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CATE

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Exercise Information

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FAO: Craven, Robert (rac101)

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For Markers only: (circle appropriate grade)

CHEW, Yihang (yc5720)	01959746	t5	2020-11-02 23:31:52	**A* A B C D E F**

Introduction to Symbolic AI : Coursework 1

1i. p - Michel is fulfilled
q - Michel is rich
r - Michel will live for another five years

$((\neg(p \vee q)) \rightarrow (\neg r))$

1ii. p - snowstorm arrives
q - Raheem will wear his boots
r - I am sure snowstorm will arrive

$(((\neg p) \vee q) \wedge r)$

1iii. p - Akira is on set
q - Toshiro is on set
r - filming will begin
s - caterers have cleared out

$((p \wedge q) \rightarrow (r \leftrightarrow s))$

1iv. p - Irad arrived
q - Sarah arrived

$((p \vee (\neg q)) \wedge (\neg(p \wedge q)))$

1v. p - Herbert heard the performance
q - Anne-Sophie heard the performance
r - Anne-Sophie answered her phone calls

$((\neg r) \rightarrow (\neg(p \wedge q)))$

2i. $h_v(A) = \text{true}$ for some v

2ii. $h_v(A) = h_v(B)$ for all v

2iii. Suppose $h_v(\neg A) = \text{true}$ for some v (i.e. $\neg A$ is satisfiable). Then, $h_v(\neg\neg A) = \text{false}$ by definition of $\neg$.
Since $h_v(T) = \text{true}$ for all v, $h_v(\neg\neg A) \not\equiv h_v(T)$ for some v.
Therefore $\neg\neg A \not\equiv T$, as required.

3.

p	q	r	$(p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \to (\neg\neg q \to r)$
t	t	t	t f f t f t f t f f t t t f t t t
t	f	t	t t t f t t f t f f t t f t f t t
f	t	t	f f f t t f f t t t f t t f t t t
f	f	t	f f t f t f f t t t f t f t f t t
t	t	f	t f f t t f t f t f t f t f t f f
t	f	f	t t t f f f t f t f t t f t f t f
f	t	f	f f f t t f t f t t f f t f t f f
f	f	f	f f t f t f t f t t f t f t f t f

Since there are cases where the formula is f, it is not valid.

4i. CNF: a
b
c
d
f
g, h

conjunction of single clause

DNF: b
d
e
h, g

a disjunction of a single conjunction

4ii. Refutation-soundness and -completeness states that if a formula S (in CNF) resolves to the empty set ϕ, then $S \models \bot$.
This means that S is satisfiable iff $S \nvdash_{res(PL)} \phi$ (i.e. there is no resolution of S that results in ϕ).
This is an important property used by many SAT-solvers (to try and resolve S to ϕ to determine satisfiability, including DP and DLL algorithms).

4iii. $\{\{p,s\},\{q,r\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\}$

pure rule on q : $\{\{p,s\},\{\neg p,\neg r,\neg s\}\}$

pure rule on $\neg r$: $\{\{p,s\}\}$

pure rule on p : $\{\} \Rightarrow$ formula is satisfiable

4iiib) $\{\{\neg p,q,r\},\{\neg q\},\{p,r,q\},\{\neg r,q\}\}$

unit propagation on $\{\neg q\}$: $\{\{\neg p,r\},\{p,r\},\{\neg r\}\}$

unit propagation on $\{\neg r\}$: $\{\{\neg p\},\{p\}\}$

unit propagation on $\{p\}$: $\{\{\}\} \Rightarrow$ formula is unsatisfiable

5. p - I am going

q - You are going

r - Tara is going

Formula: $(p \to \neg q) \land (\neg q \to \neg r) \land (r \lor \neg p) \land (r \lor p) \models q$

Need to prove ~~premise is~~ unsatisfiable: $(p \to \neg q) \land (\neg q \to \neg r) \land (r \lor \neg p) \land (r \lor p) \land \neg q$

CNF: $\{\{\neg p,\neg q\},\{q,\neg r\},\{r,\neg p\},\{r,p\},\{\neg q\}\}$

~~resolve on p : $\{\{\neg q,r\},\{r\},\{q,\neg r\}\}$~~

~~unit propagation on $\{r\}$: $\{\{q\}\}$~~

~~unit propagation on $\{q\}$~~

~~resolve on r : $\{\{\neg q,q\},\{q\}\}$~~

~~resolve on q : $\{\{q\}\}$~~

~~remove tautology $\{\neg q,q\}$: $\{\{q\}\}$~~

~~unit propagation on q : $\{\} \Rightarrow$ satisfiable~~

unit propagation on $\{\neg q\}$: $\{\{\neg r\},\{r,\neg p\},\{r,p\}\}$

unit propagation on $\{\neg r\}$: $\{\{\neg p\},\{p\}\}$

unit propagation on $\{p\}$: $\{\{\}\} \Rightarrow$ unsatisfiable

$\therefore$ The argument is valid

6i. aunt(X,Y) (X is aunt of Y) $\quad P_2 = \{$aunt, cake$\}$

andrea(X) (X is Andrea) $\quad c =$ andrea $\quad P_1 = \{$andrea$\}$

cake(X,Y) (X gave cupcake to Y)

$\forall Z \exists W \forall X(\text{aunt}(Z,c) \land \text{aunt}(X,Z) \land \text{cake}(X,W) \land \neg\,\text{andrea}(W))$

bii. computer (X) (X is computer) $\quad P_1 = \{computer\}$

connected (X,Y) (X is connected to Y) $\quad P_2 = \{connected\}$

$$\forall Y \exists X (computer(X) \land connected(X,Y) \land computer(Y) \land \neg connected(Y,Y))$$

biii. painting (X,Y) (X is painting by Y) ~~hanging in gallery Y, in room Z~~ $\quad P_2 = \{painting, gallery, room\}$

~~PK(X) (X is Paul Klee)~~ pk - Paul Klee $\quad P_1 = \{british, hang\}$

~~K(X) (X is Kandinsky)~~ k - Kandinsky $\quad C = \{pk, k\}$

gallery (X,Y) (X is in gallery Y) room (X,Y) (X is in room Y)

british (X) (X is British) ~~hang~~ hang (X) (X is hanging)

$$\exists Z \exists Y \forall V \forall W (painting(W,pk) \land gallery(W,Y) \land british(Y) \land hang(W) \land painting(V,k) \land gallery(V,Y) \land hang(V) \land room(W,Z) \land room(V,Z))$$

biv. loves (X,Y) (X loves Y) $\quad P_1 = \{loves\}$

$$\exists X \neg \exists Y \; loves(X,Y) \rightarrow \forall X \exists Y \neg loves(X,Y)$$

7i. False. We can see from the $\mathcal{L}$-structure that the antecedent $a(k,j) = true$ in the antecedent (i.e. only j fulfills the condition). However, the consequent is false for $X = j$. Therefore, the formula is false. ~~we can see that~~

7ii) We can see that the antecedent $c(l) = true$. Hence, the consequent has to be true for the formula to be true.

We can see that the consequent is true for $X = j$.

Hence, the formula is true.

7iii. True. This means that there is at least one object that does not have a directed arrow from itself to some other object other than itself. This is satisfied by objects □ and ■. ~~objects j, k, l and □ have such arrows.~~

7iv. False. This means that there all the objects that are not squares have a directed arrow from itself to at least 1 black circle. We can see that object j does not satisfy this condition as it does not have a directed arrow to a black circle.

7v. False. This means that ~~for~~ all objects that have a directed arrow to an object other than itself, also have a two-way directed arrow from itself to some object. We can see that, for objects j and k, the antecedent is true, but the consequent is false (i.e. $a(k,j)$ = true BUT $a(j,k)$ = false). Hence, the formula is false. (i.e. there is an arrow from k to j, but k does not have a two-way arrow with any object.

7vi. True. This means that all objects that ~~are connected~~ have directed arrows to j, also have directed arrows between them (either direction). We can see that only objects k and l satisfy the antecedent. We can plainly see that there is an arrow between them (from l to k). Hence, the formula is true.