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Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

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Symbolic AI Coursework I: Logic

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1 Formalization

(i) 'If Michel isn't either fulfilled or rich, he won't live another five years.' Let p_1 = 'Michel is fulfilled', p_2 = 'Michel is rich' and p_3 = 'Michel will live another 5 years'. Then:

$$((\neg(p_1 \vee p_2)) \rightarrow p_3) \tag{1}$$

(ii) 'Unless the snowstorm doesn't arrive, Raheem will wear his boots; but I'm sure it will arrive.' Let q_1 = 'The snowstorn arrives', q_2 = 'Raheem will wear his boots' and q_3 = 'I'm sure the snowstorm will arrive'. Then:

$$(((\neg q_1) \vee q_2) \wedge q_3) \tag{2}$$

(iii) 'If Akira and Toshiro are on set, then filming will begin if and only if the caterers have cleared out.' Let r_1 = 'Akira is on set', r_2 = 'Toshiro is on set', r_3 = 'Filming will begin' and r_4 = 'The caterers have cleared out'. Then:

$$((r_1 \wedge r_2) \rightarrow (r_3 \iff r_4)) \tag{3}$$

(iv) 'Either Irad arrived, or Sarah didn't: but not both!' Let s_1 = 'Irad arrived', s_2 = 'Sarah arrived'. Then:

$$((s_1 \vee (\neg s_2)) \wedge (\neg(s_1 \wedge (\neg s_2)))) \tag{4}$$

(v) 'It's not the case both that Herbert heard the performance and Anne-Sophie did, if the latter didn't answer her phone calls.' Let p_1 = 'Anne-Sophie answered her phone calls', p_2 = 'Herbert heard the performance' and p_3 = 'Anne-Sophie heard the performance'. Then:

$$((\neg p_1) \rightarrow (\neg(p_2 \wedge p_3))) \tag{5}$$

2 Definitions

(i) A propositional formula $A \in fmlas_{\mathcal{A}}$ is **satisfiable**, if there exists some truth function $v : fmlas_{\mathcal{A}} \rightarrow \{\mathbf{t}, \mathbf{f}\}$ such that for the propositional evaluation function h_v based on v, we have $h_v(A) = \mathbf{t}$.
(ii) Two propositional formulas A and B in $fmlas_{\mathcal{A}}$ are called **logically equivalent** if for every truth function $v : fmlas_{\mathcal{A}} \rightarrow \{\mathbf{t}, \mathbf{f}\}$ and the propositional evaluation function h_v based on v, we have $h_v(A) = h_v(B)$.
(iii) To show: a propositional formula $\neg A$ is satisfiable if and only if $\neg\neg A \not\equiv \top$

Proof. Suppose first, that $\neg A \in fmlas_{\mathcal{A}}$ is satisfiable, then that means that there exists an atomic evaluation function $v^* : fmlas_{\mathcal{A}} \rightarrow \{\mathbf{t}, \mathbf{f}\}$, such that $h_{v^*}(\neg A) = \mathbf{t}$ for the associated propositional evaluation function. By the definition of the negation, that entails that $h_{v^*}(\neg\neg A) = \mathbf{f}$. But $h_{v^*}(\top) = \mathbf{t}$, so we have found an atomic evaluation function for which the propositional evaluation function evaluates differently on $\neg\neg A$ and on $\top$. Both formulas are thus not equivalent. Conversely, if it's not the case that $\neg\neg A \equiv \top$, then there exists an atomic evaluation function $v^{**} : fmlas_{\mathcal{A}} \rightarrow \{\mathbf{t}, \mathbf{f}\}$ such that $h_{v^{**}}(\neg\neg A) \neq h_{v^{**}}(\top) = \mathbf{t}$. So, $h_{v^{**}}(\neg\neg A) = \mathbf{f}$. $\neg\neg\neg A$ is clearly logically equivalent to $\neg A$, and, by the definition of the negation, $h_{v^{**}}(\neg A) = h_{v^{**}}(\neg\neg\neg A) = \mathbf{t}$. But that is exactly the definition of satisfiability. $\square$

3 Truth Tables

p	q	r	$(p$	$\wedge$	$\neg q$	$\leftrightarrow$	$\neg$	$(\neg r$	$\vee$	$\neg p))$	$\rightarrow$	$(\neg$	$\neg q$	$\rightarrow$	$r)$
T	T	T	T	F	F	F	T	F	F	F	**T**	T	F	T	T
T	T	F	T	F	F	T	F	T	T	F	**F**	T	F	F	F
T	F	T	T	T	T	T	T	F	F	F	**T**	F	T	T	T
T	F	F	T	T	T	F	F	T	T	F	**T**	F	T	T	F
F	T	T	F	F	F	T	F	F	T	T	**T**	T	F	T	T
F	T	F	F	F	F	T	F	T	T	T	**F**	T	F	F	F
F	F	T	F	F	T	T	F	F	T	T	**T**	F	T	T	T
F	F	F	F	F	T	T	F	T	T	T	**T**	F	T	T	F

The truth table shows that the formula is not valid/tautological, since there are atomic truth functions for which the formula is not true.

4 CNF and DNF

(i) a. CNF, b. CNF and DNF, c. CNF, d. CNF and DNF, e. DNF, f. neither, g. CNF and DNF, h. CNF and DNF

(ii) The property of refutation-soundness and -completeness of a resolution derivation says that, for a propositional formula S in conjunctive normal form, $S \vdash_{\text{res(PL)}} \emptyset$ (i.e. from S you can derive the empty clause) iff $S \vDash \bot$. This property is important because we can use it to show that a propositional formula is satisfiable. We already know that S is satisfiable iff $S \nvDash \bot$, so if S does not resolve to the empty set, we know that S is satisfiable.

(iii)

(a) $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$ can be resolved to $\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$ by the pure rule applied to q. This, in turn, can be resolved on p to $\{\{\neg r\}\}$, removing tautologous clauses. $\{\{\neg r\}\}$ can be resolved to $\{\}$, i.e. satisfiable.

(b) $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$ can be resolved to $\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$ by unit propagation on $\neg q$, which in turn can be resolved by unit propagation to $\{\{\neg p\}, \{p\}\}$ on $\neg r$, which resolves to $\{\{\}\}$, i.e. not satisfiable.

5 Davis-Putnam

I want to determine whether the following argument is valid:
If I'm going, then you aren't.
If you're not going, then neither is Tara.
Either Tara's going or I'm not.
Tara's going unless I am.
So, you're going.

Define p = I'm going, q = You are going, r = Tara is going. Then we can write the argument as

$p \rightarrow \neg q$
$\neg q \rightarrow \neg r$
$r \vee \neg p$
$r \vee p$
$\vDash q$

Because $p_1 \rightarrow p_2$ is logically equivalent to $\neg p_1 \vee p_2$, we can can check whether the following CNF is satisfiable in order to check whether the original argument was valid: $\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$. Unit propagation applied to $\neg q$ gives us $\{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$. Unit propagation on $\neg r$ then lets us arrive at $\{\{\neg p\}, \{p\}\}$, which resolves to $\{\{\}\}$, so the CNF is unsatisfiable and thus the argument is valid.

6 FOL Translation

(i) All of Andrea's aunts' aunts gave a cupcake to someone other than Andrea.
$\mathcal{C} = \{andrea\}$
$\mathcal{P}_1 = \{cupcake, human\}$
$\mathcal{P}_2 = \{aunt\}$, where *aunt*(X,Y) stands for 'X is the aunt of Y'
$\mathcal{P}_3 = \{give\}$, where *give*(X,Y,Z) stands for 'X gives Y to Z'

$\forall X\ \forall Y\ (aunt(X,Y) \wedge aunt(Y, andrea) \rightarrow \exists Z\ \exists W\ (cupcake(Z) \wedge human(W) \wedge \neg(W = andrea) \wedge give(X,Z,W)))$

(ii) There's a computer connected to every computer which isn't connected to itself.
$\mathcal{C} = \{\}$
$\mathcal{P}_1 = \{computer\}$
$\mathcal{P}_2 = \{connected\}$, where *connected*(X,Y) stands for 'X is connected to Y'

$\forall X\ (computer(X) \wedge \neg\ connected(X,X) \rightarrow \exists Y\ (computer(Y) \wedge connected(Y,X))$

(iii) Any painting by Paul Klee in a British gallery hangs in a room where all Kandinsky paintings in that gallery hang.
$\mathcal{C} = \{paulklee, britishgallery, kandinsky\}$
$\mathcal{P}_1 = \{painting, gallery, room\}$
$\mathcal{P}_2 = \{painted, hang\}$