In this particular File the codes and commands are mentioned for following mentioned Components : -

(1) Christoffel Symbols
(2) Riemann Curvature Tensor (Mix Form)
(3) Riemann Curvature Tensor (Covariant Form)
(4) Riemann Curvature Tensor (Contravariant Form)
(5) Ricci Tensor
(6) Einstein Tensor
(7) Weyl Tensor
(8) Geodesic's Equation

In[ ]:=
```
Clear[coord, metric, inversemetric, affine, riemann, lriemann,
  uriemann, ricci, scalar, einstein, weyl, geodesic, R, G, τ, i, j, k, l, s];
Clear[r, θ, ϕ, t, χ, a, m, M]; coord = {t, r, θ, ϕ};
n = Length[coord];
```

$$\text{metric} = \Big\{\Big\{w^2\, r^{(2\gamma-2)}\, a^{(4-2\gamma)}\, e^{\left(\frac{-r}{a}\right)}\, \text{Sin}[\theta]^2\, \text{Sin}[\phi]^2\, \text{Cos}[2\,w\,t],\ 0,\ 0,\ 0\Big\},$$
$$\Big\{0,\ -r^{(2\gamma-4)}\, a^{(4-2\gamma)}\, e^{\left(\frac{-r}{a}\right)}\, \text{Sin}[\theta]^2\, \text{Sin}[\phi]^2 \left(\gamma-1-\frac{r}{2\,a}\right)^2 \text{Cos}[2\,w\,t],\ 0,\ 0\Big\},$$
$$\Big\{0,\ 0,\ -r^{(2\gamma-2)}\, a^{(4-2\gamma)}\, e^{\left(\frac{-r}{a}\right)}\, \text{Cos}[\theta]^2\, \text{Sin}[\phi]^2\, \text{Cos}[2\,w\,t],\ 0\Big\},$$
$$\Big\{0,\ 0,\ 0,\ -r^{(2\gamma-2)}\, a^{(4-2\gamma)}\, e^{\left(\frac{-r}{a}\right)}\, \text{Sin}[\theta]^2\, \text{Cos}[\phi]^2\, \text{Cos}[2\,w\,t]\Big\}\Big\}$$

Out[ ]=

$$\Big\{\Big\{a^{4-2\gamma}\, e^{-\frac{r}{a}}\, r^{-2+2\gamma}\, w^2\, \text{Cos}[2\,t\,w]\, \text{Sin}[\theta]^2\, \text{Sin}[\phi]^2,\ 0,\ 0,\ 0\Big\},$$
$$\Big\{0,\ -a^{4-2\gamma}\, e^{-\frac{r}{a}}\, r^{-4+2\gamma}\left(-1-\frac{r}{2\,a}+\gamma\right)^2 \text{Cos}[2\,t\,w]\, \text{Sin}[\theta]^2\, \text{Sin}[\phi]^2,\ 0,\ 0\Big\},$$
$$\Big\{0,\ 0,\ -a^{4-2\gamma}\, e^{-\frac{r}{a}}\, r^{-2+2\gamma}\, \text{Cos}[2\,t\,w]\, \text{Cos}[\theta]^2\, \text{Sin}[\phi]^2,\ 0\Big\},$$
$$\Big\{0,\ 0,\ 0,\ -a^{4-2\gamma}\, e^{-\frac{r}{a}}\, r^{-2+2\gamma}\, \text{Cos}[2\,t\,w]\, \text{Cos}[\phi]^2\, \text{Sin}[\theta]^2\Big\}\Big\}$$

In[ ]:=
```
inversemetric = Simplify[Inverse[metric]]
```

Out[ ]=

$$\Big\{\Big\{\frac{a^{-4+2\gamma}\, e^{\frac{r}{a}}\, r^{2-2\gamma}\, \text{Csc}[\theta]^2\, \text{Csc}[\phi]^2\, \text{Sec}[2\,t\,w]}{w^2},\ 0,\ 0,\ 0\Big\},$$
$$\Big\{0,\ -\frac{4\,a^{-2+2\gamma}\, e^{\frac{r}{a}}\, r^{4-2\gamma}\, \text{Csc}[\theta]^2\, \text{Csc}[\phi]^2\, \text{Sec}[2\,t\,w]}{(r-2\,a\,(-1+\gamma))^2},\ 0,\ 0\Big\},$$
$$\Big\{0,\ 0,\ -a^{-4+2\gamma}\, e^{\frac{r}{a}}\, r^{2-2\gamma}\, \text{Csc}[\phi]^2\, \text{Sec}[2\,t\,w]\, \text{Sec}[\theta]^2,\ 0\Big\},$$
$$\Big\{0,\ 0,\ 0,\ -a^{-4+2\gamma}\, e^{\frac{r}{a}}\, r^{2-2\gamma}\, \text{Csc}[\theta]^2\, \text{Sec}[2\,t\,w]\, \text{Sec}[\phi]^2\Big\}\Big\}$$

In[ ]:=
```
Print["-----------------------------------------------------------------------"];
Print["The Manifold has dimension   n= ", n, "\nCoordinate system:", coord];
Print["x⁰= t, x¹= r, x²= θ, x³= ϕ"]
Print["-----------------------------------------------------------------------"];
Print["gμν=", metric // MatrixForm];
Print["gμν=", inversemetric // MatrixForm];
Print[
  "g  = - w²r^(8γ-10) a^(16-8γ) e^(-4r/a) (γ-1-r/(2a))² Sin[θ]⁶ Sin[ϕ]⁶ Cos[θ]² Cos[ϕ]² Cos[2wt]⁴"];
Print[" √(-g) = w r^(4γ-5) a^(8-4γ) e^(-2r/a) (γ-1-r/(2a)) Sin[θ]³ Sin[ϕ]³ Cos[θ] Cos[ϕ] Cos[2wt]²"]
```

```
-----------------------------------------------------------------------
The Manifold has dimension   n= 4
Coordinate system:{t, r, θ, ϕ}
x⁰= t, x¹= r, x²= θ, x³= ϕ
-----------------------------------------------------------------------
```

$$g_{\mu\nu}=\begin{pmatrix} a^{4-2\gamma}\, e^{-\frac{r}{a}}\, r^{-2+2\gamma}\, w^2 \operatorname{Cos}[2\,t\,w]\operatorname{Sin}[\theta]^2 \operatorname{Sin}[\phi]^2 & 0 \\ 0 & -a^{4-2\gamma}\, e^{-\frac{r}{a}}\, r^{-4+2\gamma}\left(-1-\frac{r}{2a}+\gamma\right)^2 \operatorname{Cos}[2\,t\,w]\operatorname{Sin}[\theta]^2\operatorname{Sin}[\phi]^2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$g^{\mu\nu}=\begin{pmatrix} \frac{a^{-4+2\gamma}\, e^{\frac{r}{a}}\, r^{2-2\gamma}\operatorname{Csc}[\theta]^2\operatorname{Csc}[\phi]^2\operatorname{Sec}[2\,t\,w]}{w^2} & 0 & 0 \\ 0 & -\frac{4\,a^{-2+2\gamma}\, e^{\frac{r}{a}}\, r^{4-2\gamma}\operatorname{Csc}[\theta]^2\operatorname{Csc}[\phi]^2\operatorname{Sec}[2\,t\,w]}{(r-2a(-1+\gamma))^2} & 0 \\ 0 & 0 & -a^{-4+2\gamma}\, e^{\frac{r}{a}}\, r^{2-2\gamma}\operatorname{Csc}[\phi]^2\operatorname{Sec}[2\,t\,w]\operatorname{Sec} \\ 0 & 0 & 0 \end{pmatrix}$$

$$g = -\, w^2 r^{8\gamma-10}\, a^{16-8\gamma}\, e^{-\frac{4r}{a}}\left(\gamma-1-\frac{r}{2a}\right)^2 \operatorname{Sin}[\theta]^6\operatorname{Sin}[\phi]^6\operatorname{Cos}[\theta]^2\operatorname{Cos}[\phi]^2\operatorname{Cos}[2wt]^4$$

$$\sqrt{-g} = w\, r^{4\gamma-5}\, a^{8-4\gamma}\, e^{-\frac{2r}{a}}\left(\gamma-1-\frac{r}{2a}\right)\operatorname{Sin}[\theta]^3\operatorname{Sin}[\phi]^3\operatorname{Cos}[\theta]\operatorname{Cos}[\phi]\operatorname{Cos}[2wt]^2$$

In[ ]:=
```
affine := affine = FullSimplify[Table[
     (1 / 2) * Sum[(inversemetric[[i, s]]) *
        (D[metric[[s, j]], coord[[k]] ] +
          D[metric[[s, k]], coord[[j]] ] - D[metric[[j, k]], coord[[s]] ]),
       {s, 1, n}], {i, 1, n}, {j, 1, n}, {k, 1, n}] ];
listaffine := Table[ If[UnsameQ[affine[[i, j, k]], 0],
     {Subscript[Superscript[Γ, i - 1], j - 1, k - 1], affine[[i, j, k]]} ] ,
    {i, 1, n}, {j, 1, n}, {k, 1, j}];
Print["Christoffel Symbols:"];
Print[
 TableForm[Partition[DeleteCases[Flatten[listaffine], Null], 2], TableSpacing → {3, 5}]]
```

```
Table[Table[If[affine[[i, j, k]] =!= 0, {Γ^(i-1)_(j-1,k-1), affine[[i, j, k]]}], {i, 1, n}],
 Table[{j, 1, n}, {k, 1, j}]]
```

Christoffel Symbols:

$\Gamma^{0}{}_{0,0}$	$-w\,\mathrm{Tan}[2\,t\,w]$
$\Gamma^{0}{}_{1,0}$	$-\frac{2a+r-2a\gamma}{2ar}$
$\Gamma^{0}{}_{1,1}$	$-\frac{\left(-1-\frac{r}{2a}+\gamma\right)^2\mathrm{Tan}[2\,t\,w]}{r^2\,w}$
$\Gamma^{0}{}_{2,0}$	$\mathrm{Cot}[\theta]$
$\Gamma^{0}{}_{2,2}$	$-\frac{\mathrm{Cot}[\theta]^2\,\mathrm{Tan}[2\,t\,w]}{w}$
$\Gamma^{0}{}_{3,0}$	$\mathrm{Cot}[\phi]$
$\Gamma^{0}{}_{3,3}$	$-\frac{\mathrm{Cot}[\phi]^2\,\mathrm{Tan}[2\,t\,w]}{w}$
$\Gamma^{1}{}_{0,0}$	$-\frac{2\,a\,r\,w^2}{r-2a(-1+\gamma)}$
$\Gamma^{1}{}_{1,0}$	$-w\,\mathrm{Tan}[2\,t\,w]$
$\Gamma^{1}{}_{1,1}$	$-\frac{1}{2a}+\frac{-2+\gamma}{r}+\frac{1}{2a+r-2a\gamma}$
$\Gamma^{1}{}_{2,1}$	$\mathrm{Cot}[\theta]$
$\Gamma^{1}{}_{2,2}$	$\frac{2\,a\,r\,\mathrm{Cot}[\theta]^2}{r-2a(-1+\gamma)}$
$\Gamma^{1}{}_{3,1}$	$\mathrm{Cot}[\phi]$
$\Gamma^{1}{}_{3,3}$	$\frac{2\,a\,r\,\mathrm{Cot}[\phi]^2}{r-2a(-1+\gamma)}$
$\Gamma^{2}{}_{0,0}$	$w^2\,\mathrm{Tan}[\theta]$
$\Gamma^{2}{}_{1,1}$	$-\frac{\left(-1-\frac{r}{2a}+\gamma\right)^2\mathrm{Tan}[\theta]}{r^2}$
$\Gamma^{2}{}_{2,0}$	$-w\,\mathrm{Tan}[2\,t\,w]$
$\Gamma^{2}{}_{2,1}$	$-\frac{2a+r-2a\gamma}{2ar}$
$\Gamma^{2}{}_{2,2}$	$-\mathrm{Tan}[\theta]$
$\Gamma^{2}{}_{3,2}$	$\mathrm{Cot}[\phi]$
$\Gamma^{2}{}_{3,3}$	$-\mathrm{Cot}[\phi]^2\,\mathrm{Tan}[\theta]$
$\Gamma^{3}{}_{0,0}$	$w^2\,\mathrm{Tan}[\phi]$
$\Gamma^{3}{}_{1,1}$	$-\frac{\left(-1-\frac{r}{2a}+\gamma\right)^2\mathrm{Tan}[\phi]}{r^2}$
$\Gamma^{3}{}_{2,2}$	$-\mathrm{Cot}[\theta]^2\,\mathrm{Tan}[\phi]$
$\Gamma^{3}{}_{3,0}$	$-w\,\mathrm{Tan}[2\,t\,w]$
$\Gamma^{3}{}_{3,1}$	$-\frac{2a+r-2a\gamma}{2ar}$
$\Gamma^{3}{}_{3,2}$	$\mathrm{Cot}[\theta]$
$\Gamma^{3}{}_{3,3}$	$-\mathrm{Tan}[\phi]$

```
In[ ]:= riemann :=
  riemann = Simplify[Table[ D[affine[[i, l, j]], coord[[k]] ] - D[affine[[i, k, j]], coord[[l]]] +
      Sum[affine[[i, k, s]] × affine[[s, l, j]] - affine[[i, l, s]] × affine[[s, k, j]] ,
       {s, 1, n}], {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}] ];
listriemann := Table[If[UnsameQ[riemann[[i, j, k, l]], 0],
    {Subscript[Superscript[R, i - 1], j - 1, k - 1, l - 1], riemann[[i, j, k, l]]}] ,
   {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, k - 1}];
Print["-----------------------------------------------------------------------"];
Print["Riemann Tensor Mix Form:"];
Print[
 TableForm[Partition[DeleteCases[Flatten[listriemann], Null], 2], TableSpacing → {4, 4}]]
```

Riemann Tensor Mix Form:

$R^{0}{}_{1,1,0}$	$\frac{(r-2 a (-1+\gamma))^2 \left(1+\mathrm{Sec}[2 t w]^2\right)}{2 a^2 r^2}$
$R^{0}{}_{1,2,0}$	$\frac{(r-2 a (-1+\gamma)) \mathrm{Cot}[\theta]}{2 a r}$
$R^{0}{}_{1,2,1}$	$\frac{\left(-1-\frac{r}{2 a}+\gamma\right)^2 \mathrm{Cot}[\theta] \mathrm{Tan}[2 t w]}{r^2 w}$
$R^{0}{}_{1,3,0}$	$\frac{(r-2 a (-1+\gamma)) \mathrm{Cot}[\phi]}{2 a r}$
$R^{0}{}_{1,3,1}$	$\frac{\left(-1-\frac{r}{2 a}+\gamma\right)^2 \mathrm{Cot}[\phi] \mathrm{Tan}[2 t w]}{r^2 w}$
$R^{0}{}_{2,1,0}$	$\frac{(r-2 a (-1+\gamma)) \mathrm{Cot}[\theta]}{2 a r}$
$R^{0}{}_{2,2,0}$	$(3+\mathrm{Cos}[4 t w]) \mathrm{Cot}[\theta]^2 \mathrm{Sec}[2 t w]^2$
$R^{0}{}_{2,2,1}$	$\frac{(r-2 a (-1+\gamma)) \mathrm{Cot}[\theta]^2 \mathrm{Tan}[2 t w]}{2 a r w}$
$R^{0}{}_{2,3,0}$	$-\mathrm{Cot}[\theta] \mathrm{Cot}[\phi]$
$R^{0}{}_{2,3,2}$	$\frac{\mathrm{Cot}[\theta]^2 \mathrm{Cot}[\phi] \mathrm{Tan}[2 t w]}{w}$
$R^{0}{}_{3,1,0}$	$\frac{(r-2 a (-1+\gamma)) \mathrm{Cot}[\phi]}{2 a r}$
$R^{0}{}_{3,2,0}$	$-\mathrm{Cot}[\theta] \mathrm{Cot}[\phi]$
$R^{0}{}_{3,3,0}$	$(3+\mathrm{Cos}[4 t w]) \mathrm{Cot}[\phi]^2 \mathrm{Sec}[2 t w]^2$
$R^{0}{}_{3,3,1}$	$\frac{(r-2 a (-1+\gamma)) \mathrm{Cot}[\phi]^2 \mathrm{Tan}[2 t w]}{2 a r w}$
$R^{0}{}_{3,3,2}$	$-\frac{\mathrm{Cot}[\theta] \mathrm{Cot}[\phi]^2 \mathrm{Tan}[2 t w]}{w}$
$R^{1}{}_{0,1,0}$	$2 w^2 \left(1+\mathrm{Sec}[2 t w]^2\right)$

$R^1{}_{0,2,0}$	$\frac{2\,a\,r\,w^2\,\mathrm{Cot}[\theta]}{r-2\,a\,(-1+\gamma)}$
$R^1{}_{0,2,1}$	$w\,\mathrm{Cot}[\theta]\,\mathrm{Tan}[2\,t\,w]$
$R^1{}_{0,3,0}$	$\frac{2\,a\,r\,w^2\,\mathrm{Cot}[\phi]}{r-2\,a\,(-1+\gamma)}$
$R^1{}_{0,3,1}$	$w\,\mathrm{Cot}[\phi]\,\mathrm{Tan}[2\,t\,w]$
$R^1{}_{2,1,0}$	$-w\,\mathrm{Cot}[\theta]\,\mathrm{Tan}[2\,t\,w]$
$R^1{}_{2,2,0}$	$-\frac{2\,a\,r\,w\,\mathrm{Cot}[\theta]^2\,\mathrm{Tan}[2\,t\,w]}{r-2\,a\,(-1+\gamma)}$
$R^1{}_{2,2,1}$	$\mathrm{Cos}[4\,t\,w]\,\mathrm{Cot}[\theta]^2\,\mathrm{Sec}[2\,t\,w]^2$
$R^1{}_{2,3,1}$	$-\mathrm{Cot}[\theta]\,\mathrm{Cot}[\phi]$
$R^1{}_{2,3,2}$	$-\frac{2\,a\,r\,\mathrm{Cot}[\theta]^2\,\mathrm{Cot}[\phi]}{r-2\,a\,(-1+\gamma)}$
$R^1{}_{3,1,0}$	$-w\,\mathrm{Cot}[\phi]\,\mathrm{Tan}[2\,t\,w]$
$R^1{}_{3,2,1}$	$-\mathrm{Cot}[\theta]\,\mathrm{Cot}[\phi]$
$R^1{}_{3,3,0}$	$-\frac{2\,a\,r\,w\,\mathrm{Cot}[\phi]^2\,\mathrm{Tan}[2\,t\,w]}{r-2\,a\,(-1+\gamma)}$
$R^1{}_{3,3,1}$	$\mathrm{Cos}[4\,t\,w]\,\mathrm{Cot}[\phi]^2\,\mathrm{Sec}[2\,t\,w]^2$
$R^1{}_{3,3,2}$	$\frac{2\,a\,r\,\mathrm{Cot}[\theta]\,\mathrm{Cot}[\phi]^2}{r-2\,a\,(-1+\gamma)}$
$R^2{}_{0,1,0}$	$\frac{w^2\,(r-2\,a\,(-1+\gamma))\,\mathrm{Tan}[\theta]}{2\,a\,r}$
$R^2{}_{0,2,0}$	$w^2\,(3+\mathrm{Cos}[4\,t\,w])\,\mathrm{Sec}[2\,t\,w]^2$
$R^2{}_{0,2,1}$	$\frac{w\,(r-2\,a\,(-1+\gamma))\,\mathrm{Tan}[2\,t\,w]}{2\,a\,r}$
$R^2{}_{0,3,0}$	$-w^2\,\mathrm{Cot}[\phi]\,\mathrm{Tan}[\theta]$
$R^2{}_{0,3,2}$	$w\,\mathrm{Cot}[\phi]\,\mathrm{Tan}[2\,t\,w]$
$R^2{}_{1,1,0}$	$\frac{w\,\left(-1-\frac{r}{2\,a}+\gamma\right)^2\,\mathrm{Tan}[2\,t\,w]\,\mathrm{Tan}[\theta]}{r^2}$
$R^2{}_{1,2,0}$	$\frac{w\,(r-2\,a\,(-1+\gamma))\,\mathrm{Tan}[2\,t\,w]}{2\,a\,r}$
$R^2{}_{1,2,1}$	$-\frac{(r-2\,a\,(-1+\gamma))^2\,\mathrm{Cos}[4\,t\,w]\,\mathrm{Sec}[2\,t\,w]^2}{4\,a^2\,r^2}$
$R^2{}_{1,3,1}$	$\frac{\left(-1-\frac{r}{2\,a}+\gamma\right)^2\,\mathrm{Cot}[\phi]\,\mathrm{Tan}[\theta]}{r^2}$
$R^2{}_{1,3,2}$	$\frac{(r-2\,a\,(-1+\gamma))\,\mathrm{Cot}[\phi]}{2\,a\,r}$

$R^2{}_{3,2,0}$	$-w \,\mathrm{Cot}[\phi]\, \mathrm{Tan}[2\,t\,w]$
$R^2{}_{3,2,1}$	$-\frac{\mathrm{Cot}[\phi]}{2\,a}+\frac{(-1+\gamma)\,\mathrm{Cot}[\phi]}{r}$
$R^2{}_{3,3,0}$	$w\,\mathrm{Cot}[\phi]^2\,\mathrm{Tan}[2\,t\,w]\,\mathrm{Tan}[\theta]$
$R^2{}_{3,3,1}$	$\frac{(r-2\,a\,(-1+\gamma))\,\mathrm{Cot}[\phi]^2\,\mathrm{Tan}[\theta]}{2\,a\,r}$
$R^2{}_{3,3,2}$	$\mathrm{Cos}[4\,t\,w]\,\mathrm{Cot}[\phi]^2\,\mathrm{Sec}[2\,t\,w]^2$
$R^3{}_{0,1,0}$	$\frac{w^2\,(r-2\,a\,(-1+\gamma))\,\mathrm{Tan}[\phi]}{2\,a\,r}$
$R^3{}_{0,2,0}$	$-w^2\,\mathrm{Cot}[\theta]\,\mathrm{Tan}[\phi]$
$R^3{}_{0,3,0}$	$w^2\,(3+\mathrm{Cos}[4\,t\,w])\,\mathrm{Sec}[2\,t\,w]^2$
$R^3{}_{0,3,1}$	$\frac{w\,(r-2\,a\,(-1+\gamma))\,\mathrm{Tan}[2\,t\,w]}{2\,a\,r}$
$R^3{}_{0,3,2}$	$-w\,\mathrm{Cot}[\theta]\,\mathrm{Tan}[2\,t\,w]$
$R^3{}_{1,1,0}$	$\frac{w\,\left(-1-\frac{r}{2\,a}+\gamma\right)^2\,\mathrm{Tan}[2\,t\,w]\,\mathrm{Tan}[\phi]}{r^2}$
$R^3{}_{1,2,1}$	$\frac{\left(-1-\frac{r}{2\,a}+\gamma\right)^2\,\mathrm{Cot}[\theta]\,\mathrm{Tan}[\phi]}{r^2}$
$R^3{}_{1,3,0}$	$\frac{w\,(r-2\,a\,(-1+\gamma))\,\mathrm{Tan}[2\,t\,w]}{2\,a\,r}$
$R^3{}_{1,3,1}$	$-\frac{(r-2\,a\,(-1+\gamma))^2\,\mathrm{Cos}[4\,t\,w]\,\mathrm{Sec}[2\,t\,w]^2}{4\,a^2\,r^2}$
$R^3{}_{1,3,2}$	$-\frac{\mathrm{Cot}[\theta]}{2\,a}+\frac{(-1+\gamma)\,\mathrm{Cot}[\theta]}{r}$
$R^3{}_{2,2,0}$	$w\,\mathrm{Cot}[\theta]^2\,\mathrm{Tan}[2\,t\,w]\,\mathrm{Tan}[\phi]$
$R^3{}_{2,2,1}$	$\frac{(r-2\,a\,(-1+\gamma))\,\mathrm{Cot}[\theta]^2\,\mathrm{Tan}[\phi]}{2\,a\,r}$
$R^3{}_{2,3,0}$	$-w\,\mathrm{Cot}[\theta]\,\mathrm{Tan}[2\,t\,w]$
$R^3{}_{2,3,1}$	$-\frac{\mathrm{Cot}[\theta]}{2\,a}+\frac{(-1+\gamma)\,\mathrm{Cot}[\theta]}{r}$
$R^3{}_{2,3,2}$	$-\mathrm{Cos}[4\,t\,w]\,\mathrm{Cot}[\theta]^2\,\mathrm{Sec}[2\,t\,w]^2$

```
In[•]:= lriemann := lriemann = Simplify[Table[
      Sum[metric⟦i, ii⟧ × riemann⟦ii, j, k, l⟧, {ii, 1, n}],
      {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
  listlriemann := Table[If[UnsameQ[lriemann⟦i, j, k, l⟧, 0],
      {Subscript[R, i - 1, j - 1, k - 1, l - 1], lriemann⟦i, j, k, l⟧}] ,
     {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
  Print["-----------------------------------------------------------------------"];
  Print["Covariant Riemann Tensor:"];
  Print[
   TableForm[Partition[DeleteCases[Flatten[listlriemann], Null], 2], TableSpacing → {2, 2}]]
```

Covariant Riemann Tensor:

$R_{1,0,1,0}$	$-\frac{1}{4}a^{2-2\gamma}e^{-\frac{r}{a}}r^{-4+2\gamma}w^2(r-2a(-1+\gamma))^2(3+\mathrm{Cos}[4tw])\mathrm{Sec}[2tw]\mathrm{Sin}[\theta]^2\mathrm{Sin}[\phi]^2$
$R_{1,0,2,0}$	$\frac{1}{2}a^{3-2\gamma}e^{-\frac{r}{a}}r^{-3+2\gamma}w^2(-r+2a(-1+\gamma))\mathrm{Cos}[2tw]\mathrm{Cos}[\theta]\mathrm{Sin}[\theta]\mathrm{Sin}[\phi]^2$
$R_{1,0,2,1}$	$-\frac{1}{4}a^{2-2\gamma}e^{-\frac{r}{a}}r^{-4+2\gamma}w(r-2a(-1+\gamma))^2\mathrm{Cos}[\theta]\mathrm{Sin}[2tw]\mathrm{Sin}[\theta]\mathrm{Sin}[\phi]^2$
$R_{1,0,3,0}$	$\frac{1}{2}a^{3-2\gamma}e^{-\frac{r}{a}}r^{-3+2\gamma}w^2(-r+2a(-1+\gamma))\mathrm{Cos}[2tw]\mathrm{Cos}[\phi]\mathrm{Sin}[\theta]^2\mathrm{Sin}[\phi]$
$R_{1,0,3,1}$	$-\frac{1}{4}a^{2-2\gamma}e^{-\frac{r}{a}}r^{-4+2\gamma}w(r-2a(-1+\gamma))^2\mathrm{Cos}[\phi]\mathrm{Sin}[2tw]\mathrm{Sin}[\theta]^2\mathrm{Sin}[\phi]$
$R_{2,0,1,0}$	$-\frac{1}{2}a^{3-2\gamma}e^{-\frac{r}{a}}r^{-3+2\gamma}w^2(r-2a(-1+\gamma))\mathrm{Cos}[2tw]\mathrm{Cos}[\theta]\mathrm{Sin}[\theta]\mathrm{Sin}[\phi]^2$
$R_{2,0,2,0}$	$-a^{4-2\gamma}e^{-\frac{r}{a}}r^{-2+2\gamma}w^2(3+\mathrm{Cos}[4tw])\mathrm{Cos}[\theta]^2\mathrm{Sec}[2tw]\mathrm{Sin}[\phi]^2$
$R_{2,0,2,1}$	$-\frac{1}{2}a^{3-2\gamma}e^{-\frac{r}{a}}r^{-3+2\gamma}w(r-2a(-1+\gamma))\mathrm{Cos}[\theta]^2\mathrm{Sin}[2tw]\mathrm{Sin}[\phi]^2$
$R_{2,0,3,0}$	$a^{4-2\gamma}e^{-\frac{r}{a}}r^{-2+2\gamma}w^2\mathrm{Cos}[2tw]\mathrm{Cos}[\theta]\mathrm{Cos}[\phi]\mathrm{Sin}[\theta]\mathrm{Sin}[\phi]$
$R_{2,0,3,2}$	$-a^{4-2\gamma}e^{-\frac{r}{a}}r^{-2+2\gamma}w\,\mathrm{Cos}[\theta]^2\mathrm{Cos}[\phi]\mathrm{Sin}[2tw]\mathrm{Sin}[\phi]$
$R_{2,1,1,0}$	$-\frac{1}{4}a^{2-2\gamma}e^{-\frac{r}{a}}r^{-4+2\gamma}w(r-2a(-1+\gamma))^2\mathrm{Cos}[\theta]\mathrm{Sin}[2tw]\mathrm{Sin}[\theta]\mathrm{Sin}[\phi]^2$
$R_{2,1,2,0}$	$-\frac{1}{2}a^{3-2\gamma}e^{-\frac{r}{a}}r^{-3+2\gamma}w(r-2a(-1+\gamma))\mathrm{Cos}[\theta]^2\mathrm{Sin}[2tw]\mathrm{Sin}[\phi]^2$
$R_{2,1,2,1}$	$\frac{1}{4}a^{2-2\gamma}e^{-\frac{r}{a}}r^{-4+2\gamma}(r-2a(-1+\gamma))^2\mathrm{Cos}[4tw]\mathrm{Cos}[\theta]^2\mathrm{Sec}[2tw]\mathrm{Sin}[\phi]^2$
$R_{2,1,3,1}$	$-\frac{1}{4}a^{2-2\gamma}e^{-\frac{r}{a}}r^{-4+2\gamma}(r-2a(-1+\gamma))^2\mathrm{Cos}[2tw]\mathrm{Cos}[\theta]\mathrm{Cos}[\phi]\mathrm{Sin}[\theta]\mathrm{Sin}[\phi]$
$R_{2,1,3,2}$	$-\frac{1}{2}a^{3-2\gamma}e^{-\frac{r}{a}}r^{-3+2\gamma}(r-2a(-1+\gamma))\mathrm{Cos}[2tw]\mathrm{Cos}[\theta]^2\mathrm{Cos}[\phi]\mathrm{Sin}[\phi]$
$R_{3,0,1,0}$	$-\frac{1}{2}a^{3-2\gamma}e^{-\frac{r}{a}}r^{-3+2\gamma}w^2(r-2a(-1+\gamma))\mathrm{Cos}[2tw]\mathrm{Cos}[\phi]\mathrm{Sin}[\theta]^2\mathrm{Sin}[\phi]$
$R_{3,0,2,0}$	$a^{4-2\gamma}e^{-\frac{r}{a}}r^{-2+2\gamma}w^2\mathrm{Cos}[2tw]\mathrm{Cos}[\theta]\mathrm{Cos}[\phi]\mathrm{Sin}[\theta]\mathrm{Sin}[\phi]$
$R_{3,0,3,0}$	$-a^{4-2\gamma}e^{-\frac{r}{a}}r^{-2+2\gamma}w^2(3+\mathrm{Cos}[4tw])\mathrm{Cos}[\phi]^2\mathrm{Sec}[2tw]\mathrm{Sin}[\theta]^2$
$R_{3,0,3,1}$	$-\frac{1}{2}a^{3-2\gamma}e^{-\frac{r}{a}}r^{-3+2\gamma}w(r-2a(-1+\gamma))\mathrm{Cos}[\phi]^2\mathrm{Sin}[2tw]\mathrm{Sin}[\theta]^2$
$R_{3,0,3,2}$	$a^{4-2\gamma}e^{-\frac{r}{a}}r^{-2+2\gamma}w\,\mathrm{Cos}[\theta]\mathrm{Cos}[\phi]^2\mathrm{Sin}[2tw]\mathrm{Sin}[\theta]$
$R_{3,1,1,0}$	$-\frac{1}{4}a^{2-2\gamma}e^{-\frac{r}{a}}r^{-4+2\gamma}w(r-2a(-1+\gamma))^2\mathrm{Cos}[\phi]\mathrm{Sin}[2tw]\mathrm{Sin}[\theta]^2\mathrm{Sin}[\phi]$

$$R_{3,1,2,1} \quad -\frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} (r-2a(-1+\gamma))^2 \,\mathrm{Cos}[2tw]\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\phi]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi]$$

$$R_{3,1,3,0} \quad -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} w (r-2a(-1+\gamma))\,\mathrm{Cos}[\phi]^2\,\mathrm{Sin}[2tw]\,\mathrm{Sin}[\theta]^2$$

$$R_{3,1,3,1} \quad \frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} (r-2a(-1+\gamma))^2\,\mathrm{Cos}[4tw]\,\mathrm{Cos}[\phi]^2\,\mathrm{Sec}[2tw]\,\mathrm{Sin}[\theta]^2$$

$$R_{3,1,3,2} \quad -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} (-r+2a(-1+\gamma))\,\mathrm{Cos}[2tw]\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\phi]^2\,\mathrm{Sin}[\theta]$$

$$R_{3,2,2,0} \quad -a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} w\,\mathrm{Cos}[\theta]^2\,\mathrm{Cos}[\phi]\,\mathrm{Sin}[2tw]\,\mathrm{Sin}[\phi]$$

$$R_{3,2,2,1} \quad -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} (r-2a(-1+\gamma))\,\mathrm{Cos}[2tw]\,\mathrm{Cos}[\theta]^2\,\mathrm{Cos}[\phi]\,\mathrm{Sin}[\phi]$$

$$R_{3,2,3,0} \quad a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} w\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\phi]^2\,\mathrm{Sin}[2tw]\,\mathrm{Sin}[\theta]$$

$$R_{3,2,3,1} \quad -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} (-r+2a(-1+\gamma))\,\mathrm{Cos}[2tw]\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\phi]^2\,\mathrm{Sin}[\theta]$$

$$R_{3,2,3,2} \quad a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma}\,\mathrm{Cos}[4tw]\,\mathrm{Cos}[\theta]^2\,\mathrm{Cos}[\phi]^2\,\mathrm{Sec}[2tw]$$

In[∘]:=
```
uriemann := uriemann = FullSimplify[Table[Sum[inversemetric[[j, jj]] ×
        inversemetric[[k, kk]] × inversemetric[[l, ll]] × riemann[[i, jj, kk, ll]],
       {jj, 1, n}, {kk, 1, n}, {ll, 1, n}
      ], {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listuriemann := Table[If[UnsameQ[uriemann[[i, j, k, l]], 0],
     {Superscript[Superscript[Superscript[Superscript[R, i - 1], j - 1], k - 1], l - 1],
      uriemann[[i, j, k, l]]}
    ] , {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["----------------------------------------------------------------------"];
Print["Contravariant Riemann Tensor:"];
Print[
 TableForm[Partition[DeleteCases[Flatten[listuriemann], Null], 2], TableSpacing → {2, 2}]]
```

Contravariant Riemann Tensor:

$$R^{1010} \quad -\frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} (3+\mathrm{Cos}[4tw])\,\mathrm{Csc}[\theta]^6\,\mathrm{Csc}[\phi]^6\,\mathrm{Sec}[2tw]^5}{w^2 (r-2a(-1+\gamma))^2}$$

$$R^{1020} \quad -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma}\,\mathrm{Csc}[\theta]^5\,\mathrm{Csc}[\phi]^6\,\mathrm{Sec}[2tw]^3\,\mathrm{Sec}[\theta]}{w^2 (r-2a(-1+\gamma))}$$

$$R^{1021} \quad \frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma}\,\mathrm{Csc}[\theta]^5\,\mathrm{Csc}[\phi]^6\,\mathrm{Sec}[2tw]^3\,\mathrm{Sec}[\theta]\,\mathrm{Tan}[2tw]}{w (r-2a(-1+\gamma))^2}$$

$$R^{1030} \quad -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma}\,\mathrm{Csc}[\theta]^6\,\mathrm{Csc}[\phi]^5\,\mathrm{Sec}[2tw]^3\,\mathrm{Sec}[\phi]}{w^2 (r-2a(-1+\gamma))}$$

$$R^{1031} \quad \frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma}\,\mathrm{Csc}[\theta]^6\,\mathrm{Csc}[\phi]^5\,\mathrm{Sec}[2tw]^3\,\mathrm{Sec}[\phi]\,\mathrm{Tan}[2tw]}{w (r-2a(-1+\gamma))^2}$$

$$R^{2010} \quad -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma}\,\mathrm{Csc}[\theta]^5\,\mathrm{Csc}[\phi]^6\,\mathrm{Sec}[2tw]^3\,\mathrm{Sec}[\theta]}{w^2 (r-2a(-1+\gamma))}$$

$$R^{2020} \quad -\frac{a^{6(-2+\gamma)} e^{\frac{3r}{a}} r^{6-6\gamma} (3+\mathrm{Cos}[4tw])\,\mathrm{Csc}[\theta]^4\,\mathrm{Csc}[\phi]^6\,\mathrm{Sec}[2tw]^5\,\mathrm{Sec}[\theta]^2}{w^2}$$

$$R^{2021} \quad \frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma}\,\mathrm{Csc}[\theta]^4\,\mathrm{Csc}[\phi]^6\,\mathrm{Sec}[2tw]^3\,\mathrm{Sec}[\theta]^2\,\mathrm{Tan}[2tw]}{w (r-2a(-1+\gamma))}$$

$$R^{2030} \quad \frac{a^{6(-2+\gamma)} e^{\frac{3r}{a}} r^{6-6\gamma}\,\mathrm{Csc}[\theta]^5\,\mathrm{Csc}[\phi]^5\,\mathrm{Sec}[2tw]^3\,\mathrm{Sec}[\theta]\,\mathrm{Sec}[\phi]}{w^2}$$

R^{2032} $\frac{a^{6(-2+\gamma)} e^{\frac{3r}{a}} r^{6-6\gamma} \text{Csc}[\theta]^4 \text{Csc}[\phi]^5 \text{Sec}[2tw]^3 \text{Sec}[\theta]^2 \text{Sec}[\phi] \text{Tan}[2tw]}{w}$

R^{2110} $\frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \text{Csc}[\theta]^5 \text{Csc}[\phi]^6 \text{Sec}[2tw]^3 \text{Sec}[\theta] \text{Tan}[2tw]}{w(r-2a(-1+\gamma))^2}$

R^{2120} $\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \text{Csc}[\theta]^4 \text{Csc}[\phi]^6 \text{Sec}[2tw]^3 \text{Sec}[\theta]^2 \text{Tan}[2tw]}{w(r-2a(-1+\gamma))}$

R^{2121} $\frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \text{Cos}[4tw] \text{Csc}[\theta]^4 \text{Csc}[\phi]^6 \text{Sec}[2tw]^5 \text{Sec}[\theta]^2}{(r-2a(-1+\gamma))^2}$

R^{2131} $-\frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \text{Csc}[\theta]^5 \text{Csc}[\phi]^5 \text{Sec}[2tw]^3 \text{Sec}[\theta] \text{Sec}[\phi]}{(r-2a(-1+\gamma))^2}$

R^{2132} $-\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \text{Csc}[\theta]^4 \text{Csc}[\phi]^5 \text{Sec}[2tw]^3 \text{Sec}[\theta]^2 \text{Sec}[\phi]}{r-2a(-1+\gamma)}$

R^{3010} $-\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \text{Csc}[\theta]^6 \text{Csc}[\phi]^5 \text{Sec}[2tw]^3 \text{Sec}[\phi]}{w^2(r-2a(-1+\gamma))}$

R^{3020} $\frac{a^{6(-2+\gamma)} e^{\frac{3r}{a}} r^{6-6\gamma} \text{Csc}[\theta]^5 \text{Csc}[\phi]^5 \text{Sec}[2tw]^3 \text{Sec}[\theta] \text{Sec}[\phi]}{w^2}$

R^{3030} $-\frac{a^{6(-2+\gamma)} e^{\frac{3r}{a}} r^{6-6\gamma} (3+\text{Cos}[4tw]) \text{Csc}[\theta]^6 \text{Csc}[\phi]^4 \text{Sec}[2tw]^5 \text{Sec}[\phi]^2}{w^2}$

R^{3031} $\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \text{Csc}[\theta]^6 \text{Csc}[\phi]^4 \text{Sec}[2tw]^3 \text{Sec}[\phi]^2 \text{Tan}[2tw]}{w(r-2a(-1+\gamma))}$

R^{3032} $-\frac{a^{6(-2+\gamma)} e^{\frac{3r}{a}} r^{6-6\gamma} \text{Csc}[\theta]^5 \text{Csc}[\phi]^4 \text{Sec}[2tw]^3 \text{Sec}[\theta] \text{Sec}[\phi]^2 \text{Tan}[2tw]}{w}$

R^{3110} $\frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \text{Csc}[\theta]^6 \text{Csc}[\phi]^5 \text{Sec}[2tw]^3 \text{Sec}[\phi] \text{Tan}[2tw]}{w(r-2a(-1+\gamma))^2}$

R^{3121} $-\frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \text{Csc}[\theta]^5 \text{Csc}[\phi]^5 \text{Sec}[2tw]^3 \text{Sec}[\theta] \text{Sec}[\phi]}{(r-2a(-1+\gamma))^2}$

R^{3130} $\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \text{Csc}[\theta]^6 \text{Csc}[\phi]^4 \text{Sec}[2tw]^3 \text{Sec}[\phi]^2 \text{Tan}[2tw]}{w(r-2a(-1+\gamma))}$

R^{3131} $\frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \text{Cos}[4tw] \text{Csc}[\theta]^6 \text{Csc}[\phi]^4 \text{Sec}[2tw]^5 \text{Sec}[\phi]^2}{(r-2a(-1+\gamma))^2}$

R^{3132} $-\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \text{Csc}[\theta]^5 \text{Csc}[\phi]^4 \text{Sec}[2tw]^3 \text{Sec}[\theta] \text{Sec}[\phi]^2}{-r+2a(-1+\gamma)}$

R^{3220} $\frac{a^{6(-2+\gamma)} e^{\frac{3r}{a}} r^{6-6\gamma} \text{Csc}[\theta]^4 \text{Csc}[\phi]^5 \text{Sec}[2tw]^3 \text{Sec}[\theta]^2 \text{Sec}[\phi] \text{Tan}[2tw]}{w}$

R^{3221} $-\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \text{Csc}[\theta]^4 \text{Csc}[\phi]^5 \text{Sec}[2tw]^3 \text{Sec}[\theta]^2 \text{Sec}[\phi]}{r-2a(-1+\gamma)}$

R^{3230} $-\frac{a^{6(-2+\gamma)} e^{\frac{3r}{a}} r^{6-6\gamma} \text{Csc}[\theta]^5 \text{Csc}[\phi]^4 \text{Sec}[2tw]^3 \text{Sec}[\theta] \text{Sec}[\phi]^2 \text{Tan}[2tw]}{w}$

R^{3231} $-\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \text{Csc}[\theta]^5 \text{Csc}[\phi]^4 \text{Sec}[2tw]^3 \text{Sec}[\theta] \text{Sec}[\phi]^2}{-r+2a(-1+\gamma)}$

R^{3232} $a^{6(-2+\gamma)} e^{\frac{3r}{a}} r^{6-6\gamma} \text{Cos}[4tw] \text{Csc}[\theta]^4 \text{Csc}[\phi]^4 \text{Sec}[2tw]^5 \text{Sec}[\theta]^2 \text{Sec}[\phi]^2$

```
In[ ]:= r2 = FullSimplify[Sum[lriemann[[i, j, k, l]] × uriemann[[i, j, k, l]],
          {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
        Print["----------------------------------------------------------------------"];
        Print["R²= ", r2]
```

```
---------------------------------------------------------------------
```

$R^2 = 12\, a^{-8+4\gamma}\, e^{\frac{2r}{a}}\, r^{4-4\gamma}\, (11 + 8\, \text{Cos}[4\, t\, w] + 2\, \text{Cos}[8\, t\, w])\, \text{Csc}[\theta]^4\, \text{Csc}[\phi]^4\, \text{Sec}[2\, t\, w]^6$

In[◦]:=
```
ricci :=
  ricci = FullSimplify[Table[Sum[riemann[[i, j, i, l]], {i, 1, n}], {j, 1, n}, {l, 1, n}] ];
listricci := Table[If[UnsameQ[ricci[[j, l]], 0],
    {Subscript[R, j - 1, l - 1], ricci[[j, l]]}] , {j, 1, n}, {l, 1, j}];
Print["---------------------------------------------------------------------"];
Print["Ricci Tensor:"];
Print[
 TableForm[Partition[DeleteCases[Flatten[listricci], Null], 2], TableSpacing → {4, 4}]]
```

```
---------------------------------------------------------------------
```

Ricci Tensor:

$R_{0,0}$	$6\, w^2 \left(1 + \text{Sec}[2\, t\, w]^2\right)$
$R_{1,0}$	$\frac{w\, (r - 2\, a\, (-1+\gamma))\, \text{Tan}[2\, t\, w]}{a\, r}$
$R_{1,1}$	$-\frac{3\, (r - 2\, a\, (-1+\gamma))^2}{2\, a^2\, r^2}$
$R_{2,0}$	$-2\, w\, \text{Cot}[\theta]\, \text{Tan}[2\, t\, w]$
$R_{2,1}$	$\left(-\frac{1}{a} + \frac{2\, (-1+\gamma)}{r}\right) \text{Cot}[\theta]$
$R_{2,2}$	$-6\, \text{Cot}[\theta]^2$
$R_{3,0}$	$-2\, w\, \text{Cot}[\phi]\, \text{Tan}[2\, t\, w]$
$R_{3,1}$	$\left(-\frac{1}{a} + \frac{2\, (-1+\gamma)}{r}\right) \text{Cot}[\phi]$
$R_{3,2}$	$2\, \text{Cot}[\theta]\, \text{Cot}[\phi]$
$R_{3,3}$	$-6\, \text{Cot}[\phi]^2$

In[◦]:=
```
scalarricci = FullSimplify[Sum[inversemetric[[i, j]] × ricci[[i, j]], {i, 1, n}, {j, 1, n}] ];
Print["---------------------------------------------------------------------"];
Print["Curvature Scalar:"];
Print["R = ", scalarricci]
```

```
---------------------------------------------------------------------
```

Curvature Scalar:

$R = 6\, a^{-4+2\gamma}\, e^{\frac{r}{a}}\, r^{2-2\gamma}\, \text{Csc}[\theta]^2\, \text{Csc}[\phi]^2\, \text{Sec}[2\, t\, w] \left(4 + \text{Sec}[2\, t\, w]^2\right)$

In[∘]:=
```
einstein := einstein = FullSimplify[ricci - (1 / 2) scalarricci * metric];
listeinstein := Table[If[UnsameQ[einstein⟦j, l⟧, 0],
    {Subscript[G, j - 1, l - 1], einstein⟦j, l⟧}] , {j, 1, n}, {l, 1, j}];
Print["----------------------------------------------------------------------"];
Print["Einstein Tensor:"];
Print[
 TableForm[Partition[DeleteCases[Flatten[listeinstein], Null], 2], TableSpacing → {4, 4}]]
```

Einstein Tensor:

$G_{0,0}$	$3 w^2 \left(-2 + \text{Sec}[2 t w]^2\right)$
$G_{1,0}$	$\frac{w (r - 2 a (-1+\gamma)) \text{Tan}[2 t w]}{a r}$
$G_{1,1}$	$\frac{3 (r - 2 a (-1+\gamma))^2 \left(2+\text{Sec}[2 t w]^2\right)}{4 a^2 r^2}$
$G_{2,0}$	$-2 w \text{Cot}[\theta] \text{Tan}[2 t w]$
$G_{2,1}$	$\left(-\frac{1}{a} + \frac{2 (-1+\gamma)}{r}\right) \text{Cot}[\theta]$
$G_{2,2}$	$3 \text{Cot}[\theta]^2 \left(2 + \text{Sec}[2 t w]^2\right)$
$G_{3,0}$	$-2 w \text{Cot}[\phi] \text{Tan}[2 t w]$
$G_{3,1}$	$\left(-\frac{1}{a} + \frac{2 (-1+\gamma)}{r}\right) \text{Cot}[\phi]$
$G_{3,2}$	$2 \text{Cot}[\theta] \text{Cot}[\phi]$
$G_{3,3}$	$3 \text{Cot}[\phi]^2 \left(2 + \text{Sec}[2 t w]^2\right)$

In[∘]:=
```
mixeinstein := mixeinstein =
   FullSimplify[Table[inversemetric⟦μ, ν⟧ * einstein⟦μ, ν⟧, {μ, 1, n}, {ν, 1, n}]];
listmixeinstein := Table[If[UnsameQ[mixeinstein⟦μ, ν⟧, 0],
    {Subscript[Superscript[G, μ - 1], ν - 1], mixeinstein⟦μ, ν⟧}] ,
   {μ, 1, n}, {ν, 1, n}];
Print[
  "----------------------------------------------------------------------"];
Print["Einstein Tensor Mix Form:"];
Print[TableForm[
  Partition[DeleteCases[Flatten[listmixeinstein], Null], 2], TableSpacing → {4, 4}]]
```

```
----------------------------------------------------------------------
Einstein Tensor Mix Form:
```

$G^{0}{}_{0}$ $\quad -3\, a^{-4+2\gamma}\, e^{\frac{r}{a}}\, r^{2-2\gamma}\, \text{Cos}[4\, t\, w]\, \text{Csc}[\theta]^2\, \text{Csc}[\phi]^2\, \text{Sec}[2\, t\, w]^3$

$G^{1}{}_{1}$ $\quad -3\, a^{-4+2\gamma}\, e^{\frac{r}{a}}\, r^{2-2\gamma}\, (2+\text{Cos}[4\, t\, w])\, \text{Csc}[\theta]^2\, \text{Csc}[\phi]^2\, \text{Sec}[2\, t\, w]^3$

$G^{2}{}_{2}$ $\quad -3\, a^{-4+2\gamma}\, e^{\frac{r}{a}}\, r^{2-2\gamma}\, (2+\text{Cos}[4\, t\, w])\, \text{Csc}[\theta]^2\, \text{Csc}[\phi]^2\, \text{Sec}[2\, t\, w]^3$

$G^{3}{}_{3}$ $\quad -3\, a^{-4+2\gamma}\, e^{\frac{r}{a}}\, r^{2-2\gamma}\, (2+\text{Cos}[4\, t\, w])\, \text{Csc}[\theta]^2\, \text{Csc}[\phi]^2\, \text{Sec}[2\, t\, w]^3$

```
weyl :=
  weyl = FullSimplify[Table[lriemann[[σ, μ, ν, λ]] + 1/(n - 2) (metric[[μ, ν]] × ricci[[σ, λ]] - metric[[
          μ, λ]] × ricci[[σ, ν]] + metric[[σ, λ]] × ricci[[μ, ν]] - metric[[σ, ν]] × ricci[[μ, λ]]) +
        1/((n - 1) (n - 2)) * scalarricci *
         (metric[[σ, ν]] × metric[[μ, λ]] - metric[[σ, λ]] × metric[[μ, ν]]),
      {σ, 1, n}, {μ, 1, n}, {ν, 1, n}, {λ, 1, n}]];
listweyl := Table[If[UnsameQ[weyl[[σ, μ, ν, λ]], 0], {Subscript[C, σ - 1, μ - 1, ν - 1, λ - 1],
      weyl[[σ, μ, ν, λ]]}] , {σ, 1, n}, {μ, 1, n}, {ν, 1, n}, {λ, 1, n}];
Print["----------------------------------------------------------------------"];
Print["Weyl Tensor:"];
Print[
  TableForm[Partition[DeleteCases[Flatten[listweyl], Null], 2], TableSpacing → {2, 2}]];
```

Print["C_{0121} =0
C_{0221} =0
C_{3121} =0
C_{0131} =0
C_{3221} =0
C_{0332} =0
C_{0302} =0
C_{0201} =0
C_{2121} =0
C_{3131} =0
C_{0101} =0
C_{0331} =0
C_{0303} =0
C_{0301} =0
C_{3232} =0
C_{0202} =0
C_{3231} =0"]

```
------------------------------------------------------------------------
Weyl Tensor:
{}
C0121 =0;
C0221 =0;
C3121 =0;
C0131 =0;
C3221 =0;
C0332 =0;
C0302 =0;
C0201 =0;
C0123 =0;
```

```
In[∘]:= geodesic := geodesic = FullSimplify[Table[-Sum[affine[[i, j, k]] × u[j] × u[k], {j, 1, n},
            {k, 1, n}], {i, 1, n}]];
        listgeodesic := Table[{"d/dτ" ToString[u[i]], "=", geodesic[[i]]}, {i, 1, n}];
        Print["------------------------------------------------------------------------"];
        Print["Geodesic Equations:"];
        TableForm[listgeodesic, TableSpacing → {2}]
```

```
------------------------------------------------------------------------
Geodesic Equations:
```

Out[∘]//TableForm=

d/dτ u[1]	=	$\frac{4\,a\,r\,w\,u[1]\,((r-2\,a\,(-1+\gamma))\,u[2]-2\,a\,r\,(\mathrm{Cot}[\theta]\,u[3]+\mathrm{Cot}[\phi]\,u[4]))+\mathrm{Tan}[2\,t\,w]\,(4\,a^2\,r^2\,w^2\,u[1]^2+(r-2\,a\,(-1+\gamma))^2}{4\,a^2\,r^2\,w}$
d/dτ u[2]	=	$\frac{2\,a\,r\,w^2\,u[1]^2}{r-2\,a\,(-1+\gamma)}+2\,w\,\mathrm{Tan}[2\,t\,w]\,u[1]\times u[2]-\left(-\frac{1}{2\,a}+\frac{-2+\gamma}{r}+\frac{1}{2\,a+r-2\,a\,\gamma}\right)u[2]^2-2\,\mathrm{Cot}[\theta]\,u$
d/dτ u[3]	=	$u[3]\,\left(2\,w\,\mathrm{Tan}[2\,t\,w]\,u[1]+\frac{u[2]}{a}-\frac{2\,(-1+\gamma)\,u[2]}{r}-2\,\mathrm{Cot}[\phi]\,u[4]\right)+\frac{1}{4}\,\mathrm{Tan}[\theta]\,\left(-4\,w^2\,u\right.$
d/dτ u[4]	=	$\left(2\,w\,\mathrm{Tan}[2\,t\,w]\,u[1]+\frac{u[2]}{a}-\frac{2\,(-1+\gamma)\,u[2]}{r}-2\,\mathrm{Cot}[\theta]\,u[3]\right)u[4]+\frac{1}{4}\,\mathrm{Tan}[\phi]\,\left(-4\,w^2\,u\right.$