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Exercise Information

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Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-10-27 14:11:17

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Coursework 1 – Introduction to Symbolic AI

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Propositional Logic

1. (i) Michel is fulfilled as p, Michel is rich as q, Michel will live another five years as r:
$$(\neg(p \lor q) \rightarrow (\neg r))$$

   (ii) Raheem will wear his boots unless the snowstorm doesn't arrive, but I'm sure it will arrive. Raheem will wear his boots as p, the snowstorm arrives as q:
$$(((\neg q) \lor p) \land q)$$

   (iii) Akira on set as p, Toshiro on set as q, filming will begin as r, caterers have cleaned out as s:
$$((p \land q) \longrightarrow (r \leftrightarrow s))$$

   (iv) Irad arrived as p, Sarah arrived as q:
$$((p \land q) \lor ((\neg p) \land (\neg q)))$$

   (v) Herbert heard the performance as p, Anne-Sophie heard the performance as q, Anne-Sophie didn't answer phone calls as r:
$$(r \longrightarrow (\neg(p \land q)))$$

2. (i) A propositional formula A is said to be satisfiable if there exists at least one atomic evaluation function v for which the corresponding propositional evaluation function applied to A, $h_v(A)$, evaluates to true.

   (ii) Two propositional formulas A and B are said to be logically equivalent if, for all possible atomic evaluation functions v, $h_v(A) = h_v(B)$.

   (iii) If we assume that there is an atomic evaluation formula v such that $h_v(\neg\neg A)$!= $\top$, then using the definition of negation, $\neg\neg A = A$ evaluates under the same v to $\bot$. As a result, $\neg A$ evaluated under the same v is $h_v(\neg A) = \top$. Given there is at least one atomic evaluation function for which $\neg A$ evaluates to true, then the propositional formula $\neg A$ is satisfiable.

3.

p	q	r	$(p$	$\land$	$\neg q$	$\leftrightarrow$	$\neg$	$(\neg r$	$\lor$	$\neg p))$	$\rightarrow$	$(\neg\neg q$	$\rightarrow$	$r)$
T	T	T	T	F	F	F	T	F	F	F	T	T	T	T
T	T	F	T	F	F	T	F	T	T	F	F	T	F	F
T	F	T	T	T	T	T	T	F	F	F	T	F	T	T
F	T	T	F	F	F	T	F	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F	T	T	F	T	F	T	F
F	T	F	F	F	F	T	F	T	T	T	F	T	F	F
F	F	T	F	F	T	T	F	F	T	T	T	F	T	T
F	F	F	F	F	T	T	F	T	T	T	T	F	T	F

Since the truth values in the 'central' Boolean connective's column (the material conditional, shaded in green) are not all 'T', then it follows that the propositional formula is **not** valid for all truth values of p, q, and r.

SAT Solving

4. (i) CNF: a, b, d, f, g, h DNF: b, d, e, g, h

   (ii) Refutation-soundness and -completeness is a theorem that states that, for a CNF S, when a propositional resolution derivation of S results in an empty set clause at the root node of the created tree, then S is semantically implied to be false or $\bot$. This is important because, as a result of this theorem, for any CNF S, S can be shown the be satisfiable iff it is not possible to derive the empty set from a resolution derivation. SAT solvers work by trying to construct a resolution derivation of the empty set – if they fail, S is satisfiable. If they succeed, S is not satisfiable.

   (iii) a)

   $$\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$$

   Applying pure rule on $\{q\}$:

   $$\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$$

   b)

   $$\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$$

   Applying unit propagation on $\{\neg q\}$:

   $$\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$$

   Applying unit propagation on $\{\neg r\}$:

   $$\{\{\neg p\}, \{p\}\}$$

   Applying unit propagation on $\{p\}$:

   $$\{\{\}\}$$

   (Since the CNF only contains an empty clause, it is unsatisfiable).

5. With I'm going as p, you are going as q, Tara is going as r, we have the clauses P

   $$p \longrightarrow \neg q \equiv (\neg p \vee \neg q)$$
   $$\neg q \longrightarrow \neg r \equiv (q \vee \neg r)$$
   $$(r \vee \neg p)$$
   $$(r \vee p)$$

   and conclusion Q

   $$q$$

   the argument is valid if the conjunction of the clauses P and negation of the conclusion Q is unsatisfiable:

   $$P \wedge \neg Q$$

   In CNF form this is

   $$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$$

   Applying unit propagation on $\{\neg q\}$:

   $$\{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$$

   Applying unit propagation on $\{\neg r\}$:

   $$\{\{\neg p\}, \{p\}\}$$

   Applying unit propagation on $\{p\}$:

   $$\{\{\}\}$$

   Since we have derived the empty clause, $P \wedge \neg Q$ is unsatisfiable, so the original argument is valid.

First-Order Logic

6. (i) $C = \{\text{Andrea}\}$, $P_2 = \{cupcake\}$, $P_1 = \{aunt\}$, where $cupcake(X,\ Y)$ means X gave a cupcake to Y, and $aunt(X)$ refers to the aunt of X.
$$\forall X(X = aunt(aunt(Andrea)) \rightarrow \exists Y(cupcake(X,Y) \wedge \neg(Y = Andrea))$$

   (ii) $P_2 = \{connected\}$, where $connected(X,\ Y)$ means computer X is connected to computer Y.
$$\forall Y \exists X(\neg connected(Y,X) \rightarrow connected(X,Y))$$

   (iii) $C = \{\text{Klee, Kandinsky}\}$, $P_1 = \{gallery\}$, $P_2 = \{painted,\ in,\ hang,\ room\}$ where $gallery(X)$ means X is a British gallery, $room(X,\ Y)$ means X is a room in Y, $painted(X,\ Y)$ means X was painted by Y, $in(X,\ Y)$ means X is located in Y, and $hang(X,\ Y)$ means X hangs in Y.
$$\forall Y \forall X \exists Z(painted(X,Klee) \wedge painted(Y,Kandinsky) \wedge gallery(Z) \wedge in(X,Z) \wedge in(Y,Z)$$
$$\rightarrow \exists R\big(room(R,Z) \wedge hang(X,R) \wedge hang(Y,R)\big))$$

   (iv) $P_2 = \{loves\}$ where $loves(X,\ Y)$ means X loves Y.
$$\exists X \exists Y(\neg loves(X,Y) \rightarrow \neg \forall Z\big(loves(Z,Y)\big))$$

7. (i) For all X where $a(k,X)$ is false, e.g. when $\sigma(X) = l$ or k, then the conditional is trivially true. So, considering all the values of X that make the antecedent true, we only need to show the consequent is true for the entire statement to be true in the structure. $a(k,X)$ is true if $\sigma(X) = j$, meaning that the consequent $\neg(X = j)$ is false; hence, the statement is **false** in the structure.

   (ii) $c(l)$ is true, so we require the consequent to be true. For this to be the case $\sigma(X) = k$ or $\sigma(X) = j$, so there is at least one object which makes the conditional is true, meaning that the statement is **true** in the structure.

   (iii) Translating this as: For some X there is no Y such that $(\neg(X = Y) \wedge a(X,Y))$ – i.e. no directed arrows exist between any two different objects. Since for $\sigma(X) = l$ and $\sigma(Y) = k$ we can satisfy the conjunction, it follows that the statement is **false** in the structure.

   (iv) As in (i), when X is such that the antecedent is false, then the conditional is trivially true. So, considering all the X for which the antecedent is true, the consequent must be correspondingly true for the conditional to evaluate to true. The antecedent is true if $\sigma(X) = k, j, l$. Considering $\sigma(X) = j$, there is no Y such that $c(Y) \wedge b(Y) \wedge a(X,Y)$ evaluates to true. As a result, the conditional is not true for all values of X, so the statement is **false** in the structure.

   (v) Again, considering the X such that the antecedent is true, we require the consequent to be true for all the corresponding X. For the antecedent $\exists Y(\neg(X = Y) \wedge a(X,Y))$, we want to find all the values of X for which there is some Y which is connected to X and does not equal X. This is satisfied by $\sigma(X) = k, j, l$. Considering $\sigma(X) = k$ with the consequent $\exists Y(a(X,Y) \wedge a(Y,X))$, there is no Y such that k is connected to Y and Y is connected to k; hence the statement is **false**.

   (vi) Considering the X and Y that make the antecedent true, we can have possible pairs $\sigma(X), \sigma(Y) = [(k,\ k),\ (l,\ l),\ (k,\ l),\ (l,\ k)]$. Evaluating the consequent for each of these possible combinations of X and Y, we get false, false, true, true (respectively). As a result of the consequent not being true for all combinations of X and Y that make the antecedent true, the material conditional does not always evaluate to true, so the entire statement is **false** in the structure.