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Department of Computing

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Question 1

i. $((\neg(p \wedge q)) \longrightarrow (\neg r))$

p: Michael is fulfilled.

q: Michael is rich.

r: Michael will live another 5 years.

ii. $(((\neg p) \vee q) \wedge r)$

p: the snowstorm arrive.

q: Raheem will wear his boots

r: ~~the catere~~ I'm sure it will arrive.

iii. $(p \longrightarrow (q \longleftrightarrow r))$

p: Akira and Toshiro are on set.

q: Fileming will begin.

r: the caterers have cleared out.

iv. $(p \vee (\neg q))$

p: Irad arrived.

q: Sarah arrived

v. $((\neg p) \longrightarrow (\neg(q \wedge r)))$

p: ~~Irad~~ Anne-Sophie answered her plone call

q: Herbert heard the performance

r: Anne-Sophie heard the performance

Question 2.

i. A propositional formula A is satisfiable if there is some v such that $h_v(A) = t$

ii. Two propositional formula A, B are logically equivalent if, for every v, $h_v(A) = h_v(B)$

iii. $\neg A$ is satisfiable means there are some v such that $h_v(\neg A) = t$ for some v such that ~~$h_v(A) = f$, so~~ $h_v(\neg A) = f$.
so $h_v(\neg\neg A) = t$ for some v. but $h_v(\top) = t$ for all v.
Thus ~~$h_v(\neg\neg A) \neq$~~ $\neg\neg A$ and $\top$ are not logically equivalent.
~~Conversely, $\neg\neg$~~ $\neg\neg A \not\equiv \top$.
Conversely, since $\neg\neg A \not\equiv \top$, and $h_v(\top) = t$ for all v, so there is some v such that $h_v(\neg\neg A) = f$. so $h_v(\neg A) = t$ for these v. Thus, formula $\neg A$ is satisfiable.

Question 3.

p	q	r	(p	∧	¬q	↔	¬	(¬r	∨	¬p))	→	(¬¬q	→	r)
t	t	t	t	f	f	f	t	f	f	f	t	t	t	t
t	t	f	t	f	f	t	f	t	t	f	f	t	f	f
t	f	t	t	t	t	t	t	f	f	f	t	f	t	t
t	f	f	t	t	t	f	f	t	t	f	t	f	t	f
f	t	t	f	f	f	t	f	f	t	t	t	t	t	t
f	t	f	f	f	f	t	f	t	t	t	f	t	f	f
f	f	t	f	f	t	t	f	f	t	t	t	f	t	t
f	f	f	f	f	t	t	f	t	t	t	t	f	t	f

Since $h_v((p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \rightarrow (\neg\neg q \rightarrow r)) = t$ not for all v, so this formula is not valid.

Question 4

i. ~~a~~ CNF : a. c. f. g

~~b~~ DNF : e. h.

ii. a. Let S be in CNF. $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$.

b. Because if it's impossible to derive $\emptyset$ from S by a resolution ~~then~~ derivation, then S is satisfiable. ~~Th~~ This is important for many SAT-solvers.

iii. a. $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$.

$\Rightarrow \{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$. [q was pure].

$\Rightarrow \{\{p, s\}\}$ [$\neg r$ was pure]

$\Rightarrow \{\ \}$ [p was pure]

b. $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$.

$\Rightarrow \{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$. [unit propagation by unit clause $\{\neg q\}$]

$\Rightarrow \{\{\neg p\}, \{p\}\}$ [unit propagation by unit clause $\{\neg r\}$]

$\Rightarrow \{\{\}\}$ [unit propagation by unit clause $\{p\}$].

Question 5

We formalize it as : $p \to \neg q$, $\neg q \to \neg r$, $r \vee \neg p$, $p \vee r$, therefore q .

p : I'm going

q : You're going .

r : Tara's going .

So we must check whether $p \to \neg q$, $\neg q \to \neg r$, $r \vee \neg p$, $p \vee r \models q$.

We know that , in general $A_1, \cdots, A_n \models B$ if $A_1 \wedge \cdots A_n \wedge \cancel{B} \neg B$ is unsatisfiable.

So we check whether $(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (p \vee r) \wedge (\neg q)$ is satisfiable .

We first it to clausal-form CNF

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{p, r\}, \{\neg q\}\}$.

Now applying DP .

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{p, r\}, \{\neg q\}\}$.

$\Rightarrow$ $\{\{\neg r\}, \{r, \neg p\}, \{p, r\}\}$. [unit propagation by unit clause $\{\neg q\}$]

$\Rightarrow$ $\{\{\neg p\}, \{p\}\}$ [unit propagation by unit clause $\{\neg r\}$]

$\Rightarrow$ $\{\{\}\}$. [unit propagation by unit clause $\{p\}$]

$\Rightarrow$ unsatisfiable . [since ϕ is in the set].

Since the CNF is unsatisfiable , the original argument is propositionally valid .

Question 6

i. $C = \{Andrea\}$

$P_1 = \{cupcakes\}$

$F_1 = \cancel{ant}\{aunts\}$

$\forall X \exists Y (X = aunts(aunts(Andrea)) \rightarrow cupcak(X, Y) \wedge (Y \neq Andrea))$

$X = aunts(aunts(Andrea))$: X is Andrea's aunt's aunt.

$cupcake(X, Y)$: X gives a cupcake to Y.

$Y \neq Andrea$: Y is not Andrea.

ii. $P_1 = \{computer\}$.

$P_2 = \{connect\}$.

$\exists X \forall Y (computer(X) \rightarrow computer(Y) \wedge (X \neq Y) \wedge connect(X, Y))$

$computer(X)$: There's a computer X.

$computer(Y)$: Every computer Y.

$connect(X, Y)$: X connect to Y.

$X \neq Y$: X is not Y, this means isn't connected to itself.

iii. $C = \{Paul\ Klee,\ Kandinsky\}$

$P_1 = \{gallery,\ roomA,\ hang\}$.

$P_2 = \{paint\}$

$$\forall X \forall Y (\cancel{P}paint(Paul\ Klee, X) \rightarrow gallery(X) \wedge hang(X) \wedge roomA(X) \wedge \cancel{Pa}\ paint(Kandinsky, \cancel{X}Y) \wedge gallery(Y) \wedge hang(Y) \wedge roomA(Y))$$

$paint(Paul\ Klee, X)$: Paul Klee painted X.

$paint(Kandinsky, Y)$: Kandinsky painted Y.

$gallery(X)$: X in a British gallery.

$gallery(Y)$: Y in a British gallery.

$hang(X)$: hang X

$hang(Y)$: hang Y.

$roomA(X)$: X is in the room A.

$room\cancel{B}A(Y)$: Y is in the room A.

iv. $P_1 = \{love\}$.

$\exists X \forall Y (\neg love(X, Y) \rightarrow \neg love(Y, X))$

$love(X, Y)$: X loves Y.

$\neg love(X, Y)$: X doesn't love Y.

$love(Y, X)$: Y loves X.

$\neg love(Y, X)$: ~~Y loves~~ It's false that Y loves X.

Question 7.

i. False

There is only one directed arrow from K to X, so X is j, this ~~is~~ contradict with $\neg(X = j)$. so false.

ii. True

X is black and circular, so the arrow from j to X, X is K or j.
This satisfies the argument.

iii. True

$\exists X \neg \exists Y\ (\neg(X = Y) \land a(X, Y))$ is equivalent to
$\exists X\ \forall Y\ ((X = Y) \lor a(X, Y))$.
From the diagram, it's easy to find there is ~~not~~ an X that can point to any one or equal to anyone.

iv. False.

For all circular X, if X is j, we cannot find a black and circular object that X can point to.

v. False.

For all X, we cannot find a Y that ~~they point each other and X~~ $X \neq Y$ and they point each other. such as $X = K$.

vi. True

For all X and Y that can point to j, they are K and L, and there is an arrow from j to K. This satisfies the argument.