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## Exercise Information

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## Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-11-02 19:30:55

**For Markers only:** (circle appropriate grade)

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# Introduction to Symbolic AI: Coursework 1- Logic

1)

i)

$\neg(p \vee q) \longrightarrow \neg r$

p: Michel is fulfilled
q: Michel is rich
r: Michel will live another 5 years

ii)

$(p \longrightarrow q) \wedge r$

p: The snowstorm does arrive
q: Raheem will wear his boots
r: I'm sure the snowstorm will arrive

iii)

$p \wedge q \longrightarrow (a \leftrightarrow b)$

p: Akira is on set
q: Toshiro is on set
a: filming will begin
b: the caterers have cleared out

iv)

$(p \vee \neg q) \wedge \neg(p \wedge \neg q)$
p: Irad has arrived
q: Sarah has arrived
note: here assumed both meant both p and ¬q scenarios not occurring together, as opposed to both not arriving.

v)

$\neg a \longrightarrow \neg(b \wedge c)$
a: Anne-Sophie did answer her phone calls
b: Herbert heard the performance
c: Anne-Sophie heard the performance

2)

i)
A propositional formula A is satisfiable if there exists some atomic valuation v such that for a valuation function h (mapping to 1 (True) or 0 (False)), the function maps A to true at v i.e. $h_v(A) = 1$ exists.

ii)
Two propositional formulas A and B are logically equivalent if for every atomic valuation v, the valuation function h (mapping to 1 (True) or 0 (False)), maps A and B to the same value. i.e. $h_v(A) = h_v(B)$

iii)

First assume that $\neg\neg A \equiv \top$. i.e $\neg\neg A$ and $\top$ are logically equivalent.

Hence it follows that for any atomic valuation v, $h_v(\neg\neg A) = \boldsymbol{t}$ $= \top$.

Therefore, by the property of $\neg$, it follows that $h_v(\neg A) = \boldsymbol{f} = \neg\top = \bot$.

Hence for any atomic valuation v, $h_v(\neg A) = \boldsymbol{f} = \bot$ i.e. it is not possible for $\neg A$ to be satisfiable.

Therefore, if it is impossible for $\neg A$ to be satisfiable when $\neg\neg A \equiv \top$, by contradiction it must be the case that $\neg\neg A \not\equiv \top$, for $\neg A$ to be satisfiable.

3)

Note in following truth table: 1 and 0 are equivalent to True and False, respectively.

| p | q | r | $\neg p$ | $\neg q$ | $\neg r$ | $\neg\neg q$ | $p \land \neg q$ | $\neg r \lor \neg p$ | $\neg(\neg r \lor \neg p)$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

| $\neg\neg q \longrightarrow r$ | $p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p)$ | $\big(p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p)\big) \longrightarrow (\neg\neg q \longrightarrow r)$ |
|---|---|---|
| 1 | 0 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

From the truth table, we may conclude that the proposition $\big(p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p)\big) \longrightarrow (\neg\neg q \longrightarrow r)$ is not valid. This can be seen by looking at the final row of the truth table.

For example, for a certain valuation function v such that **v(p)=0, v(q)=1 and v(r)=0**
v$\Big(\big(p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p)\big) \longrightarrow (\neg\neg q \longrightarrow r)\Big) = 0$

Or for a certain valuation function v such that **v(p)=1, v(q)=1 and v(r)=0**
v$\Big(\big(p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p)\big) \longrightarrow (\neg\neg q \longrightarrow r)\Big) = 0$

Hence the proposition is invalid in both these cases.

4)

i)

a) In CNF

b) In both CNF and DNF

c) Neither CNF nor DNF

d) Neither CNF nor DNF

e) In DNF

f) Neither CNF nor DNF

g) In both CNF and DNF

h) In both CNF and DNF

ii)

Effectively, for S in CNF, it is possible to derive Ø via resolution (i.e. $S\vdash_{res(PL)}$ Ø) if and only if S|=⊥.

This is important since it allows derivation of the Corollary which states that, for S in CNF, S is satisfiable if and only if it is not possible to derive Ø from S via resolution.

Hence this allows us to check whether S in CNF is satisfiable by just checking whether it resolves down to Ø or not (i.e. an empty clause would be taken as false whereas an empty conjunction would be taken as true).

Note: Ø above corresponds to the situation of an 'empty clause'.

iii)

a)

$\{\{\mathrm{p},\mathrm{s}\},\{\mathrm{q},\mathrm{r}\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\}$

| | |
|---|---|
| $\{\{p,s\},\{\neg p,\neg r,\neg s\}\}$ | pure rule on $q$ |
| $\{\{p,s\}\}$ | pure rule on $\neg r$ |
| {} | pure rule on $s$ |

b)

$\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

$\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$ unit propagation on $\neg q$

$\{\{\neg p\}, \{p\}\}$ unit propagation on $\neg r$

$\{\{\}\}$ unit propagation on $p$

5)

First formalize the argument:

$p \longrightarrow \neg q, \neg q \longrightarrow \neg r, \neg r \longrightarrow \neg p, \neg p \longrightarrow r \models q$

p: I'm going
q: you're going
r: Tara is going

In general $A_1, \ldots, A_n \models B$ iff $A_1 \wedge \cdots \wedge A_n \wedge \neg B$ is unsatisfiable

Hence can check whether following is satisfiable:

$(p \longrightarrow \neg q) \wedge (\neg q \longrightarrow \neg r) \wedge (\neg r \longrightarrow \neg p) \wedge (\neg p \longrightarrow r) \wedge \neg q$

Convert to clauses using fact that $\neg q \longrightarrow p$ is equivalent to $p \vee q$:

$(\neg p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee r) \wedge \neg q$

Convert to CNF set notation:

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{p, r\}, \{\neg q\}\}$

$\{\{\neg r\}, \{r, \neg p\}, \{p, r\}\}$ unit propagation on $\neg q$

$\{\{\neg p\}, \{p\}\}$ unit propagation on $\neg r$

$\{\{\}\}$ unit propagation on $\neg p$

$\Rightarrow$ unsatisfiable since $\emptyset$ is in the set

**Hence since the CNF is unsatisfiable, the original argument is propositionally valid.**

6)

i)

$$\exists X \exists Y \left( Andrea(X) \land givecup\big(s\_aunts\big(s\_aunts(X)\big), Y\big) \land \neg Andrea(Y) \right)$$

$$\mathcal{P}_1 = \{Andrea\}$$

$$\mathcal{P}_2 = \{givecup\}$$

$$\mathcal{F}_1 = \{s\_aunts\}$$

Where, Andrea(X) is read as X is Andrea; givecup(X,Y) is read as X gives a cupcake to Y; s_aunts(X) is read as X's aunts.

ii)

$$\exists Y \forall X (computer(Y) \land computer(X) \land connected(Y, X) \land \neg connected(Y, Y))$$

$$\mathcal{P}_1 = \{computer\}$$

$$\mathcal{P}_2 = \{connected\}$$

Where, computer(X) is read as X is a computer; connected (X,Y) is read as X is connected to Y.

iii)

$$\forall X \forall Y (paul\_klee(X) \land british\_gallery(X) \land kandinsky(Y) \land british\_gallery(Y)) \longrightarrow hang(X, Y)$$

$$\mathcal{P}_1 = \{paul\_klee, british\_gallery, kandinsky\}$$

$$\mathcal{P}_2 = \{hang\}$$

Where paul_klee(X) is read as X is a Paul Klee painting; british_gallery(X) is read as X is in a British Gallery; kandinsky(X) is read as X is a Kandinsky painting; hang(X,Y) is read as X and Y hang in same room in the gallery.

iv)

$$\exists Y \forall X \neg loves(Y, X) \longrightarrow \neg(\forall X \exists Y\ loves(Y, X))$$

$$\mathcal{P}_2 = \{loves\}$$

Where loves(X,Y) is read as X loves Y.

7)

i) **False**: $\forall X(a(k, X) \rightarrow \neg(X = j))$

a(k,X) refers to an object X to which k connects. However, k is only connected to j; whereas the proposition claims that for all object to which k connects none of them are j i.e. $\neg(X = j)$.

ii) **True**: $c(l) \rightarrow \exists X(b(X) \land c(X) \land a(l, X))$

c(l) is true since object l is circular

$\exists X(b(X) \land c(X) \land a(l, X))$ is true since object j is black (i.e. b(j) is valid) and is circular (i.e. c(j) is valid) and l connect to j (i.e. a(l,j) is valid).

iii) **False**: $\exists X \neg \exists Y(\neg(X=Y) \land a(X,Y))$

Proposition is false since there is an object in ϕ(s) and ϕ(b) i.e. the black square which is not connected to any other object, but is connected to itself. Hence for that object's case $\neg(X=Y)$ would be invalid, but a(X,Y) would be valid where X=Y due to self connection.

iv) **False**: $\forall X(\neg s(X) \rightarrow \exists Y(c(Y) \land b(Y) \land a(X,Y)))$

Does not hold for all X, since in the case where X is j then $\neg s(X)$ holds since j is a circle; but then $\exists Y(c(Y) \land b(Y) \land a(X,Y))$ is false, since there is no such object Y such that Y is a circle and black and j connects to Y (e.g. k is black circle but j does not connect to it, whereas l is w white circle).

v) **False**: $\forall X(\exists Y(\neg(X = Y) \land a(X,Y)) \rightarrow \exists Y(a(X,Y) \land a(Y,X)))$

Does not hold for all X since in the case where X=k, then $\exists Y(\neg(X = Y) \land a(X,Y))$ would hold for Y equal to j or l; but $\exists Y(a(X,Y) \land a(Y,X))$ would fail to hold as there are no two way connections from k.

vi) **False**: $\forall X \forall Y(a(X,j) \land a(Y,j) \rightarrow (a(X,Y) \lor a(Y,X)))$

This is true in all cases where X=l and Y=k or when X=k and Y=l since both l and k are connected to j and k and l are connected.

However, the proposition is false in the case where X=Y=k or X=Y=l since there is no self connection present in either k or l .i.e**. ¬(X=Y) was not specified**.