Basic Calculations

To solve the problems based on data quickly, you need to master the basic calculations. Most of the DI problems involve mathematical calculations only. Hence, we start with the basic calculations. Here, we will be discussing the quicker techniques for the following:

Addition	Multiplication	Division
Squaring	Square Root	Cube
Cube Roots of Exact Cubes	Comparison of Fractional Numbers	

Addition

In the problem of addition we have two main factors (*speed* and *accuracy*) under consideration. We will discuss a method of addition which is faster than the method used by most people and also has a higher degree of accuracy. In the latter part of this chapter we will also discuss a method of checking and double-checking the results.

In using conventional method of addition, the average man cannot always add a fairly long column of figures without making a mistake. We shall learn how to check the work by individual columns, without repeating the addition. This has several advantages:

1) We save the labour of repeating all the work;
2) We locate the error, if any, in the column where it occurs; and
3) We are *certain* to find error, which is not necessary in the conventional method.

This last point is something that most people do not realise. Each one of us has his own weaknesses and own kind of proneness to commit error. One person may have the tendency to say that 9 times 6 is 56. If you ask him directly he will say "54", but in the middle of a long calculation it will slip out as "56". If it is his favourite error, he would be likely to repeat it when he checks by repetition.

Totalling in columns

As in the conventional method of addition, we write the figures to be added in a column, and under the bottom figure we draw a line, so that the total will be under the column. When writing them we remember that the mathemetical rule for placing the numbers is to align the right-hand-side digits (when there are whole numbers) and the decimal points (when there are decimals). For example:

Right-hand-side-digits alignment	**Decimal alignment**
4 2 3 4	13.05
8 2 3 8	2.51
6 4 6	539.652
5 3 2 1	2431.0003
3 5 0	49.24
9 9 8 9	

The conventional method is to add the figures down the right-hand column, 4 plus 8 plus 6, and so on.You can do this if you wish in the new method, but it is not compulsory; you can begin working on any column. But for the sake of convenience, we will start on the right-hand column.

We add as we go down, but we "never count higher than 10". That is, when the running total becomes greater than 10, we reduce it by 10 and go ahead with the reduced figure. As we do so, we make a small tick or check-mark beside the number that made our total higher than 10. For example:

4

8 4 plus 8,12 : this is more than 10, so we subtract 10 from 12. Mark a tick and start adding again.

6 6 plus 2, 8

1 1 plus 8, 9

0 0 plus 9, 9

9 9 plus 9, 18: mark a tick and reduce 18 by 10, say 8.

The final figure, 8, will be written under the column as the “running total”.

Next we count the ticks that we have just made as we dropped 10’s. As we have 2 ticks, we write 2 under the column as the “tick figure”. The example now looks like this:

$$\begin{array}{lr}
 & 4234 \\
 & 8238' \\
 & 646 \\
 & 5321 \\
 & 350 \\
 & 9989' \\
\hline
\text{running total:} & 8 \\
\text{ticks:} & 2
\end{array}$$

If we repeat the same process for each of the columns we reach the result:

$$\begin{array}{lr}
 & 4234 \\
 & 8'238' \\
 & 6'4'6 \\
 & 5321 \\
 & 350 \\
 & 9'9'8'9' \\
\hline
\text{running total:} & 6558 \\
\text{ticks:} & 2222
\end{array}$$

Now we arrive at the final result by adding together the running total and the ticks in the way shown in the following diagram.

$$\begin{array}{lcccccc}
\text{running total:} & 0 & 6 & 5 & 5 & 8 & 0 \\
\text{ticks:} & 0 & 2 & 2 & 2 & 2 & 0 \\
\hline
\text{Total:} & & 2 & 8 & 7 & 7 & 8
\end{array}$$

Save more time : We observe that the running total is added to the ticks below in the immediate right column. This addition of the ticks with immediate left column can be done in single step. That is, the number of ticks in the first column from right is added to the second column from right, the number of ticks in the 2nd column is added to the third column, and so on. The whole method can be understood in the following steps.

Step I.

$$\begin{array}{r}
4\,2\,3\,4 \\
8\,2\,3\,8' \\
6\,4\,6 \\
5\,3\,2\,1 \\
3\,5\,0 \\
9\,9\,8\,9' \\
\hline
\text{Total}\;\; 8
\end{array}$$

[4 plus 8 is 12, mark a tick and add 2 to 6, which is 8; 8 plus 1 is 9; 9 plus 0 is 9; 9 plus 9 is 18, mark a tick and write down 8 in the first column of total-row.]

Step II.

$$\begin{array}{r} 4\,2\,3\,4 \\ 8\,2\,3\,8' \\ 6\,4'\,6 \\ 5\,3\,2\,1 \\ 3\,5\,0 \\ 9\,9\,8'\,9' \\ \hline \text{Total}\ \ 7\,8 \end{array}$$

[3 plus 2 (number of ticks in first column) is 5; 5 plus 3 is 8; 8 plus 4 is 12, mark a tick and carry 2; 2 plus 2 is 4; 4 plus 5 is 9; 9 plus 8 is 17, mark a tick and write down 7 in 2nd column of total-row.]

In a similar way we proceed for 3rd and 4th columns.

$$\begin{array}{rr} & 4\ \ 2\ \ 3\ \ 4 \\ & 8'\,2\ \ 3\ \ 8' \\ & 6'\,4'\,6 \\ & 5\,3\ \ 2\ \ 1 \\ & 3\ \ 5\ \ 0 \\ & 9'9'\,8'\,9' \\ \cline{2-2} \text{Total} & 2\,8\,7\ \ 7\ \ 8 \end{array}$$

Note: We see that in the leftmost column we are left with 2 ticks. Write down the number of ticks in a column left to the leftmost column. Thus we get the answer a little earlier than the previous method.

One more illustration:

Q: 707.325 + 1923.82 + 58.009 + 564.943 + 65.6 = ?

Solution :

$$\begin{array}{rl} & 707.325 \\ & 19'23'.8'2 \\ & 58'.009' \\ & 5'6'4.9'43 \\ & 6'5.6 \\ \cline{2-2} \text{Total:} & 3319.697 \end{array}$$

You may raise a question: is it necessary to write the numbers in column-form? The answer is ‘no’. You may get the answer without doing so. Question written in a row-form causes a problem of alignment. If you get command over it, there is nothing better than this. For initial stage, we suggest you a method which would bring you out of the alignment problem.

Step I. “Put zeros to the right of the last digit after decimal to make the no. of digits after decimal equal in each number.”

For example, the above question may be written as

707.325 + 1923.820 + 58.009 + 564.943 + 65.600

Step II. Start adding the last digit from right. Strike off the digit which has been dealt with. If you don’t cut, duplication may occur. During running total, don’t exceed 10. That is, when we exceed 10, we mark a tick anywhere near about our calculation. Now, go ahead with the number exceeding 10.

$$707.32\cancel{5} + 1923.82\cancel{0} + 58.00\cancel{9} + 564.94\cancel{3} + 65.60\cancel{0} = \_\,\_\,\_\,\_\,.\,\_\,\_\,7$$

5 plus 0 is 5; 5 plus 9 is 14, mark a tick in rough area and carry over 4; 4 plus 3 is 7; 7 plus 0 is 7, so write down 7. During this we strike off all the digits which are used. It saves us from confusion and duplication.

Step III. Add the number of ticks (in rough) with the digits in 2nd places. and erase that tick from rough.

$$707.3\cancel{2}\cancel{5} + 1923.8\cancel{2}\cancel{0} + 58.0\cancel{0}\cancel{9} + 564.9\cancel{4}\cancel{3} + 65.6\cancel{0}\cancel{0} = \_\,\_\,\_\,\_\,.\,\_\,97$$

1 (number of tick) plus 2 is 3; 3 plus 2 is 5; 5 plus 0 is 5; 5 plus 4 is 9 and 9 plus 0 is 9; so write down 9 in its place.

Step IV.

$$707.325 + 1923.820 + 58.009 + 564.943 + 65.600 = \_\_\_\_.697$$

3 plus 8 is 11; mark a tick in rough and carry over 1; 1 plus 0 is 1; 1 plus 9 is 10, mark another tick in rough and carry over zero; 0 plus 6 is 6, so put down 6 in its place.

Step V.

Last Step : Following the same way get the result:

$$707.325 + 1923.820 + 58.009 + 564.943 + 65.600 = 3319.697$$

Addition of numbers (without decimals) written in a row form

Q. 53921 + 6308 + 86 + 7025 + 11132 = ?

Soln: Step I: 53921 + 6308 + 86 + 7025 + 11132 = _ _ _ _2

Step II : 53921 + 6308 + 86 + 7025 + 11132 = _ _ _ 72

Step III: 53921 + 6308 + 86 + 7025 + 11132 = _ _ 472

Step IV: 53921 + 6308 + 86 + 7025 + 11132 = _ 8472

Step V: 53921 + 6308 + 86 + 7025 + 11132 = 78472

Note : One should get good command over this method because it is very much useful and fast-calculating. If you don't understand it, try again and again.

Addition and subtraction in a single row

Ex. 1: 412 - 83 + 70 = ?

Step I: For units digit of our answer add and subtract the digits at units places according to the sign attached with the respective numbers. For example, in the above case the unit place of our temporary result is

2 – 3 + 0 = -1

So, write as:

412 - 83 + 70 = __ __ (-1)

Similarly, the temporary value at tens place is

1 - 8 + 7 = 0. So, write as:

412 - 83 + 70 = __ (0) (-1)

Similarly, the temporary value at hundreds place is 4. So, we write as:

412 - 83 + 70 = (4) (0) (-1)

Step II: Now, the above temporary figures have to the changed into real value. To replace (-1) by a +ve digit we borrow from digits at tens or hundreds.

As the digit at tens is zero, we will have to borrow from hundreds.

We borrow 1 from 4 (at hundreds) which becomes 10 at tens leaving 3 at hundreds. Again we borrow 1 from tens which becomes 10 at units place, leaving 9 at tens. Thus, at units place 10 - 1 = 9. Thus our final result = 399.

The above explanation can be represented as

(-1)	(10) (-1)	(10)
(4)	(0)	(-1)
(3)	(9)	(9)

Note: The above explanation is easy to understand. And the method is more easy to perform. If you practise well, the two steps (I & II) can be performed simultaneously.
The second step can be performed in another way like:
(4) (0) (-1) = 400 - 1 = 399

Ex. 2: 5124 - 829 + 731 - 435
Soln: According to step I, the temporary figure is:
(5) (-4) (0) (-9)
Step II: Borrow 1 from 5. Thousands place becomes 5 - 1 = 4. 1 borrowed from thousands becomes 10 at hundreds. Now, 10 - 4 = 6 at hundreds place, but 1 is borrowed for tens. So digit at hundreds becomes 6 - 1 = 5. 1 borrowed from hundreds becomes 10.
Again we borrow 1 from tens for units place, after which the digit at tens place is 9. Now, 1 borrowed from tens becomes 10 at units place. Thus the result at units place is 10 - 9 = 1.
Our required answer = 4591
Note: After step I we can perform like:
5 (-4) (0) (-9) = 5000 - 409 = 4591
But this method can't be combined with step I to perform simultaneously. So, we should try to understand steps I & II well so that in future we can perform them simultaneously.

Ex. 3: 73216 - 8396 + 3510 - 999 = ?
Soln: Step I gives the result as:
(7) (-2) (-5) (-16) (-9)
Step II: Units digit = 10 - 9 = 1 [1 borrowed from (-16) results - 16 - 1 = -17]
Tens digit = 20 - 17 = 3 [2 borrowed from (-5) results -5 - 2 = -7]
Hundreds digit = 10 - 7 = 3 [1 borrowed from -2 results -2 - 1 = - 3]
Thousands digit = 10 - 3 = 7 [1 borrowed from 7 results 7 - 1 = 6]
So, the required value is 67331.
The above calculations can also be started from leftmost digit as done in last two examples. We have started from rightmost digit in this case. The result is the same in both cases. But for the combined operation of two steps you will have to start from rightmost digit (i.e. units digit). See Ex. 4.
Note: Other method for step II: (-2) (-5) (-16) (-9) = (-2) (-6) (-6) (-9) = -(2669)
$\therefore$ Ans = 70000 - (2669) = 67331

Ex. 4: 89978 - 12345 - 36218 = ?
Soln: Step I: (4) (1) (4) (2) (-5)
Step II: 4 1 4 1 5

Single-step solution:

Now, you must learn to perform the two steps simultaneously. This is the simplest example to understand the combined method.
At units place: 8 - 5 - 8 = (-5). To make it positive we have to borrow from tens. You should remember that we can't borrow from -ve value i.e., from 12345. **We will have to borrow from positive value i.e. from 89978.** So, we borrowed 1 from 7 (tens digit of 89978):

$8\ 9\ 9\ \overset{(-1)}{7}\ 8$ - 12345 - 36218 = _ _ _ _ 5
Now digit at tens: (7 - 1 =) 6 - 4 - 1 = 1
Digit at hundreds: 9 - 3 - 2 = 4
Digit at thousands: 9 - 2 - 6 = 1
Digit at ten thousands: 8 - 1 - 3 = 4
$\therefore$ the required value = 41415

Ex. 5: 28369 + 38962 - 9873 = ?

Soln: Single-step solution: Units digit = 9 + 2 - 3 = 8
Tens digit = 6 + 6 - 7 = 5 Hundreds digit = 3 + 9 - 8 = 4
Thousands digit = 8 + 8 - 9 = 7
Ten thousands digit = 2 + 3 = 5 $\therefore$ required value = 57458

Ex. 6: Solve Ex. 2 by single-step method.

Soln: 5124 - 829 + 731 - 435 =
Units digit: 4 - 9 + 1 - 5 = (-9). Borrow 1 from tens digit of the positive value. Suppose we borrowed from 3 of 731. Then

$5124 - 829 + 7\overset{-1}{3}1 - 435 =$ _ _ _1

Tens digit: 2 - 2 + 2 - 3 = (-1). Borrow 1 from hundreds digit of +ve value. Suppose we borrowed from 7 of 731. Then

$5124 - 829 + \overset{-1}{7}\ \overset{-1}{3}\ 1 - 435 =$ _ _ 9 1

Hundreds digit: 1 - 8 + 6 - 4 = (-5). Borrow 1 from thousands digit of +ve value. We have only one such digit, i.e. 5 of 5124. Then

$\overset{-1}{5}\,1\,2\,4 - 829 + \overset{-1}{7}\ \overset{-1}{3}\ 1 - 435 = 4591$

(Thousands digit remains as 5 - 1 = 4)
Now you can perform the whole calculation in a single step without writing anything extra.

Ex. 7: Solve Ex. 3 in a single step without writing anything other than the answer. Try it yourself. Don't move to next example until you can confidently solve such questions within seconds.

Ex. 8: 10789 + 3946 - 2310 - 1223 = ?

Soln: Whenever we get a value more than 10 after addition of all the units digits, we will put the units digit of the result and carry over the tens digit. **We add the tens digit to +ve value, not to the -ve value.**
Similar method should be adopted for all digits.

$1\ \overset{+1}{0}\ \overset{+1}{7}\ \overset{+1}{8}\ \ 9 + 3946 - 2310 - 1223 = 11202$

Note: 1. We put +1 over the digits of +ve value 10789. It can also be put over the digits of 3946. But it can't be put over 2310 and 1223.
2. In the exam when you are free to use your pen on question paper you can alter the digit with your pen instead of writing +1, +2, -1, -2 .... over the digits. Hence, instead of writing 8, you should write 9 over 8 with your pen. Similarly, write 8 in place of $\overset{+1}{7}$.

Ex. 9: 765.819 - 89.003 + 12.038 - 86.89 = ?

Soln: First, equate the number of digits after decimals by putting zeroes at the end. So, ? = 765.819 - 89.003 + 12.038 - 86.890
Now, apply the same method as done in Ex. 4, 5, 6, 7 & 8.

$\overset{-1}{7}\ \overset{-1}{6}\ \overset{-1}{5}.\overset{-1}{8}\ \overset{+1}{1}\ \ 9 - 89.003 + 12.038 - 86.890 = 601.964$

Method of checking the calculation : Digit-sum Method

This method is also called the **nines-remainder method.** The concept of digit-sum consists of this :

I. We get the digit-sum of a number by "adding across" the number. For instance, the digit-sum of 13022 is 1 plus 3 plus 0 plus 2 plus 2 is 8.

II. We always reduce the digit-sum to a single figure if it is not already a single figure. For instance, the digit-sum of 5264 is 5 plus 2 plus 6 plus 4 is 8 (17, or 1 plus 7 is 8).

III. In "adding across" a number, we may drop out 9's. Thus, if we happen to notice two digits that add up to 9, such as 2 and 7, we ignore both of them; so the digit-sum of 990919 is 1 at a glance. (If we add up 9's we get the same result.)

IV. Because "nines don't count" in this process, as we saw in III, a digit-sum of 9 is the same as a digit-sum of zero. The digit-sum of 441, for example, is zero.

Quick Addition of Digit-sum : When we are "adding across" a number, as soon as our running total reaches two digits we add these two together, and go ahead with a single digit as our new running total.

For example : To get the digit-sum of 886542932851 we do like: 8 plus 8 is 16, a two-figure number. We reduce this 16 to a single figure: 1 plus 6 is 7. We go ahead with this 7 ; 7 plus 6 is 4 (13, or 1+3=4), 4 plus 5 is 9, forget it. 4 plus 2 is 6. Forget 9 .... Proceeding this way we get the digit-sum equal to 7.

For decimals we work exactly the same way. But we don't pay any attention to the decimal point. The digit-sum of 6.256, for example, is 1.

Note: It is not necessary in a practical sense to understand why the method works, but you will see how interesting this is. The basic fact is that the reduced digit-sum is the same as the remainder when the number is divided by 9.

For example : Digit-sum of 523 is 1. And also when 523 is divided by 9, we get the remainder 1.

Checking of Calculation

Basic rule: Whatever we do to the numbers, we also do to their digit-sum; then the result that we get from the digit-sum of the numbers must be equal to the digit-sum of the answer.

For example :

The number : 23 + 49 + 15 + 30 = 117

The digit-sum : 5 + 4 + 6 + 3 = 0

Which reduces to : 0 = 0

This rule is also applicable to subtraction, multiplication and upto some extent to division also. These will be discussed in the coming chapters. We should take another example of addition.

1.5 + 32.5 + 23.9 = 57.9

digit-sum: 6 + 1 + 5 = 3

or, 3 = 3

Thus, if we get LHS = RHS we may conclude that our calculation is correct.

Sample Question : Check for all the calcutions done in this chapter.

Note : Suppose two students are given to solve the following question: 1.5+ 32.5 + 23.9 = ?

One of them gets the solution as 57.9. Another student gets the answer 48.9. If they check their calculation by this method, both of them get it to be correct. Thus this method is not always fruitful. If our luck is against us, we may approve our wrong answer also.

Addition of mixed numbers

Q. $3\frac{1}{2}+4\frac{4}{5}+9\frac{1}{3}=?$

Solution : A conventional method for solving this question is by converting each of the numbers into pure fractional numbers first and then taking the LCM of denominators. To save time, we should add the whole numbers and the fractional values separately. Like here,

$$3\frac{1}{2}+4\frac{4}{5}+9\frac{1}{3}=(3+4+9)+\left(\frac{1}{2}+\frac{4}{5}+\frac{1}{3}\right)=16+\frac{15+24+10}{30}=16+1\frac{19}{30}$$

$$= (16+1) + \frac{19}{30} = 17 + \frac{19}{30} = 17\frac{19}{30}$$

Q. $5\frac{2}{3} - 4\frac{1}{6} + 2\frac{3}{4} - 1\frac{1}{4}$

Soln : $(5 - 4 + 2 - 1) + \left(\frac{2}{3} - \frac{1}{6} + \frac{3}{4} - \frac{1}{4}\right)$

$$= 2 + \left(\frac{8-2+9-3}{12}\right) = 2 + \frac{12}{12} = 2 + 1 = 3$$

Multiplication

Special Cases

We suggest you to remember the tables up to 30 because it saves some valuable time during calculation. Multiplication should be well commanded, because it is needed in almost every question of our concern.

Let us look at the case of multiplication by a number more than 10.

MULTIPLICATION BY 11

Step I: The last digit of the multiplicand (number multiplied) is put down as the right-hand figure of the answer.

Step II: Each successive digit of the multiplicand is added to its neighbour at the right.

Ex.1. Solve $5892 \times 11 = ?$

Soln: Step I: Put down the last figure of 5892 as the right hand figure of the answer:

$$\frac{5892 \times 11}{2}$$

Step II: Each successive figure of 5892 is added to its right-hand neighbour. 9 plus 2 is 11, put 1 below the line and carry over 1. 8 plus 9 plus 1 is 18, put 8 below the line and carry over 1. 5 plus 8 plus 1 is 14, put 4 below the line and carry over 1.

$\frac{5892 \times 11}{12}$ (9+2 = 11, put 1 below the line and carry over 1)

$\frac{5892 \times 11}{812}$ (8+9+1 =18, put 8 below the line and carry over 1)

$\frac{5892 \times 11}{4812}$ (5+8+1 = 14, put 4 below the line and carry over 1)

Step III: The first figure of 5892, 5 plus 1, becomes the left-hand figure of the answer:

$$\frac{5892 \times 11}{64812}$$

The answer is 64812.

As you see, each figure of the long number is used twice. It is first used as a "number", and then, at the next step, it is used as a neighbour. Looking carefully, we can use just one rule instead of three rules, And this only rule can be called as **"add the right neighbour"** rule.

We must first write a zero in front of the given number, or at least imagine a zero there.

Then we apply the idea of adding the neighbour to every figure of the given number in turn:

$\underline{05892 \times 11}$
2 As there is no neighbour on the right, so we add nothing.

$\underline{05892 \times 11}$
4812 _ _ _ _ _ As we did earlier

$\underline{05892 \times 11}$
64812 _ _ _ _ _ _ zero plus 5 plus carried-over 1 is 6

This example shows why we need the zero in front of the multiplicand. It is to remind us not to stop too soon. Without the zero in front, we might have neglected the last 6, and we might then have thought that the answer was only 4812. The answer is longer than the given number by one digit, and the zero in front takes care of that.

Sample Problems : Solve the following :

1) 111111 × 11 2) 23145 × 11 3) 89067 × 11 4) 5776800 × 11
5)1122332608 × 11

Ans: 1) 1222221 2) 254595 3) 979737 4) 63544800
5) 12345658688

MULTIPLICATION BY 12

To multiply any number by 12, we

"Double each digit in turn and add its neighbour".

This is the same as multiplying by 11 except that now we double the "number" before we add its "neighbour".

For example :

Ex 1: Solve : 5324 × 12

Soln : Step I. $\underline{05324 \times 12}$
8 (double the right hand figure and add zero, as there is no neighbour)

Step II. $\underline{05324 \times 12}$
88 (double the 2 and add 4)

Step III. $\underline{05324 \times 12}$
888 (double the 3 and add 2)

Step IV. $\underline{05324 \times 12}$
3888 (double the 5 and add 3 (=13), put 3 below the line and carry over 1)

Last Step. $\underline{05324 \times 12}$
63888 (zero doubled is zero, plus 5 plus carried-over 1)

The answer is 63,888. If you go through it yourself you will find that the calculation goes very fast and is very easy.

Practice Question

Solve the following:

1)35609 × 12 2)11123009 × 12 3)456789 × 12 4)22200007 × 12 5)444890711 × 12

Ans: 1) 427308 2) 132476108 3) 5481468 4) 266400084 5) 5338688532

MULTIPLICATION BY 13

To multiply any number by 13, we

“Treble each digit in turn and add its right neighbour”.
This is the same as multiplying by 12 except that now we “treble” the “number” before we add its “neighbour”.
If we want to multiply 9483 by 13, we proceed like this :

Step I. $\underline{09483 \times 13}$
9 (treble the right hand figure and write it down as there is no neighbour on the right)

Step II. $\underline{09483 \times 13}$
79 (8 × 3 + 3 = 27, write down 7 and carry over 2)

Step III. $\underline{09483 \times 13}$
279 (4 × 3 + 8 + 2 = 22, write down 2 and carry over 2)

Step IV. $\underline{09483 \times 13}$
3279 (9 × 3 + 4 + 2 = 33, write down 3 and carry over 3)

Last Step. $\underline{09483 \times 13}$
123279 (0 × 3 + 9 + 3 = 12, write it down)

The answer is 1,23,279.

In a similar way, we can define rules for multiplication by 14,15,.... But, during these multiplications we will have to get four or five times of a digit, which is sometimes not so easy to carry over. We have an easier method of multiplication for those large values.

Can you get similar methods for multiplication by 21 and 31? It is not very tough to define the rules. Try it.

MULTIPLICATION BY 9

Step I: Subtract the right-hand figure of the long number from 10. This gives the right-hand figure of the answer.
Step II: Taking the next digit from right, subtract it from 9 and add the neighbour on its right.
Step III: At the last step, when you are under the zero in front of the long number, subtract one from the neighbour and use that as the left-hand figure of the answer.

Ex 1: 8576 × 9 = ?
Soln: 08576 × 9

77184

Step I. Subtract the 6 of 8576 from 10, and we have 4 of the answer.
Step II. Subtract the 7 from 9 (we have 2) and add the neighbour 6; the result is 8.
Step III. (9-5)+7 = 11; put 1 under the line and carry over 1.
Step IV. (9-8)+5+1 (carried over) = 7, put it down.
Step V (Last step). We are under the left-hand zero, so we reduce the left-hand figure of 8576 by one, and 7 is the left-hand figure of the answer.

Thus answer is 77184.

Here are a few questions for you.

1) 34 × 9 = ? 2) 569 × 9? 3) 1328 × 9 = ? 4) 56493 × 9 = ? 5) 89273258 × 9 = ?

Answers

1) 306 2) 5121 3) 11952 4) 508437 5) 803459322

We don’t suggest you to give much emphasis on this rule. Because it is not very much easy to use. Some times it proves very lengthy also.

Another method :
Step I : Put a zero at the right end of the number; ie, write 85760 for 8576.
Step II: Subtract the original number from that number. Like 85760 – 8576 = 77184

MULTIPLICATION BY 25

Suppose you are given a large number like 125690258. And someone asks you to multiply that number by 25. What will you do? Probably you will do nothing but go for simple multiplication. Now, we suggest you to multiply that number by 100 and then divide by 4.

To do so remember the two steps :

Step I: Put two zeroes at the right of the number (as it has to be multiplied by 100).

Step II: Divide it by 4.

So, your answer is 12569025800 ÷ 4 = 3142256450. Is it easier than your method?

General Rule for Multiplication

Having dealt in fairly sufficient detail with the application of special cases of multiplication, we now proceed to deal with the "General Formula" applicable to all cases of multiplication. It is sometimes not very convenient to keep all the above cases and their steps in mind, so all of us should be very much familiar with "General Formula" of multiplication.

Multiplication by a two-digit number

Ex. 1 Solve (1) 12 × 13 = ? (2) 17 × 18 = ? (3) 87 × 92 = ?

Soln: (1) 12 × 13 = ?

Step I. Multiply the right-hand digits of multiplicand and multiplier (unit-digit of multiplicand with unit-digit of the multiplier).

$$\begin{array}{cc} 1 & 2 \\ 1 & 3 \\ \hline & 6 \end{array} \quad (2 \times 3)$$

Step II. Now, do cross-multiplication, ie, multiply 3 by 1 and 1 by 2. Add the two products and write down to the left of 6.

$$\begin{array}{cc} 1 & 2 \\ 1 & 3 \\ \hline 5 & 6 \end{array} \quad (3 \times 1 + 2 \times 1)$$

Step III. In the last step we multiply the left-hand figures of both multiplicand and multiplier.

$$\begin{array}{ccc} & 1 & 2 \\ & 1 & 3 \\ \hline 1 & 5 & 6 \end{array} \quad (2 \times 3)$$

(2) 17 × 18 = ?

Step I.

$$\begin{array}{cc} 1 & 7 \\ 1 & 8 \\ \hline & 6 \end{array} \quad (7 \times 8 = 56, \text{ write down 6 and carry over 5})$$

Step II.

$$\begin{array}{cc} 1 & 7 \\ 1 & 8 \\ \hline 0 & 6 \end{array} \quad (1 \times 8 + 7 \times 1 + 5 = 20, \text{ write down 0 and carry over 2})$$

Step III.

$$\begin{array}{ccc} & 1 & 7 \\ & 1 & 8 \\ \hline 3 & 0 & 6 \end{array} \quad (1 \times 1 + 2 = 3, \text{ write it down})$$

(3) 87 × 92 = ?

Step I.

```
8   7
9   2
-----
    4   (7 × 2 = 14, write down 4 and carry over 1)
```

Step II.

```
8   7
9   2
-----
   04 (8 × 2 + 9 × 7 + 1 = 80, write down 0 and carry over 8)
```

Step III.

```
8   7
9   2
-----
 8004   (8 × 9 + 8 = 80)
```

Practice questions

1) 57 × 43 2) 51 × 42 3) 38 × 43 4) 56 × 92 5) 81 × 19
6) 23 × 99 7) 29 × 69 8) 62 × 71 9) 17 × 37 10) 97 × 89

ANSWERS

1) 2451 2) 2142 3) 1634 4) 5152 5) 1539 6) 2277 7) 2001 8) 4402 9) 629 10) 8633

Ex 2. Solve: (1) 325 × 17 = ? (2) 4359 × 23 = ?

Soln: (1). Step I.

```
3   2   5

    1   7
---------
        5   (5 × 7 = 35, put down 5 and carry over 3)
```

Step II.

```
3   2   5

    1   7
---------
      2 5   (2 × 7 + 5 × 1 + 3 = 22, put down 2 and carry over 2)
```

Step III.

```
3   2   5

    1   7
---------
    5 2 5   (3 × 7 + 2 × 1 + 2 = 25, put down 5 and carry over 2)
```

Note: Repeat the cross-multiplication untill all the consecutive pairs of digits exhaust. In step II, we cross-multiplied 25 and 17 and in step III, we cross-multiplied 32 and 17.

Step IV.

```
3   2   5

    1   7
---------
  5 5 2 5   (3 × 1 + 2 = 5, put it down)
```

(2). Step I.

```
4   3   5   9
        2   3
-------------
            7
```
(9 × 3 = 27, put down 7, carry over 2)

Step II.

```
4   3   5   9
        2   3
-------------
        5   7
```
(5 × 3 + 9 × 2 + 2 = 35, put down 5 and carry over 3)

Step III.

```
4   3   5   9
        2   3
-------------
    2   5   7
```
(3 × 3 + 5 × 2 + 3 = 22, put down 2, carry over 2)

Step IV.

```
4   3   5   9
        2   3
0   2   5   7
```
(4 × 3 + 3 × 2 + 2 = 20, put down 0, carry over 2)

Step V.

```
    4   3   5   9
            2   3
-----------------
1 0 0   2   5   7
```
(4 × 2 + 2 = 10, put it down)

We can write all the steps together :

```
                 4        3         5          9
                          2         3
-------------------------------------------------
4 × 2/ 4 × 3 + 2× 3 /3 × 3 + 5 × 2/ 5 × 3 + 9 × 2/ 9 × 3
```
$= 10 \quad {}_{2}0 \quad {}_{2}2 \quad {}_{3}5 \quad {}_{2}7$

= 100257

Or, we can write the answer directly without writing the intermediate steps. The only thing we should keep in mind is the "carrying numbers".

```
4  3  5  9               4  3  5  9
      2  3      or,            2  3
-----------            -------------
```
$10\,{}_{2}0\,{}_{2}2\,{}_{3}5\,{}_{2}7$ 1 0 0 2 5 7

Note: You should try for this direct calculation. It saves a lot of time. It is a very systematic calculation and is very easy to remember. Watch the above steps again and again until you get that systematic pattern of cross-multiplication.

Multiplication by a 3-digit number

Ex: 1. Solve (1) $321 \times 132 = ?$ (2) $4562 \times 345 = ?$ (3) $69712 \times 641 = ?$

Soln: (1). Step I.

$$\begin{array}{ccc} 3 & 2 & 1 \\ 1 & 3 & 2 \\ \hline & & 2 \end{array} \quad (1 \times 2 = 2)$$

Step II.

$$\begin{array}{ccc} 3 & 2 & 1 \\ 1 & 3 & 2 \\ \hline & 7 & 2 \end{array} \quad (2 \times 2 + 3 \times 1 = 7)$$

Step III.

$$\begin{array}{ccc} 3 & 2 & 1 \\ 1 & 3 & 2 \\ \hline 3 & 7 & 2 \end{array}$$

($2 \times 3 + 3 \times 2 + 1 \times 1 = 13$, write down 3 and carry over 1)

Step IV.

$$\begin{array}{cccc} & 3 & 2 & 1 \\ & 1 & 3 & 2 \\ \hline 2 & 3 & 7 & 2 \end{array}$$

($3 \times 3 + 1 \times 2 + 1 = 12$, write down 2 and carry over 1)

Step V.

$$\begin{array}{ccccc} & 3 & 2 & & 1 \\ & 1 & 3 & & 2 \\ \hline 4 & 2 & 3 & 7 & 2 \end{array} \quad (1 \times 3 + 1 = 4)$$

or,

$$\begin{array}{ccccc} & & 3\ 2\ 1 & & \\ & & 1\ 3\ 2 & & \\ \hline 1\times3 & /\ 3\times3+1\times2\ / & 2\times3+3\times2+1\times1\ / & 2\times2+3\times1\ / & 1\times2 \end{array}$$

$= 4 \quad {}_{1}2 \quad {}_{1}3 \quad 7 \quad 2$

$= 42372$

(2) 4 5 6 2
3 4 5

4×3 / 4×4+3×5 / 5×4+4×5+3×6 / 5×5+4×6+3×2 / 5×6+4×2 / 2×5
= 15 $_{3}7$ $_{6}3$ $_{5}8$ $_{3}9$ $_{1}0$
= 1573890

(3) 6 9 7 1 2
6 4 1

6×6 / 4×6+6×9 / 1×6+4×9+6×7 / 1×9+4×7+6×1 / 1×7+4×1+6×2 / 1×1+4×2 / 1×2
= 44 $_{8}6$ $_{8}8$ $_{4}5$ $_{2}3$ 9 2
= 44685392

Note: Did you get the clear concept of cross-multiplication and carrying-cross-multiplication? Did you mark how the digits in cross-multiplication increase, remain constant, and then decrease? Take a sharp look at question (3). In the first row of the answer, if you move from right to left, you will see that there is only one multiplication (1×2) in the first part. In the second part there are two (1×1 and 4×2), in the 3rd part three (1×7, 4×1 and 6×2), in the 4th part three (1×9, 4×7 and 6×1), in the 5th part again three (1×6, 4×9 and 6×7), in the 6th part two (4×6 and 6×9) and in the last part only one (6×6) multiplication. The participation of digits in cross-multiplication can be shown by the following diagrams.

I. 6 9 7 1 2
6 4 1

II. 6 9 7 1 2
6 4 1

III. 6 9 7 1 2
6 4 1

IV. 6 9 7 1 2
6 4 1

V. 6 9 7 1 2
6 4 1

VI. 6 9 7 1 2
6 4 1

VII. 6 9 7 1 2
6 4 1

For each of the groups of figures, you have to cross-multiply.

Multiplication by a 4-digit number

Example:

Solve 1) 4325 × 3216 2) 646329 × 8124

Soln: 1) 4325 × 3216 = ?

Step I.

4 3 2 5
3 2 1 6
0 (5 × 6 = 30, write down 0 and carry over 3)

Step II.

4 3 2 5
3 2 1 6
0 0 (2 × 6 + 1 × 5 + 3 = 20, write down 0 and carry over 2)

Step III.

    4 3 2 5

    3 2 1 6
    ----
      2 0 0   (3 × 6 + 2 × 1 + 5 × 2 + 2 = 32, write down 2 and carry over 3)

Step IV.

    4 3 2 5

    3 2 1 6
    ----
    9 2 0 0   (4 × 6 + 3 × 1 + 2 × 2 + 5 × 3 + 3 = 49, write down 9 and carry over 4)

Step V.

      4 3 2 5

      3 2 1 6
    ----
    0 9 2 0 0   (4 × 1 + 3 × 2 + 2 × 3 + 4 = 20, write down 0 and carry over 2)

Step VI.

        4 3 2 5

        3 2 1 6
    ----
    9 0 9 2 0 0   (4 × 2 + 3 × 3 + 2 = 19, write down 9 and carry over 1)

Step VII.

            4 3 2 5

            3 2 1 6
    ----
    1 3 9 0 9 2 0 0   (4 × 3 + 1 = 13, write it down)

Ans : 13909200

2. 646329 × 8124 = ?

Try this question yourself and match your steps with the given diagrammatic presentation of participating digits.

Step I.

    6 4 6 3 2 9

          8 1 2 4

Step II.

    6 4 6 3 2 9

          8 1 2 4

Step III.

    6 4 6 3 2 9

          8 1 2 4

Step IV.

    6 4 6 3 2 9

          8 1 2 4

Step V.

6 4 6 3 2 9
8 1 2 4

Step VI.

6 4 6 3 2 9
8 1 2 4

Step VII.

6 4 6 3 2 9
8 1 2 4

Step VIII.

6 4 6 3 2 9
8 1 2 4

Step IX.

6 4 6 3 2 9
8 1 2 4

Practice Problems: Solve the following :

1) 234 × 456 2)336 × 678 3) 872 × 431 4) 2345 × 67
5) 345672 × 456 6) 569 × 952 7) 458 × 908 8) 541 × 342
9) 666 × 444 10) 8103 × 450 11) 56321 × 672 12) 1278 × 569
13) 5745 × 562 14) 4465 × 887 15) 8862 × 341

ANSWERS

1) 106704 2) 227808 3) 375832 4) 157115 5) 157626432
6) 541688 7) 415864 8) 185022 9) 295704 10) 3646350
11) 37847712 12) 727182 13) 3228690 14) 3960455 15) 3021942

CHECKING OF MULTIPLICATION

Ex. 1: 15 × 13 = 195
digit-sum: 6 × 4 = 6
or, 24 = 6
or, 6 = 6. Thus, our calculation is correct.

Ex. 2: 69712 × 641 = 44685392
digit-sum: 7 × 2 = 5
or, 14 = 5
or, 5 = 5 Therefore, our calculation is correct.

Ex. 3: 321 × 132 = 42372
digit sum: 6 × 6 = 0
or, 36 = 0
or, 0 = 0 Thus, our calculation is correct.

But if someone gets the answer 43227, and tries to check his calculation with the help of digit-sum rule, see what happens :

321 × 132 = 43227
digit sum: 6 × 6 = 0
or, 36 = 0
or, 0 = 0

This shows that our answer is correct. But it is not true. Thus we see that if our luck is very bad, we can aprove our wrong answer.

Division

We now go on to the Quicker Maths of at-sight division which is based on long-established Vedic process of mathematical calculations. Different from "for the special cases", it is capable of immediate application to all cases and it can be described as the "crowning gem of all" for the universality of its applications.

To understand the at-sight mental one-line method of division, we should take an example and its explanation.

DIVISION BY A 2-DIGIT NUMBER

Ex 1. Divide 38982 by 73.

Soln:

Step I. Out of the divisor 73, we put down only the first digit, ie, 7 in the divisor-column and put the other digit, ie, 3 "on top of the flag", as shown in the chart below.

$$7^{3} \quad 38 \quad 9 \quad 8 \quad 2$$

The entire division will be by 7.

Step II. As one digit (3) has been put on top, we allot one place at the right end of the dividend to the remainder position of the answer and mark it off from the digits by a vertical line.

$$\begin{array}{c|ccc|c} 7^{3} & 38 & 9 & 8 & 2 \\ \hline & & & & \end{array}$$

Step III. As the first digit from the left of dividend (3) is less than 7, we take 38 as our first dividend. When we divide 38 by 7, we get 5 as the quotient and 3 as the remainder. We put 5 down as the first quotient-digit and just prefix the remainder 3 before the 9 of the dividend.

$$\begin{array}{c|ccc|c} 7^{3} & 38 & {}_{3}9 & 8 & 2 \\ \hline & 5 & & & \end{array}$$

Step IV. Now our dividend is 39. From this we, however, deduct the product of the indexed 3 and the first quotient-digit (5), ie, 3 × 5 = 15. The remainder 24 is our actual net-dividend. It is then divided by 7 and gives us 3 as the second quotient-digit and 3 as the remainder, to be placed in their respective places as was done in third step.

$$\begin{array}{c|ccc|c} 7^{3} & 38 & {}_{3}9 & {}_{3}8 & 2 \\ \hline & 5 & 3 & & \end{array}$$

Step V. Now our dividend is 38. From this we subtract the product of the index (3) and the 2nd quotient-digit (3), ie, 3 × 3 = 9. The remainder 29 is our next actual dividend and divide that by 7. We get 4 as the quotient and 1 as the remainder. We put them at their respective places.

$$\begin{array}{c|ccc|c} 7^{3} & 38 & {}_{3}9 & {}_{3}8 & {}_{1}2 \\ \hline & 5 & 3 & 4 & \end{array}$$

Step VI. Our next dividend is 12 from which, as before, we deduct 3 × 4 the third quotient-digit, ie, 12 and obtain 0 an the remainder

$$\begin{array}{c|ccc|c} 7^{3} & 38 & {}_{3}9 & {}_{3}8 & {}_{1}2 \\ \hline & 5 & 3 & 4 & 0 \end{array}$$

Thus we say : Quotient (Q) is 534 and Remainder (R) is 0.

And thus finishes the whole procedure; and all of it is one-line mental arithmetic in which all the actual division is done by the single-digit divisor 7.

The procedure is very simple and needs no further exposition and explanation. A few more illustrations with running comments will be found useful and helpful and are therefore given below :

Ex. 2: Divide 16384 by 128 (As 12 is a small number to handle with, we can treat 128 as a two-digit number).

Soln:

$$\begin{array}{c|ccc|c} 12^{8} & 16 & {}_{4}3 & {}_{11}8 & {}_{6}4 \\ \hline & 1 & 2 & 8 & 0 \end{array}$$

Step I. We divide 16 by 12. Q = 1 & R = 4.

Step II. 43 – 8 × 1 = 35 is our next devidend. Dividing it by 12, Q = 2, R = 11.

Step III. 118 – 8 × 2 = 102 is our next dividend. Dividing it by 12, Q = 8, R=6

Step IV. 64 – 8 × 8 = 0

Then our final quotient = 128 & remainder = 0

Ex 3: Divide 601325 by 76.

Soln:

$$\begin{array}{c|cccc|c} 7^{6} & 60 & {}_{11}1 & {}_{6}3 & {}_{2}2 & {}_{2}5 \\ \hline & 7 & 9 & 1 & 2 & 13 \end{array} \ (= 25 - 6 \times 2)$$

Step I. Here, in the first division by 7, we can put 8 down as the first quotient-digit, but the remainder then left will be too small for the subtraction expected at the next step. So, we take 7 as the quotient-digit and prefix the remainder 11 to the next dividend-digit.

All the other steps are similar to the previously mentioned steps in Ex 1 & 2.

Our final quotient is 7912 and remainder is 13.

If we want the values in decimal, we go on dividing as per rule instead of writing down the remainder. Such as;

$$\begin{array}{c|cccc|ccc} 7^{6} & 60 & 1 & 3 & 2 & 5 & {}_{6}0 & {}_{5}0 \\ \hline & 7 & 9 & 1 & 2 & 1 & 7 & 1 \end{array}$$

Ans = 7912.171

Note: The vertical line separating the remainder from the quotient part may be the demarcating point for decimal.

Ex. 4: Divide 7777777 by 38

Soln:

$$\begin{array}{c|cccccc|c} 3^{8} & 7 & {}_{1}7 & {}_{1}7 & {}_{5}7 & {}_{7}7 & {}_{8}7 & {}_{7}7 \\ \hline & 2 & 0 & 4 & 6 & 7 & 8 & 13 \end{array} \ (= 77 - 8 \times 8)$$

Q = 204678, R = 13

You must go through all the steps of the above solution. Try to solve it. Did you find some difference?

Ex. 5: Divide 8997654 by 99. Try it step by step.

Ex. 6:(i) Divide 710.014 by 39 (to 4 places of decimals)

(ii) 718.589 ÷ 23 = ? (iii) 718.589 ÷ 96 = ?

Soln. (i) Since there is one flag-digit the vertical line is drawn such that one digit before the decimal comes under remainder portion.

$$\begin{array}{c|cc|ccccc} 3^{9} & 7 & {}_{4}1 & {}_{8}0. & {}_{2}0 & {}_{2}1 & {}_{6}4 & {}_{7}0 \\ \hline & 1 & 8 & . & 2 & 0 & 5 & 4 \end{array}$$

For the last section, we had 64-45 = 19 as our dividend, divided by 3 we choose 4 as our suitable quotient. If we take 5 as a quotient it leaves 4 as remainder (19-15). Now the next dividend will be 40-9 × 5 = –5, which is not acceptable.

The vertical line separating the remainder from quotient part may be demarcation point for decimal.

Therefore, ans = 18.2054

(ii)

2^3	$7_1 1$	$_0 8.$	$_1 5$	$_1 8$	$_0 9$
	31.	2	4	3	0

Ans = 31.243

(iii)

9^6	7 1	$_8 8\,.$	$_{10} 5$	$_9 8$	$_5 9$	$_2 0$	$_2 0$
	7.	4	8	5	3	0	2

Ans = 7.4853

Ex. 7: Divide 7.3 by 53.

Soln.

5^3	7.	$_2 3$	$_5 0$	$_6 0$	$_4 0$	$_4 0$	
		1	3	7	7	3	6

Ans = 0.137736

DIVISION BY A 3-DIGIT NUMBER

Ex 8: Divide 7031985 by 823.

Soln: Step I. Here the divisor is of 3 digits. All the difference which we make is to put the last two digits(23) of divisor on top. As there are two flag-digits (23), we will separate two digits (85) for remainder.

8^{23}	70	3	1	9	85

Step II. We divide 70 by 8 and put down 8 and 6 in their proper places.

8^{23}	70	$_6 3$	1	9	85
	8				

Step III. Now, our gross dividend is 63. From that we subtract 16, the product of the tens of the flag-digits, ie 2, and the first quotient-digit, ie 8, and get the remainder 63 - 16 = 47 as the actual dividend. And, dividing it by 8, we have 5 and 7 as Q & R respectively and put them at their proper places.

8^{23}	70	$_6 3$	$_7 1$	9	85
	8	5			

Step IV. Now our gross dividend is 71, and we deduct the cross-products of two flag-digits 23 and the two quotient digits (8 & 5) ie 2 × 5 + 3 × 8 = 10 +24 = 34; and our remainder is 71-34 = 37. We then continue to divide 37 by 8 . We get Q = 4 & R = 5

8^{23}	70	$_6 3$	$_7 1$	$_5 9$	85
	8	5	4		

Step V. Now our gross dividend is 59. And actual dividend is equal to 59 minus cross-product of 23 and 54, ie, 59 - (2 × 4 + 3 × 5) = 59-23 = 36. Dividing 36 by 8, our Q = 4 and R = 4.

8^{23}	70	${}_{6}3$	${}_{7}1$	${}_{5}9$	${}_{4}85$
	8	5	4	4	

Step VI. Actual dividend = 48 – (3 × 4 + 2 × 4) = 48 – 20 = 28.
Dividing it by 8, our Q = 3 & R = 4

8^{23}	70	${}_{6}3$	${}_{7}1$	${}_{5}9$	${}_{4}8$	${}_{4}5$
	8	5	4	4	3	

Step VII. Actual dividend = 45 – (3 × 4 + 2 × 3) = 45-18 = 27. Dividing 27 by 8, we have Q = 3 & R = 3.

8^{23}	70	${}_{6}3$	${}_{7}1$	${}_{5}9$	${}_{4}8$	${}_{4}5$ ${}_{3}$
	8	5	4	4	3	3

The vertical line separating the remainder from the quotient part may be a demarcation point for decimal.

Ans = 8544.33

Our answer can be 8544.33, but if we want the quotient and remainder, the procedure is somewhat different. In that case, we do not need the last two steps, ie, the calculation upto the stage

8^{23}	70	${}_{6}3$	${}_{7}1$	${}_{5}9$	${}_{4}8$ 5
	8	5	4	4	

is sufficient to answer the question.
Quotient = 8544;
Remainder = 485 - 10 × (Cross multiplication of 23 and 44)* - unit digit of flagged number ×unit digit of quotient.
= 485 - 10 (4 × 2 + 4 × 3) – 3 × 4 = 485 – 200 – 12 = 273
* Cross-multiplication of the two flag-digits and last two digits of quotient.

Ex. 9: Divide 1064321 by 743 (to 4 places of decimals). Also find the remainder.
Soln:

7^{43}	10	${}_{3}6$	${}_{4}4$	${}_{4}3$	${}_{5}2$ 1
	1	4	3	2	

Q = 1432, Remainder = 521 -10 (cross-multiplication of 43 & 32) - 3 × 2 = 521 – 10 × 17 – 6 = 345
For decimals :

7^{43}	10	${}_{3}6$	${}_{4}4$	${}_{4}3$	${}_{5}2$	${}_{7}1$	${}_{7}0$	${}_{6}0$	${}_{5}0$
	1	4	3	2		4	6	4	3

Ans = 1432.4643

Ex. 10: Divide 888 by 672 (to 4 places of decimals).

6^{72}	8	${}_{2}8$	${}_{3}8$	${}_{3}0$	${}_{4}0$	${}_{5}$
	1.	3	2	1	4	

Ans = 1.3214

Note: Vertical line separating the remainder from the quotient part is the demarcation point for decimal.

Can you find the quotient and remainder? Try it.

Ex. 11: Divide 4213 by 1234 to 4 places of decimals. Also find quotient and remainder.

Soln: Although 1234 is a four-digit number, we can treat it as a 3-digit number because 12 is small enough to handle with.

$$\begin{array}{r|c|l} 12^{34} & 42 & {}_{6}1\ 3 \\ \hline & 3 & \end{array}$$

Q = 3, R = 613-10 (cross multiplication of 03 and 34) – 4 × 3

= 613 - 90 - 12 = 511

$$\begin{array}{r|c|llllll} 12^{34} & 42 & {}_{6}1 & {}_{4}3 & {}_{7}0 & {}_{3}0 & {}_{2} \\ \hline & 3. & 4 & 1 & 4 & 1 & \end{array}$$

Ans = 3.4141

DIVISION BY A 4-DIGIT NUMBER

Ex. 12: Divide 703195 by 8231.

Soln: As there are 3 flag-digits, three digits are separated by vertical line.

$$\begin{array}{r|lll|llll} 8^{231} & 7 & 0 & {}_{6}3 & {}_{7}1 & {}_{5}9 & {}_{4}5 & {}_{6} \\ \hline & 8 & & 5 & 4 & 3 & 2 & \end{array}$$

Answer = 85.432

Step I. First dividend = 70, Q = 8, R = 6

Step II. 2nd real dividend = 63 -16 (=2 × 8) = 47, Q = 5, R = 7

Step III. 3rd real dividend = 71- (cross-multiplication [C.M.]of 23 and 85 = 34) = 37, Q = 4, R=5

Step IV. 4th real dividend = 59 - (C.M. of 231 and 854) = 59 – (1 × 8 + 3 × 5 + 2 × 4)

= 59 – 31 = 28,

Q = 3, R = 4

Step V. 5th real dividend = 45 - (C.M. of 231 and 543)

= 45 – (1 × 5 + 3 × 4 + 2 × 3) = 45 - 23 = 22, Q = 2, R = 6

But if we want to get the quotient and the remainder we do the following.

$$\begin{array}{r|lll|l} 8^{231} & 7 & 0 & {}_{6}3 & {}_{7}195 \\ \hline & & 8 & 5 & \end{array}$$

Q = 85

R = 7195 -100 (C.M. of 231 and 085) - 10 (C.M. of 31 and 85) – 1 × 5

= 7195 - 3400 - 230 - 5 = 7195 - 3635 = 3560

Ex 13: Divide 41326 by 31046 (to 5 decimal places).

Soln:

$$\begin{array}{r|l|lllll} 3^{1046} & 4 & {}_{1}1 & {}_{1}3 & {}_{1}2 & {}_{2}6 & {}_{4}0 \\ \hline & 1. & 3 & 3 & 1 & 1 & 2.. \end{array}$$

Ans = 1.33112

Step I. 1st real dividend 4, Q = 1, R = 1.
Step II. 2nd real dividend = 11 – 1 × 1 = 10, Q = 3, R = 1.
Step III. 3rd real dividend = 13 – (C.M. of 10 & 13) = 13 – (0 × 1 + 1 × 3) = 10, Q = 3. R = 1.
Step IV. 4th real dividend = 12 – (C.M. of 104 & 133) =12 – 7 = 5, Q = 1, R = 2
Step V. 5th real dividend = 26 – (C.M. of 1046 and 1331) = 26 – 19
= 7, our convenient Q = 1, R = 4
Step VI. 6th real dividend = 40 - (C.M. of 1046 & 3311) = 40 - 31
= 9, convenient Q = 2, R = 4.
Note: 1. C.M. = Cross-Multiplication
2. C.M. of 1046 & 1331 = 6 × 1 + 4 × 3 + 0 × 3 + 1 × 1 = 19

Ex. 14: Divide 20014 by 137608 (to 5 decimal places).
Soln:

$$\begin{array}{r|c|cccccc} 1^{37608} & 0 & 2 & {}_{1}0 & {}_{3}0 & {}_{6}1 & {}_{8}4 & {}_{9} \\ \hline & 0 & 1 & 4 & 5 & 4 & 4\ldots & \end{array}$$

Ans. = 0.14544.

The method is similar in this case also. The only remarkable point is that since there are five digits in flagged number all the five digits of the dividend come in remainder section.

Ex. 15: Divide 8905 by 769315 (to 4 decimal places)
Soln :

$$\begin{array}{r|c|ccccc} 7^{69315} & 0 & 0 & 8 & {}_{1}9 & {}_{6}0 & {}_{10}5 \\ \hline & 0. & 0 & 1 & 1 & 5 & 7 \end{array}$$

Ans. = 0.0115

It is better to calculate up to 5 decimal places if you are asked to calculate upto 4 places, because without doing so we can't conclude that the further valid calculations exist.

The dividend has only four digits, so we put a zero before 8 to make the number of digits equal to that in flagged-number (69315). During calculation, this zero is copied down and doesn't participate in any type of calculation.

Ex. 16: Divide .0034147 by 81.4256321 (to 6 decimal places).
Soln: Change the form of the number because this form may puzzle you over the placement of decimal.
Divide 34147 by 814256321 (shifted the decimal to 7 position right). Now,

$$\begin{array}{r|c|l} 8^{14256321} & 0 & 0\ 0\ 0\ 3\ {}_{3}4\ {}_{2}1\ {}_{9}4\ {}_{5}7 \\ \hline & 0. & 0\ 0\ 0\ 0\ \ 4\ \ 1\ \ 9 \end{array}$$

Ans = 0.000041
Step I. Put 3 zeros before the number in the remainder section because there are 8 flagged digits.
Step II. Copy down all the 3 zeros.
Step III. 3 is less than 8. In usual cases we have been taking first two digits at a time when first digit was less than the prime divisor, but as in this case we are after decimal, we can't do so. Therefore, in this case, 3 is treated as the first dividend; and we get Q = 0 and R = 3. Further, we proceed in the same way as discussed earlier.

Now you must have seen all the possible cases which you may come across in mathematical division. Don't escape any of the examples discussed above. Having a broad idea of at-sight mathematical division, you should solve as many questions yourself as you can.

Checking of Calculation

We will check the calculations of first seven examples given in the beginning of this chapter. It can be seen that rule of digit-sum fails in some cases here.

Ex. 1: 38982 ÷ 73 = 534 (Since remainder is zero)

digit sum : 3 ÷ 1 = 3

or, 3 = 3

Therefore, our calculation is correct.

Ex. 2: Check yourself.

Ex. 3: 601325 ÷ 76 gives quotient = 7912 and remainder = 13

We may write the above calculation as

7912 × 76 + 13 = 601325

Now, check the correctness.

digit sum : 1 × 4 + 4 = 8

or 4 + 4 = 8 or, 8 = 8

Therefore, our calculation is correct.

Ex. 4: Check yourself.

Ex. 5: Check yourself.

Ex. 6: (i) 710.014 ÷ 39 = 18.2054...

We can't apply the digit-sum method to check our calculation because our calculation is not complete.

(ii) 718.589 ÷ 23 = 31.2430

This calculation is complete because in the end we get 0 as quotient and remainder. We can apply the digit-sum rule in this case.

digit-sum = 2 ÷ 5 = 4

or, 0.4 = 4 or, 4 = 4

Our calculation is correct.

Note: During digit-sum (forget-nine method) we don't take decimals into account.

Ex. 7: Tell whether digit-sum rule is applicable to this calculation or not.

Note: Thus, we see the limitation of digit-sum rule.

Practice Problems

Q. 1: Divide 'x' by 'y' where

a) x = 135921 & y = 89
b) x = 64932 & y = 99
c) x = 8899 & y = 101
d) x = 9995 & y =122
e) x = 13596289 & y = 76
f) x = 89325 & y = 132
g) x = 89 & y = 71
h) x = 96 & y = 95
i) x=53 & y = 83
j) x = 93 & y = 109

Q. 2: Divide 'x' by 'y' where

a) x = 359281 & y = 567
b) x = 8932 & y = 981
c) x = 99899 & y = 789
d) x = 1053 & y = 989
e) x = 738 & y = 895
f) x = 13569 & y = 1051
g) x = 69325 & y = 1163
h) x = 935 & y = 1259
i) x = 100002 & y = 777
j) x = 12345 & y = 567

Q.3: Divide 'x' by 'y' where

a) x = 3961256 & y = 6539
b) x = 698956 & y = 5783
c) x = 8889 & y = 6798
d) x = 889 & y 9834
e) x = .00532 & y 89.325168
f) x = .056 & y = .893
g) x = 12 & y = 998563
h) x = .867 & y = 93.834516
i) x = 1.932 & y = 105.63513
j) x = 0.00035 & y 78.53

ANSWERS

1. a) 1527.2022 b) 655.87878 c) 88.10891 d) 81.926229
e) 178898.539 f) 676.7045 g) 1.2535 h) 1.0105

i) 0.6385 j) 0.8532

2. a) 633.6525 b) 9.1049 c) 126.6147 d) 1.0647
e) 0.8245 f) 12.9105 g) 59.6087 h) 0.7426
i) 128.7027 j) 21.7724

3. a) 605.7892 b) 120.8639 c) 1.30759 d) 0.0904
e) 0.0000595 f) 0.0627 g) 0.000012 h) 0.0092396
i) 0.0182893 j) 0.0000044

Squaring

Squaring of a number is largely used in mathematical calculations. There are so many rules for special cases. But we will discuss a general rule for squaring which is capable of universal application.

This method is intimately connected with a procedure known as the "Duplex Combination" process and is of still greater importance and utility at the next step on the ladder, namely, the easy and facile extraction of square roots. We now go on to a brief study of this procedure.

I. Duplex Combination Process

The first one is by squaring; and the second one is by cross-multiplication. In the present context, it is used in both senses (a^2 and 2*ab*).

In the case of a single central digit, the square is meant; and in the case of an even number of diits equidistant from the two ends, double the cross-product is meant. A few examples will elucidate the procedure.

Ex. 1: For 2, Duplex (D) = $2^2 = 4$
Ex. 2: For 8, D = $8^2 = 64$
Ex. 3: For 34, D = 2 × (3 × 4) = 24
Ex. 4: For 79, D = 2 × (7 × 9) = 126
Ex. 5: For 103, D = 2(1 × 3) + 0^2 = 6
Ex. 6: For 346, D = 2(3 × 6) + 4^2 = 52
Ex. 7: For 096, D = 2(0 × 6) + 81 = 81
Ex. 8: For 1342, D = 2(1 × 2) + 2(3 × 4) = 28
Ex. 9: For 7358, D = 2(56) + 2(15) = 142
Ex. 10: For 23564, D = 2(2 × 4) + 2(3 × 6) + 5^2 = 77
Ex. 11: For 123456, D = 2(1 × 6) + 2(2 × 5) + 2(3 × 4) = 56

Now, we see the method of squaring in the following examples.

Ex. 1: $207^2 = ?$

Soln: $2^2 / 2(2 \times 0) / 2(2 \times 7) + 0^2 / 2(0 \times 7) / 7^2$

$= 4 \;/\; 0 \;/\; 28 \;/\; 0 \;/\; 49$

$= 4 \;/\; 0 \;/\; {}_2 8 \;/\; 0 \;/\; {}_4 9$

$= 4 \;/\; 0 + 2 \;/\; 8 \;/\; 0 + 4 \;/\; 9$

$= 42849$

If you have understood the duplex method and its use in squaring, you may get the answer in a line. For example, $207^2 = 42_2 84_4 9$

Explanations:

1. Square the last digit (7). Put the first digit (9) of square in answer line and carry the other (4).
2. 2 × 0 × 7 + 4 (carried) = 4; write it down at 2nd position.
3. 2 × 2 × 7 + 0^2 = 28; write down 8 and carry over 2.
4. 2 × 2 × 0 + 2 (carried over) = 2; write it down.
5. $2^2 = 4$; write it down.

Note: (1) If there are *n* digits in a number, the square will have either 2n or 2n – 1 digits.
(2) Participation of digits follows the same systematic pattern as in multiplication.

Ex. 2: $(897)^2 = 81_{16}4_{20}6_{13}0_49 = 804609$

Explanation: 1. $7^2 = 49;$ write down 9 and carry over 4
2. 2 × 9 × 7 + 4 (carried) = 130; write down 0 and carry over 13.
3. 2 × 8 × 7 + 9^2 + 13 = 206; write down 6 and carry over 20.
4. 2 × 8 × 9 + 20 = 164; write down 4 and carry over 16.
5. $8^2 + 16 = 80;$ write it down.

Ex. 3: $(1432)^2 = 2_10_25_30_26_124 = 2050624$

Explanation:

1. $2^2 = 4$; write it down.
2. 2 × (3 × 2) = 12; write down 2 and carry over 1.
3. 2 × (4 × 2) + $3^2 + 1 = 26$; write down 6 and carry over 2.
4. 2(1 × 2) + 2(4 × 3) + 2 = 30; write down 0 and carry over 3.
5. 2(1 × 3) + $4^2 + 3$ = 25; write down 5 and carry over 2.
6. 2(1 × 4) + 2 = 10; write down 0 and carry over 1.
7. $1^2 + 1 = 2$; write it down.

Ex. 4: $(73214)^2 = 53_46_40_32_68_29_179_16 = 5360289796$

Ex. 5: $(5432819)^2 = 29_{45}51_55_{11}5_{10}2_{16}2_{12}2_{12}8_66_{14}7_26_81 = 29515522286761$

Practice Problem:

Q. : Find the squares of the following numbers
1) 835 2) 8432 3) 45321 4) 530026 5) 73010932

Answers
1) 597225 2) 7109824 3) 2053993041 4) 280927560676
5) 5330596191508624

Note: To find the square of a fractional (decimal) number, we square the number without looking at decimal. After that we count the number of digits after the decimal in the original value. In the square value, we place the decimal after double the number of digits after decimal in the original value. For example,

$(12.46)^2 = 15_15_32_45_51_36 = 155.2516$

To check the calculation

We use the digit-sum method for checking calculations in squaring. For example:

In Ex. 1: $(207)^2 = 42849$

digit-sum: $(0)^2 = (0)^2$

Hence, our calculation is correct.

In Ex. 2: $(897)^2 = 804609$

digit-sum $(6)^2 = 18$
or, 36 = 18 or, 0 = 0
Thus, our calculation is correct.

In Ex. 3: $(1432)^2 = 2050624$

digit-sum: $1^2 = 1$ or, 1 = 1
Thus, our calculation is correct.

Note: 1. Follow the "forget-nine" rule during the calculation of digit-sum.
2. Check all the calculations mentally.
3. Check the correctness of calculations in Ex. 4 & Ex. 5 without using pen.

II. Squaring of the numbers having repeated unit or repeated digit

$(1)^2 = 1$

$(11)^2 = 121$

$(111)^2 = 12321$

$(1111)^2 = 1234321$

$(11111)^2 = 123454321$

.... and so on

Explanation: See the trend carefully. From the above trend we can calculate square of the given number as follows.

Write down the natural number starting from 1 as many times as 1 appears in the given number. Thereafter write down the natural numbers in the descending order up to 1.

This trend follows up to squaring of (111111111).

$(111111111)^2 = 12345678987654321$.

Ex. 1: Find the value of $\sqrt{1234567654321}$? **CAT 1999**

Soln: As from the above method,

$\Rightarrow \sqrt{1234567654321} = 1111111$

Ex. 2: Find $(6666)^2$.

Soln: Now, $(6666)^2 = (6 \times 1111)^2 = 1234321 \times 36 = 44435556$

Ex. 3: Find $(7777)^2$.

Soln: $\Rightarrow (7777)^2 = (7 \times 1111)^2 = 1234321 \times 49 = 60481729$.

Square Root

The method to get the square root of any number is similar to the procedure of "Division". The method of "Duplex" is also used in the process of square-rooting. So, we suggest you to go through the "Duplex" process again. Before jumping on to the process of square root, we must know some fundamental principles related to square root.

1) The given number is first arranged in two-digit groups from right to left; and a single digit if any left over at the left-hand-end is counted as a single group by itself.
2) The number of digits in a square root will be the same as the number of digit-groups in the given number itself including a single digit, if any. Thus <u>25</u> will be counted as one group, <u>1</u> <u>69</u> as two groups and <u>10</u> <u>24</u> as two groups.
3) If the square root contains n digits, the square must consist of $2n$ or $2n - 1$ digits and conversely, if the given number has n digits, the square root will contain $\frac{n}{2}$ or $\frac{n+1}{2}$ digits.
4) But, in cases of a pure decimals, the number of digits in the square is always double that in the square root.
5) The squares of first nine natural numbers are 1, 4, 9, 16, 25, 36, 49, 64 and 81 respectively. This simply means
   a) that an exact square cannot end in 2, 3, 7 or 8;

b) (i) that a complete square ending in 1 must have either 1 or 9, mutual complements from 10, as the last digit of its square root;
(ii) that a square can end in 4, only if the square root ends in 2 or 8, a complement from 10;
(iii) that the ending of a square is 5 or 10 means that its square root too ends in 5 or 0 respectively;
(iv) that a square ending in 6 must have 4 or 6, a complement from 10, as the last digit in its square root; and
(v) that the termination of an exact square in 9 is possible, if and only if square root ends in 3 or 7, a complement from 10.

In brief it can be said

(i) that 1, 5, 6 and 0 at the end of a number reproduce themselves as the last digits in their squares.

(ii) that squares of complements from 10 have the same last digit. Thus 1^2 and 9^2; 2^2 and 8^2; 3^2 and 7^2; 4^2 and 6^2; 5^2 (and 5^2); and 0^2 and 10^2 have the same ending, namely 1, 4, 9, 7, 5 and 0 respectively; and

(iii) that 2, 3, 7 and 8 are out of course altogether as the final digit of a perfect square.

After having a brief knowledge regarding the square root principles now, we shift to the calculating procedure of square root.

Step I: First arrange in two-digit groups as suggested earlier in principle 1.
Step II: First group from left should be separated from othr groups by a vertical line.
Step III: Write down the digital square-root value in answr row. For example.

$$\begin{array}{c|c|l} & 61 & 13\ 64 \\ \hline & 7 & \end{array}$$

Answer row

Here, 7 is written because 8^2 is greater than 61.

Other examples,

$$\begin{array}{c|c|l} & 73 & 60\ 84 \\ \hline & 8 & \end{array} \qquad \begin{array}{c|c|l} & 8 & 56\ 78 \\ \hline & 2 & \end{array}$$

Here, 8 and 2 are written because 9^2 is greater than 73 and 3^2 is greater than 8.

Step IV: Now, write down the double of first answer digit at the place of divisor. For example

$$\begin{array}{c|c|l} 14 & 61 & 13\ 64 \\ \hline & 7 & \end{array} \qquad \text{and} \qquad \begin{array}{c|c|l} 16 & 73 & 60\ 84 \\ \hline & 8 & \end{array}$$

Step V: WRite down the remainder after substraction of the square of the first answer-digit from the first group as a prefix to the next dividend-digit. For example:

$$\begin{array}{c|c|l} 6 & 11 & {}_{2}9\ \ 7\ \ 1\ \ 6 \\ \hline & 8 & \end{array}$$

$11 - 3^2 = 2$, which is written as a prefix of 9, the next dividend-digit.

Step VI: Our next gross diidend-number is thus 29. Without subtracting anything from it, we simply divide 29 by the divisor 6 and put down the second quotient-digit 4 and the second remainder 5 in their proper places as usual.

$$\begin{array}{c|c|l} 6 & 11 & {}_{2}9\ \ {}_{5}7\ \ 1\ \ 6 \\ \hline & 3 & 4 \end{array}$$

Step VII: Thus our next gross dividend is 57. From this we subtract 16, the square of the second quotient-digit, get 41 as the actual dividend, divide it by 6 and set down the quotient 6 and remainder 5 in their proper places as usual.

$$\begin{array}{r|c|l} 6 & 11 & {}_{2}9\ \ {}_{5}7\ \ {}_{5}1\ \ 6 \\ \hline & 3 & 4\ \ 6 \end{array}$$

Step VIII: Our fourth gross dividend-number is 51. From this we subtract the "Duplex" of 46, the two quotient obtained earlier, which comes to [2 (4 × 6) =]48. We get 3 as remainder, divide it by 6 and set down the quotient (0) and the remainder (3) in their proper places.

$$\begin{array}{r|c|l} 6 & 11 & {}_{2}9\ \ {}_{5}7\ \ {}_{5}1\ \ {}_{3}6 \\ \hline & 3 & 4\ \ 6\ \ \ 0 \end{array}$$

Step IX: This gives us 36, our last gross dividend-number. From this we subtract 36, the "Duplex" of 460 ($2 \times 4 \times 0 + 6^2 = 36$). We get 0 as remainder which when divided by 6 gives 0 as quotient and remainder.

$$\begin{array}{r|c|l} 6 & 11 & {}_{2}9\ \ {}_{5}7\ \ {}_{5}1\ \ {}_{3}6\ \ {}_{0} \\ \hline & 3 & 4\ \ 6\ \ \ 0\ \ \ 0 \end{array}$$

Note: 1. As the number (119716) has 6 digits, its square root will have 3 digits, so we put decimal after three digits.

2. For a complete square number when the process is continued into decimal part, all the quotient-digits in the decimal part are found to be zeroes and the remainders too are all zeroes.

Some more examples:

Ex. 1: $\sqrt{529}$

$$\begin{array}{r|c|l} 4 & 5 & {}_{1}2\ \ {}_{0}9\ \ {}_{0} \\ \hline & 2 & 3.0\ \ \ = 23 \end{array}$$

As there are 3 (odd) digits in the number, the number of digits in its square root would be $\frac{3+1}{2} = 2$; so we put decimal after 2 digits of answer.

Ex. 2: $\sqrt{4096}$

$$\begin{array}{r|c|l} 12 & 40 & {}_{4}9\ \ {}_{1}6\ \ {}_{0} \\ \hline & 6 & 4.0\ \ \ = 64 \end{array}$$

Ex. 3: $\sqrt{16384}$

$$\begin{array}{r|c|l} 2 & 1 & {}_{0}6\ \ {}_{2}3\ \ {}_{3}8\ {}_{6}4\ {}_{0} \\ \hline & 1 & 2\ \ 8.\ \ 0\ \ 0 \end{array}$$

The above example is different from others. It needs some explanation. Upto the stage

$$\begin{array}{r|c|l} 2 & 1 & {}_{0}6\ \ 3\ \ 8\ \ 4 \\ \hline & 1 & \end{array}$$

there should not be any confusion.

Now, our gross dividend is 6. If we divide it by 2, we can get 3 as quotient and 0 as remainder. In that case, 3 is our next gross dividend which after subtracting 3^2 gives a negative value; thus we march towards an incalculative goal. A similar condition arises when we are left with $(23 - 2^2 =)19$ as a real dividend. When we divide it by 2, we get 9 as quotient and 1 as remainder. But exactly the same condition is got when we are left with $(38 - 2 \times 2 \times 8 = 6)$ as our real dividend. (Try to solve it yourself.)

Ex. 4: $\sqrt{53163214}$

14	53	${}_{4}1$	${}_{13}6$	${}_{6}3$	${}_{13}2$	${}_{5}1$	${}_{7}4$	
	7	2	9	1.	3	1...		(incomplete)

Ex. 5: $\sqrt{738915489}$

4	7	${}_{3}3$	${}_{5}8$	${}_{5}9$	${}_{13}1$	${}_{6}5$	${}_{7}4$	${}_{4}8$	${}_{0}9$	${}_{0}$
	2	7	1	8	3.	0	0	0	0	

Ex. 6: $\sqrt{.0009240160}$ (to 6 decimal places)

6	.00	0	9	${}_{0}2$	${}_{2}4$	${}_{6}0$	${}_{6}1$	${}_{10}6$	${}_{16}0$
	. 0		3	0	3	9	7	6	

Ans = .030397

Note: 1. Note that our first calculative group is "09" and not "00".

2. If you are required to find the answer up to 6 decimal places, we suggest you to find the 7th place also because it check the further viability of calculations.

Ex. 7: $\sqrt{27}$ (to 6 places of decimal)

Soln:

10	27	${}_{2}0$	${}_{10}0$	${}_{9}0$	${}_{12}0$	${}_{17}0$	${}_{10}0$	${}_{16}0$
	5.	1	9	6	1	5	2	4

Ans = 5.1961524

Ex. 8: Find the square root of 0.000000831 (to 8 places of decimal)

Soln:

18	.000000	83	${}_{2}1$	${}_{3}0$	${}_{11}0$	${}_{18}0$	${}_{7}0$	${}_{6}0$
	. 0 01 0	9	1	1	5	9	2	

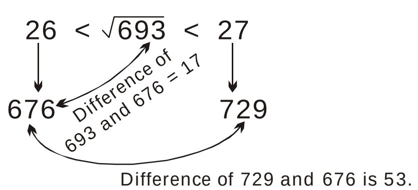
Ans = .00091159

Alternative Method for Finding Approximate Value of Square Root of a Given Number

We can better understand the method from the example given below.

Suppose we have to find the square root of 693.

Since 693 lies between 676 $\left(26^2\right)$ and 729 $(27)^2$



$\because$ For the difference of 53, difference in the square root is 1

$\therefore$ For the difference of 1, difference in the square root is $\dfrac{1}{53}$

$\therefore$ For the difference of 17, difference of the square root is $\dfrac{17}{53} \approx 0.327$

$\Rightarrow \sqrt{693} = 26 + 0.327 = 26.327$

Ex. 1: $\sqrt{1759}$ = ?

Soln:

$$\begin{array}{ccccc} 41 & < & \sqrt{1759} & < & 42 \\ \downarrow & & & & \downarrow \\ 1681 & & & & 1764 \end{array}$$

$\Delta_1 = 78$

$\Delta_2 = 83$

As per the method discussed above

$$\Rightarrow \sqrt{1759} = 41 + \frac{78}{83} \approx 41 + 0.94 \approx 41.94$$

Ex. 2: $\sqrt{2.71} = ?$

Soln: $\sqrt{2.71} = \sqrt{\frac{271}{100}} = \frac{\sqrt{271}}{10}$

And,

$$\begin{array}{ccccc} 16 & < & \sqrt{271} & < & 17 \\ \downarrow & & & & \downarrow \\ 256 & & & & 289 \end{array}$$

$\Delta_1 = 15$

$\Delta_2 = 33$

As per the method discussed above

$$\sqrt{271} = 16 + \frac{15}{33} \approx 16 + 0.454 \approx 16.454$$

$$\therefore \sqrt{2.71} = \frac{\sqrt{271}}{10} = \frac{16.454}{10} = 1.6454$$

Cube

Cubes of large numbers are rarely used. During our mathematical calculations, we sometimes need the cube value of two-digit numbers. So, an easy rule for calculating the cubes of 2-digit numbers is being given. In its process the cube values of the "first ten natural numbers", ie, 1 to 10, are used. Readers are suggested to remember the cubes of only these "first ten natural numbers."

$1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$ $6^3 = 216$, $7^3 = 343$,

$8^3 = 512$, $9^3 = 729$ and $10^3 = 1000$.

To calculate the cube value of two-digit numbers we proceed like this:

Step I: The first thing we have to do is to put down the cube of the tens-digit in a row of 4 figures. The other three numbers in the row of answer should be written in a geometrical ratio in the exact proportion which is there between the digits of the given number.

Step II: The second step is to put down, under the second and third numbers, just two times of second and third number. Then add up the two rows.

For example :

Ex 1. $12^3 = ?$

Soln. Step I: We see that the ten-digit in the number is 1, so we write the cube of 1. And also as the ratio between 1and 2 is 1 : 2, the next digits will be double the previous one. So, the first row is

1 2 4 8

Step II: In the above row our 2nd and 3rd digits (from right) are 4 and 2 respectively. So, we write down 8 and 4 below 4 and 2 respectively. Then add up the two rows.

$$\begin{array}{cccc} 1 & 2 & 4 & 8 \\ & & 4 & 8 \\ \hline 1 & 7 & {}_{1}2 & 8 \end{array} = 1728$$

Ex 2 : 11^3 = ? (Solve it yourself.)

Ex 3: 16^3 = ?

Soln :

$$\begin{array}{cccc} 1 & 6 & 36 & 216 \\ & 12 & 72 & \\ \hline 4 & {}_{3}0 & {}_{12}9 & {}_{21}6 \end{array} = 4096$$

Explanations : 1^3 (from $\underline{1}6$ = 1. So, 1 is our first digit in the first row. Digits of 16 are in the ratio 1 : 6, hence our other digits should be 1 × 6=6, 6 × 6 = 36, 36 × 6 = 216. In the second row, double the 2nd and 3rd number is written. In the third row, we have to write down only one digit below each column (except under the last column which may have more than one digit). So, after putting down the unit-digit, we carry over the rest to add up with the left-hand column. Here,

i) Write down 6 of 216 and carry over 21.

ii) 36 + 72 + 21 (carried) = 129, write down 9 and carry over 12.

iii) 6 + 12 + 12 (carried) = 30, write down 0 and carry over 3.

iv) 1 + 3 (carried) = 4, write down 4.

Ex 4: 18^3 = ?

Soln :

$$\begin{array}{cccc} 1 & 8 & 64 & 512 \\ & 16 & 128 & \\ \hline 5 & {}_{4}8 & {}_{24}3 & {}_{51}2 \end{array} = 5832$$

i) Write down 2 and carry over 51 of 512.

ii) 64+128+51 = 243, write down 3 and carry over 24.

iii) 8+16+24 = 48, write down 8 and carry over 4.

iv) 1+4 = 5 write it down.

Ex 5: 17^3 = ? (Solve it yourself)

Ex 6: 19^3 = ? (Solve it yourself)

Ex 7: 21^3 = ?

Soln :

$$\begin{array}{cccc} 8 & 4 & 2 & 1 \\ & 8 & 4 & \\ \hline 9 & {}_{1}2 & 6 & 1 \end{array} = 9261$$

[8 = 2^3 , 8 ÷ 2 = 4, 4 ÷ 2 = 2, 2 ÷ 2 = 1, since ratio is 2:1]

[4 × 2 = 8, 2 × 2 = 4, double is written below]

Do you mark the difference? If no, go through the following explanations.

Step I: i) 2^3 = 8 is the first figure (from left) in the first row.

ii) Ratio between the two digits is 2:1, ie, the number should be halved subsequently. Therefore, the next three numbers in the first row should be 4, 2 & 1.

Step II: It should be clear to all of you because it has nothing new.

Ex 8 : 23^3 = ?

Soln :

$$\begin{array}{cccc} 8 & 12 & 18 & 27 \\ & 24 & 36 & \\ \hline 12 & {}_{4}1 & {}_{5}6 & {}_{2}7 \end{array} = 12167$$

Explanations:

Step I: i) $2^3 = 8$ ———— the first figure (from left) in the first row.

ii) 2:3 $\Rightarrow$ the next numbers should be 3^2 of the previous ones. So, we have

$8 \times \frac{3}{2} = 12$, $12 \times \frac{3}{2} = 18$, $18 \times \frac{3}{2} = 27$.

Ex 9: 33^3 = ?

Soln :

7	27	27	27	
	54	54		
35	$_{8}9$	$_{8}3$	$_{2}7$	= 35937

Explanations:

Step i) 3^3 = 27 ——— the first figure in first row.

ii) 3 : 3 = 1 : 1 $\Rightarrow$ the subsequent numbers should be the same.

Ex 10: 34^3 = ?

Soln :

27	36	48	64	
	72	96		
39	$_{12}3$	$_{15}0$	$_{6}4$	= 39304

Explanations:

i) 3^3 = 27 ———— the first figure (from left) in the first row.

ii) Ratio is 3:4, ie, the next numbers should be 4^3 of their previous ones.

Here, 27 × $\frac{4}{3}$ = 36, 36 × $\frac{4}{3}$ = 48, 48 × $\frac{4}{3}$ = 64.

Ex 11: 93^3 = ?

Soln :

729	243	81	27	
	486	162		
804	$_{75}3$	$_{24}5$	$_{2}7$	= 804357

Explanations:

i) 9^3 = 729 ———— the first figure (from left) in the first row,

ii) 9 : 3 $\Rightarrow$ 3 : 1 $\Rightarrow$ the subsequent figures should be $\frac{1}{3}$ of their previous ones.

Ex 12: 97^3 = ?

Soln :

729	567	441	343	
	1134	882		
912	$_{183}6$	$_{135}7$	$_{34}3$	= 912673

Explanations:

i) 9^3= 729 ——— first figure (from left) in the first row.

ii) Ratio = 9 : 7 $\Rightarrow$ Next numbers should be $\frac{7}{9}$ of the previous ones. Therefore, 729 × $\frac{7}{9}$ = 567,

567 × $\frac{7}{9}$ = 441, 441 × $\frac{7}{9}$ = 343.

Practice Problems.

Q. Find the cubes of the following numbers.

1. 17	**2.** 26	**3.** 27	**4.** 32	**5.** 41	**6.** 43	**7**. 49
8. 51	**9.** 53	**10.**55	**11.** 57	**12.** 64	**13.** 67	**14.** 69
15. 73	**16.**77	**17.** 88	**18.** 92	**19.** 95	**20.** 99.	

ANSWERS

1) 4913	2) 17576	3) 19683	4) 32768	5) 68921	6) 79507
7) 117649	8) 132651	9) 148877	10) 166375	11) 185193	12) 262144
13) 300763	14) 328509	15) 389017	16) 456533	17) 681472	18) 778688
19) 857375	20) 970299				

Note: Don't use this method for getting the cubes of 20, 30, 40,.... Do you know the other quick method?

Checking the correctness (with the help of digit-sum)

Ex. 1: $12^3 = 1728$

digit sum : $(3)^3 = 0$ (7+2 = 9, forget it. 1+8 = 9, forget it.)

or, $0 = 0$, thus the cube value is correct.

Ex 2: $16^3 = 4096$

digit-sum : $7^3 = 1$

or, $(49)\ 7 = 1$

or, $4 \times 7 = 1$

or, $28 = 1$ or, $1 = 1$, Thus cube value is correct.

Practice-Problem: Check all the calculations (from Ex 2 to Ex 12) done in this chapter.

Cube Roots of Exact Cubes

In this chapter, the quick method for getting the cube roots of exact cubes is being given. This method is completely at-sight method for upto 6-digit numbers. You don't need to write anything to get the answer. The method is totally based on inspection. Before jumping on to the procedure, we must know some well-known principles.

1) The lowest cubes, ie, the cubes of the first nine natural numbers, are 1, 98, 27, 64, 125, 216, 343, 512 and 729 (Get it by heart).
2) Thus, they all have their own distinct endings; and there is no possibility of overlapping or doubt as in the case of squares.
3) Therefore, the last digit of the cube root of an exact cube is obvious:
   (i) Cube ends in 1; then cube root ends in 1.
   (ii) Cube ends in 2; then cube root ends in 8.
   (iii) Cube ends in 3; then cube root ends in 7.
   (iv) Cube ends in 4; then cube root ends in 4.
   (v) Cube ends in 5; then cube root ends in 5.
   (vi) Cube ends in 6; then cube root ends in 6.
   (vii) Cube ends in 7; then cube root ends in 3.
   (viii) Cube ends in 8; then cube root ends in 2.
   (ix) Cube ends in 9; then cube root ends in 9.
4) In other words
   (i) 1, 4, 5, 6, 9 and 0 repeat themselves in the cube-ending; and
   (ii) 2, 3, 7 and 8 have an interplay of complements from 10.
5) The number of digits in a cube root is the same as the number of 3-digit groups in the original cube including a single-digit or a double-digit group if there is any.

6) The first digit (from left) of the ube-root will always be obvious from the first group in the cube.

7) Thus, the number of digits, the first digit and the last digit of the cube root of an exact cube are the data with which we start when we begin the work of extracting the cube root of an exact cube.

For example

Ex. 1: Find the cube roots of the following values.

i) 1728 ii) 2744 iii) 3375 iv) 15625

v) 912673 vi) 35973 vii) 804357 viii) 13824

Soln: i) Step I. Make groups of 3-digits sarting from right. Thus 1 728

Step II. As there are two groups, the cube-root will have two digits.

Thus n = 2.

Step III. The right-hand group (728) ends in 8, thus the cube-root will end in 2.

Step IV. The left-hand group (1), when cube-rooted, gives 1.

Thus we conclude; n = 2, Left-digit = 1, Right-digit = 2. Therefore answer is 12.

ii) Step I. 2 744

Step II. Because there are only two groups n = 2

Step III. Right-hand group ends in 4, thus R-digit 4.

Step IV. Left-hand group (2) is less than 2^3 and more than 1^3, thus L-digit = 1.

L-digit can't be 2 or more (why ?). Thus the value is 14.

iii) 3 375: Here, n = 2 R-digit = 5 L-digit = 1,

Thus answer = 15.

iv) 15 625

Here, n = 2, R-digit = 5, L-digit = 2 (as 15 is more than 2^3 and less than 3^3).

$\therefore$ answer = 25

v) 912 673: Thus, n = 2, R-digit = 7, L-digit = 9

$\left(\text{as } 9^3 < 912 < 10^3\right)$

vi), vii) and viii): Solve yourself.

Cube roots of numbers having 7, 8 or 9 digits

When we have a number with 7, 8 or 9 digits, the cube-root of the number will have 3 digits (as there will be 3 groups of 3 digits taken at a time). In this case, we are required to do minor calculation in written. If we denote the left digit, the right diit and the middle digit by L, R and M respectively then a relation exists between them which is used thereafter.

For example

Ex. 1: Find the cube root of 33,076,161

Soln: Here R = 1, L = 3, n = 3

We need only "M" to get the exact value.

Step I: Subtract R^3 from the number of forget the last digit (0) obtained after subtraction. Like

$$\begin{array}{r} 33\ 076\ 161 \\ 1 \\ \hline 33\ 076\ 16 \end{array} \qquad (\text{As, } R^3 = 1^3 = 1)$$

Step II: Middle digit of the cube-root is obtained by $3R^2M$.

$3R^2M = 3 \times 1^2 \times M = 3M$.

Now, we are supposed that this $3R^2M$ is contained in 3307616.

Since, $3R^2M$ = 3M is contained in 3307616 we may say that both should end with the same digit. Last digit of 3307616 is 6 and 3M should end in 6, hence M = 2.

Thus, we get all the three digits, L = 3, M = 2 & R = 1.

Therefore, cube root of 33 076 161 is 321.

Ex. 2: Find the cube root of 84604519.
Soln: n = 3, R = 9, L = 4, M = ?

Step I: Subtract R^3 from the number and forget the 'zero'. As R^3 = 729,

$$\begin{array}{r} 84604519 \\ 729 \\ \hline 8460379 \end{array}$$

Step II: $3R^2M = 3 \times 81 \times M = 243M$

As 8460379 is ending in 9, M = 3

Thus, we get all the three digits: L = 4, M = 3 and R = 9

Therefore, the cube root of the given number is 439.

Ex. 3: Find the cube root of the exact cube 248858189.
Soln: Here, n = 3, R = 9, L = 6 and M = ?
Step I:

$$\begin{array}{r} 248858189 \\ 729 \\ \hline 24885746 \end{array} \qquad (\text{As } R^3 = 729)$$

Step II: $3R^2M = 3 \times 81 \times M = 243M$

As 24885746 is ending in 6, M = 2.

Thus, the cube root is 629.

Ex. 4: Find the cube root of the exact cube = 143055667.
Soln: L = 5, R = 3, N = 3 and M = ?
Step I:

$$\begin{array}{r} 143055667 \\ 27 \\ \hline 14305564 \end{array}$$

Step II: $3R^2M = 3 \times 9 \times M = 27M$

As 14305564 is ending in 4, M = 2

Thus, the cube root = 523.

Ex. 5: Find the cube root of the exact cube 105823.817.
Soln: Since the above value is the exact cube, our answer will be of 3 digits with decimal after one digit from right. To calculate the cube root, we don't need to carry the decimal. After calculating L, R and M, we will put the decimal.

Here, L = 4, R = 3, N = 3 and M = ?
Step I:

$$\begin{array}{r} 105823817 \\ 27 \\ \hline 10582379 \end{array} \qquad (\text{As } R^3 = 27)$$

Step II: $3R^2M = 3 \times 9 \times M = 27M$

As 10582379 is ending in 9, M = 7

Thus, the cube root of the given value = 47.3

Ex. 6: Find cube root of the exact cube 83.453453. (Solve it yourself.)
Note: We don't need to go for the number which has more than 9 digits because it is rarely used. Cube-rooting has also a general rule (for all numbers) but it is useless to discuss it here as it needs some complex calculations.

Alternative Method for Finding Approximate Value of Cube Root of the Given Number

We can better understand the method from the example given below:

Suppose we have to find the cube root of 179.

Since 179 lies between 125 (ie 5^3) and 216 $\left(\text{ie } 6^3\right)$.

$$\begin{tikzcd}
\Rightarrow & 5 \arrow[d] & < & \sqrt[3]{179} & < & 6 \arrow[d] \\
& (5^3)\ 125 \arrow[urr, bend left, "\Delta_1 \text{ ie difference of } 179 \text{ and } 125 = 54"'] \arrow[rrrr, leftrightarrow, bend right, "\Delta_2 \text{ ie difference of } 216 \text{ and } 125 = 91"'] & & & & 216\ (6^3)
\end{tikzcd}$$

$\because$ For the difference of 91, difference in the cube root is 1

$\therefore$ For the difference of 1, difference in the cube root is $\frac{1}{91}$

$\therefore$ For the difference of 54, difference in the cube root is $\frac{54}{91}$

ie $\sqrt[3]{179} = 5 + \frac{54}{91} \approx 5 + 0.594 \approx 5.594$

Ex. 7: $\sqrt[3]{457} = ?$

Soln: As per the method mentioned above,

$$\begin{tikzcd}
7 \arrow[d] & < & \sqrt[3]{457} & < & 8 \arrow[d] \\
(7^3)\ 343 \arrow[urr, bend left, "\Delta_1 = 114"'] \arrow[rrrr, leftrightarrow, bend right, "\Delta_2 = 169"'] & & & & 512\ (8^3)
\end{tikzcd}$$

$\Rightarrow \sqrt[3]{457} = 7 + \frac{114}{169} \approx 7.68$

Ex. 8: $\sqrt[3]{1.56} = ?$

Soln: $\sqrt[3]{1.56} = \sqrt[3]{\frac{1560}{1000}} = \frac{\sqrt[3]{1560}}{10}$

(1000 is given in the denominator because finding cube root of 1000 is easier)

and,

$$\begin{tikzcd}
11 \arrow[d] & < & \sqrt[3]{1560} & < & 12 \arrow[d] \\
(11^3)\ 1331 \arrow[urr, bend left, "229"'] \arrow[rrrr, leftrightarrow, bend right, "397"'] & & & & 1728\ (12^3)
\end{tikzcd}$$

$\therefore \sqrt[3]{1560} = 11 + \frac{229}{397}$ (For finding the value of $\frac{229}{397}$ quickly we can do the following:

$$\frac{229}{397} \approx \frac{229+3}{397+3}, \ \frac{232}{400} = 0.58\,)$$

$\approx 11.576 \approx 11.58$

$\Rightarrow \sqrt[3]{1.56} = \frac{\sqrt[3]{1560}}{10} = \frac{11.576}{10} = 1.1576$ or 1.158.

Comparison of Fractional Numbers

Comparison of fractional numbers in the least possible time is very necessary to solve many Data Interpretation problems. The problems in which we have to find whether percentage increase/ decrease is maximum/minimum need comparison of fractional numbers.

The following are the methods for comparison of fractional numbers.
Method I: By Rough Decimal Expression
Method II: By Exact Decimal Expression
Method III: By Cross-Multiplication
Method IV: By Formulae

Method I: By Rough Decimal Expression

Suppose we have to compare $\frac{2}{3}$ and $\frac{5}{7}$.

Roughly, we can say that $\frac{2}{3}$ is less than 0.7 and $\frac{5}{7}$ is greater than 0.7.

$\therefore \frac{2}{3} < \frac{5}{7}$

Again, suppose we have to compare $\frac{328}{546}$ and $\frac{393}{625}$.

Roughly, we can say that $\frac{328}{546}$ is just greater than 0.6 but $\frac{393}{625}$ is quite greater than 0.6.

Therefore, $\frac{328}{546} < \frac{393}{625}$

Method II: By Exact Decimal Expression

This method is suggestive for comparison of those fractional numbers in which numerators and denominators are not very large.

Ex. 1: Compare $\frac{6}{11}$ and $\frac{9}{17}$.

Soln: $\frac{6}{11} = 0.55$ and $\frac{9}{17} \approx 0.53$

$\Rightarrow \frac{6}{11} > \frac{9}{17}$

Ex. 2: Compare $\frac{83}{91}$ and $\frac{67}{83}$.

Soln: $\frac{83}{91} \approx 0.91$ and $\frac{67}{83} \approx 0.80$

$\Rightarrow \frac{83}{91} > \frac{67}{83}$

In this case, method I is suggested.

Clearly $\frac{83}{91}$ is greater than 0.9 but $\frac{67}{83}$ is less than 0.9.

Ex. 3: Compare $\frac{338}{461}$ and $\frac{473}{542}$.

Soln: Exact decimal expression of the above fractional number is not possible to get quickly.

$\frac{338}{461} \approx \frac{34}{46}$ (Two-digit conversion of 338 and 461 is 34 and 46 respectively.)

Similarly, $\frac{473}{542} \approx \frac{47}{54}$

$\frac{34}{46}$ is less than 0.8 but $\frac{47}{54}$ is greater than 0.8.

$\Rightarrow \frac{338}{461} < \frac{473}{542}$

However, getting the exact decimal expression of the fractional numbers $\frac{338}{461}$ and $\frac{473}{542}$ after two-digit conversion will take time.

Method III: By Cross-Multiplication

Suppose we have to compare $\frac{a}{b}$ and $\frac{c}{d}$.

$\frac{a}{b} \quad \frac{c}{d}$

Cross-multiplication (ad) is corresponding to $\frac{a}{b}$.

Cross-multiplication (bc) is corresponding to $\frac{c}{d}$

(i) If ad > bc, then $\frac{a}{b} > \frac{c}{d}$

(ii) If ad < bc, then $\frac{a}{b} < \frac{c}{d}$

Ex. 4: Compare $\frac{13}{19}$ and $\frac{21}{26}$.

Soln: Since (13 × 26 = 338) < (21 × 19 = 399)

$\Rightarrow \frac{13}{19} < \frac{21}{26}$

Ex. 5: Compare $\frac{18}{37}$ and $\frac{47}{58}$.

Soln: By actual cross-multiplication

$\frac{18}{37} \quad \frac{47}{58}$

$\because$ 18 × 58 < 47 × 37

$\Rightarrow \frac{18}{37} < \frac{47}{58}$

By rough cross-multiplication

$$\frac{18}{37} \quad \frac{47}{58}$$

(18 for 18
6 for 58)

(47 for 47
3.5 for 37)

$18 \times 6 < 47 \times 3.5$

$\Rightarrow \frac{18}{37} < \frac{47}{58}$

This method is very helpful. It requires one or two digit conversion of large numbers, because it is not always possible to find the actual cross-multiplication. Students are advised to learn this method.

Ex. 6: Compare $\frac{576}{789}$ and $\frac{699}{892}$.

Soln:

$$\frac{576}{789} \quad \frac{699}{892}$$

(58 for 576
9 for 892)

(70 for 699
8 for 789)

$58 \times 9 < 70 \times 8$

$\Rightarrow \frac{576}{789} < \frac{699}{892}$

Ex. 7: Compare $\frac{757}{479}$ and $\frac{951}{563}$.

Soln:

$$\frac{757}{479} \quad \frac{951}{563}$$

(76 for 757
4.8 for 479)

(95 for 951
5.6 for 563)

$\Rightarrow 76 \times 4.8 < 95 \times 5.6$

$\Rightarrow \frac{757}{479} < \frac{951}{563}$

Method IV: By Formulae

***Theorem (i)**: If $\frac{a}{b}$ and $\frac{a+x}{b+x}$ are two fractional numbers such that*

(a) $a < b$
(b) *a, b and x are positive integers*

then $\frac{a}{b} < \frac{a+x}{b+x}$

Ex. 8: Compare $\frac{76}{89}$ and $\frac{89}{102}$.

Soln: Difference of numerator and numerator equals to the difference of denominator and denominator. Also, numerator is less than denominator. So, as per the theorem (i),

$\frac{76}{89} < \frac{89}{102}$ $\left(\text{ie } \frac{76+13}{89+13}\right)$

Ex. 9: Compare $\frac{576}{783}$ and $\frac{687}{894}$.

Soln: The difference of numerator and numerator and that of denominator and denominator is equal to 111.

Therefore, $\frac{576}{783} < \frac{687}{894}$

Theorem (ii): *If* $\frac{a}{b}$ *and* $\frac{a+x}{b+x}$ *are two fractional numbers such that*

(a) $a > b$
(b) *a, b and x are positive integers*

then $\frac{a}{b} > \frac{a+x}{b+x}$

Ex. 10: Compare $\frac{78}{49}$ and $\frac{91}{62}$.

Soln: Difference of numerator and numerator and difference of denominator and denominator is 13. Also numerator is greater than denominator.

Therefore, $\frac{78}{49} > \frac{91}{62}$

Theorem (iii): *If* $\frac{a}{b}$ *and* $\frac{a+x}{b+y}$ *are two fractional numbers such that*

(a) $a < b$
(b) *a, b, x and y are positive integers*
(c) $x \geq y$

then $\frac{a}{b} < \frac{a+x}{b+y}$

In other words, the difference of numerator and numerator is greater than or equal to the difference of denominator and denominator.

Ex. 11: Compare $\frac{373}{486}$ and $\frac{282}{413}$.

Soln: The difference of 373 and 282 is 91 and the difference of 486 and 413 is 73. Since 91 > 73, ie difference of numerator and numerator is greater than or equal to the difference of denominator and denominator.

$\Rightarrow \frac{373}{486} > \frac{282}{413}$

Theorem (iv): *If* $\frac{a}{b}$ *and* $\frac{a+x}{b+y}$ *are two fractional numbers such that*

(a) $a > b$
(b) *a, b, x and y are positive integers.*
(c) $x \leq y$ *then,*

$$\frac{a}{b} > \frac{a+x}{b+y}$$

Ex. 12: Compare $\frac{579}{392}$ and $\frac{662}{483}$.

Soln: Numerator is greater than denominator. The difference of numerator and numerator (662 – 579 = 83) is less than the difference of denominator and denominator (483 – 392 = 91).

$$\therefore \frac{579}{392} > \frac{662}{483}$$

Theorem (v): If $\frac{a}{b}$ and $\frac{a+x}{b+y}$ are two fractional numbers such that

(a) $a < b$
(b) a, b, x and y are positive integers
(c) $x < y$

then we can't conclude anything without actual comparison.

Theorem (vi): If $\frac{a}{b}$ and $\frac{a+x}{b+y}$ are two fractional numbers such that

(a) $a > b$
(b) a, b, x and y are positive integers
(c) $x > y$

then we can't conclude anything, without actual comparison. It means that any of the following is possible.

(i) $\frac{a}{b} < \frac{a+x}{b+y}$ or (ii) $\frac{a}{b} > \frac{a+x}{b+y}$ or (iii) $\frac{a}{b} = \frac{a+x}{b+y}$