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Exercise Information

Module: 70051 Introduction to Symbolic Artificial Intelligence (MSc AI)
Exercise: 2 (CW)
Title: Logic
FAO: Craven, Robert (rac101)

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- I acknowledge the following people for help through our original discussions:

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Signed: (electronic signature) Date: 2020-11-03 17:31:48

For Markers only: (circle appropriate grade)

KONCHWALLA, Zubair (mk3320)	01221267	t5	2020-11-03 17:31:48	**A* A B C D E F**

1)

i) p = Michael is fulfilled. q = Michael is rich
r = he will live another 5 years.

$\neg(p \wedge q) \rightarrow \neg r$

ii) p · the snow storm arrives
q Raheem will wear his boots

$\neg p \vee q$ other bit has no Truth value.

iii) p: Akira is on set
q: Toshira are on set,
r: filming will begin
s: the caterers have cleared out

$(p \wedge q) \rightarrow (r \leftrightarrow s)$ technically not needed due to right associativity

iv) p: Irad has arrived
q: Sarah has arrived

$(p \vee \neg q) \wedge \neg(p \wedge \neg q)$
$= (p \vee \neg q) \wedge (\neg p \vee q)$

v) p: Herbert heard performance

q: Anne Sophie heard performance.

r: Anne sophie answered her phone.

$\neg r \rightarrow \neg(p \wedge q)$

2) i) A propositional formula, A, is satisfiable if there is some atomic evaluation function V s.t $h_V(A) = t$.

ii) Two proposition formulas are logically equivalent if for every V, $h_V(A) = h_V(B)$.

iii) Lets try to show

$\neg A$ is satisfiable $\longrightarrow$ $\neg(\neg A) \not\equiv T$

Let $B = \neg A$

Note: $\neg B \not\equiv T$ $\quad\quad \neg(\neg B \equiv T)$

$\therefore$

B is satisfiable $\Rightarrow \exists\, V'$ s.t $h_{V'}(B) = T$

So using

3) 1 → 2 → 3 → 4 → 5

p	q	r	$(p$	$\wedge$	$\neg q$	$\leftrightarrow$	$\neg($	$\neg r$	$\vee$	$\neg p))$	$\rightarrow$	$(\neg\neg q$	$\rightarrow$	$r)$
			1	2	1	4	3	1	2	1	5	1	2	1
T	T	T	T	F	F	F	T	F	F	F	T	T	T	T
T	T	F	T	F	F	T	F	T	T	F	F	T	F	F
T	F	T	T	T	T	T	T	F	F	F	T	F	T	T
T	F	F	T	T	T	F	F	T	T	F	T	F	T	F
F	T	T	F	F	F	T	F	F	T	T	T	T	T	T
F	T	F	F	F	F	T	F	T	T	T	F	T	F	F
F	F	T	F	F	T	F	F	F	T	T	T	F	T	T
F	F	F	F	F	T	F	F	T	T	T	T	F	T	F

The expression is not valid as for $V(p) = V(q) = T$ and $V(r) = F$,

$h_V(A) = f$

4) a) CNF

b) CNF and DNF

c) Neither

d) T is not a literal ∴ Neither

e) DNF

f) $\neg\neg p$ is not a literal ∴ neither

g) CNF and DNF

h) CNF and DNF

ii)

Refutation soundness and completeness.

Let S be in CNF. $S \vdash_{res(PL)} \phi$ iff $S \models \bot$

This property provides a useful way to show satisfiability as we can determine if S is satisfiable iff we can't derive the empty set from by our resolution derivation

iii) a) $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

Applying pure rule to q, delete any clauses with q

$\Rightarrow$ $\{\{p, s\}, \{\neg p, \neg r, \neg s\}$

r never appears so apply pure rule to $\neg r$

$\{\{p, s\}\}$

$\Rightarrow$ $\{\ \}$ s is pure

$\Rightarrow$ Satisfiable since no conditions for further applications apply

b) $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

$=$ $\neg q$ is a unit, delete everything with $\neg q$ and remove literal q.

$\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$

$\Rightarrow$ $\{\{\neg p\}, \{p\}\}$ $\neg r$ is a unit,

$\Rightarrow$ $\{\{\}\}$ $\therefore$ Not satisfiable

5) p: I am going, q: You are going.
r: Tara is going.

$p \to \neg q,\ \neg q \to \neg r,\ r \vee \neg p,\ r \vee p \models q$

$A_1, \dots A_n \models B$ iff $A_1 \wedge \dots A_n \wedge \neg B$ is unsatisfiable.

$\therefore$ Lets check satisfiablility

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$

$\neg q$ is a unit,

$\Rightarrow$ $\{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$

$\Rightarrow$ $\{\{\neg p\}, \{p\}\}$ [$\neg r$ is a unit]

$\Rightarrow$ $\{\emptyset\}$

$\Rightarrow$ unsatisfiable as $\emptyset$ is in the set.

$\Rightarrow$ Argument is valid.

They could just ask nicely instead.

6) i) - $C = \{Andrea\}$

- $P_1 = \{cupcake\}$ where $cupcake(x)$ is 'x is a cupcake'
- $P_2 = \{aunt\}$ where $aunt(x, y)$ is 'x is aunt of y'
- $P_3 = \{gave\}$ where $gave(x, y, z)$ is x gave y to z

$\Rightarrow$

$$\forall w, \forall x, \exists y \exists z(gave(x, aunt(w, Andrea)), cupcake(y), z)$$
$$\wedge \neg gave(x, aunt(w, Andrea)), cupcake(y), Andrea)$$

ii) $P_1 = \{Computer\}$ where $computer(x)$ is x is a computer
$P_2 = \{connected\}$ where $connected(x, y)$ is x is connected to y

$$\Rightarrow \exists y \forall x(connected(computer(y), computer(x))$$
$$\wedge \neg connected(computer(y), computer(y))$$

If the computer connects to every computer in general but isn't connected to itself. Otherwise replace with x, x as y connects to all computers who cant connect to themselves

iii) $C = \{ \text{paul klee}, \text{kandinsky} \}$

$P_1 = \{ \text{british} \}$ where british(x) is 'x is british'

$P_2 = \{ \text{Painting}, \text{hangs}, \text{gallery} \}$

and in location

where Painting (x, y) is 'x is a painting by y'

hangs (x, y) is 'x is hung in y'

gallery (x, y) is 'x is in gallery y'

$$\Rightarrow \forall x \, \exists y \, (\text{british} (\text{gallery} (\text{Painting} (x, \text{Paul klee}), y)$$
$$\longrightarrow \exists z \, \forall w (\text{hangs}(x, z) \wedge \text{hangs}(w, \text{room}(z))$$
$$\wedge \text{gallery} (\text{Painting} (w, \text{kandinsky}), y))))$$

iv) $P_1 = \{ \text{loves} \}$ where love(x, y): 'x loves y'

$$\exists x \, \nexists y \quad \text{loves}(x, y) \longrightarrow \neg \forall z \, \exists w \quad \text{loves}(z, w)$$

7) i) False

ii) False

iii) True

iv) False

v) False

vi) False