Mathematical Models and Methods in Applied Sciences
(2020)

DOI: 10.1142/S0218202520020017

Active particles methods and challenges in behavioral systems

N. Bellomo*

Departamento de Matemática Aplicada,
University of Granada, Spain

Politecnico of Torino and IMATI CNR Pavia, Italy
nicola.bellomo@polito.it, bellomo@imati.cnr.it

F. Brezzi

IMATI - CNR, Via A. Ferrata 5/A, 27100 Pavia, Italy
brezzi@imati.cnr.it

J. Soler

Departamento de Matemática Aplicada,
University of Granada, Research Unit "Modeling Nature" (MNat),
18071 Granada, Spain
jsoler@ugr.es

Received 19 January 2020
Accepted 24 January 2020
Published 10 March 2020

This paper first provides an introduction to the mathematical approach to the modeling, qualitative analysis, and simulation of large systems of living entities, specifically self-propelled particles. Subsequently, a presentation of the papers published in this special issue follows. Finally, a critical analysis of the overall contents of the issue is proposed, thus leading to define some challenging research perspectives.

Keywords: Complexity; active particles; swarms; nonlinear interactions; immune competition; control theory; exotic applications.

AMS Subject Classification: 35B40, 35L60, 35K55, 35Q70, 35Q91, 35Q92

1. Introduction

This special issue is devoted to research contributions to the modeling, qualitative analysis, and simulation of large systems of self-propelled particles viewed as living entities carriers of social states. This topic is one of the key streams of the journal *Mathematical Models and Methods in the Applied Sciences* as documented in its recent special issues that have been devoted to closely related topics.[9,10] Indeed, the presentation will also refer to previously published issues with the aim of testing how this research activity has progressed over time.

*Corresponding author.

The interest of mathematicians in this specific field of research is growing rapidly due to the joint action of the stimuli exerted by highly challenging problems and by the related interest in applications that have, in several cases, a significant impact on safety and wellbeing of society. Applications are everywhere, as reported for instance in Ref. 7, where the complexity features of living systems are enlightened referring to a variety of case studies from vehicular traffic to networks.

An additional important feature to consider in the modeling of living systems is the fact that these systems operate far from equilibrium, as critically analyzed in Ref. 6 devoted to the applications of the methods of kinetic theory in biology.

As shown in the previous special issues,[9,10] different mathematical tools can be developed to describe collective behaviors from the underlying dynamics of a few entities. For instance, non Maxwellian kinetic theory,[31] the kinetic theory for active particles that combines theoretical tools of stochastic game theory,[13] and the classical theory of swarms which, after the pioneering paper by Cucker and Smale,[16] has been developed by various authors.[2]

All methods can be referred to the framework of the so-called *active particles approach* that is currently used to denote a dynamic that goes somehow beyond the classical deterministic interactions, while the rules of behavior appear in entities that interact dynamically in large living systems.

The contents are presented in Sec. 2, which outlines the contributions of the papers published in the issue that covers a variety of topics, specifically various analytic problems arising for post-Cucker–Smale models, control theory methods associated with swarm problems for dispersed systems, social dynamics focusing on wealth distribution, and dynamics of multicellular systems focusing on the immune competition. Finally, Sec. 3 is devoted to research perspectives drawn from a critical analysis of the contents of this special issue.

2. Presentation of the Special Issue

This section provides a brief description of the papers published in this special issue that are focused on the modeling of a variety of living systems and on the mathematical tools to be used in their qualitative and quantitative studies. The presentation refers to the literature appeared in the preceding issues with the aim of enlightening the evolution of the specific topic treated in each paper. First, we focus on applications in life sciences and social sciences, and subsequently the presentation goes to the theory of swarms.

An approach from kinetic theory methods to modeling a specific feature of the immune competition between tumor and immune cells is treated in Ref. 19. This paper focuses on the dynamics of macrophages with a continuum of diverse functional states, ranging from pro-tumor to anti-tumor states. The interaction is with tumor cells having a variety of progression (i.e. mutation) states. Therefore, the authors consider heterogeneity of the biological function expressed by cells, namely progression for cancer cells and clonal heterogeneity, as well as phenotypic heterogeneity activated by interactions. In this case, the approach that naturally accounts for the aforementioned features is that of the kinetic theory of active particles.[13]

The dynamics of social systems is reviewed and critically analyzed in Ref. 22 based on a Fokker–Plank approach specifically developed in the study of wealth distribution. The analysis is based on tools reviewed in Ref. 21 that are focused on opinion formation, as well as on the appearance of Log-Normal distributions in various phenomena of economics and social sciences.[23] The interested reader can find in Ref. 31 a presentation of the mathematical framework for the tools of the kinetic theory approach as well as a variety of applications.

Control problems for multiagent systems are studied in Ref. 27, in which the authors analyze a herding problem for two interacting populations, where the drivers try to steer the evaders' trajectories while the evaders always move away from the drivers. The authors first prove the well posedness and the long-time behavior of the one-driver and one-evader model under the assumption that the friction coefficients coincide, subsequently the model is extended to the multi-driver and multi-evader case, while numerical simulations explore the nature of controlled dynamics in various scenarios. This paper opens research perspectives to the study of control problems on swarms undergoing complex interactions as it happens, for instance, in predators–prey systems,[18] see also Ref. 20. An additional field of applications can be identified in the dynamics of crowds, where this type of complex interactions appear.[2]

The following two papers are devoted to the qualitative analysis of descriptive properties of swarms models. This is a topic that has been intensively treated in preceding special issues on active particle methods in this journal, see Refs. 8, 25, 28 and 33. More specifically, a flocking model, including a stochastic Justh–Krishnaprasad type model, to describe the interactions among individual entities in a two-dimensional space is proposed in Ref. 24. The authors first show, using a classical Lyapunov functional approach, that the nematic alignment state is nonlinearly stable. Subsequently, the stability properties are investigated in the case of turning off of the noise.

The effect of linear diffusion and the interactions with the boundary of the domain is studied on swarm equilibria by a detailed analysis of the critical points of the associated energy functional.[29] The achievements of the qualitative analysis are: (1) unboundedness from below of the energy due to an imbalance between diffusive and aggregative forces depending explicitly on a certain volume filling property of the domain and (2) metastable mass translation that occurs in domains under certain symmetry properties. The authors also develop some numerical tools to compute critical points of the energy.

3. From the Special Issue to Research Perspectives

The papers of this special issue have the additional relevance of motivating a wide variety of research perspectives. Some of these perspectives have been illustrated by the authors of these papers. We know that some of them are already the subject of a research plan. We would like to raise, in the hope of doing a useful service, five key questions, mainly focused on aspects of modeling, related to the four types of documents that appear in the special issue. The questions will be followed by some addressing to the bibliography.

• *How Darwinist mutations and selection characterize the immune competition?*

It is worth mentioning that recent studies on the immune therapy (which was initiated, as reported in Ref. 19, in the last years of the past century) have been motivated and revitalized by the achievements by the Nobel laureates James Allison and Tasuku Honjo.[5,26]

The complex dynamics of the immune competition in various biological events is presented in Ref. 30 referred to the foundations of immunology. This "on line" book is periodically updated. The dynamics depends on the complex dialogue between cells within a collective learning dynamics.

• *Can models of social dynamics account for unethical behaviors?*

A recent literature has enlightened how the dynamics of social and economic systems can be influenced by unethical behaviors which can have a remarkable influence on the collective behaviors.[32,34] This relevant aspect of the modeling approach is referred to mathematical tools of active particles as well as to applications in Refs. 1 and 11. Collective learning dynamics contributes to the propagation of different social behaviors.[15]

• *How the classical Cucker–Smale approach can go beyond the modeling of animal swarms, say toward exotic applications?*

A hint toward the study of this type of problems is given in Ref. 14, where, as remarked in Sec. 2, the authors introduce a complex individual-based dynamics where interactions include learning and repulsion dynamics. Useful concepts on interaction dynamics related to coordinations are given in Ref. 17. Complex interaction dynamics have been widely treated in kinetic theory model, for instance in crowd dynamics[2] even accounting for the role of social variables in the modeling of the interaction rules.[12] Some general concepts might be transferred to "*exotic*" applications.

• *How far the approach to mathematical control problems can be addressed to exotic applications and multiscale problems?*

Let us make precise this question referring to the contents of this special issue. What we have called exotic applications have been treated in this special issue in Refs. 19 and 22, while the mathematical control problem treated in Ref. 27 for swarms (where interactions mimic a hiding chasing dynamics) provides new ideas to modeling crowd dynamics where complex social interactions can appear.[3,12]

• *How collective learning dynamics can affect the modeling approach?*

It can be remarked that all case studies in this issue refer to a dynamics where the strategy expressed by the interacting entities, namely the active particles, is generated by the dialogue developed among them in various ways. Therefore, collective learning dynamics appears to be as the first step in the development of the interactive dynamics of living systems.

Arguably the topics emerging from the five key questions that have been posed above might capture the attention of mathematicians involved in the study of behavioral systems. And each topic might be viewed as a research perspective worth to be developed.

References

1. G. Ajmone Marsan, N. Bellomo and L. Gibelli, Stochastic evolutionary differential games toward a systems theory of behavioral social dynamics, *Math. Models Methods Appl. Sci.* **26** (2016) 1051–1093.
2. G. Albi, N. Bellomo, L. Fermo, S.-Y. Ha, J. Kim, L. Pareschi, D. Poyato and J. Soler, Traffic, crowds and swarms. From kinetic theory and multiscale methods to applications and research perspectives, *Math. Models Methods Appl. Sci.* **29** (2019) 1901–2005.
3. G. Albi, M. Bongini, F. Rossi and F. Solombrino, Leader formation with mean-field birth and death models, *Math. Models Methods Appl. Sci.* **29** (2019) 633–679.
4. G. Albi, Y.-P. Choi and A.-S. Häck, Pressureless Euler alignment system with control, *Math. Models Methods Appl. Sci.* **28** (2018) 1635–1664.
5. J. P. Allison, Nobel lecture, immune checkpoint blockade in cancer therapy: New insights, opportunities and prospects for cures (2019), https://www.nobelprize.org/prizes/medicine/2018/allison/lecture/.
6. V. V. Aristov, Biological systems as nonequilibrium structures described by kinetic methods, *Results in Phys.* **13** (2019), Article no. 102232.
7. P. Ball *Why Society is a Complex Matter* (Springer-Verlag, 2012).
8. M. Bastan and J. A. Carillo, Reduced fluid models for self-propelled particles interacting through alignment, *Math. Models Methods Appl. Sci.* **27** (2017) 1255–1290.
9. N. Bellomo and F. Brezzi, Challenges in active particles methods: Theory and applications, *Math. Models Methods Appl. Sci.* **28** (2018) 1627–1633.
10. N. Bellomo and F. Brezzi, Towards a multiscale vision of active particles, *Math. Models Methods Appl. Sci.* **29** (2019) 581–588.
11. N. Bellomo, F. Colasuonno, D. Knopoff and J. Soler, From a systems theory of sociology to modeling the onset and evolution of criminality, *Netw. Heterogeneous Media* **10** (2015) 421–441.
12. N. Bellomo, L. Gibelli and N. Outada, On the interplay between behavioral dynamics and social interactions in human crowds, *Kinetic Relat. Models* **12** (2019) 397–409.
13. N. Bellomo, D. Knopoff and J. Soler, On the difficult interplay between life "complexity" and mathematical sciences, *Math. Models Methods Appl. Sci.* **23** (2013) 1861–1913.
14. U. Biccari, D. Ko and E. Zuazua, Dynamics and control for multiagent networked systems: A finite difference PDE approach, *Math. Models Methods Appl. Sci.* **29** (2019) 755–790.

15. D. Burini and S. De Lillo, On the complex interaction between collective learning and social dynamics, *Symmetry* **11**, 967 (2019), doi:10.3390/sym11080967.
16. F. Cucker and S. Smale, Emergent behavior in flocks, *IEEE Trans. Autom. Control* **52** (2007) 852–862.
17. P. Degond, A. Frouvelle and S. Merino-Aceituno, A new flocking model through body attitude coordination, *Math. Models Methods Appl. Sci.* **27** (2017) 1005–1049.
18. M. Di Francesco and S. Fagioli, A nonlocal swarm model for predators-prey interactions, *Math. Models Methods Appl. Sci.* **26** (2016) 319–355.
19. R. Eftimie and L. Gibelli, A kinetic theory approach for modelling tumour and macrophages heterogeneity and plasticity during cancer progression, *Math. Models Methods Appl. Sci.* **30** (2020), doi:10.1142/S0218202520400011.
20. S. Fagioli and E. Radici, Solutions to aggregation-diffusion equations with nonlinear mobility constructed via a deterministic particle approximation, *Math. Models Methods Appl. Sci.* **28** (2018) 1801–1829.
21. G. Furioli, A. Pulvirenti, E. Terraneo and G. Toscani, Fokker–Planck equations in the modeling of socio-economic phenomena, *Math. Models Methods Appl. Sci.* **27** (2017) 115–158.
22. G. Furioli, A. Pulvirenti, E. Terraneo and G. Toscani, Non-Maxwellian kinetic equations modeling the dynamics of wealth distribution, *Math. Models Methods Appl. Sci.* **30** (2020), doi:10.1142/S0218202520400023.
23. S. Gualandi and G. Toscani, Human behavior and lognormal distribution. A kinetic description, *Math. Models Methods Appl. Sci.* **29** (2019) 717–753.
24. S.-Y. Ha, D. Ko, W. Shim and H. Yu, Stochastic persistency of nematic alignment state for the Justh–Krishnaprasad model with additive white noises, *Math. Models Methods Appl. Sci.* **30** (2020), doi:10.1142/S0218202520400035.
25. S.-Y. Ha and X. Zhang, Uniform-in-time transition from discrete dynamics to continuous dynamics in the Cucker–Smale flocking, *Math. Models Methods Appl. Sci.* **28** (2018) 1699–1735.
26. T. Honjo, Nobel lecture, serendipities of acquired immunity (2019), https://www.youtube.com/watch?v=9gx05bZIMc8.
27. D. Ko and E. Zuazua, Asymptotic behavior and control of a guidance by a repulsion model, *Math. Models Methods Appl. Sci.* **30** (2020), doi:10.1142/S0218202520400047.
28. M. Lachowicz, H. Leszczynski and M. Parisot, Blow-up and global existence for a kinetic equation of swarm formation, *Math. Models Methods Appl. Sci.* **27** (2017) 1153–1175, doi:10.1142/S0218202520400059.
29. D. Messenger and R. C. Fetecau, Equilibria of an aggregation model with linear diffusion in domains with boundaries, *Math. Models Methods Appl. Sci.* **30** (2020).
30. P. Musiani and G. Forni, *Basic Immunology 2019* (Issue, 2019), https://issue.com/guidoforni5/docs/2019o.
31. L. Pareschi and G. Toscani, *Interacting Multiagent Systems: Kinetic Equations and Monte Carlo Methods* (Oxford Univ. Press, 2013).
32. P. K. Piff, D. M. Stancato, S. Coté, R. Mendoza-Denton and D. Keltner, Higher social class predicts increased unethical behavior, *Proc. Nat. Acad. Sci.* **109** (2014) 4086–4091.
33. D. Poyato and J. Soler, Euler-type equations and commutators in singular and hyperbolic limits of kinetic Cucker–Smale models, *Math. Models Methods Appl. Sci.* **27** (2017) 1089–1152.
34. S. Salvi, Corruption corrupts: Society-level rule violations affect individuals' intrinsic honesty, *Nature* **531** (2016) 456–457.