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Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

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Introduction to Symbolic AI Coursework 1: logic

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I Question 1

i. $\neg(p \vee q) \to \neg r$
   p: Michel is fulfilled.
   q: Michel is rich.
   r: Michel will live for another five years.

ii. $(\neg p \vee q) \wedge p$
   p: The snowstorm arrives.
   q: Raheem will wear his boots.

iii. $(r \wedge s) \to (p \leftrightarrow q)$
   r: Akira is on set.
   s: Toshiro is on set.
   p: The filming will begin.
   q: The caterers have cleared out.

iv. $(p \vee \neg q) \wedge \neg(p \vee q)$
   p: Irad arrived.
   q: Sarah arrived.

v. $\neg r \to \neg(p \wedge q)$
   r: Anne Sophie answered her phone calls.
   p: Herber heard the performance.
   q: Anne Sophie heard the performance.

II Question 2

i. A propositional formular A is *satisfiable* if there is some ν such that $h_\nu(A) = t$.

ii. Two propositional formulas A,B are *logically equivalent* if, for every ν, $h_\nu(A) = h_\nu(B)$.

iii. Prove that a propositional formular $\neg A$ is satisfiable iff $\neg\neg A \not\equiv \top$

Proof. To prove that $\neg A$ is satisfiable $\leftrightarrow \neg\neg A \not\equiv \top$, we should prove this on both side.

a) First we prove that $\neg A$ is satisfiable $\rightarrow \neg\neg A \not\equiv \top$. Assume that $\neg A$ is satisfiable, then there is some ν such that $h_\nu(\neg A) = t$, which means $h_\nu(\neg\neg A) = f$. Therefore, $\neg\neg A \not\equiv \top$.

b) Then we prove that $\neg\neg A \not\equiv \top \rightarrow \neg A$ is satisfiable. $\neg\neg A \not\equiv \top$ means that $\neg A \not\equiv \bot$ and further $\neg A \equiv \top$ .Then $h_\nu(\neg A) = t$, which means that $\neg A$ is satisfiable.

Hence proved.

III Question 3

Applying ***de morgan's law***, we can reduce the formular $(p \wedge \neg q \leftrightarrow \neg(\neg r \wedge \neg p)) \rightarrow (\neg\neg q \rightarrow r)$ to $(p \wedge \neg q \leftrightarrow r \wedge p) \rightarrow (q \rightarrow r)$. Therefore, determining the validity of original formular is logically equivalent to determine the validity of the new one.

p	q	r	$(p \wedge \neg q \leftrightarrow r \wedge p) \rightarrow (q \rightarrow r)$
t	t	t	t f f f t t t **t** t t t
t	t	f	t f f t t f f **f** t f f

For a formular A to be valid, then $\forall\nu$, $h_\nu(A) = t$. There are 8 possible combinations of the truth-value of tuple (p, q, r), we derive them sequentially.

However, we found that, when $\nu(p) = t$, $\nu(q) = t$ and $\nu(r) = f$, we got $h_\nu(A) = f$. So we stop deriving the truth-table, and we know that the formular is invalid.

IV Question 4

i. In CNF: a,b,d,g.

In DNF: b,d,e,h.

ii. The property of *refutation-soundness and -completeness*:
Let S be in CNF. $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$.
This property is important because it implies that for SAT and resolution, S is satisfiable iff $S \nvdash_{res(PL)} \emptyset$. Therefore, if it impossible to derive $\emptyset$ from S by a resolution derivation, then S is satisfiable.

iii. Apply unit propogation and the pure rule repeatedly to reduce the CNF.

a. $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$
$\Rightarrow \{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$ [q is pure]
$\Rightarrow \{\{p, s\}\}$ [$\neg r$ is pure]
$\Rightarrow \{\}$ [p is pure]

b. $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$
$\Rightarrow \{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$ [unit propogation by clause $\{\neg q\}$]
$\Rightarrow \{\{\neg p\}, \{p\}\}$ [unit propogation by clause $\{\neg r\}$]
$\Rightarrow \{\{\}\}$ [unit propogation by clause $\{p\}$]

V Question 5

Formalize the argument as: $a \to \neg b$, $\neg b \to \neg c$, $c \vee \neg a$, $c \vee a$, therefore, b.
a: I'm going.
b: You are going.
c: Tara is going.

Then we should check whether $a \to \neg b$, $\neg b \to \neg c$, $c \vee \neg a$, $c \vee a \models b$.
We know that, $A_1 ... A_n \models B$ iff $A_1 \wedge ... \wedge A_n \wedge \neg B$ is unsatisfiable.
So we should check whether $(a \to \neg b) \wedge (\neg b \to \neg c) \wedge (c \vee \neg a) \wedge (c \vee a) \wedge (\neg b)$ is satisfiable.
First convert it to CNF: $\{\{\neg a, \neg b\}, \{b, \neg c\}, \{c, \neg a\}, \{c, a\}\}$

Now apply DP:
$\{\{\neg a, \neg b\}, \{b, \neg c\}, \{c, \neg a\}, \{c, a\}, \{\neg b\}$
$\Rightarrow \{\{\neg c\}, \{c, \neg a\}, \{c, a\}\}$ [unit propogation by clause $\{\neg b\}$]
$\Rightarrow \{\{\neg a\}, \{a\}\}$ [unit propogation by clause $\{\neg c\}$]
$\Rightarrow \{\{\}\}$ [unit propogation by clause $\{c\}$]
$\Rightarrow$ UNSATISFIABLE [since $\emptyset$ is in the set].
Since CNF is unsatifiable, the original argument is propositionally valid.

VI Question 6

i. $\forall X(X = aunt(aunt(Andrea)) \rightarrow \exists Y(\neg(GiveCupcakeTo(X,Y) \wedge (Y = Andrea))))$

- $\mathcal{C} = \{Andrea\}$
- $\mathcal{P}_2 = \{GiveCupcakeTo\}$, where the binary predicate $GiveCupcakeTo(X,Y)$ means that object X gives a cupcake to Y.
- $\mathcal{F}_1 = \{aunt\}$

ii. $\exists X \forall Y(computer(X) \wedge computer(Y) \wedge connect(X,Y) \wedge \neg(connect(Y,Y))$

- $P_1 = \{computer\}$
- $P_2 = \{connect\}$, where the binary predicate $connect(X,Y)$ means that object X is connected to object Y.

iii. $\forall X \forall Y \forall A \forall B((painting(X) \wedge paint(PaulKlee, X) \wedge BritishGallary(X) \wedge hang(X,A) \wedge room(A)) \wedge (painting(Y) \wedge paint(Kandinsky, Y) \wedge BritishGallary(Y) \wedge hang(Y,B) \wedge room(B)) \rightarrow A = B)$

- $\mathcal{C} = \{PaulKlee, Kandinsky\}$
- $\mathcal{P}_1 = \{painting, room, BrtishGallary\}$, where the unary predicate $painting(X)$ means that object X is a painting; $room(X)$ means that object X is a room; $BrtishGallary(X)$ means that object X is in the BrtishGallary.
- $\mathcal{P}_2 = \{paint, hang\}$, where the binary predicate $paint(X,Y)$ means that object X paints object Y; $hang(X,Y)$ means X is hangs in place Y.

iv. $\exists X \neg \exists Y(love(X,Y)) \rightarrow \neg(\forall X \exists Y(love(X,Y)))$

- $\mathcal{P}_2 = \{love\}$,where the binary predicate $love(X,Y)$ means that object X loves Y.

VII Question 7

i. False. This says that all objects accessible from k cannot be j. However, the graph shows that only j is accessible from k. let σ be such that $(\sigma(k), \sigma(X)) \in \varphi_M(a)$. Then plainly $\sigma(X) = j$, and $\neg(X = j)$ is false.

ii. True. This says that at least one black and circular object is accessible from l. For this to be true, $b(\sigma(X)) \in \varphi_M(b)$, $c(\sigma(X)) \in \varphi_M(c)$, and $(\sigma(l), \sigma(X)) \in \varphi_M(a)$. Then plainly it is true considering $\sigma(X) = j$ or $\sigma(X) = k$.

iii. True. This says that for at least one object X, there's no other Y accessible from X. Let $\sigma(X)$=the black square object, or $\sigma(X)$=the black white object, and $(\sigma(X), \sigma(Y)) \in \varphi_M(a)$. Then plainly $\sigma(X) = \sigma(Y)$. Hence, the argument is true.

iv. False. This implies that for all objects that are not square, there is at least one accessible circular black object. Let σ be such that $s(\sigma(X)) \notin \varphi_M(s)$, then $\sigma(X)$ could be k or l or j. Assume there is Y such that $c(\sigma(Y)) \in \varphi_M(c)$, $b(\sigma(Y)) \in \varphi_M(b)$ and $(\sigma(X), \sigma(Y)) \in \varphi_M(a)$. Let $\sigma(X) = j$, clearly this is not the case.

v. False. This says that, for all object X, if there is some other object Y accessible from X, then the two objects communicate with each other. Let $\sigma(X) = k$ and $\sigma(Y) = j$. Clearly $\sigma(X) = k \neq \sigma(Y) = j$ and $(\sigma(k), \sigma(j)) \in \varphi_M(a)$. However, $(\sigma(j), \sigma(k)) \notin \varphi_M(a)$. Therefore, it is clearly false.

vi. False. This says that all objects arrow to j mutually communicate. Let $\sigma(X) = k$ and $\sigma(Y) = l$. Clearly, $(\sigma(k), \sigma(j)) \in \varphi_M(a)$ and $(\sigma(j), \sigma(j)) \in \varphi_M(a)$. However, $(\sigma(k), \sigma(j)) \notin \varphi_M(a)$. Therefore, it is clearly false.