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Department of Computing
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CATE

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| 70051 rac101 2<br>t5 gts20 v1<br>0012000015175 | Electronic submission<br>**Mon - 02 Nov 2020**<br>**22:17:58**<br>gts20 |

## Exercise Information

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| **Module:** | 70051 Introduction to Symbolic Artificial Intelligence (MSc AI) | **Issued:** | Tue - 20 Oct 2020 |
| **Exercise:** | 2 (CW) | **Due:** | Tue - 03 Nov 2020 |
| **Title:** | Logic | **Assessment:** | Individual |
| **FAO:** | Craven, Robert (rac101) | **Submission:** | Electronic |

## Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-11-02 22:09:21

**For Markers only:** (circle appropriate grade)

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| **SARAPATA, Grzegorz (gts20)** | 01995527 | t5 | 2020-11-02 22:09:21 | A* A B C D E F |

Coursework 1: Logic

1.

i) p - Michel is fulfilled

q - Michel is rich

r - Michel will live another five years

$$((\neg(p \vee q)) \rightarrow (\neg r))$$

ii) p - ~~[illegible]~~ Unless the snowstorm doesn't arrive, Raheem will wear his boots; but I'm sure it will arrive.

$$p$$

iii) p - Akira and Toshiro are on set

q - filming will begin

r - the caterers have cleared out

$$(p \rightarrow (q \leftrightarrow r))$$

iv) p - Irad arrived

q - Sarah arrived

$$(\neg(p \wedge (\neg q)))$$

v) p - Herbert heard the performance
$q_h$ - Anne-Sophie heard the performance
r - Anne-Sophie answered her phone calls

$((\neg r) \to (\neg(p \wedge q_h)))$

2. i) A propositional formula A is satisfiable if there is some valuation v such that $h_v(A) = t$. ie. there is some truth assignment of atoms that constitute A, that makes the formula evaluate to t.

ii) Two propositional formulas A and B are logically equivalent if for any valuation v; $h_v(A) = h_v(B)$ ie. ~~for any~~ for any atomic evaluation v, the two formulas evaluate to the same outcome based on the v.

iii) Let's prove both directions, first ~~if~~
Let's assume $\neg\neg A \not\equiv T$.
If it was the case that $\neg\neg A \equiv T$ then we would have $h_v(\neg\neg A) = h_v(T) = t$ for any v. Since it is not the case that means there is some valuation v' that makes $h_{v'}(\neg\neg A) = f$

If we discard one negation to obtain $\neg A$ we have that $h_{v'}(\neg A) = t$, so $\neg A$ is SAT.

Now, the other direction.

if $\neg A$ is satisfiable, then for some $v''$ we have $h_{v''}(\neg A) = t$

if we 'add' the negation we get $h_{v''}(\neg\neg A) = f$ so there is ~~sa~~ a valuation $v''$ that makes it $f$, hence $\neg\neg A \not\equiv \top$ (since $h_v(\top) = t$ for all $v$).

4. a) CNF
   b) both
   c) neither
   d) neither
   e) DNF
   f) neither
   g) can be both
   h) can be both

ii) Let $S$ be in CNF. There is a derivation of $\emptyset$ by propositional resoultion from $S$ if and only if $S$ is not satisfiable

$S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$

This property is used by SAT solvers, if it is impossible to derive $\emptyset$ from $S$ by resolution der., then $S$ is satisfiable - this can reduce the computational time when solving for satisfiability.

iii)

a) $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

$\Rightarrow \{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$ q was pure

$\Rightarrow \{\{p, s\}\}$ $\neg r$ was pure

$\Rightarrow \{\}$ p was pure

b) $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

$\Rightarrow \{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$ unit prop. by $\{\neg q\}$

$\Rightarrow \{\{\neg p\}, \{p\}\}$ unit propagation by $\{\neg r\}$

$\Rightarrow \{\{\}\}$ unit propagation by $\{p\}$

5. $p$ - I am going
$q$ - you are going
$r$ - Tara is going

$p \to \neg q$, $\neg q \to \neg r$, $r \vee \neg p$, $\neg p \to r$

so $q$

We check whether $p \to \neg q$, $\neg q \to \neg r$, $r \vee \neg p$, $\neg p \to r$ $\models q$

We will check if $(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (\neg p \to r) \wedge (\neg q)$ is satisfiable:

We convert to CNF

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{\neg r, \neg p\}, \{p, r\}, \{\neg q\}\}$

$\Rightarrow \{\{\neg r\}, \{\neg r, \neg p\}, \{p, r\}\}$ unit prop. by $\{\neg q\}$

$\Rightarrow \{\{p\}\}$ unit prop. by $\{\neg r\}$

$\Rightarrow \{\}$ unit prop. by $\{p\}$

$\Rightarrow$ SATISFIABLE (since $\emptyset$ not in the set)

we know that in general $A_1, \dots, A_n \models B$ iff
$A_1 \wedge \dots \wedge A_n \wedge \neg B$ is unsatisfiable

So the argument is NOT valid.

6 i)

$\forall X(\exists Y\, aunt(X, aunt(Y, Andrea)) \rightarrow \exists Z\,(gives(X,Z) \wedge \neg(Z = Andrea)))$

$C = \{Andrea\}$

$P_2 = \{aunt, gives\}$

ii) $\exists X\, \forall Y\,(computer(X) \wedge computer(Y) \wedge connected(X,Y) \wedge \neg connected(X,X))$

$P_1 = \{computer\}$
$P_2 = \{connected\}$

iii) $\forall X((Klee(X) \wedge in\_BG(X)) \rightarrow \forall Y(Kandinsky(Y) \wedge in\_BG(Y) \wedge hangs(X, room(Y))))$

$P_1 = \{Klee, Kadinsky, in\_BG\}$
$P_2 = \{hangs\}$
$F_1 = \{room\}$

iv) $\exists X (\neg \exists Y \, loves(X,Y)) \rightarrow \neg (\forall X \exists Y \, loves(X,Y))$

$P_2 = \{loves\}$

7.

i) False, ~~only~~ k points only to j
so $\neg(X = j)$ is false for all X. such that $a(k, X)$

ii) True, both k and j 'meet' requirements for X,
~~and~~

iii) ~~False~~, True, the black filled square points only to itself

iv) False, j is not a square and it doesn't point to any circular and filled objects.

v) False, k points to ~~other~~ say j, but there is no object such that k points to it and that object points to k.

vi) False, if we take X and Y to both be k say, then the conjunction is true, but the disjunction is not, as k doesn't point to itself.