GHAFFARI, Behzad (bg916)

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GHAFFARI, Behzad (bg916)	01206822	c4	2020-02-19 04:59:49	A* A B C D E F

COURSEWORK

IMPERIAL COLLEGE LONDON

DEPARTMENT OF COMPUTING

499 - Modal Logic for Strategic Reasoning in AI

Author:
Behzad Ghaffari (CID: 01206822)

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1.

a)

$(M,\pi) \models \rho R \psi$ iff either for some $i \geq 0$ $(M,\pi[i..\infty]) \models \rho$ and for all $0 \leq j \leq i$ $(M,\pi[0..j]) \models \psi$,
or for all $k \geq 0$ $(M,\pi[k..\infty]) \models \psi$

b)

$$a \models \rho R \psi \iff (\psi U \rho) \lor (\psi \land \rho) \lor (\psi \land X(\psi))$$

c)

$(M,\pi) \models \rho R \psi \iff (\psi U \rho) \lor (\psi \land \rho) \lor (\psi \land X(\psi))$ (by definition)

$\iff (\psi U \rho) \lor (\psi \land \rho) \lor G(\psi)$ (expansion (slides p.94) and logic*)

$\iff$ there exists $i \geq 0$ such that $(M,\pi[i..\infty]) \models \rho$, and for all $0 \leq j < i, (M,\pi[j..\infty]) \models \psi$
or $(M,\pi[0..\infty]) \models \psi$ and $(M,\pi[0..\infty]) \models \rho$
or for all $k \geq 0$ $(M,\pi[k..\infty]) \models \psi$

$\iff$ there exists $i \geq 0$ such that $(M,\pi[i..\infty]) \models \rho$, and for all $0 \leq j \leq i, (M,\pi[j..\infty]) \models \psi$
or for all $k \geq 0$ $(M,\pi[k..\infty]) \models \psi$ ($< i$ and $= i$ mean $\leq i$)

d)

$(M,\pi) \models \perp R \psi$

$\iff$ there exists $i \geq 0$ such that $(M,\pi[i..\infty]) \models \perp$, and for all $0 \leq j \leq i, (M,\pi[j..\infty]) \models \psi$
or for all $k \geq 0$ $(M,\pi[k..\infty]) \models \psi$ ($< i$ and $= i$ mean $\leq i$)

$\iff \perp$ and for all $0 \leq j \leq i, (M,\pi[j..\infty]) \models \psi$
or for all $k \geq 0$ $(M,\pi[k..\infty]) \models \psi$ (first part not satisfiable)

$\iff$ for all $k \geq 0$ $(M,\pi[k..\infty]) \models \psi$ ($\perp \land p \equiv \perp$)

$\iff G\psi$ (by definition of G)

*: if α is such that $\alpha \equiv \psi \land X\alpha$ then $\alpha \implies G\psi$

2.

$(M,q) \models EF\Phi \iff E(trueU\Phi)$ (abbreviations in CTL)
$\iff$ for some path λ starting from $q, (M,\lambda) \models trueU\Phi$ (definition of E)
$\iff$ for some path λ starting from $q, (M,\lambda[j]) \models \Phi$ for some j ≥ 0,
and $(M,\lambda[i] \models true$ for all $0 \leq i < j$ (definition of U)
$\iff$ for some path λ starting from $q, (M,\lambda[j]) \models \Phi$ for some j ≥ 0 (true is true everywhere) $\blacksquare$

$(M,q) \models AF\Phi \iff A(trueU\Phi)$ (abbreviations in CTL)
$\iff$ for every path λ starting from $q, (M,\lambda) \models trueU\Phi$ (definition of A)
$\iff$ for every path λ starting from $q, (M,\lambda[j]) \models \Phi$ for some j ≥ 0,
and $(M,\lambda[j] \models true$ for all $0 \leq i < j$ (definition of U)
$\iff$ for every path λ starting from $q, (M,\lambda[j]) \models \Phi$ for some j ≥ 0 (true is true everywhere) $\blacksquare$

$(M,q) \models EG\Phi$
$\iff$ for some path λ starting from $q, (M,\lambda) \models G\Phi$ (definition of E)
$\iff$ for some path λ starting from $q, (M,\lambda[j]) \models \Phi$ for all j ≥ 0, (definition of G)

$(M,q) \models AG\Phi$
$\iff$ for every path λ starting from $q, (M,\lambda) \models G\Phi$ (definition of A)
$\iff$ for every path λ starting from $q, (M,\lambda[j]) \models \Phi$ for all j ≥ 0, (definition of G)

3.

a)

Prove by induction of structure of CTL, we show each construct in CTL is defined also in CTL* by the given definition:

atoms, are also state formulas in CTL*.
negation of a state formulae is also a state formula in CTL*.
conjunction of state formulae is a state formula in CTL*.
$E\phi$ is also a state formula in CTL*
$A\phi$ is also a state formula in CTL*
$X\psi$ is also a path formula in CTL*
$\phi U\psi$ is also a path formula in CTL*

Therefore all formulas in CTL are defined in CTL*. (CTL* is only a superset with addition of conjunction, negation and path formulas)

b)

Everybody will always be safe, from some moment on:
in CTL*: $AFG(\Lambda_{i\in people}\text{safe}_i))$
This is not defined in CTL because we cannot have FG (path formula followed by path formula)

4.

To do this we can remove the rules for (conjunction, negation and path formulas) from our state formulas, then we would be removing the rules for $(M,\pi) \models \Phi, (M,\pi) \models \neg\psi$ and$(M,\pi) \models \phi \wedge \phi\prime$ and are exactly left with the definition with def 1.7 and 1.8 in the slides.

5.

a)

This proof is similar to 3. Suppose $(M,s) \models \Phi$ in CTL. Then Φ must be a state formula and is either an atom, negation of a state formula, conjunction of two state formulae, E of a path formula or A of a path formula. Then Φ is also expressed using the corresponding rules in CTL* as the rules in CTL* are a superset of the rules in CTL (as seen in 4.). Then we also have $(M,s) \models \Phi'$ with $\Phi' = \Phi$ in CTL*.

b)

$\Psi \implies G\Phi \equiv \neg(\Psi \wedge \neg G\Phi)$
is not expressible in CTL because we cannot express a condition of a state with a path at the same time. ($\Psi \wedge \neg G\Phi$ is not expressible in CTL).

6.

Assume (M, t) and $(M, t\prime)$ are bisimilar

Then, we assume $(M, t) \models \Phi$ for each state formulaΦ and show $(M, t\prime) \models \Phi$

Case $(M, t) \models p$

$\iff t \in V(p)$ (by definition 2)

$\iff t\prime \in V\prime(p)$ (by bisimilarity of (M, t) and (M', t') and condition a of definition 3)

$\iff (M\prime, t\prime) \models p$ (by definition 2)

Case $(M, t) \models \neg\Phi$

Suppose $(M\prime, t\prime) \models \Phi$

$\implies (M, t) \models \Phi$ (by bisimilarity of (M, t) and (M', t') and condition c (back) of definition 3)

$\perp$ (by $(M, t) \models \neg\Phi$ and $(M, t) \models \Phi$)

Then it must be that $(M\prime, t\prime) \models \neg\Phi$ by contradiction

Case $(M, t) \models \Phi \wedge \Phi\prime$

$\iff (M, t) \models \Phi$ and $(M, t) \models \Phi\prime$ (by definition 2)

$\iff (M\prime, t\prime) \models \Phi$ and $(M\prime, t\prime) \models \Phi\prime$ (by bisimilarity of (M, t) and (M', t') and condition b (forth) of definition 3 for both $(M\prime, t\prime) \models \Phi$ and $(M\prime, t\prime) \models \Phi\prime$)

$\iff (M\prime, t\prime) \models \Phi \wedge \Phi\prime$ (by definition 2)

Case $(M, t) \models E\psi$

$\iff$ for some path π starting from t, $(M, \pi) \models \psi$ (by definition 2)

$\iff$ for some path $\pi\prime$ starting from t', $(M, \pi\prime) \models \psi$

(by bisimilarity of (M, t) and (M', t') and definition 3: (M, π) and $(M\prime, \pi\prime)$ are bisimilar if for every $i \geq 0$ $(M, \pi[i])$ and $(M\prime, \pi\prime[i])$ are bisimilar)

$\iff (M\prime, t\prime) \models E\psi$ (by definition 2)

Case $(M, t) \models A\psi$

$\iff$ for every path π starting from t, $(M, \pi) \models \psi$ (by definition 2)

$\iff$ for every path $\pi\prime$ starting from t', $(M, \pi\prime) \models \psi$

(by bisimilarity of (M, t) and (M', t') and definition 3: (M, π) and $(M\prime, \pi\prime)$ are bisimilar if for every $i \geq 0$ $(M, \pi[i])$ and $(M\prime, \pi\prime[i])$ are bisimilar)

$\iff (M\prime, t\prime) \models A\psi$ (by definition 2)

Assume (M, π) and $(M, \pi\prime)$ are bisimilar
Now for path formulae again assume $(M, \pi) \models \psi$ for some arbitrary state formula ψ and show $(M, \pi\prime) \models \psi$

Case $(M, \pi) \models \Phi$
$\iff (M, \pi[0]) \models \Phi$ where $\pi[0]$ is the initial state of path π (by definition 2)
$\iff (M\prime, \pi\prime[0]) \models \Phi$ where $\pi\prime[0]$ is the initial state of path $\pi\prime$
(We have $(M, \pi) \approx (M\prime, \pi\prime)$ Then by Definition 3, for every $i \geq 0$ $(M, \pi[i])$ and $(M\prime, \pi\prime[i])$ are bisimilar. Namely, at i=0 $(M, \pi[0]) \approx (M\prime, \pi\prime[0])$ then we can use the previous proof on state formulae)
$\iff (M\prime, \pi\prime) \models \Phi$ (by definition 2)

Case $(M, \pi) \models \neg\psi$
Suppose $(M\prime, \pi\prime) \models \neg\psi$
$\implies (M, \pi) \models \psi$ (by bisimilarity of (M, π) and (M', π') and definition 3c (back))
$\perp$ (by contradiction of $(M, \pi) \models \neg\psi$ and $(M, \pi) \models \psi$)
Then it must be that $(M\prime, t\prime) \models \neg\psi$ by contradiction (as required)

Case $(M, \pi) \models \psi \wedge \neg\psi$
$\iff (M, \pi) \models \psi$ and $(M, \pi) \models \psi\prime$ (by definition 2)
$\iff (M\prime, \pi\prime) \models \psi$ and $(M\prime, \pi\prime) \models \psi\prime$ (by bisimilarity of (M, π) and (M', π') and condition b (forth) of definition 3 for both $(M\prime, \pi\prime) \models \psi$ and $(M\prime, \pi\prime) \models \psi$
$\iff (M\prime, \pi\prime) \models \psi \wedge \psi\prime$ (by definition 2)

Case $(M, \pi) \models X\psi$
$\iff (M, \pi[1..\infty]) \models \psi$ (by definition 2)
$\iff (M\prime, \pi\prime[1..\infty]) \models \psi$ (by bisimilarity of (M, π) and (M', π') and definition 3)
$\iff (M\prime, \pi\prime) \models X\psi$

Case $(M, \pi) \models \psi U \psi\prime$
$\iff (M, \pi[1..\infty]) \models \psi\prime$ for some $i \geq 0$, and
$(M, \pi[j..\infty]) \models \psi$ for all $0 \leq j < i$ (by definition 2)
$\iff (M\prime, \pi\prime[1..\infty]) \models \psi\prime$ for some $i \geq 0$, and
$(M\prime, \pi\prime[j..\infty]) \models \psi$ for all $0 \leq j < i$ $((M, \pi) \approx (M\prime, \pi\prime))$
$\iff (M', \pi') \models \psi U \psi\prime$ (by definition 2)

we have shown that $(M, t) \models \Phi iff (M, t') \models \Phi$ and $(M, t) \models \psi iff (M, t') \models \psi$. This means if a state formula Φ or path formula ψ is true on one side of the bisimulation it must be true on the other side as well (by def of iff).

7.

condition a) t, t' satisfy the same formulas in CTL. Therefore, they must also satisfy the same atoms.

condition b) Assume there is a state t mapped to t' (B(t, t')) and a state u such that $R(t,u)$. Now assume there is no u' such that B(u, u') and $R(t',u')$. As $S' = \{x' \in St' | R(t',x')\}$ cannot be empty (a single world is not defined in kripke semantics) and is finite, for all states in St' there must be a formula such that $(M,t) \models \psi_i$ and $(M',t') \not\models \psi_i$. Therefore we have $(M,t) \models E(\Lambda\psi_i)$ but $(M',t') \not\models E(\Lambda\psi_i)$. However, this cannot be true as t and t' are equivalent so we have a contradiction.

condition c) Assume there is a state t', mapped to t (B(t, t')) and a state u' such that $R(t',u')$. Now assume there is no u such that B(u, u') and $R(t,u)$. As $S = \{x \in St | R(t,x)\}$ cannot be empty (a single world is not defined in kripke semantics) and is finite, for all states in St there must be a formula such that $(M',t') \models \psi_i$ and $(M,t) \not\models \psi_i$ Therefore we have $(M',t') \models E(\Lambda\psi_i)$ but $(M,t) \not\models E(\Lambda\psi_i)$. However, this cannot be true as t and t' are equivalent so we have a contradiction.

8.

if (M, t) and (M’, t’) satisfy the same formulas in CTL, by (7.) we know that they are bisimilar. Then by (6) we know that the truth of formulas in bisimilar relations in CTL* are preserved. Therefore, they satisfy the same formulas in CTL*.

Now if (M, t) and (M’, t’) satisfy the same formulas in CTL*, then by (6) they are bisimilar. So they satisfy the same formulae in CTL.

(5.b) shows that CTL* is more expressive that CTL which means we can express formulae in CTL* that we cannot express in CTL. However, here we are not saying that formulae that are expressible in CTL* must have a corresponding formula in CTL to satisfy. Only that if t, t’ satisfy formulae in CTL* they must satisfy same formulas in CTL (if such correspondence exists). However, the opposite holds always as all formulae in CTL are expressible in CTL*.