Measuring Modes on Strings: Part 1

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1 Introduction

This report details an investigation into the phenomena observed on a vibrating string. Several experiments took place, using a guitar, as guitars feature strings under tension that can freely vibrate. The frequency and amplitude of vibrations can be easily and precisely adjusted on a well-intonated guitar by fretting the strings along the fingerboard. Results are tabulated and plotted on graphs using MATLAB.

2 Theory

A string vibrates with a fundamental, or in music theory tonic, frequency. This fundamental frequency is related to the length L of a vibrating string:

$$f_1 = \frac{c}{2L} \tag{1}$$

The mass of a string and its length can be used to find a mass per unit length M. This mass per unit length can then be formulated with it's tension T to find the speed of a transverse wave on a string:

$$c = \sqrt{\frac{T}{M}} \tag{2}$$

Equation 2 allows us to formulate:

$$f_1 = \frac{\sqrt{\frac{T}{M}}}{2L} \tag{3}$$

If the ends of a string under tension are fixed, the fundamental frequency will have a wavelength equal to half the string's length. The frequencies of higher harmonics of the fundamental frequency are integer multiples of the fundamental frequency:

$$f_n = \frac{c}{2L}n \tag{4}$$

3 Experiment One

3.1 Methodology

In experiment one, the relationship between resonant frequency and string length were investigated. Preliminary measurements of mass and length were taken in order to find a mass per unit length $M = 0.012618\mathrm{kg\,m^{-1}}$. Then the low E string was played in an open position, thus at it's maximum string length and at a frequency of 83Hz. Using equation 4 the higher harmonics $n = 1, 2, 3...$ are predicted for the vibrating open E string. The sound of the vibrating string was captured using a microphone and passed into a Simulink frequency spectrum analyser. From this spectrum analyser the frequencies of the fundamental tone and it's higher harmonics could be ascertained. However, not every frequency in the harmonic series had a sufficient amplitude to be declared a peak by the Simulink spectrometer, meaning that the recorded harmonic series will have gaps compared to the array of predicted harmonic frequencies. Rearranging equation 4 allowed a wave speed c_n to be found for each harmonic number, for both the predicted and recorded harmonics. To compare the error between the predicted and observed harmonic wave speeds, a mean wave speed was taken for the observed values. The difference between this mean wave speed c and each predicted harmonic wave speed c_n was then taken. These differences were then summed and divided by the number of harmonics to give a mean difference and thus average error:

$$\Delta c = \frac{1}{N}\sum_{n=1}^{N} |c_n - c| \tag{5}$$

The array of predicted wave speeds for harmonic numbers up to harmonic number $n = 6$ were tabluated: