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Exercise Information

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Exercise: 2 (CW)
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FAO: Craven, Robert (rac101)

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- I acknowledge the following people for help through our original discussions:

Yaniel Cabrera (yc7620)

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For Markers only: (circle appropriate grade)

CABRERA, Yaniel (yc7620)	02001245	t5	2020-10-28 22:53:09	**A* A B C D E F**

① (i) p = Michael is either fulfilled or rich
q = Michael will live another 5 years

$((\neg p) \rightarrow (\neg r))$

(ii) p = storm arrives
q = Raheem will wear boots
r = I'm sure it will arrive

$((p \rightarrow q) \wedge r)$

(iii) p = Akira is on set

σ = Toshiro is on set
q = filming will begin
r = caterers have cleared out

$((p \wedge \sigma) \rightarrow (q \leftrightarrow r))$

(iv) p = Irad arrived
q = Sarah arrived

$((p \vee (\neg q)) \wedge (\neg (p \wedge q)))$

(v) p = Herbert heard the performance

q = Anne-Sophie heard the performance

r = Anne-Sophie answered her phone calls

$$((\neg r) \rightarrow (\neg(p \wedge q)))$$

I wrote the above formula by reading the sentence in question as

if not(Anne-Sophie answered her calls)

then not(both Herbert and Anne-Sophie heard the performance)

(2) (i) A propositional formula A is satisfiable if there exists some evaluation v such that $h_v(A) = t$.

(ii) Two propositional formulas A, B are logically equivalent if

$$h_v(A) = h_v(B)$$

for all evaluations v.

(iii) To show $\neg A$ is satisfiable iff $\neg\neg A \not\equiv T$.

Sufficiency:

Let $\neg A$ be satisfiable. $\Rightarrow \exists\, v_*$ such that

$$h_{v_*}(\neg A) = t$$

$$\Rightarrow \quad h_{v_*}(\neg\neg A) = f$$

$$\Rightarrow \quad h_v(\neg\neg A) \neq h_v(T) \quad \forall\, v$$

$$\Rightarrow \quad \neg\neg A \not\equiv T$$

Necessity:

Let $\neg\neg A \not\equiv T$

$$\Rightarrow \quad h_v(\neg\neg A) \neq h_v(T) \quad \text{for some } v$$

$$\Rightarrow \quad h_v(\neg\neg A) = f$$

$$\Rightarrow \quad h_v(\neg A) = t$$

$\Rightarrow$ $\neg A$ is satisfiable

③ Use truth table to determine validity

p	q	r	$(p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \rightarrow (\neg\neg q \rightarrow r)$
t	t	f	t f f t f t t f f t f f f

$\Rightarrow$ The formula is not valid because it evaluates to false under $(v(p)=t,\ v(q)=t,\ v(r)=f)$

④ (i)

(a) $p \wedge (\neg q \vee r)$ CNF

(b) $\neg p$ CNF, DNF

(c) $p \wedge (q \vee (p \wedge r))$ none

(d) $\top$ CNF, DNF

(e) $(p \wedge q) \vee (p \wedge q)$ DNF

(f) $\neg\neg p \wedge (q \vee p)$ none ($\neg\neg p$ is not a literal)

(g) $p \wedge q$ CNF, DNF

(h) $p \vee q$ DNF, CNF

(ii) Let S be in CNF. $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$

The importance of the above property derives from the fact if we cannot derive $\emptyset$ from S, then S is satisfiable.

(iii) Reduce the following

(a) $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

$\Rightarrow \{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$, pure rule on q

$\Rightarrow \{\{p, s\}\}$, pure rule on $\neg r$

$\Rightarrow \{\}$, pure rule on p

(b) $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

$\Rightarrow \{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$, unit clause propagation $\{\neg q\}$

$\Rightarrow \{\{\neg p\}, \{p\}\}$, propagation on unit clause $\{\neg r\}$

$\Rightarrow \{\{\}\}$, " " " $\{p\}$

(5) p = I'm going

q = you're going

r = Tara is going

$$p \to \neg q,\ \neg q \to \neg r,\ r \vee \neg p,\ r \vee p \models q$$

$\Leftrightarrow$

$$S = (p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (r \vee p) \wedge \neg q$$

Where we used $A_1, \ldots, A_n \models B \Leftrightarrow \bigwedge_i A_i \wedge \neg B$ unsat

Using $a \to b \equiv \neg a \vee b$ we obtain the following clausal form:

$$S = \{ \{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\} \}$$

No tautologies

$\Rightarrow \{ \{\neg r\}, \{r, \neg p\}, \{r, p\} \}$, unit clause $\{\neg q\}$

$\Rightarrow \{ \{\neg p\}, \{p\} \}$, " " $\{\neg r\}$

$\Rightarrow \{\emptyset\}$, " " $\{\neg p\}$

$\Rightarrow \emptyset \in S \Rightarrow$ unsat

$\Rightarrow$ The argument is valid

(6) $C = \{\text{Andrea}, \text{cupcake}\}$

(i) $P_2 = \{\text{aunt}\}$, $P_3 = \{\text{give}\}$,

$P_i = \phi \quad \forall i > 3$

$F_i = \phi \quad \forall i$

where $\text{aunt}(X, Y) =$ "X is Y's aunt"

$\text{give}(X, Y, Z) =$ "X gives Z to Y"

$$\forall X \forall Y (\text{aunt}(X, \text{andrea}) \wedge \text{aunt}(Y, X) \longrightarrow$$
$$\exists Z (\text{give}(Y, Z, \text{cupcake}) \wedge \neg (Z = \text{andrea})))$$

(ii) $P_1 = \{\text{computer}\}$, $P_2 = \{\text{connected}\}$

$P_i = \phi, \; i > 2$

$F_i = \phi \quad \forall i$

$$\forall Y \left(computer(Y) \wedge \neg connected(Y,Y) \rightarrow \exists X \; computer(X) \wedge connected(X,Y) \right)$$

(iii) $P_1 = \{ byKlee, byKand, inBG \}$

$P_2 = \{ share\_gal, share\_room \}$

$P_i = \phi \;\forall i > 2$, $F_i = \phi \;\forall i$

$share\_gal(X,Y)$ = "X and Y ~~share~~ in some gallery"

$share\_room(X,Y)$ = "X and Y in some room"

$byKlee(X)$ = "X is painting by klee"

$byKand(X)$ = "X is painting by Kandisky"

$inBG(X)$ = "X is in British gallery"

$$\forall X \forall Y (byKlee(X) \wedge byKand(Y) \wedge share\_gal(X,Y) \wedge inBG(X) \wedge inBG(Y) \longrightarrow share\_room(X,Y))$$

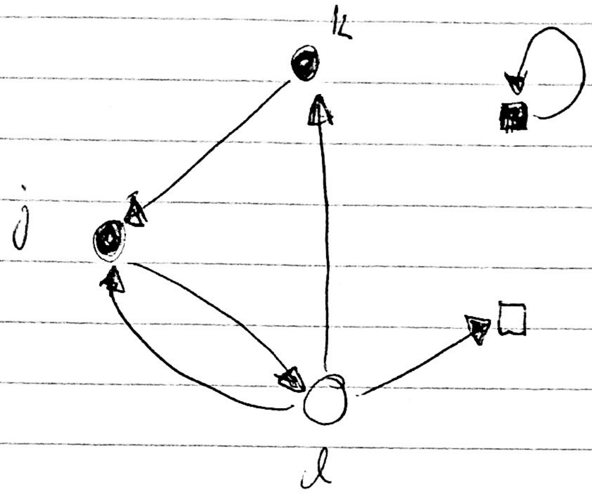
(iv)

$P_2 = \{ love \}$

$love(X,Y)$ = "X loves Y"

$\exists X \forall Y (\neg love(X,Y)) \longrightarrow \neg \forall X \exists Y \, love(X,Y)$

(7)



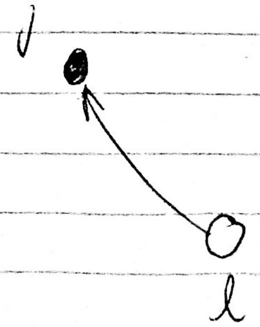
(i) $\forall X (a(k,X) \longrightarrow \neg(X=j))$

False. The figure shows $(k,X) \in \varphi(a)$ only if $X=j$

(ii) $c(l) \longrightarrow \exists X (b(X) \wedge c(X) \wedge a(l, X))$

True.

We have:



For $\sigma(X) = j$

we have an arrow going from circle l to black circle j.

(iii) $\exists X \neg \exists Y \left(\neg (X = Y) \wedge a(X,Y) \right)$

True

Consider the assignment

$$\ell' := \sigma(X)$$

$$\Rightarrow \exists Y \left(\neg (\ell' = Y) \wedge a(\ell', Y) \right) \quad \text{is false}$$

since the arrow only goes to ℓ itself

$$\Rightarrow \neg \exists Y \left(\neg (\ell' = Y) \wedge a(\ell, Y) \right) \quad \text{is True}$$

(iv) $\forall X \left(\neg s(X) \rightarrow \exists Y \left(c(Y) \wedge b(Y) \wedge a(X,Y) \right) \right)$

False

Consider the assignment $\sigma(X) = j$

$\Rightarrow \neg s(j)$ is true yet $\nexists$ black circle receiving an arrow from j.

(v) $\forall X \left(\exists Y \left(\neg (X = Y) \wedge a(X,Y) \right) \longrightarrow \exists Y \left(a(X,Y) \wedge a(Y,X) \right) \right)$

False

Consider the assignment $\sigma(X) = k$

Then it is true $\exists Y (\neg (X = Y) \wedge a(X,Y))$

namely let $\sigma(Y) = j$.

Yet there are no objects ~~and~~ such th

$a(k, Y) \wedge a(Y, k)$ is true.

(vi) $\forall X \forall Y (a(X,j) \wedge a(Y,j) \rightarrow$

$(a(X,Y) \vee a(Y,X)))$

True

Consider $\sigma(X) = k$, $\sigma(Y) = l$

$\Rightarrow a(k,j) \wedge a(l,j)$ is true

and so is $a(k,l) \vee a(l,k)$.