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CATE

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Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
Exercise:	2 (CW)	**Due:**	Tue - 03 Nov 2020
Title:	Logic	**Assessment:**	Individual
FAO:	Craven, Robert (rac101)	**Submission:**	Electronic

Student Declaration - Version 1

- I acknowledge the following people for help through our original discussions:

Nikolas Theodosiou (nt220)

Signed: (electronic signature) Date: 2020-11-02 16:22:06

For Markers only: (circle appropriate grade)

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Introduction to Symbolic AI CW1

1) i) $\neg(p \vee q) \rightarrow \neg r$

where the atoms:

p: Michael is fullfilled

q: Michael is rich

r: Michael will live 5 years

ii) $(\neg p \vee q) \wedge r$

p: the sandstorm arrives

q: Rakeem will wear his boots

r: Im sure the storm will arrive

iii) $(p \wedge q) \rightarrow (r \leftrightarrow s)$

p: Akira is on set

q: Toshiro is on set

r: filming will begin

s: caterers have cleared out

iv) $(p \wedge q) \vee (\neg p \wedge \neg q)$ (XOR)

p: Irad Arrived

q: Sarah Arrived

v.) $\neg p \rightarrow \neg(q \wedge r)$

p: sophie-Anne answered the phone

q: Herberd heard the performane

r: Sophie Anne heard the performance.

2) i) A propositional formula A is satisfiable if there exists some validation v such that $h_v(A) = t$

ii) Two propositional formulas A and B are logically equivalent if for every validation v
$h_v(A) = h_v(B)$

iii) First for the if direction. Assume that $\neg A$ is satisfiable. Then, there exists an arbitrary validation v such that $h_v(\neg A) = t$, but since $h_v(\neg A) = t$ $h_v(\neg\neg A) = f$ (by the meaning of $\neg$)

Now suppose that $h_v(\neg\neg A) \not\equiv T$
Then there exists v such that $h_v(\neg\neg A) = f$
but then, by the definition of $\neg$ $h_v(\neg A) = t$ for some v. Hence $\neg A$ is satisfiable

We draw the truth table of the expression

p	q	r	$(p \wedge \neg q$	$\leftrightarrow$	$\neg(\neg r \vee \neg p))$	$\to$	$(\neg\neg p \to r)$
T	T	T	T F F	F	T F F F	T	T T T
T	T	F	T F F	T	F	F	

so for the second validation the formula is not satisfiable

4) a) This is in CNF
b) as $\neg p$ is a literal this is in CNF and DNF
c) CNF
d) This is both CNF & DNF
e) DNF
f) since $\neg\neg p$ isn't a literal this is in CNF
g) CNF and CNF
h) This is DNF but not full DNF.
This is also in CNF

ii) Refutation-soundness & completeness:
Let S be in CNF. Then $S \vdash res(b2) Q$ if and only if $S \models \bot$. That is, let S be in CNF, then there is no derivation by propositional logic iff S is not satisfiable.
This is important, because this way we can show that S is satisfiable through resolution derivation iff it is impossible to derive $\emptyset$ from S

iii) a) q is a pure clause in a) so we can remove the clauses that include the pure literal q, ending up with
$\{\{p,s\}, \{\neg p, \neg r, \neg s\}\}$
Here $\neg r$ is pure, so we can remove the second clause ending up with $\{\{p s\}\}$.
Again, since p,s has p which is a pure ~~claus~~ literal we can remove the entire clause
so we are left with $\{\}$ so this outputs satisfiable.

b) sine $\neg q$, is a unit clause we can start by resolving all the ways on $\{\neg q\}$
we are left with $\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$
Now we repeat the same step for $\{\neg r\}$
So we are left with $\{\{\neg p\}\{p\}\}$
which results to $\{\{\}\}$
This is UNSATISFIABLE since $\emptyset$ is in the set

5) For all of the below we formalise the arguments

$p \to \neg q$	If im going then you aren't
$\neg q \to \neg r$	if you're not going then neither is Tara
$r \vee \neg p$	Either tara is going or I'm not
$\neg r \vee p$	Tara's going unless I am
~~So you~~	
q	So you're going

Where p: I am going
q: You are going
r: Tara is going
Since $A \to B \equiv \neg A \vee B$
we have $\neg p \vee \neg q, q \vee \neg r, r \vee \neg p, \neg r \vee p \models q$

we convert to clausal form

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{\neg r, p\}\}$

We have no tautologies ✓

we can resolve all the ways on p

$\{\{\neg q\}, \{q, \neg r\}, \{r\}, \{\neg r\}\}$

we perform unit propagation of r:

$\{\{\neg q\} \{q\}\}$

and after unit propagation of q we are left with $\{\{\}\}$ so UNSATISFIABLE

6) i) First I define the predicates

P_2: aunt (x, y) x is an aunt of y

capcake(x, y) x gives cupkake to y

constants: andrea

For all aunts of the aunts of Andrea an aunt gave a cupkake to someone that is not Andrea:

$\forall x, \forall y\ (aunt(x, y) \wedge aund(y, andrea) \rightarrow \exists z\ (cupcake(x, z) \wedge \neg(z = andrea))$

ii) P_1 (x) $\Rightarrow$ computer(x) x is a computer

P_2 $\Rightarrow$ connected (x, y) x is conected to y

$\forall x\ (computer(x) \wedge \neg connected(x, x) \rightarrow \exists y\ (computer(y) \wedge connected(y, x))$

7 i) For all objects x such that x is connected to k x is not equal to j

False, since the only object connected to k is j

ii) True, for c(l) there exists a black circular object that can be reached from l ($x = j$)

iii) For some x not equal to y there is y reachable from x.
i.e there is ~~only~~ on x that can be reached by itself.
True, the black square on the top right

iv) for all x that is not square, ∃ a circular black y that can be reached from x

False, The only two circular black objects are reachable only from l which is circular.

v) For all x and y that point to j either x points to y or y points to x

l and k point to j. l points to k
so the condition is satisfied and this is True

iii) P_1: Paul(x) X was painted by Paul Klee

Kandinsky(x) X was painted by Kandinsky

P_2: British Gallery(x) X is in the Br. Gallery

room(x, y) x hangs in the same room as y

$$\forall x\,(Paul(x) \wedge BritishGallery(x) \wedge \forall y\,(Kandisky(y) \wedge BritishGallery(y))) \rightarrow room(x,y)$$

iv) P_1: $P(x)$ x is a Person

P_2 $\lambda(x,y)$ x loves y

there is somebody that loves nobody:

$$\exists x\,(P(x) \wedge \exists y\, P(y)) \rightarrow \neg(\lambda(x,y))$$

everybody loves somebody

$$\forall x \exists y\,(P(x) \wedge P(y) \rightarrow \lambda(x,y))$$

Combining the above we get

$$(\exists x\,((P(x) \wedge \exists y P(y) \rightarrow \neg(\lambda(x,y)))) \rightarrow \neg(\forall x \exists y\,(P(x) \wedge P(y) \rightarrow \lambda(x,y)))$$