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Student Declaration - Version 1

- I acknowledge the following people for help through our original discussions:

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For Markers only: (circle appropriate grade)

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Symbolic AI Coursework 1

Victoria Derks

October 2020

1 Question 1

Formalize each of the following in propositional logic, including all brackets required by the strict definition of a propositional formula (remember to give the correspondence between the basic sentences of the original and the propositional atoms):

i If Michel isn't either fulfilled or rich, he won't live another five years.

ii Unless the snowstorm doesn't arrive, Raheem will wear his boots; but I'm sure it will arrive.

iii If Akira and Toshiro are on set, then filming will begin if and only if the caterers have cleared out.

iv Either Irad arrived, or Sarah didn't: but not both!

v It's not the case both that Herbert heard the performance and Anne-Sophie did, if the latter didn't answer her phone calls.

1.1 Answers to Q1

i $((\neg p) \vee (\neg q) \rightarrow \neg r)$
p : Michael is fulfilled
q: Michael is rich
r: Michael will live another five years

ii $(\neg(\neg p) \rightarrow q) \wedge r)$
p: The snowstorm arrives
q: Raheem will wear his boots
r: I'm sure it will arrive

iii $((p \wedge q \rightarrow r) \leftrightarrow s)$
p: Akira is on set
q: Toshiro is on set
r: Filming will begin
s: The caterers have cleared out

iv $((p \land \neg q) \lor ((\neg p) \land q))$ (shows exclusive disjunction)
p: Irad arrived
q: Sarah arrived

v $((\neg r) \to \neg(p \land q))$
r: Anne-Sophie answered her phone calls
p: Herbert heard the performance
q: Anne-Sophie heard the performance

2 Question 2

i What is the definition of the satisfiability of a propositional formula, A?

ii What is the definition of the logical equivalence of two propositional formulas A and B?

iii Prove that a propositional formula $\neg A$ is satisfiable iff $\neg\neg A \not\equiv \top$ (i.e., iff it's not the case that $\neg\neg A \equiv \top$)

2.1 Answers to Q2

i A propositional formula A is satisfiable if it evaluates to true in some cases i.e. it is possible to find a certain configuration of propositional atoms that makes it true. The formula A can evaluate to false, but there have to be one or more cases in which A is evaluated as true for it to be satisfiable.

ii Propositional formulas A and B are logically equivalent, if, when A is evaluated to be true, B is as well, and when B is true then A is as well. Similarly, when A is false, B should be false as well and vice-versa for equivalence to hold.

iii First, let's assume A evaluates to true. In this case, $\neg A$ would evaluate to false. $\neg\neg A$ evaluates to true again. $\top$ always has a truth-value of true. Therefore, if $\neg A$ is false, then $\neg\neg A \equiv \top$.
However, if we take A to be false, $\neg A$ evaluates to true. $\neg\neg A$ then evaluates to false. In this case, $\neg\neg A \not\equiv \top$. We see that $\neg\neg A \not\equiv \top$ if and only if $\neg A$ is evaluated to true.
In conclusion, iff $\neg\neg A \not\equiv \top$ then $\neg A$ is satisfiable, as per the definition of satisfiability: $\neg A$ is satisfiable if it evaluates to true at least once, which happens in the case of $\neg\neg A \not\equiv \top$.

3 Question 3

Use truth-tables to determine whether the following is valid or not: $(p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p)) \to (\neg\neg q \to r)$.

3.1 Answers to Q3

A propositional formula is valid if it is true in all configurations of the atoms (t/f), i.e. if it always evaluates to true.

p	q	r	$(p$	$\wedge$	$\neg$	q	$\leftrightarrow$	$\neg$	$(\neg$	r	$\vee$	$\neg$	$p))$	$\to$	$(\neg$	$\neg$	q	$\to$	$r)$
t	t	t	t	**f**	f	t	**f**	t	f	t	**f**	f	t	$\underline{\textbf{t}}$	t	f	t	**t**	t
t	t	f	t	**f**	f	t	**t**	f	t	f	**t**	f	t	$\underline{\textbf{f}}$	t	f	t	**f**	f
t	f	f	t	**t**	t	f	**f**	f	t	f	**t**	f	t	$\underline{\textbf{t}}$	f	t	f	**t**	f
f	t	t	f	**f**	f	t	**t**	f	f	t	**t**	t	f	$\underline{\textbf{t}}$	t	f	t	**t**	t
f	f	t	f	**f**	t	f	**t**	f	f	t	**t**	t	f	$\underline{\textbf{t}}$	f	t	f	**t**	t
f	t	f	f	**f**	f	t	**t**	f	t	f	**t**	t	f	$\underline{\textbf{f}}$	t	f	t	**f**	f
t	f	t	t	**t**	t	f	**t**	t	f	t	**f**	f	t	$\underline{\textbf{t}}$	f	t	f	**t**	t
f	f	f	f	**f**	t	f	**t**	f	t	f	**t**	t	f	$\underline{\textbf{t}}$	f	t	f	**t**	f

From the first two lines in the table we can already see that the formula does not always evaluate to true. For the sake of completeness I have written out the entire table. Nonetheless, the formula is not valid.

4 Question 4

i Which of the following are in CNF? Which are in DNF?

(a) $p \wedge (\neg q \vee r)$

(b) $\neg p$

(c) $p \wedge (q \vee (p \wedge r))$

(d) $\top$

(e) $(p \wedge q) \vee (p \wedge q)$

(f) $\neg\neg p \wedge (q \vee p)$

(g) $p \wedge q$

(h) $p \vee q$

ii Define the property of the refutation-soundness and -completeness of a resolution derivation. Why is this property important?

iii Apply unit propagation and the pure rule repeatedly, in order to reduce the following to their simplest forms (stating which rule you're applying, and indicate the literal involved):

(a) $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

(b) $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

4.1 Answers to Q4

i (a) CNF, this is equal to $(p \vee \neg q) \wedge (p \vee r)$

(b) CNF and DNF, this is equal to $\neg p \wedge \neg p$ and also $\neg p \vee \neg p$ (conjunction/disjunction of a literal)

(c) DNF, this is equal to $p \vee (q \wedge p) \vee (q \wedge r)$

(d) CNF or DNF, this is equal to $\top \wedge \top$ and $\top \vee \top$ (conjunction/disjunction of a literal)

(e) This is a CNF formula written in DNF, therefore it is in DNF.

(f) Neither, $(\neg\neg p)$ is not a literal and therefore is an invalid clause

(g) This is written in DNF and CNF.

(h) This is written in CNF and DNF.

ii Refutation-soundness and -completeness means that the resolvent of a certain clause C_1 and clause C_2 on a literal p is logically equivalent to $\{C_1, C_2\}$, i.e. $\{C_1, C_2\} \vDash \mathit{resolvent}(C_1, C_1, p)$. Put simply, it indicates that the resolvent combines C_1 and C_2 on p without losing information or changing the meaning of the original formula that contains clauses C_1 and C_2. This is important, because resolvents are used when checking the satisfiability of a certain formula. If the resolvents would change the original formula, using them would not be a good way to determine whether the original formula is satisfiable. Refutation-soundness and -completeness ensures that we can state something meaningful about the satisfiability of the original formula after reducing the formula to its simplest form.

iii (a) Set is: $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$
Applying the pure rule on q first: $\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$
Pure rule on $\neg r$: $\{\{p, s\}\}$
Pure rule on p: $\{\}$
The empty set is not in the set, so the formula is satisfiable.

(b) Set is: $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$
Unit propagation on $\neg q$: $\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$
Unit propagation on $\neg r$: $\{\{\neg p\}, \{p\}\}$
Unit propagation on $\neg p$: $\{\{\}\}$
The empty set is in the set, so the formula is unsatisfiable.

5 Question 5

Use DP to determine whether the following argument is valid or not:
If I'm going, then you aren't.
If you're not going, then neither is Tara.
Either Tara's going or I'm not.
Tara's going unless I am.
So, you're going.

5.1 Answers to Q5

Assign the text to literals as follows:
p: I'm going
q: You're going
r: Tara is going
We can now convert the sentences to propositional logic:

$p \to \neg q$
$\neg q \to \neg r$
$r \lor \neg p$
$p \to \neg r$
q

To determine whether the argument is valid, we check the following:
$p \to \neg q$, $\neg q \to \neg r$, $r \lor \neg p$, $p \to \neg r \vDash q$. $A_1, ..., A_n \vDash B$ is valid if and only if $A_1 \land ... \land A_n \land \neg B$ is unsatisfiable. Therefore, we check whether $p \to \neg q$, $\neg q \to \neg r$, $r \lor \neg p$, $p \to \neg r \vDash \neg q$ is (un)satisfiable.
To start off, we need to convert the clauses to CNF and put them in a set: $\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{\neg p, \neg r\}, \{\neg q\}\}$. We now use DP to check whether it is satisfiable.

Unit propagation on $\neg q$: $\{\neg r\}, \{r, \neg p\}, \{\neg p, \neg r\}$
Unit propagation on $\neg r$: $\{\neg p\}$
Unit propagation on $\neg p$: $\{\}$

The CNF is satisfiable, therefore the original argument is invalid.

6 Question 6

Translate into first-order logic, giving as much logical structure as possible. Be sure to specify the signature for each part.

i All of Andrea's aunts' aunts gave a cupcake to someone other than Andrea.

ii There's a computer connected to every computer which isn't connected to itself.

iii Any painting by Paul Klee in a British gallery hangs in a room where all Kandinsky paintings in that gallery hang.

iv If there's somebody who loves nobody, then it's false that everybody loves somebody.

6.1 Answers to Q6

i $C = \{cupcake\}$
$P_1 = \{Andrea\}$ - $Andrea$(X) means X is Andrea
$P_2 = \{aunt\}$ - $aunt$(X, Y) means X is an aunt of Y
$P_3 = \{give\}$ - $give$(X, Y, Z) means X gives Z to Y
$\forall X \forall Y \exists Z(aunt(X,Y) \land aunt(Y,Z) \land Andrea(Z)) \land \exists A(give(X, A, cupcake) \land \neg(Andrea(A)))$

ii $P_1 = \{computer\}$ - $computer$(X) means X is a computer
$P_2 = \{connected\}$ - $connect$(X, Y) means X connected to Y
$\exists X \forall Y (computer(X) \wedge computer(Y) \wedge connected(X, Y) \wedge \neg connected(X, X))$

iii $C = \{Paul\ Klee,\ Kandinsky\}$
$P_1 = \{room\}$ - $room$(X) means X is a room in a British gallery
$P_2 = \{painting,\ hangs\}$ - $painting$(X, Y) means X is a painting by Y, $hangs$(X, Y) means X hangs in location Y
$\forall X \exists Y (painting(X, Paul\ Klee) \wedge hangs(X, room(Y)) \rightarrow \exists Z (painting(Z, Kadinsky) \rightarrow hangs(Z, Y)$

iv $P_2 = \{loves\}$ - $loves$(X, Y) means X loves to Y
$\exists X \neg \forall Y (loves(X, Y) \rightarrow \neg loves(\neg Y, X)$

7 Question 7

Let L be a signature containing just four unary predicate symbols b, w, s and c, and a single binary relation symbol a; and three constants j, k and l. Consider the L-structure (D, ϕ) in the coursework specs. Further:
$\phi(b)$ is the set of filled ('black') objects
$\phi(w)$ is the set of unfilled ('white') objects
$\phi(s)$ is the set of square objects
$\phi(c)$ is the set of circular objects.
$\phi(a)$ is the set of pairs (x, y) such that there is a directed arrow from x to y
Determine, for each of the following, whether it is true or false, and provide a justification in each case.

i $\forall X (a(k, X) \rightarrow \neg(X = j))$

ii $c(l) \rightarrow \exists X (b(X) \wedge c(X) \wedge a(l, X))$

iii $\exists X \neg \exists Y (\neg(X = Y) \wedge a(X, Y))$

iv $\forall X (\neg s(X) \rightarrow \exists Y (c(Y) \wedge b(Y) \wedge a(X, Y)))$

v $\forall X (\exists Y (\neg(X = Y) \wedge a(X, Y)) \rightarrow \exists Y (a(X, Y) \wedge a(Y, X)))$

vi $\forall X \forall Y (a(X, j) \wedge a(Y, j) \rightarrow (a(X, Y) \vee a(Y, X)))$

7.1 Answers to Q7

i False. There is one directed arrow that goes from object k to object j. This is the only directed arrow from k. Therefore, the statement "for all objects X that receive an arrow from k, those objects X are not j" is false.

ii True. The statement says that if there is an object l that is a circle, then there is some circular black object X that receives an arrow from l. Looking at the diagram, we see that this is accurate and that the indicated black circular object X is equal to object j.

iii True. There exists an X such that it is false that X is not some object Y and X, Y are connected. This states that there must be an object that is connected to itself. This is the case, namely the black square in the upper corner of the diagram.

iv False. The sentence states that, for all circular (non-square) objects X, there is some black circular object Y that receives a line from X. This holds true for circles l and k which send arrows to black circle j, but j only sends an arrow to object l which is a white circle, not a black circle.

v False. The sentence states that, for all objects X where an arrow goes from X to object Y and Y is not the same as the object X, then there is an arrow back from the object Y to the object X. This is false. Object k for example is connected to object j with a directed arrow and j is not the same as k, but there is no arrow back from j to k.

vi False. If there is an arrow from object X to j and an arrow from object Y to j, then those two are either connected with an arrow from X to Y or from Y to X. This holds true for objects k and l in the diagram. Both are connected to j and there is an arrow from l to k. However, it could also be the case that X = Y - this is not excluded. In this case, l needs to be connected to l and k needs to be connected to k. This isn't the case, there are no connections from k to itself or l to itself. Therefore the evaluation results in false.