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Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
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Title:	Logic	**Assessment:**	Individual
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Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

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Cynthuja Ramanan

Introduction to Symbolic AI

Coursework 1 : Logic

1.i) $(((\neg p) \vee (\neg q)) \rightarrow (\neg r))$

p: Michel is fulfilled

q: Michel is rich

r: Michel will live another five years

ii) $((\neg(\neg p)) \rightarrow q)$

p: The snowstorm does arrive

q: Raheem wears his boots

iii) $((p \wedge q) \rightarrow (r \leftrightarrow s))$

p: Akira is on set

q: Toshiro is on set

r: filming begins

s: caterers have cleared out

iv) $((p \vee (\neg q)) \wedge (\neg (p \wedge (\neg q))))$

p: Irad arrived

q: Sarah arrived

v) $((\neg p) \rightarrow (\neg (q \wedge r)))$

p: Anne-Sophie answered her phone calls

q: Herbert heard the performance

r: Anne-Sophie heard the performance

2 i) A propositional formula, A, is satisfiable if there is some v such that the propositional evaluation function based on v, $h_v(A) = t$

ii) Two propositional formulas A and B are logically equivalent if for every v, $h_v(A) = h_v(B)$

2 iii) To prove: $\neg A$ is satisfiable iff $\neg\neg A \not\equiv T$

$\neg A$	$\neg$	$(\neg A)$
t	f	t
f	t	f

From the truth table, $h_v(\neg A)$ is only true when $h_v(\neg\neg A) = f$.

Therefore it can be said that $\neg$ is satisfiable iff $\neg\neg A \not\equiv T$

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p	q	r	(p	$\wedge$	$\neg q$	$\leftrightarrow$	$\neg$	$(\neg r$	$\vee$	$\neg p))$	$\rightarrow$	$(\neg\neg q$	$\rightarrow$	r)
t	t	t	t	f	f	f	t	f	f	f	t	t	t	t
t	t	f	t	f	f	t	f	t	t	f	f	t	f	f
t	f	t	t	t	t	t	t	f	f	f	t	f	t	t
t	f	f	t	t	t	f	f	t	t	f	t	f	t	f
f	t	t	f	f	f	t	f	f	t	t	t	t	t	t
f	t	f	f	f	f	t	f	t	t	t	f	t	f	f
f	f	t	f	f	t	t	f	f	t	t	t	f	t	t
f	f	f	f	f	t	t	f	t	t	t	t	f	t	f

Since $h_v((p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \rightarrow (\neg\neg q \rightarrow r)) \neq t$ for any valuation v, the formula is not valid.

4i) a) CNF (a conjunction of clauses p and $(\neg q \vee r)$)

b) CNF and DNF ("conjunction" with just one conjunct)

c) neither (due to nested conjuncts and disjuncts)

d) neither

e) DNF

f) CNF

g) CNF and DNF } (conjunction with just one conjunct)

h) DNF and CNF

ii) refutation - soundness and - completeness:

Let S be in CNF. $S \vdash_{RES(PL)} ()$ iff $S \models \bot$

This important because it follows on from this that S is satisfiable iff $S \nvdash_{RES(PL)} ()$, as defined in corollary 2.8 (SAT and resolution). This corollary allows determination of the satisfiability of S, where S is satisfiable if it is impossible to derive $()$ from it.

iii a) $\{\{p,s\}, \{q,r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

$\Rightarrow \{\{p,s\}, \{r\}, \{\neg s\}, \{\neg p, \neg r, \neg s\}\}$ [pure rule: q was pure]

$\Rightarrow \{\{p\}, \{r\}, \{\neg p, \neg r\}\}$ [unit propagation by unit clause $\{\neg s\}$]

$\Rightarrow \{\{p\}, \{\neg p\}\}$ [unit propagation by unit clause $\{r\}$]

$\Rightarrow \{\{\}\}$ [unit propagation by unit clause $\{p\}$]

b) $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

$\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$ [unit propagation by unit clause $\{\neg q\}$]

$\{\{\neg p\}, \{p\}\}$ [unit propagation by unit clause $\{\neg r\}$]

$\{\{\}\}$ [unit propagation by unit clause $\{p\}$]

5 formalizing the argument: $p \to \neg q$, $\neg q \to \neg r$, $r \vee \neg p$, $\neg p \to r$, therefore q

p: I'm going, q: you're going, r: Tara's going

We must check whether $p \to \neg q, \neg q \to \neg r, r \vee \neg p, \neg p \to r \models q$

In general, $A_1, \dots A_N \models B$ iff $A_1 \wedge \dots \wedge A_n \wedge \neg B$ is unsatisfiable. Therefore we can check whether $(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (\neg p \to r) \wedge q$ is unsatisfiable.

conversion to clausal-form CNF:

$\{\{p, \neg q\}, \{\neg q, \neg r\}, \{r, \neg p\}, \{\neg p, r\}, \{q\}\}$

$\Rightarrow \{\{p\}, \{\neg r\}, \{r, \neg p\}, \{\neg p, r\}\}$ [unit clause $\{q\}$]

$\Rightarrow \{\{\neg r\}, \{r\}, \{r\}\}$ [unit clause $\{p\}$]

$\Rightarrow \{\{\}\}$ [unit clause $\{\neg r\}$]

$\Rightarrow$ unsatisfiable [since $\{\}$ is in the set]

since the CNF is unsatisfiable, the original argument is propositionally valid.

6 i) $\forall X (X = aunt(aunt(andrea))) \land \exists Y (\neg (Y = andrea)) \rightarrow givescupcake(X, Y)$

$C = \{andrea\}$

$P = \{givescupcake\}$

$\mathcal{F} = \{aunt\}$

ii) $\forall X (computer(X) \land \neg(connected(X, X))) \longrightarrow \exists Y (computer(Y) \land connected(Y, X))$

$P_1 = \{computer\}$

$P_2 = \{connected\}$

iii) $\forall X (painting(X) \land painted(paulklee, X) \rightarrow \forall Y (painting(Y) \land painted(kandinsky, Y) \land hangs(Y, X))$

$C = \{paulklee, kandinsky\}$

$P_1 = \{painting\}$

$P_2 = \{hangs\}$

$P_3 = \{painted\}$

iv) $\exists X\ loves(X, nobody) \longrightarrow \forall Y \exists Z (\neg(loves(Y, Z)))$

$C = \{nobody\}$

$P_1 = \{loves\}$

7i) For all X, if X has a directed arrow from k, then X is not j.

Since j is the only object with an arrow directed from K, this is false

ii) If circular object l, then there is some X such that it is black, circular, and has a directed arrow from L.

Since k and j are black circular objects with an arrow directed from L to them, this is true.

iii) There is some X and not some Y such that X is not equal to Y and there is an arrow directed from X to Y

This is false since the white square object, k, j and l have arrows directed to them from other objects.

iv) For all X, such that if X is not a square object, then there is some Y such that Y is circular and black and there is an arrow directed from X to Y

In the case of j, there is no arrow pointing to a black object, therefore this is false

v) For all X, and some Y, if X is not equal to Y and an arrow points from X to Y, then there is some Y such that an arrow points from X to Y and an arrow from Y to X

The second part of the formula is only true if Y = j or l, therefore this is true

vi) For all X and all Y such that an arrow points from X to j and an arrow points from Y to j, then an arrow points from X to Y or an arrow points from Y to X.

The only objects with arrows pointing to j are k and l, and since an arrow points from l to k, this is true