

**In this particular File the codes and commands are mentioned for following mentioned Components :-**

- (1) *Christoffel Symbols*
- (2) *Riemann Curvature Tensor (Mix Form)*
- (3) *Riemann Curvature Tensor (Covariant Form)*
- (4) *Riemann Curvature Tensor (Contravariant Form)*
- (5) *Ricci Tensor*
- (6) *Einstein Tensor*
- (7) *Weyl Tensor*
- (8) *Geodesic's Equation*

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In[*]:= Clear[coord, metric, inversemetric, affine, riemann, lriemann,
      uriemann, ricci, scalar, einstein, weyl, geodesic, R, G, τ, i, j, k, l, s];
Clear[r, θ, ϕ, t, χ, a, m, M]; coord = {t, r, θ, ϕ};
n = Length[coord];
metric = {{w^2 r^(2 γ-2) a^(4-2 γ) e^(r/a) Sin[θ]^2 Sin[ϕ]^2 Cos[2 w t], 0, 0, 0},
      {0, -r^(2 γ-4) a^(4-2 γ) e^(r/a) Sin[θ]^2 Sin[ϕ]^2 (γ-1 - r/(2 a))^2 Cos[2 w t], 0, 0},
      {0, 0, -r^(2 γ-2) a^(4-2 γ) e^(r/a) Cos[θ]^2 Sin[ϕ]^2 Cos[2 w t], 0},
      {0, 0, 0, -r^(2 γ-2) a^(4-2 γ) e^(r/a) Sin[θ]^2 Cos[ϕ]^2 Cos[2 w t]}}
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Out[*]= {{a^(4-2 γ) e^(r/a) r^(2+2 γ) w^2 Cos[2 t w] Sin[θ]^2 Sin[ϕ]^2, 0, 0, 0},
      {0, -a^(4-2 γ) e^(r/a) r^(4+2 γ) (-1 - r/(2 a) + γ)^2 Cos[2 t w] Sin[θ]^2 Sin[ϕ]^2, 0, 0},
      {0, 0, -a^(4-2 γ) e^(r/a) r^(2+2 γ) Cos[2 t w] Cos[θ]^2 Sin[ϕ]^2, 0},
      {0, 0, 0, -a^(4-2 γ) e^(r/a) r^(2+2 γ) Cos[2 t w] Cos[ϕ]^2 Sin[θ]^2}}
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In[*]:= inversemetric = Simplify[Inverse[metric]]
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Out[*]= {{a^(-4+2 γ) e^(r/a) r^(2-2 γ) Csc[θ]^2 Csc[ϕ]^2 Sec[2 t w], 0, 0, 0},
      {0, -4 a^(-2+2 γ) e^(r/a) r^(4-2 γ) Csc[θ]^2 Csc[ϕ]^2 Sec[2 t w], 0, 0},
      {0, 0, -a^(-4+2 γ) e^(r/a) r^(2-2 γ) Csc[ϕ]^2 Sec[2 t w] Sec[θ]^2, 0},
      {0, 0, 0, -a^(-4+2 γ) e^(r/a) r^(2-2 γ) Csc[θ]^2 Sec[2 t w] Sec[ϕ]^2}}
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In[ ]:= Print["-----"];
Print["The Manifold has dimension   n= ", n, "\nCoordinate system:", coord];
Print["x0= t, x1= r, x2=  $\theta$ , x3=  $\phi$ "]
Print["-----"];
Print["g $\mu\nu$ ", metric // MatrixForm];
Print["g $\mu\nu$ ", inversemetric // MatrixForm];
Print[
  "g  = - w2 r8 $\gamma$ -10 a16-8 $\gamma$  e $-\frac{4r}{a}$  ( $\gamma-1-\frac{r}{2a}$ )2 Sin[ $\theta$ ]6 Sin[ $\phi$ ]6 Cos[ $\theta$ ]2 Cos[ $\phi$ ]2 Cos[2wt]4"];
Print[" $\sqrt{-g}$  = w r4 $\gamma$ -5 a8-4 $\gamma$  e $-\frac{2r}{a}$  ( $\gamma-1-\frac{r}{2a}$ ) Sin[ $\theta$ ]3 Sin[ $\phi$ ]3 Cos[ $\theta$ ] Cos[ $\phi$ ] Cos[2wt]2"]

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The Manifold has dimension   n= 4
Coordinate system: {t, r,  $\theta$ ,  $\phi$ }
x0= t, x1= r, x2=  $\theta$ , x3=  $\phi$ 

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$$g_{\mu\nu} = \begin{pmatrix} a^{4-2\gamma} e^{-\frac{r}{a}} r^{2-2\gamma} w^2 \cos[2tw] \sin[\theta]^2 \sin[\phi]^2 & 0 & 0 & 0 \\ 0 & -a^{4-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} \left(-1 - \frac{r}{2a} + \gamma\right)^2 \cos[2tw] \sin[\theta]^2 \sin[\phi]^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$


$$g^{\mu\nu} = \begin{pmatrix} \frac{a^{-4+2\gamma} e^{\frac{r}{a}} r^{2-2\gamma} \csc[\theta]^2 \csc[\phi]^2 \sec[2tw]}{w^2} & 0 & 0 & 0 \\ 0 & -\frac{4 a^{-2+2\gamma} e^{\frac{r}{a}} r^{4-2\gamma} \csc[\theta]^2 \csc[\phi]^2 \sec[2tw]}{(r-2a(-1+\gamma))^2} & 0 & 0 \\ 0 & 0 & -a^{-4+2\gamma} e^{\frac{r}{a}} r^{2-2\gamma} \csc[\phi]^2 \sec[2tw] \sec & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$


$$g = - w^2 r^{8\gamma-10} a^{16-8\gamma} e^{-\frac{4r}{a}} \left(\gamma-1-\frac{r}{2a}\right)^2 \sin[\theta]^6 \sin[\phi]^6 \cos[\theta]^2 \cos[\phi]^2 \cos[2wt]^4$$


$$\sqrt{-g} = w r^{4\gamma-5} a^{8-4\gamma} e^{-\frac{2r}{a}} \left(\gamma-1-\frac{r}{2a}\right) \sin[\theta]^3 \sin[\phi]^3 \cos[\theta] \cos[\phi] \cos[2wt]^2$$


In[ ]:= affine := affine = FullSimplify[Table[
  (1/2) * Sum[(inversemetric[[i, s]]) *
    (D[metric[[s, j]], coord[[k]]] +
      D[metric[[s, k]], coord[[j]]] - D[metric[[j, k]], coord[[s]]]),
    {s, 1, n}], {i, 1, n}], {j, 1, n}], {k, 1, n}]];
listaffine := Table[ If[UnsameQ[affine[[i, j, k]], 0],
  {Subscript[Superscript[ $\Gamma$ , i-1], j-1, k-1], affine[[i, j, k]]} ],
  {i, 1, n}], {j, 1, n}], {k, 1, n}]];
Print[
  "-----"];
Print["Christoffel Symbols:"];
Print[
  TableForm[Partition[DeleteCases[Flatten[listaffine], Null], 2], TableSpacing -> {3, 5}]]

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**Christoffel Symbols:**

$$\Gamma_{0,0}^0 -w \tan [2 t w]$$

$$\Gamma_{1,0}^0 -\frac{2 a+r-2 a \gamma}{2 a r}$$

$$\Gamma_{1,1}^0 -\frac{\left(-1-\frac{r}{2 a}+\gamma\right)^2 \tan [2 t w]}{r^2 w}$$

$$\Gamma_{2,0}^0 \cot [\theta]$$

$$\Gamma_{2,2}^0 -\frac{\cot [\theta]^2 \tan [2 t w]}{w}$$

$$\Gamma_{3,0}^0 \cot [\phi]$$

$$\Gamma_{3,3}^0 -\frac{\cot [\phi]^2 \tan [2 t w]}{w}$$

$$\Gamma_{0,0}^1 -\frac{2 a r w^2}{r-2 a(-1+\gamma)}$$

$$\Gamma_{1,0}^1 -w \tan [2 t w]$$

$$\Gamma_{1,1}^1 -\frac{1}{2 a}+\frac{-2+\gamma}{r}+\frac{1}{2 a+r-2 a \gamma}$$

$$\Gamma_{2,1}^1 \cot [\theta]$$

$$\Gamma_{2,2}^1 \frac{2 a r \cot [\theta]^2}{r-2 a(-1+\gamma)}$$

$$\Gamma_{3,1}^1 \cot [\phi]$$

$$\Gamma_{3,3}^1 \frac{2 a r \cot [\phi]^2}{r-2 a(-1+\gamma)}$$

$$\Gamma_{0,0}^2 w^2 \tan [\theta]$$

$$\Gamma_{1,1}^2 -\frac{\left(-1-\frac{r}{2 a}+\gamma\right)^2 \tan [\theta]}{r^2}$$

$$\Gamma_{2,0}^2 -w \tan [2 t w]$$

$$\Gamma_{2,1}^2 -\frac{2 a+r-2 a \gamma}{2 a r}$$

$$\Gamma_{2,2}^2 -\tan [\theta]$$

$$\Gamma_{3,2}^2 \cot [\phi]$$

$$\Gamma_{3,3}^2 -\cot [\phi]^2 \tan [\theta]$$

$$\Gamma_{0,0}^3 w^2 \tan [\phi]$$

$$\Gamma_{1,1}^3 -\frac{\left(-1-\frac{r}{2 a}+\gamma\right)^2 \tan [\phi]}{r^2}$$

$$\Gamma_{2,2}^3 -\cot [\theta]^2 \tan [\phi]$$

$$\Gamma_{3,0}^3 -w \tan [2 t w]$$

$$\Gamma_{3,1}^3 -\frac{2 a+r-2 a \gamma}{2 a r}$$

$$\Gamma_{3,2}^3 \cot [\theta]$$

$$\Gamma_{3,3}^3 -\tan [\phi]$$

```

In[ ]:= riemann :=
  riemann = Simplify[Table[D[affine[[i, 1, j]], coord[[k]]] - D[affine[[i, k, j]], coord[[1]]] +
    Sum[affine[[i, k, s]] × affine[[s, 1, j]] - affine[[i, 1, s]] × affine[[s, k, j]],
      {s, 1, n}], {i, 1, n}, {j, 1, n}, {k, 1, n}, {1, 1, n}]];
listriemann := Table[If[UnsameQ[riemann[[i, j, k, 1]], 0],
  {Subscript[Superscript[R, i - 1], j - 1, k - 1, 1 - 1], riemann[[i, j, k, 1]]},
  {i, 1, n}, {j, 1, n}, {k, 1, n}, {1, 1, k - 1}]];
Print["-----"];
Print["Riemann Tensor Mix Form:"];
Print[
  TableForm[Partition[DeleteCases[Flatten[listriemann], Null], 2], TableSpacing → {4, 4}]]

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Riemann Tensor Mix Form:

$$R^0_{1,1,0} = \frac{(r-2a(-1+\gamma))^2 (1+\sec[2tw]^2)}{2a^2 r^2}$$

$$R^0_{1,2,0} = \frac{(r-2a(-1+\gamma)) \cot[\theta]}{2ar}$$

$$R^0_{1,2,1} = \frac{\left(-1-\frac{r}{2a}+\gamma\right)^2 \cot[\theta] \tan[2tw]}{r^2 w}$$

$$R^0_{1,3,0} = \frac{(r-2a(-1+\gamma)) \cot[\phi]}{2ar}$$

$$R^0_{1,3,1} = \frac{\left(-1-\frac{r}{2a}+\gamma\right)^2 \cot[\phi] \tan[2tw]}{r^2 w}$$

$$R^0_{2,1,0} = \frac{(r-2a(-1+\gamma)) \cot[\theta]}{2ar}$$

$$R^0_{2,2,0} = (3 + \cos[4tw]) \cot[\theta]^2 \sec[2tw]^2$$

$$R^0_{2,2,1} = \frac{(r-2a(-1+\gamma)) \cot[\theta]^2 \tan[2tw]}{2arw}$$

$$R^0_{2,3,0} = -\cot[\theta] \cot[\phi]$$

$$R^0_{2,3,2} = \frac{\cot[\theta]^2 \cot[\phi] \tan[2tw]}{w}$$

$$R^0_{3,1,0} = \frac{(r-2a(-1+\gamma)) \cot[\phi]}{2ar}$$

$$R^0_{3,2,0} = -\cot[\theta] \cot[\phi]$$

$$R^0_{3,3,0} = (3 + \cos[4tw]) \cot[\phi]^2 \sec[2tw]^2$$

$$R^0_{3,3,1} = \frac{(r-2a(-1+\gamma)) \cot[\phi]^2 \tan[2tw]}{2arw}$$

$$R^0_{3,3,2} = -\frac{\cot[\theta] \cot[\phi]^2 \tan[2tw]}{w}$$

$$R^1_{0,1,0} = 2w^2 (1 + \sec[2tw]^2)$$

$$\begin{aligned}
R^1_{0,2,0} &= \frac{2 a r w^2 \cot[\theta]}{r-2 a (-1+\gamma)} \\
R^1_{0,2,1} &= w \cot[\theta] \tan[2 t w] \\
R^1_{0,3,0} &= \frac{2 a r w^2 \cot[\phi]}{r-2 a (-1+\gamma)} \\
R^1_{0,3,1} &= w \cot[\phi] \tan[2 t w] \\
R^1_{2,1,0} &= -w \cot[\theta] \tan[2 t w] \\
R^1_{2,2,0} &= -\frac{2 a r w \cot[\theta]^2 \tan[2 t w]}{r-2 a (-1+\gamma)} \\
R^1_{2,2,1} &= \cos[4 t w] \cot[\theta]^2 \sec[2 t w]^2 \\
R^1_{2,3,1} &= -\cot[\theta] \cot[\phi] \\
R^1_{2,3,2} &= -\frac{2 a r \cot[\theta]^2 \cot[\phi]}{r-2 a (-1+\gamma)} \\
R^1_{3,1,0} &= -w \cot[\phi] \tan[2 t w] \\
R^1_{3,2,1} &= -\cot[\theta] \cot[\phi] \\
R^1_{3,3,0} &= -\frac{2 a r w \cot[\phi]^2 \tan[2 t w]}{r-2 a (-1+\gamma)} \\
R^1_{3,3,1} &= \cos[4 t w] \cot[\phi]^2 \sec[2 t w]^2 \\
R^1_{3,3,2} &= \frac{2 a r \cot[\theta] \cot[\phi]^2}{r-2 a (-1+\gamma)} \\
R^2_{0,1,0} &= \frac{w^2 (r-2 a (-1+\gamma)) \tan[\theta]}{2 a r} \\
R^2_{0,2,0} &= w^2 (3 + \cos[4 t w]) \sec[2 t w]^2 \\
R^2_{0,2,1} &= \frac{w (r-2 a (-1+\gamma)) \tan[2 t w]}{2 a r} \\
R^2_{0,3,0} &= -w^2 \cot[\phi] \tan[\theta] \\
R^2_{0,3,2} &= w \cot[\phi] \tan[2 t w] \\
R^2_{1,1,0} &= \frac{w \left(-1 - \frac{r}{2 a} + \gamma\right)^2 \tan[2 t w] \tan[\theta]}{r^2} \\
R^2_{1,2,0} &= \frac{w (r-2 a (-1+\gamma)) \tan[2 t w]}{2 a r} \\
R^2_{1,2,1} &= -\frac{(r-2 a (-1+\gamma))^2 \cos[4 t w] \sec[2 t w]^2}{4 a^2 r^2} \\
R^2_{1,3,1} &= \frac{\left(-1 - \frac{r}{2 a} + \gamma\right)^2 \cot[\phi] \tan[\theta]}{r^2} \\
R^2_{1,3,2} &= \frac{(r-2 a (-1+\gamma)) \cot[\phi]}{2 a r}
\end{aligned}$$

$$R^2_{3,2,0} = -w \cot[\phi] \tan[2tw]$$

$$R^2_{3,2,1} = -\frac{\cot[\phi]}{2a} + \frac{(-1+\gamma) \cot[\phi]}{r}$$

$$R^2_{3,3,0} = w \cot[\phi]^2 \tan[2tw] \tan[\theta]$$

$$R^2_{3,3,1} = \frac{(r-2a(-1+\gamma)) \cot[\phi]^2 \tan[\theta]}{2ar}$$

$$R^2_{3,3,2} = \cos[4tw] \cot[\phi]^2 \sec[2tw]^2$$

$$R^3_{0,1,0} = \frac{w^2 (r-2a(-1+\gamma)) \tan[\phi]}{2ar}$$

$$R^3_{0,2,0} = -w^2 \cot[\theta] \tan[\phi]$$

$$R^3_{0,3,0} = w^2 (3 + \cos[4tw]) \sec[2tw]^2$$

$$R^3_{0,3,1} = \frac{w (r-2a(-1+\gamma)) \tan[2tw]}{2ar}$$

$$R^3_{0,3,2} = -w \cot[\theta] \tan[2tw]$$

$$R^3_{1,1,0} = \frac{w \left(-1 - \frac{r}{2a} + \gamma\right)^2 \tan[2tw] \tan[\phi]}{r^2}$$

$$R^3_{1,2,1} = \frac{\left(-1 - \frac{r}{2a} + \gamma\right)^2 \cot[\theta] \tan[\phi]}{r^2}$$

$$R^3_{1,3,0} = \frac{w (r-2a(-1+\gamma)) \tan[2tw]}{2ar}$$

$$R^3_{1,3,1} = -\frac{(r-2a(-1+\gamma))^2 \cos[4tw] \sec[2tw]^2}{4a^2 r^2}$$

$$R^3_{1,3,2} = -\frac{\cot[\theta]}{2a} + \frac{(-1+\gamma) \cot[\theta]}{r}$$

$$R^3_{2,2,0} = w \cot[\theta]^2 \tan[2tw] \tan[\phi]$$

$$R^3_{2,2,1} = \frac{(r-2a(-1+\gamma)) \cot[\theta]^2 \tan[\phi]}{2ar}$$

$$R^3_{2,3,0} = -w \cot[\theta] \tan[2tw]$$

$$R^3_{2,3,1} = -\frac{\cot[\theta]}{2a} + \frac{(-1+\gamma) \cot[\theta]}{r}$$

$$R^3_{2,3,2} = -\cos[4tw] \cot[\theta]^2 \sec[2tw]^2$$

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In[*]:= lriemann := lriemann = Simplify[Table[
  Sum[metric[[i, ii]] × riemann[[ii, j, k, l]], {ii, 1, n}],
  {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listlriemann := Table[If[UnsameQ[lriemann[[i, j, k, l]], 0],
  {Subscript[R, i - 1, j - 1, k - 1, l - 1], lriemann[[i, j, k, l]]} ,
  {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}];
Print["-----"];
Print["Covariant Riemann Tensor:"];
Print[
  TableForm[Partition[DeleteCases[Flatten[listlriemann], Null], 2], TableSpacing → {2, 2}]]

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Covariant Riemann Tensor:

$$\begin{aligned}
 R_{1,0,1,0} &= -\frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} w^2 (r - 2a(-1 + \gamma))^2 (3 + \cos[4tw]) \sec[2tw] \sin[\theta]^2 \sin[\phi]^2 \\
 R_{1,0,2,0} &= \frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} w^2 (-r + 2a(-1 + \gamma)) \cos[2tw] \cos[\theta] \sin[\theta] \sin[\phi]^2 \\
 R_{1,0,2,1} &= -\frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} w (r - 2a(-1 + \gamma))^2 \cos[\theta] \sin[2tw] \sin[\theta] \sin[\phi]^2 \\
 R_{1,0,3,0} &= \frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} w^2 (-r + 2a(-1 + \gamma)) \cos[2tw] \cos[\phi] \sin[\theta]^2 \sin[\phi] \\
 R_{1,0,3,1} &= -\frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} w (r - 2a(-1 + \gamma))^2 \cos[\phi] \sin[2tw] \sin[\theta]^2 \sin[\phi] \\
 R_{2,0,1,0} &= -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} w^2 (r - 2a(-1 + \gamma)) \cos[2tw] \cos[\theta] \sin[\theta] \sin[\phi]^2 \\
 R_{2,0,2,0} &= -a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} w^2 (3 + \cos[4tw]) \cos[\theta]^2 \sec[2tw] \sin[\phi]^2 \\
 R_{2,0,2,1} &= -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} w (r - 2a(-1 + \gamma)) \cos[\theta]^2 \sin[2tw] \sin[\phi]^2 \\
 R_{2,0,3,0} &= a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} w^2 \cos[2tw] \cos[\theta] \cos[\phi] \sin[\theta] \sin[\phi] \\
 R_{2,0,3,2} &= -a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} w \cos[\theta]^2 \cos[\phi] \sin[2tw] \sin[\phi] \\
 R_{2,1,1,0} &= -\frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} w (r - 2a(-1 + \gamma))^2 \cos[\theta] \sin[2tw] \sin[\theta] \sin[\phi]^2 \\
 R_{2,1,2,0} &= -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} w (r - 2a(-1 + \gamma)) \cos[\theta]^2 \sin[2tw] \sin[\phi]^2 \\
 R_{2,1,2,1} &= \frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} (r - 2a(-1 + \gamma))^2 \cos[4tw] \cos[\theta]^2 \sec[2tw] \sin[\phi]^2 \\
 R_{2,1,3,1} &= -\frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} (r - 2a(-1 + \gamma))^2 \cos[2tw] \cos[\theta] \cos[\phi] \sin[\theta] \sin[\phi] \\
 R_{2,1,3,2} &= -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} (r - 2a(-1 + \gamma)) \cos[2tw] \cos[\theta]^2 \cos[\phi] \sin[\phi] \\
 R_{3,0,1,0} &= -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} w^2 (r - 2a(-1 + \gamma)) \cos[2tw] \cos[\phi] \sin[\theta]^2 \sin[\phi] \\
 R_{3,0,2,0} &= a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} w^2 \cos[2tw] \cos[\theta] \cos[\phi] \sin[\theta] \sin[\phi] \\
 R_{3,0,3,0} &= -a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} w^2 (3 + \cos[4tw]) \cos[\phi]^2 \sec[2tw] \sin[\theta]^2 \\
 R_{3,0,3,1} &= -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} w (r - 2a(-1 + \gamma)) \cos[\phi]^2 \sin[2tw] \sin[\theta]^2 \\
 R_{3,0,3,2} &= a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} w \cos[\theta] \cos[\phi]^2 \sin[2tw] \sin[\theta] \\
 R_{3,1,1,0} &= -\frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} w (r - 2a(-1 + \gamma))^2 \cos[\phi] \sin[2tw] \sin[\theta]^2 \sin[\phi]
 \end{aligned}$$



$$\begin{aligned}
R_{3,1,2,1} &= -\frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} (r-2a(-1+\gamma))^2 \cos[2tw] \cos[\theta] \cos[\phi] \sin[\theta] \sin[\phi] \\
R_{3,1,3,0} &= -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} w (r-2a(-1+\gamma)) \cos[\phi]^2 \sin[2tw] \sin[\theta]^2 \\
R_{3,1,3,1} &= \frac{1}{4} a^{2-2\gamma} e^{-\frac{r}{a}} r^{-4+2\gamma} (r-2a(-1+\gamma))^2 \cos[4tw] \cos[\phi]^2 \sec[2tw] \sin[\theta]^2 \\
R_{3,1,3,2} &= -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} (-r+2a(-1+\gamma)) \cos[2tw] \cos[\theta] \cos[\phi]^2 \sin[\theta] \\
R_{3,2,2,0} &= -a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} w \cos[\theta]^2 \cos[\phi] \sin[2tw] \sin[\phi] \\
R_{3,2,2,1} &= -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} (r-2a(-1+\gamma)) \cos[2tw] \cos[\theta]^2 \cos[\phi] \sin[\phi] \\
R_{3,2,3,0} &= a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} w \cos[\theta] \cos[\phi]^2 \sin[2tw] \sin[\theta] \\
R_{3,2,3,1} &= -\frac{1}{2} a^{3-2\gamma} e^{-\frac{r}{a}} r^{-3+2\gamma} (-r+2a(-1+\gamma)) \cos[2tw] \cos[\theta] \cos[\phi]^2 \sin[\theta] \\
R_{3,2,3,2} &= a^{4-2\gamma} e^{-\frac{r}{a}} r^{-2+2\gamma} \cos[4tw] \cos[\theta]^2 \cos[\phi]^2 \sec[2tw]
\end{aligned}$$

```

In[*]:= uriemann := uriemann = FullSimplify[Table[Sum[inversemetric[[j, jj]] ×
    inversemetric[[k, kk]] × inversemetric[[l, ll]] × riemann[[i, jj, kk, ll]],
    {jj, 1, n}, {kk, 1, n}, {ll, 1, n}
], {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]];
listuriemann := Table[If[UnsameQ[uriemann[[i, j, k, l]], 0],
    {Superscript[Superscript[Superscript[Superscript[R, i - 1], j - 1], k - 1], l - 1],
    uriemann[[i, j, k, l]]}
], {i, 1, n}, {j, 1, i - 1}, {k, 1, n}, {l, 1, k - 1}]];
Print["-----"];
Print["Contravariant Riemann Tensor:"];
Print[
    TableForm[Partition[DeleteCases[Flatten[listuriemann], Null], 2], TableSpacing → {2, 2}]]

```

Contravariant Riemann Tensor:

$$\begin{aligned}
R^{1010} &= -\frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} (3+\cos[4tw]) \csc[\theta]^6 \csc[\phi]^6 \sec[2tw]^5}{w^2 (r-2a(-1+\gamma))^2} \\
R^{1020} &= -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^5 \csc[\phi]^6 \sec[2tw]^3 \sec[\theta]}{w^2 (r-2a(-1+\gamma))} \\
R^{1021} &= \frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \csc[\theta]^5 \csc[\phi]^6 \sec[2tw]^3 \sec[\theta] \tan[2tw]}{w (r-2a(-1+\gamma))^2} \\
R^{1030} &= -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^6 \csc[\phi]^5 \sec[2tw]^3 \sec[\phi]}{w^2 (r-2a(-1+\gamma))} \\
R^{1031} &= \frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \csc[\theta]^6 \csc[\phi]^5 \sec[2tw]^3 \sec[\phi] \tan[2tw]}{w (r-2a(-1+\gamma))^2} \\
R^{2010} &= -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^5 \csc[\phi]^6 \sec[2tw]^3 \sec[\theta]}{w^2 (r-2a(-1+\gamma))} \\
R^{2020} &= -\frac{a^6 (-2+\gamma) e^{\frac{3r}{a}} r^{6-6\gamma} (3+\cos[4tw]) \csc[\theta]^4 \csc[\phi]^6 \sec[2tw]^5 \sec[\theta]^2}{w^2} \\
R^{2021} &= \frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^4 \csc[\phi]^6 \sec[2tw]^3 \sec[\theta]^2 \tan[2tw]}{w (r-2a(-1+\gamma))} \\
R^{2030} &= \frac{a^6 (-2+\gamma) e^{\frac{3r}{a}} r^{6-6\gamma} \csc[\theta]^5 \csc[\phi]^5 \sec[2tw]^3 \sec[\theta] \sec[\phi]}{w^2}
\end{aligned}$$

$$\begin{aligned}
R^{2032} &= \frac{a^6 (-2+\gamma) e^{\frac{3r}{a}} r^{6-6\gamma} \csc[\theta]^4 \csc[\phi]^5 \sec[2tw]^3 \sec[\theta]^2 \sec[\phi] \tan[2tw]}{w} \\
R^{2110} &= \frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \csc[\theta]^5 \csc[\phi]^6 \sec[2tw]^3 \sec[\theta] \tan[2tw]}{w (r-2a (-1+\gamma))^2} \\
R^{2120} &= \frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^4 \csc[\phi]^6 \sec[2tw]^3 \sec[\theta]^2 \tan[2tw]}{w (r-2a (-1+\gamma))} \\
R^{2121} &= \frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \cos[4tw] \csc[\theta]^4 \csc[\phi]^6 \sec[2tw]^5 \sec[\theta]^2}{(r-2a (-1+\gamma))^2} \\
R^{2131} &= -\frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \csc[\theta]^5 \csc[\phi]^5 \sec[2tw]^3 \sec[\theta] \sec[\phi]}{(r-2a (-1+\gamma))^2} \\
R^{2132} &= -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^4 \csc[\phi]^5 \sec[2tw]^3 \sec[\theta]^2 \sec[\phi]}{r-2a (-1+\gamma)} \\
R^{3010} &= -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^6 \csc[\phi]^5 \sec[2tw]^3 \sec[\phi]}{w^2 (r-2a (-1+\gamma))} \\
R^{3020} &= \frac{a^6 (-2+\gamma) e^{\frac{3r}{a}} r^{6-6\gamma} \csc[\theta]^5 \csc[\phi]^5 \sec[2tw]^3 \sec[\theta] \sec[\phi]}{w^2} \\
R^{3030} &= -\frac{a^6 (-2+\gamma) e^{\frac{3r}{a}} r^{6-6\gamma} (3+\cos[4tw]) \csc[\theta]^6 \csc[\phi]^4 \sec[2tw]^5 \sec[\phi]^2}{w^2} \\
R^{3031} &= \frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^6 \csc[\phi]^4 \sec[2tw]^3 \sec[\phi]^2 \tan[2tw]}{w (r-2a (-1+\gamma))} \\
R^{3032} &= -\frac{a^6 (-2+\gamma) e^{\frac{3r}{a}} r^{6-6\gamma} \csc[\theta]^5 \csc[\phi]^4 \sec[2tw]^3 \sec[\theta] \sec[\phi]^2 \tan[2tw]}{w} \\
R^{3110} &= \frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \csc[\theta]^6 \csc[\phi]^5 \sec[2tw]^3 \sec[\phi] \tan[2tw]}{w (r-2a (-1+\gamma))^2} \\
R^{3121} &= -\frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \csc[\theta]^5 \csc[\phi]^5 \sec[2tw]^3 \sec[\theta] \sec[\phi]}{(r-2a (-1+\gamma))^2} \\
R^{3130} &= \frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^6 \csc[\phi]^4 \sec[2tw]^3 \sec[\phi]^2 \tan[2tw]}{w (r-2a (-1+\gamma))} \\
R^{3131} &= \frac{4 a^{-10+6\gamma} e^{\frac{3r}{a}} r^{8-6\gamma} \cos[4tw] \csc[\theta]^6 \csc[\phi]^4 \sec[2tw]^5 \sec[\phi]^2}{(r-2a (-1+\gamma))^2} \\
R^{3132} &= -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^5 \csc[\phi]^4 \sec[2tw]^3 \sec[\theta] \sec[\phi]^2}{-r+2a (-1+\gamma)} \\
R^{3220} &= \frac{a^6 (-2+\gamma) e^{\frac{3r}{a}} r^{6-6\gamma} \csc[\theta]^4 \csc[\phi]^5 \sec[2tw]^3 \sec[\theta]^2 \sec[\phi] \tan[2tw]}{w} \\
R^{3221} &= -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^4 \csc[\phi]^5 \sec[2tw]^3 \sec[\theta]^2 \sec[\phi]}{r-2a (-1+\gamma)} \\
R^{3230} &= -\frac{a^6 (-2+\gamma) e^{\frac{3r}{a}} r^{6-6\gamma} \csc[\theta]^5 \csc[\phi]^4 \sec[2tw]^3 \sec[\theta] \sec[\phi]^2 \tan[2tw]}{w} \\
R^{3231} &= -\frac{2 a^{-11+6\gamma} e^{\frac{3r}{a}} r^{7-6\gamma} \csc[\theta]^5 \csc[\phi]^4 \sec[2tw]^3 \sec[\theta] \sec[\phi]^2}{-r+2a (-1+\gamma)} \\
R^{3232} &= a^6 (-2+\gamma) e^{\frac{3r}{a}} r^{6-6\gamma} \cos[4tw] \csc[\theta]^4 \csc[\phi]^4 \sec[2tw]^5 \sec[\theta]^2 \sec[\phi]^2
\end{aligned}$$

```

In[*]:= r2 = FullSimplify[Sum[lriemann[[i, j, k, 1]] * uriemann[[i, j, k, 1]],
  {i, 1, n}, {j, 1, n}, {k, 1, n}, {1, 1, n}]];
Print["-----"];
Print["R^2= ", r2]

```

$$R^2 = 12 a^{-8+4\gamma} e^{\frac{2r}{a}} r^{4-4\gamma} (11 + 8 \cos[4 t w] + 2 \cos[8 t w]) \csc[\theta]^4 \csc[\phi]^4 \sec[2 t w]^6$$

```

In[*]:= ricci :=
  ricci = FullSimplify[Table[Sum[riemann[[i, j, i, l]], {i, 1, n}], {j, 1, n}, {l, 1, n}]];
  listricci := Table[If[UnsameQ[ricci[[j, l]], 0],
    {Subscript[R, j - 1, l - 1], ricci[[j, l]]}, {j, 1, n}, {l, 1, j}]];
  Print["-----"];
  Print["Ricci Tensor:"];
  Print[
    TableForm[Partition[DeleteCases[Flatten[listricci], Null], 2], TableSpacing -> {4, 4}]]

```

Ricci Tensor:

$$R_{0,0} = 6 w^2 (1 + \sec[2 t w]^2)$$

$$R_{1,0} = \frac{w (r - 2 a (-1 + \gamma)) \tan[2 t w]}{a r}$$

$$R_{1,1} = -\frac{3 (r - 2 a (-1 + \gamma))^2}{2 a^2 r^2}$$

$$R_{2,0} = -2 w \cot[\theta] \tan[2 t w]$$

$$R_{2,1} = \left(-\frac{1}{a} + \frac{2 (-1 + \gamma)}{r}\right) \cot[\theta]$$

$$R_{2,2} = -6 \cot[\theta]^2$$

$$R_{3,0} = -2 w \cot[\phi] \tan[2 t w]$$

$$R_{3,1} = \left(-\frac{1}{a} + \frac{2 (-1 + \gamma)}{r}\right) \cot[\phi]$$

$$R_{3,2} = 2 \cot[\theta] \cot[\phi]$$

$$R_{3,3} = -6 \cot[\phi]^2$$

```

In[*]:= scalarricci = FullSimplify[Sum[inversemetric[[i, j]] * ricci[[i, j]], {i, 1, n}, {j, 1, n}]];
  Print["-----"];
  Print["Curvature Scalar:"];
  Print["R = ", scalarricci]

```

Curvature Scalar:

$$R = 6 a^{-4+2\gamma} e^{\frac{r}{a}} r^{2-2\gamma} \csc[\theta]^2 \csc[\phi]^2 \sec[2 t w] (4 + \sec[2 t w]^2)$$

```

In[*]:= einstein := einstein = FullSimplify[ricci - (1/2) scalarricci*metric];
listeinstein := Table[If[UnsameQ[einstein[[j, 1]], 0],
  {Subscript[G, j - 1, 1 - 1], einstein[[j, 1]]}, {j, 1, n}, {1, 1, j}]];
Print["-----"];
Print["Einstein Tensor:"];
Print[
  TableForm[Partition[DeleteCases[Flatten[list Einstein], Null], 2], TableSpacing -> {4, 4}]

```

Einstein Tensor:

$$G_{0,0} \quad 3 w^2 \left( -2 + \sec[2 t w]^2 \right)$$

$$G_{1,0} \quad \frac{w (r - 2 a (-1 + \gamma)) \tan[2 t w]}{a r}$$

$$G_{1,1} \quad \frac{3 (r - 2 a (-1 + \gamma))^2 (2 + \sec[2 t w]^2)}{4 a^2 r^2}$$

$$G_{2,0} \quad -2 w \cot[\theta] \tan[2 t w]$$

$$G_{2,1} \quad \left( -\frac{1}{a} + \frac{2 (-1 + \gamma)}{r} \right) \cot[\theta]$$

$$G_{2,2} \quad 3 \cot[\theta]^2 (2 + \sec[2 t w]^2)$$

$$G_{3,0} \quad -2 w \cot[\phi] \tan[2 t w]$$

$$G_{3,1} \quad \left( -\frac{1}{a} + \frac{2 (-1 + \gamma)}{r} \right) \cot[\phi]$$

$$G_{3,2} \quad 2 \cot[\theta] \cot[\phi]$$

$$G_{3,3} \quad 3 \cot[\phi]^2 (2 + \sec[2 t w]^2)$$

```

In[*]:= mixeinstein := mixeinstein =
  FullSimplify[Table[inversemetric[[μ, ν]] * einstein[[μ, ν]], {μ, 1, n}, {ν, 1, n}]];
listmixeinstein := Table[If[UnsameQ[mixeinstein[[μ, ν]], 0],
  {Subscript[Superscript[G, μ - 1], ν - 1], mixeinstein[[μ, ν]]},
  {μ, 1, n}, {ν, 1, n}]];
Print[
  "-----"];
Print["Einstein Tensor Mix Form:"];
Print[TableForm[
  Partition[DeleteCases[Flatten[listmixeinstein], Null], 2], TableSpacing -> {4, 4}]

```

-----

Einstein Tensor Mix Form:

$$G^0_0 = -3 a^{-4+2\gamma} e^{\frac{r}{a}} r^{2-2\gamma} \cos[4 t w] \csc[\theta]^2 \csc[\phi]^2 \sec[2 t w]^3$$

$$G^1_1 = -3 a^{-4+2\gamma} e^{\frac{r}{a}} r^{2-2\gamma} (2 + \cos[4 t w]) \csc[\theta]^2 \csc[\phi]^2 \sec[2 t w]^3$$

$$G^2_2 = -3 a^{-4+2\gamma} e^{\frac{r}{a}} r^{2-2\gamma} (2 + \cos[4 t w]) \csc[\theta]^2 \csc[\phi]^2 \sec[2 t w]^3$$

$$G^3_3 = -3 a^{-4+2\gamma} e^{\frac{r}{a}} r^{2-2\gamma} (2 + \cos[4 t w]) \csc[\theta]^2 \csc[\phi]^2 \sec[2 t w]^3$$

weyl :=

$$\begin{aligned} \text{weyl} = & \text{FullSimplify}\left[\text{Table}\left[\text{Riemann}[\sigma, \mu, \nu, \lambda] + \frac{1}{n-2} (\text{metric}[\mu, \nu] \times \text{ricci}[\sigma, \lambda] - \text{metric}[\mu, \lambda] \times \text{ricci}[\sigma, \nu] + \text{metric}[\sigma, \lambda] \times \text{ricci}[\mu, \nu] - \text{metric}[\sigma, \nu] \times \text{ricci}[\mu, \lambda]) + \right. \right. \\ & \left. \frac{1}{(n-1)(n-2)} * \text{scalarricci} * \right. \\ & \left. (\text{metric}[\sigma, \nu] \times \text{metric}[\mu, \lambda] - \text{metric}[\sigma, \lambda] \times \text{metric}[\mu, \nu]), \right. \\ & \left. \{\sigma, 1, n\}, \{\mu, 1, n\}, \{\nu, 1, n\}, \{\lambda, 1, n\}\right]; \end{aligned}$$

listweyl := Table[If[UnsameQ[weyl[[σ, μ, ν, λ]], 0], {Subscript[C, σ - 1, μ - 1, ν - 1, λ - 1], weyl[[σ, μ, ν, λ]]}, {σ, 1, n}, {μ, 1, n}, {ν, 1, n}, {λ, 1, n}];

Print["-----"];

Print["Weyl Tensor:"];

Print[

TableForm[Partition[DeleteCases[Flatten[listweyl], Null], 2], TableSpacing -> {2, 2}]];

Print["C<sub>0121</sub> = 0

C<sub>0221</sub> = 0

C<sub>3121</sub> = 0

C<sub>0131</sub> = 0

C<sub>3221</sub> = 0

C<sub>0332</sub> = 0

C<sub>0302</sub> = 0

C<sub>0201</sub> = 0

C<sub>2121</sub> = 0

C<sub>3131</sub> = 0

C<sub>0101</sub> = 0

C<sub>0331</sub> = 0

C<sub>0303</sub> = 0

C<sub>0301</sub> = 0

C<sub>3232</sub> = 0

C<sub>0202</sub> = 0

C<sub>3231</sub> = 0"]

-----

Weyl Tensor:

{}

$C_{0121} = 0;$

$C_{0221} = 0;$

$C_{3121} = 0;$

$C_{0131} = 0;$

$C_{3221} = 0;$

$C_{0332} = 0;$

$C_{0302} = 0;$

$C_{0201} = 0;$

$C_{0123} = 0;$

```
In[*]:= geodesic := geodesic = FullSimplify[Table[-Sum[affine[[i, j, k]] × u[j] × u[k], {j, 1, n},
      {k, 1, n}], {i, 1, n}]];
listgeodesic := Table[{"d/dτ" ToString[u[i]], "=", geodesic[[i]]}, {i, 1, n}];
Print["-----"];
Print["Geodesic Equations:"];
TableForm[listgeodesic, TableSpacing → {2}]
```

-----

Geodesic Equations:

Out[\*]//TableForm=

$$\begin{aligned} d/d\tau u[1] &= \frac{4 a r w u[1] ((r-2 a (-1+\gamma)) u[2]-2 a r (\cot[\theta] u[3]+\cot[\phi] u[4]))+\tan[2 t w] (4 a^2 r^2 w^2 u[1]^2+(r-2 a (-1+\gamma))^2}{4 a^2 r^2 w} \\ d/d\tau u[2] &= \frac{2 a r w^2 u[1]^2}{r-2 a (-1+\gamma)}+2 w \tan[2 t w] u[1] \times u[2]-\left(-\frac{1}{2 a}+\frac{-2+\gamma}{r}+\frac{1}{2 a+r-2 a \gamma}\right) u[2]^2-2 \cot[\theta] u \\ d/d\tau u[3] &= u[3] \left(2 w \tan[2 t w] u[1]+\frac{u[2]}{a}-\frac{2 (-1+\gamma) u[2]}{r}-2 \cot[\phi] u[4]\right)+\frac{1}{4} \tan[\theta] \left(-4 w^2 u \right. \\ d/d\tau u[4] &= \left(2 w \tan[2 t w] u[1]+\frac{u[2]}{a}-\frac{2 (-1+\gamma) u[2]}{r}-2 \cot[\theta] u[3]\right) u[4]+\frac{1}{4} \tan[\phi] \left(-4 w^2 u \right. \end{aligned}$$