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Academic Year **2020-2021**
Page created Tue Nov 3 23:15:03 GMT 2020

CATE

70051 rac101 2
t5 jcr20 v1

0 0 1 2 0 0 0 1 4 2 1 4

Electronic submission

Mon - 02 Nov 2020 20:07:54

jcr20

Exercise Information

Module: 70051 Introduction to Symbolic Artificial Intelligence (MSc AI)
Exercise: 2 (CW)
Title: Logic
FAO: Craven, Robert (rac101)

Issued: Tue - 20 Oct 2020
Due: Tue - 03 Nov 2020
Assessment: Individual
Submission: Electronic

Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-10-30 11:53:08

For Markers only: (circle appropriate grade)

REILLY, Johnny (jcr20)	01928630	t5	2020-10-30 11:53:08	**A* A B C D E F**

Intro to Symbolic AI Coursework 1

Johnny Reilly (jcr20)

October 2020

Exercise 1

RETURN TO CHECK ALL

i p - Michael is either fulfilled or rich
r - Michael will live a fulfilled life

$$((\neg p) \to (\neg r))$$

ii p - The snowstorm does arrive
q - Raheem will wear his boots
r - I'm sure the snowstorm will arrive

$$(((\neg p) \lor q) \land r)$$

iii p - Akira is on set
q - Toshiro is on set
r - Filming will begin
s - The caterers have cleared out

$$((p \land q) \to (r \leftrightarrow s))$$

iv p - Irad arrived
q - Sarah arrived

$$((p \lor \neg q) \land (\neg(p \land \neg q)))$$

v p - Herbert heard the performance
q - Anne-sophie heard the performance
r - Anne-sophie answered her phone calls

$$((\neg r) \to (\neg(p \land q)))$$

Exercise 2

i A is satisfiable if there is some atomic evaluation function $v : \mathcal{A} \to \{\mathbf{t}, \mathbf{f}\}$ such that $h_v(A) = \mathbf{t}$ where $\mathcal{A}$ is the set of propositional atoms in A and h_v is the propositional evaluation function based on v

i.e A evaluates to true is true for some choice of truth values of its propositional atoms.

ii A and B are logically equivalent if for all possible v, $h_v(A) = h_v(B)$. i.e You cannot distinguish between A and B based on their truth values given any evaluation of their propositional atoms.

iii $\neg A$ is satisfiable iff there is an atomic evaluation function v such that $h_v(\neg A) = \mathbf{t}$. Following the rule for negation in Definition 1.5, we see that this is true iff $h_v(\neg\neg A) = \mathbf{f}$, which is equivalent to saying $\neg\neg A \not\equiv \top$ since $h_v(\top) = \mathbf{t}$.

Exercise 3

p	q	r	(p	$\wedge$	$\neg q$	$\leftrightarrow$	$\neg$	$(\neg r$	$\vee$	$\neg p))$	$\to$	$(\neg\neg q$	$\to$	r)
t	t	t	t	f	f	f	t	f	f	f	t	t	t	t
t	t	f	t	f	f	t	f	t	t	f	f	t	f	f
t	f	t	t	t	t	t	t	f	f	f	t	f	t	t
t	f	f	t	t	t	f	f	t	t	f	t	f	t	f
f	t	t	f	f	f	t	f	f	t	t	t	t	t	t
f	t	f	f	f	f	t	f	t	t	t	f	t	f	f
f	f	t	f	f	t	t	f	f	t	t	t	t	f	f
f	f	f	f	f	t	t	f	t	t	t	t	f	t	f

The principal connective is shown seperated by double vertical lines, there are two f's in that column so the formula is not valid.

Exercise 4

i CNF: a, b, d, g, h
DNF: b, d, e, g, h

ii let S be a formula in CNF, refutation-soundness and -completeness states that S is unsatisfiable if and only if there is a resolution derivation of $\emptyset$ from S ($S \vdash_{res(PL)} \emptyset$). This means that, we can replace pairs of clauses in S by their resolvents and preserve satisfiability. If we can derive $\emptyset$ then S is satisfiable, if we do not reach $\emptyset$ and can no longer resolve any clauses, then S is unsatisfiable. (This is much quicker than building a truth table for S).

iii

a

$$
\begin{array}{rr}
\{\{p,s\},\{q,r\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\} & \\
\{\{p,s\},\{\neg p,\neg r,\neg s\}\} & \text{pure rule: } q \\
\{\{p,s\}\} & \text{pure rule: } \neg r \\
\{\} & \text{pure rule: } p
\end{array}
$$

b

$$
\begin{array}{rr}
\{\{\neg p,q,r\},\{\neg q\},\{p,r,q\},\{\neg r,q\}\} & \\
\{\{\neg p,r\},\{p,r\},\{\neg r\}\} & \text{unit prop.: } \neg q \\
\{\{\neg p\},\{p\}\} & \text{unit prop.: } \neg r \\
\{\{\}\} & \text{unit prop.: } p
\end{array}
$$

Exercise 5

Using the atoms:

$$p - \text{I'm going},\ q - \text{You're going},\ r - \text{Tara's going}$$

we can write the argument as

$$(\neg p \vee \neg q)\ \wedge (q \vee \neg r)\ \wedge (r \vee \neg p)\ \wedge (r \vee \neg p) \text{ therefore } q$$

Recall that an argument $A_1, ..., A_n$ therefore B is valid iff $A_1 \wedge ... \wedge A_n \wedge \neg B$ is unsatisfiable. So we need to determine the satisfiability of the following CNF:

$$S = \{\{\neg p,\neg q\},\{q,\neg r\},\{r,\neg p\},\{\neg q\}\}$$

$$
\begin{array}{rr}
\{\{\neg r\},\{r,\neg p\}\} & \text{unit prop.: } \neg q \\
\{\{\neg r\},\{r\}\} & \text{pure rule: } \neg p \\
\{\{\}\} & \text{unit prop.: } r
\end{array}
$$

The presence of $\{\}$ in the last line indicates that S is unsatisfiable, therefor the argument is valid.

Exercise 6

i Constants = {andrea}
Predicates$_2$ = $\{aunt, cupcake\}$.
aunt(X,Y) reads 'X is Y's aunt', cupcake(X,Y): 'X gave a cupcake to Y'.

$$\forall X(aunt(X,a) \rightarrow \exists Y(\neg(Y = andrea) \wedge cupcake(X,Y)))$$

ii 1-Predicates = {computer}
2-Predicates = {connected}
computer(X) reads 'X is a computer, connected(X, Y): 'X is connected to Y'

$\forall X(computer(X) \wedge \neg connected(X,X) \rightarrow \exists Y(computer(Y) \wedge connected(Y,X)))$

iii Constants = {klee, kandinsky}
1-Predicates = {painting, british}
2-Predicates = {sameroom}
1-Functions = {gallery, artist}
gallery(X) is the gallery X is in, artist(X) is the artist who made X, painting(X): X is a painting, british(X): X is british, sameroom(X, Y): X and Y are in the same room.

$$\forall X(painting(X) \wedge (artist(X) = klee) \wedge british(gallery(X))$$
$$\rightarrow \forall Y(painting(Y) \wedge (artist(Y) = kandinsky) \wedge (gallery(Y) = gallery(X)) \rightarrow sameroom(X,Y)))$$

iv 1-Predicates = {body}
2-Predicates = {loves}
body(X) reads 'X is somebody', loves(X,Y) reads 'X loves Y'

$\exists X(body(X) \wedge \neg \exists Y(body(Y) \wedge loves(X,Y))) \rightarrow \neg \forall Z(body(Z) \rightarrow \exists W(body(W) \wedge loves(Z,W)))$

Exercise 7

For simplicity, if a(X,Y) then we say that X 'connects to' Y (via an arrow).

i 'Everything that k connects to (via an arrow) is not j'.
False because $a(k,j)$

ii 'If l is a circle then l connects to a black circle'
True because l is a circle and connects to j which is a black circle.

iii 'Something does not connect to something other than itself'
True, both unlabelled squares do not connect to anything other than themselves.

iv 'Everything that is not square connnects to a black circle'
False because j is not square but only connects to l which is not black.

v 'Everything that connects to something other than itself connects to something that connects to it'
False, k connects to j which is not k, but j does not connect to k

vi 'Every (perhaps not distinct) pair of things that both connect to j connect to each other'

False, $a(k, j) \wedge a(k, j)$ but $\neg a(k, k)$