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- I declare that this final submitted version is my unaided work.

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Introduction to Symbolic AI - Coursework 1

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1 Formalize each of the following in propositional logic

- **If Michel isn't either fulfilled or rich, he won't live another five years.**

  $((\neg(p \vee q)) \to (\neg r))$

  $p =$ Michel fulfiiled

  $q =$ Michel rich

  $q =$ Michel live another five years

- **Unless the snowstorm doesn't arrive, Raheem will wear his boots; but I'm sure it will arrive**

  $(((\neg p) \vee q) \wedge (r \vee q))$

  $p =$ snowstorm arrives

  $q =$ Raheem wear his boots

  $q =$ I'm sure

- **If Akira and Toshiro are on set, then filming will begin if and only if the caterers have cleared out**

  $(((p \vee q) \to (r)) \leftrightarrow s)$

  $p =$ Akira on set

  $q =$ Toshiro on set

  $r =$ film will begin

  $s =$ caterers have cleaned out

- **Either Irad arrived, or Sarah didn't: but not both**

  $((p \vee (\neg q)) \wedge (\neg(p \wedge q)))$

  $p =$ Irad arrived

  $q =$ Sarah arrived

- **It's not the case both that Herbert heard the performance and Anne-Sophie did, if the latter didn't answer her phone calls**

  $((\neg(p \wedge q)) \vee r)$

  $p =$ Herbert heard the performance

  $q =$ Anne-Sophie heard the performance

  $r =$ Anne-Sophie answered her phone calls

2 Answer the questions

2.1 What is the definition of the satisfiability of a propositional formula, $\boldsymbol{A}$?

A propositional formula A is satisfiable if there is some v such that $h_v(A) = \top$

2.2 What is the definition of the logical equivalence of two propositional formulas $\boldsymbol{A}$ and $\boldsymbol{B}$?

Two propositional formulas A and B are logically equivalent if, for every v, $h_v(A) = h_v(B)$

2.3 Prove that a propositional formula $\neg A$ is satisfiable iff $\neg\neg A \not\equiv \top$ (i.e., iff it's not the case that $\neg\neg A \equiv \top$)

($\Rightarrow$) If $\neg A$ is satisfiable, then there exists at least one v such that $h_v(\neg A) = \top$. Which implies for the same v, $h_v(\neg\neg A) = \perp$, and thus $\neg\neg A \not\equiv \top$.

($\Leftarrow$) If $\neg\neg A \not\equiv \top$, i.e., $\neg\neg A$ is not valid, then there exists at least one v suth that $h_v(\neg\neg A) = h_v(A) = \perp$. Then for the same v, we have $h_v(\neg A) = \top$, which means $\neg A$ is satisfiable.

3 Use truth-tables to determine whether the following is valid or not: $(p \wedge q \leftrightarrow \neg(\neg r \vee \neg p)) \rightarrow (\neg\neg q \rightarrow r)$

p	q	r	$(p$	$\wedge$	$\neg q$	$\leftrightarrow$	$\neg$	$(\neg r$	$\vee$	$\neg p))$	$\rightarrow$	$(\neg\neg q$	$\rightarrow$	$r)$
t	t	f	t	f	f	$\mathbf{t}$	f	t	t	f	$\mathbf{f}$	t	$\mathbf{f}$	f

Here we can see, for the evaluation function v such that $v(p) = \mathbf{t}$,$v(q) = \mathbf{t}$,$v(r) = \mathbf{f}$, we will have $h_v(p \wedge q \leftrightarrow \neg(\neg r \vee \neg p)) \rightarrow (\neg\neg q \rightarrow r) = \bot$. Thus the formula is not valid.

4 Answer the Question

4.1 Which of the following are in CNF? Which are in DNF?

CNF: a, b, f, g

DNF: b, e, h

4.2 Define the property of the refutation-soundness and -completeness of a resolution derivation. Why is this property important?

Let S be in Conjuctive Normal Form (CNF). $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$ is the property of the refutation-soundness and -completeness of a resolution derivation.

This property is important because it shows the connection between resolution derivation and satisfiability, which implies a logic to derive satisfiability from CNF.

4.3 Apply unit propagation and the pure rule repeatedly, in order to reduce the following to their simplist forms

4.3.1 $\{\{p, s\}, \{q.r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

$\Longrightarrow$ $\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$(q is pure)

$\Longrightarrow$ $\{\{p, s\}\}$($\neg r$ is pure)

$\Longrightarrow$ $\{\}$ $\Longrightarrow$ **SATISFIABLE**

4.3.2 $\{\{\neg p, q, r\}, \{\neg q\}, \{p, q, r\}, \{\neg r, q\}\}$

$\Longrightarrow$ $\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$(unit clause $\{\neg q\}$)

$\Longrightarrow$ $\{\{\neg p\}, \{p\}\}$(unit clause $\{\neg r\}$)

$\Longrightarrow$ $\{\{\}\}$(unit clause $\{p\}$) $\Longrightarrow$ **UNSATISFIABLE** ($\emptyset$ is in the set)

5 Use DP to determine whether the argument is valid or not

If we define $p =$ I'm going, $q =$ You're going and $r =$ Tara's going, then we can formalize the statement to be the following:

$$\begin{aligned} \textbf{Premises:} & \neg p \vee \neg q \\ & q \vee \neg r \\ & r \vee \neg p \\ & p \vee r \\ \textbf{Conclusion:} & \quad q \end{aligned}$$

To check the validity of $A_1, \ \dots \ , A_n \models B$, is equivalent to check the satisfiability of $(\bigwedge A_i) \wedge \neg B$. Which means, if $(\bigwedge A_i) \wedge \neg B$ is unsatisfiable, then we can conclude $A_1, \ \dots \ , A_4 \models B$ is valid. So to check the validity of $(\neg p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee r) \models q$, we can approve this by checking the satisfiability of $(\neg p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee r) \wedge (\neg q)$. Converting it to CNF and applying DP, we get:

$$\begin{aligned} & \{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{p, r\}, \{\neg q\}\} \\ \Longrightarrow \ & \{\{\neg r\}, \{r, \neg p\}, \{p, r\}\} (\text{unit clause } \{\neg q\}) \\ \Longrightarrow \ & \{\{\neg p\}, \{p\}\} (\text{unit clause } \{\neg r\}) \\ \Longrightarrow \ & \{\{\}\} (unitclause \{\neg p\}) \quad \Longrightarrow \textbf{UNSATISFIABLE} \end{aligned}$$

and thus we can conclude that the original argument is valid.

6 Translate into first-order logic.

- **All of Andrea's aunts' aunts gave a cupcake to someone other than Andrea**

$$\forall X (X = aunt(aunt(Andrea))) \rightarrow \exists Y (\neg (Y = Andrea) \wedge cupcake(X, Y))$$

$$C = \{Andrea\}$$
$$F_1 = \{aunt\}$$
$$P_2 = \{cupcake\}$$
$$L = Tuple(C, P_2, F_1)$$

- **There's a computer connected to every computer which isn't connected to itself**

$$\exists X \forall Y (computer(X) \wedge (computer(Y) \wedge \neg (X = Y) \wedge connect(X, Y))$$

$$P_1 = \{computer\}$$
$$P_2 = \{connect\}$$
$$L = Tuple(P_1, P_2)$$

- **Any painting by Paul Klee in a British gallery hangs in a room where all Kandinsky paintings in that gallery hang**

$\forall G \exists Z \forall Y \forall X (room(Z) \land in(Z, Britishgallery(G)) \land ((Y = painting(Kandinsky)) \to in(Y, Z)) \to (X = painting(PaulKlee)) \to in(X, Z))$

$$C = \{PaulKlee, Kandinsky\}$$
$$P_1 = \{room, Britishgallery\}$$
$$P_2 = \{in\}$$
$$F_1 = \{painting\}$$
$$L = Tuple(P_1, P_2, F_1)$$

- **If there's somebody who loves nobody, then it's false that everybody loves somebody**

$$\exists X(\neg(\exists Y love(X, Y))) \to \neg(\forall Y \exists X love(X, Y))$$

$$P_2 = \{love\}$$
$$L = Tuple(P_2)$$

7 Determine true or false, and provide a justification in each case.

- $\forall X a(k, X) \to \neg(X = j)$

False. Assume for any X, $a(k, X)$, we can show that $j = X$, contradict with $\neg(X = j)$, so the argument is false.

- $c(l) \to \exists X(b(X) \land c(X) \land a(l, X))$

True, for $X = k$.

- $\exists X \neg \exists Y(\neg(X = Y) \land a(Y, X))$

True, for X to be the black square.

- $\forall X(\neg s(X) \to \exists Y(c(Y) \land b(Y) \land a(X, Y)))$

False, for $X = j$.

- $\forall X(\exists Y(\neg(X = Y) \land a(X, Y)) \to \exists Y(a(X, Y) \land a(Y, X)))$

True, for $Y = k$.

- $\forall X \forall Y(a(X, j) \land a(Y, j) \to (a(X, Y) \lor a(Y, X))$

False, for $X = k, Y = k$.