Task Allocation Without Communication Based on Incomplete Information Game Theory for Multi-robot Systems

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Task Allocation without Communication Based on Incomplete Information Game Theory for Multi-robot Systems

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Abstract In the task allocation of multi-robot system, the communication is an important condition to ensure global consistency. Unfortunately, with the popularity of WLAN, the congestion and interference among the bands are particularly severe, making traditional task allocation methods which rely on communication can not work. In this paper, the task allocation without communication is considered to be an incomplete information game, where game theory is employed to realize the task allocation and cooperation of soccer robots. Firstly, the joint probability distribution of the robot type is established according to the distance information, which can be employed to solve the incomplete information static game. Then the ball velocity information is added to modify the joint probability distribution and the incomplete information dynamic game can be solved in the same way. The experimental results show that the success rate of task allocation without communication can be improved effectively using the proposed method.

Keywords Task allocation · Without communication · Incomplete information game · Soccer robots

1 Introduction

Compared to single robot systems, multi-robot systems have the ability to accomplish more complex tasks, where the communication is a key element to make the system run effectively [2]. With the deepening of the research on multi-robot systems, task allocation has become a hot and difficult topic in this field [16]. Currently, task allocation in most multi-robot systems relies on the communication among the agents [27]. Furthermore, WLAN (wireless local area network) is

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the best choice for the communication due to its low cost and high bandwidth. However, as the working environment of the robots becomes more and more complicated, the amount of data transmission increases along with the increase of the number, type and precision of the robot's sensors. The huge communication burden makes the communication among robots extremely unreliable, which affects the task allocation of multi-robot systems. At the same time, in some complex hostile environment, the fragile communication system takes the lead in bear the brunt of the environmental influences. If the multi-robot system is over-relied on the communication, it will be easy to paralyze.

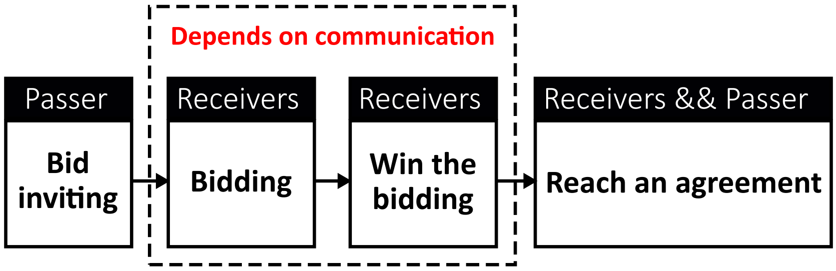

Fig. 1 The process of PRC using market-based task allocation

RoboCup [14] is a standard test-bed for multi-robot cooperation and coordination. According to the current rule of RoboCup Middle Size League (MSL), a goal is valid only after the ball has been passed between two robot teammates. So ball passing and receiving cooperation (PRC) is an important ability for a robot soccer team to win the game. In this paper, the PRC among three robots will serve as the research background. The traditional task allocation strategy can be regarded as an optimal problem for a predefined goal. With reliable communication, a typical multi-robot task allocation method based on the market mechanism has been widely utilized in the field of soccer robots [13,10]. In this case, the process of PRC is illustrated in Fig.1, where the procedure in the dashed box is communication dependent.

However, when the communication cannot work, incomplete information will lead to a dilemma in the action choice of each robot. As a result, the multi-robot system cannot to reach a global consistent task allocation result. Therefore, this paper attempts to use the incomplete information game theory to solve the communication-less task allocation problem of the multi-robot system. Particularly, for static expanded games, the establishment of joint probability distribution is simply based on the distance information among robots and ball or obstacles. The Bayesian Nash Equilibrium of the expanded game is the task allocation result, and the final success rate is higher than the random allocation method, but it is still not ideal. In the subsequent dynamic expanded game, the joint probability distribution is modified by the ball velocity. In this case, the Perfect Bayesian Nash Equilibrium is the new result of the task allocation, and the final success rate is further improved.

In the existing studies, there was no shortage of precedent which used the game theory to solve the task allocation. Furthermore, the game theory is very suitable for solving coordination among heterogeneous nodes in task allocation. And the incomplete information game theory is often used to solve the problem of task allocation under unreliable communication as [6,20]. To the best of our knowledge, we believe there is not any research on task allocation without communication based on incomplete information game theory. This paper uses this method to solve a simple cooperation, but also to lay a foundation for the more complex cooperation in future research.

The remainder of this paper is organized as follows. Section 2 presents an overview of related research on communication improvement and some novel methods of task allocation with unreliable communication in multi-agent systems (MAS). A brief background about incomplete information game theory is given in Section 3. Then in Section 4, a game-theoretic model of multi-robot task allocation without communication is established based on the distance and ball velocity information. The solutions of this model in different circumstances are addressed in Section 5. Section 6 presents three experiments to validate the proposed model and analyses the performance. Finally, the paper is concluded in Section 7.

2 Related Work

To reduce the impact of communication problems, researchers generally use two kinds of solutions. One is to enhance the communication capacity and reliability by improving the communication mode [35] or framework [32]. Another method is to reduce the dependence of the multi-robot system on the communication.

2.1 Improving the communication

Reviewing the history of the multi-robot system, the classic CEBOT (Cellular robotic system) is the earliest attempt to realize multi-robot cooperation using communication information in 1990 [9]. In 2008, the MinDART (Minnesota Distributed Autonomous Robotic Team) multi-robot system tries to communicate implicitly with semaphores, and finally completes the search and rescue tasks [26]. With the popularization of WLAN, most multi-robot systems use it to realize information exchange. Tiderko et al. proposed a combination of the new wireless communication protocol WNet [1] and a series of software services to realize the information exchange among robots. In [28], the multicast communication is applied instead of the traditional unicast communication, therefore the communication burden can be reduced and the communication delay can be lowered to a certain extent. In [22], a model to exchange sensing, hearing and visual information in real environments with an efficient manner is proposed which improves the communication efficiency for the MAS. In a recent study, machine learning is applied to improve the communication between agents in the MAS [7], where the authors try to establish a system that enables agents to learn new concepts form several different agents at the same time.

2.2 Reducing communication dependency

On the other hand, in some large environments, such as the complex outdoor environment with a lot of obstacles which not only block the action of the robots, but also reduce the reliability of the communication. In this case, the key issue is to reduce the multi-robot system's dependence on the communication. In [24], the space is explored with a team of mobile robots, where two robots communicate only when they are within the line of sight of each other. A Boustrophedon decomposition method [3] is proposed to improve the coverage efficiency for the multi-robot system while reducing the repetitive rate. In [19], researches find that when the human is lost, they will go to a pre-agreed place to meet and re-establish the contact. Similarly, when the robot is lost, the visual system can guide the robot to move to a rally point which is appointed in advance thus re-enter the communication range [21,25]. In 2011, Wang et al. presented an intelligent communication strategy, which combines explicit and implicit communication via predicting the robot's behavior [30,31]. Kobayashi and Hosoe designed a decentralized capturing behavior for multiple mobile robots, which decides how to move only based on the available local information from the omnidirectional cameras [15]. Gunn et al. proposed a framework of task allocation for coordinating a varying collection of heterogeneous robots, operating in complex and dynamic environments with unreliable communication such as disaster zones [11]. In 2016, my research [4] attempted to use the front-camera to recognize the teammate and designed a heuristic algorithm based on this information to carry out the cooperation without communication. Although the traditional market-based algorithms is extremely dependent on communication, Oh et al. proposed a new market-based decentralized algorithms for the task allocation of multiple UAV (unmanned aerial vehicles) in dynamic environments with a limited communication range [23].

The consequence caused by terrible communication is that the information update is delayed or incomplete. As a result, the robots involved cannot gather the whole situation of the tasks. Considering this incomplete information condition, researchers attempt to use the game theory to deal with the multi-robot task allocation problem. In 1985, researchers studied the game with incomplete information from a point of view which emphasizes the empirical predictions arising from a game-theoretic model [20]. In 2016, Eksin and Ribeiro studied the optimal behavior problem of multi-robot cooperation under the condition of incomplete information, where each robot could make the same decision in the absence of the information to realize multi-robot cooperation [6]. In [29], the researchers found that the game theory is able to re-produce the decision process of humans more accurately than a decision model that predicts humans individually. The robots could use the game model to understand the human avoidance behaviors, and make human-like decisions during the navigation. However, the methods mentioned above cannot get rid of the dependence on the communication thoroughly.

3 Incomplete Information Game Theory

If an agent involved in the game holds its private information which other agents do not know, such a game is called an incomplete information game, also known as the Bayesian game [12]. In the incomplete game theory, the "private information" is presented as the agent's type. So incomplete information game also means that each agent does not know the types of other agents.

Table 1 The notations used in game theory

Agent (i)	the agent involved in the game
Type (θ)	the private information owned by an agent is its "type", θ_i denotes the type of agent i, and H_i denotes the set of all possible types of an agent ($\theta_i \in H_i, i = 0, 1, \ldots, n$). Accordingly, H_{-i} denotes the set of all possible types of other agents.
Action (a)	every agent i has different action a_i, and $A_i(\theta_i)$ denotes the set of all possible actions of the agent ($a_i \in A_i(\theta_i)$)
Payoffs (u)	$u_i((a_i\|\theta_i), (a_{-i}\|\theta_{-i}))$is the payoff of agent i when the type of agent i is θ_i and it selects the action a_i. Meanwhile, the types of other agents are θ_{-i}, and they select the actions a_{-i}. The payoff represents the reasonable degree of the current selection

3.1 How to obtain the Nash Equilibrium

The notations used in the game theory are listed in Table 1. The following Example 1 illustrates how to obtain the Nash Equilibrium.

Example 1 $N = \{Agent1, Agent2\}, H_1 = \{\dot{\theta}_1, \ddot{\theta}_1\}, H_2 = \{\dot{\theta}_2, \ddot{\theta}_2\}, A_1(\dot{\theta}_1) = A_1(\ddot{\theta}_1) = \{\dot{a}_1, \ddot{a}_1\}, A_2(\dot{\theta}_2) = A_2(\ddot{\theta}_2) = \{\dot{a}_2, \ddot{a}_2\}$, then we get the partial payoffs as shown in Table 2. where the payoffs satisfy $u_{11} > u_{13}, u_{12} < u_{14}, u_{21} > u_{22}, u_{23} > u_{24}, u_{15} > u_{17}, u_{16} < u_{18}, u_{25} < u_{26}, u_{27} < u_{28}$.

When the action of one agent is known, another agent prefers the action $\widehat{a}_i$ with a larger payoff:

$$\widehat{a}_i \in \arg \max_{a_i \in A_i(\theta_i)} u_i((a_{-i}|\theta_{-i}), (a_i|\theta_i)) \quad i = 1, 2. \tag{1}$$

When the type of Agent1 is $\dot{\theta}_1$ and the action a_1 is selected, the larger payoff $u_{21} > u_{22}$ is marked. When the type of Agent2 is $\dot{\theta}_2$ and the action a_2 is selected, the larger payoff $u_{11} > u_{13}$ is marked. Through this method, we assume u_{11}, u_{14} and u_{23}, u_{21} are marked shown with underlines in Table 2.

Table 2 The partial payoffs, and $u_{11} > u_{13}, u_{12} < u_{14}, u_{21} > u_{22}, u_{23} > u_{24}, u_{15} > u_{17}, u_{16} < u_{18}, u_{25} < u_{26}, u_{27} < u_{28}$

Action|Type	$\dot{a}_2\|\dot{\theta}_2$	$\ddot{a}_2\|\dot{\theta}_2$	$\dot{a}_2\|\ddot{\theta}_2$	$\ddot{a}_2\|\ddot{\theta}_2$
$\dot{a}_1\|\dot{\theta}_1$	$(\underline{u_{11}}, \underline{u_{21}})$	(u_{12}, u_{22})	/	/
$\ddot{a}_1\|\dot{\theta}_1$	$(u_{13}, \underline{u_{23}})$	$(\underline{u_{14}}, u_{24})$	/	/
$\dot{a}_1\|\ddot{\theta}_1$	/	/	$(\underline{u_{15}}, u_{25})$	$(u_{16}, \underline{u_{26}})$
$\ddot{a}_1\|\ddot{\theta}_1$	/	/	(u_{17}, u_{27})	$(\underline{u_{18}}, \underline{u_{28}})$

According to Table 2, the (1) can lead to the following conclusion:

$$\begin{aligned}\max_{a_1\in A_1(\dot\theta_1)} u_1((a_1|\dot\theta_1),(\dot a_2|\dot\theta_2)) &= u_1((\dot a_1|\dot\theta_1),(\dot a_2|\dot\theta_2)) = u_{11} \quad \widehat{a}_1=\dot a_1,\\ \max_{a_2\in A_2(\dot\theta_2)} u_2((\dot a_1|\dot\theta_1),(a_2|\dot\theta_2)) &= u_2((\dot a_1|\dot\theta_1),(\dot a_2|\dot\theta_2)) = u_{21} \quad \widehat{a}_2=\dot a_2.\end{aligned} \tag{2}$$

As a result, $a^* = (\widehat{a}_1, \widehat{a}_2) = (\dot a_1, \dot a_2)$ is defined as the Nash Equilibrium of this game. In other words, the Nash Equilibrium is the cell whose payoffs are all underlined in Table 2. And a^* satisfies:

$$a_i^* \in \arg\max_{a_i\in A_i(\theta_i)} u_i((a_{-i}^*|\theta_{-i}),(a_i|\theta_i)) \quad i=1,2. \tag{3}$$

It means neither agent can increase its payoff through changing its own action. (All the values marked with underline in the following table of the paper are obtained through this method)

3.2 Pure Strategy Nash Equilibrium and Bayesian Nash Equilibrium

In the complete information game, when the types of all involved agents are known and shared, their actions are also determined immediately as (2). The unique equilibrium point $(\dot a_1, \dot a_2)$ is called Pure Strategy Nash Equilibrium (PSNE). But the type is private information in the incomplete information game. The uncertain types of agents may lead to a dilemma in the action selection, such as $a^* = (\dot a_1, \dot a_2)$ when $\theta_1 = \dot\theta_1$ and $\theta_2 = \dot\theta_2$, but $a^* = (\ddot a_1, \ddot a_2)$ when $\theta_1 = \ddot\theta_1$ and $\theta_2 = \ddot\theta_2$ according to Table 2.

Before the game, the type of agent i is inferred to be θ_i based on the prior information, known as the prior probability $P(\theta_i)$. Such a prior information may be derived from the experience, or it can be calculated from known information. Under this premise, the probability of other agents select type set θ_{-i} is $P(\theta_{-i}|\theta_i)$. It is a conditional probability that satisfies the Bayesian formula:

$$P(\theta_{-i}|\theta_i) = \frac{P(\theta_i,\theta_{-i})}{P(\theta_i)} = \frac{P(\theta_i,\theta_{-i})}{\sum_{\theta_{-i}\in H_{-i}} P(\theta_{-i},\theta_i)}. \tag{4}$$

Because agent i cannot confirm the types θ_{-i} of other agents, we use the expected payoff to describe the payoff under uncertain conditions:

$$\sum_{\theta_{-i}} P(\theta_{-i}|\theta_i) u_i((a_{-i}|\theta_{-i}),(a_i|\theta_i)). \tag{5}$$

Assume $a^* = (a_1^*,\ldots,a_i^*,\ldots,a_n^*)$ is a set of optimal action choice with the current types, where a_i^* satisfies:

$$a_i^* \in \arg\max_{a_i\in A_i(\theta_i)} \sum_{\theta_{-i}} P(\theta_{-i}|\theta_i) u_i((a_{-i}^*|\theta_{-i}),(a_i|\theta_i)) \quad i=1,\ldots,n, \tag{6}$$

It is called the Bayesian Nash Equilibrium (BNE) [8]. In this application, in order to reduce the complexity, it is assumed that each agent has the same model about the distribution of the type $(\theta_1,\ldots,\theta_n)$ before the game. In other words, the priori probability $P(\theta_i)$ is the public awareness for all agents, which is known as Harsanyi axiom. It indicates that all agents' beliefs about the cooperation are the same.

3.3 The static game and dynamic game

The majority of the game in the society are incomplete information based, such as the Chicken Game [5], which is also known as the Coward Game. Another typical example is the single-plank bridge game, where two people located on each side of the bridge do not know each other's current decision. If both or neither of them step on the bridge at the same time, no one could cross the bridge. Only when one person goes ahead, and another waits on the other side, they can reach the destination successfully.

The incomplete information game can be further categorized into dynamic game and static game. In the dynamic case, if one agent executes its action in advance, other agents can observe the action and correct the corresponding probability distribution of the optional type. Afterwards, the agents choose their own actions according to the new situation of the game. The new equilibrium of the incomplete information dynamic game is called the Perfect Bayesian Nash Equilibrium (PBNE). Obviously, the PRC between soccer robots can be considered as a typical dynamic game. The ball passing action is always ahead of the ball receiving action. The equilibriums of different games are summarized in Table 3.

Table 3 The equilibriums of different games

Abbreviation	**Full name**	**The type of game**
PSNE	Pure Strategy Nash Equilibrium	complete information game
BNE	Bayesian Nash Equilibrium	incomplete information static game
PBNE	Perfect Bayesian Nash Equilibrium	incomplete information dynamic game

4 Establishing the Game Model of PRC Task Allocation without Communication

In a normal soccer robot competition, with the help of the communication, the whole world model is known and unified for each robot. Under this condition, the multi-robot cooperation can be regarded as a complete information game. However, when the communication mechanism is manually restricted or becomes unreliable, the robot can only obtain part of the information with their own sensors, resulting an incomplete information game.

4.1 The acquired information without communication

Without communication, robots can only obtain information using their own sensors, particularly the visual system. In this paper, we use our RoboCup MSL robot [34] as the research platform, which is named NuBot and illustrated in Fig 2. As can be seen, the robot has an omnidirectional vision system and a RGB-D vision system for visual perception. The omnidirectional vision system is the major sensor to realize self-localization, ball and obstacles recognition and localization [17]. It is worth mentioning that, the main body of the robots and obstacles are in black. So the teammates and obstacles are all indiscriminate black objects in the omnidirectional vision system [18]. With the communication, each robot can get the teammate's absolute position, so the black objects except teammates are obstacles. But this method is not applicable in a communication-less environment. In this condition, the RGB-D (Kinect) camera is used to distinguish teammates and obstacles by recognizing the robot's color marker (the same color for each team) and get the teammate's relative position.

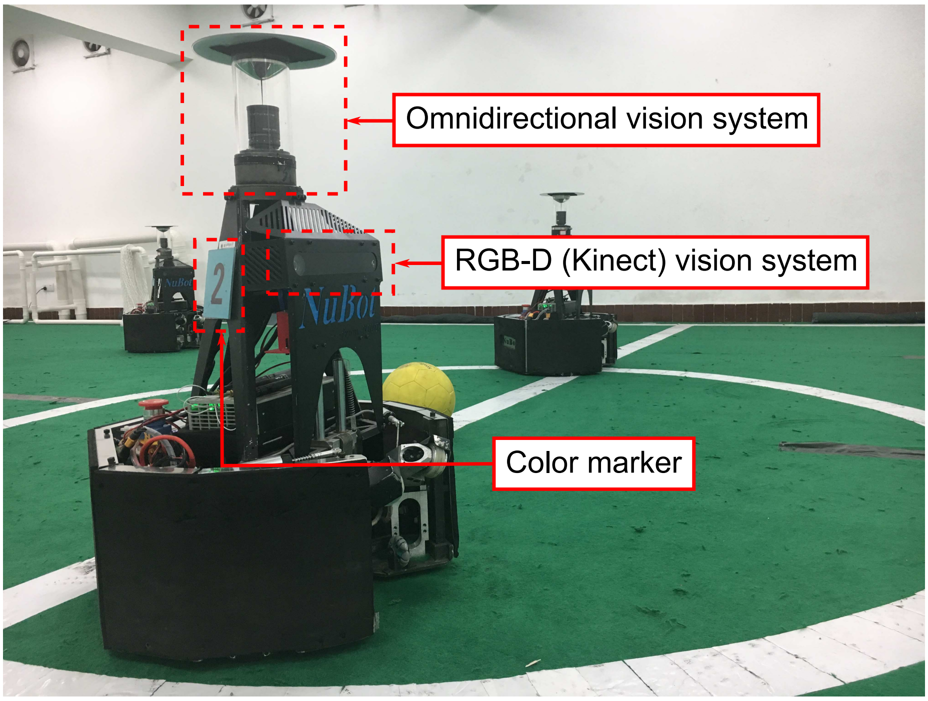

Fig. 2 The vision systems equipped by our MSL robot

In this study, the robots cannot share their own localization, type and action information through the communication. Each robot makes its own action selection purely depending on limited environmental information and combining the inference of other robots' types and actions. Thus, a typical incomplete information game model can be established.

4.2 Basic situation of incomplete information game

There are three robots $N = \{Agent0, Agent1, Agent2\}$ for PRC task allocation without communication. Agent0 is the passing robot (Passer), Agent1 and Agent2 are the receiving robots (Receiver). An incomplete information game is established between the Passer and each Receiver. The relationship of the three robots involved in the PRC is displayed in Fig 3.

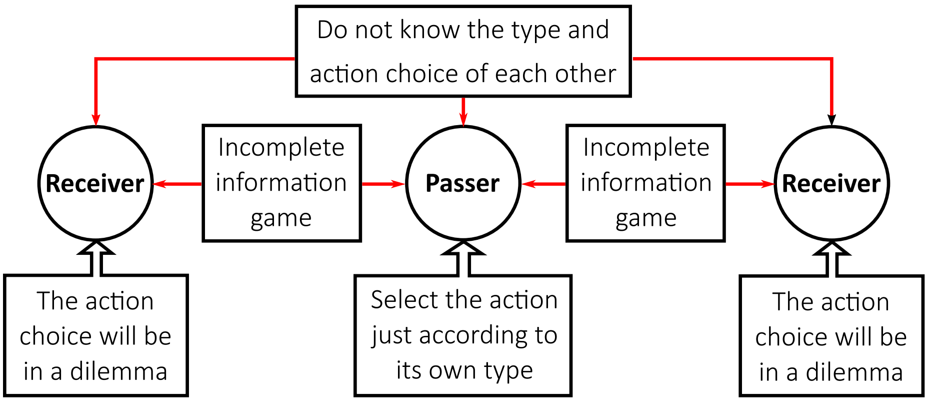

Fig. 3 The relationship of the three robots involved in PRC

According to the different situations, the task will be entirely different. Take one couple (one Passer and one Receiver) of them for analysis, the Passer will be aggressive if the situation is suitable to attack, or be conservative if the situation is not suitable to attack. The Receiver will be assist or defense in different positions. The task with confirmed type can be denoted as:

$$task = t((a_0|\theta_0), (a_i|\theta_i)) \quad i = 1, 2. \tag{7}$$

When the type of the Passer is aggressive, it will tend to attack and pass the ball forward to the assist to form scoring opportunities. On the contrary, when the type of the Passer is conservative, it will tend to defense and pass the ball backward to the defense, waiting for the next opportunity to attack. The Receiver can choose whether to receive the ball according to its own type and the Passer's type. The types and actions of each robot are illustrated in Table 4.

Table 4 The types and actions of each robot

Agent(i)	**Passer(0)**	**Receiver(1&2)**
Types(θ_i)	aggressive(a)	assist(A)
	conservative(c)	defense(D)
Actions(a_i)	pass forward(f)	receive(y)
	pass backward(b)	not receive(n)

At first, we analyze how the Receiver can make the correct action choice when its own type is assist. The basic situation of the incomplete information game is shown in Table 5.

Table 5 Basic situation of the Receiver (assist) in the game

Passer $(a_0\|\theta_0)$		$f\|a$	$b\|a$	$f\|c$	$b\|c$
Receiver	$y\|A$	**($\underline{9}$,$\underline{5}$)**	(5,$\underline{3}$)	($\underline{4}$,2)	(3,3)
$(a_1\|\theta_1)$	$n\|A$	(3,1)	($\underline{4}$,2)	(4,$\underline{3}$)	**($\underline{7}$,$\underline{5}$)**

The Passer and Receiver have different payoffs when they choose different actions under different types, which are denoted as:

$$\begin{cases} \text{Passer:} & u_0((a_0|\theta_0),(a_i|\theta_i)), \\ \text{Receiver:} & u_i((a_0|\theta_0),(a_i|\theta_i)). \end{cases} \quad i=1,2 \tag{8}$$

In the game model, the payoffs represent the rationality of the action choice. According to the reasonable degree, the payoff is set from 1 to 9. For example, when the allocated task is $t((f|a),(y|A))$, the payoffs (u_0,u_i) are (9, 5), where the front value is the Passer's payoff in this task, and the second value is the Receiver's payoff. As the current choice is more reasonable, the payoffs are relatively large. On the contrary, the (3, 1) are the least payoffs for the unreasonable choice $t((f|a),(n|A))$, and other cases are similar. The payoffs in Table 5 are not fixed, but they should be reasonable according to the actual situations. For example, the payoff of the Receiver choosing to receive should be higher than that choosing not to receive when its type is assist and the aggressive Passer passes forward. The current payoffs in Table 5 are set empirically. When the types of the Passer and Receiver are determined, they can choose their actions according to the payoffs. Each underlined value in Table 5 is the preferred action choice with larger payoff when the other agent's action is confirmed. For instance, when the Receiver action is receive (y), the payoffs (u_0,u_i) are (9, 5) when the Passer action is pass forward (f) or (5, 3) when the Passer action is pass backward (b). So the Passer's prefer action is pass forward with $u_0 = 9 > 5$ when the Receiver action is receive. The allocated task with chosen action is called the PSNE of the game. Through the analysis of the basic situation, two circumstances can be found.

Circumstance 1: when the type of the Passer is aggressive, and the Receiver is assist, $t((f|a),(y|A))$ is the PSNE, and the payoffs are (9, 5).

Circumstance 2: when the type of the Passer is conservative, and the Receiver is assist, $t((b|c),(n|A))$ is the PSNE, and the payoffs are (7, 5).

Such incomplete information leads to the difficulty of the Receiver in deciding whether to receive the ball when the type of the Passer is unknown. So the Receiver needs to deduce the probability of the Passer choosing the aggressive or conservative type, and then obtain correct action choice based on it.

Similarly, the basic situation is shown in Table 6 when the Receiver is defense, and the only difference lies on the payoffs. Because the offensive desire of the defense is low, it tends to receive the ball in the backcourt.

However, the Passer can obtain an unique action choice whether its own type is aggressive or conservative, and the type of the Receiver is assist or defense. From Table 5 and Table 6, it can be found that when the type of the Passer is aggressive,

Table 6 Basic situation of the Receiver (defense) in the game

Passer $(a_0\|\theta_0)$		$f\|a$	$b\|a$	$f\|c$	$b\|c$
Receiver	$y\|D$	(3,3)	($\underline{4}$,2)	(5,$\underline{3}$)	**($\underline{9}$,$\underline{5}$)**
$(a_1\|\theta_1)$	$n\|D$	**($\underline{7}$,$\underline{5}$)**	(4,$\underline{3}$)	($\underline{4}$,2)	(3,1)

it will pass forward regardless of the type of the Receiver. On the contrary, the Passer will pass backward when its type is conservative. So the following analysis will focus on the Receivers.

4.3 Joint probability distributions of robot types based on distance information

Compared with the standard incomplete information game problem, the multi-robot cooperation without communication is more complicated. In the classic incomplete information game, each agent has a different type that other agents do not know. But its probability distribution is known by all agents, which is the key to solve the problem. Without communication, no effective information can be transmitted among the agents, including the probability distribution. So the joint probability distribution of their types is difficult to be unified.

In order to solve this problem, based on the limited environmental information that a single robot can obtain, we propose a joint probability distribution of robot types based on two kinds of distance information. Assuming that the Passer gets the ball, thus the Passer and the ball are in the same position. Two distances are defined in Fig 4.

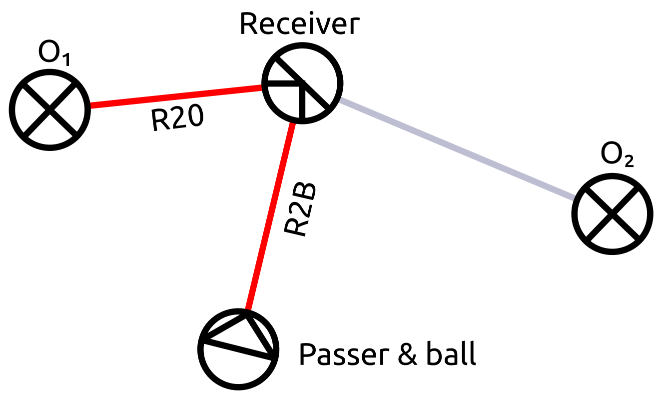

Fig. 4 The definition of two distances

Definition 1 *Receiver2Ball (Passer) (R2B):* the distance between the Receiver and the ball (Passer).

Definition 2 *Receiver2Obstacle (R2O):* the distance between the Receiver and the nearest obstacle.

For the Receiver, the positions of the ball (Passer is dribbling) and the obstacles can be easily obtained by the omnidirectional vision system, thus the two distances can be directly calculated. But for the Passer, the relative position of the Receiver can only be obtained by the RGB-D camera as illustrated in Section 4.1.

$$\begin{cases} P(A,a) = R2O/R2B, \\ P(A,c) = R2B/R2O, \end{cases} \quad if(\text{Receiver=assist}) \\ \begin{cases} P(D,a) = R2B/R2O, \\ P(D,c) = R2O/R2B. \end{cases} \quad if(\text{Receiver=defense}) \tag{9}$$

(9) defines the joint probability distribution (without normalization) and the results are shown in Table 7. $P(A,a)$ represents the probability of that the type of the Receiver is assist, meanwhile the type of the Passer is aggressive. Similarly, $P(A,c), P(D,a), P(D,c)$ of the Passer and Receiver are defined under different combinations of robot types.

Table 7 Joint probability distribution based on the distance information

Passer		aggressive	conservative
Receiver	assist	$P(A,a)$	$P(A,c)$
	defense	$P(D,a)$	$P(D,c)$

The Passer and Receiver will choose their own types according to this joint probability distribution in different situations, and deduce each other's probable types under this premise. Taking the Receiver as an example, the prior probability of the Receiver's initial type being assist is:

$$P(A) = P(A,a) + P(A,c). \tag{10}$$

On this premise, the probability of the Passer's type is inferred based on conditional probability:

$$P(a|A) = P(A,a)/(P(A,a) + P(A,c)), \tag{11}$$

$$P(c|A) = P(A,c)/(P(A,a) + P(A,c)). \tag{12}$$

In the same way, the Receiver can get the inference about the type of the Passer when its own type is defense, and the Passer can also obtain the inference about the type of the Receiver when its own type is selected in advance.

4.4 Joint probability distributions of robot types modified by the ball velocity

Each robot in the field can estimate the velocity of the ball after obtaining the position of the ball. So the Receiver can further deduce the type of the Passer according to the information of ball velocity after the Passer passes the ball, and modify the joint probability distribution of its type. This game is defined as incomplete information dynamic game model. The velocity includes the speed and direction, and the ball can be considered to move with constant speed in the PRC, so the direction is an important reference information that Receivers can use to deduce the type of the Passer.

Definition 3 *Angle α:* the angle between the direction of ball velocity and the line from the ball to the Receiver. $(0 \leq \alpha \leq \pi)$

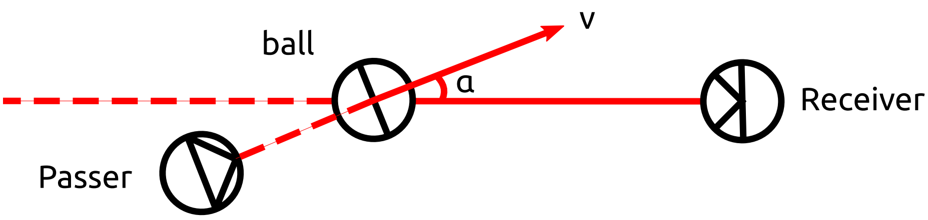

Fig. 5 Angle α of ball velocity

For various reasons (self-localization or object localization error, kicking accuracy, etc), the passing line cannot point to the Receiver accurately, so we define the angle α as shown in Fig 5 to describe the angle error. The coefficient λ is defined as the modifying factor when the Receiver is assist or defense, as shown in (13).

$$\begin{cases} \lambda(A,a) = (\pi - \alpha)/\pi, & \multirow{2}{*}{$if(\text{Receiver=assist})$} \\ \lambda(A,c) = \alpha/\pi, & \\ \lambda(D,a) = \alpha/\pi, & \multirow{2}{*}{$if(\text{Receiver=defense})$} \\ \lambda(D,c) = (\pi - \alpha)/\pi. & \end{cases} \tag{13}$$

The modified joint probability distributions of robot types (without normalization) are listed in Table 8.

Table 8 The modified joint probability distributions based on the information of the ball velocity

Passer		aggressive	conservative
Receiver	assist	$P^*(A,a) = P(A,a) * \lambda(A,a)$	$P^*(A,c) = P(A,c) * \lambda(A,c)$
	defense	$P^*(D,a) = P(D,a) * \lambda(D,a)$	$P^*(D,c) = P(D,c) * \lambda(D,c)$

4.5 The conversion of incomplete information game model

When an agent has private information that the other agents do not know, the basic situation of incomplete information game is uncertain and the unique PSNE cannot be obtained. This problem was solved till Harsanyi introduced the virtual Agent "Nature" to deal with the incomplete information game. Based on the joint probability distribution we defined above, the incomplete information game can be transformed to be a completely imperfect information expanded game by Harsanyi transformation.

In our problem, the "Nature" initializes the type of the Receiver based on the prior probability $(P(\theta_i) \quad i = 1, 2)$, and then the Receiver deduces the type of the

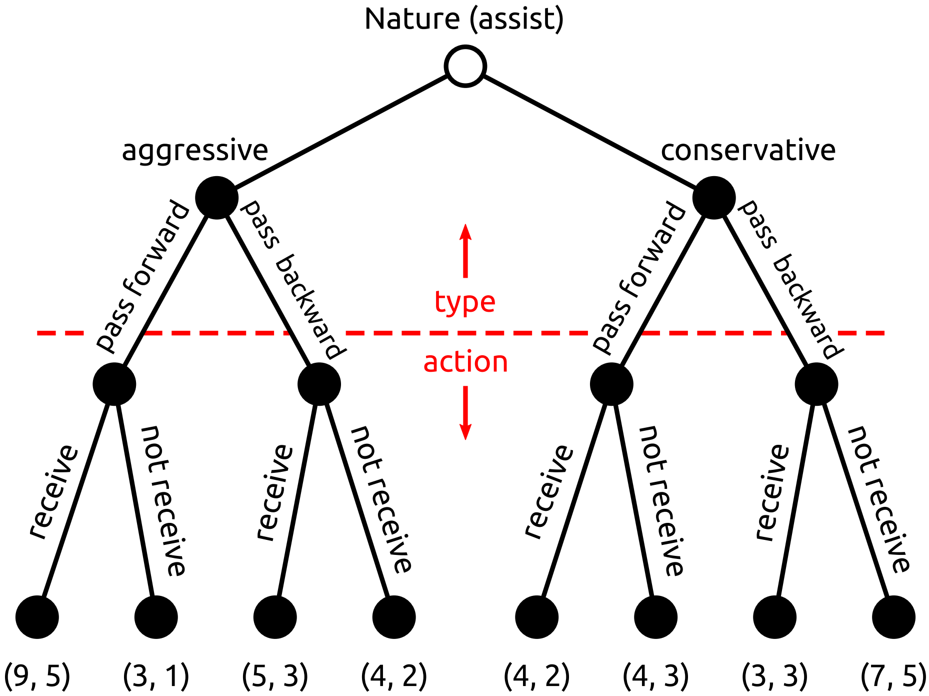

Fig. 6 Game tree of the Receiver (initial type: assist)

Passer based on its own initial type (assist), so the basic situation of the Receiver in the game (Table 5) can be presented as a game tree shown in Fig 6.

There are three kinds of nodes in the game tree, and every path from the root node to a leaf node is a kind of allocated task:

- ***Root node:*** it represents the "Nature", and is denoted by "∘" in the figure. All the nodes except the root node have and only have one parent node;
- ***Decision node:*** all the nodes except the root node are decision nodes. They indicate the action selections in the game process, and are denoted by "•" in the figure;
- ***Leaf node:*** it represents the Receiver's action selection, and it is also a decision node.

The Harsanyi transformation transforms the action choice under the uncertain agents' types into the action choice under a probability distribution of types. For example, when the Receiver's initial type is assist, the type of Passer is uncertain, but the joint probability distribution of Passers' types is known, then we could obtain the following conversion as shown in Fig 7.

5 The Solution of PRC Task Allocation Game Model With Incomplete Information Game Theory

In the PRC, the final solution is an allocated task related to the types of robots. Taking one Receiver as an example, it only knows its own type θ_1, but does not

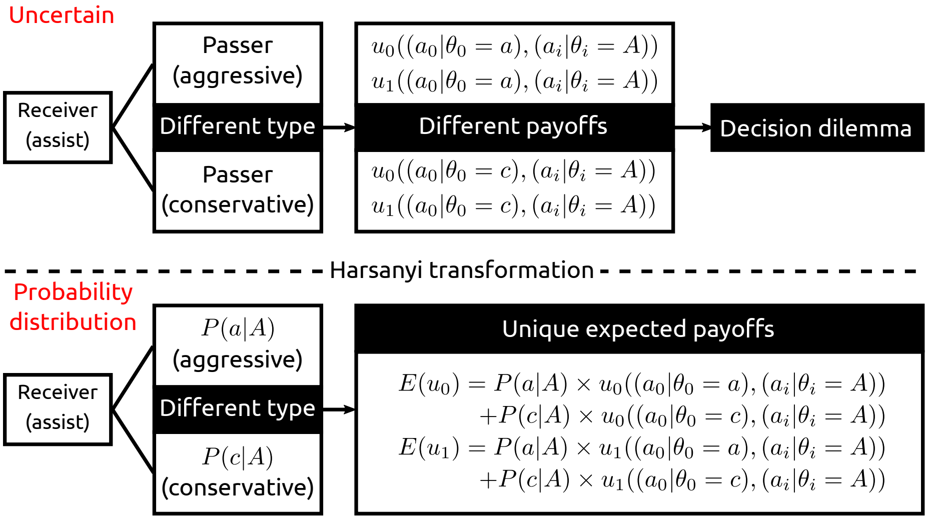

Fig. 7 The comparison before and after Harsanyi transformation

know the true type of the Passer θ_0, so the Receiver will choose the action a_1^* according to the probable action of the Passer a_0 to maximize its expected payoff. The task with the actions a^* is called BNE for the static expanded game or PBNE for the dynamic expanded game.

5.1 The solution of the expanded game

We consider the Receiver(assist) as an example, and establish a corresponding static expanded game model without considering the modification of joint probability distribution based on the direction of the ball velocity. For different situations on the field, three expanded game situations are listed as follows. Then we present a dynamic expanded game to compare with the static expanded game. The task of the expanded game is ulteriorly denoted as:

$$task = t((a_0|\theta_0 = a, a_0|\theta_0 = c), (a_1|\theta_1 = A)). \tag{14}$$

Circumstance 1: *unique BNE in static expanded game*

First, we assume that the R2B is 2m, and the R2O is 0.8m, then the conditional probability $P(a|A) = 0.14, P(c|A) = 0.86$ can be obtained according to (9)(11)(12). As the Receiver does not know the type of the Passer, we take the task $t((f|a, f|c), (y|A))$ as an example, which means that the Passer chooses to pass forward whether the type is aggressive or conservative, and the Receiver (assist) decides to receive the ball. According to the basic situation Table 5, the expected payoffs of the Passer and Receiver are computed as follows:

$$\begin{aligned} E(u_0) &= P(a|A) \times u_0((f|a), (y|A)) + P(c|A) \times u_0((f|c), (y|A)) \\ &= 0.14 \times 9 + 0.86 \times 4 = 4.70, \end{aligned} \tag{15}$$

$$
\begin{aligned}
E(u_1) &= P(a|A) \times u_1((f|a),(y|A)) + P(c|A) \times u_1((f|c),(y|A)) \\
&= 0.14 \times 5 + 0.86 \times 2 = 2.42.
\end{aligned}
\tag{16}
$$

Similarly, the expected payoffs of other tasks can also be computed. The situation of the static expanded game is shown in Table 9.

Table 9 Static expanded game (R2B: 2m, R2O: 0.8m)

Receiver $(a_1\|\theta_1)$		$y\|A$	$n\|A$
Passer $(a_0\|\theta_0, a_0\|\theta_0)$	$(f\|a, f\|c)$	($\underline{4.70}$,2.42)	(3.86,$\underline{2.72}$)
	$(f\|a, b\|c)$	(3.83,3.28)	(6.45,$\underline{4.45}$)
	$(b\|a, f\|c)$	(4.14,2.14)	(4.00,$\underline{2.86}$)
	$(b\|a, b\|c)$	(3.28,3.00)	**($\underline{6.59}$,$\underline{4.59}$)**

The Unique BNE of this game is $t((b|a, b|c),(n|A))$ on the basis of Section 3, and the expected payoffs are (6.59, 4.59). In this case, regardless of the type of the Passer being aggressive or conservative, the Receiver will not receive the ball in order to get the maximum payoffs.

If the two distances are changed, such as the R2B becoming 1.5m and R2O becoming 2.5m, then we can obtain a new expanded game situation, as shown in Table 10.

Table 10 Static expanded game (R2B: 1.5m, R2O: 2.5m)

Receiver $(a_1\|\theta_1)$		$y\|A$	$n\|A$
Passer $(a_0\|\theta_0, a_0\|\theta_0)$	$(f\|a, f\|c)$	**($\underline{7.67}$,$\underline{4.21}$)**	(3.26,1.53)
	$(f\|a, b\|c)$	(7.41,$\underline{4.47}$)	(4.06,2.06)
	$(b\|a, f\|c)$	(4.74,$\underline{2.74}$)	(4.00,2.26)
	$(b\|a, b\|c)$	(4.47,$\underline{3.00}$)	($\underline{4.79}$,2.79)

In this case, the BNE becomes $t((f|a, f|c),(y|A))$, and the payoffs are (7.67, 4.21). Comparing the two allocated tasks of the Receiver, we can find that if the Receiver is far away from the ball and closer to the obstacle, it is not suitable to receive the ball, and vice versa. It is considered that the Receiver makes the correct action selection in these two situations.

Circumstance 2: *two BNEs with obviously different expected payoffs in static expanded game*

When the position of the Receiver is special, e.g., it is much closer to the obstacle or the ball, there will be one BNE. However, when the R2B is 1.5m, and the R2O is 1m, the difference between two distances is not large, and the result of

Table 11 Static expanded game (R2B: 1.5m, R2O: 1m)

Receiver $(a_1\|\theta_1)$		$y\|A$	$n\|A$
Passer $(a_0\|\theta_0, a_0\|\theta_0)$	$(f\|a, f\|c)$	**(<u>5.54</u>,<u>2.92</u>)**	(3.69,2.38)
	$(f\|a, b\|c)$	(4.85,3.62)	(5.77,<u>3.77</u>)
	$(b\|a, f\|c)$	(4.31,2.31)	(4.00,<u>2.69</u>)
	$(b\|a, b\|c)$	(3.62,3.00)	**(<u>6.08</u>,<u>4.08</u>)**

the game becomes complicated. The situation of the expanded game is shown in Table 11.

In this situation, the Receiver is close to the ball and obstacle, so the two actions are both seemingly reasonable. The two BNEs are $t((f|a, f|c),(y|A))$ with expected payoffs (5.54, 2.92), and $t((b|a, b|c),(n|A))$ with expected payoffs (6.08, 4.08) respectively. But it can be found that there is a significant gap between the expected payoffs of the two BNEs. The expected payoffs of the latter are greater than those of the former for the Passer and Receiver. So we consider that the BNE with greater expected payoffs (6.08, 4.08) is better, and it is the reasonable task allocation of this game.

Circumstance 3: *two BNEs with similar expected payoffs in static expanded game*

This situation is the worst in the game, such as when the R2B is 2.5m, and the R2O is 2m. The situation of the expanded game is shown in Table 12.

Table 12 Static expanded game (R2B: 2.5m, R2O: 2m)

Receiver $(a_1\|\theta_1)$		$y\|A$	$n\|A$
Passer $(a_0\|\theta_0, a_0\|\theta_0)$	$(f\|a, f\|c)$	**(<u>5.95</u>,<u>3.17</u>)**	(3.61,2.22)
	$(f\|a, b\|c)$	(4.85,<u>3.78</u>)	(5.44,3.44)
	$(b\|a, f\|c)$	(4.39,2.39)	(4.00,<u>2.61</u>)
	$(b\|a, b\|c)$	(3.78,3.00)	**(<u>5.83</u>,<u>3.83</u>)**

In this case, the two distances are moderate. Two BNEs are $t((f|a, f|c),(y|A))$ with expected payoffs (5.95, 3.17), and $t((b|a, b|c),(n|A))$ with expected payoffs (5.83, 3.83). There is no obvious difference between the expected payoffs of the two BNEs, and each has its own advantage. Since we only discuss the action selection of the Receiver, the BNE with higher expected payoff ($3.83 > 3.17$) of the Receiver is chosen as the allocated task of this game.

Circumstance 4: *PBNE in dynamic expanded game*

In the static expanded game situation of the Receiver, only the distances of R2B and R2O are considered. We cannot obtain the unique BNE in most cases, although the better action choice is confirmed according the analysis mentioned above. In the dynamic expanded game, the direction of the ball velocity is taken into account, so the action choice will be more reasonable based on the modified joint probability distributions. On the basis of the Circumstance 3, assuming the Passer passes the ball to the assist with an angle error of α (random value between 0 and 10 degrees), the situation of the dynamic expanded game is shown in Table 13. We can find that because the ball moves toward the assist, the probability of the receiving action is enhanced, which results in a unique PBNE.

Table 13 Dynamic expanded game (R2B: 2.5m, R2O: 2m)

Receiver $(a_1\|\theta_1)$		$y\|A$	$n\|A$
Passer	$(f\|a, f\|c)$	**($\underline{\mathbf{3.50}}$,$\underline{\mathbf{1.94}}$)**	(1.18,0.41)
$(a_0\|\theta_0, a_0\|\theta_0)$	$(f\|a, b\|c)$	(3.49,$\underline{1.95}$)	(1.21,0.42)
	$(b\|a, f\|c)$	(1.96,$\underline{1.17}$)	(1.57,0.79)
	$(b\|a, b\|c)$	(1.95,$\underline{1.18}$)	($\underline{1.59}$,0.81)

5.2 Algorithms for the task allocation with game theory

The whole process of the task allocation without communication using the game theory is summarized in the following Algorithm 1.

Algorithm 1 Task Allocation with Game Theory

Input:
u : The payoffs of robots' actions with different types
Output:
t : The task allocation of robots

```
Obtain the self-localization l_R, locations of the ball l_B and obstacles l_O, the velocity of
  the ball v_B
Calculate the R2B and R2O
Establish the joint probability distributions of robot types based on distance information
if Dynamic game then
    Modify the joint probability distributions based on the v_B
end if
Initialize the the type θ
Transform the incomplete information game to a completely imperfect information ex-
  panded game by Harsanyi transformation
Find the BNE (static game) or PBNE (dynamic game) of the expanded game, which is
  the result of the task allocation
```

6 Experiments and Analysis

In this section, we conduct experiments to verify the feasibility of the proposed method. In the experiment, there are three robots (one Passer and two Receivers) on the field without communication, and they rely on their own visual systems to get the limited environmental information.

For the three robots in the PRC, each cooperation requires only two robots, so the task allocation about "who"and "how" to cooperate becomes the biggest problem. The dimension of the RoboCup MSL field is $18m \times 12m$ as shown in Fig 8. The Passer (ball dribbling) will pass on any point of the field, and the obstacles will be placed on the field randomly. In order to simplify the process of the cooperation, and avoid the role conflict, the types of two Receivers are predetermined. For better attack and defense, one is assist ($\theta_1 = A$) on the frontcourt, another is defense ($\theta_2 = D$) on the backcourt, and only one Receiver on each half of the field. Once the half where the Receiver locates has changed, its type will change correspondingly.

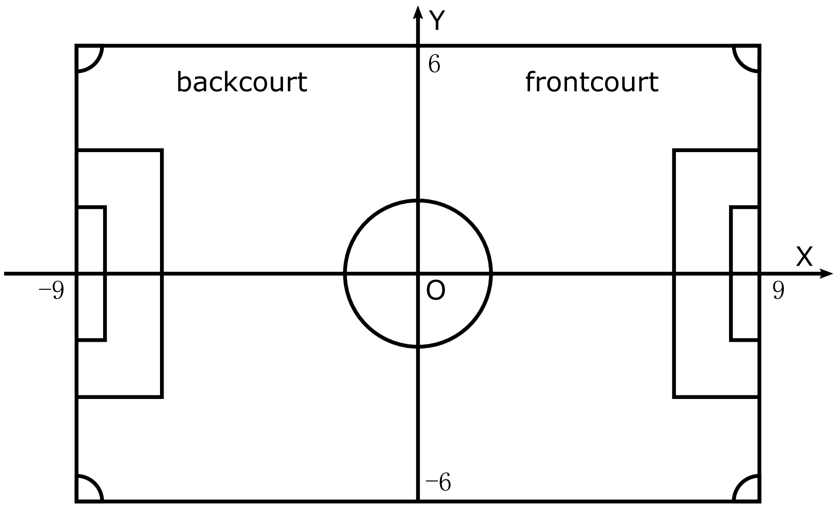

Fig. 8 The field of RoboCup MSL competition (unit: m)

6.1 Task allocation based on static expanded game in Matlab simulation

The assist and defense are both Receivers, but have different types, so their situations of incomplete information game are different. We analyze the action selection of Receivers based on the distance information, where the joint probability distribution is not modified according to the ball velocity. Because the Passer doesn't have the choice dilemma, it selects the action according to its own type with largest probability. Meanwhile, the Receivers select the action based on the solution of game model. According to the different action choices of the Passer and Receivers, there are eight cases in the task allocation of three robots ($task = t(a_0, a_1, a_2) \quad \theta_1 = A, \theta_2 = D$) as shown in Table 14. Because the success of task allocation is determined by the action choices of robots, the description of the task allocation has omitted the type.

At first, we assume that the recognition of the ball and obstacles, as well as the self-localization results are accurate and consistent. Then we perform 1000 independent experiments with the proposed method, where the success rate of

Table 14 The eight cases in the task allocation

Successful	Failed		
$t(f,y,n)$	$t(f,n,y)$	$t(f,n,n)$	$t(f,y,y)$
$t(b,n,y)$	$t(b,y,n)$	$t(b,n,n)$	$t(b,y,y)$

task allocation is shown in Fig 10. As contrast, if the three robots follow the probability of 1/2 to select the action randomly, the theoretical success rate of task allocation will converge to 1/4 as Fig 9 illustrates.

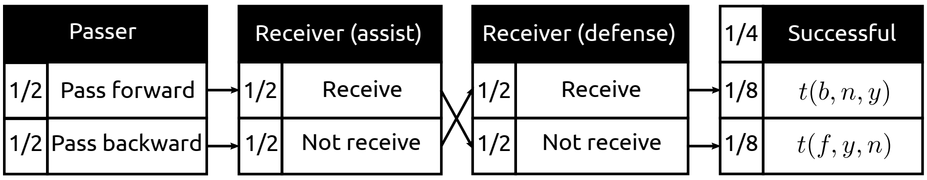

Fig. 9 Three robots follow the probability of 1/2 to select the action randomly

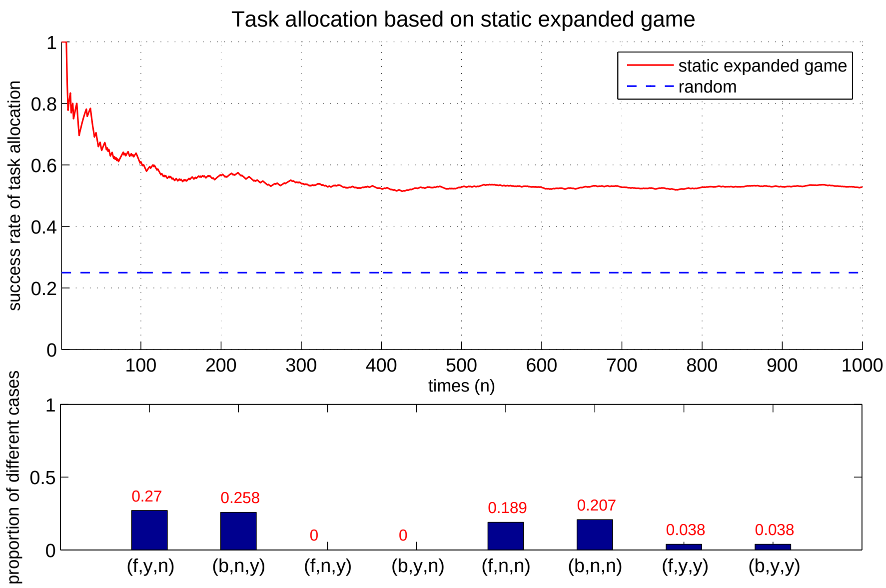

Fig. 10 Success rate of task allocation based on static expanded game

It can be seen from Fig 10 that the action choice based on the static expanded game makes the final success rate stable at about 0.5. Although the final success rate is still low, it can be concluded that the task allocation based on game theory without communication is much better than randomly choosing actions. On the distribution of the failure, the majorities are $t(f, n, n)$ and $t(b, n, n)$, on the other hand, the cases $t(f, n, y)$ and $t(b, y, n)$ do not occur in the 1000 independent experiments.

Next, we add the appropriate noise to the object recognition and self-localization (the noise is a random value between -0.1m and 0.1m on each axis), and perform the above experiments again. The result is shown in Fig 11.

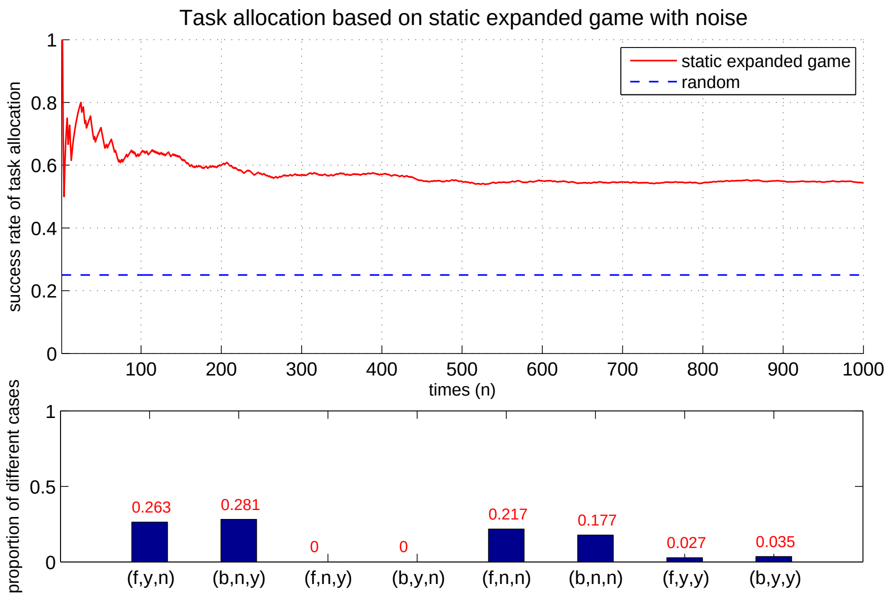

Fig. 11 Success rate of task allocation based on static expanded game with added noise

From Fig 10 and Fig 11, the results are similar. The success of the task allocation mainly depends on the distribution of the three robots and the obstacles. The erroneous allocation is often due to the proximity of the two Receivers, and the noise may make the distance between the two Receivers closer or further. So the noise has little influence on the success rate on the basis of a large number of experiments.

6.2 Task allocation based on dynamic expanded game in Matlab simulation

In this experiment, the modification of the joint probability distribution based on the ball velocity is added (angle α is a random value between 0 and 10 degrees). We also perform 1000 independent experiments with added noise. The success rate of task allocation is shown in Fig 12.

It can be found that the modification of the joint probability distribution based on the ball velocity is effective, and the success rate of task allocation is improved to be higher than 0.75. The main reason is that the modification reduces the proportion of $t(f, n, n)$ and $t(b, n, n)$ significantly. At the same time, the proportion

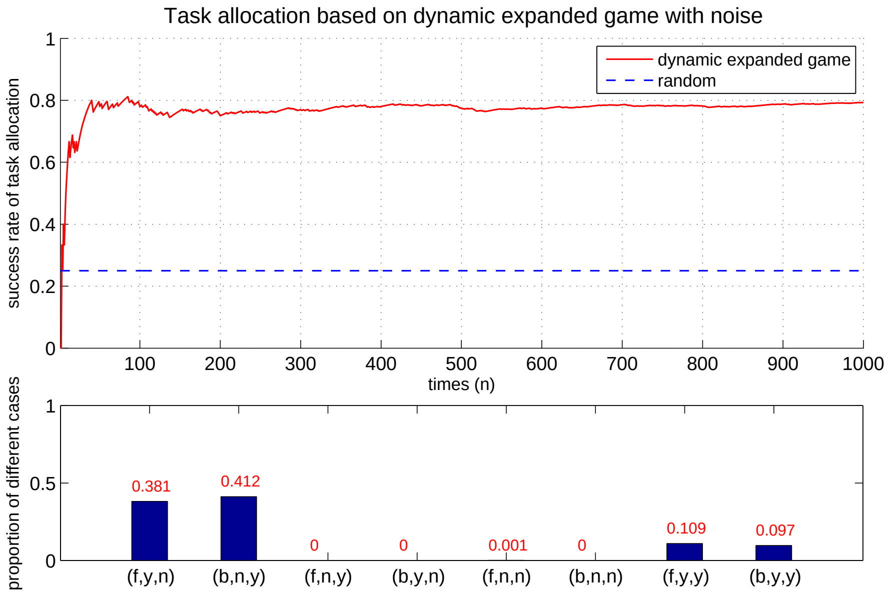

Fig. 12 Success rate of task allocation based on dynamic expanded game with added noise

of $t(f, y, y)$ and $t(b, y, y)$ will increase slightly. These results can be explained as follows: with the modification based on the ball velocity, the Receiver at the direction of the velocity is more likely to receive the ball, so the probability of neither to receive is reduced greatly. On the other hand, if the two Receivers are both near the passing path, the modification based on the ball velocity may increase the probability of both to receive.

6.3 Comparing the performance of task allocation based on different methods in Gazebo

In the above two experiments, the Matlab simulation environment is too ideal. Furthermore, the 1000 independent experiments were performed using each method, and we did not compare the performance of three methods in the same scene. So we build a simulation system [1] of PRC task allocation based on Gazebo and ROS [36,33], to further compare the performance of the three methods. The simulation system is shown in Fig 13, where the left part is the terminal of task allocation, which controls the process of simulation and displays the current information of robots (position, type, action), ball (position, velocity) and obstacles (position) in real time. The right part is the main interface to show the simulation process. In addition, the Gazebo simulation environment also has the following characteristics:

1. It is possible to complete a larger number of independent experiments incessantly without human supervision.

[1] The simulation system has been open source on github: https://github.com/nubot-nudt/task_allocation_gazebo

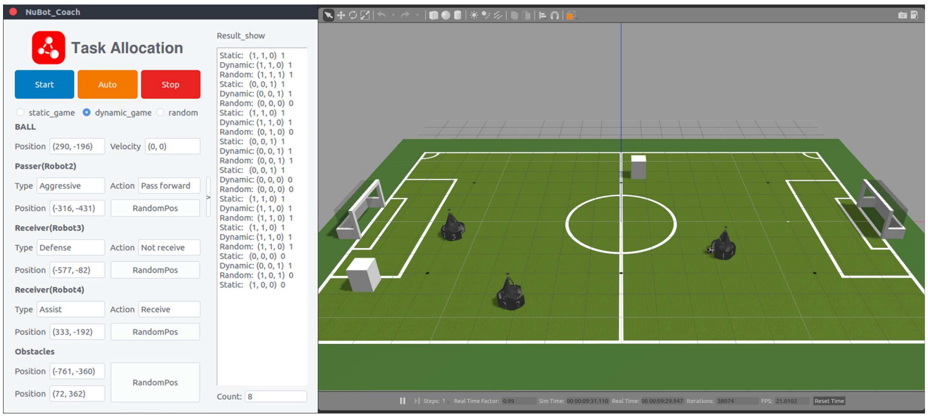

Fig. 13 The simulation system based on Gazebo and ROS

2. In order to make the simulation closer to the real situation, the Gaussian noise is added in the localization of all objects (robots, ball and obstacles).
3. In the Gazebo, each object is no longer a point, but has the different collision volume and inertia characteristics.
4. In the Gazebo, because of the robots' collision volume, the distance between them cannot be too close, so the random selection of the robots' positions will filter out the points whose distance is within two meters.

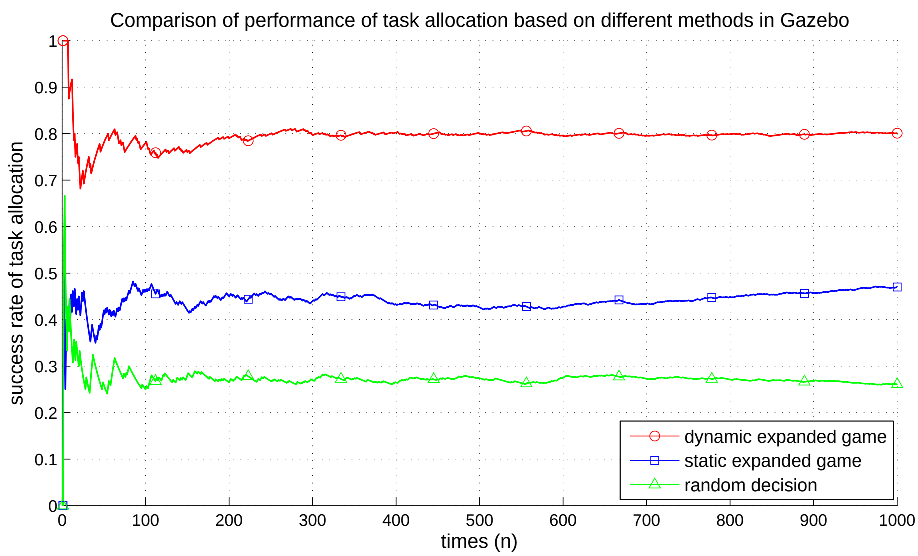

Fig. 14 Comparison of performance of task allocation based on different methods in Gazebo

Similarly, we also carry out 1000 independent experiments, where task allocation is performed in each scene with three different methods (random decision, static expanded game, dynamic expanded game) and the positions of Passer, Receivers and obstacles are changed randomly before each experiment. After about 20 hours of simulation, the statistical results are obtained as shown in Fig 14.

By comparing the results, we can find that the success rates of task allocation based on dynamic expanded game and random decision are almost the same as the previous experiments in Section 6.2. But the success rate of task allocation based on static expanded game is reduced to below 0.5, which is slightly lower than the previous experiment in Section 6.1. The reason is as follows: in the short distance PRC, the correct allocation can almost be obtained based on the distance information in the static expanded game. However, in this experiment, the situation of short distance PRC is eliminated in order to avoid collisions as the characteristic 4 stated, so the success rate is reduced. The allocations based on the dynamic expanded game with velocity information or random decision are little influenced.

6.4 Task allocation based on game theory for real soccer robots

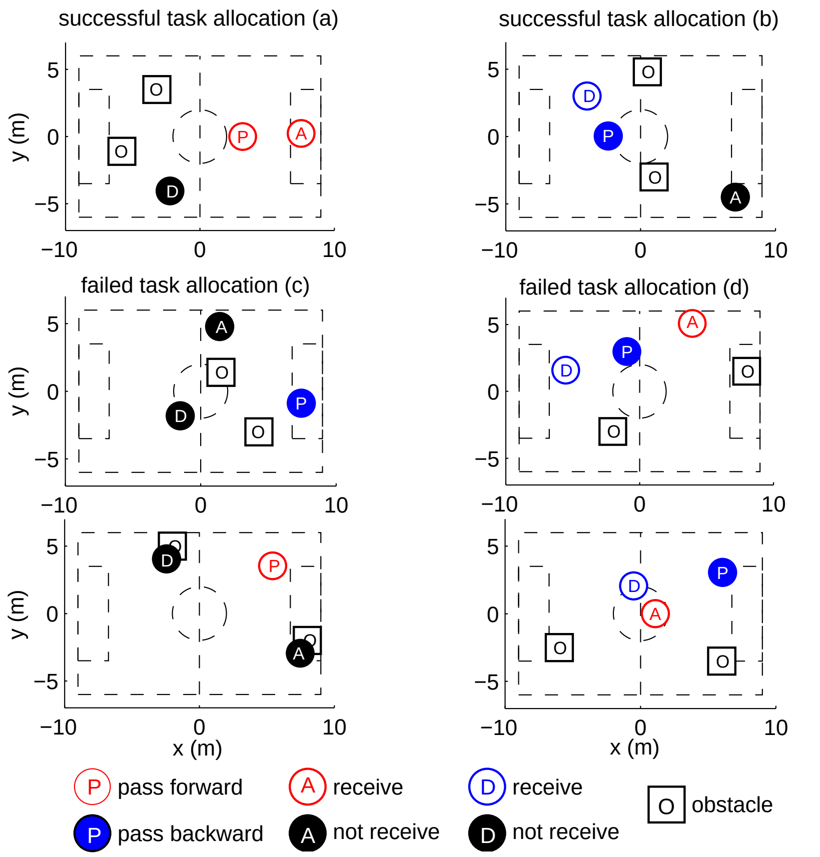

Fig. 15 Typical task allocations in static game. The circle represents the robot, the square represents the obstacle, and the P is Passer, A is assist, D is defense

After the simulation experiments, we use our RoboCup MSL soccer robots (NuBot as shown in Fig 2) to verify the feasibility of the proposed algorithm.

Since the actual system experiment is time-consuming, we can not carry out a large number of experiments and get the statistical results as in the simulation system. We perform four typical task allocations in the static expanded game as shown in Fig 15. In case (a), the Passer is closer to the assist, and there is no obstacle around the assist. As a result, the Passer passes forward, the assist decides to receive, and the defense decides not to receive. The situation shown in case (b) is similar, and successful task allocation is realized in these two cases. On the contrary, in case (c) and (d), the task allocation will fail because both or neither of the two Receivers receive the ball at the same time (there are two examples in each case). There is no instance for $t(f, n, y)$ and $t(b, y, n)$, because the two cases rarely appear in the expanded game according to the result of Section 6.1.

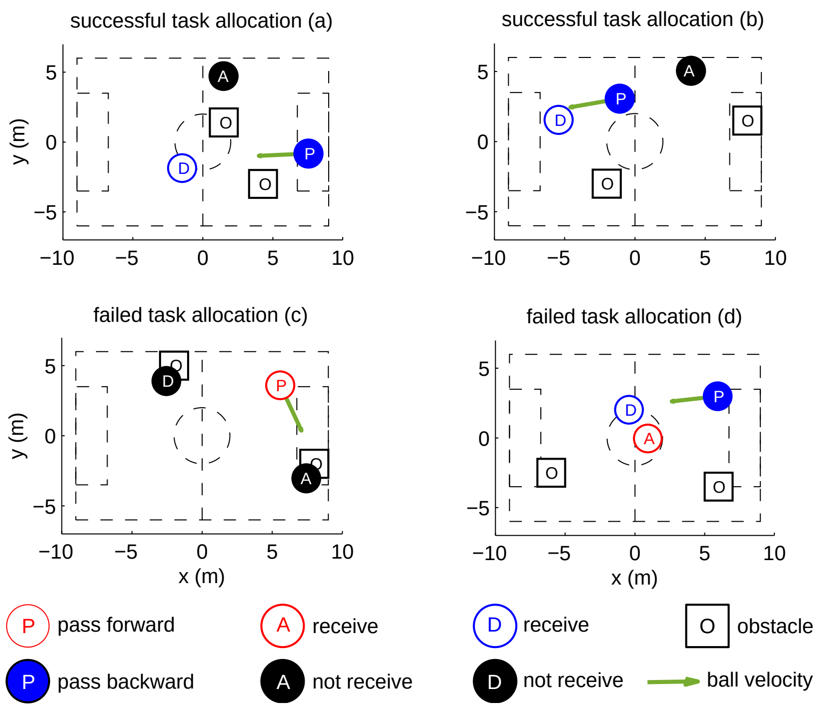

Fig. 16 Typical task allocations in dynamic game. The circle represents the robot, the square represents the obstacle, and the P is Passer, A is assist, D is defense

The failed cases are retested in the dynamic expanded game. As Fig 16 shows, the first two cases will be successful, but the task allocation will still fail in the two latter cases even with the modification of the joint probability distribution based on the ball velocity. Because there are obstacles around the Receivers in case (c) or the angles α of the defense and assist are similar in case (d). Fortunately, according to Fig 12, the proportion of case (c) is 0.001 (0.001+0), and the proportion of case (d) is 0.206 (0.109+0.097), so they will appear only in some very special circumstances as Fig 16 demonstrates.

7 Concluding Remarks

According to the results of the previous four experiments, it can be found that the success rate of the static expanded game is not good enough. The main reason is that the lack of the communication leads to very limited information that the participating robots can use. When the joint probability distribution of each robot's type is established using the distances between robots and ball or obstacles, the action selection depends too much on the distance information. So when the distribution of two Receivers are excessively similar, the task allocation in this cooperation is failure-prone. After the ball velocity information is added to modify the joint probability distribution, the success rate of the dynamic expanded game is improved significantly. But if both of the Receivers are near to the path of ball passing or obstructed by the obstacles, the dynamic expanded game also can not guarantee a correct task allocation result.

If each robot knows others' strategy and has a full understanding of the cooperation situation, it can establish a certain rule to choose the rational action. Unfortunately, it is difficult to realize in the multi-robot systems without communication. Therefore, the key to improve the success rate of task allocation is to use a more reasonable way to establish a joint probability distribution, such as the distance combined with velocity information is a good choice.

No communication is a situation of extreme lack of information, and the robots simply rely on their own sensors to obtain the limited information. They cannot have a full understanding of the world model. Each robot only has its local information, which leads to the difficulty to obtain a consensus on the task allocation. Incomplete information game model overcomes these shortcomings, where the limited but common information about the ball, teammates and obstacles can be used to link all the robots involved in the cooperation to describe their types. So task allocation for each robot is rational, and not generated randomly.

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