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Exercise Information

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Exercise:	2 (CW)	**Due:**	Tue - 03 Nov 2020
Title:	Logic	**Assessment:**	Individual
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Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-11-03 12:25:52

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Introduction to Symbolic AI

Coursework 1: Logic

1

i) $((\neg(f \vee r)) \rightarrow (\neg l))$

f: Michael is filled
r: Michael is rich
l: Michael will live another five years

ii) $((b \vee (\neg s)) \wedge s)$

b: Raheem will wear boots
s: Snowstorm will arrive

iii) $((a \wedge t) \rightarrow (f \leftrightarrow c))$

a: Akira is on set, t: Toshiro is on set
f: filming will begin, c: caterers have cleared out.

iv) $((i \wedge s) \vee ((\neg i) \wedge (\neg s)))$

i: Ivad arrived
s: sora arrived

v) $((\neg a) \rightarrow (\neg(h \wedge q)))$

a: Anne-Sophie answered her phone calls.
h: Herbert heard the performance
q: Anne-Sophie heard the performance.

2

i) A propositional formula A is satisfiable if there is some valuation v for which $h_v(A) = \text{true}$

ii) Two propositional formulae are logically equivalent if for every v, $h_v(A) = h_v(B)$. (often written $A \equiv B$)

iii) Assume $\neg\neg A \equiv T$, then $\neg A \equiv \neg T$, then $\neg A \equiv F$ which means that formula $\neg A$ is logically equivalent to false, i.e.: there is no valuation v for which $h_v(\neg A) = \text{true}$.
By definition in i, this means that $\neg A$ is not satisfiable. Hence, $\neg A$ is satisfiable iff $\neg\neg A \equiv T$ does not hold, namely iff $\neg\neg A \not\equiv T$.

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p	q	r	$((p$	$\wedge$	$\neg q)$	$\leftrightarrow$	$\neg($	$\neg r$	$\vee$	$\neg p))$	$\rightarrow$	$(\neg\neg q$	$\rightarrow$	$r)$
t	t	t	t	f	f	f	t	f	f	f	t	t	t	t
t	t	f	t	f	f	t	f	t	t	f	f	t	f	f
t	f	t	t	t	t	t	t	f	f	f	t	f	t	t
t	f	f	t	t	t	f	f	t	t	f	t	f	t	f
f	t	t	f	f	f	t	f	f	t	t	t	t	t	t
f	t	f	f	f	f	t	f	t	t	t	f	t	f	f
f	f	t	f	f	t	t	f	f	t	t	t	f	t	t
f	f	f	f	f	t	t	f	t	t	t	t	f	t	f

The propositional formula is not true regardless of the evaluation function (not true in all cases) so it is <u>not</u> valid.

4

i) a) CNF b) both CNF and DNF c) None

d) None e) DNF f) None g) Both CNF and DNF

h) Both CNF and DNF

"in set notation this is $\{\emptyset\}$

ii) Let propositional formula S be in CNF. $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$.
This is important because it means that if, through PL resolutions we can reduce a formula in CNF to $\emptyset$ (the empty set), then the formula is <u>un</u>satisfiable.
Otherwise, it's satisfiable.

iii) a) $\{\{p,s\},\{q,r\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\}$

pure rule on q

$\{\{p,s\},\{\neg p,\neg r,\neg s\}\}$

pure rule on $\neg r$

$\{\{p,s\}\}$

pure rule on either p or s

$\emptyset$

so satisfiable

b) $\{\{\neg p,q,r\},\{\neg q\},\{p,r,q\},\{\neg r,q\}\}$

unit propagation of $\neg q$

$\{\{\neg p,r\},\{p,r\},\{\neg r\}\}$

unit prop. of $\neg r$

$\{\{\neg p\},\{p\}\}$

unit prop of p (or $\neg p$)

$\{\emptyset\}$

so unsatisfiable

5. Formalise it as:

as there isn't an "but not both"

i: I'm going
t: Tara is going
u: You are going

$(i \to \neg u),\ (\neg u \to \neg t),\ (t \vee \neg i),\ (\neg i \to t)$

therefore u

using $(A \to B) \equiv (\neg A \vee B)$ and knowing

$A_1, \dots, A_n \models B$ iff $A_1 \wedge \dots \wedge A_n \wedge \neg B$ is unsatisfiable

$(\neg i \vee \neg u) \wedge (u \vee \neg t) \wedge (t \vee \neg i) \wedge (i \vee t) \wedge u$

noticing this is in CNF, use set notation

$\{\{\neg i, \neg u\}, \{u, \neg t\}, \{t, \neg i\}, \{i, t\}, \{u\}\}$

unit prop. of u

$\{\{\neg i\}, \{t, \neg i\}, \{i, t\}\}$

unit prop. of $\neg i$

$\{\{t\}\}$

unit prop. of t

$\emptyset$

So satisfiable, so argument is <u>not</u> valid.

6

i) Signature $\mathcal{L}$ contains:

$C = \{andrea\}$

$P_1 = \{cupcake\}$: cupcake(x) means X is a cupcake

$P_2 = \{aunt\}$: aunt(x, y) means X is Y's aunt

$P_3 = \{gave\}$: gave(x, y, z) means X gave to Y a Z

$$\forall X(\forall Y(aunt(X,Y) \land aunt(Y, andrea) \to \exists Z(\exists W(cupcake(W) \land gave(X,Z,W)))))$$

ii) Signature $\mathcal{L}$ contains

$P_1 = \{computer\}$: computer(x) means X is a computer

$P_2 = \{connected\}$: connected(x, y) means X is connected to Y

$$\forall X(computer(X) \land \neg connected(X,X) \to \exists Y(connected(Y,X)))$$

iii) Signature $\mathcal{L}$ contains:

$C = \{paulKlee, Kandinsky\}$

$P_1 = \{british, gallery, room\}$: british(x) means X is British.

gallery(x) means X is a gallery

room(x) means X is a room

$P_2 = \{painting, hangs, in\}$: painting(x,y) means X is a painting by Y.

hangs(x,y) means X hangs in Y

in(x,y) means X is in Y.

$$\forall W(\forall X(\forall Y(\forall Z(painting(X, paulKlee) \land gallery(Y) \land british(Y) \land room(Z) \land in(Z,Y) \land painting(W, Kandinsky) \land in(W,Y) \land hangs(X,Z) \rightarrow hangs(W,Z))))) \quad *$$

iv) Signature $\mathcal{L}$ contains:

$P_2 = \{loves\}$: loves(x,y) means X loves Y

$$\forall X(\neg(\exists Y(loves(Y,X))) \rightarrow \neg\exists Z(loves(X,Z)))$$

* The way I have translated it, in case it is hard to see is:

If: (There is a gallery which is British and has a room in it, in which room a painting by Paul Klee hangs, and there is also a painting by Kandinsky in that gallery), then the latter painting also hangs in that same room.

7 (ie. false)

i) Not true because $a(k,x)$ is true iff x represents the object $\varphi(j)$, in which case $\neg(x=j)$ is false.

ii) $c(l)$ is true

$\exists x(b(x) \wedge c(x) \wedge a(l,x))$ is true for $x = k$, $x = j$ $\Rightarrow$

$\Rightarrow$ whole argument is true

iii) if x is l and y is k, then:

$(\neg(x=y) \wedge a(x,y))$ is true so the whole argument that there is not some y for some x to satisfy the above is false

iv) for $\neg s(x)$ to be true x is either one of j, k, l:

if x is l, there is indeed a black circle to which x is related

if x is j, there is only a pointer from it to l, which is not a black dot so the argument sinks for all x

so, false

v) if $x = k$: $\exists y(\neg(x=y) \wedge a(x,y))$ is true for $y = j$

$\exists y(a(x,y) \wedge a(y,x))$ is false so false

vi) if $x = k$ and $y = k$: $a(x,j) \wedge a(y,j)$ is true

$a(x,y) \vee a(y,x)$ is false

so false