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Introduction to Symbolic AI
CW1

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1.

i. $\neg(p \vee q) \rightarrow \neg r$
p: Michel is fulfilled.
q: Michel is rich.
r: Michel will live another five years.

ii. $(p \rightarrow q) \wedge r$
p: The snowstorm arrives.
q: Raheem will wear his boots.
r: I'm sure the snowstorm will arrive.

iii. $p \wedge q \rightarrow (r \leftrightarrow t)$
p: Akira is on set.
q: Toshiro is on set.
r: Filming will begin.
t: The caterers have cleared out.

iv. $(p \vee \neg q) \wedge \neg(p \wedge \neg q)$
p: Irad arrived.
q: Sarah arrived.

v. $\neg r \rightarrow \neg(p \wedge q)$
p: Herbert heard the performance.
q: Anne-Sophie heard the performance.
r: Anne-Sophie answered her phone calls.

2.

i. If there are some atomic evaluation functions v that the propositional evaluation function based on v, $h_v(A) = \mathbf{t}$, then the propositional formula A is satisfiable.

ii. For every v, if $h_v(A) = h_v(B)$, then two propositional formulas A and B are logically equivalent.

iii. According to the definition of satisfiability, we know that ¬A is satisfiable $\leftrightarrow$ $\exists v, h_v(\neg A) = \mathbf{t}$.

Then, according to the definition of logical equivalence,

$$\neg\neg A \not\equiv \top \leftrightarrow \neg(\neg\neg A \equiv \top) \leftrightarrow \neg(\forall v, h_v(\neg\neg A) = \mathbf{t})$$

$$\leftrightarrow \exists v, h_v(\neg\neg A) = \mathbf{f}$$

$$\leftrightarrow \exists v, h_v(\neg A) = \mathbf{t}$$

Therefore, a propositional formula $\neg A$ is satisfiable iff $\neg\neg A \not\equiv \top$.

3.

p	q	r	$p \wedge \neg q$	$\neg(\neg r \vee \neg p)$
t	t	t	t $\boldsymbol{f}$ f	$\mathbf{t}$ f f f

According to the truth-table, we can find that $p \wedge \neg q \nleftrightarrow \neg(\neg r \vee \neg p)$.
Therefore, $\big(p \wedge \neg q \leftrightarrow (\neg r \vee \neg p)\big) \rightarrow (\neg\neg q \rightarrow r)$ is not valid.

4.

i.

CNF: a. b. g.
DNF: e. h.

ii. Refutation-soundness and –completeness:

Let S be in CNF. $S \vdash_{res(PL)} \emptyset \;\; iff \;\; S \vDash \bot$.

According to this theorem, we can get the following corollary:

Let S be in CNF. S is satisfiable iff $S \nvdash_{res(PL)} \emptyset$.

This corollary is at the heart of many SAT-solvers. In addition, we can use this theorem to build all resolution-derivations from S, which can be used to check SAT.

iii.

a.
At first, we apply pure rule on q: $\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$.
Then, we apply pure rule on $\neg r$, and we get the simplest form: $\{p, s\}$.

b. We first apply unit propogation on $\neg q$: $\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$.
Then, we apply unit propogation on $\neg r$: $\{\{\neg p\}, \{p\}\}$.
Finally, we apply unit propogation on p, and the simplest form is {}.

5.

p: I'm going
q: You're going.
r: Tara is going.
We formalize the argument as: $p \rightarrow \neg q,\ \neg q \rightarrow \neg r, r \vee \neg p, r \vee p, so\ q.$
We must check whether $p \rightarrow \neg q,\ \neg q \rightarrow \neg r, r \vee \neg p, r \vee p \vDash q.$

So we can check whether $(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (r \vee p) \wedge \neg q$ is satisfiable.

The clausal-form CNF is: $\{\{p, \neg q\},\ \{\neg q, \neg r\},\ \{r, \neg p\},\ \{r, p\},\ \{\neg q\}\}$.

Then we apply DP:

unit propogation on $\neg q$: $\{\{r, \neg p\}, \{r, p\}\}$.

pure rule on r : $\{\{\}\}$

unsatisfiable since $\emptyset$ is in the set

Therefore, the argument is not valid.

6.

i. aint(X) (X's aunt)

cupcake(X,Y) (X gives Y a cupcake)

$(\exists Z cupcake(aunt(aunt(Andrea)),\ Z) \wedge$
$\neg cupcake(aunt(aunt(Andrea)),\ Andrea))$

signature $L = (\{Andrea\}, \{aunt\}, \{cupcake\})$

ii. connect(X,Y) (X connects to Y)

$\exists X \forall Y (connect(X, Y) \wedge \neg connect(X, X)$

signature $L = (\emptyset, \emptyset, \{connect\})$

iii. PaulKlee(X) (Paul Klee's painting)

Kandinsky(X) (Kandinsky's painting)

hang(X, Y) (X hangs in the room Y of a British gallery)

$\forall X \exists Y \forall Z (hang(PaulKlee(X), Y)) \wedge hang(Kandinsky(Z), Y))$

signature $L = (\emptyset, \{PaulKlee, Kandinsky\}, \{hang\})$

iv. $love(X, Y)$ (X loves Y)

$\exists X \forall Y \neg love(X, Y) \to \forall Z love(Z, X) = \perp$

signature $L = (\emptyset, \emptyset, \{love\})$

7.

i.

$\forall X (a(k, X) \to \neg(X = j))$ is true, since $\forall X, a(k, X)$ is false.

ii. This is true.

Firstly, $c(l)$ is true because l is drawn circular.

Secondly, $\varphi(k)$ is in $\varphi(b), \varphi(c)\ and\ \varphi(a(l))$, since k is drawn filled, circular, and connected by l.

Therefore, $\exists X (b(X) \wedge c(X) \wedge a(l, X))$ is true.

So, $c(l) \to \exists X (b(X) \wedge c(X) \wedge a(l, X))$ is true.

iii. This is false.

Let $X = l, Y = k$, then $\exists X \exists Y (\neg(X = Y) \wedge a(X, Y))$ is true.

Therefore, $\exists X \neg \exists Y (\neg(X = Y) \wedge a(X, Y))$

iv. This is false.

Let $X = j$, then $\neg s(j)$ is true. But there is only l that $a(j, l)$ is true, and l is not filled ($b(l)$ is false).

Therefore, $\exists Y(c(Y) \land b(Y) \land a(X,Y))$ is false.
So, $\forall X(\neg s(X) \rightarrow \exists Y\big(c(Y) \land b(Y) \land a(X,Y)\big))$ is false.

v. This is true.
If we let X = the black square at top right, then $\exists Y(\neg(X = Y) \land a(X,Y))$ is false.

vi. This is true.
If we let X = the black square at top right, then $a(X,j)$ is false. So $a(X,j) \land a(Y,j)$ is false.