Chaos

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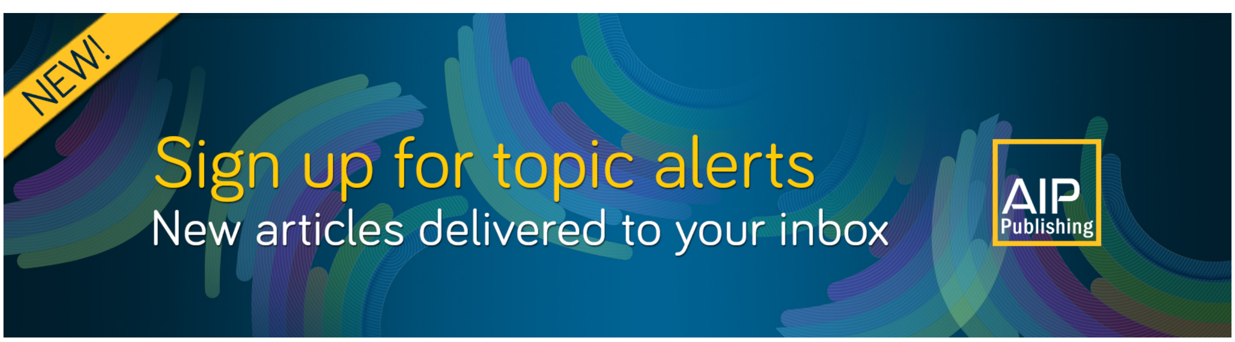

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ABSTRACT

The network of self-sustained oscillators plays an important role in exploring complex phenomena in many areas of science and technology. The aging of an oscillator is referred to as turning non-oscillatory due to some local perturbations that might have adverse effects in macroscopic dynamical activities of a network. In this article, we propose an efficient technique to enhance the dynamical activities for a network of coupled oscillators experiencing aging transition. In particular, we present a control mechanism based on delayed negative self-feedback, which can effectively enhance dynamical robustness in a mean-field coupled network of active and inactive oscillators. Even for a small value of delay, robustness gets enhanced to a significant level. In our proposed scheme, the enhancing effect is more pronounced for strong coupling. To our surprise even if all the oscillators perturbed to equilibrium mode were delayed negative self-feedback is able to restore oscillatory activities in the network for strong coupling strength. We demonstrate that our proposed mechanism is independent of coupling topology. For a globally coupled network, we provide numerical and analytical treatment to verify our claim. To show that our scheme is independent of network topology, we also provide numerical results for the local mean-field coupled complex network. Also, for global coupling to establish the generality of our scheme, we validate our results for both Stuart–Landau limit cycle oscillators and chaotic Rössler oscillators.

Studying a network of coupled oscillators provides a great insight to unravel self-organizing activities in various complex systems in physical biological and engineering sciences. Examples include neural networks, circadian rhythms, and power grids. In such systems, robust oscillatory dynamics are a prerequisite for their proper functioning. Emergent rhythmic activities of such a large-scale system should be stable against local perturbations. In the recent past, researchers have spent a considerable amount of time investigating the robustness of rhythmic activities in a network of coupled oscillators when a fraction of the dynamical components are deteriorated or functionally degraded but not removed. In such a scenario, the regular functioning of many natural and man-made systems may face severe disruption. Therefore, it has substantial practical importance to present a control mechanism that will help to maintain regular oscillatory dynamics in a network of coupled oscillators that is experiencing local degradation of dynamical activities. In the present study, we demonstrate that self-feedback delay very effectively redeems the oscillatory dynamics in a mean-field coupled damaged network. Our proposed mechanism is independent of coupling topology as well as local subsystems. Our investigation provides significant new insight into the role of self-feedback delay to inflate the oscillatory dynamics in a damaged network of oscillators, which will have a significant impact in invoking broad interest in the community of nonlinear studies.

I. INTRODUCTION

Exploring emergent dynamics of a vast network of coupled oscillators got considerable attention in recent years due to its applicability to understand various self-organized complex systems.[1–3] The emergent behaviors of such complex systems depend on the individual dynamics of local sub-units as well as the network topology. The normal functioning of many natural systems requires stable and robust oscillatory dynamics. Therefore, the macroscopic dynamics of such a large-scale system should exhibit resilience against local perturbations.

In recent times, much attention has been invested in understanding the dynamical robustness, which is defined as the ability of a network of oscillators to regulate its rhythmic activity when a fraction of the dynamical units is malfunctioning.[4] Physically, this situation can be modeled as a network of coupled oscillators where oscillatory nodes switch to fixed points progressively.[5] If the number of nodes which perturbed to fixed point reaches a critical level, the normal oscillatory activities of such systems will hamper and might have sudden phase transition to non-oscillatory state. This catastrophic emergent phenomenon is described as aging transition.[4–13] The aging transition might have catastrophic effects in many natural and real-world systems such as metapopulation dynamics in ecology, neuronal dynamics in brain, cardiac oscillations, and power-grid network.[14–17] Therefore, it is of great practical significance to propose some remedial measures or control mechanisms to enhance the dynamical robustness of the coupled systems against the aging or deterioration of the individual unit. Till now, researchers mainly studied aging transition not only considering different network topologies but also using different coupling functions.[4,18,19] Some recent efforts have been directed to explore the possible mechanisms to enhance the dynamical robustness to avoid aging transition. Liu *et al.*[10] proposed a mechanism for robustness enhancement that involves an additional parameter to control the diffusion rate. Kundu *et al.*[20,21] have shown that the robustness can be enhanced by a positive feedback mechanism as well as asymmetric couplings. Bera[22] has employed low pass filtering mechanism for enhancing dynamical robustness. Despite such attempts, enhancement mechanisms are yet to be explored fully and deserve significant attention.

All the previous studies on the aging transition considered nearest neighbor diffusive coupling, which effectively depicts many real-world complex systems. However, for many biological and physical systems, the mean-field coupling is also relevant. It has been shown that the diffusion process for a network of genetic oscillators can be modeled as mean-field coupling through a regulation known as quorum-sensing.[23,24] A network of mean-field coupled oscillators can model the dynamics of suprachiasmatic nucleus (SCN) neurons,[25] which produces circadian oscillations. In this article, we explore a possible mechanism to augment the dynamical robustness of a mean-field coupled oscillators.

The feedback mechanism is considered as one of the main themes of scientific understanding of the last century and has been widely used in control theory.[26] The positive feedback which favors the system's instability in the dynamical system has been used extensively in neural networks, genetic networks, etc.[27] In the context of dynamical systems, positive feedback with time delay plays a crucial role and has been used by Pyragas to control chaotic dynamics through the stabilization of unstable periodic orbits[28] embedded in the chaotic attractor. Pyragas[28] proposed a method based on the feedback perturbation constructed in the form of the difference between the delayed output and the output signal. The stabilization is achieved by adjusting the weight of perturbation upto the one period of the unstable periodic orbit so that the control signal vanishes when the stabilization of the desired orbit is attained. Pyragas controller consists of both negative feedback of the output signal as well as delayed positive feedback of the output signal. Hövel and Schöll[29] implemented this controller successfully to control unstable steady state. On the other hand, negative feedback generally promotes stability.[30–32] But the negative feedback with time delay favors oscillatory dynamics in the system,[33,34] a scenario that we explore in this work as well.

It is a well-established fact that time delays are an essential part of many natural systems.[33,35] In recent years, more and more systems are being recognized to be influenced by or to be describable via a delayed coupling. Time delay due to the finite propagation speed of the external signal, which is described as propagation delay, has been widely used to control the dynamics of coupled nonlinear systems.[36,37] The internal self-feedback delay appears because sometimes the system needs a finite time to process the received signal. It is demonstrated that such type of local self-feedback delay plays a crucial role in reviving oscillations or amplitude death and oscillation death.[38] In comparison to propagation delay, the effects of self-feedback delay in coupling are very less explored. There are only a few instances where systems with self-feedback delay have been investigated.

In the context of coupled oscillators, feedback has been widely used for control of network dynamics and synchronization.[39,40] However, the effects of feedback in the aging transition have not been well explored. Only recently, Kundu *et al.*[20] bring out an in-depth study on the effects of external positive feedback to increase the dynamical persistence of a network of oscillators. In this work, we study the ability of negative self-feedback with a time delay to enhance the dynamical persistence of a network that is experiencing an aging transition. We have shown that delayed self-feedback is an effective control mechanism to enhance the dynamical robustness for global as well as local mean-field coupled network. Even when the amount of the local self-feedback delays is minimal, it effectively enhances the dynamical robustness of the network. We present an in-depth study of the global network. For a global network to show that our enhancement mechanism is independent of the model, we provide results for the Stuart–Landau limit cycle system as well as the chaotic Rössler system. We elucidate our results, both analytically and numerically. To show that our method is independent of coupling topology, we also present numerical results for mean-field coupled oscillators interacting via complex network topology.

II. GLOBAL MEAN-FIELD COUPLED NETWORK

In this section, we present an extensive study of the effect of negative self-feedback with a time delay to strengthen the dynamical robustness in a global mean-field coupled network. Here, global coupling signifies that all the dynamical units have equal contribution to the mean-field, which acts equally on all the units. We consider N mean-field coupled Stuart–Landau oscillators with timed delayed self-feedback. Mathematically, one can write the governing equation of motion as

$$\dot{z}_j(t) = (\rho_j + i\omega - |z_j(t)|^2)z_j(t) + k[\bar{z} - z_j(t-\tau)], \tag{1}$$

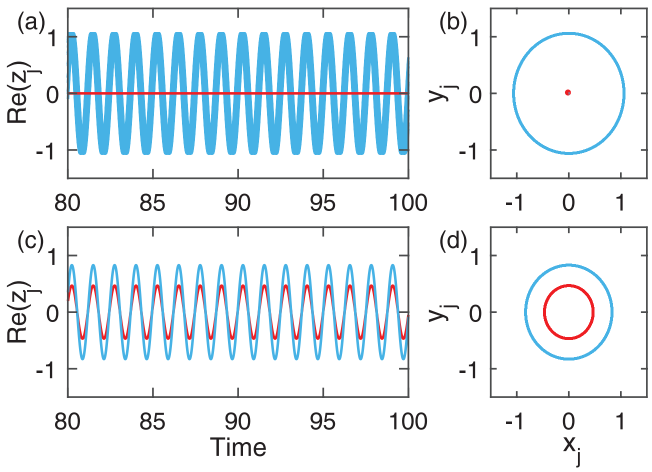

FIG. 1. Time series of real part of z_j and phase portrait of $N = 500$ mean-field coupled Stuart–Landau oscillators consisting of active nodes (blue) and inactive nodes (red) are plotted with (a) and (b) $k = 0$ and (c) and (d) $k = 2$ at the inactivation ratio $p = 0.2$.

for $j = 1, 2, \ldots, N$, where $z_j = x_j + iy_j$ is the complex amplitude of the jth oscillator and $\bar{z}$ is the average mean-field interaction, which describes the influx of the mean-field in dynamical systems. Here, τ accounts for the delay in the negative self-feedback term $z_j(t - \tau)$, which basically acts as a controller to enhance dynamical robustness. Here, ω $(= 5)$ is the internal frequency of each oscillators and k for the coupling strength. ρ_j is the bifurcation parameter of the jth oscillator and gives rise to a supercritical Hopf bifurcation at $\rho_j = 0$. In isolation ($k = 0$), the jth Stuart–Landau oscillator displays a stable sinusoidal oscillation for $\rho_j = a(> 0)$ and converges to stable fixed point $z_j = 0$ for $\rho_j = b(< 0)$.

Here, aging implies an active oscillator with $\rho_j = a > 0$ turns into inactive mode with $\rho_j = b < 0$ through some perturbations. Without loss of generality, one can set the group of oscillators $j \in \{1, 2, \ldots, [N(1 - p)]\}$ as active ones and the remaining oscillators $j \in \{[N(1 - p) + 1], \ldots, N\}$ as inactive. Here, N is the total number of oscillators in the network, and p is the fraction of inactive oscillators. As the inactivation ratio p reaches a critical value, the global oscillation of the network dies out. Following Daido and Nakanishi,[5] we define an order parameter $Z = \frac{|Z(p)|}{|Z(0)|}$ to quantify the dynamics of the system, which defines the average magnitude of global oscillation in the network, where $|Z(p)| = N^{-1} \sum_{l=1}^{N} |z_l|$. We consider the aging transition, when Z is less than 10^{-4}. We have taken the network size $N = 500$ (results are also valid for larger network size) and consider the value of $a = 1$ for active and $b = -3$ for inactive oscillators. The numerical integrations in this whole work were performed for random initial conditions by means of the modified RK4 method for delay differential equation with time steps 0.002.[41] In the absence of any coupling ($k = 0$), the dynamics of all the nodes divide into two groups of phase synchronized oscillation and stable fixed points. For $p = 0.2$, the time series and phase portrait of z_j is shown in Figs. 1(a) and 1(b). As we incorporate the coupling in the network, the inactive oscillators start oscillation under the influence of active oscillators. Both groups of oscillators show the synchronized motion with a different amplitude which is shown in Figs. 1(c) and 1(d). In Fig. 2(a), we have plotted Z against the inactivation ratio p for different values of local self-feedback delay τ for a fixed coupling

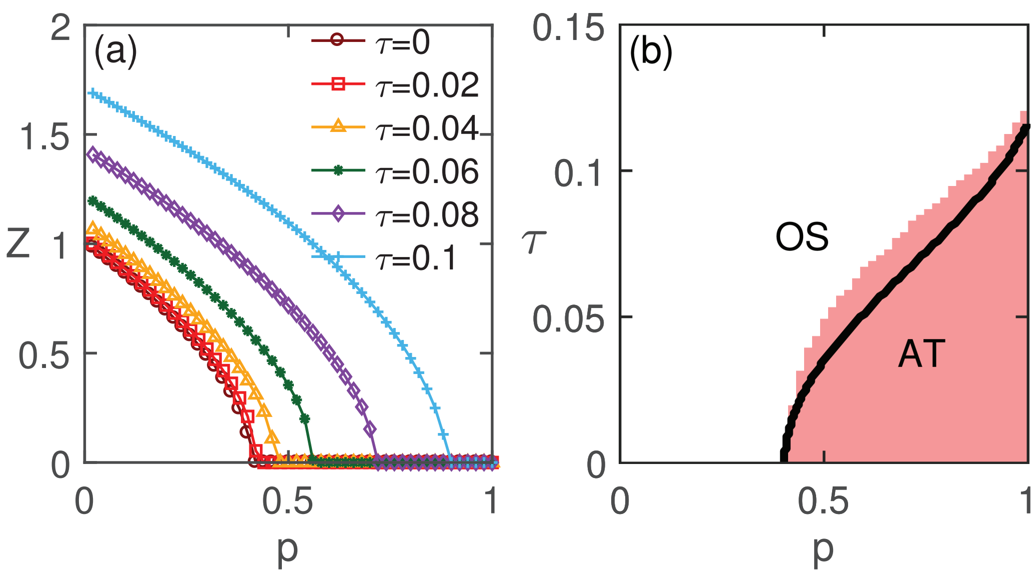

FIG. 2. (a) The order parameter Z as a function of the inactivation ratio p for various values of τ in a mean-field coupled network of $N = 500$ Stuart–Landau oscillators for $a = 1, b = -3, \omega = 5$, and $k = 5$. (b) The phase diagram of the coupled oscillator in the $(p - \tau)$ parameter plane at fixed $k = 5$, where white region belongs to the oscillatory state (OS), light red domain is the region of stable fixed point dynamics, and black line is obtain from the critical value of aging transition p_c obtained from Eq. (6).

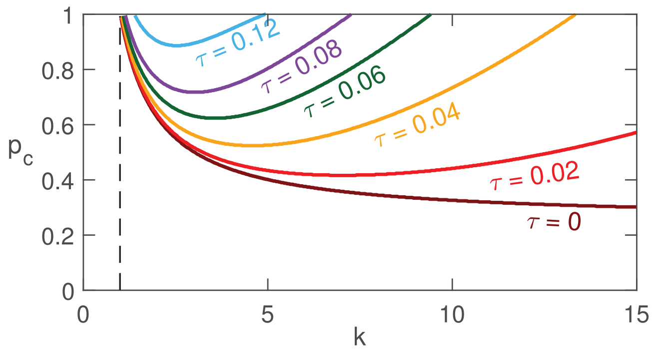

FIG. 3. The critical value of aging transition p_c as a function of coupling strength k for different values of local self-delay τ in the mean-field coupled oscillators, where p_c monotonically decreases for increasing of k.

strength $k = 5$. One can observe that in the absence of local self-feedback delay τ, the order parameter Z vanishes at a lower value of $p = p_c$. This implies that the aging transition takes place much faster. As we increase the local self-feedback delay τ, the aging transition can be observed at a higher value of $p = p_c$. So one can conclude that local delayed self-feedback plays an important role in enhancing the dynamical robustness. For a better understanding of the effect of local self delay feedback in the coupled oscillators, we have shown in Fig. 2(b) the phase transition diagram in the plane $\tau - p$ for fixed $k = 5$. In this figure, region OS and AT denote the oscillatory state and death state (where $Z = 0$), respectively. It is evident from the figure that when τ is a minimal change in the p_c value insignificant. But, as we increase the value of τ on higher side, the critical value of p_c increases and reach to $p_c = 1$ for $\tau = 0.12$. It shows that the local self-feedback delay τ dominates the aging transition in the coupled oscillator and enhances the dynamical robustness of mean-field coupled oscillators.

Next, we find the critical value p_c analytically. The global oscillation collapses at p_c during aging transition and the fixed point $z_j = 0$ is stabilized. We assume that the coupled system build of two groups, where all nodes have synchronized activity in each group which permits us to reformulate the system in such a way. By setting $z_j = A$ for the active group and $z_j = I$ for the inactive group of oscillators, the original equation (1) reduces to the following coupled systems:[5]

$$\begin{aligned}\dot{A}(t) &= (a + i\omega + kq - |A(t)|^2)A(t) - kA(t-\tau) + kpI(t),\\ \dot{I}(t) &= (b + i\omega + kp - |I(t)|^2)I(t) - kI(t-\tau) + kqA(t),\end{aligned} \tag{2}$$

where $q = 1 - p$. Now, we carry out linear stability analysis to reduce Eq. (2) around the fixed point $A = I = 0$. In order to find the critical value of p_c. The origin ($A = I = 0$) is stabilized if the real parts of the eigenvalues become negative. Therefore, the critical inactivation ratio p_c can be derived when two complex conjugate eigenvalues of above equations intersect the imaginary axis. A linear stability analysis around the origin now gives a characteristic equation of the form,

$$\begin{aligned}&(a + i\omega + qk - k\,e^{-\lambda\tau} - \lambda) \times (b + i\omega + pk - k\,e^{-\lambda\tau} - \lambda)\\ &\quad - pqk^2 = 0.\end{aligned} \tag{3}$$

Here, $\lambda = \lambda_R + i\lambda_I$. By taking the real part of the eigenvalue equal to zero ($\lambda_R = 0$) and separate the real and imaginary part of Eq. (3), we

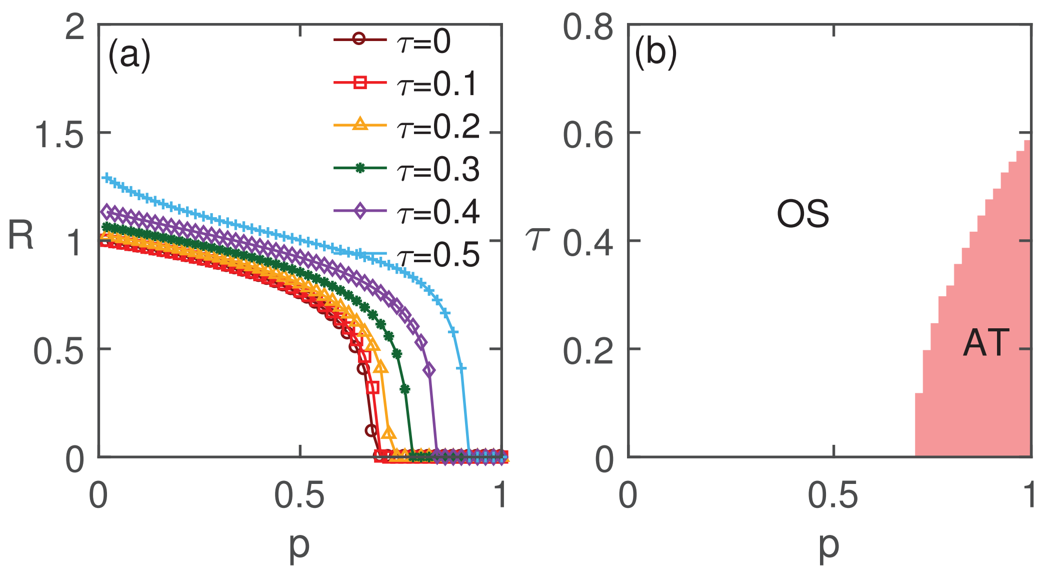

FIG. 4. (a) The order parameter R is plotted as a function of the inactivation ratio p at the various values of τ in mean-field coupled network of $N = 500$ Rössler oscillators for $e = 5.7$ and $k = 0.2$. (b) The phase diagram of coupled oscillators in the $(p - \tau)$ parameter plane at fixed $k = 0.2$, where the white and light red region belongs to the oscillatory state (OS) and the aging transition (AT) region.

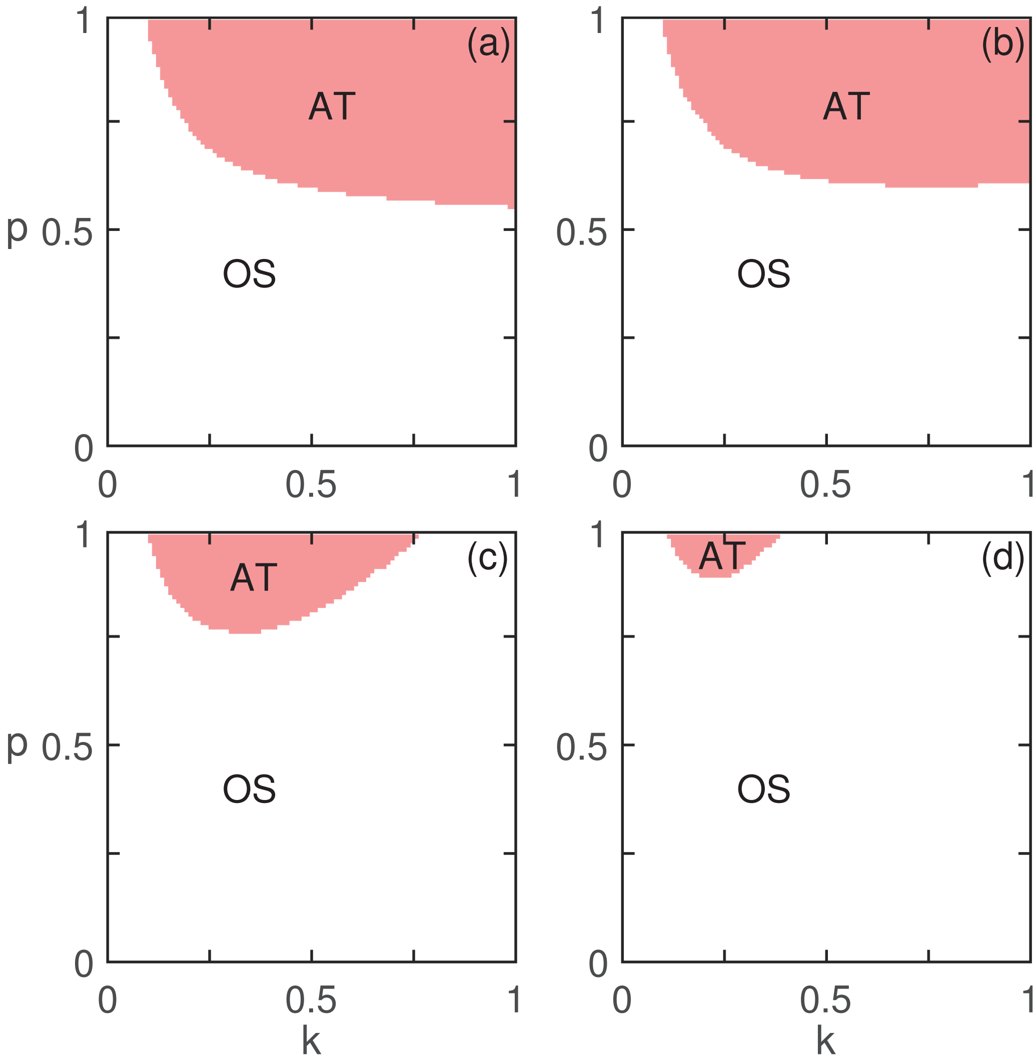

FIG. 5. The phase diagram in the (p, k) parameter plane for the network of mean-field coupled Rössler oscillators. The aging transition (AT) and oscillatory state (OS) regions are characterized by the light red and white color for different values of local self-delay (a) $\tau = 0$, (b) $\tau = 0.2$, (c) $\tau = 0.4$, and (d) $\tau = 0.6$.

obtain the following equations:

$$[a + qk - k\cos(\lambda_I\tau)][b + pk - k\cos(\lambda_I\tau)] = (\omega - \lambda_I + k\sin(\lambda_I\tau))^2 + pqk^2, \tag{4}$$

$$[a + b + k(p + q) - 2k\cos(\lambda_I\tau)][\omega - \lambda_I + k\sin(\lambda_I\tau)] = 0, \tag{5}$$

where $p + q = 1$. Solving the above equations, we get the critical value of inactivation ratio p_c as

$$p_c = \frac{-ab + k(a+b)\beta + k^2\beta - kb - k^2\beta^2}{k(a-b)}, \tag{6}$$

where $\beta = \cos(\alpha\tau)$ and $\alpha = \omega + k\sqrt{1 - \left(\frac{a+b+k}{2k}\right)^2}$.

The occurrence of the aging transition is featured by the existence of critical parameter p_c. The analytical value of p_c (black solid line) also has a good agreement with the numerical result (shaded region) for the aging transition as shown in Fig. 2(b). There is a tiny disagreement due to the fact that the reduced system is actually an approximation of the network of coupled oscillators.

When the local self-feedback delay $\tau = 0$, the aging transition takes place for all $k > 1$ and p_c decreases as coupling strength k increases. To explore the impact of k on the parameter p_c, we plot p_c as a function of coupling strength k for different τ values in Fig. 3. Surprisingly, we observe that for non-zero τ, the aging transition exists only for a finite interval of coupling strength k. Within this range, p_c gradually decreases till it reaches its minimum value and then again monotonically increases to unity. The above

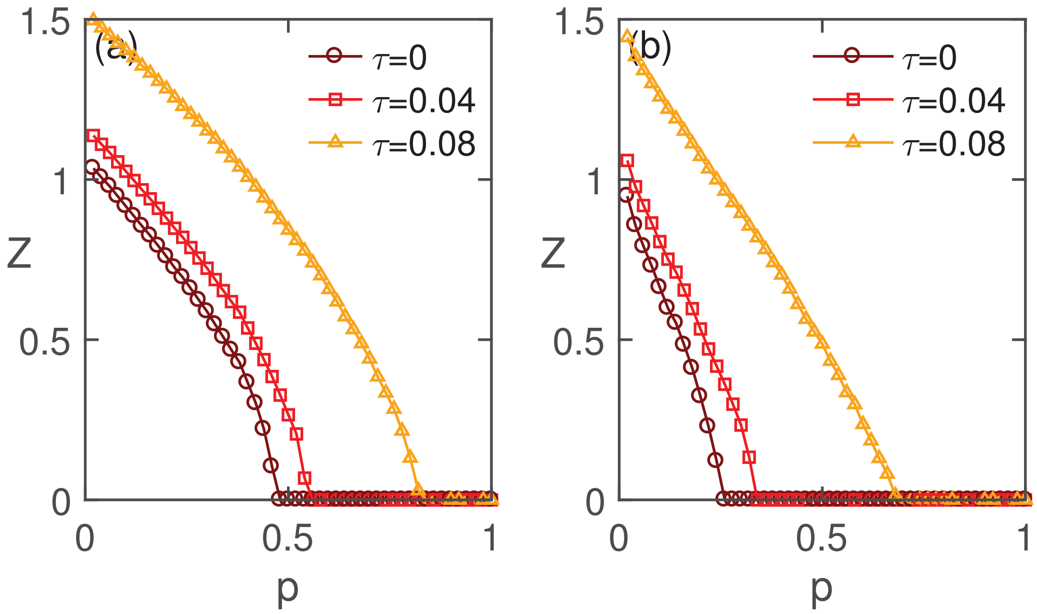

FIG. 6. The order parameter Z as a function of the inactivation ratio p for various values of τ in complex network of $N = 500$ Stuart–Landau oscillators for $a = 1, b = -3, \omega = 5$, and $k = 5$ for (a) the random network for probability of connecting two distinct pair of nodes $\gamma = 0.02$ and (b) the scale-free network of average degree $\langle m \rangle$=8, and the the power-law exponent $\eta = 3.0$.

observation implies that strong coupling favors dynamical resilience of the network against aging when feedback delay τ is large enough.

To demonstrate that our enhancement method is independent of the dynamical behavior, we choose a chaotic Rössler system.[42] The governing equation of N mean-field coupled Rössler oscillators with negative self-feedback delay can be written as

$$\begin{aligned} \dot{x}_j(t) &= -y_j(t) - z_j(t) + k[\bar{x} - x_j(t-\tau)], \\ \dot{y}_j(t) &= x_j(t) + r_j y_j(t) + k[\bar{y} - y_j(t-\tau)], \\ \dot{z}_j(t) &= r_j + z_j(t)(x_j(t) - e) + k[\bar{z} - z_j(t-\tau)], \end{aligned} \tag{7}$$

where $j = 1, 2, \ldots N$. $r_j, e(= 5.7)$ are the intrinsic parameters of Rössler oscillator. $\bar{x}, \bar{y}$, and $\bar{z}$ are the mean-field term of x, y, z variables. r_j is the bifurcation parameter of the jth Rössler oscillator which gives rise to a supercritical Hopf bifurcation at $r_j = 0$. With further increase of parameter r_j one can observe period doubling route to chaos. We set $r_j = -0.2$ for $j = 1, \ldots, pN$ for the inactive oscillators falls into a fixed point and $r_j = 0.2$ for $j = pN + 1, \ldots, N$ for active oscillators which shows the the chaotic oscillations. To study the aging transition in chaotic oscillators, we measure the amplitude of oscillation $R = \frac{M(p)}{M(0)}$, where M is define as[5]

$$M = \sqrt{\langle (X_c - \langle X_c \rangle)^2 \rangle}, \tag{8}$$

$X_c = N^{-1} \sum_{j=1}^{N} (x_j, y_j, z_j)$ is the centroid and $\langle . \rangle$ means it calculated for long time average ($\approx 10^5$ iterations). We consider the coupled system goes through the aging transition, when M is less than 10^{-4}.

We have plotted order parameter R in Fig. 4(a) against the inactivation ratio p for the fixed value $k = 0.2$ at different values of τ. In the absence of local self-delay $\tau = 0$, the brown line of R demarcated the critical aging transition point at $p_c = 0.72$, and when τ is increased, the transition threshold point p_c increase to the higher value. It shows that local delayed self-feedback effectively enhances the robustness. In Fig. 4(b), we have shown the phase diagram in $\tau - p$ parameter plane for a fixed coupling strength $k = 0.2$, where two regions are observed, oscillatory state (OS) and aging transition (AT). The critical value p_c of aging transition from oscillatory to fixed point (where $R = 0$) increases with τ gradually. For Rössler model also we investigate the effects of coupling strength on robustness. We have shown the phase diagram in $(p - k)$ parameter space for different values of τ in Fig. 5. The light pink region in the parameter plane shows the aging transition where order parameter R falls to zero. It clearly shows that the AT region decreases in size for the higher values of τ, which is quite similar to the results we have obtained for Stuart–Landau oscillators. It confirms that self-feedback delay τ is also very useful in inflating the dynamical robustness of a coupled chaotic oscillator network.

III. COMPLEX NETWORK TOPOLOGY

Next, we demonstrate the effectiveness of our enhancement technique when the oscillators are interacting through a complex network topology.[43,44] We Consider N mean-field coupled Stuart–Landau oscillators interacting via a complex bidirectional network topology. The mathematical model of the coupled system is given by

$$\dot{z}_j(t) = (\rho_j + i\omega - |z_j(t)|^2)z_j(t) + k\left(\frac{\sum_{l=1}^{N} A_{jl} z_l(t)}{d_j} - z_j(t-\tau)\right), \quad (9)$$

for $j = 1, 2, \ldots, N$ and $N = 500$. Here, A_{jl} is the adjacency matrix of the connection in the complex network, i.e., $A_{jl} = 1$ if jth and lth nodes are connected and zero otherwise. Here, we assume that all coupled oscillators are interacting with each other through a specific network topology and all the connections between them have the same coupling strength. d_j is the degree of node j, and k is the coupling strength.

In this present study, we have considered two different complex networks, namely, random network and scale-free network.[45] Probability degree distribution of a node in a random network follows a binomial or Poisson distribution. In contrast, a scale-free networks obey a heavy-tailed degree distribution that can be approximated by a power law [$P(k) \approx k^{-\eta}$, where $P(k)$ is the probability of having a node of degree m and η is the power-law exponent].

First, we generate the random network using the Erdös–Rényi algorithm for $N = 500$, where the probability of connecting two distinct pair of nodes $\gamma = 0.02$.[46] In Fig. 6(a), we have plotted the order parameter Z as a function of inactivation ratio p for different values of self-feedback delay term τ. It demonstrates the fact the p_c value increases as we increase τ. Even for a small amount of delay τ, dynamical robustness gets enhanced to a significant amount. We have similar results for the scale-free network, which is generated by using the B-A algorithm for $\eta = 3$.[47] In Fig. 6(b), we have shown how p_c value increases as we increase τ. Here, we have considered only random inactivation of nodes. If we consider targeted inactivation,[4] then we will get similar qualitative behavior, which means that due to the effects of negative self-feedback delay robustness will increase even though p_c value might change. These outcomes show that local self-feedback with time delay is very efficient in enhancing the dynamical robustness when the oscillators are interacting through a complex network topology. It also establishes the fact that our enhancement technique is independent of coupling topology.

IV. CONCLUSION

In this paper, we have demonstrated that the introduction of a negative self-feedback with a delay into the mean-field coupled oscillators can effectively increase the dynamical robustness, which is exhibited by enhancing the endurance of the oscillatory dynamics of the network against the aging of the individual nodes. We have demonstrated that our enhancement technique is applicable for both global as well as local mean-field coupling. We have shown that the critical vale p_c at which the aging transition takes place is positively correlated with the small delay τ, while for higher value aging transition does not take place. It means the network becomes more resilient to the effect of the local deterioration of oscillatory nodes with increases of τ.

We have done a comprehensive, detailed study for the global network. We have seen that the enhancing effect of negative self-feedback with delay is more prominent for a strong coupling term k. To our surprise, we have found that for a non-zero τ, the aging transition exists for a finite length of coupling strength k. Within this range, p_c first diminishes from maximum to a minimum value and then again increases to the maximum. The global oscillations can be restored by local self-feedback with delays in coupled networks of purely non-oscillatory units. The introduction of delayed self-feedback in the coupling provides a straightforward but highly valuable technique for recovering dynamical activities in the network, whose oscillatory behavior has been weakened due to the aging of some elements. We have delineated the results using numerical simulations as well as analytical findings. To state that our scheme is independent of local subsystems, we have successfully employed our control mechanism to a network of coupled chaotic Rössler oscillators.

We have also studied the role of negative self-feedback with a time delay to enhance dynamical robustness when the oscillators interact through a complex network topology. The results demonstrate the fact that our enhancement technique is effective in different types of network topology.

Our study might have applicability in enhancing the resilience of several natural systems like chemical oscillators,[48] electro–optic oscillator[49] that can experience aging in its local dynamical units. Finally, our proposed framework widens the understanding of the roles of negative self-feedback delay in regulating oscillatory dynamics to design network oscillators which can resist the aging transition.

DATA AVAILABILITY

The data and codes that support the findings of this study are available from the corresponding author upon reasonable request.

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