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FAO: Craven, Robert (rac101)

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Introduction to Symbolic AI
Coursework 1: Logic

① (i) p: Michael is fulfilled
q: Michael is rich
r: Michael will live another 5 years

$((\neg(p \vee q)) \rightarrow (\neg r))$ ← strict def

unambiguous abreviation
$\neg(p \vee q) \rightarrow \neg r$

(ii) p: the snowstorm arrives
q: Raheen will wear his boots
r: I'm sure it will arrive

$(((\neg p) \vee q) \wedge r)$

(iii) p: Akira is on set
q: Toshiro is on set
r: Filming will begin
s: The caterers have cleared out

$(((p \wedge q) \rightarrow r) \leftrightarrow s)$

(iv) p: Irad arrived
q: Sarah arrived

$((p \vee (\neg q)) \wedge (\neg(p \wedge (\neg q))))$

(v) p: Herbert heard the performance
q: Ann-Sophie heard the performance
r: Ann-Sophie answered her phone calls

$((\neg r) \rightarrow (\neg(p \wedge q)))$

② (i) A propositional formula A is satisfiable if there is some v such that $h_v(A) = t$

(ii) Two propositional formulas A and B are logically equivalent if, for every v, $h_v(A) = h_v(B)$

(iii) <u>Step 1</u> Assume $\neg A$ is satisfiable. Want to show that $\neg\neg A \not\equiv T$

There is some v s.t $h_v(\neg A) = t$ (by definition of satisfiability). Then by the meaning of $\neg$, $h_v(\neg\neg A) = f$ (for given v). Then ~~for given~~ $\neg\neg A \not\equiv T$ [Since there is a v, s.t $h_v(\neg\neg A) = f$ but $h_v(T) = t$ (since $h_v(T)$ is always true)]

<u>Step 2</u> Assume $\neg\neg A \not\equiv T$. Want to show $\neg A$ is satisfiable.

$\neg\neg A \not\equiv T$, then there is some v s.t $h_v(\neg\neg A) \neq h_v(T)$. Since $h_v(T)$ is alway true, there is some v s.t $h_v(\neg\neg A) = f$. Then by meaning of $\neg$, for given v, $h_v(\neg A) = t$. So $\neg A$ is satisfiable since there exists a v s.t $h_v(\neg A) = t$

$\square$

③

p	q	r	p	$\wedge$	$\neg q$	$\Leftrightarrow$	$\neg$	$(\neg r$	$\vee$	$\neg p)$	$\rightarrow$	$\neg\neg q$	$\rightarrow$	r
t	t	t	t	f	f	f	t	f	f	f	t	t	t	t
t	t	f	t	f	f	t	f	t	t	f	f	t	f	f
t	f	t	t	t	t	t	t	f	f	f	t	f	t	t
t	f	f	t	t	t	f	f	t	t	f	t	f	t	f
f	f	f	f	f	t	t	f	t	t	t	t	f	t	f
f	f	t	f	f	t	t	f	f	t	t	t	f	t	t
f	t	f	f	f	f	t	f	t	t	t	f	t	f	f
f	t	t	f	f	f	t	f	f	t	t	t	t	t	t
											main			

It is NOT valid, since it's not true for all valuations v.

④ (i) a) CNF
b) CNF & DNF
c) Neither
d) CNF & DNF
e) DNF
f) Neither, since $\neg\neg p$ is not a literal
g) CNF & DNF
h) DNF & CNF

(ii) Let S be in CNF. $S \vdash_{res} (PL)\ \emptyset$ iff $S \models \bot$.
It's an important property because it ~~[illegible]~~ shows that S is satisfiable iff $S \nvdash res_{(PL)}\ \emptyset$. S is satisfiable if it's impossible to derive the empty set from S. It gives a good method to check the satisfiability of S

(iii) a) $\{\{p,s\},\{q,r\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\}$
$\Rightarrow \{\{p,s\},\{\neg p,\neg r,\neg s\}\}$ [q was pure]
$\Rightarrow \{\{p,s\}\}$ [$\neg r$ was pure]
$\Rightarrow \{\ \}$ [p was pure]
This is is not the empty set.

b) $\{\{\neg p,q,r\},\{\neg q\},\{p,r,q\},\{\neg r,q\}\}$
$\Rightarrow \{\{\neg p,r\},\{p,r\},\{\neg r\}\}$ [unit propagation by unit clause $\{\neg q\}$]
$\Rightarrow \{\{\neg p\},\{p\}\}$ [unit propagation by unit clause $\{\neg r\}$]
$\Rightarrow \{\{\}\}$ [unit propagation by unit clause $\{p\}$]
$\Rightarrow$ unsatisfiable

5. p : I am going

q : you are going

r : Tara is going

Formalize as: $p \to \neg q, \neg q \to \neg r, r \vee \neg p, \neg p \to r \models q$

The above is valid iff $(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (\neg p \to r) \wedge \neg q$ is ~~(satisfiable)~~ unsatisfiable

In CNF: $\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{p, r\}, \{\neg q\}\}$

$\Rightarrow \{\{\neg r\}, \{r, \neg p\}, \{p, r\}\}$ [unit propagation by unit clause $\{\neg q\}$]

$\Rightarrow \{\{\neg p\}, \{p\}\}$ [unit propagation by unit clause $\{\neg r\}$]

$\Rightarrow \{\{\}\}$ [unit propagation by unit clause $\{p\}$]

$\Rightarrow$ unsatisfiable

Since ~~the CNF~~ it's unsatisfiable, the argument is $\boxed{\text{Valid}}$

6. (i) $aunt(X) \in \mathcal{F}_1$ (maps to X's aunt)

$cupcake(X) \in \mathcal{P}_1$ ('X is a cupcake')

'Andrea' $\in C$

$give(X, Y, Z) \in \mathcal{P}_3$ ('X gives Y to Z')

$C = \{\text{'Andrea}\}$

$\mathcal{P}_1 = \{cupcake\}$

$\mathcal{P}_3 = \{give\}$

$\forall X\,(X = aunt(aunt(Andrea)) \to \exists Y \exists Z((Y \neq Andrea) \wedge cupcake(Z)$

$\forall X(X = aunt(aunt(Andrea)) \to \exists Y \exists Z((Y \neq Andrea) \wedge cupcake(Z) \wedge give(X, Z, Y)))$

(ii) $computer(X) \in \mathcal{P}_1$ ('X is a computer')

$connected(X, Y) \in \mathcal{P}_2$ ('X is connected to Y')

$\mathcal{P}_1 = \{computer\}$

$\mathcal{P}_2 = \{connected\}$

$\forall X\,(computer(X) \wedge \neg connected(X, X) \to \exists Y(computer(Y) \wedge connected(Y, X)))$

(6) (iii) painting $(X) \in P_1$ ('X is a painting')

painted $(X,Y) \in P_2$ ('X was painted by Y')

hang$(X) \in F_1$ ('maps towhere X is hanged')

room $(X) \in P_1$ ('X is a room')

'Paul Klee', 'Kandinsky' $\in C$

BGallery $(X) \in P_1$ ('X is a Brithis library')

inBGallery $(X,Y) \in P_2$ ('X is in Y')

$C = \{$'Paul Klee', 'Kandinsky'$\}$

$P_1 = \{$painting, room, BGallery

$P_2 = \{$painted, inBGallery

$F_1 = \{$hang$\}$

$$\forall X \forall Y (\text{painting}(X) \wedge (\text{painted}(X, \text{'Paul Klee'}) \vee \text{painted}(X, \text{Kandinsky})) \wedge \text{BGallery}(Y) \wedge \text{inBGallery}(X,Y))$$
$$\rightarrow \exists Z (\text{room}(Z) \wedge \text{inBGallery}(Z,Y) \wedge Z = \text{hang}(X)))$$

(iv) loves $(X,Y) \in P_2$ ('X loves Y') $\quad P_2 = \{$loves$\}$

$$(\exists X (\neg \exists Y (\text{loves}(X,Y)))) \rightarrow \neg (\forall X \exists Y (\text{loves}(X,Y)))$$

7 (i) $\boxed{\text{False}}$ Since $a(k,x)$ is true for $\sigma(X)=j$, it is not true that for all X, if $a(k,x)$ then $X \neq j$

(ii) $\boxed{\text{True}}$ $c(l)$ is true, since l is a circle

$\exists X(b(X) \wedge c(X) \wedge a(l,X))$, for this to be true, there has to be at least 1 object s.t the object is black, a circle, and there is a directed arrow from l to the object. Let that object be $\phi(j)$, so the statement is true

Then the full statement is true.

(iii) $\boxed{\text{True}}$ Let $\sigma(X)$ = black square, then there is not an object $\sigma(Y)$ s.t $\sigma(Y) \neq \sigma(X)$ and $a(\sigma(X), \sigma(Y))$, since the black square only has an arrow to itself

(iv) $\boxed{\text{False}}$ As a counterexample let $\sigma(X)=j$, then j is not a square but there isn't an object $\sigma(Y)$ s.t $\sigma(Y)$ is black, is a circle, and j has a direct arrow to such an object

(v) $\boxed{\text{False}}$ Let $X = \phi(k)$, then there exists $\sigma(Y)$ s.t $X \neq \sigma(Y)$ and $a(X, \sigma(Y))$ (let $\sigma(Y)=j$), but it doesn't exist a $\sigma(Y)$ s.t $a(k, \sigma(Y))$ and $a(\sigma(Y), k)$

(vi) $\boxed{\text{False}}$ As a counterexample, let $\sigma(X)=l$ and $\sigma(Y)=l$, then $a(l,j) \wedge a(l,j)$ is true, but $a(l,l) \vee a(l,l)$ is false (since l doesn't have an arrow to itself)