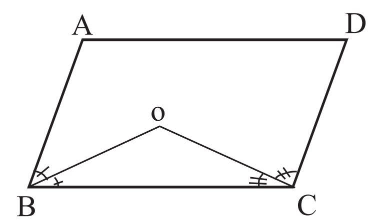
2) Bisectors of the angles of a parallelogram form a rectangle.
3) Sum of any two consecutive angles are supplementary.



$\angle A + \angle B = \angle B + \angle C = \angle C + \angle D =$

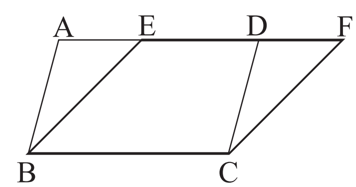
$\angle D + \angle A = 180^0$

4) Bisector of any two consecutive angles intersect at 90^0 .



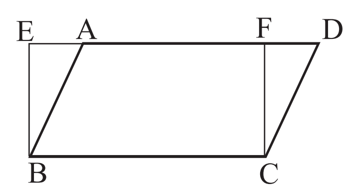
$\angle BOD = 90^0$

5) Two parallelograms having same base and between same parallel equal in area.



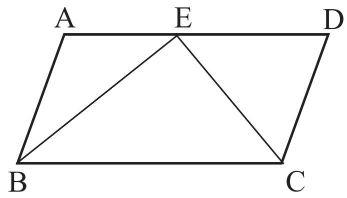
Area of | | gm ABCD = Area of | |gm EBCF

6) Area of a parallelogram and a rectangle having same base and between same parallel equal in area.



Area of | | gm ABCD = Area of rectangle EBCF

7) Area of a triangle is equal to half of area of a | | gm having same base and between same parallel.

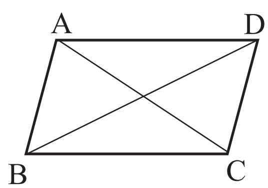


Area of $\Delta BEC = \frac{1}{2}$ Area of ||gm ABCD

8) Parallelogram inscribed in circle is rectangle or square.

9) Parallelogram circumscribed is circle is rhombus or square.

10) Sum of squares of sides is equal to sum of squares of diagonal..



(i) $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

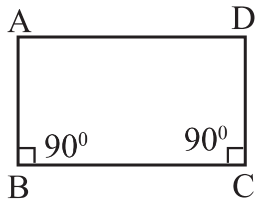
(ii) $AC^2 + BD^2 = 2(AB^2 + BC^2)$

11) Area of parallelogram = Base x Height

12) If the diagonals of a parallelogram are equal then all its angles are right angles that means it is a rectangle or square.

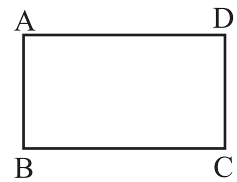
RECTANGLE

Rectangle is a parallelogram in which each angle is equal to 90⁰.



Properties of Rectangle

1) Pair of opposite sides are equal.



AD = BC & AB = CD

2) Each angle is equal to 90^0

3) Diagonals are equal.

4) Diagonal bisect each other but not perpendicularly.

5) Diagonals are not angle bisector.

6) Line segment joining midpoints of rectangle forms a rhombus

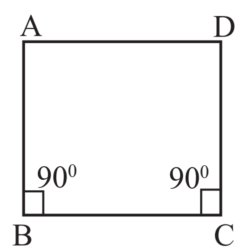
7) Area of rectangle = Length x Breadth

8) Perimetre of rectangle = 2 (ℓ + b)

9) Diagonal rectangle = $\sqrt{l^2 + b^2}$

SQUARE

Square is a parallelogram in which all sides are equal and each angle is equal to 90^0.



AB = BC = CD = DA

Properties of Square

1) All sides are equal

2) Each angles is equal to 90^0 .

3) Diagonals are equal.

4) Diagonals bisect each-other perpendicularly.

5) Diagonals are angle bisector.

6) Area = $(\text{Side})^2$

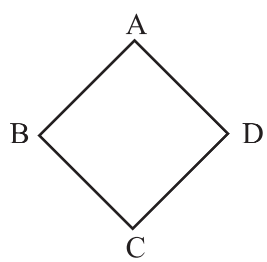
7) Perimetre = 4 x side

8) Diagonal = $\sqrt{2} \times \text{side}$

9) Line segment joining midpoints of sides is form a square.

RHOMBUS

Rhombus is a parallelogram in which all sides are equal.



AB = BC = CD = DA

Properties of Rhombus

1) All sides are equal.

2) Pair of opposite angles are equal.

3) Diagonals are not equal.

4) Diagonals bisect each other perpendicularly.

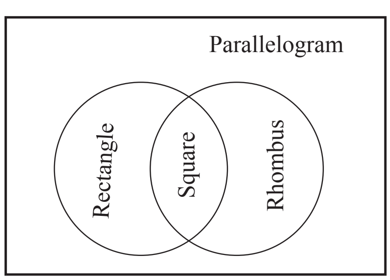
5) Diagonals are angle bisector

6) Area = $\frac{1}{2} \times$ product of diagonals

7) Perimetre = 4 x side

8) Line segment joining midpoints of sides forms a rectangle.

Relation between Parallelogram, Rectangle, Square & Rhombus

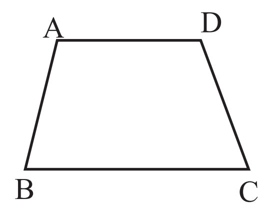


Diagonal side & Angle properties of all Parallelogram

Diagonal side & Angle properties	Parallelogra	Rectangle	Rhombus	Square
1) Diagonal bisect each other	✓	✓	✓	✓
2) Diagonals are equal	×	✓	×	✓
3) Diagonal is angle bisector	×	×	✓	✓
4) Diagonals are perpendicular to each other	×	×	✓	✓
5) Diagonal makes 4 congruent triangle	×	×	✓	✓
6) All sides are equal	×	×	✓	✓
7) All angle are Right Angle.	×	✓	✓	×

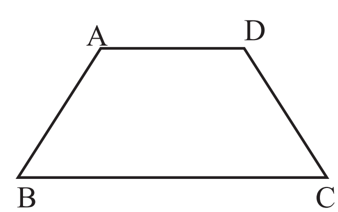
Trapezium (समलम्ब चतुर्भुज)

A quadrilateral in which one pair of opposite sides are parallel.



AD | | BC

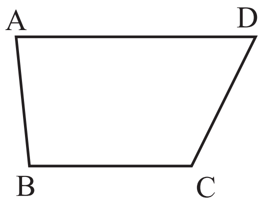
If non-parallel sides are equal then it is called isosceles trapezium.



AD | | BC & AB = CD

Properties related to trapezium

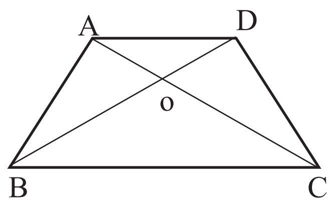
1) Consecutive angles along both parallel sides are supplementary.



AD | | BC then

$\angle A + \angle B = \angle D + \angle C = 180^0$

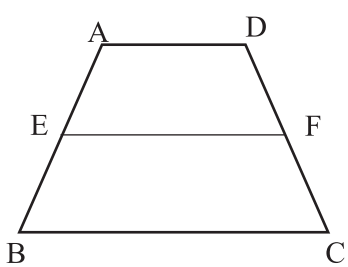
2) Diagonal of trapezium intersect each other proportionally.



$$\frac{AO}{OC} = \frac{OD}{OB}$$

3) If the diagonal of a quadrilateral divide each other proportionally, then it is a trapezium.

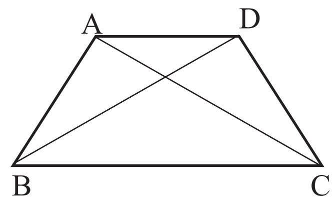
4) Any line parallel to the parallel side of a trapezium divides the non-parallel sides proportionally.



AD | | EF | BC then

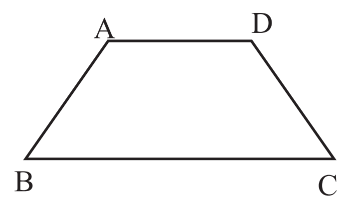
$$\frac{AE}{EB} = \frac{DF}{FC}$$

5) In Isosceles trapezium diagonals are equal.



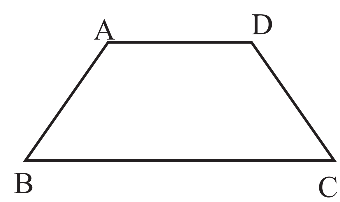
If AD | | BC & AB = CD then
AC = BD

6) In Isosceles trapezium consecutive angles along each parallel sides are equal.



$\angle B = \angle C$ & $\angle A = \angle D$

7) In Isosceles trapezium pair of opposite angles are supplementary.



$\angle A + \angle C = \angle B + \angle D = 180^0$

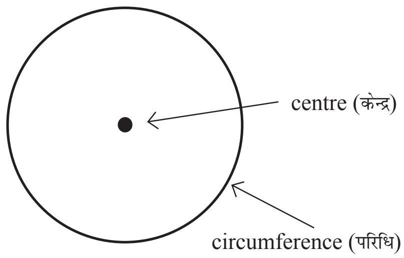
8) Vertices of Isosceles trapezium are concyclic.

9) Area of trapezium =

$\frac{1}{2}$ (sum of parallel sides) x height

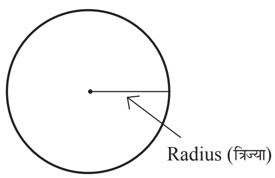
CIRCLE (वृत्त)

A circle is a simple closed curve, all the points of which are at the same distance from a given fixed point. The fixed point is called centre of the circle.

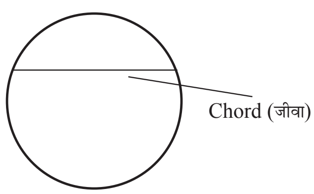


<u>Terms related to Circle</u>

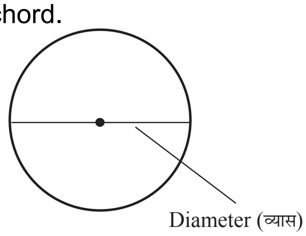
1) **Radius (त्रिज्या) :** Line segment joining centre and any point of circle.



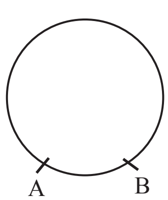
2) **Chord (जीवा) :** A line segment joining any two points on a circle is called chord of the circle.



3) **Diameter (व्यास) :** A chord passing through the centre of a circle is known as its diameter. Diameter is longest chord.



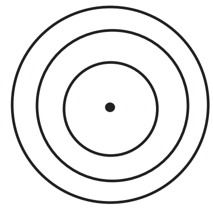
4) **Arc of a circle (चाप) :** A continuous piece of a circle is called an arc.



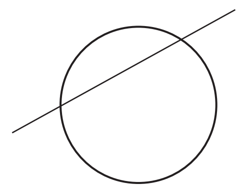
Arc is denoted by counter clockwise direction.

Minor arc => $\overset{\frown}{AB}$ | Major arc => $\overset{\frown}{BA}$

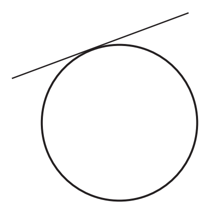
5) **Concentric Circles (सकेन्द्रियवृत्त) :** Circles having the same centre are said to be concentric circles.



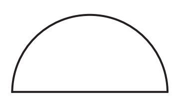
6) **Secant of a circle (प्रतिच्छेदीरेखा) :** A straight line intersecting the circle at two points, is called a secant.



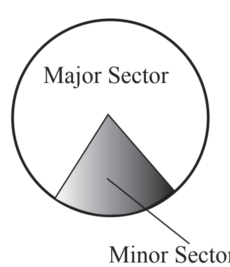
7) **Tangent of a circle (स्पर्शरेखा) :** A straight line touching the circle at one point only is called a tangent.



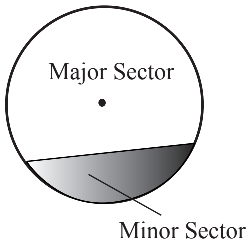
8) **Semicircle (अर्द्धवृत्त) :** A diameter of a circle divides the circumference of the circle into two equal arcs and each of these arcs is known as a semicircle.



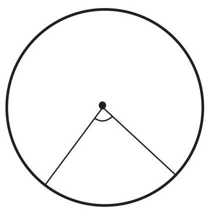
9) **Sector of a circle (त्रिज्यखण्ड) :** The part of a circle enclosed by an arc and two radii is called a sector.



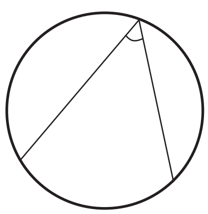
10) **Segment of a circle (वृत्तखण्ड) :** The part of the circular region enclosed by an arc and the chord joining the end points of the arc is called a segment of the circle.



11) **Central Angle (केन्द्रीयकोण) :** An angle subtended by an arc (or a chord) at the centre is called a central angle.

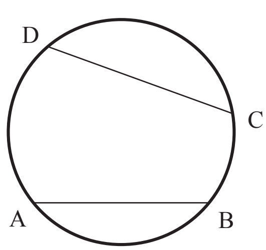


12) **Inscribed Angle (परिधिकोण) :** An angle, whose vertex lies on the circumference of a circle and the two arms are the chords of the circle, is called an inscribed angle.



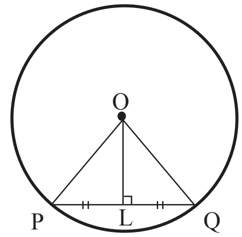
<u>Properties related to Circle</u>

1. *If to arcs of a circle are congruent then corresponding chords are equal.*



If $\overset{\frown}{AB} = \overset{\frown}{CD}$ then AB = CD

2. *The perpendicular from the centre of a circle to a chord bisects the chord.*



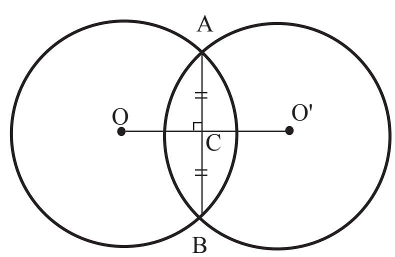
If OL $\perp$ PQ then PL = LQ

3. *The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.*

4. *Perpendicular bisector of a chord passing through the centre.*

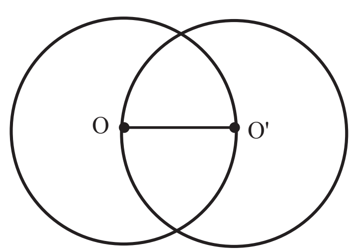
5. *Perpendicular bisectors of two or more chords intersect at its centre.*

6. *If two circles intersect at two points then the line segment joining their centre is perpendicular bisector of common chord.*

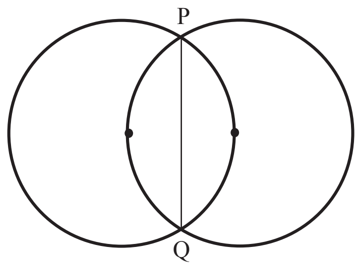


AC = BC and OC $\perp$ AB

7. *If two circle intersect each other and they are passing through each other centre then circles are congruent that means they have equal radius.*



8. *If two circles intersect each other and they are passing through each other centre then length of common chord is equal to* $\sqrt{3}r$

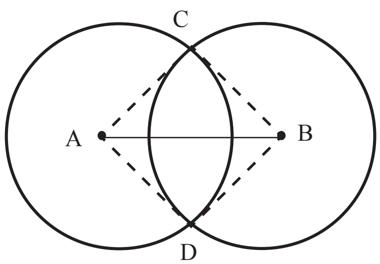


$PQ = \sqrt{3}\ r$

9. *Bisectors of two or more parallel chords passes through the centre.*

10. *If a diameter of a circle bisects each of the two chords of a circle then the chords are parallel.*

11. *If two circles intersect each other than the line segment joining their centre make equal angle at point of intersection.*

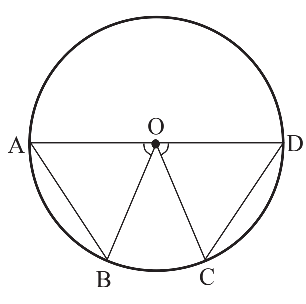


$\angle ACB = \angle ADB$

12. *Equal chords are equidistant from the centre.*

13. *Chords of a circle which are equidistant from the centre are equal.*

14. *Equal chords of a circle subtend equal angle at the centre.*

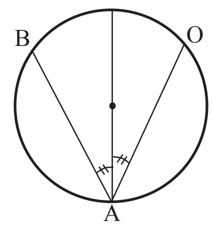


If AB = CD then $\angle AOB = \angle COD$

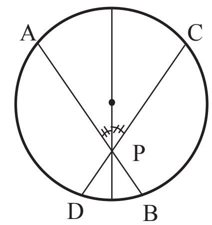
15. *If the angles subtended by two chords of a circle at the centre are equal, the chords are equal.*

16. *If two chords are unequal then larger chord is nearer to the centre.*

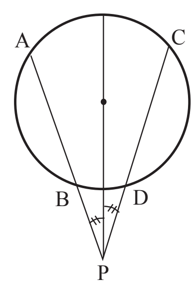
17. *If two chords of a circle are equally inclined to the diameter through their point of intersection then the chords are equal and their segments are equal.*



AB = AC



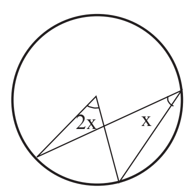
AB = CD
AP = CP
PD = PB



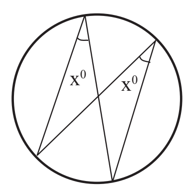
AB = CD
AP = CP
BP = DP

18. *If two chords of a circle bisect one another they must be diameters.*

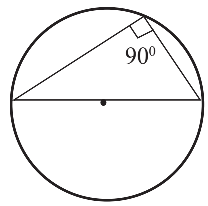
19. *The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.*



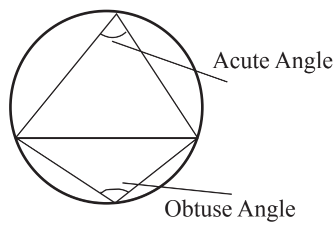
20. *Angles in the same segment of a circle are equal.*



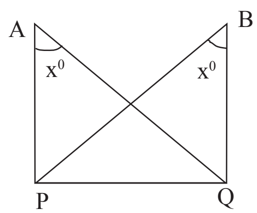
21. *The angle in a semi-circle is a right angle.*



22. *Angle made by a chord in minor segment is obtuse and in major segment is acute.*

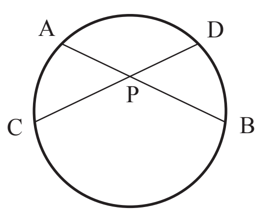


23. *If a line segment makes equal angle at two points than two end points of line segment and two point that means all the four points are concyclic.*

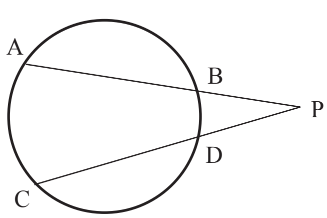


A, P, B, Q are concyclic

24. *If two chords intersect each other internally or externally then product of their segments are equal.*

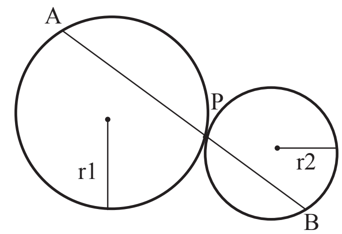


PA x PB = PC x PD



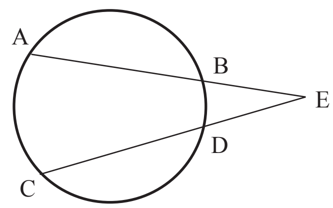
PA x PB = PC x PD

25. *If two circle touches each other then point of contact divide the line segment passing through their point of contact such that it touch circumference of both the circle, in the ratio of their radius.*



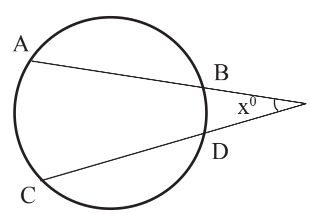
$$\frac{PA}{PB} = \frac{r1}{r2}$$

26. *Two equal chords AB and CD of a circle with centre O, when produced to meet at point E, then BE = DE that means AE = CE*



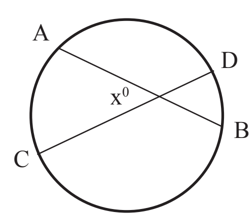
If AB = CD, then BE= DE & AC = CE

27. *If two chords intersect each other then angle made by them on point of intersection –*



$x = \frac{1}{2} \times$ *angle by*

(arc AC – arc BD) at the centre

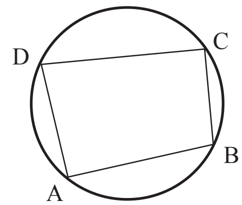


$x = \frac{1}{2} \times$ *angle by*

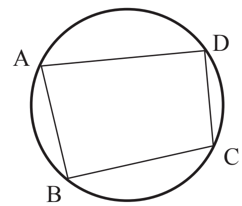
(arc AC + arc BD) at centre

Cyclic Properties of Circle

Cyclic quadrilateral (चक्रियचतुर्भुज) : A cyclic quadrilateral is called cyclic quadrilateral if its all vertices lie on a circle.



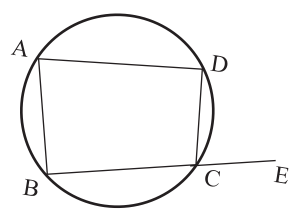
1. The sum of either pair or opposite angles of a cyclic quadrilateral is 180^0.



$\angle A + \angle C = 180^0$ and $\angle B + \angle D = 180^0$

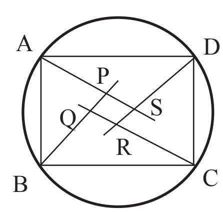
2. If the sum of any pair of opposite angles of a quadrilateral is 180^0, then the quadrilateral is cyclic.

3. If one side of a cyclic quadrilateral is produced, then the exterior angle is equal to interior opposite angle.



$\angle CDE = \angle A$

4. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

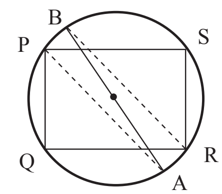


PQRS is cyclic quadrilateral

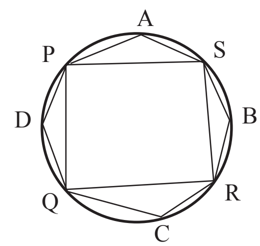
5. If two sides of a cyclic quadrilateral are parallel then the remaining two sides are equal and diagonal are also equal.

6. If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel.

7. If the bisectors of the opposite angles $\angle P$ and $\angle R$ of a cyclic quadrilateral PQRS intersect the corresponding circle at A and B respectively, then AB is a diameter of the circle.

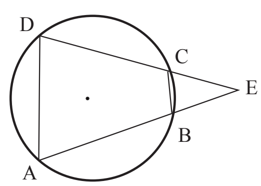


8. The sum of the angles in the four segments exterior to a cyclic quadrilateral is equal to 6 right angles.

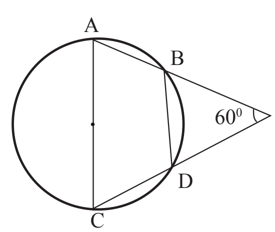


$\angle A + \angle B + \angle C + \angle D = 90^0 \times 6 = 540^0$

9. ABCD is a cyclic quadrilateral. AB and DC are produced to meet in E, then $\Delta EBC \sim \Delta EDA$.

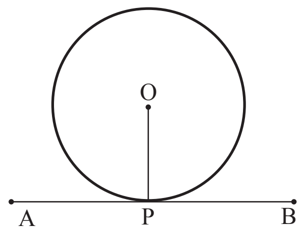


10. AB is diameter of a circle. Chord CD is equal to radius. If AC and BD when produced intersect at P, then $\angle APB = 60^0$



TANGENT AND ITS PROPERTIES

1) A tangent to a circle is perpendicular to the radius through the point of contact.



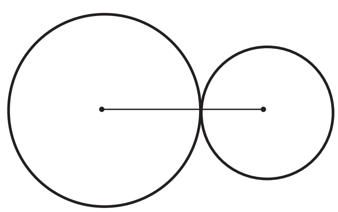
$OP \perp AB$

2) A line drawn through the end point of a radius and perpendicular to it is a tangent to the circle.

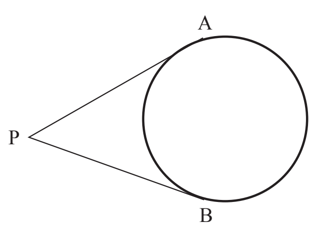
3) One and only one tangent can be drawn to a circle at a given point on the circumference.

4) The perpendicular to a tangent through its point of a contact passes through the centre of the circle.

5) If two circles touches each-other, the point of contact lies on the straight line joining their centers.

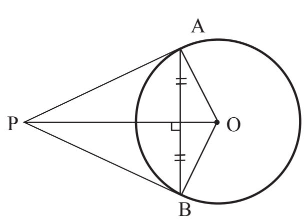


6) From any point outside a circle two tangents can be drawn to it and they are equal in length.



PA = PB

7)



(i) PA = PB

(ii) $\Delta PAO \cong \Delta PBO$

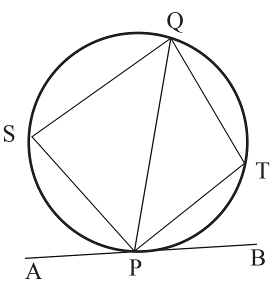
(iii) $\angle P + \angle O = 180^0$

(iv) PO is a angle bisector of $\angle P$ & $\angle O$

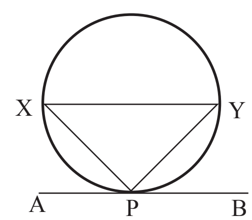
(v) OP is perpendicular bisector of AB

(vi) $\overset{\frown}{AB} < \overset{\frown}{BA}$

8) If a line touches a circle and from the point of contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segment.

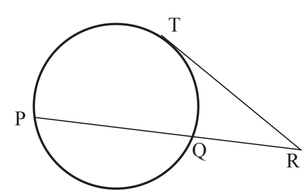


$\angle QPB = \angle PSQ$ & $\angle QPA = \angle PTQ$



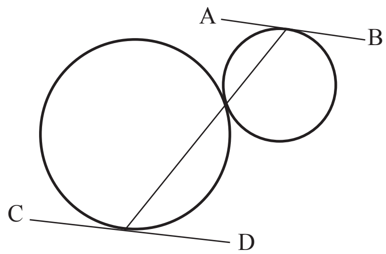
$\angle APX = \angle PYX$ & $\angle BPY = \angle PXY$

9) If a chord and a tangent intersect externally then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.



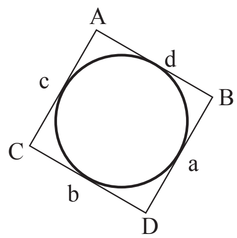
$PR \times RQ = TR^2$

10) Two circles touch externally and through the point of contact a straight line is drawn, touches the circumference of both circle, then the tangent at its extremities are parallel.



AB | | CD

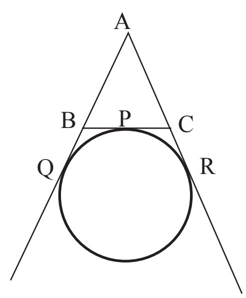
11) If a circle touches all the four sides of a quadrilateral then the sum of opposite pair of sides are equal.



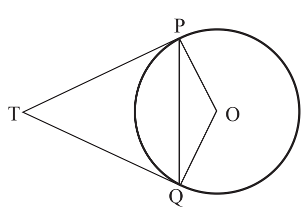
AB + DC = BC + DA

Area = $\sqrt{s(s-a)(s-b)(s-c)(s-d)}$

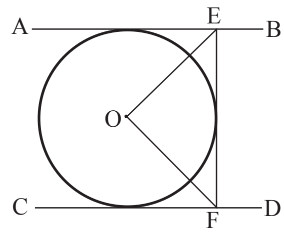
12) A circle touching the side BC of ΔABC at P and touching AB and AC produced at Q and R respectively then AQ = $\frac{1}{2}$ (perimeter of ΔABC)



13) Two tangents TP and TQ are drawn to a circle with centre O from an external point T, then ∠PTQ = 2 ∠OPQ .

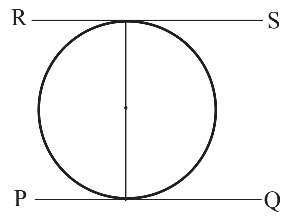


14)



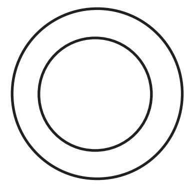
If AB | | CD than ∠EOF = 90^0

15) Tangents at the end point of a diameter of a circle are parallel.

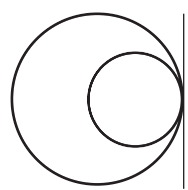


PQ | | RS

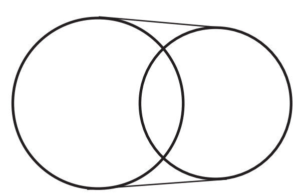
16) Common tangents to two circle



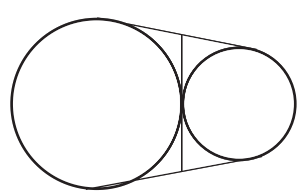
No common tangent



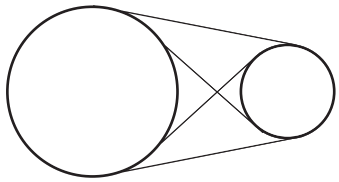
One common tangent



Two common tangent

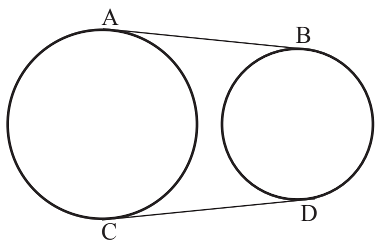


Three common tangent



Four common tangent

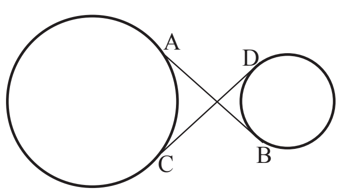
17) The two direct common tangent (अनु स्पर्श रेखा) drawn to two circles are equal in length.



AB = CD

18) The length of a direct common to two circles is $\sqrt{d^2 - (r_1 - r_2)^2}$, where d is the distance between the centres of the circles, and r1 and r2 are the radii of given circles.

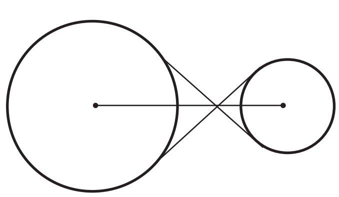
19) The two transverse common tangent (अनुप्रस्थ स्पर्श रेखा) drawn to two circles are equal in length.



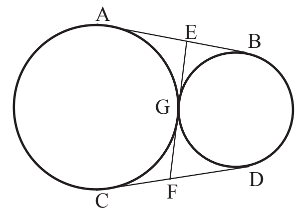
AB = CD

20) The length of a transverse common tangent to two circles is $\sqrt{d^2 - (r_1 + r_2)^2}$

21) The transverse common tangents drawn to two circles intersect on the line drawn through the centres of the circles.



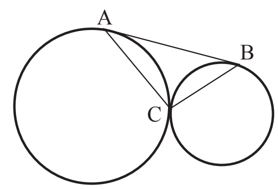
22)



(i) AB = CD = EF

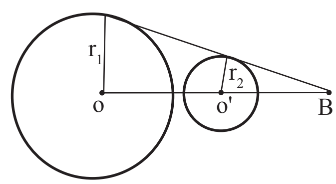
(ii) AE = EB = EG = GF = CF = FD

23)



$\angle$ ACB = 90^0

24) If direct common tangent of two circle and the line segment joining their centres intersect each-other at a point. Then point of intersection divide the line segment joining their centre externally into the ratio of their radius.



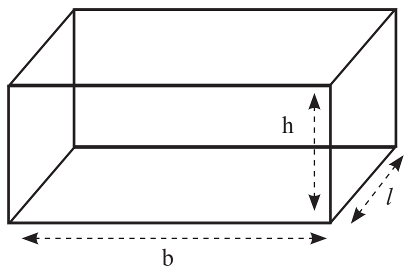
$$\frac{BO}{BO'} = \frac{r_1}{r_2}$$

<u>Area and Perimetre of Circle</u>

1. Area of circle = πr^2
2. Perimetre of circle = $2\pi r$
3. Area of semicircle = $\frac{1}{2}\pi r^2$
4. Perimeter of semicircle = $(\pi + 2)r$
5. Area of a quadrant of a circle = $\frac{1}{4}\pi r^2$
6. Perimeter of a quadrant of a circle = $\left(\frac{\pi}{2} + 2\right)r$
7. Area of a sector of a circle = $\frac{\theta}{360^\circ} \times \pi r^2$
8. Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r$

MENSURATION (3-D)

CUBOID (Parallelepiped) घनाभ (समांतर षट्फलक)



1) Volume (आयतन) = Area of base x height

2) Volume = l x b x h

3) Volume = $\sqrt{A_1 \times A_2 \times A_3}$ where A_1, A_2 & A_3 are area of three adjacent faces.

4) Diagonal (विकर्ण) = $\sqrt{l^2 + b^2 + h^2}$

5) Lateral surface Area or Area of four walls (पार्श्वीय सतह का क्षेत्रफल या चारों दीवारों का क्षेत्रफल) = Perimeter of base x height

6) Lateral surface Area = 2 (l + b) h

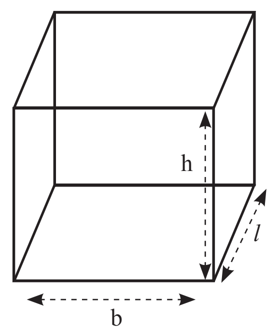
7) Total surface area (सम्पूर्ण सतह का क्षेत्रफल)
= 2 (lb + bh + hl)

8) Total surface Area =
$(l + b + h)^2 - (\text{diagonal})^2$

9) For a box having closed top (ढक्कनदार बॉक्स)
(i) Internal length (भीतरी लम्बाई) =
External length – 2 (thickness of material)
(ii) External length = Internal length +
2 (thickness of material)
(iii) Internal breadth = External breadth -
2 (thickness of material)
(iv) External breadth= Internal breadth+
2 (thickness of material)
(v) Internal height = External height –
2(thickness of material)
(vi) External height = Internal height +
2 (thickness of material)

10) A box having open top (बिना ढक्कन का बॉक्स)
(i) Internal length (भीतरी लम्बाई) =
External length – 2 (thickness of material)
(ii) External length = Internal length +
2 (thickness of material)
(iii) Internal breadth = External breadth -
2 (thickness of material)
(iv) External breadth= Internal breadth+
2 (thickness of material)
(v) Internal height = External height –
(thickness of material)
(vi) External height = Internal height +
(thickness of material)

CUBE (घन / समषट्फलक)



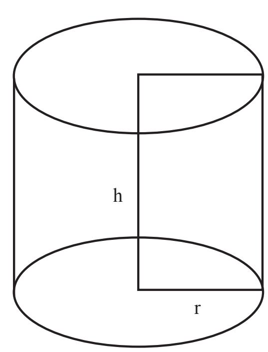
1) Volume = a^3 (a = length of side)

2) Lateral surface Area = $4a^2$

3) Total surface Area = $6a^2$

4) Diagonal = $\sqrt{3}$ a

Right Circular cylinder (लम्ब वृत्तीय बेलन)



1) Volume = Area of base x height

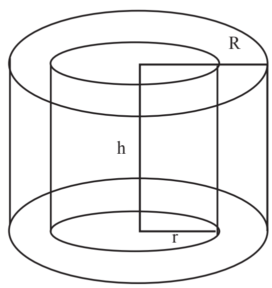
2) Volume = $\pi r^2 h$

3) Curved surface Area (वक्र पृष्ठ क्षेत्रफल) = Perimeter of base x height

4) Curved surface Area = $2\pi rh$

5) Total surface Area = $2\pi rh + 2\pi r^2$
$= 2\pi r(h + r)$

Hollow Cylinder (खोखला बेलन)



1) Thickness of material = R – r

2) Area of each end = $\pi(R^2 - r^2)$

3) External surface Area = $2\pi Rh$

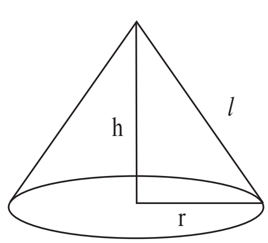
4) Internal surface Area = $2\pi rh$

5) Curved surface Area = $2\pi Rh + 2\pi rh$
$= 2\pi(R + r)h$

6) Total surface Area = $2\pi RH + 2\pi rh$
$+ 2(\pi R^2 - \pi r^2)$
$= 2\pi(R + r)(R - r + h)$

7) Volume of material =
External volume – Internal Volume
$= \pi R^2 h - \pi r^2 h = \pi(R^2 - r^2)h$

Right Circular Cone (लंब वृत्तीय शंकु)



h = height of cone
ℓ = slant height (तिरछी ऊँचाई) of cone
r = radius of cone

1) Slant height = $\sqrt{h^2 + r^2}$

2) Volume = $\frac{1}{3} \times$ Area of base x height

3) Volume = $\frac{1}{3}\pi r^2 h$

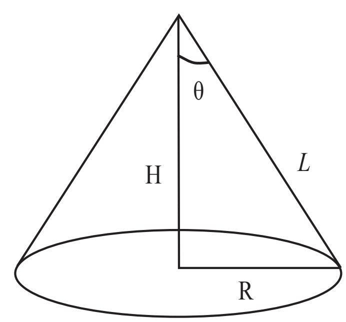
4) Curved surface Area = $\frac{1}{2} \times$ Perimeter of base x slant height $= \pi r\ell$

5) Total surface Area = $\pi r\ell + \pi r^2$
$= \pi r(\ell + r)$

6) If a cone is formed by sector of a circle then
(i) Slant height of cone
= Radius of sector
(ii) Circumference of base of cone
= length of arc of sector

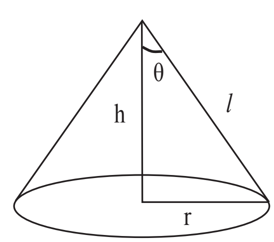
7) Two cones having equal vertex angle

Cone – I



Volume of cone-I = A
Curved surface Area of cone-I = B

Cone – II



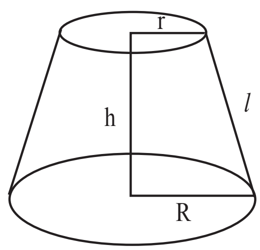
Volume of cone-II = a
Curved surface Area of cone-II = b

(i) $\frac{H}{h} = \frac{L}{\ell} = \frac{R}{r}$

(ii) $\frac{A}{a} = \frac{H^3}{h^3} = \frac{L^3}{l^3} = \frac{R^3}{r^3}$

(iii) $\frac{B}{b} = \frac{H^2}{h^2} = \frac{L^2}{\ell^2} = \frac{R^2}{r^2}$

Frustum of Cone (छिन्नक)



1) Slant height of frustum = $\sqrt{h^2 + (R-r)^2}$

2) Volume = $\frac{1}{3} \times \pi (R^2 + r^2 + R \cdot r) h$

3) Volume = $\frac{h}{3} \left(A_1 + A_2 + \sqrt{A_1 A_2} \right)$

Where A_1 & A_2 are area of base and top.

4) Curved surface Area = $\pi (R + r) \ell$

5) Total surface Area = $\pi (R + r) \ell + \pi R^2 + \pi r^2 = \pi [(R+r) \ell + R^2 + r^2]$

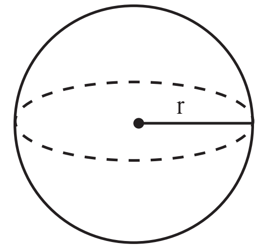
6) Height of cone of which frustum is a part = $\frac{hR}{R-r}$

7) Slant height of cone of which frustum is a part = $\frac{\ell R}{R-r}$

8) Height of cone of upper part of frustum = $\frac{hr}{R-r}$

9) Slant height of cone of upper part of frustum = $\frac{\ell R}{R-r}$

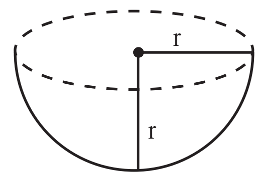
SPHERE (गोला)



1) Volume = $\frac{4}{3} \pi r^3$

2) Surface Area = $4 \pi r^2$

HEMISPHERE (अर्द्धगोला)

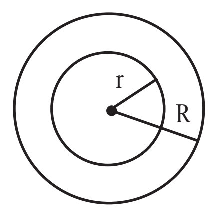


1) Volume = $\frac{2}{3} \pi r^3$

2) Curved surface Area = $2 \pi r^2$

3) Total surface Area = $3 \pi r^2$

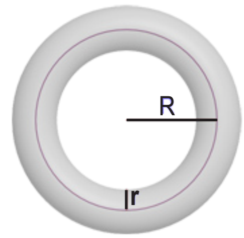
SPHERICAL SHELL (गोलाकार खोल)



1) Volume of material = $\frac{4}{3} \pi (R^3 - r^3)$

2) Outer surface Area = $4 \pi R^2$

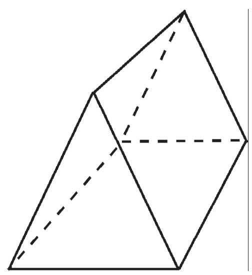
TORUS

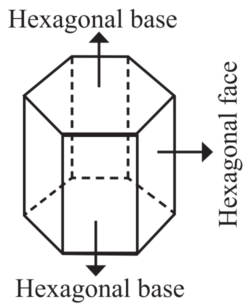


1) Volume = $2 \times \pi^2 \times R \times r^2$

2) Surface Area = $4 \times \pi^2 \times R \times r$

PRISM (प्रिज्म)



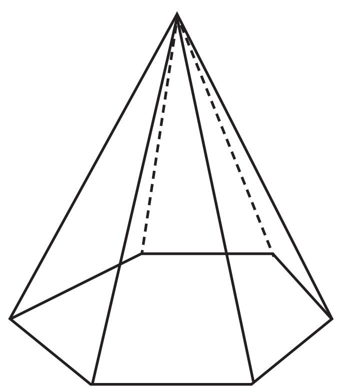


1) Volume = Area of base x height

2) Lateral surface Area = Perimeter of base x height

3) Total surface Area = Lateral surface area + 2 x Area of base

PYRAMID

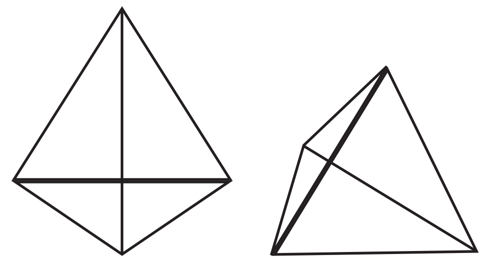


1) Volume = $\frac{1}{3} \times$ Area of base x height

2) Lateral surface Area = $\frac{1}{2} \times$ Perimeter of base x slant height

3) Total surface Area = Lateral surface Area + Area of base

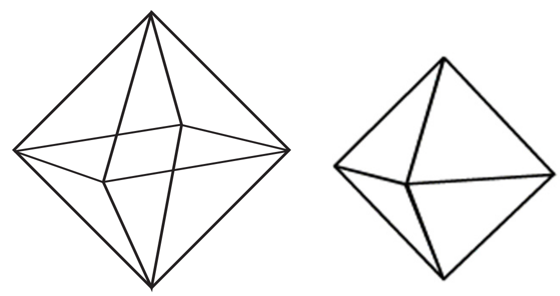
TETRAHEDRON (समचतुष्फलक)



1) Volume = $\frac{\sqrt{2}}{12} a^3$

2) Total surface Area = $\sqrt{3}\, a^2$

OCTAHEDRON (समअष्टफलक)



1) Volume = $\frac{\sqrt{2}}{3} a^3$

2) Total surface Area = $2\sqrt{3}\, a^2$