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Academic Year 2019-2020
Page created Thu Feb 20 02:15:22 GMT 2020

CATE

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Exercise Information

Module:	499 Modal Logic for Strategic Reasoning in AI	**Issued:**	Wed - 05 Feb 2020
Exercise:	6 (CW)	**Due:**	Wed - 19 Feb 2020
Title:	Coursework2	**Assessment:**	Individual
FAO:	Belardinelli, Francesco (fbelard)	**Submission:**	Electronic

Student Declaration - Version 1

For Markers only: (circle appropriate grade)

MAHAJAN, Avishkar (apm316)	00930960	t5	2020-02-19 18:01:12	A* A B C D E F

~~(a) (M, Ψ ⊨ Ψ~~

(a) $(M, \pi) \models \varphi R \psi$ iff
$\nexists j$ st. $(M, \pi[j]) \models \varphi$ ~~and~~ and $\forall 0 \leq i \ (M, \pi[i]) \models \psi$ OR
$\exists j \geq 0$ st. $(M, \pi[j]) \models \varphi$ and $\forall 0 \leq i \leq j \ (M, \pi[i]) \models \psi$

b) $\left[\left[(X\varphi) \cup \varphi \right] \wedge \psi \right] \vee \left(\neg (\text{true} \cup \neg \psi) \right)$

c) $(M, \pi) \models \neg (\text{true} \cup \neg \psi)$
   iff
   $\forall j \geq 0$
   $(M, \pi[j]) \not\models \neg \psi$
   iff
   $\forall j \geq 0$
   $(M, \pi[j]) \models \psi$ (□)

   $(M, \pi) \models \left[\left[(X\varphi) \cup \varphi \right] \wedge \psi \right]$
   iff
   $\exists \ j \geq 0$ st.
   $(M, \pi[j]) \models \varphi$ and $\forall 0 \leq i < j$ ~~(M, π[i]) ⊨ Xφ   and~~
                                                   $(M, \pi[i]) \models X\varphi$
                                                        and
                                                   $(M, \pi) \models \psi$
   iff
   $\exists j \geq 0$ st.
   $(M, \pi[j]) \models \varphi$ and $\forall \ 0 \leq i \leq j \ (M, \pi[i]) \models \psi$
                                                                               (Δ)
   Putting (Δ), (□) together we see that these

are the same conditions as in (a)

d) ~~GY~~ $(M, \pi) \models G\psi$ iff $\forall i \geq 0 \quad (M, \pi[i]) \models \psi$
$(M, \pi) \models \bot R \psi$ iff $\forall i \geq 0 \quad (M, \pi[i]) \models \psi$
Since $\not\exists j \geq 0$ st. $(M, \pi[j]) \models \bot$

$\therefore G\psi \equiv \bot R \psi$

2a) $(M, q) \models EF\phi$ iff for some path λ from q
$(M, \lambda) \models$ true $U \phi$
iff
$\exists j \geq 0$ st.
$(M, \lambda[j]) \models \phi$

$\therefore (M, q) \models EF\phi$ if for some path λ from q and
some $j \geq 0 \quad (M, \lambda[j]) \models \phi$

$(M, q) \models AF\phi$
iff
$\forall$ paths λ from ~~q~~ q we have
$(M, \lambda) \models$ true $U \phi$

iff

for all paths λ from q, $\exists j \geq 0$ st.
$(M, \lambda[j]) \models \phi$

$(M,q) \models EG\phi$ iff

$(M,q) \models \neg AF\neg\phi$

  iff

$\exists$ a path λ from q st.

  $(M,\lambda) \models \neg F\neg\phi$

   iff

   $\not\exists j \geq 0$ st. $(M,\lambda[j]) \models \neg\phi$

    iff

    $\forall j \geq 0$

    $(M,\lambda[j]) \models \phi$

$\therefore (M,q) \models EG\phi$ iff for some path λ from q, for all $j \geq 0$

  $(M,\lambda[j]) \models \phi$

$(M,q) \models AG\phi$ iff $(M,q) \models \neg EF\neg\phi$

   iff

for all paths λ from q

  we have

    $(M,\lambda) \models \neg F\neg\phi$

   iff

   $\forall j \geq 0$ we have

    $(M,\lambda[j]) \models \phi$

    so $(M,q) \models AG\phi$ iff for all paths λ from q

      for all $j \geq 0$ ~~(M,q)~~ $(M,\lambda[j]) \models \phi$

3a) If ϕ is a formula of CTL, then either

ϕ is an atom or:

ϕ is of the form $\neg\psi$ where ψ is a state formula

or ϕ is the conjunction of 2 state formulas.

or

ϕ is of the form $E\psi$ or $A\psi$ for some path

formula ψ.

If ϕ is a path formula then ϕ is of the form

$X\psi$ or $\psi U\psi$ for state formulas ψ, ψ

So Any CTL formula fits into the BNF for

CTL* so any ~~CTL*~~ CTL formula is a CTL* formula

b) $AXX\phi$ is a CTL* formula but not a CTL

formula because in a CTL formula, every

temporal operator must be immediately preceeded

by exactly one path quantifier. In the formula

$AXX\phi$ we have 2 consecutive X operators.

4.) The definition of satisfaction for state formulas is exactly the same in Def 2 and Def 1.7, 1.8.

For path formulas for CTL in Def 1.8, we have

$(m, \lambda) \models X\phi$ iff $(m, \lambda[1]) \models \phi$

where ϕ is a state fmla.

In definition 2, we have

$(m, \lambda) \models X\psi$ iff $(m, \lambda[1, \ldots \infty]) \models \psi$

where ψ is a path fmla

However by Part 6 of def 2

Satisfaction of state formulas on paths is defined in the semantics of CTL^*

So by defn 2 $(m, \lambda) \models X\phi$ iff

$(m, \lambda[1, \ldots, \infty]) \models \phi$

iff

$(m, \lambda[1, \ldots, \infty][0]) \models \phi$

iff

$(m, \lambda[1]) \models \phi$ by part 6 (here ϕ is a state fmla)

So we recover the conditions of 1.8

Similary by definition 2 we have

$(M, \lambda) \models \phi \cup \psi$

iff

$(M, \lambda[i...\infty]) \models \psi$ for some $i \geq 0$ and

$(M, \lambda[j...\infty]) \models \phi \quad \forall \quad 0 \leq j < 1$ (where ϕ, ψ are state fmlas)

Like before using def satisfiability of
state fmlas on paths
we get by part 6 of definition 2

$(M, \lambda[i...\infty]) \models \psi \Longleftrightarrow$
$(M, \lambda[i...\infty][0]) \models \psi$

$\Longleftrightarrow$

$(M, \lambda[i]) \models \psi$

and similarly for ϕ
so by definition 2 we have

$(M, \lambda) \models \phi \cup \psi \Longleftrightarrow$

$\exists i$ st. $(M, \lambda[i]) \models \psi$ and $\forall \quad 0 \leq j < i$

$(M, \lambda[j]) \models \phi$.

this is the ~~des~~ condition of
1.8

5a) We know that every formula in CTL is a formula in CTL' by (3a). Moreover the semantics of CTL^* reduce to the semantics of CTL when restricted to CTL formulas by (4). So for a CTL formula ϕ, setting $\phi' = \phi$ we get the desired result.

b) Consider the CTL* formula $AFGa$.

we know that for any state formula ϕ

$$AF\phi \equiv \phi \vee AXAF\phi$$

so $\phi \equiv AF\phi \wedge \neg AXAF\phi$

so if $AFGa$ can be written as a CTL formula ψ

then $\exists$ a CTL fmla $\chi \equiv \psi \wedge (\neg AX\psi)$

where $\chi \equiv Ga$

so $\exists$ a CTL fmla ψ st.

for (m, s_0)

$(m, s_0) \models Ga$ iff $(m, s_0) \models \chi$

but $(m, s_0) \models Ga$ iff $(m, s_0) \models AGa$

so $\psi \equiv AGa$

but we know from lectures that

$AFAGa \not\equiv AFGa$. Consider the model

we have $s_0 \models AFGa$ but $s_0 \not\models AFAGa$

since

$(s_0)^\omega \not\models FAGa$ because $s_0 \not\models AGa$

because $s_1 \not\models a$

so $AFGa$ cannot be written as a CTL fmla.

6) We prove by induction on connectives, quantifiers and operators

Let ϕ be a state fmla

Say ϕ is an ~~fm~~ atom P.

then
$(M,t) \models P$ iff $(M',t') \models P$ since

$t \in V(P)$ iff $t' \in V'(P)$ due to bisimilarity

If ϕ is of the form $\neg \psi$
Assume $(M,t) \models \psi$ iff $(M',t') \models \psi$
then $(M,t) \models \neg \psi$ iff

$(M,t) \not\models \psi$ iff $(M',t') \not\models \psi$ iff

$(M',t') \models \neg \psi$

If ϕ is of the form $\psi \wedge \chi$
Assume $(M,t) \models \psi$ iff $(M',t') \models \psi$ and likewise for χ.
$(M,t) \models \psi \wedge \chi$ iff $\left[(M,t) \models \psi \text{ and } (M,t) \models \chi \right]$ iff

$\left[(M',t') \models \psi \text{ and } (M',t') \models \chi \right]$ iff

$(M',t') \models \psi \wedge \chi$

Say
ϕ is of the form $E\psi$, where ψ is a path formula

$(M,t) \models E\psi \implies \exists$ a path π starting at t st.

$(M,\pi) \models \psi$

~~Then~~ We have $R(t, \pi[1])$, then by the forth condition
of a bisimulation, we know that
$\exists t_1 \in M'$ st. $R'(t', t_1')$ and $B(\pi[1], t_1')$

Again $R(\pi[1], \pi[2])$ so by the forth condition
$\exists \ t_2' \in M'$ st. $R'(t_1', t_2')$ and $B(\pi[2], t_2')$
we can continue this to obtain a path
~~$t' t'$~~ $t', t_1', t_2' \ldots = \pi'$
~~st. π is b~~ π is bisimilar to π'
similarly by using the back condition repeatedly
for every path in M' ~~we~~ starting from t' we
can obtain a bisimilar path in M, starting from t.

It remains to prove that the truth of
path formulae is preserved on bisimilar paths,
we do that first, before going back to state
formulae.
We do this by induction on the number of
connectives and temporal operators.

Say a path formula x is of the
form $X\psi$
and we know that given bisimilar paths
π, π' in M and M' resp.

$(M, \pi) \models \psi$ iff $(M', \pi') \models \psi$

~~Say (~~

so
$(M, \pi) \models X\psi$ iff $(M, \pi[1\ldots\infty]) \models \psi$

iff
$(M', \pi'[1\ldots,\infty]) \models \psi$ (since $\pi[1\ldots,\infty]$ is bisimilar to
$\qquad\qquad\qquad\qquad \pi'[1\ldots,\infty]$)

$[\Rightarrow)$

$(M', \Pi') \models X \psi$

Say ϕ^x is of the form $\phi U \psi$
Assume $(M, \Pi) \models \phi$ iff $(M', \Pi') \models \phi$ and likewise for ψ
$(M, \Pi) \models \phi U \psi$

iff

$\exists j \geq 0$ st. $(M, \Pi[j \ldots, \infty]) \models \psi$

and $\forall \, 0 \leq i < j$

$\qquad (M, \Pi[i \ldots, \infty]) \models \phi$

iff

$(M', \Pi'[j \ldots, \infty]) \models \psi$

and

$\forall \, 0 \leq i < j$

$(M', \Pi'[i \ldots, \infty]) \models \phi$

iff

$(M', \Pi') \models \phi U \psi$

The case for conjunction, negation of path fmlas is similar to state fmlas
so bisimilar paths preserve truth of path
formulas.

Now considering state fmlas again,

Say we have $(M, t) \models E \psi$ and $B(t, t')$

$\Rightarrow \exists$ a path π starting at t s.t.

$\quad (M, \pi) \models \psi$

we know $\exists$ a bisimilar path π' starting at t' in M'

so

$\quad (M', \pi') \models \psi$

$\quad \Rightarrow (M', t') \models E\psi$

and the converse direction follows similarly from the fact that every path in M' has a bisimilar path in M

If ϕ is of the form $A\psi$ then:

$\quad \cancel{M, t \models A\psi} \quad (M, t) \models A\psi$

$\quad \Longleftrightarrow \cancel{M \not\models E\neg\psi} \quad (M, t) \not\models E\neg\psi$

$\quad \Longleftrightarrow (M', t') \not\models E\neg\psi$

$\quad \Longleftrightarrow (M', t') \models A\psi$

So the truth of CTL formulas is preserved by bisimulations.

7.) Say $t \in M$, $t' \in M'$ are CTL equivalent.

then clearly for ~~all atoms p~~ any arbitrary
atom p

$\quad (M, t) \models P$ iff $(M, t') \models P$

$\quad \quad -: t \in V(P)$ iff $t' \in V'(P)$

~~Say for a t~~

Define the relation $B \in St \times St'$ by
~~B(t,t)~~ $B(a, a')$ iff a, a' are CTL equiv.
We show that B is a bisimulation
and $B(t, t')$.
we have already verified condition 1.
For the forth condition
Say for a contradiction

$R(t, u)$ but $\not\exists$ $u' \in M'$ st. $R(t', u')$ and
$B(u, u')$

Let $S' = \{ w' \in M' \mid R(t', w') \}$
then for every $w'_i \in S'$, $\exists$ a CTL fmla ψ_i

st. $(M, u) \models \psi_i$ but $(M', w'_i) \not\models \psi_i$

$\Rightarrow (M, t) \models EX(\psi_1 \wedge \cdots \wedge \psi_n)$ but $(M', t') \not\models EX(\psi_1 \wedge \cdots \psi_n)$

This contradicts the fact that (t, t')
are CTL equiv.

Similarly for the back condition, ~~say we~~
say for a condriction

$R'(t', v')$ and $B(t, t')$
but $\not\exists$ v st. $R(t, v)$ and $B(v, v')$

Let $S' = \{w \in M \mid R(t, w)\}$
then $\forall w_i \in S'$ $\exists$ a CTL fmla ψ_i st.

$(m', v') \models \psi_i$ but $(m, w_i) \not\models \psi_i$

$\therefore (m', t') \models EX(\psi_1 \wedge \psi_2 \dots \wedge \psi_n)$

but $(m, t) \not\models EX(\psi_1 \wedge \psi \dots \wedge \psi_n)$

this contradicts the fact that t, t' are
CTL equiv.

Hence the back condition is also · verified

so $(M, t), (m', t')$ are CTL equivalent

$\Rightarrow t, t'$ are bisimilar

8.) $(M, t), (m', t')$ ~~are~~ are CTL equiv iff they are
Bisimilar because bisimilarity preserves the truth of
CTL* fmlas (Hence CTL fmlas) and CTL equiv $\Rightarrow$ bisimilarity
by (7)

$(m, t), (m' t')$ are bisimilar $\Rightarrow (m, t), (m', t')$ are CTL* equiv
by (6)

$(M, t), (M', t')$ are CTL^* equiv

$\Rightarrow$ they are CTL equiv

$\Rightarrow$ they are bisimilar by (7)

So.

$(M^\circ, t), (M', t')$ satisfy the same fmlas of CTL

iff

they are bisimilar

iff

$(M, t), (M', t')$ are CTL^* equiv

so we have CTL equivalence iff we have CTL^* equivalence.

This is akin to how a Polynomial over $\mathbb{R}$ can be uniquely specified by a sequence of real numbers, but to write down its roots you need complex numbers which is a more expressive language.

So in the domain of Poly's over $\mathbb{R}$

$\mathbb{R}$ and $\mathbb{C}$ have the same distinguishing power even though $\mathbb{C}$ is more expressive than $\mathbb{R}$

Out of **49**

1			
a/**2**	b/**2**	c/**3**	d/**3**
Solution almost correct but the 'there does not exist a j' statement is not required	Solution could have been simplified futher		Equivalence shown but the proof itself is lacking any explanation of the steps
1	1	3	2

2			
a/**2**	b/**2**	c/**2**	d/**2**
2	2	2	2

3	
a/**3**	b/**2**
3	2

4
/**5**
5

5	
a/**2**	b/**2**
2	2

6	7	8
/**6**	/**6**	/**5**
		An interesting, and apt analogy! Well written
6	6	5