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Department of Computing
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Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
Exercise:	2 (CW)	**Due:**	Tue - 03 Nov 2020
Title:	Logic	**Assessment:**	Individual
FAO:	Craven, Robert (rac101)	**Submission:**	Electronic

Student Declaration - Version 1

- I acknowledge the following people for help through our original discussions:

Hudson Yeo (hcy20)

Signed: (electronic signature) Date: 2020-10-28 18:49:45

For Markers only: (circle appropriate grade)

YEO, Hudson (hcy20)	01944572	t5	2020-10-28 18:49:45	**A* A B C D E F**

NO.: Intro to Symbolic AI CID: 01944572 Date:

Coursework 1: Logic

1i p: Michael is either fulfilled or rich

~~q: Michael is rich~~

q: Michael will live another 5 years.

$\neg p \rightarrow \neg q$

ii p: snowstorm arrives

q: Raheem wears his boots

r: I'm sure it will arrive

$((\neg p) \vee q) \wedge r$

iii p: Akira on set, q: Toshino on set

r: filming will begin

s: caterers have cleared out

$(p \wedge q) \rightarrow (r \leftrightarrow s)$

iv p: Irad arrived

q: Sarah arrived

$(p \vee \neg q) \wedge \neg(p \wedge q)$

v p: Herbert heard the performance

q: Anne-Sophie did hear the performance

r: Anne-sophie answered her phone calls.

$\neg r \longrightarrow \neg(p \wedge q)$

2i A propositional formula, A, is satisfiable if there is some v such that $h_v(A) = t$

ii Two propositional formulas are logically equivalent if for every v, $h_v(A) = h_v(B)$

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2iii suppose $\neg A$ satisfiable

for some v, we have $h_v(\neg A) = t$

hence $h_v(\neg\neg A) = f$

but $h_v(T) = t$

so $h_v(\neg\neg A) \neq h_v(T)$

thus $\neg\neg A \not\equiv T$

suppose $\neg\neg A \not\equiv T$

$h_v(T) = t$ for some v

so $h_v(\neg\neg A) \neq t$

and $h_v(\neg\neg A) = f$

hence $h_v(\neg A) = t$

and $\neg A$ is satisfiable

3

p	q	r	$(p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p))$	$\longrightarrow$	$(\neg\neg q \longrightarrow r)$
t	t	t	t f f t f t f t f f t	t	t f t t t
t	t	f	t f f t t f t f t f t	f	t f t f f

Not valid.

When $v(p) = t$, $v(q) = t$ and $v(r) = f$, the overall propositional formula is evaluated as false

4i a In CNF, not in DNF

b In CNF, in DNF

c Not in CNF, not in DNF

d Not in CNF, not in DNF

e Not in CNF, in DNF

f Not in CNF, not in DNF

g In CNF, in DNF

h In CNF, in DNF

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4ii

Let S be in CNF.

$S \vdash_{res(PL)} \emptyset$ iff $S \models \perp$

This property is important because it is used in many SAT-solvers.
The property is also ~~equ~~ the same as:
Let S be in CNF
$S \nvdash_{res(PL)} \emptyset$ iff S is satisfiable
If it is impossible to derive $\emptyset$ from S by a resolution derivation, then S is satisfiable.

iiia Apply unit prop and pure rule:

$\{\{p,s\},\{q,r\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\}$

$\Rightarrow \{\{p,s\},\{\neg p,\neg r,\neg s\}\}$ (q pure)

$\Rightarrow \{\{p,s\}\}$ ($\neg r$ pure)

$\Rightarrow \{\ \}$ (p pure)

satisfiable

b Apply unit prop and pure rule:

$\{\{\neg p, q, r\},\{\neg q\},\{p,r,q\},\{\neg r,q\}\}$

$\Rightarrow \{\{\neg p,r\},\{p,r\},\{\neg r\}\}$ (unit propagation of $\neg q$)

$\Rightarrow \{\{\neg p\},\{p\}\}$ (unit propagation of $\neg r$)

$\Rightarrow \{\{\ \}\}$ (unit propagation of p)

unsatisfiable

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5 p: I am going

q: you are going

r: Tara is going

$p \to \neg q \equiv \neg p \vee \neg q$

$\neg q \to \neg r \equiv q \vee \neg r$

$r \vee \neg p$

$r \vee p$

hence check if:

$p \to \neg q, \neg q \to \neg r, r \vee \neg p, r \vee p \vDash q$

using the logical equivalent, check if following is satisfiable

$(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (r \vee p) \wedge \neg q$

Applying DP to CNF form,

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$

$\Rightarrow \{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$ (unit propagation of $\neg q$)

$\Rightarrow \{\{\neg p\}, \{p\}\}$ (unit propagtion of $\neg r$)

$\Rightarrow \{\{\ \}\}$ (unit propagation of p)

UNSATISFIABLE

Since the CNF is unsatisfiable, the original argument is propositionally valid

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6i $\mathcal{L}$: $C = \{andrea\}$
$P_1 = \{cupcake\}$
$P_2 = \{aunt\}$
$P_3 = \{gives\}$

$\forall A \forall B \exists C \exists D((aunt(A,B) \wedge aunt(B, andrea)) \rightarrow (gives(A,C,D) \wedge cupcake(c) \wedge \neg(D = andrea)))$

ii $\mathcal{L}$: $P_1 = \{computer\}$
$P_2 = \{connected\}$
$\forall X \exists Y((computer(X) \wedge \neg connected(X,X)) \rightarrow (computer(Y) \wedge connected(Y,X)))$

iii $\mathcal{L}$: $C = \{Paul Klee, Kandinsky\}$
$P_1 = \{painting, British Gallery, room\}$
$P_2 = \{painter, hangs in\}$

$\forall p \exists G((painting(p) \wedge painter(Paul Klee, p) \wedge British Gallery(G) \wedge hangs in(p,G)) \rightarrow (\forall k (painting(k) \wedge painter(Kandinsky, k) \wedge hangs in(k,G)) \rightarrow (\exists r (room(r) \wedge hangs in(p,r) \wedge hangs in(k,r)))))$

iv $\mathcal{L}$: $P_2 = \{loves\}$
$(\exists X \neg \exists Y(loves(X,Y))) \rightarrow \neg(\forall X \exists Y(loves(X,Y)))$

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7 $C = \{j, k, l\}$

$P_1 = \{b, w, s, c\}$

i False.

$a(k, X)$ is satisfied when there is an arrow from k to X, and this is only true when $\sigma(X) = j$. But then $\neg(X=j)$ is false. So it is False that $\forall X\,(a(k,X) \rightarrow \neg(X=j))$ since there is one assignment for X that makes it false.

ii True

$c(l)$ is true since l is circular

consider k in the domain.

$b(k)$ true, since k is black

$c(k)$ true, since k is circle

and $a(l, k)$ true, since there is an arrow from l to k

so $b(k) \wedge c(k) \wedge a(l, k)$ is true

and $c(l) \longrightarrow \exists X\,(b(X) \wedge c(X) \wedge a(l, X))$ is true

iii False

consider when $\sigma(X) = l$, $\sigma(Y) = k$

then $\neg(X=Y)$ is true, $a(X,Y)$ true and $(\neg(X=Y) \wedge a(X,Y))$

thus $\exists X\, \exists Y\,(\neg(X=Y) \wedge a(X,Y))$ so the original statement is false

iv False

$\neg s(X)$ refers to all non-square objects, or j, k, l.

consider when $\sigma(X) = j$, then $a(X,Y)$ true only if $\sigma(Y) = l$

but l is not black, ie $b(Y)$ is false

then $\exists Y\,(c(Y) \wedge b(Y) \wedge a(X,Y))$ is false

and hence overall statement is false since consequent false

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7 v False

consider the antecedent when $\sigma(X) = k$ and $\sigma(Y) = j$
then $\neg(X = Y) \wedge a(X, Y)$ is true

so the antecedent is true

consider the consequent for the same $X = k$
$a(X, Y)$ is true only when $Y = j$
but $a(Y, X)$ is false since there is no arrow from j to k

vi True

the antecedent is true only when X, Y each have an arrow to j. This occurs when $\sigma(X) = k, \sigma(Y) = l$ or $\sigma(X) = l, \sigma(Y) = k$

When $\sigma(X) = k, \sigma(Y) = l$, $a(X, Y) \vee a(Y, X)$ is true since $a(Y, X)$ true
When $\sigma(X) = l, \sigma(Y) = k$, $a(X, Y) \vee a(Y, X)$ is true since $a(X, Y)$ true
Thus for all cases that the antecedent is true, the consequent is also true. Hence overall true.