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Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
Exercise:	2 (CW)	**Due:**	Tue - 03 Nov 2020
Title:	Logic	**Assessment:**	Individual
FAO:	Craven, Robert (rac101)	**Submission:**	Electronic

Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-11-03 18:49:54

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CW 1 - Logic

① i) p = Michael is fulfilled
q = Michael is rich.
r = Michael will live another 5 years.

$$\neg(p \vee q) \rightarrow \neg r.$$

ii) p = Snowstorm arrives
q = Raheem wears his boots

$$\begin{aligned}(\neg p \vee q) \wedge p &\equiv p \wedge (\neg p \vee q) \\ &\equiv \underbrace{(p \wedge \neg p)}_{\text{false}} \vee \cancel{p(\wedge c} \, (p \wedge q). \\ &\equiv p \wedge q.\end{aligned}$$

iii) p = Akira and Toshiro are on set
q = filming will begin

r = caterers have cleared out

$$p \rightarrow (q \leftrightarrow r)$$

iv) p = Irad arrived
q = Sarah arrived

$$(p \vee \neg q) \wedge \neg(p \wedge \neg q)$$

v) p = Hebert heard the performance
q = Anne-Sophie heard the performance
r = Anne-Sophie answered her phone calls.

$$\neg r \rightarrow \neg(p \wedge q)$$

② i) A propositional formula A is satisfiable if there is some v s.t. $h_v(A) = t$.

$v : A \rightarrow \{t, f\}$ is the atomic evaluation function for A.

$h_v : \text{fmlas}_A \rightarrow \{t, f\}$ is the propositional evaluation function based on v.

ii) Two propositional formulas A and B are logically equivalent if, for every v, $h_v(A) = h_v(B)$.

v is the atomic evaluation function for A & B.

$h(v)$ is the propositional evaluation function based on v. (As previously defined).

iii) Assume $\neg A$ is satisfiable.
Then there exists some atomic evaluation function $v : A \to \{t, f\}$ such that $h_v(\neg A) = t$ [$h_v : A \to \{t, f\}$ is the propositional evaluation function based on v]

~~$h_v(\neg A) = t \Rightarrow h_v(A) = h_v(T)$ is false.~~

~~Since it~~

$h_v(\neg A) = t$ implies that it is not true that for every v, $h_v(A) = h_v(T)$.

Therefore,

$$A \not\equiv T$$
$$\Rightarrow \neg\neg A \not\equiv T$$

Now Assume $\neg A$ is unsatisfiable.
Then there is no v s.t. $h_v(\neg A) = t$
$\Rightarrow h_v(\neg A) = f$
$\Rightarrow h_v(A) = t$ (by meaning of $\neg$)

$\therefore$ $A \equiv T$
$\Rightarrow \neg\neg A \equiv T$. ☒

③

p	q	r	$((p$	$\neg q$	$\wedge)$	$\leftrightarrow$	$\neg($	$\neg r$	$\vee$	$\neg p))$	$\longrightarrow$	$(\neg\neg q$	$\to$	$r)$
t	t	t	t	f	f	f	t	f	f	f	t	t	t	t
t	t	f	t	f	f	t	f	t	t	f	f	t	f	f
t	f	t	t	t	t	t	t	f	f	f	t	f	t	t
t	f	f	t	t	t	f	f	t	t	f	t	f	t	f
f	t	t	f	f	f	t	f	f	t	t	t	t	t	t
f	t	f	f	f	f	t	f	t	t	t	f	t	f	f
f	f	t	f	f	t	t	f	f	t	t	t	f	t	t
f	f	f	f	f	t	t	f	t	t	t	t	f	t	f

It is not true that $h_v(((p \wedge \neg q) \to \neg(\neg r \vee \neg p)) \to (\neg\neg q \to r)) = t$ for any valuation v. Therefore it is <u>not</u> valid.

4 i) a) CNF

b) CNF

c) $(p \wedge q) \vee (p \wedge (p \wedge r))$

$\equiv (p \wedge q) \vee (p \wedge r)$

DNF

d) CNF

e) DNF

f) DNF

g) CNF

h) DNF

ii) Refutation-soundness and completeness:

Let S be in CNF. $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$

$S \vdash_{res(PL)} \emptyset$ means that there is such a resolution derivation of $\emptyset$ from S.

This property is important for SAT solving. The corollary of this is: Let S be in CNF, S is satisfiable iff $S \nvdash_{res(PL)} \emptyset$.

iii) a) $\{\{p,s\}, \{q,r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$.

$= \{\{p,s\}, \{\neg p, \neg r, \neg s\}\}$ [q was pure]

$= \{\{p,s\}\}$ [$\neg r$ was pure]

b) $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

$= \{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$ [unit propagation by unit clause $\{\neg q\}$].

$= \{\{\neg p\}, \{p\}\}$ [unit prop. by clause $\{\neg r\}$]

$= \{\{\}\}$

(5) p: I'm going
q: You're going
r: Tara is going.

$p \to \neg q$, $\neg q \to \neg r$, $(r \vee \neg p) \cancel{\wedge (r}$, $(r \vee p) \wedge \neg(r \wedge p)$, $\neg q$.

Check if $(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (r \vee p) \wedge (\neg r \vee \neg p) \wedge (\neg q)$ is satisfiable.

convert to CNF:

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg r, \neg p\}, \{\neg q\}\}$

$= \{\{\neg p, \neg q\}, \{\neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg r, \neg p\}\}$ [unit prop $\{\neg q\}$]

$= \{\{\neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg r, \neg p\}\}$ [pure rule $\{\neg q\}$]

$= \{\{\neg p\}, \{p\}\}$ [unit prop $\{\neg r\}$]

$= \{\{\}\}$ [unit prop. $\{p\}$]

$\therefore$ unsatisfiable.

$\therefore$ Argument valid

(6) i) $\forall$ aunt(aunt(Andrea)) $\wedge$ gave (cupcake, $\neg$ Andrea))

F_1 = aunt.
F_2 = gave.

~~ii) ¬ connected (comput~~

~~ii) ∃X ∀Y ¬connected (computer(Y), computer(Y))~~

~~ii) ∃X∀Y compu~~

ii) $\exists X \forall Y$ $\neg$connected (computer(Y), computer(Y)) $\to$ connected (computer(X), computer(Y))

P_1 = {computer}

F_2 = {connected}

in signature L.

iii)* $\forall x$ painting(Paul, x) $\wedge$ gallery(x)

iv) ~~$\forall$~~ $\exists x \neg \exists y$ loves(x, y) $\rightarrow$ $\forall x \exists y \neg$ loves(x, y)

$F_2 = \{$loves$\}$ in signature L.

(7) i) False

There is only one directed arrow from k to j. Hence, $X = j$.

ii) True.

$X = k$

iii) False.

Take $X = k$ and $Y = l$.

$\neg(X = Y) \wedge a(k, l)$ is true.

Hence $\exists X \exists Y (\neg(X = Y) \wedge a(X, Y))$

iv) False. Take $X = j$

$\neg(S X)$ is true, but $\exists Y (c(Y) \wedge b(Y) \wedge a(X, Y))$ is fase.

v) Fase.

Since not true for $\varphi(b) \wedge \varphi(s)$ ~~doesn't~~

vi) True.

Take $X = k$, $Y = l$

$a(k, j) \wedge a(l, j)$ is true.

$a(k, l) \vee a(l, k)$ also true since $a(l, k)$ is true.