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Preface

About The Book...

Shortcuts in Quantitative Aptitude (with e-books) by Disha Publications is the most comprehensive book that focuses the fundamentals of quantitative aptitude with simple chapters, an array of examples and quick exercises. It is one of the best-selling reference guides that will help you get your elementary mathematics right. The book has been designed to cover each topic that is part of the Quantitative Ability syllabus. Shortcuts in Quantitative Aptitude is exceptional in a way that it conceives the short cut methods with which one can solve problems in no time with scientific yet student-friendly approach to discussing the topic. Hence, it not only enhances your efficiency but also helps you master the subject.

Each chapter of Shortcuts in Quantitative Aptitude covers basic theory followed by shortcut approaches and formulae and will help in learning various tips and tricks of Quantitative Aptitude. It has been a bestseller in this segment for many years and is heralded as 'a must read' book on this topic. The book is an attempt by Disha Publications to provide quality material to aspirants at a throwaway price.

Shortcuts in Quantitative Aptitude is also supported by ample practice material through e-books which cover:

* Chapter-wise Solved Examples

* Chapter-wise Practice Exercises with Hints and Solutions

* Chapter-wise Tests

* Past Solved Papers (IBPS PO/Clerk, SBI PO/Clerk, SSC, CDS exams etc)

We hope the book will prove to be an asset for all those aspiring for competitive examinations like UPSC (IAS Prelim), Banking, SSC, Insurance, Railway Recruitment Board Examinations, CDS, CBI, MBA, Sub-Inspectors of Police, CPO and various other competitive examinations.

Deepak Agarwal

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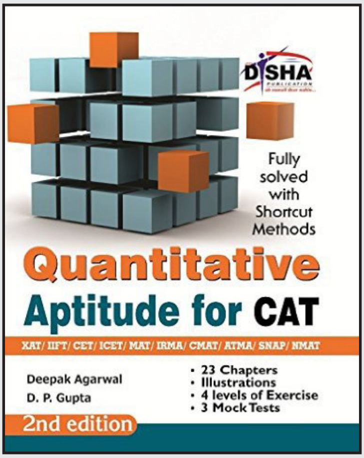

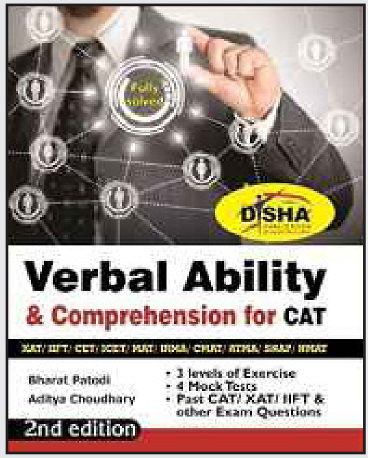

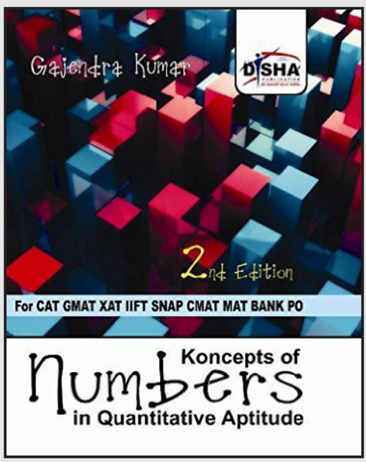

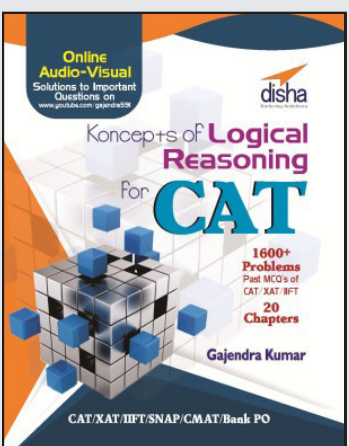

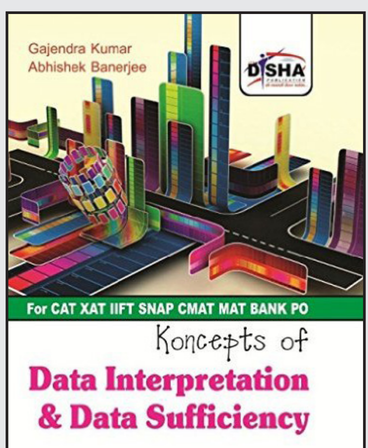

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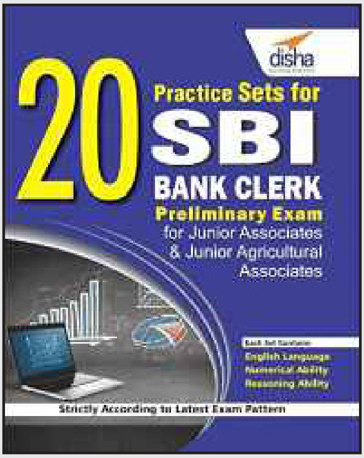

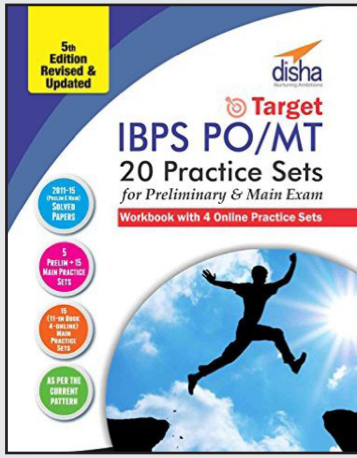

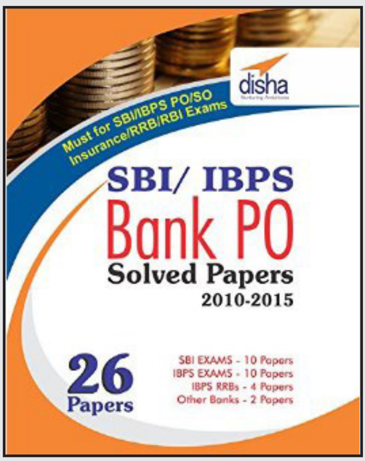

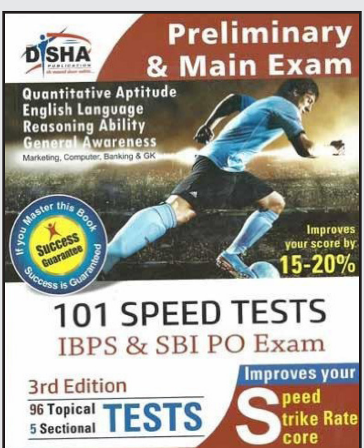

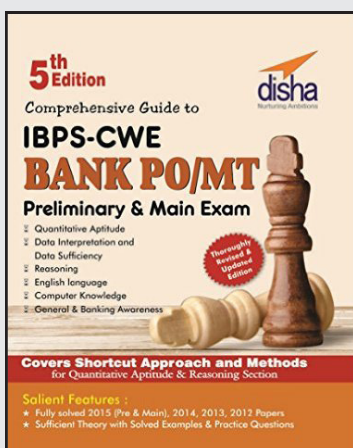

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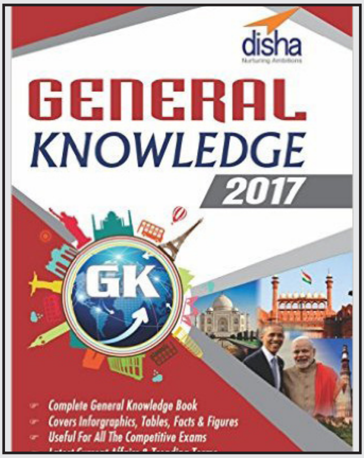
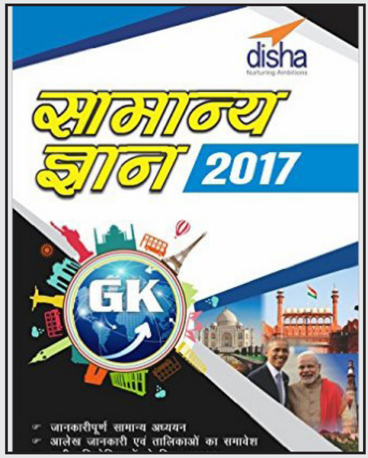
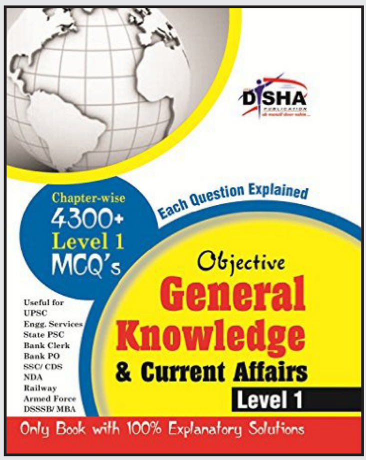
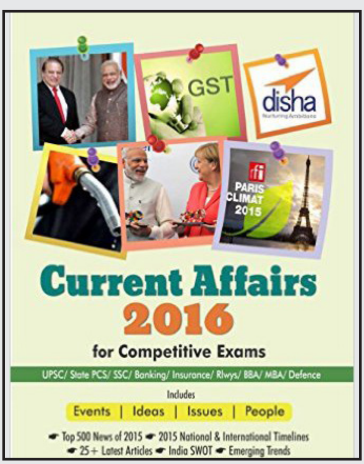
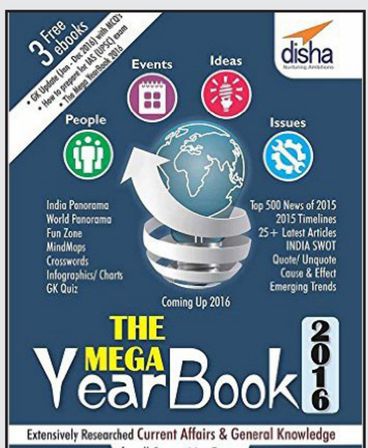








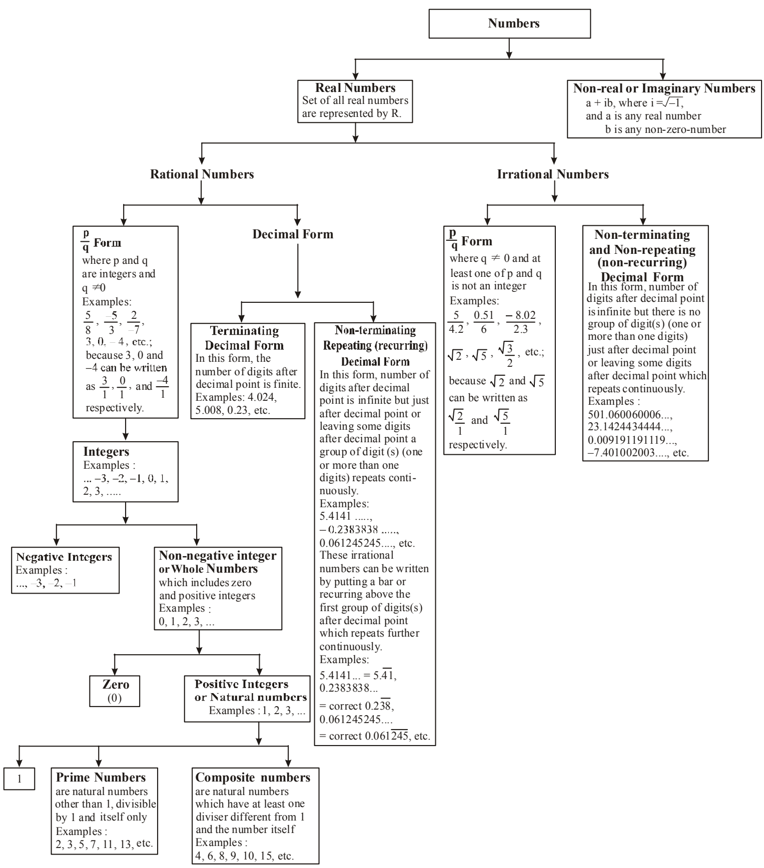


Chapter

1

Number System & Simplification

INTRODUCTION



REMEMBER

- ★ The ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called digits.
- ★ 1 is neither prime nor composite.
- ★ 1 is an odd integer.
- ★ 0 is neither positive nor negative.
- ★ 0 is an even integer.
- ★ 2 is prime & even both.
- ★ All prime numbers (except 2) are odd.

Natural Numbers :

These are the numbers (1, 2, 3, etc.) that are used for counting.
It is denoted by N.
There are infinite natural numbers and the smallest natural number is one (1).

Even numbers :

Natural numbers which are divisible by 2 are even numbers.
It is denoted by E.
E = 2, 4, 6, 8,...
Smallest even number is 2. There is no largest even number.

Odd numbers :

Natural numbers which are not divisible by 2 are odd numbers.
It is denoted by O.
O = 1, 3, 5, 7, ...
Smallest odd number is 1.
There is no largest odd number.

Based on divisibility, there could be two types of natural numbers : Prime and Composite.

Prime Numbers :

Natural numbers which have exactly two factors, i.e., 1 and the number itself are called prime numbers.
The lowest prime number is 2.
2 is also the only even prime number.

Composite Numbers :

It is a natural number that has atleast one divisor different from unity and itself.

Every composite number can be factorised into its prime factors.

For Example : $24 = 2 \times 2 \times 2 \times 3$. Hence, 24 is a composite number.

The smallest composite number is 4.

Twin-prime Numbers:

Pairs of such prime numbers whose difference is 2.

Example: 3 and 5, 11 and 13, 17 and 19.

How to check whether a given number is prime or not ?

Steps : (i) Find approximate square root of the given number.

(ii) Divide the given number by every prime number less than the approximate square root.

(iii) If the given number is exactly divisible by atleast one of the prime numbers, the number is a composite number otherwise a prime number.

Example : Is 401 a prime number?

Sol. Approximate square root of 401 is 20.

Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19

401 is not divisible by 2, 3, 5, 7, 11, 13, 17 or 19.

$\therefore$ 401 is a prime number.

(**Hint :** Next prime number after 19 and 23, which is greater than 20, so we need not check further.)

Co-prime Numbers : Co-prime numbers are those numbers which are prime to each other i.e., they don't have any common factor other than 1. Since these numbers do not have any common factor, their HCF is 1 and their LCM is equal to product of the numbers.

Note : Co-prime numbers can be prime or composite numbers. Any two prime numbers are always co-prime numbers.

Example 1 : 3 and 5 : Both numbers are prime numbers.

Example 2 : 8 and 15 : Both numbers are composite numbers but they are prime to each other i.e., they don't have any common factor.

Face value and Place value :

Face Value is absolute value of a digit in a number.

Place Value (or Local Value) is value of a digit in relation to its position in the number.

Example : Face value and Place value of 9 in 14921 is 9 and 900 respectively.

Whole Numbers :

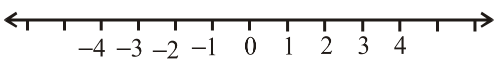
The natural numbers along with zero (0), form the system of whole numbers.
It is denoted by W.
There is no largest whole number and
The smallest whole number is 0.

Integers :

The number system consisting of natural numbers, their negative and zero is called integers.
It is denoted by Z or I.
The smallest and the largest integers cannot be determined.

The Number Line :

The number line is a straight line between negative infinity on the left to positive infinity on the right.



Rational numbers : Any number that can be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.

- It is denoted by Q.
- Every integer is a rational number.
- Zero (0) is also a rational number. The smallest and largest rational numbers cannot be determined. Every fraction (and decimal fraction) is a rational number.

$$Q = \frac{p \quad (\text{Numerator})}{q \quad (\text{Denominator})}$$

REMEMBER

★ If x and y are two rational numbers, then $\frac{x+y}{2}$ is also a rational number and its value lies between the given two rational numbers x and y.

★ An infinite number of rational numbers can be determined between any two rational numbers.

Irrational numbers : The numbers which are not rational or which cannot be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called irrational number.

It is denoted by Q' or Q^c.

$\sqrt{2}, \sqrt{3}, \sqrt{5}, 2+\sqrt{3}, 3-\sqrt{5}, 3\sqrt{3}$ are irrational numbers.

NOTE :

(i) *Every positive irrational number has a negative irrational number corresponding to it.*

(ii) $\sqrt{2}+\sqrt{3} \neq \sqrt{5}$

$\sqrt{5}-\sqrt{3} \neq \sqrt{2}$

$\sqrt{3}\times\sqrt{2} = \sqrt{3\times 2} = \sqrt{6}$

$\sqrt{6} \div \sqrt{2} = \sqrt{\frac{6}{2}} = \sqrt{3}$

(iii) Some times, product of two irrational numbers is a rational number.

For example : $\sqrt{2}\times\sqrt{2} = \sqrt{2\times 2} = 2$

$(2+\sqrt{3})\times(2-\sqrt{3}) = (2)^2 - (\sqrt{3})^2 = 4-3 = 1$

(iv) *π is an irrational number. π : approximately equal to $\frac{22}{7}$ or 3.14.*

Real Numbers :

All numbers that can be represented on the number line are called real numbers.

It is denoted by R.

R^+ : denotes the set of all positive real numbers and

R^- : denotes the set of negative real numbers.

Both rational and irrational numbers can be represented in number line.

Every real number is either rational or irrational.

FRACTIONS

A fraction is a quantity which expresses a part of the whole.

$$\boxed{\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}}$$

TYPES OF FRACTIONS :

1. **Proper fraction :** If numerator is less than its denominator, then it is a proper fraction.

   For example : $\frac{2}{5}, \frac{6}{18}$

2. **Improper fraction :** If numerator is greater than or equal to its denominator, then it is a improper fraction.

   For example : $\frac{5}{2}, \frac{18}{7}, \frac{13}{13}$

NOTE : *If in a fraction, its numerator and denominator are of equal value then fraction is equal to unity i.e. 1.*

---

3. **Mixed fraction :** It consists of an integer and a proper fraction.

   For example : $1\frac{1}{2},\ 3\frac{2}{3},\ 7\frac{5}{9}$

NOTE : *Mixed fraction can always be changed into improper fraction and vice versa.*

For example : $7\frac{5}{9} = \frac{7\times 9+5}{9} = \frac{63+5}{9} = \frac{68}{9}$

and $\frac{19}{2} = \frac{9\times 2+1}{2} = 9+\frac{1}{2} = 9\frac{1}{2}$

---

4. **Equivalent fractions or Equal fractions :** Fractions with same value.

   For example : $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}\left(=\frac{2}{3}\right).$

NOTE : *Value of fraction is not changed by multiplying or dividing both the numerator or denominator by the same number.*

For example :

(i) $\frac{2}{5}=\frac{2\times5}{5\times5}=\frac{10}{25}$ So, $\frac{2}{5}=\frac{10}{25}$

(ii) $\frac{36}{16}=\frac{36\div4}{16\div4}=\frac{9}{4}$ So, $\frac{36}{16}=\frac{9}{4}$

5. **Like fractions:** Fractions with same denominators.

   For example : $\frac{2}{7},\frac{3}{7},\frac{9}{7},\frac{11}{7}$

6. **Unlike fractions :** Fractions with different denominators.

   For example : $\frac{2}{5},\frac{4}{7},\frac{9}{8},\frac{9}{2}$

NOTE : *Unlike fractions can be converted into like fractions.*

For example : $\frac{3}{5}$ and $\frac{4}{7}$

$\frac{3}{5}\times\frac{7}{7}=\frac{21}{35}$ and $\frac{4}{7}\times\frac{5}{5}=\frac{20}{35}$

7. **Simple fraction :** Numerator and denominator are integers.

   For example : $\frac{3}{7}$ and $\frac{2}{5}$.

8. **Complex fraction :** Numerator or denominator or both are fractional numbers.

   For example : $\frac{\frac{2}{5}}{7},\frac{2\frac{1}{3}}{5\frac{2}{3}},\frac{2+\frac{1+\frac{2}{7}}{3}}{2}$

9. Decimal fraction : Denominator with the powers of 10.

For example : $\frac{2}{10} = (0.2), \frac{9}{100} = (0.09)$

10. Comparison of Fractions

Comparison of two faction can be easily understand by the following example:

To compare two fraction $\frac{3}{5}$ and $\frac{7}{9}$, multiply each fraction by the LCM (45) of their denominators 5 and 9.

$$\frac{3}{5} \times 45 = 3 \times 9 = 27$$

$$\frac{7}{9} \times 45 = 7 \times 5 = 35$$

Since $27 < 35$

$$\therefore \quad \frac{3}{5} < \frac{7}{9}$$

<u>SHORT CUT METHOD</u>

$$\frac{3}{5} \quad \frac{7}{9}$$

[Write the each product on their numerator side]

$$27 < 35$$

$$\therefore \quad \frac{3}{5} < \frac{7}{9}$$

ADDITION OF MIXED FRACTIONS

You can easily understand the addition of mixed fractions by the following example:

$$1\frac{3}{5} + 1\frac{8}{9} + 2\frac{4}{9} = \frac{8}{5} + \frac{17}{9} + \frac{14}{5}$$

$$= \frac{72 + 85 + 126}{45} = \frac{283}{45} = 6\frac{23}{45}$$

<u>SHORT CUT METHOD</u>

$$1\frac{3}{5} + 1\frac{8}{9} + 2\frac{4}{5} = (1 + 1 + 2) + \left(\frac{3}{5} + \frac{8}{9} + \frac{4}{5}\right)$$

$$= 4 + \frac{27 + 40 + 36}{45}$$

$$= 4 + \frac{103}{45} = 4 + 2\frac{13}{45} = 6\frac{13}{45}$$

Rounding off (Approximation) of Decimals :

There are some decimals in which numbers are found upto large number of decimal places.

For example : 3.4578, 21.358940789.

But many times we require decimal numbers upto a certain number of decimal places. Therefore,

If the digit of the decimal place is five or more than five, then the digit in the preceding decimal place is increased by one and if the digit in the last place is less than five, then the digit in the precedence place remains unchanged.

CONVERSION OF RATIONAL NUMBER OF THE FORM NON-TERMINATING RECURRING DECIMAL INTO THE RATIONAL NUMBER OF THE FORM $\frac{p}{q}$

First write the non-terminating repeating decimal number in recurring form i.e., write

64.20132132132..... as $64.20\overline{132}$

Then using formula given below we find the required $\frac{p}{q}$ form of the given number.

Rational number in the form $\frac{p}{q}$

$$= \frac{\begin{bmatrix}\text{Complete number neglecting} \\ \text{the decimal and bar over} \\ \text{repeating digit (s)}\end{bmatrix} - \begin{bmatrix}\text{Non-recurring part of} \\ \text{the number neglecting} \\ \text{the decimal}\end{bmatrix}}{m \text{ times } 9 \text{ followed by } n \text{ times } 0}$$

where $\quad m$ = number of recurring digits in decimal part

and $\quad n$ = number of non-recurring digits in decimals part

Thus, $\frac{p}{q}$ form of $64.20\overline{132} = \frac{6420132 - 6420}{99900}$

$$= \frac{6413712}{99900} = \frac{534476}{8325}$$

In short; $0.\overline{a} = \frac{a}{9}, 0.\overline{ab} = \frac{ab}{99}, 0.\overline{abc} = \frac{abc}{999},$ etc. and

$$0.a\overline{b} = \frac{ab - a}{90},\ 0.a\overline{bc} = \frac{abc - a}{990},\ 0.ab\overline{c} = \frac{abc - ab}{900},$$

$$0.ab\overline{cd} = \frac{abcd - ab}{9900},\ ab.c\overline{de} = \frac{abcde - abc}{990}, \text{etc.}$$

PROPERTIES OF OPERATIONS :

The following properties of addition, subtraction and multiplication are valid for real numbers a , b and c.

(a) Commutative property of addition :

$a + b = b + a$

(b) Associative property of addition :

$(a + b) + c = a + (b + c)$

(c) Commutative property of multiplication:

$a \times b = b \times a$

(d) Associative property of multiplication :

$(a \times b) \times c = a \times (b \times c)$

(e) Distributive property of multiplication with respect to addition :

$(a + b) \times c = a \times c + b \times c$

DIVISIBILITY RULES

Divisibility by 2 :

A number is divisible by 2 if its unit's digit is even or 0.

Divisibility by 3 :

A number is divisible by 3 if the sum of its digits are divisible by 3.

Divisibility by 4 :

A number is divisible by 4 if the last 2 digits are divisible by 4, or if the last two digits are 0's.

Divisibility by 5 :

A number is divisible by 5 if its unit's digit is 5 or 0.

Divisibility by 6 :

A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

Divisiblity by 7 :

A number is divisible by 7 if unit's place digit is multiplied by 2 and subtracted from the remaining digits and the number obtained is divisible by 7.

For example,

$$1680\boxed{7} = 1680 - 7 \times 2 = 1666$$

It is difficult to decide whether 1666 is divisible by 7 or not. In such cases, we continue the process again and again till it become easy to decide whether the number is divisible by 7 or not.

$$166\boxed{6} \longrightarrow 166 - 6 \times 2 = 154$$

Again $15\boxed{4} \longrightarrow 15 - 4 \times 2 = 7$, divisible by 7

Hence 16807 is divisible by 7.

Divisibility by 8 :

A number is divisible by 8 if the last 3 digits of the number are divisible by 8, or if the last three digits of a number are zeros.

Divisibility by 9 :

A number is divisible by 9 if the sum of its digits is divisible by 9.

Divisibility by 10 :

A number is divisible by 10 if its unit's digit is 0.

Divisibility by 11 :

A number is divisible by 11 if the sum of digits at odd and even places are equal or differ by a number divisible by 11.

Divisibility by 12 :

A number is divisible by 12 if the number is divisible by both 4 and 3.

Divisibility by 13 :

A number is divisible by 13 if its unit's place digit is multiplied by 4 and added to the remaining digits and the number obtained is divisible by 13.

For example,

$219\boxed{7} \longrightarrow 219 + 7 \times 4 = 247$

Again $24\boxed{7} \longrightarrow 24 + 7 \times 4 = 52$, divisible by 13.

Hence 2197 is divisible by 13.

Divisibility by 14 :

A number is divisible by 14 if the number is divisible by both 2 and 7.

Divisibility by 15 :

A number is divisible by 15 if the number is divisible by both 3 and 5.

Divisibility by 16 :

A number is divisible by 16 if its last 4 digits is divisible by 16 or if the last four digits are zeros.

Divisibility by 17 :

A number is divisible by 17 if its unit's place digit is multiplied by 5 and subtracted from the remaining digits and the number obtained is divisible by 17.

For example,

$491\boxed{3} \longrightarrow 491 - 3 \times 5 = 476$

Again, $47\boxed{6} \longrightarrow 47 - 6 \times 8 = 17,$ divisible by 17.

Hence 4913 is divisible by 17.

Divisibility by 18 :

A number is divisible by 18 if the number is divisible by both 2 and 9.

Divisibility by 19 :

A number is divisible by 19 if its unit's place digit is multiplied by 2 and added to the remaining digits and the number obtained is divisible by 19.

For example,

$4873\boxed{7} \longrightarrow 4873 + 7 \times 2 = 4887$

$488\boxed{7} \longrightarrow 488 + 7 \times 2 = 502$

$50\boxed{2} \longrightarrow 50 + 2 \times 2 = 54$ not divisible by 19.

Hence 48737 is not divisible by 19.

Properties of Divisibility

(i) The product of 3 consecutive natural numbers is divisible by 6.

(ii) The product of 3 consecutive natural numbers, the first of which is even, is divisible by 24.

(iii) Difference between any number and the number obtained by writing the digits in reverse order is divisible by 9.

(iv) Any number written in the form $(10^n - 1)$ is divisible by 3 and 9.

(v) Any six-digits, twelve-digits, eighteen-digits or any such number with number of digits equal to multiple of 6, is divisible by each of 7, 11 and 13 if all of its digits are same.

For example 666666, 888888, 333333333333 are all divisible by 7, 11 and 13.

(vi) Any number in the form abcabc (a, b, c are three different digits) is divisible by 1001.

(vii) (a) $(a^n - b^n)$ is divisible both by $(a + b)$ and $(a - b)$, when n is even.
(b) $(a^n - b^n)$ is divisible only by $(a - b)$, when n is odd.

DIVISION ALGORITHM :

Dividend = (Divisor × Quotient) + Remainder

where, Dividend = The number which is being divided

Divisor = The number which performs the division process

Quotient = Greatest possible integer as a result of division

Remainder = Rest part of dividend which cannot be further divided by the divisor.

Complete remainder :

A complete remainder is the remainder obtained by a number by the method of successive division.

Complete remainder = [I divisor × II remainder] + I remainder

$$\boxed{\begin{array}{l} C.R. = d_1 r_2 + r_1 \\ C.R. = d_1 d_2 r_3 + d_1 r_2 + r_1 \end{array}}$$

Shortcut Approach

Two different numbers x and y when divided by a certain divisor D leave remainder r_1 and r_2 respectively. When the sum of them sis divided by the same divisor, the remainder is r_3. Then,

divisor $D = r_1 + r_2 - r_3$

See Example : Refer ebook Solved Examples/Ch-1

Method to find the number of different divisors (or factors) (including 1 and itself) of any composite number N :

STEP I : Express N as a product of prime numbers as

$N = x^a \times y^b \times z^c$ ....

STEP II : Number of different divisors (including 1 and itself)

$= (a + 1)(b + 1)(c + 1)$ ....

HIGHEST COMMON FACTOR (HCF) OR GREATEST COMMON DIVISOR (GCD)

The highest (i.e. largest) number that divides two or more given numbers is called the highest common factor (HCF) of those numbers.

Methods to Find The HCF or GCD

There are two methods to find HCF of the given numbers

(i) Prime Factorization Method

When a number is written as the product of prime numbers, then it is called the prime factorization of that number. For example, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$. Here, $2 \times 2 \times 2 \times 3 \times 3$ or $2^3 \times 3^2$ is called prime factorization of 72.

To find the HCF of given numbers by this methods, we perform the prime factorization of all the numbers and then check for the common prime factors. For every prime factor common to all the numbers, we choose the least index of that prime factor among the given numbers. The HCF is the product of all such prime factors with their respective least indices.

(ii) Division Method

To find the HCF of two numbers by division method, we divide the larger number by the smaller number. Then we divide the smaller number by the first remainder, then first remainder by the second remainder.. and so on, till the remainder becomes 0. The last divisor is the required HCF.

☞ ***Shortcut Approach***

To find the HCF of any number of given numbers, first find the difference between two nearest given numbers. Then find all factors (or divisors) of this difference. Highest factor which divides all the given numbers is the HCF.

See Example : Refer ebook Solved Examples/Ch-1

LEAST COMMON MULTIPLE (LCM)

The least common multiple (LCM) of two or more numbers is the lowest number which is divisible by all the given numbers.

Methods to Find The LCM

There are two methods to find the LCM.

(i) Prime Factorization Method

After performing the prime factorization of all the given numbers, we find the highest index of all the prime numbers among the given numbers. The LCM is the product of all these prime numbers with their respective highest indices because LCM must be divisible by all of the given numbers.

(ii) Division Method

To find the LCM of 5, 72, 196 and 240, we use the division method in the following way:

Check whether any prime number that divides at least two of all the given numbers. If there is no such prime number, then the product of all these numbers is the required LCM, otherwise find the smallest prime number that divides at least two of the given numbers. Here, we see that smallest prime number that divides at least two given numbers is 2.

Divide those numbers out of the given numbers by 2 which are divisible by 2 and write the quotient below it. The given number(s) that are not divisible by 2 write as it is below it and repeat this step till you do not find at least two numbers that are not divisible by any prime number.

2	5,	72,	196,	240
2	5,	36,	98,	120
2	5,	18,	49,	60
3	5,	9,	49,	30
5	5,	3,	49,	10
	1,	3,	49,	2

After that find the product of all divisors and the quotient left at the end of the division. This product is the required LCM.

Hence, LCM of the given numbers = product of all divisors and the quotient left at the end.

$$= 2 \times 2 \times 2 \times 3 \times 5 \times 3 \times 49 \times 2 = 35280$$

☞ Shortcut Approach

Using idea of co-prime, you can find the LCM by the following shortcut method:

LCM of 9, 10, 15 and 36 can be written directly as $9 \times 10 \times 2$.

The logical thinking that behind it is as follows:

Step 1: If you can see a set of 2 or more co-prime numbers in the set of numbers of which you are finding the LCM, write them down by multiply them.

In the above situation, since we see that 9 and 10 are co-prime to each other, we start off writing the LCM by writing 9×10 as the first step.

Step 2: For each of the other numbers, consider what prime factor(s) of it is/are not present in the LCM (if factorised into primes) taken in step 1. In case you see some prime factors of each of the other given numbers separately are not present in the LCM (if factorised into primes) taken in step 1, such prime factors will be multiplied in the LCM taken in step 1.

Prime factorisation of $9 \times 10 = 3 \times 3 \times 2 \times 5$

Prime factorisation of $15 = 3 \times 5$

Prime factorisation of $36 = 2 \times 2 \times 3 \times 3$

Here we see that both prime factors of 15 are present in the prime factorisation of 9×10 but one prime factor 2 of 36 is not present in the LCM taken in step 1. So to find the LCM of 9, 10, 15 and 36; we multiply the LCM taken in step 1 by 2.

Thus required LCM $= 9 \times 10 \times 2 = 180$

See Example : Refer ebook Solved Examples/Ch-1

RULE FOR FINDING HCF AND LCM OF FRACTIONS

(I) HCF of two or more fractions

$$= \frac{\text{HCF of numerator of all fractions}}{\text{LCM of denominator of all fractions}}$$

(II) LCM of two or more fractions

$$= \frac{\text{LCM of numerator of all fractions}}{\text{HCF of denominator of all fractions}}$$

SIMPLIFICATION

FUNDAMENTAL OPERATIONS :

1. Addition :

(a) Sum of two positive numbers is a positive number.
For example : $(+5) + (+2) = +7$

(b) Sum of two negative numbers is a negative number.
For example : $(-5) + (-3) = -8$

(c) Sum of a positive and a negative number is the difference between their magnitudes and give the sign of the number with greater magnitude.
For example : $(-3) + (+5) = 2$ and $(-7) + (+2) = -5$

2. Subtractions :

Subtraction of two numbers is same as the sum of a positive and a negative number.

For Example :

$$(+9) - (+2) = (+9) + (-2) = 7$$
$$(-3) - (-5) = (-3) + 5 = +2.$$

NOTE : *In subtraction of two negative numbers, sign of second number will change and become positive.*

3. Multiplication :

(a) Product of two positive numbers is positive.

(b) Product of two negative numbers is positive.

(c) Product of a positive number and a negative number is negative.

(d) Product of more than two numbers is positive or negative depending upon the presence of negative quantities.

If the number of negative numbers is even then product is positive and if the number of negative numbers is odd then product is negative.

For Example :

$$(-3) \times (+2) = -6$$
$$(-5) \times (-7) = +35$$
$$(-2) \times (-3) \times (-5) = -30$$
$$(-2) \times (-3) \times (+5) = +30$$

4. Division :

(a) If both the dividend and the divisor are of same sign, then quotient is always positive.

(b) If the dividend and the divisor are of different sign, then quotient is negative,

For Example :

$$(-36) \div (+9) = -4$$
$$(-35) \div (-7) = +5$$

Brackets :

Types of brackets are :

(i) Vinculum or bar –

(ii) Parenthesis or small or common brackets : ( )

(iii) Curly or middle brackets : { }

(iv) Square or big brackets : [ ]

The order for removal of brackets is (), {}, []

NOTE : *If there is a minus (–) sign before the bracket then while removing bracket, sign of each term will change.*

'BODMAS' RULE

Now a days it becomes 'VBODMAS' where,

'V' stands for "Vinculum"

'B' stands for "Bracket"

'O' stands for "Of"

'D' stands for "Division"

'M' stands for "Multiplication"

'A' stands for "Addition"

'S' stands for "Subtraction"

Same order of operations must be applied during simplification.

☞ ***Shortcut Approach***

To simplify an expression, add all the positive numbers together and all the negative numbers separately and add or subtract the resulting numbers as the case will.

See Example : Refer ebook Solved Examples/Ch-1

POWERS OR EXPONENTS

When a number is multiplied by itself, it gives the square of the number. i.e., $a \times a = a^2$ (Example $5 \times 5 = 5^2$)

If the same number is multiplied by itself twice we get the cube of the number i.e., $a \times a \times a = a^3$ (Example $4 \times 4 \times 4 = 4^3$)

In the same way $a \times a \times a \times a \times a = a^5$

and $a \times a \times a \times \ldots$ upto n times $= a^n$

There are five basic rules of powers which you should know:

If a and b are any two real numbers and m and n are positive integers, then

(i) $a^m \times a^n = a^{m+n}$ (Example: $5^3 \times 5^4 = 5^{3+4} = 5^7$)

(ii) $\frac{a^m}{a^n} = a^{m-n}$, if $m > n$ $\left(\text{Example}: \frac{6^5}{6^2} = 6^{5-2} = 6^3\right)$

$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, if $m < n$ $\left(\text{Example}: \frac{4^3}{4^8} = \frac{1}{4^{8-3}} = \frac{1}{4^5}\right)$

and $\frac{a^m}{a^n} = a^0 = 1$, if $m = n$ $\left(\text{Example}: \frac{3^4}{3^4} = 3^{4-4} = 3^0 = 1\right)$

(iii) $(a^m)^n = a^{mn} = (a^n)^m$ (Example: $(6^2)^4 = 6^{2\times 4} = 6^8 = (6^4)^2$)

(iv) (a) $(ab)^n = a^n \cdot b^n$ (Example: $(6\times 4)^3 = 6^3 \times 4^3$)

(b) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ $\left(\text{Example}: \left(\frac{5}{3}\right)^4 = \frac{5^4}{3^4}\right)$

(v) $a^{-n} = \frac{1}{a^n}$ $\left(\text{Example}: 5^{-3} = \frac{1}{5^3}\right)$

(vi) For any real number a, $a^0 = 1$

ALGEBRIC IDENTITIES

Standard Identities

(i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a+b)(a-b)$

(iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Some More Identities

We have dealt with identities involving squares. Now we will see how to handle identities involving cubes.

(i) $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$\Rightarrow (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(ii) $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$\Rightarrow (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(iii) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

(iv) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(v) $a^3 + b^3 + c^3 - 3abc$

$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

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Chapter 2

Algebraic Expression & Inequalities

VARIABLE

An unknown quantity used in any equation may be constant known as variable. Variables are generally denoted by the last English alphabet x, y, z etc.
An equation is a statement of equality of two algebraic expressions, which involve one or more variables.

LINEAR EQUATION

An equation in which the highest power of variables is one, is called a linear equation. These equations are called linear because the graph of such equations on the x–y cartesian plane is a straight line.

Linear Equation in one variable

A linear equation which contains only one variable is called **linear equation in one variable**.
The general form of such equations is ax + b = c, where a, b and c are constants and $a \neq 0$.
All the values of x which satisfy this equation are called its solution(s).

NOTE : *An equation satisfied by all values of the variable is called an identity. For example :* $2x + x = 3x$.

Linear equation in two variables

General equation of a linear equation in two variables is ax + by + c = 0, where a, $b \neq 0$ and c is a constant, and x and y are the two variables.
The sets of values of x and y satisfying any equation are called its solution(s).
Consider the equation 2x + y = 4. Now, if we substitute x = –2 in the equation, we obtain 2.(–2) + y = 4 or – 4 + y = 4 or y = 8. Hence (–2, 8) is a solution. If we substitute x = 3 in the equation, we obtain 2.3 + y = 4 or 6 + y = 4 or y = –2

Hence (3, –2) is a solution. The following table lists six possible values for x and the corresponding values for y, i.e. six solutions of the equation.

x	–2	–1	0	1	2	3
y	8	6	4	2	0	–2

Systems of Linear equation

Consistent System : A system (of 2 or 3 or more equations taken together) of linear equations is said to be consistent, if it has at least one solution.
Inconsistent System: A system of simultaneous linear equations is said to be inconsistent, if it has no solutions at all.

e.g. $X + Y = 9;$ $3X + 3Y = 8$

Clearly there are no values of X & Y which simultaneously satisfy the given equations. So the system is inconsistent.

REMEMBER

★ The system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ has :

- a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.
- Infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
- No solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

★ The homogeneous system $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ has the only solution $x = y = 0$ when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

★ The homogeneous system $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ has a non-zero solution only when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, and in this case, the system has an infinite number of solutions.

QUADRATIC EQUATION

An equation of the degree two of one variable is called quadratic equation.
General form : $ax^2 + bx + c = 0$...........(1) where a, b and c are all real number and $a \neq 0$.

For Example :

$2x^2 - 5x + 3 = 0$; $2x^2 - 5 = 0$; $x^2 + 3x = 0$

If $b^2 - 4ac \geq 0$, then the quadratic equation gives two and only two values (either same or different) of the unknown variable and both these values are called the roots of the equation.

The roots of the quadratic equation (1) can be evaluated using the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{...(2)}$$

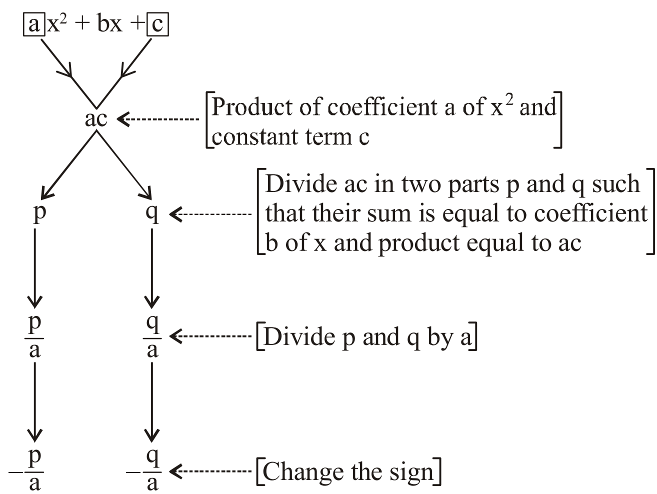
The above formula provides both the roots of the quadratic equation, which are generally denoted by α and β,

say $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

The expression inside the square root $b^2 - 4ac$ is called the DISCRIMINANT of the quadratic equation and denoted by D. Thus, Discriminant (D) = $b^2 - 4ac$.

☞ *Shortcut Approach*

Shortcut Approach to solve Quadratic equation $ax^2 + bx + c = 0$, if $b^2 - 4ac \geq 0$,



Here $\dfrac{-p}{a}$ and $\dfrac{-q}{a}$ are two roots or solutions of quadratic equation

$ax^2 + bx + c = 0$ i.e. $x = -\dfrac{p}{a}$ or $-\dfrac{q}{a}$.

See Example : Refer ebook Solved Examples/Ch-2

Nature of Roots

The nature of roots of the equation depends upon the nature of its discriminant D.

1. If $D < 0$, then the roots are non-real complex, Such roots are always conjugate to one another. That is, if one root is $p + iq$ then other is $p - iq$, $q \neq 0$.
2. If $D = 0$, then the roots are real and equal. Each root of the equation becomes $-\frac{b}{2a}$. Equal roots are referred as repeated roots or double roots also.
3. If $D > 0$ then the roots are real and unequal.

Sign of Roots:

Let α, β are real roots of the quadratic equation $ax^2 + bx + c = 0$ that is $D = b^2 - 4ac \geq 0$. Then

1. Both the roots are positive if a and c have the same sign and the sign of b is opposite.
2. Both the roots are negative if a, b and c all have the same sign.
3. The Roots have opposite sign if sign of a and c are opposite.
4. The Roots are equal in magnitude and opposite in sign if $b = 0$ [that is its roots α and $-\alpha$]
5. The roots are reciprocal if $a = c$.

[that is the roots are α and $\frac{1}{\alpha}$]

Symmetric Functions of Roots :

An expression in α, β is called a symmetric function of α, β if the function is not affected by interchanging α and β. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$ then,

Sum of roots : $\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

and Product of roots : $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Formation of quadratic Equation with Given Roots:

➢ An equation whose roots are α and β can be written as $(x - \alpha)(x - \beta) = 0$ or $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ or $x^2 -$ (sum of the roots) $x +$ product of the roots $= 0$.

➢ Further if α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then

$$ax^2 + bx + c = a(x - \alpha)(x - \beta) \text{ is an identity.}$$

INEQUATIONS :

A statement or equation which states that one thing is not equal to another, is called an inequation.

Symbols :

'$<$' means "is less than"

'$>$' means "is greater than"

'$\leq$' means "is less than or equal to"

'$\geq$' means "is greater than or equal to"

For example :

(a) $x < 3$ means x is less than 3.

(b) $y \geq 9$ means y is greater than or equal to 9.

Properties

1. Adding the same number to each side of an inequation does not effect the sign of inequality, i.e. if $x > y$ then, $x + a > y + a$.
2. Subtracting the same number to each side of an inequation does not effect the sign of inequaltiy, i.e., if $x < y$ then, $x - a < y - a$.
3. Multiplying each side of an inequality with same positive number does not effect the sign of inequality, i.e., if $x \leq y$ then $ax \leq ay$ (where, $a > 0$).
4. Multiplying each side of an inequality with a negative number reverse the sign of inequality i.e., if $x < y$ then $ax > ay$ (where $a < 0$).
5. Dividing each side of an inequation by a positive number does not effect the sign of inequality, i.e., if $x \leq y$ then $\frac{x}{a} \leq \frac{y}{a}$ (where $a > 0$).
6. Dividing each side of an inequation by a negative number reverses the sign of inequality, i.e., if $x > y$ then $\frac{x}{a} < \frac{y}{a}$ (where $a < 0$).

REMEMBER

★ If a > b and a, b, n are positive, then $a^n > b^n$ but $a^{-n} < b^{-n}$.
For example 5 > 4; then $5^3 > 4^3$ or 125 > 64, but

$5^{-3} < 4^{-3}$ or $\frac{1}{125} < \frac{1}{64}$.

★ If a > b and c > d, then (a + c) > (b + d).

★ If a > b > 0 and c > d > 0, then ac > bd.

★ If the signs of all the terms of an inequality are changed, then the sign of the inequality will also be reversed.

MODULUS :

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

1. If a is positive real number, x and y be the fixed real numbers, then

(i) $|x-y| < a \Leftrightarrow y-a < x < y+a$

(ii) $|x-y| \leq a \Leftrightarrow y-a \leq x \leq y+a$

(iii) $|x-y| > a \Leftrightarrow x > y+a$ or $x < y-a$

(iv) $|x-y| \geq a \Leftrightarrow x \geq y+a$ or $x \leq y-a$

2. Triangle inequality :

(i) $|x+y| \leq |x|+|y|, \ \forall x, y \in R$

(ii) $|x-y| \geq |x|-|y|, \ \forall x, y \in R$

Applications Formulation of Equations/ Expressions:

A formula is an equation, which represents the relations between two or more quantities.

For example :

Area of parallelogram (A) is equal to the product of its base (b) and height (h), which is given by

$A = b \times h$

or $A = bh$.

Perimeter of triangle (P),

$P = a + b + c$, where a, b and c are length of three sides.

More Applications of Equations :

Problems on Ages can be solved by linear equations in one variable, linear equations in two variables, and quadratic equations.

Shortcut Approach

- If present age of the father is F times the age of his son. T years hence, the father's age become Z times the age of son then present age of his son is given by $\frac{(Z-1)T}{(F-Z)}$

- If T_1 years earlier the age of the father was n times the age of his son, T_2 years hence, the age of the father becomes m times the age of his son then his son's age is given by

$$\text{Son's age} = \frac{T_2(n-1)+T_1(m-1)}{n-m}$$

- Present age of Father : Son = a : b
  After / Before T years = m : n

  Then son's age $= b \times \frac{T(m-n)}{an-bm}$

  and Father's age $= a \times \frac{T(m-n)}{an-bm}$

See Example : Refer ebook Solved Examples/Ch-2

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Chapter 3

Average

AVERAGE

'Average' is a very simple but effective way of representing an entire group by a single value.

$$\textbf{Average or Mean} = \frac{\text{Sum of given quantities}}{\text{Number of quantities}}$$

To calculate the sum of quantities, they should be in the same unit.

☞ Shortcut Approach

If X is the average of $x_1, x_2, x_3 \ldots. x_n$ then

(a) The average of $x_1 + a, x_2 + a, x_3 + a, \ldots. x_n + a$ is $X + a$.

(b) The average of $x_1 - a, x_2 - a, x_3 - a \ldots. x_n - a$ is $X - a$

(c) The average of $ax_1, ax_2, \ldots. ax_n$ is aX, provied $a \neq 0$

(d) The average of $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots \frac{x_n}{a}$ is $\frac{x}{a}$, provided $a \neq 0$

See Example : Refer ebook Solved Examples/Ch-3

Average of a group consisting two different groups when their averages are known :

Let Group A contains m quantities and their average is a and Group B contains n quantities and their average is b, then **average of group C** containing m + n quantities $= \frac{\text{ma} + \text{mb}}{\text{m} + \text{n}}$

WEIGHTED AVERAGE

If we have two or more groups of members whose individual averages are known, then combined average of all the members of all the groups is known as weighted average. Thus if there are k groups having member of number $n_1, n_2, n_3, \ldots\ldots. n_k$ with averages $A_1, A_2,$

A_3........A_k respectively then weighted average.

$$A_w = \frac{n_1 A_1 + n_2 A_2 + n_3 A_3 + ... + n_k A_k}{n_1 + n_2 + n_3 + ... + n_k}$$

☞ **Shortcut Approach**

If, in a group, one or more new quantities are added or excluded, then the new quantity or sum of added or excluded quantities = [Change in no. of quantities × original average] ± [change in average × final no. of quantities]

Take +ve sign if quantities added and

take –ve sign if quantities removed.

See Example : Refer ebook Solved Examples/Ch-3

AVERAGE SPEED IF EQUAL DISTANCES ARE TRAVELLED BY TWO DIFFERENT SPEEDS

If a car travels at a speed S_1 from A to B and at a speed S_2 from B to A. Then

$$\boxed{\text{Average speed} = \frac{2\,S_1 \,.\, S_2}{S_1 + S_2}}$$

The above formula can be found out as follows:

If distance between A and B is d, then

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2d}{\dfrac{d}{S_1} + \dfrac{d}{S_2}} = \frac{2}{\dfrac{1}{S_1} + \dfrac{1}{S_2}} = \frac{2\,S_1 \,.\, S_2}{S_2 + S_1}$$

AVERAGE SPEED IF EQUAL DISTANCES ARE TRAVELLED BY THREE DIFFERENT SPEEDS

$$\text{Average speed } \frac{3xyz}{xy + yz + zx}$$

Where x, y and 2 are these different speeds.

REMEMBER

- ★ Average of first n natural numbers $= \frac{(n+1)}{2}$
- ★ Average of first n consecutive × 2 even numbers = (n + 1)
- ★ Average of first n consecutive × 2 odd numbers = n
- ★ Average of consecutive numbers $= \frac{\text{First number} + \text{Last number}}{2}$
- ★ Average of 1 to n odd numbers $= \frac{\text{Last odd number} + 1}{2}$
- ★ Average of 1 and n even numbers $= \frac{\text{Last even number} + 2}{2}$
- ★ Average of squares of first n natural numbers $= \frac{(n+1)(2n+1)}{6}$
- ★ Average of the cubes of first n natural numbers $= \frac{n(n+1)^2}{4}$
- ★ Average of n multiples of any number $= \frac{\text{Number} \times (n+1)}{2}$
- ★ If *n* is odd: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the middle number.
- ★ If *n* is even: The average of *n* consecutive numbers, consecutive even numbers or consecutive odd numbers is always the average of the middle two numbers.
- ★ The average of squares of first *n* consecutive even number is $\frac{2(n+1)(2n+1)}{3}$.
- ★ The average of squares of consecutive even numbers till *n* is $\frac{(n+1)(n+2)}{3}$.
- ★ The average of square of consecutive odd numbers till *n* is $\frac{n(n+2)}{3}$.
- ★ If the average of *n* consecutive numbers is *m*, then the difference between the smallest and the largest number is $2(n-1)$.

If a person or a motor car covers three equal distances at the speed of x km/h, y km/h and z km/h, respectively, then for the entire journey average speed of the person or motor car is $\left(\frac{3xzy}{xy + yz + zx}\right)$ km/h.

☞ Shortcut Approach

- If average of n observations is a but the average becomes b when one observation is eliminated, then
  Value of eliminated observation = n (a – b) + b
- If average of n observations is a but the average becomes b when a new observation is added, then
  Value of added observation = n (b – a) + b. We have n observations out of which some observations (a_1, a_2, a_3, ...) are replaced by some other new observations in this way, if the average increases or decreases by b, then
  Value of new observations = a ± nb
  where, $a = a_1 + a_2 + a_3 + ...$

See Example : Refer ebook Solved Examples/Ch-3

NOTE : *In this formula, the signs of '+' and '–' depend upon the increment or decrement in the average.*

Chapter 4

Percentage

PER CENT

The word "per cent" is derived from the latin words "per centum", which means "per hundred".

A **percentage** is a fraction with denominator hundred.

It is denoted by the symbol %.

Numerator of the fraction is called the **rate per cent**.

VALUE OF PERCENTAGE

Value of percentage always depends on the quantity to which it refers.

Consider the statement :

"65% of the students in this class are boys". From the context, it is understood that boys form 65% of the total number of students in the class. To know the value of 65% of the total number of students in the class, the value of the total number of boys student should be known.

If the total number of students is 200, then, the number of boys

$= \frac{200 \times 65}{100} = 130$; It can also be written as $(200) \times (0.65) = 130$.

If the total number of students is 500, then the number of boys

$$= \frac{500 \times 65}{100} = 325$$

NOTE *that the expressions 6%, 63%, 72%, 155% etc. do not have any value to themselves. Their values depend on the quantities to which they refer.*

Some Quick Results:

5% of a number $= \frac{\text{Number}}{20}$, 10% of a number $= \frac{\text{Number}}{10}$

$12\frac{1}{2}\%$ of a number $= \frac{\text{Number}}{8}$, 20% of a number $= \frac{\text{Number}}{5}$

25% of a number $= \frac{\text{Number}}{4}$, 50% of a number $= \frac{\text{Number}}{2}$

To express the fraction equivalent to % :

Express the fraction with the denominator 100, then the numerator is the answer.

Fractional Equivalents of % (Percentage)

$1\% = \frac{1}{100}$	$33\frac{1}{3}\% = \frac{1}{3}$
$2\% = \frac{1}{50}$	$40\% = \frac{2}{5}$
$4\% = \frac{1}{25}$	$50\% = \frac{1}{2}$
$5\% = \frac{1}{20}$	$66\frac{2}{3}\% = \frac{2}{3}$
$6\frac{1}{4}\% = \frac{1}{16}$	$60\% = \frac{3}{5}$
$10\% = \frac{1}{10}$	$75\% = \frac{3}{4}$
$11\frac{1}{3}\% = \frac{17}{150}$	$80\% = \frac{4}{5}$
$12\frac{1}{2}\% = \frac{1}{8}$	$96\% = \frac{24}{25}$
$16\% = \frac{4}{25}$	$100\% = 1$
$16\frac{2}{3}\% = \frac{1}{6}$	$115\% = \frac{23}{20}$
$20\% = \frac{1}{5}$	$133\frac{1}{3}\% = \frac{4}{3}$
$25\% = \frac{1}{4}$	

EXPRESSING ONE QUANTITY AS A PER CENT WITH RESPECT TO OTHER

To express a quantity as a per cent with respect to other quantity following formula is used.

$$\frac{\text{The quantity to be expressed in per cent}}{\text{2nd quantity (in respect of which the per cent has to be obtained)}}\times 100\%$$

Note : *To apply this formula, both the quantities must be in same unit.*

PERCENTAGE INCREASE OR DECREASE OF A VALUE

$$\boxed{\text{Increase }\% = \frac{\text{Increase value}}{\text{Original value}}\times 100\%}$$

$$\boxed{\text{Decrease }\% = \frac{\text{Decrease value}}{\text{Original value}}\times 100\%}$$

☞ ***Shortcut Approach***

When a number x is increased or decreased by y%, then the new number will be $\frac{100 \pm y}{100}\times x$.

NOTE : 1. '+' *sign is used in case of increase.*

2. '–' *sign is used in case of decrease.*

- If *x* is a% more than *y*, then *y* is $\left(\frac{a}{100+a}\times 100\right)$% less than *x*.
- If *x* is a% less than *y*, then *y* is $\left(\frac{a}{100-a}\times 100\right)$% more than x.

See Example : Refer ebook Solved Examples/Ch-4

☞ Shortcut Approach

- If A's income is r % more than that of B, then B's income is less than that of A by

$$\left(\frac{r}{100+r}\times 100\right)\%$$

- If A's income is r % less than that of B, then B's income is more than that of A by

$$\left(\frac{r}{100-r}\times 100\right)\%$$

☞ Shortcut Approach

- If A is x % of C and B is y % of C, then A is $\frac{x}{y}\times 100$ % of B.
- x % of a quantity is taken by the first, y % of the remaining is taken by the second and z % of the remaining is taken by third person. Now, if A is left in the fund, then the initial amount

$$= \frac{A\times 100\times 100\times 100}{(100-x)(100-y)(100-z)} \text{ in the begining.}$$

- x % of a quantity is added. Again, y % of the increased quantity is added. Again z % of the increased quantity is added. Now it becomes A, then the initial amount

$$= \frac{A\times 100\times 100\times 100}{(100+x)(100+y)(100+z)}$$

☞ Shortcut Approach

- If the value of a number is first increased by a% and later decreased by a%, then the net effect is always a decrease which is equal to a% of a and is written as $\frac{a^2}{100}$% or $\left(\frac{a^2}{10}\right)$%.

See Example : Refer ebook Solved Examples/Ch-4

Shortcut Approach

- If the price of a commodity increases by r %, then reduction in consumption, so as not to increase the expenditure is $\left(\frac{r}{100+r}\times 100\right)\%$.
- If the price of a commodity decreases by r %, then the increase in consumption so as not to decrease the expenditure is $\left(\frac{r}{100-r}\times 100\right)\%$

Shortcut Approach

- If due to r% decrease in the price of an item, a person can buy A kg more in ₹ x, then

Actual price of that item = ₹ $\frac{rx}{(100-r)A}$ per kg

Shortcut Approach

Population Formula

- If the original population of a town is P, and the annual increase is r %, then the population after n years is $P\left(1+\frac{r}{100}\right)^n$ and

population before n years $= \frac{P}{\left(1+\frac{r}{100}\right)^n}$

- If the annual decrease be r %, then the population after n years is $P\left(1-\frac{r}{100}\right)^n$ and

Population before n years $= \frac{P}{\left(1-\frac{r}{100}\right)^n}$

See Example : Refer ebook Solved Examples/Ch-4

Shortcut Approach

First Increase and then decrease :

If the value is first increased by x % and then decreased by y % then there is $\left(x - y - \frac{xy}{100}\right)$% increase or decrease, according to the + ve or – ve sign obtained respectively.

Average percentage rate of change over a period.

$$= \frac{(\text{New Value} - \text{Old Value})}{\text{Old Value}} \times \frac{100}{n} \% \text{ where } n = \text{period.}$$

The percentage error $= \frac{\text{The Error}}{\text{True Value}} \times 100\%$

Shortcut Approach

Successive increase or decrease

If the value is increased **successively by x % and y %** then the final **increase** is given by

$$\left(x + y + \frac{xy}{100}\right)\%$$

If the value is decreased **successively by x % and y %** then the final **decrease** is given by

$$\left(-x - y + \frac{xy}{100}\right)\%$$

Shortcut Approach

Student and Marks

The percentage of passing marks in an examination is x%. If a candidate who scores y marks fails by z marks, then the maximum marks $M = \frac{100(y+z)}{x}$

A candidate scoring x % in an examination fails by 'a' marks, while another candidate who scores y% marks gets 'b' marks more than the minimum required passing marks. Then the maximum marks $M = \frac{100(a+b)}{y-x}$.

See Example : Refer ebook Solved Examples/Ch-4

☞ Shortcut Approach

- If two candidates contested in an election and one candidate got x% of total votes and still lose by y votes, then
- Total number of votes casted = $\frac{100 \times y}{100 - 2x}$

2–dimensional figure and area

- If the sides of a triangle, square, rhombus or radius of a circle are increased by a%, its area is increased by $\frac{a(a+200)}{100}\%$

See Example : Refer ebook Solved Examples/Ch-4

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Chapter

5 Ratio & Proportion

RATIO

Ratio is strictly a mathematical term to compare two similar quantities expressed in the same units.

The ratio of two terms 'x' and 'y' is denoted by x : y.

In general, the ratio of a number x to a number y is defined as the quotient of the numbers x and y.

COMPARISON OF TWO OR MORE RATIOS

Two or more ratios may be compared by reducing the equivalent fractions to a common denominator and then comparing the magnitudes of their numerator. Thus, suppose 2 : 5, 4 : 3 and 4 : 5 are three ratios to be compared then the fractions $\frac{2}{5}, \frac{4}{3}$ and $\frac{4}{5}$ are reduced to equivalent fractions with a common denominator. For this, the denominator of each is changed to 15 equal to the L.C.M. their denominators Hence the given ratios are expressed $\frac{6}{15}, \frac{20}{15}$ and $\frac{12}{15}$ or 2 : 5, 4 : 3, 4 : 5 according to magnitude.

REMEMBER

- ★ In the ratio of two quantities the two quantities must be of the same kind and in same unit.
- ★ The ratio is a pure number, i.e., without any unit of measurement.
- ★ The ratio would stay unaltered even if both the numerator and the denominator are multiplied or divided by the same number.

COMPOUND RATIO

Ratios are compounded by multiplying together the numerators for a new denominator and the denominator for a new denominator.

The compound ratio of a : b and c : d is $\frac{a \times c}{b \times d}$, i.e., ac : bd.

Shortcut Approach

The duplicate ratio of x : y is $x^2 : y^2$.
The triplicate ratio of x : y is $x^3 : y^3$.

The subduplicate ratio of x : y is $\sqrt{x} : \sqrt{y}$.

The subtriplicate ratio of x : y is $\sqrt[3]{x} : \sqrt[3]{y}$.

Reciprocal ratio of a : b is $\frac{1}{a} : \frac{1}{b}$ or b : a

Inverse ratio

Inverse ratio of x : y is y : x.

See Example : Refer ebook Solved Examples/Ch-5

PROPERTIES OF RATIOS

1. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$, i.e., the inverse ratios of two equal ratios are equal. The property is called **Invertendo**.

2. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$, i.e., the ratio of antecedents and consequents of two equal ratios are equal. This property is called **Alternendo**.

3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$. This property is called **Componendo**.

4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$. This property is called **Dividendo**.

5. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. This property is called **Componendo and Dividendo.**

6. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots\ldots\ldots$ Then,

   Each ratio $= \frac{\text{sum of Numerators}}{\text{sum of Denominators}}$

   i.e. $\frac{a}{b} = \frac{c}{d} = \frac{a+c+e+\ldots}{b+d+f+\ldots}$

If we have two equations containing three unknowns as

$a_1x + b_1y + c_1z = 0$ and ... (i)

$a_2x + b_2y + c_2z = 0$ ... (ii)

then, the values of x, y and z cannot be resolved without having a third equation.

However, in the absence of a third equation, we can find the ratio x : y : z.

This will be given by

$b_1c_2 - b_2c_1 : c_1a_2 - c_2a_1 : a_1b_2 - a_2b_1$.

Shortcut Approach

To divide a given quantity into a given ratio.

Suppose any given quantity a, is to be divided in the ratio m : n.

Let one part of the given quantity be x then the other part will be a – x.

$$\therefore \quad \frac{x}{a-x} = \frac{m}{n} \quad \text{or} \quad nx = ma - mx \quad \text{or} \quad (m+n)x = ma$$

$$\therefore \quad \text{one part is } \frac{ma}{m+n} \text{ and the other part will be}$$

$$a - \frac{ma}{m+n} = \frac{na}{m+n}$$

If A : B = a : b and B : C = m : n, then A : B : C = am : mb : nb and A : C = am : bn

If A : B = a : b, B : C = c : d and C : D = e : f, then A : B : C : D = ace : bce : bde : bdf

Two numbers are in ratio a : b and x is subtracted from the numbers, then the ratio becomes c : d. The two numbers will be $\frac{xa(d-c)}{ad-bc}$ and $\frac{xb(d-c)}{ad-bc}$, respectively.

See Example : Refer ebook Solved Examples/Ch-5

☞ Shortcut Approach

- In any 2-dimensional figures, if the corresponding sides are in the ratio x : y, then their areas are in the ratio $x^2 : y^2$.
- In any two 3-dimensional similar figures, if the corresponding sides are in the ratio x : y, then their volumes are in the ratio $x^3 : y^3$.

☞ Shortcut Approach

- If the ratio between two numbers is a : b and if each number is increased by x, the ratio becomes c : d. Then, two numbers are given as $\frac{xa(c-d)}{ad-bc}$ and $\frac{xb(c-d)}{ad-bc}$
- If the sum of two numbers is A and their difference is a, then the ratio of numbers is given by A + a : A – a.

☞ Shortcut Approach

- Let a vessel contains Q unit of mixture of ingredients A and B. From this, R unit of mixture is taken out and replaced by an equal amount of ingredient B only.
  If this process is repeated n times, then after n operations
  $$\frac{\text{Quantity of A left}}{\text{Quantity of A originally present}} = \left(1 - \frac{R}{Q}\right)^n$$
  and Quantity of B left = Q – Quantity of A Left
- In a container, milk and water are present in the ratio a : b. If x L of water is added to this mixture, the ratio becomes a : c. Then,
- Quantity of milk in original mixture $= \frac{ax}{c-b}$ L
  and quantity of water in original mixture $= \frac{bx}{c-b}$ L
- A container has milk and water in the ratio a : b, a second container has milk and water in the ratio c : d. If both the mixtures are emptied into a third container, then the ratio of milk of water in third container is given by

$$\left(\frac{a}{a+b}+\frac{c}{c+d}\right):\left(\frac{b}{a+b}+\frac{d}{c+d}\right)$$

See Example : Refer ebook Solved Examples/Ch-5

PROPORTION

When two ratios are equal, the four quantities composing them are said to be in proportion.

If $\frac{a}{b}=\frac{c}{d}$, then a, b, c, d are in proportions.

This is expressed by saying that 'a' is to 'b' as 'c' is to 'd' and the proportion is written as

$a : b :: c : d$ or $a : b = c : d$

The terms a and d are called the extremes while the terms b and c are called the means.

REMEMBER

★ If four quantities are in proportion, the product of the extremes is equal to the product of the means.
Let a, b, c, d be in proportion, then
$\frac{a}{b}=\frac{c}{d} \Rightarrow ad = bc.$

★ If three quantities a, b and c are in continued proportion, then $a : b = b : c$
$\therefore \quad ac = b^2$
b is called mean proportional.

DIRECT PROPORTION

If on the increase of one quantity, the other quantity increases to the same extent or on the decrease of one, the other decreases to the same extent, then we say that the given two quantities are directly proportional. If A and B are directly proportional then we denote it by $A \propto B$.

Some Examples :

1. Work done $\propto$ number of men
2. Cost $\propto$ number of Articles
3. Work $\propto$ wages
4. Working hour of a machine $\propto$ fuel consumed
5. Speed $\propto$ distance to be covered

INDIRECT PROPORTION (OR INVERSE PROPORTION)

If on the increase of one quantity, the other quantity decreases to the same extent or vice versa, then we say that the given two quantities are indirectly proportional. If A and B are indirectly proportional then we denote it by $A \propto \frac{1}{B}$.

Also, $A = \frac{k}{B}$ (k is a constant)

$\Rightarrow \quad AB = k$

If b_1, b_2 are the values of B corresponding to the values a_1, a_2 of A respectively, then

$$a_1b_1 = a_2b_2$$

Some Examples :

1. More men, less time
2. Less men, more time
3. More speed, less taken time to be covered distance

RULE OF THREE

In a problem on simple proportion, usually three terms are given and we have to find the fourth term, which we can solve by using Rule of three. In such problems, two of given terms are of same kind and the third term is of same kind as the required fourth term.

First of all we have to find whether given problem is a case of direct proportion or indirect proportion.

For this, write the given quantities under their respective headings and then mark the arrow in increasing direction. If both arrows are in same direction then the relation between them is direct otherwise it is indirect or inverse proportion. Proportion will be made by either head to tail or tail to head.

The complete procedure can be understand by the examples.

PARTNERSHIP

A partnership is an association of two or more persons who invest their money in order to carry on a certain business.

A partner who manages the business is called the **working partner** and the one who simply invests the money is called the **sleeping partner.**

Partnership is of two kinds :

(i) Simple (ii) Compound.

Simple partnership :

If the capitals is of the partners are invested for the same period, the partnership is called simple.

Compound partnership :

If the capitals of the partners are invested for different lengths of time, the partnership is called compound.

Shortcut Approach

- If the period of investment is the same for each partner, then the profit or loss is divided in the ratio of their investments.
  If A and B are partners in a business, then

$$\frac{\text{Investment of A}}{\text{Investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}} = \frac{\text{Loss of A}}{\text{Loss of B}}$$

- If A, B and C are partners in a business, then
  Investment of A : Investment of B : Investment of C
  = Profit of A : Profit of B : Profit of C, or
  = Loss of A : Loss of B : Loss of C

- When the amount of capital invested by different partners is same (say ₹ x) for differend time periods, t_1, t_2, t_3, ..., then
- Ratio of profit/loss = Ratio of time period for which the capital is invested

$$P_1 : P_2 : P_3 : \ldots = t_1 : t_2 : t_3 : \ldots$$

See Example : Refer ebook Solved Examples/Ch-5

MONTHLY EQUIVALENT INVESTMENT

It is the product of the capital invested and the period for which it is invested.
If the period of investment is different, then the profit or loss is divided in the ratio of their Monthly Equivalent Investment.

$$\frac{\text{Monthly Equivalent Investment of A}}{\text{Monthly Equivalent Investment of B}}$$

$$= \frac{\text{Profit of A}}{\text{Profit of B}} \text{ or } \frac{\text{Loss of A}}{\text{Loss of B}}$$

i.e., $\dfrac{\text{Investment of A} \times \text{Period of Investment of A}}{\text{Investment of B} \times \text{Period of Investment of B}}$

$$= \frac{\text{Profit of A}}{\text{Profit of B}} \text{ or } \frac{\text{Loss of A}}{\text{Loss of B}}$$

Shortcut Approach

- If A, B and C are partners in a business, then
  Monthly Equivalent Investment of A : Monthly Equivalent Investment of B : Monthly Equivalent Investment of C
  = Profit of A : Profit of B : Profit of C.
  = Loss of A : Loss of B : Loss of C.
- When capital invested by the partners is givne as $X_1, X_2, X_3, \ldots$ for different time period $t_1, t_2, t_3, \ldots$ in a business, then
  Ratio of their profits $P_1 : P_2 : P_3 : \ldots = X_1t_1 : X_2t_2 : X_3t_3 : \ldots$
- If $P_1 : P_2 : P_3$: ---- is the ratio of profit and $t_1 : t_2 : t_3 : \ldots$ is the ratio of time $\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3}$ ..... periods, then ratio of investments is given by

See Example : Refer ebook Solved Examples/Ch-5

MIXTURE

Simple Mixture : When two different ingredients are mixed together, it is known as a simple mixture.

Compound Mixture : When two or more simple mixtures are mixed together to form another mixture, it is known as a compound mixture.

Alligation : Alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together.

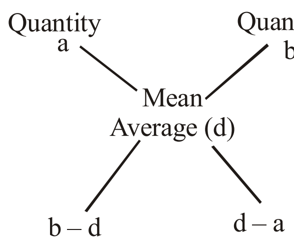
The word 'Alligation' literally means 'linking'.

ALLIGATION RULE

It states that when different quantities of the same or different ingredients of different costs are mixed together to produce a mixture of a mean cost, the ratio of their quantities is inversely proportional to the difference in their cost from the mean cost.

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Price of Dearer} - \text{Mean Price}}{\text{Mean Price} - \text{Price of Cheaper}}$$

Graphical representation of Alligation Rule :



$$\frac{\text{Quantity of a}}{\text{Quantity of b}} = \frac{b-d}{d-a}$$

Applications of Alligation Rule :

(i) To find the mean value of a mixture when the prices of two or more ingredients, which are mixed together and the proportion in which they are mixed are given.

(ii) To find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price.

☞ ***Shortcut Approach***

Price of the Mixture :

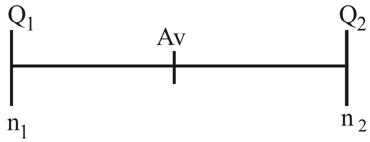
When quantities Q_i of ingredients M_i's with the cost C_i's are mixed then cost of the mixture C_m is given by

$$C_m = \frac{\sum C_i Q_i}{\sum Q_i}$$

See Example : Refer ebook Solved Examples/Ch-5

STRAIGHT LINE APPROACH OF ALLIGATION

Let Q_1 and Q_2 be the two quantities, and n_1 and n_2 are the number of elements present in the two quantities respectively,



where Av is the average of the new group formed then

n_1 corresponds to $Q_2 - Av$, n_2 corresponds to $Av - Q_1$ and $(n_1 + n_2)$ corresponds to $Q_2 - Q_1$.
Let us consider the previous example.

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Chapter 6

Pofit and Loss

INTRODUCTION

Cost Price

The amount paid to purchase an article or the price at which an article is made, is known as its cost price.

The cost price is abbreviated as C.P.

Selling Price

The price at which an article is sold, is known as its selling price.

The selling price is abbreviated as S.P.

Profit

If the selling price (S.P.) of an article is greater than the cost price (C.P.), then the difference between the selling price and cost price is called profit.

Thus, If S.P. > C.P., then

$$\text{Profit} = \text{S.P.} - \text{C.P.}$$

$$\Rightarrow \quad \text{S.P.} = \text{C.P.} + \text{Profit}$$

$$\Rightarrow \quad \text{C.P.} = \text{S.P.} - \text{Profit.}$$

Loss

If the selling price (S.P.) of an article is less than the cost price (C.P.), then the difference between the cost price (C.P.) and the selling price (S.P.) is called loss.

Thus, if S.P. < C.P., then

$$\text{Loss} = \text{C.P.} - \text{S.P.}$$

$$\Rightarrow \quad \text{C.P.} = \text{S.P.} + \text{Loss}$$

$$\Rightarrow \quad \text{S.P.} = \text{C.P.} - \text{Loss}$$

Profit and Loss percentage

The profit per cent is the profit that would be obtained for a C.P. of ₹ 100. Similarly, the loss per cent is the loss that would be made for a C.P. of ₹ 100.

$$\text{Profit per cent} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$\text{Loss per cent} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

REMEMBER

★ $\text{Profit} = \dfrac{\text{C.P.} \times \text{Profit \%}}{100}$

★ $\text{Loss} = \dfrac{\text{C.P.} \times \text{Loss \%}}{100}$

★ $\text{S.P.} = \left(\dfrac{100 + \text{Profit\%}}{100}\right) \times \text{C.P.}$

★ $\text{S.P.} = \left(\dfrac{100 - \text{Loss\%}}{100}\right) \times \text{C.P.}$

★ $\text{C.P.} = \dfrac{100 \times \text{S.P.}}{100 + \text{Profit \%}}$

★ $\text{C.P.} = \dfrac{100 \times \text{S.P.}}{100 - \text{Loss \%}}$

NOTE

(i) If an article is sold at a certain gain (say 45%), then SP = 145% of CP

(ii) If an article is sold at certain loss (say 25%), then SP = 75% of CP.

☞ Shortcut Approach

Dishonest dealing:

$$\text{Gain }\% = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

$$\frac{\text{True Scale}}{\text{False Scale}} = \frac{100 + \text{gain}\%}{100 - \text{loss}\%}$$

☞ Shortcut Approach

Real Profit/Loss percentage :

If the profit or loss is calculated on S.P., then it is not actual profit or loss.

Real profit (loss)% is the profit (loss)% on C.P.

$$\text{Real Profit }\% = \frac{\%\text{ profit on S.P.}}{100 - \%\text{ profit on S.P.}} \times 100$$

☞ Shortcut Approach

Goods passing through successive hands

- When there are two successive profits of a% and b%, then the resultant profit per cent is given by

$$\left(a + b + \frac{ab}{100}\right)\%$$

- When there are two successive loss of a% and b%, then the resultant loss per cent is given by $\left(-a - b + \frac{ab}{100}\right)\%$
- When there is a profit of a% and loss by b% in a transaction, then the resultant profit or loss per cent is given by $\left(a - b - \frac{ab}{100}\right)\%$, according to the +ve or –ve sign respectively.

See Example : Refer ebook Solved Examples/Ch-6

When cost price and selling price are reduced by the same amount (A) and profit increases then cost price (C.P.)

$$= \frac{[\text{Initial profit}\% + \text{Increase in profit}\%] \times A}{\text{Increase in profit}\%}$$

Shortcut Approach

- If cost price of x articles is equal to the selling price of y articles, then profit/loss percentage $= \frac{x-y}{y} \times 100\%$, according to +ve or –ve sign respectively.
- A man purchases a certain number of articles at x a rupee and the same number at y rupee. He mixes them together and sells them at z rupee. Then his gain or loss %

  $= \left[\frac{2xy}{z(x+y)} - 1\right] \times 100$ according as the sign is +ve or –ve.
- If two items are sold, each at ₹. x, one at a gain of p% and the other at a loss of p%, there is an overall loss given by $\frac{p^2}{100}$%. The absolute value of the loss is given by $\frac{2p^2 x}{100^2 - p^2}$.
- If CP of two items is the same and % Loss and % Gain on the two items are equal, then net loss or net profit is zero.

See Example : Refer ebook Solved Examples/Ch-6

Shortcut Approach

A businessman sells his items at a profit/loss of a%. If he had sold it for ₹ R more, he would have gained/lost b%. Then,

$$\text{CP of items} = \frac{R}{b \pm a} \times 100$$

'–' = When both are either profit or loss
'+' = When one is profit and other is loss

If A sold an article to B at a profit (loss) of r_1 % and B sold this article to C at a profit (loss) of r_2 %, then cost price of article for C is given by (cost price for A) $\times \left(1 \pm \frac{r_1}{100}\right)\left(1 \pm \frac{r_2}{100}\right)$.

If a man purchases m items for ₹ x and sells n items for ₹ y, then

Profit or loss per cent is given by $\frac{my - nx}{nx} \times 100\%$

[Positive result means profit and negative result means loss].

See Example : Refer ebook Solved Examples/Ch-6

Marked Price

The price on the lable is called the marked price or list price.

The marked price is abbreviated as M.P.

Discount

The reduction made on the 'marked price' of an article is called the discount.

NOTE : *When no discount is given, 'selling price' is the same as 'marked price'.*

- Discount = Marked price × Rate of discount.
- S.P. = M.P. – Discount.
- $\text{Discount \%} = \frac{\text{Discount}}{\text{M.P.}} \times 100$.
- Buy x get y free i.e., if x + y articles are sold at cost price of x articles, then the percentage discount = $\frac{y}{x + y} \times 100$.

REMEMBER

★ In **successive discounts**, first discount is subtracted from the marked price to get net price after the first discount. Taking this price as the new marked price, the second discount is calculated and it is subtracted from it to get net price after the second discount. Continuing in this manner, we finally obtain the final selling price.

In case of successive discounts a% and b%, the effective discount is $\left(a+b-\frac{ab}{100}\right)\%$

NOTE : *If the list price of an item is given and discounts d_1 and d_2 are given succesively on it then,*

$$\textit{Final price} = \textit{list price}\left(1-\frac{d_1}{100}\right)\left(1-\frac{d_2}{100}\right)$$

SALES TAX

To meet government's expenditures like construction of roads, railway, hospitals, schools etc. the government imposes different types of taxes. Sales tax (S.T.) is one of these tax.

Sales tax is calculated on selling price (S.P.)

NOTE : *If discount is given, selling price is calculated first and then sales tax is calculated on the selling price of the article.*

Shortcut Approach

If 'a'th part of some items is sold at x% loss, then required gain per cent in selling rest of the items in order that there is neither gain nor loss in whole transaction, is $\frac{ax}{1-a}\%$

See Example : Refer ebook Solved Examples/Ch-6

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Chapter 7

Simple & Compound Interest

INTEREST

Interest is the fixed amount paid on borrowed money.
The sum lent is called the **Principal**.
The sum of the principal and interest is called the **Amount**.
Interest is of two kinds :

(i) Simple interest
(ii) Compound interest

(I) SIMPLE INTEREST

When interest is calculated on the original principal for any length of time, it is called simple interest.

REMEMBER

★ Simple interest $= \dfrac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$

i.e. $\text{S.I.} = \dfrac{P \times R \times T}{100}$

★ Principal (P) $= \dfrac{100 \times \text{S.I.}}{R \times T}$

★ Rate (R) $= \dfrac{100 \times \text{S.I.}}{T \times P}$

★ Time (T) $= \dfrac{100 \times \text{S.I.}}{P \times R}$

★ If rate of simple interest differs from year to year, then

$$\text{S.I.} = P \times \frac{(R_1 + R_2 + R_3 + \ldots..)}{100}$$

★ Amount = Principal + Interest

i.e. $A = P + I = P + \dfrac{PRT}{100} = P\left(1 + \dfrac{RT}{100}\right)$

Shortcut Approach

- If $\frac{1}{x}$ part of a certain sum P is lent out at R_1% SI, $\frac{1}{y}$ part is lent out at R_2 % SI and the remaining $\frac{1}{z}$ part at R_3 % SI and this way the interest received by l, then $P = \dfrac{1 \times 100}{\frac{R_1}{x} + \frac{R_2}{y} + \frac{R_3}{z}}$

- If a sum of money becomes n times in T yr at simple interest, then formula for calculating rate of interest will be given as

$$R = \frac{100(n-1)}{T}\%$$

- If a sum of money at a certain rate of interest becomes n times in T_1 yr and m times in T_2 yr, then formula for T_2 will be given as

$$T_2 = \left(\frac{m-1}{n-1}\right) \times T_1$$

See Example : Refer ebook Solved Examples/Ch-7

(II) COMPOUND INTEREST

Money is said to be lent at compound interest when at the end of a year or other fixed period, the interest that has become due is not paid to the lender, but is added to the sum lent, and the amount thus obtained becomes the principal in the next year or period. The process is repeated until the amount for the last period has been found. Hence, When the interest charged after a certain specified time period is added to form new principal for the next time period, the interest is said to be compounded and the total interest accrued is compound interest.

REMEMBER

★ $\text{C.I.} = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$;

★ $\text{Amount (A)} = P\left(1+\frac{r}{100}\right)^n$

★ If rate of compound interest differs from year to year, then

$$\text{Amount} = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right).......$$

Compound interest – when interest is compounded annually but time is in fraction

If time $= t\frac{p}{q}$ years, then

$$\boxed{A = P\left(1+\frac{r}{100}\right)^t\left(1+\frac{\frac{p}{q}r}{100}\right)}$$

Compound interest – when interest is calculated half-yearly

Since r is calculated half-yearly therefore the rate per cent will become half and the time period will become twice, i.e.,

Rate per cent when interest is paid half-yearly $= \frac{r}{2}\%$

and time = 2 × time given in years

Hence,

$$\boxed{A = P\left(1+\frac{r}{2\times100}\right)^{2n}}$$

Compound interest – when interest is calculated quarterly

Since 1 year has 4 quarters, therefore rate of interest will become $\frac{1}{4}$th of the rate of interest per annum, and the time period will be 4 times the time given in years

Hence, for quarterly interest

$$\boxed{A = P\left(1+\frac{r/4}{100}\right)^{4\times n} = P\left(1+\frac{r}{400}\right)^{4n}}$$

☞ Shortcut Approach

Difference between Compound Interest and Simple Interest

When T = 2

(i) $\text{C.I.} - \text{S.I.} = P\left(\frac{R}{100}\right)^2$

(ii) $\text{C.I.} - \text{S.I.} = \frac{R \times \text{S.I.}}{2 \times 100}$

When T = 3

(i) $\text{C.I.} - \text{S.I.} = \frac{PR^2}{10^4}\left(\frac{300 + R}{100}\right)$

(ii) $\text{C.I.} - \text{S.I.} = \frac{\text{S.I.}}{3}\left[\left(\frac{R}{100}\right)^2 + 3\left(\frac{R}{100}\right)\right]$

***NOTE:** SI and CI for one year on the same sum and at same rate are equal.*

☞ Shortcut Approach

If a certain sum at compound interest becomes x times in n_1 yr and y times in n_2 yr, then $x^{\frac{1}{n_1}} = y^{\frac{1}{n_2}}$

☞ Shortcut Approach

If the population of a city is P and it increases with the rate of R% per annum, then

(i) Population after n yr $= P\left(1 + \frac{R}{100}\right)^n$

(ii) Population n yr ago $= \frac{P}{\left(1 + \frac{R}{100}\right)^n}$

See Example : Refer ebook Solved Examples/Ch-7

Note: • If population decreases with the rate of R%, then (–) sign will be used in place of (+) in the above mentioned formula

• If the rate of growth per year is $R_1\%, R_2\%, R_3\%, \ldots\ldots., R_n\%$, then Population after n yr

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)\ldots\ldots\left(1+\frac{R_n}{100}\right)$$

(This formula can also be used, if there is increase/decrease in the price of an article.)

A computer gives the following results for various values of n.		
Interest is compounded	„	$\left(1+\frac{1}{n}\right)^n$
Annually	1	$\left(1+\frac{1}{1}\right)^2 = 2$
Semiannually	2	$\left(1+\frac{1}{2}\right)^2 = 2.25$
Quarterly	4	$\left(1+\frac{1}{4}\right)^4 = 2.4414$
Monthly	12	$\left(1+\frac{1}{12}\right)^{12} = 2.6130$

Chapter

8 Time & Work

TIME AND WORK

In most of the problems on time and work, either of the following basic parameters are to be calculated :

Shortcut Approach

- If A can do a piece of work in X days, then A's one day's work = $\frac{1}{X}$th part of whole work.
- If A's one day's work = $\frac{1}{X}$th part of whole work, then A can finish the work in X days.
- If A can do a piece of work in X days and B can do it in Y days then A and B working together will do the same work in $\frac{XY}{X+Y}$ days.
- If A, B and C can do a work in X, Y and Z days respectively then all of them working together can finish the work in

$$\frac{XYZ}{XY+YZ+XZ} \text{ days.}$$

- If (A + B) can do a piece of work in X days, (B + C) can do a place of work in Y days and (C + A) can do a piece of work in Z days. Then,

(A + B + C) can do a piece of work in $\frac{2XYZ}{XY+YZ+ZX}$ days

See Example : Refer ebook Solved Examples/Ch-8

Shortcut Approach

- If A and B together can do a piece of work in X days and A alone can do it in Y days, then B alone can do the work in

$$\frac{XY}{Y-X} \text{ days.}$$

- If (A + B + C) can do a piece of work in X days and (B + C) can do a piece of work in Y days then

A can do a piece of work $\frac{XY}{Y-X}$ days

- A and B can do a work in 'X' and 'Y' days respectively. They started the work together but A left 'a' days before completion of the work. Then, time taken to finish the work is $\frac{Y(X+a)}{X+Y}$

- If 'A' is 'a' times efficient than B and A can finish a work in X days, then working together, they can finish the work in $\frac{aX}{a+1}$ days.

- If A is 'a' times efficient than B and working together they finish a work in Z days then, time taken by A = $\frac{Z(a+1)}{a}$ days. and time taken by B = Z(a + 1) days.

- If A working alone takes 'x' days more than A and B together, and B working along takes 'y' days more than A and B together then the number of days taken by A and B working together is given by $[\sqrt{xy}]$ days.

See Example : Refer ebook Solved Examples/Ch-8

☞ Shortcut Approach

- If a_1 men and b_1 boys can complete a work in x days, while a_2 men and b_2 boys can complete the same work in y days, then

$$\frac{\text{One day work of 1 man}}{\text{One day work of 1 boy}} = \frac{(yb_2 - xb_1)}{(xa_1 - ya_2)}$$

- If n men or m women can do a piece of work in X days, then N men and M women together can finish the work in $\frac{nmX}{nM + mN}$ days.

- A and B do a piece of work in a and b days, respectively. Both begin together but after some days, A leaves off and the remaining work is completed by B in x days. Then, the time after which A left, is given by $T = \frac{(b-x)a}{a+b}$

- If 'M_1' persons can do 'W_1' works in 'D_1' days and 'M_2' persons can do 'W_2' works in 'D_2' days then
$M_1D_1W_2 = M_2D_2W_1$
If T_1 and T_2 are the working hours for the two groups then
$M_1D_1W_2T_1 = M_2D_2W_1T_2$
Similarly,
$M_1D_1W_2T_1E_1 = M_2D_2W_1T_2E_2$, where E_1 and E_2 are the efficiencies of the two groups.

- If the number of men to do a job is changed in the ratio a : b, then the time required to do the work will be in the ratio b : a, assuming the amount of work done by each of them in the given time is the same, or they are identical.

- A is K times as good a worker as B and takes X days less than B to finish the work. Then the amount of time required by A and B working together is $\frac{K \times X}{K^2 - 1}$ days.

- If A is n times as efficient as B, i.e. A has n times as much capacity to do work as B, then A will take $\frac{1}{n}$ of the time taken by B to do the same amount of work.

See Example : Refer ebook Solved Examples/Ch-8

WORK AND WAGES

Wages are distributed in proportion to the work done and in indirect proportion to the time taken by the individual.

PIPES AND CISTERNS

The same principle of Time and Work is employed to solve the problems on Pipes and Cisterns. The only difference is that in this case, the work done is in terms of filling or emptying a cistern (tank) and the time taken is the time taken by a pipe or a leak (crack) to fill or empty a cistern respectively.

Inlet : A pipe connected with a tank (or a cistern or a reservoir) is called an inlet, if it fills it.

Outlet : A pipe connected with a tank is called an outlet, if it empties it.

☞ Shortcut Approach

- If a pipe can fill a tank in x hours, then the part filled in 1 hour $=\frac{1}{x}$
- If a pipe can empty a tank in y hours, then the part of the full tank emptied in 1 hour $=\frac{1}{y}$.
- If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then the net part filled in 1 hour, when both the pipes are opened $=\left(\frac{1}{x}-\frac{1}{y}\right)$.

  ∴ Time taken to fill the tank, when both the pipes are opened $=\frac{xy}{y-x}$.
- If a pipe can fills or empties tank in x hours and another can fill or empties the same tank in y hours, then time taken to fill or empty the tank $=\frac{xy}{y+x}$, when both the pipes are opened
- If a pipe fills a tank in x hours and another fills the same tank is y hours, but a third one empties the full tank in z hours, and all of them are opened together, then net part filled in 1 hr $=\left[\frac{1}{x}+\frac{1}{y}-\frac{1}{z}\right]$

  ∴ Time taken to fill the tank $=\frac{xyz}{yz+xz-xy}$ hours.

- A pipe can fill a tank in x hrs. Due to a leak in the bottom it is filled in y hrs. If the tank is full, the time taken by the leak to empty the tank $= \frac{xy}{y-x}$ hrs.

- A cistern has a leak which can empty it in X hours. A pipe which admits Y litres of water per hour into the cistern is turned on and now the cistern is emptied in Z hours. Then the capacity of the cistern is $\frac{X+Y+Z}{Z-X}$ litres.

- A cistern is filled by three pipes whose diameters are X cm., Y cm. and Z cm. respectively (where X < Y < Z). Three pipes are running together. If the largest pipe alone will fill it in P minutes and the amount of water flowing in by each pipe is proportional to the square of its diameter, then the time in which the cistern will be filled by the three pipes is

$$\left[\frac{PZ^2}{X^2+Y^2+Z^2}\right] \text{ minutes.}$$

- If one filling pipe A is n times faster and takes X minutes less time than the other filling pipe B, then the time they will take to fill a cistern, if both the pipes are opened together, is $\left[\frac{nX}{(n^2-1)}\right]$ minutes.

  A will fill the cistern in $\left(\frac{X}{n-1}\right)$ minutes and B will take to fill the cistern $\left(\frac{nX}{n-1}\right)$ minutes.

  Here, A is the faster filling pipe and B is the slower one.

- Two filling pipes A and B opened together can fill a cistern in t minutes. If the first filling pipe A alone takes X minutes more or less than t and the second fill pipe B along takes Y minutes more or less than t minutes, then t is given by $[t = \sqrt{xy}]$ minutes.

See Example : Refer ebook Solved Examples/Ch-8

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Chapter 9

Time, Speed and Distance

TIME, SPEED AND DISTANCE

Speed

The rate at which any moving body covers a particular distance is called its speed.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}\ ; \ \text{Time} = \frac{\text{Distance}}{\text{Speed}}\ ;$$

$$\text{Distance} = \text{Speed} \times \text{time}$$

Unit :

SI unit of speed is metre per second (mps). It is also measured in kilometers per hour (kph) or miles per hour (mph).

Basic Conversions :

(i)
- 1 hour = 60 minutes = 60 × 60 seconds.
- 1 km = 1000 m
- 1 km = 0.6214 mile
- 1 mile = 1.609 km i.e. 8 km = 5 miles
- 1 yard = 3 feet
- 1 foot = 12 inches
- $1 \text{ km/h} = \frac{5}{18} \text{ m/sec,}$
- $1 \text{ m/sec} = \frac{18}{5} \text{ km/h}$
- $1 \text{ miles/hr} = \frac{22}{15} \text{ ft/sec}$

Shortcut Approach

- Average speed $= \dfrac{\text{Total Distance}}{\text{Total time}}$
- While travelling a certain distance (d), if a man changes his speed in the ratio m : n, then the ratio of time taken becomes n : m.
- If a certain distance (d), say from A to B, is covered at 'a' km/hr and the same distance is covered again say from B to A in 'b' km/hr, then the average speed during the whole journey is given by :

$$\text{Average speed} = \left(\frac{2ab}{a+b}\right) \text{ km/hr}$$

Also, if t_1 and t_2 is time taken to travel from A to B and B to A respectively, the distance 'd' from A to B is given by :

$$d = (t_1 + t_2)\left(\frac{ab}{a+b}\right)$$

$$d = (t_1 - t_2)\left(\frac{ab}{b-a}\right)$$

$$d = (b-a)\left(\frac{t_1 t_2}{t_1 - t_2}\right)$$

- If first part of the distance is covered at the rate of v_1 in time t_1 and the second part of the distance is covered at the rate of v_2 in time t_2, then the average speed is $\left(\dfrac{v_1 t_1 + v_2 t_2}{t_1 + t_2}\right)$

See Example : Refer ebook Solved Examples/Ch-9

Relative Speed

When two bodies are moving in same direction with speeds S_1 and S_2 respectively, their relative speed is the difference of their speeds.

i.e., Relative Speed $= S_1 - S_2$, If $S_1 > S_2$

$= S_2 - S_1$, if $S_2 > S_1$

When two bodies are moving in opposite direction with speeds S_1 and S_2 respectively, then their relative speed is the sum of their speeds.

i.e., Relative Speed $= S_1 + S_2$

☞ Shortcut Approach

If two persons (or vehicles or trains) start at the same time in opposite directions from two points A and B, and after crossing each other they take x and y hours respectively to complete the journey, then

$$\frac{\text{Speed of first}}{\text{Speed of second}} = \sqrt{\frac{y}{x}}$$

☞ Shortcut Approach

Usual speed : If a man changes his speed to $\frac{a}{b}$ of his usual speed, reachss his destination late/earlier by t minutes then,

$$\text{Usual time} = \frac{\text{Change in time}}{\left(\frac{b}{a} - 1\right)}$$

☞ Shortcut Approach

A man covers a certain distance D. If he moves S_1 speed faster, he would have taken t time less and if he moves S_2 speed slower, he would have taken t time more. The original speed is given by

$$\frac{2 \times (S_1 \times S_2)}{S_2 \sim S_1}$$

☞ Shortcut Approach

If a person with two different speeds U & V cover the same distance, then required distance

$$= \frac{U \times V}{U \sim V} \times \text{Difference between arrival time}$$

Also, required distance $= \text{Total time taken} \times \frac{U \times V}{U + V}$

See Example : Refer ebook Solved Examples/Ch-9

☞ Shortcut Approach

- A policemen sees a thief at a distance of d. He starts chasing the thief who is running at a speed of 'a' and policeman is chasing with a speed of 'b' (b > a). In this case, the distance covered by the thief when he is caught by the policeman, is given by $d\left(\frac{a}{b-a}\right)$.

- A man leaves a point A at t_1 and reaches the point B at t_2. Another man leaves the point B at t_3 and reaches the point A at t_4, then they will meet at

$$t_1 + \frac{(t_2 - t_1)(t_4 - t_1)}{(t_2 - t_1) + (t_4 - t_3)}$$

- Relation between time taken with two different modes of transport : $t_{2x} + t_{2y} = 2(t_x + t_y)$

  where,

  t_x = time when mode of transport x is used single way.

  t_y = time when mode of transport y is used single way.

  t_{2x} = time when mode of transport x is used both ways.

  t_{2y} = time when mode of transport y is used both ways.

See Example : Refer ebook Solved Examples/Ch-9

TRAINS

A train is said to have crossed an object (stationary or moving) only when the last coach of the train crosses the object completely. It implies that the total length of the train has crossed the total length of the object.

Shortcut Approach

- Time taken by a train to cross a pole/a standing man

$$= \frac{\text{Length of train}}{\text{Speed of train}}.$$

- Time taken by a train to cross platform/bridge etc. (i.e. a stationary object with some length)

$$= \frac{\text{length of train} + \text{length of platform/bridge etc.}}{\text{speed of train}}.$$

- When two trains with lengths L_1 and L_2 and with speeds S_1 and S_2 respectively, then
  (a) When they are moving in the same direction, time taken by the faster train to cross the slower train

$$= \frac{L_1 + L_2}{\text{difference of their speeds}}.$$

  (b) When they are moving in the opposite direction, time taken by the trains to cross each other

$$= \frac{L_1 + L_2}{\text{sum of their speeds}}.$$

- Suppose two trains or two bodies are moving in the same direction at u km/hr and v km/hr respectively such that $u > v$, then their relative speed $= (u - v)$ km/hr.
  If their lengths be x km and y km respectively, then time taken by the faster train to cross the slower train (moving in the same direction)

$$= \left(\frac{x + y}{u - v}\right) \text{hrs.}$$

- Suppose two trains or two bodies are moving in opposite directions at u km/hr and v km/hr, then their relative speed $= (u + v)$ km/hr.
  If their lengths be x km & y km, then :

  time taken to cross each other $= \left(\frac{x + y}{u + v}\right)$ hrs.

- If a man is running at a speed of u m/sec in the same direction in which a train of length L meters is running at a speed v m/sec, then (v – u) m/sec is called the speed of the train relative to man. Then the time taken by the train to cross the man = $\frac{1}{v-u}$ seconds

- If a man is running at a speed of u m/sec in a direction opposite to that in which a train of length L meters is running with a speed v m/sec, then (u + v) is called the speed of the train relative to man. Then the time taken by the train to cross the man

$$= \frac{1}{v+u} \text{ seconds.}$$

- If two trains start at the same time from two points A and B towards each other and after crossing, they take (a) and (b) hours in reaching B and A respectively. Then,

A's speed : B's speed = $\left(\sqrt{b} : \sqrt{a}\right)$.

- If a train of length L m passes a platform of x m in t_1s, then time taken t_2s by the same train to pass a platform of length y m is given as

$$t_2 = \left(\frac{L+y}{L+x}\right)t_1$$

- From stations P and Q, two trains start moving towards each other with the speeds a and b, respectively. When they meet each other, it is found that one train covers distance d more than that of another train. In such cases, distance between stations P and Q is given as $\left(\frac{a+b}{a-b}\right)\times d$.

- The distance between P and Q is (d) km. A train with (a) km/h starts from station P towards Q and after a difference of (t) hr another train with (b) km/h starts from Q towards station P, then both the trains will meet at a certain point after time T. Then,

$$T = \left(\frac{d \pm tb}{a+b}\right)$$

- If second train starts after the first train, then t is taken as positive. If second train starts before the first train, then t is taken as negative.

See Example : Refer ebook Solved Examples/Ch-9

☞ Shortcut Approach

- The distance between two stations P and Q is d km. A train starts from P towards Q and another train starts from Q towards P at the same time and they meet at a certain point after t h. If train starting from P travels with a speed of x km/h slower or faster than another train, then

  (i) Speed of faster train = $\left(\frac{d+tx}{2t}\right)$km/h

  (ii) Speed of slower train = $\left(\frac{d-tx}{2t}\right)$km/h

- A train covers distance d sbetween two stations P and Q in t_1 h. If the speed of train is reduced by (a) km/h, then the same distance will be covered in t_2 h.

  (i) Distance between P and Q is

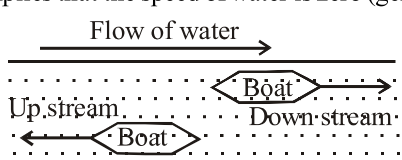
  $$d = a\left(\frac{t_1 t_2}{t_2 - t_1}\right) \text{ km}$$

  (ii) Speed of the train = $\left(\frac{at_2}{t_2 - t_1}\right)$km/h

See Example : Refer ebook Solved Examples/Ch-9

BOATS AND STREAMS

Stream : It implies that the water in the river is moving or flowing.
Upstream : Going against the flow of the river.
Downstream : Going with the flow of the river.
Still water : It implies that the speed of water is zero (generally, in a lake).



Let the speed of a boat (or man) in still water be X m/sec and the speed of the stream (or current) be Y m/sec. Then,

☞ Shortcut Approach

- Speed of boat with the stream (or downstream or D/S) $= (X + Y)$ m/sec.
- Speed of boat against the stream (or upstream or U/S) $= (X - Y)$ m/sec.
- Speed of boat in still water is

$$X = \frac{(X+Y)+(X-Y)}{2} = \frac{\text{Upstream} + \text{Downstream}}{2}$$

- Speed of the stream or current is $Y = \frac{(X+Y)-(X-Y)}{2}$

$$= \frac{\text{Downstream} - \text{Upstream}}{2}$$

☞ Shortcut Approach

- A man can row X km/h in still water. If in a stream which is flowing of Y km/h, it takes him Z hours to row to a place and back, the distance between the two places is $\frac{Z(X^2 - Y^2)}{2X}$
- A man rows a certain distance downstream in X hours and returns the same distance in Y hours. If the stream flows at the rate of Z km/h, then the speed of the man in still water is given by

$$\frac{Z(X+Y)}{Y-X} \text{km/hr}$$

- And if speed of man in still water is Z km/h then the speed of stream is given by

$$\frac{Z(Y-X)}{X+Y} \text{km/hr}$$

See Example : Refer ebook Solved Examples/Ch-9

☞ Shortcut Approach

- If speed of stream is a and a boat (swimmer) takes n times as long to row up as to row down the river, then

  Speed of boat (swimmer) in still water = $\frac{a(n+1)}{(n-1)}$

  Note: *This formula is applicable for equal distances.*

- If a man capable of rowing at the speed (u) m/sec in still water, rows the same distance up and down a stream flowing at a rate of (v) m/sec, then his average speed through the journey is

  $$= \frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}} = \frac{(u-v)\,(u+v)}{u}$$

- If boat's (swimmer's) speed in still water is a km/h and river is flowing with a speed of b km/h, then average speed in going to a certain place and coming back to starting point is given by

  $\frac{(a+b)(a-b)}{a}$ km/h.

See Example : Refer ebook Solved Examples/Ch-9

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Chapter

10 Mensuration

MENSURATION

Mensuration is the science of measurement of the lenghts of lines, areas of surfaces and volumes of solids.

Perimeter

Perimeter is sum of all the sides. It is measured in cm, m, etc.

Area

The area of any figure is the amount of surface enclosed within its boundary lines. This is measured in square unit like cm^2, m^2, etc.

Volume

If an object is solid, then the space occupied by such an object is called its volume. This is measured in cubic unit like cm^3, m^3, etc.

Basic Conversions :

I. 1 m = 10 dm

1 dm = 10 cm

1 cm = 10 mm

1 m = 100 cm = 1000 mm

1 km = 1000 m

II. 1 km $= \frac{5}{8}$ miles

1 mile = 1.6 km

1 inch = 2.54 cm

III. 100 kg = 1 quintal

10 quintal = 1 tonne

1 kg = 2.2 pounds (approx.)

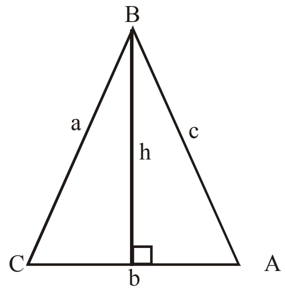
IV. 1 litre = 1000 cc

1 acre = 100 m^2

1 hectare = 10000 m^2 (100 acre)

PART I : PLANE FIGURES

TRIANGLE



Perimeter $(P) = a + b + c$

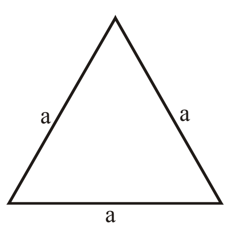
Area $(A) = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$ and a, b and c are three sides of the triangle.

Also, $A = \frac{1}{2} \times bh$; where $b \rightarrow$ base

$h \rightarrow$ altitude

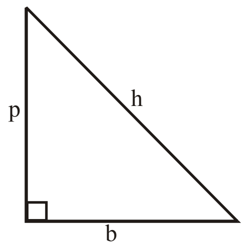
Equilateral triangle



Perimeter $= 3a$

$A = \frac{\sqrt{3}}{4} a^2$; where $a \rightarrow$ side

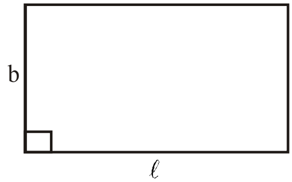
Right triangle



$A = \frac{1}{2}pb$ and $h^2 = p^2 + b^2$ (Pythagoras triplet)

where $p \rightarrow$ perpendicular
$b \rightarrow$ base
$h \rightarrow$ hypotenuse

RECTANGLE



Perimeter $= 2(\ell + b)$

Area $= \ell \times b$; where $\ell \rightarrow$ length

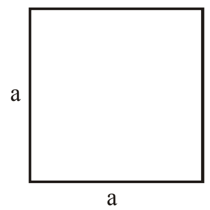
$b \rightarrow$ breadth

☞ ***Shortcut Approach***

If the length and breadth of a rectangle are increased by a% and b%, respectively, then are will be increased by $\left(a + b + \frac{ab}{100}\right)\%$.

See Example : Refer ebook Solved Examples/Ch-10

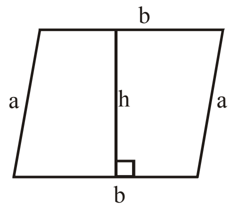
SQUARE



Perimeter = 4 × side = 4a

Area = $(\text{side})^2 = a^2$;   where a → side

PARALLELOGRAM



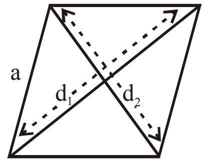
Perimeter = 2 (a + b)

Area = b × h;   where a → breadth

b → base (or length)

h → altitude

RHOMBUS

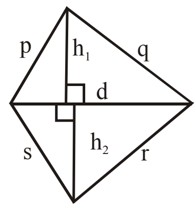


Perimeter = 4 a

Area $= \frac{1}{2} d_1 \times d_2$ where a → side and

d_1 and d_2 are diagonals.

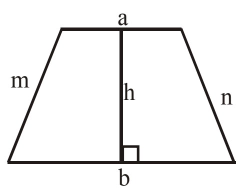
IRREGULAR QUADRILATERAL



Perimeter $= p + q + r + s$

Area $= \frac{1}{2} \times d \times (h_1 + h_2)$

TRAPEZIUM

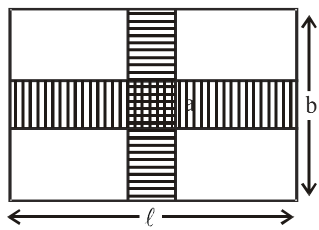


Perimeter $= a + b + m + n$

Area $= \frac{1}{2}(a + b)h$; where (a) and (b) are two parallel sides;

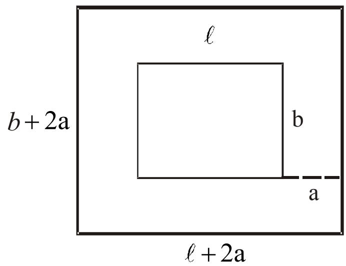
(m) and (n) are two non-parallel sides;
$h \rightarrow$ perpendicular distance between two parallel sides.

AREA OF PATHWAYS RUNNING ACROSS THE MIDDLE OF A RECTANGLE



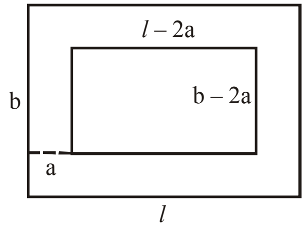
$A = a(\ell + b) - a^2$; where $\ell \rightarrow$ length
$b \rightarrow$ breadth,
$a \rightarrow$ width of the pathway.

Pathways outside



$A = (l + 2a)(b + 2a) - lb$; where $l \to$ length
$b \to$ breadth
$a \to$ width of the pathway

Pathways inside



$A = lb - (l - 2a)(b - 2a)$; where $l \to$ length
$b \to$ breadth
$a \to$ width of the pathway

☞ Shortcut Approach

- If a pathway of width x is made inside or outside a rectangular plot of length l and breadth b, then are of pathway is
  (i) $2x(l + b + 2x)$, if path is made outside the plot.
  (ii) $2x(l + b - 2x)$, if path is made inside the plot.
- If two paths, each of width x are made parallel to length (l) and breadth (b) of the rectangular plot in the middle of the plot, then area of the paths is $x(l + b - x)$

See Example : Refer ebook Solved Examples/Ch-10

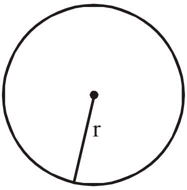
CIRCLE

Perimeter (Circumference) $= 2\pi r = \pi d$

Area $= \pi r^2$; where $r \rightarrow$ radius

$d \rightarrow$ diameter

and $\pi = \frac{22}{7}$ or 3.14



☞ Shortcut Approach

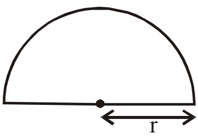
The length and breadth of a rectangle are l and b, then are of circle of maximum radius inscribed in that rectangle is $\frac{\pi b^2}{4}$.

See Example : Refer ebook Solved Examples/Ch-10

SEMICIRCLE

Perimeter $= \pi r + 2r$

Area $= \frac{1}{2} \times \pi r^2$



☞ Shortcut Approach

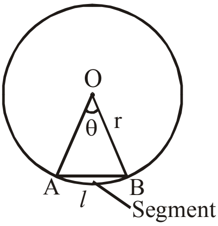
The are a of the largest triangle incribed in a semi-circle of radius r is equal to r^2.

See Example : Refer ebook Solved Examples/Ch-10

SECTOR OF A CIRCLE

Area of sector OAB $= \frac{\theta}{360} \times \pi r^2$

Length of an arc $(l) = \frac{\theta}{360} \times 2\pi r$



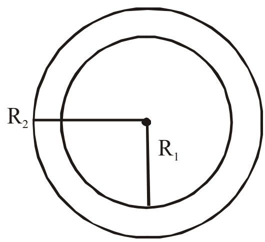
Area of segment = Area of sector – Area of triangle OAB

$$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

Perimeter of segment = length of the arc + length of segment

$$AB = \frac{\pi r\theta}{180} + 2r\sin\frac{\theta}{2}$$

RING

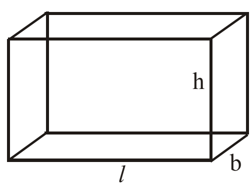


Area of ring $= \pi\left(R_2^2 - R_1^2\right)$

PART-II SOLID FIGURE

CUBOID

A cuboid is a three dimensional box.
Total surface area of a cuboid = 2 (lb + bh + lh)
Volume of the cuboid = $l$bh



Area of four walls = 2(l + b) × h

☞ ***Shortcut Approach***

If length, breadth and height of a cuboid are changed by x%, y% and z% respectively, then its volume is increased by

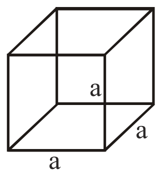
$$= \left[x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{(100)^2}\right]\%$$

See Example : Refer ebook Solved Examples/Ch-10

Note: *Increment in the value is taken as positive and decrement in value is taken as negative. Positive result shows total increment and negative result shows total decrement.*

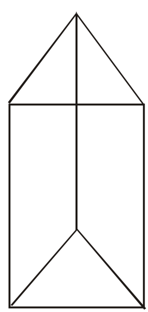
CUBE

A cube is a cuboid which has all its edges equal.
Total surface area of a cube = $6a^2$
Volume of the cube = a^3



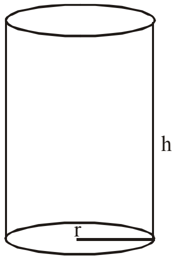
RIGHT PRISM

A prism is a solid which can have any polygon at both its ends.
Lateral or curved surface area = Perimeter of base × height
Total surface area = Lateral surface area + 2 (area of the end)
Volume = Area of base × height



RIGHT CIRCULAR CYLINDER

It is a solid which has both its ends in the form of a circle.
Lateral surface area = $2\pi rh$
Total surface area = $2\pi r (r + h)$
Volume = $\pi r^2 h$; where r is radius of the base and h is the height



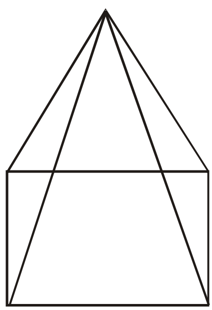
PYRAMID

A pyramid is a solid which can have any polygon at its base and its edges converge to single apex.

Lateral or curved surface area

$$= \frac{1}{2}\text{(perimeter of base)} \times \text{slant height}$$

Total surface area = lateral surface area + area of the base



$$\text{Volume} = \frac{1}{3}\text{(area of the base)} \times \text{height}$$

RIGHT CIRCULAR CONE

It is a solid which has a circle as its base and a slanting lateral surface that converges at the apex.

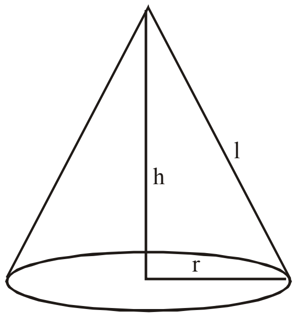
Lateral surface area = πrl

Total surface area = $\pi r(l + r)$

Volume $= \frac{1}{3}\pi r^2 h$; where r : radius of the base

h : height

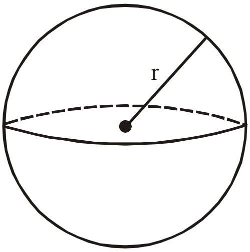
l : slant height



SPHERE

It is a solid in the form of a ball with radius r.
Lateral surface area = Total surface area = $4\pi r^2$

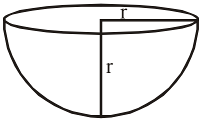
Volume $= \frac{4}{3}\pi r^3$; where r is radius.



HEMISPHERE

It is a solid half of the sphere.
Lateral surface area = $2\pi r^2$
Total surface area = $3\pi r^2$

Volume $= \frac{2}{3}\pi r^3$; where r is radius



☞ **Shortcut Approach**

If side of a cube or radius (or diameter) of sphere is increased by x%,

then its volume increases by $\left[\left(1+\frac{x}{100}\right)^3-1\right]\times 100\%$

☞ **Shortcut Approach**

If in a cylinder or cone, height and radius both change by x%, then

volume changes by $\left[\left(1+\frac{x}{100}\right)^3-1\right]\times 100\%$

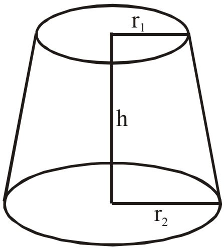
See Example : Refer ebook Solved Examples/Ch-10

FRUSTUM OF A CONE

When a cone cut the left over part is called the frustum of the cone.
Curved surface area $= \pi l\,(r_1 + r_2)$

Total surface area $= \pi l\left(r_1 + r_2\right) + \pi r_1^2 + \pi r_2^2$

where $l = \sqrt{h^2 + \left(r_1 - r_2\right)^2}$



Volume $= \frac{1}{3}\pi h\left(r_1^2 + r_1 r_2 + r_2^2\right)$

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Chapter 11

Clock and Calendar

CLOCK

Introduction

- A clock has two hands : Hour hand and Minute hand.
- The minute hand (M.H.) is also called the long hand and the hour hand (H.H.) is also called the short hand.
- The clock has 12 hours numbered from 1 to 12.

Also, the clock is divided into 60 equal minute divisions. Therefore, each hour number is separated by five minute divisions. Therefore,

Shortcut Approach

- One minute division $= \frac{360}{60} = 6°$ apart. ie. In one minute, the minute hand moves 6°.
- One hour division $= 6° \times 5 = 30°$ apart. ie. In one hour, the hour hand moves 30° apart.

  Also, in one minute, the hour hand moves $= \frac{30°}{60°} = \frac{1°}{2}$ apart.
- Since, in one minute, minute hand moves 6° and hour hand moves $\frac{1°}{2}$, therefore, in one minute, the minute hand gains $5\frac{1°}{2}$ more than hour hand.
- In one hour, the minute hand gains $5\frac{1°}{2} \times 60 = 330°$ over the hour hand. i.e. the minute hand gains 55 minutes divisions over the hour hand.

See Example : Refer ebook Solved Examples/Ch-11

Relative position of the hands

The position of the M.H. relative to the H.H. is said to be the same, whenever the M.H. is separated from the H.H. by the same number of minute divisions and is on same side (clockwise or anticlockwise) of the H.H.

Any relative position of the hands of a clock is repeated 11 times in every 12 hours.

(a) When both hands are 15 minute spaces apart, they are at right angle.

(b) When they are 30 minute spaces apart, they point in opposite directions.

(c) The hands are in the same straight line when they are coincident or opposite to each other.

- In every hour, both the hand coincide once.
- In a day, the hands are coinciding 22 times.
- In every 12 hours, the hands of clock coincide 11 times.
- In every 12 hours, the hands of clock are in opposite direction 11 times.
- In every 12 hours, the hands of clock are at right angles 22 times.
- In every hour, the two hands are at right angles 2 times.
- In every hour, the two hands are in opposite direction once.
- In a day, the two hands are at right angles 44 times.
- If both the hands coincide, then they will again coincide after $65\frac{5}{11}$ minutes. i.e. in correct clock, both hand coincide at an interval of $65\frac{5}{11}$ minutes.
- If the two hands coincide in time less than $65\frac{5}{11}$ minutes, then clock is too fast and if the two hands coincides in time more than $65\frac{5}{11}$ minutes, then the clock is too slow.

☞ *Shortcut Approach*

- **Shortcut Approach for finding degrees minutes and hours is**

  $\theta = \left(\frac{11}{2}M - 30H\right)$

  Where, M = minutes
  and, H = Hours

- When value of θ becomes more than 360, subtract 360 from the value of θ and complete the calculation.

See Example : Refer ebook Solved Examples/Ch-11

INCORRECT CLOCK

If a clock indicates 6 : 10, when the correct time is 6 : 00, it is said to be 10 minute too fast and if it indicates 5 : 50 when the correct time is 6 : 00, it is said to be 10 minute too slow.

- Also, if both hands coincide at an interval x minutes and $x < 65\frac{5}{11}$, then total time gained $= \left(\frac{65\frac{5}{11} - x}{x}\right)$ minutes and clock is said to be 'fast'.
- If both hands coincide at an interval x minutes and $x > 65\frac{5}{11}$, then total time lost $= \left(\frac{x - 65\frac{5}{11}}{x}\right)$ minutes and clock is said to be 'slow'.

CALENDAR

INTRODUCTION

An ordinary year has 365 days. Every year which is divisible by 4, is a leap year and has 366 days, But century year has 365 days except for year divisible by 400 which has 366 days.
An ordinary year contains 365 days i.e., 52 weeks + 1 day i.e. 1 odd day.
A leap year contains 366 days i.e. 52 weeks + 2 days i.e. 2 odd days.

A century (100 years) contains = 24 leap years + 76 ordinary years
$= 24 \times 2 + 76 = 124$ odd days = 17 weeks + 5 odd days
Similarly,
200 years contains $2 \times 5 - 7 = 3$ odd days
300 years contains $3 \times 5 - 14 = 1$ odd day
400 years contains $4 \times 5 + 1 - 21 = 0$ odd days
First January, 1 A.D. was Monday.
A solar year contains 365 days 5 hours 48 minutes 48 seconds.
The first day of a century must either be Monday, Tuesday, Thursday or Saturday.

Months	Odd days
January	3
February	0/1 (ordinary/leap)
March	3
April	2
May	3
June	2
July	3
August	3
September	2
October	3
November	2
December	3

To find a particular day witihout given date and day

Following steps are taken into consideration to solve such questions

Step I Firstly, you have to find the number of odd upto the date for which the day is to be determined.

Step II Your required day will be according to the following conditions

(a) If the number of odd days = 0, then required day is Sunday.
(b) If the number of odd days = 1, then required day is Monday.
(c) If the number of odd days = 2, then required day is Tuesday.
(d) If the number of odd days = 3, then required day is Wednesday.
(e) If the number of odd days = 4, then required day is Thursday.
(f) If the number of odd days = 5, then required day is Friday.
(g) If the number of odd days = 6, then required day is Saturday.

NOTE : *February in an ordinary year gives no odd days, but in a leap year gives one odd day.*

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Chapter 12

Permutation and Combination

INTRODUCTION

Factorial

The important mathematical term "Factorial" has extensively used in this chapter.

The product of first n consecutive **natural numbers** is defined as **factorial of n**. It is denoted by n! or $\underline{|n}$. Therefore,

$$\boxed{n! = 1 \times 2 \times 3 \times .... \times (n-1) \times n}$$

For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Note that :

$$\frac{n!}{r!} \neq \left(\frac{n}{r}\right)!$$

$0! = 1$

The factorials of fractions and negative integers are not defined.

Fundamental Principles of Counting

1. **Principle of Addition :** If an event can occur in 'm' ways and another event can occur in 'n' ways independent of the first event, then either of the two events can occur in (m + n) ways.
2. **Principle of Multiplication :** If an operation can be performed in 'm' ways and after it has been performed in any one of these ways, a second operation can be performed in 'n' ways, then the two operations in succession can be performed in (m × n) ways.

Method of Sampling :

Sampling process can be divided into following forms :

1. The order is IMPORTANT and the repetition is ALLOWED, each sample is then a SEQUENCE.
2. The order is IMPORTANT and the repetition is NOT ALLOWED, each sample is then a PERMUTATION.
3. The order is NOT IMPORTANT and repetition is ALLOWED, each sample is then a MULTISET.
4. The order is NOT IMPORTANT and repetition is NOT ALLOWED, each sample is then a COMBINATION.

PERMUTATION

Each of the arrangements, which can be made by taking, some or all of a number of things is called a PERMUTATION.

For Example : Formation of numbers, word formation, sitting arrangement in a row.

The number of permutations of 'n' things taken 'r' at a time is denoted by

${}^nP_r$. It is defind as, ${}^nP_r = \dfrac{n!}{(n-r)!}$.

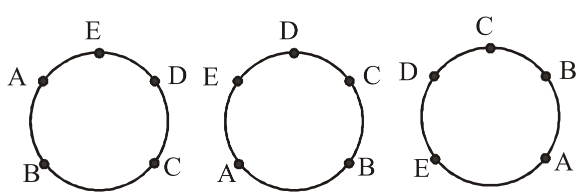
Note that:

$${}^nP_n = n!$$

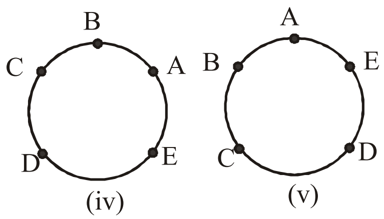
Circular permutations:

(i) Arrangements round a circular table :

Consider five persons A, B, C, D and E to be seated on the circumference of a circular table in order (which has no head). Now, shifting A, B, C, D and E one position in anticlockwise direction we will get arrangements as follows:



(i) (ii) (iii)



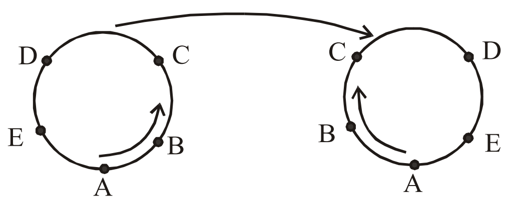
we see that arrangements in all figures are same.

$\therefore$ The number of circular permutations of n different things taken all at a time is $\frac{{}^nP_n}{n} = (n-1)\,!$, if clockwise and anticlockwise orders are taken as different.

(ii) Arrangements of beads or flowers (all different) around a circular necklace or garland:

Consider five beads A, B, C, D and E in a necklace or five flowers A, B, C and D, E in a garland etc. If the necklace or garland on the left is turned over we obtain the arrangement on the right, i.e., anticlockwise and clockwise order of arrangements are not different.

Thus the number of circular permutations of 'n' different things taken. all at a time is $\frac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken to be some.



Conditional Permutations

Number of permutations of n things taking r at a time, in which a particular thing always occurs = $r.\,{}^{n-1}P_{r-1}$.

Distinguishable Permutations

1. Suppose a set of n objects has n_1 of one kind of object, n_2 of a second kind, n_3 of a third kind, and so on, with $n = n_1 + n_2 + n_3 + \ldots + n_k$, Then the number of distinguishable permutations of the n

objects is $\dfrac{n!}{n_1!\,n_2!\,n_3!\,.....\,n_k!}$

2. Number of permutations of n things taking r at a time, in which a particular thing never occurs = ${}^{n-1}P_r$.

3. Number of permutations of n different things taking all at a time, in which m specified things always come together = $m!(n-m+1)!$.

4. Number of permutations of n different things taking all at a time, in which m specified things never come together = $n! - m!(n-m+1)!$

5. The number of permutations of 'n' things taken all at a time, when 'p' are alike of one kind, 'q' are alike of second, 'r' alike of third, and so on $= \dfrac{n!}{p!\,q!\,r!}$.

6. The number of permutations of 'n' different things, taking 'r' at a time, when each thing can be repeated 'r' times = n^r

Shortcut Approach

Number of ways to declare the result where 'n' match are played = 2^n

See Example : Refer ebook Solved Examples/Ch-12

COMBINATION

Each of the different selections that can be made with a given number of objects taken some or all of them at a time is called a COMBINATION.
The number of combinations of 'n' dissimilar things taken 'r' at a time is denoted by ${}^nC_r$ or C(n, r). It is defined as,

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

REMEMBER

- ★ ${}^nC_0 = 1,\ {}^nC_n = 1;\ {}^nP_r = r!\,{}^nC_r$
- ★ ${}^nC_r = {}^nC_{n-r}$
- ★ ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$
- ★ ${}^nC_x = {}^nC_y \Rightarrow x + y = n$
- ★ ${}^nC_r = {}^nC_{r+1} = {}^{n+1}C_r$
- ★ ${}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$
- ★ ${}^nC_r = \frac{1}{r}(n - r + 1)\,{}^nC_{r-1}$
- ★ ${}^nC_1 = {}^nC_{n-1} = n$

Conditional Combinations

1. Number of combinations of n distinct things taking $r\ (\le n)$ at a time, when $k\ (0 \le k \le r)$ particular objects always occur $= {}^{n-k}C_{r-k}$.
2. Number of combinations of n distinct objects taking $r(\le n)$ at a time, when $k\,(0 \le k \le r)$ particular objects never occur $= {}^{n-k}C_r$.
3. Number of selections of r things from n things when p particular things are not together in any selection $= {}^nC_r - {}^{n-p}C_{r-p}$
4. Number of selection of r consecutive things out of n things in a row $= n - r + 1$
5. Number of selection of r consecutive things out of n things along a circle

$$= \begin{cases} n, \text{ when } & r < n \\ 1, \text{ when } & r = n \end{cases}$$

6. The number of Combinations of 'n' different things taking some or all at a time

$$= {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots\ldots\ldots + {}^nC_n = 2^n - 1$$

NOTE : *If a person is always there then we have to select only 1 from the remaining 25 – 1 = 24*

☞ Shortcut Approach

Let there are n persons in a hall. If every person shakes his hand with every other person only once, then total number of handshakes

$$= {}^nC_2 = \frac{n(n-1)}{2}$$

Note: *If in place of handshakes each person gives a gift to another person, then formula changes to = n (n – 1)*

See Example : Refer ebook Solved Examples/Ch-12

7. The number of ways of dividing 'm + n' things into two groups containing 'm' and 'n' things respectively

$$= {}^{m+n}C_m \quad {}^nC_n = \frac{(m+n)!}{m!n!}$$

8. The number of ways of dividing 'm + n + p' things into three groups containing 'm', 'n' and 'p' things respectively

$$= {}^{m+n+p}C_m \cdot {}^{n+p}C_p = \frac{(m+n+p)!}{m!\ n!\ p!}$$

(i) If m = n = p i.e. '3m' things are divided into three equal groups then the number of combinations is $\frac{(3m)!}{m!\ m!\ m!\ 3!} = \frac{(3m)!}{(m!)^3\ 3!}$

(ii) Buf if '3m' things are to be divided among three persons, then the number of divisions is $\frac{(3m)!}{(m!)^3}$

9. If mn distinct objects are to be divided into m groups. Then, the number of combination is

$\frac{(mn)!}{m!\ (n!)^m}$, when the order of groups is not important and

$\frac{(mn)!}{(n!)^m}$, when the order of groups is important

NUMBER OF RECTANGLES AND SQUARES

(a) Number of rectangles of any size in a square of size $n \times n$ is $\sum_{r=1}^{n} r^3$ and number of squares of any size is $\sum_{r=1}^{n} r^2$.

(b) Number of rectangles of any size in a rectangle size $n \times p$ $(n < p)$ is $\frac{np}{4}(n+1)(p+1)$ and number of squares of any size is $\sum_{r=1}^{n}(n+1-r)(p+1-r)$.

☞ Shortcut Approach

- If there are n non-collinear points in a plane, then
  - (i) Number of straight lines formed = ${}^nC_2$
  - (ii) Number of triangles formed = ${}^nC_3$
  - (iii) Number of quadrilaterals formed = ${}^nC_4$
- If there are n points in a plane out of which m are collinear, then
  - (i) Number of straight lines formed = ${}^nC_2 - {}^mC_2 + 1$
  - (ii) Number of triangles formed = ${}^nC_3 - {}^mC_3$
- Number of diagonals in a polygen of n sides = ${}^nC_2 - n$

See Example : Refer ebook Solved Examples/Ch-12

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Chapter

13

Probability

INTRODUCTION

Random Experiment :

It is an experiment which if conducted repeatedly under homogeneous condition does not give the same result.

The total number of possible outcomes of an experiment in any trial is known as the **exhaustive number** of events.

For example

(i) In throwing a die, the exhaustive number of cases is 6 since any one of the six faces marked with 1, 2, 3, 4, 5, 6 may come uppermost.

(ii) In tossing a coin, the exhaustive number of cases is 2, since either head or tail may turn over.

(iii) If a pair of dice is thrown, then the exhaustive number of cases is $6 \times 6 = 36$

(iv) In drawing four cards from a well-shuffled pack of cards, the exhaustive number of cases is ${}^{52}C_4$.

Events are said to be **mutually exclusive** if no two or more of them can occur simultaneously in the same trial.

For example,

(i) In tossing of a coin the events head (H) and tail (T) are mutually exclusive.

(ii) In throwing of a die all the six faces are mutually exclusive.

(iii) In throwing of two dice, the events of the face marked 5 appearing on one die and face 5 (or other) appearing on the other are not mutually exclusive.

Outcomes of a trial are **equally likely** if there is no reason for an event to occur in preference to any other event or if the chances of their happening are equal.

For example,

(i) In throwing of an unbiased die, all the six faces are equally likely to occur.

(ii) In drawing a card from a well-shuffled pack of 52 cards, there are 52 equally likely possible outcomes.

The **favourable cases** to an event are the outcomes, which entail the happening of an event.

For example,

(i) In the tossing of a die, the number of cases which are favourable to the " appearance of a multiple of 3" is 2, viz, 3 and 6.

(ii) In drawing two cards from a pack, the number of cases favourable to "drawing 2 aces" is ${}^4C_2$.

(iii) In throwing of two dice, the number of cases favourable to "getting 8 as the sum" is 5, : (2, 6), (6, 2), (4, 4), (3, 5) (5, 3).

Events are said to be **independent if the happening** (or non-happening) of one event is not affected by the happening or non-happening of others.

CLASSICAL DEFINITION OF PROBABILITY

If there are n-mutually exclusive, exhaustive and equally likely outcomes to a random experiment and 'm' of them are favourable to an event A, then the probability of happening of A is denoted by P (A) and is defined by

$$P(A) = \frac{m}{n}.$$

$$P(A) = \frac{\text{No. of elementary events favourable to A}}{\text{Total no. of equally likely elementary events}}$$

Obviously, $0 \le m \le n$, therefore $0 \le \frac{m}{n} \le 1$ so that $0 \le P(A) \le 1$.

P(A) can never be negative.

Since, the number of cases in which the event A will not happen is 'n – m', then the probability P ($\overline{A}$) of not happening of A is given by

$$P(\overline{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$$\Rightarrow \boxed{P(A) + P(\overline{A}) = 1}$$

The **ODDS IN FAVOUR** of occurrence of A are given by

$m : (n - m)$ or $P(A) : P(\overline{A})$

The **ODDS AGAINST** the occurrence of A are given by $(n - m) : m$ or $P(\overline{A}) : P(A)$.

ALGEBRA OF EVENTS

Let A and B be two events related to a random experiment. We define

(i) The event "A or B" denoted by "$A \cup B$", which occurs when A or B or both occur. Thus,

$P(A \cup B) =$ Probability that at least one of the events occur

(ii) The event "A and B", denoted by "$A \cap B$", which occurs when A and B both occur. Thus,

$P(A \cap B) =$ Probability of simultaneous occurrence of A and B.

(iii) The event " Not - A" denoted by $\overline{A}$, which occurs when and only when A does not occur. Thus

$P(\overline{A})$ = Probability of non-occurrence of the event A.

(iv) $\overline{A} \cap \overline{B}$ denotes the " non-occurrence of both A and B".

(v) "$A \subset B$" denotes the " occurrence of A implies the occurrence of B".

For example :

Consider a single throw of die and following two events

A = the number is even = {2, 4, 6}

B = the number is a multiple of 3 = {3, 6}

Then $P(A \cup B) = \frac{4}{6} = \frac{2}{3}$, $P(A \cap B) = \frac{1}{6}$

$P(\overline{A}) = \frac{1}{2}$, $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - \frac{2}{3} = \frac{1}{3}$.

ADDITION THEOREM ON PROBABILITY

1. ADDITION THEOREM : If A and B are two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. ADDITION THEOREM FOR THREE EVENTS : If A, B, C are three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

3. If A and B are **two mutually exclusive events** and the probability of their occurence are P(A) and P(B) respectively, then probability of either A or B occuring is given by

$P(A \text{ or } B) = P(A) + P(B)$
$\Rightarrow P(A+B) = P(A) + P(B)$

CONDITIONAL PROBABILITY

Let A and B be two events associated with a random experiment. Then $P\left(\frac{A}{B}\right)$, represents the conditional probability of occurrence of A relative to B.

Also, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ and $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

For example :

Suppose a bag contains 5 white and 4 red balls. Two balls are drawn one after the other without replacement. If A denotes the event "drawing a white ball in the first draw" and B denotes the event "drawing a red ball in the second draw".

P (B/A) = Probability of drawing a red ball in second draw when it is known that a white ball has already been drawn in the first draw $= \frac{4}{8} = \frac{1}{2}$

Obviously, P (A/B) is meaning less in this problem.

MULTIPLICATION THEOREM

If A and B are two events, then

$$P(A \cap B) = P(A)\,P(B/A), \text{ if } P(A) > 0$$
$$= P(B)\,P(A/B) \text{ if } P(B) > 0$$

From this theorem we get

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \text{ and } P(A/B) = \frac{P(A \cap B)}{P(B)}$$

For example :

Consider an experiment of throwing a pair of dice. Let A denotes the event " the sum of the point is 8" and B event " there is an even number on first die"

Then A = {(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)},
B = {(2, 1), (2, 2), ..., (2, 6), (4, 1), (4, 2),.....
(4, 6), (6, 1), (6, 2),....(6, 6)}

$$P(A) = \frac{5}{36},\ P(B) = \frac{18}{36} = \frac{1}{2},\ P(A \cap B) = \frac{3}{36} = \frac{1}{12}$$

Now, P(A/B) = Prob. of occurrence of A when B has already occurred = prob. of getting 8 as the sum, when there is an even number on the first die

$$= \frac{3}{18} = \frac{1}{6} \text{ and similarly } P(B/A) = \frac{3}{5}.$$

INDEPENDENCE

An event B is said to be independent of an event A if the probability that B occurs is not influenced by whether A has or has not occurred. For two independent events A and B.

$$P(A \cap B) = P(A)\,P(B)$$

Event $A_1, A_2, \ldots\ldots A_n$ are independent if

(i) $P(A_i \cap A_j) = P(A_i)\,P(A_j)$ for all $i, j, i \neq j$, That is, the events are pairwise independent.

(ii) The probability of simultaneous occurrence of (any) finite number of them is equal to the product of their separate probabilities, that is, they are mutually independent.

For example :

Let a pair of fair coin be tossed, here S = {HH, HT, TH, TT}

A = heads on the first coin = {HH, HT}

B = heads on the second coin = {TH, HH}

C = heads on exactly one coin = {HT, TH}

Then $P(A) = P(B) = P(C) = \frac{2}{4} = \frac{1}{2}$ and

$$P(A \cap B) = P(\{HH\}) = \frac{1}{4} = P(A)\,P(B)$$

$$P(B \cap C) = P(\{TH\}) = \frac{1}{4} = P(B)\,P(C)$$

$$P(A \cap C) = P(\{HT\}) = \frac{1}{4} = P(A)\,P(C)$$

Hence the events are pairwise independent.

Also $P(A \cap B \cap C) = P(\phi) = 0 \neq P(A)P(B)P(C)$

Hence, the events A, B, C are not mutually independent.

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