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Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
Exercise:	2 (CW)	**Due:**	Tue - 03 Nov 2020
Title:	Logic	**Assessment:**	Individual
FAO:	Craven, Robert (rac101)	**Submission:**	Electronic

Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-10-31 14:55:01

For Markers only: (circle appropriate grade)

LOH, Haoyuan (hyl414)	01020498	t5	2020-10-31 14:55:01	**A***	**A**	**B**	**C**	**D**	**E**	**F**

Introduction to Symbolic AI: Coursework 1 – Logic

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Q1i)

(p V q) → r

p: Michel is not fulfilled

q: Michel is not rich

r: Michel will not live another 5 years

Q1ii)

(p V q) Λ ¬p

p: Snowstorm does not arrive

q: Raheem will wear his boots

Q1iii)

(p → q) ↔ r

p: Akira and Toshiro are on set

q: filming will begin

r: caterers have cleared out

Q1iv)

p V q

p: Irad arrived

q: Sarah did not arrive

Q1v)

¬ (p Λ q) → r

p: Herbert heard the performance

q: Anne-Sophie heard the performance

r: Anne-Sophie did not answer her phone calls

Q2i)

Definition of a Satisfiable Propositional Formula A is whereby a propositional formula A is satisfiable if there is some v such that $h_v(A) = t$, where v is an atomic evaluation function

Q2ii)

Definition of Logical Equivalence between 2 Propositional Formulas A and B is whereby they are logically equivalent if, for every v, $h_v(A) = h_v(B)$

Q2iii)

First assume that $\neg\neg A \neq$ T is satisfiable.

Take an arbitrary propositional evaluation function v. If $h_v(\neg\neg A) =$ t, then since $\neg\neg\mathrm{A} \neq \mathrm{T}$ is satisfiable, it cannot be that $h_v(\mathrm{T}) =$ t, hence, $h_v(\mathrm{T}) =$ f.

However, consider the case of $\neg\neg\mathrm{A} = \mathrm{T}$. Take an arbitrary propositional evaluation function v^1. If $h_v(\neg\neg A) =$ t, then since $\neg\neg\mathrm{A} = \mathrm{T}$ is satisfiable, it can only be that $h_v(\mathrm{T}) =$ t.

Therefore, $\neg\neg\mathrm{A} \neq \mathrm{T}$ is satisfiable iff it's not the case of $\neg\neg\mathrm{A} = \mathrm{T}$.

Q3)

$(p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p)) \rightarrow (\neg\neg q \rightarrow r)$

p	q	r		$(p$	$\land$	$\neg q$	$\leftrightarrow$	$\neg$	$(\neg$r	$\lor$	$\neg p))$	$\rightarrow$	$(\neg\neg q$	$\rightarrow$	$r)$
t	t	t		t	f	f	f	t	f	f	f	t	t	t	t
t	t	f		t	f	f	t	f	t	t	f	t	t	f	f
t	f	f		t	t	t	f	f	t	t	f	t	f	t	f
f	t	f		f	f	f	t	f	t	t	t	f	t	f	f
f	t	t		f	f	f	t	f	f	t	t	t	t	t	t
f	f	f		f	f	t	t	f	t	t	t	t	f	t	f
f	f	t		f	f	t	t	f	f	t	t	t	f	t	t
t	f	t		t	t	t	t	t	f	f	f	t	f	t	t
				1	2i)	1	4	3	1	2ii)	1	5	1	2iii)	1

Process of determining truth table is. Evaluate col. 1 first.

Then evaluate col. 2 from col. 1.

Evaluate col. 3 from col. 2i)

Evaluate col. 4 from col. 3) and col. 2i)

Evaluate col. 5 from col. 4) and col. 2iii)

Since the final evaluation of the formula given in column 5) has 7 truths and 1 falsity, we can conclude that the formula given is not valid as there is 1 falsity.

Q4ia)

Yes, this is CNF.

Q4ib)

Yes, this is CNF.

Q4ic)

Yes, this is CNF.

Q4id)

Yes. Truths are CNF.

Q4ie)

Not CNF. Disjunctions are DNF

Q4if)

No. $\neg\neg$p is not a literal.

Q4ig)

Yes. Conjunctions are CNF.

Q4ih)

Not CNF. Disjunctions are DNF.

Q4ii)

Theorem of Refutation soundness and refutation completeness.

Let S be in CNF $S \ \vdash_{res\ (PL)} \ \emptyset \ iff \ S \ \vDash \ \bot$.

This property leads immediate corollary that is the result which is at the heart of many SAT-solvers. Whereby the corollary of SAT and resolution.

Let S be in CNF. S is satisfiable iff $\vdash/_{\mathrm{res(PL)}}\ \emptyset$.

Q4iiia)

The clausal form notation is given by:

$\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

$\Rightarrow \{\{p, s\}, \{\neg s, q\}, \{\neg p, \neg s\}\}$ [by resolving in all ways on r]

$\Rightarrow \{\{\neg s, q\}\}$ [by resolving in all ways on p]

$\Rightarrow \{\{q\}\}$ [pure rule on $\neg s$]

$\Rightarrow \{\{\emptyset\}\}$ [pure rule on q]

$\Rightarrow$ Unsatisfiable [since $\emptyset$ is in the set]

Q4iiib)

The clausal form notation is given by:

$\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

$\Rightarrow \{\{\neg p, q\}, \{\neg q\}, \{p, q\}\}$ [by resolving in all ways on r]

$\Rightarrow \{\{\neg q\}\}$ [by resolving in all ways on p]

$\Rightarrow \{\{\emptyset\}\}$ [pure rule on $\neg q$]

$\Rightarrow$ Unsatisfiable [since $\emptyset$ is in the set]

Q5)

We formalise it as

$p \to q, \neg p \to r, \neg r \vee \neg p, \neg r \vee p$ therefore, p.

p: I'm going

q: You are not going

r: Tara is not going

So, we must check whether $p \to q, \neg p \to r, \neg r \vee \neg p, \neg r \vee p \vDash p$

We know that in general, A1, . . ., An |= B iff A1 ^ · · · ^ An ^ ¬B is unsatisfiable.

So, we can check whether (p → q) ∧ (¬p → r) ∧ (¬r ∨ ¬p) ∧ (¬r ∨ p) ∧ ¬p is satisfiable.

First convert it to clausal form CNF: {{¬p, q}, {p, r}, {¬r, ¬p}, {¬r , p}, {¬p }}

Now, applying DP:

{{¬p, q}, {p, r}, {¬r, ¬p}, {¬r, p}, {¬p }}

⇒ {{¬p, q}, {¬p}} [by resolving in all ways on r]

⇒ {{¬p}, {¬p}} [pure rule on q]

⇒ {{¬p}, {¬p}} [pure rule on {¬p]

⇒ {}

⇒ Satisfiable [since no conditions for further application of rules apply]

Q6i)

¬ → ↔ ∀ ⊥ T ∧ V

Predicates used are as follows:

aunts(X) ('X is aunts'), gives_cupcake(Y) ('Y is giving cupcakes'), Andrea(Z) ('Z' is Andrea)

First-order language signature is as follows:

$\forall X(Andrea(aunts(aunts(X)))) \ \rightarrow \ gives\_cupcake(Y) \ \rightarrow \ \neg Andrea(Z) \,)$

Q6ii)

Predicates used are as follows:

computer(X) ('X is a computer'), connected(X, Y) ('X is connected to Y')

First-order language signature is as follows:

∀X (computer(X) → (connected(X, Y) ∧ ¬computer(X))

Q6iii)

Predicates used are as follows:

Paul_Klee's(X) ('X is Paul Klee's Paintings'), Kandinsky's (Y) ('Y is Kandinsky's Paintings), room(Z, X) ('Z room in a British gallery containing Painting X'), room (Z, Y) ('Z room in a British gallery containing Painting Y')

First-order language signature is as follows:

$\forall X \ (room(Z,X) \ \rightarrow \ room(Z,Y) \,)$

Q6iv)

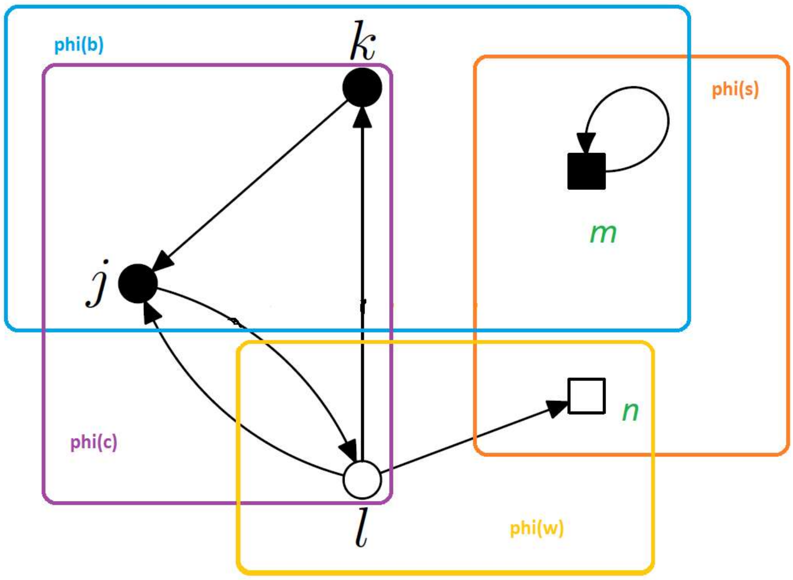
Predicates used are as follows:

person(X), loves(X, Y) ('X loves Y')

First-order language signature is as follows:

$\exists X\, \exists Y\; (\, person(X)\; \rightarrow\; \neg loves(X,Y)\,)\;\; \rightarrow\; \forall X\; (\, person\; (X)\;\; \rightarrow \exists Y(\, loves(X,Y)\,)\,)$

Q7)



The following L- structure includes the 4 objects $\varphi(\mathrm{b}), \varphi(\mathrm{c}), \varphi(\mathrm{w}), \varphi(\mathrm{s})$ as described.

2 objects have also been named as $\varphi(\mathrm{m}), \varphi(\mathrm{m})$ as shown above.

Q7i)
False.

Since $\varphi(a)$ of the set pairs of (k, X) where denotes the directional arrow from k and that j is the only symbol from *k* which has a directional arrow, the statement logic can only be true if *(X = j).*

Q7ii)
The statement logic defines that for the object *l* in the set of $\varphi(\mathrm{c})$, there is some object X such that it satisfies the following of i) being in the set of black objects, ii) circular objects and iii) having the directional arrow of *l* pointing to *n.*

However, since the directional arrow of *l* pointing to *n* is not sitting within the set of $\varphi(\mathrm{b}), \varphi(\mathrm{c})$. The statement is false.

False.

Q7iii)
The statement logic says that there is some *X* objects and all *Y* objects, where there exists some $X \neq Y$ and $\varphi(\mathrm{a})$ set where *X* object has a directional arrow to another (i.e. not back to *X* itself) *Y* object.

This statement is true since, for some objects *X* (but not all objects since m has a directional arrow back to itself), the 4 objects *j, k, l, n* has directional arrows pointing to another object.

True.

Q7iv)
True.

Outside of the set s(X), as denoted by the objects not within the orange box, there exists some Y objects in the intersection of the sets c(Y), b(Y) and a(X,Y) which has the objects j and k within the intersection that satisfies the logic statement.

Q7v)

False.

The statement suggests that for all objects *X*, if there are some objects with no directional arrows pointing to itself then there are some objects with arrows pointing to itself.

We can see that this is clearly not true as there are no objects that has directional arrows pointing away and to itself.

Q7vi)

False.

Since only objects *k* and *l* point to *j*, a counterexample of using object *m* and *n* which does not point to *j* makes this logic statement false.