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Imperial College London

Department of Computing

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Exercise Information

Module: 70051 Introduction to Symbolic Artificial Intelligence (MSc AI)
Exercise: 2 (CW)
Title: Logic
FAO: Craven, Robert (rac101)

Issued: Tue - 20 Oct 2020
Due: Tue - 03 Nov 2020
Assessment: Individual
Submission: Electronic

Student Declaration - Version 1

- I acknowledge the following people for help through our original discussions:

Salomon Lupo

Signed: (electronic signature) Date: 2020-10-30 16:23:40

For Markers only: (circle appropriate grade)

LUPO, Salomon (sl10918)	01595466	t5	2020-10-30 16:23:40	**A* A B C D E F**

Introduction to Symbolic AI

Coursework 1: Logic

Total possible: 72 marks.

1. [**10 marks**] Formalize each of the following in propositional logic, including all brackets required by the *strict* definition of a propositional formula (remember to give the correspondence between the basic sentences of the original and the propositional atoms):

   i. If Michel isn't either fulfilled or rich, he won't live another five years.

   ii. Unless the snowstorm doesn't arrive, Raheem will wear his boots; but I'm sure it *will* arrive.

   iii. If Akira and Toshiro are on set, then filming will begin if and only if the caterers have cleared out.

   iv. Either Irad arrived, or Sarah didn't: but not both!

   v. It's not the case both that Herbert heard the performance and Anne-Sophie did, if the latter didn't answer her phone calls.

2. [**8 marks**]

   i. What is the definition of the *satisfiability* of a propositional formula, A?

   ii. What is the definition of the *logical equivalence* of two propositional formulas A and B?

   iii. Prove that a propositional formula $\neg A$ is satisfiable iff $\neg\neg A \not\equiv \top$ (i.e., iff it's not the case that $\neg\neg A \equiv \top$).

3. [**8 marks**] Use truth-tables to determine whether the following is valid or not:
   $(p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \rightarrow (\neg\neg q \rightarrow r)$.

4. [**14 marks**]

   i. Which of the following are in CNF? Which are in DNF?

      a. $p \wedge (\neg q \vee r)$
      b. $\neg p$
      c. $p \wedge (q \vee (p \wedge r))$
      d. $\top$
      e. $(p \wedge q) \vee (p \wedge q)$
      f. $\neg\neg p \wedge (q \vee p)$
      g. $p \wedge q$
      h. $p \vee q$

   ii. Define the property of the *refutation-soundness and -completeness* of a resolution derivation. Why is this property important?

   iii. Apply unit propagation and the pure rule repeatedly, in order to reduce the following to their simplist forms (stating which rule you're applying, and indicate the literal involved):

      a. $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$
      b. $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

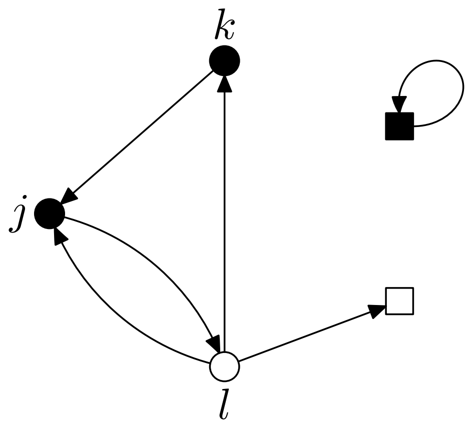
5. [**8 marks**] Use DP to determine whether the following argument is valid or not:

   If I'm going, then you aren't.
   If you're not going, then neither is Tara.
   Either Tara's going or I'm not.
   Tara's going unless I am.
   So, you're going.

6. [**12 marks**] Translate into first-order logic, giving as much logical structure as possible. Be sure to specify the signature for each part.

   i. All of Andrea's aunts' aunts gave a cupcake to someone other than Andrea.
   ii. There's a computer connected to every computer which isn't connected to itself.
   iii. Any painting by Paul Klee in a British gallery hangs in a room where all Kandinsky paintings in that gallery hang.
   iv. If there's somebody who loves nobody, then it's false that everybody loves somebody.

7. [**12 marks**] Let $\mathcal{L}$ be a signature containing just four unary predicate symbols b, w, s and c, and a single binary relation symbol a; and three constants j, k and l. Consider the following $\mathcal{L}$-structure (D, φ), containing five objects:



The objects $\varphi(j)$, $\varphi(k)$ and $\varphi(l)$ are shown (indicated by the relevant letters). Further:

$\varphi(b)$ is the set of filled ('black') objects
$\varphi(w)$ is the set of unfilled ('white') objects
$\varphi(s)$ is the set of square objects
$\varphi(c)$ is the set of circular objects.
$\varphi(a)$ is the set of pairs (x, y) such that there is a directed arrow from x to y

For example, the object $\varphi(k)$, to the top in the diagram, is in $\varphi(b)$ and $\varphi(c)$, since it is drawn filled and circular.

Determine, for each of the following, whether it is true or false, and provide a justification in each case.

   i. $\forall X(a(k, X) \to \neg(X = j))$
   ii. $c(l) \to \exists X(b(X) \wedge c(X) \wedge a(l, X))$
   iii. $\exists X \neg \exists Y(\neg(X = Y) \wedge a(X, Y))$
   iv. $\forall X(\neg s(X) \to \exists Y(c(Y) \wedge b(Y) \wedge a(X, Y)))$
   v. $\forall X(\exists Y(\neg(X = Y) \wedge a(X, Y)) \to \exists Y(a(X, Y) \wedge a(Y, X)))$
   vi. $\forall X \forall Y(a(X, j) \wedge a(Y, j) \to (a(X, Y) \vee a(Y, X)))$

1. [**10 marks**] Formalize each of the following in propositional logic, including all brackets required by the *strict* definition of a propositional formula (remember to give the correspondence between the basic sentences of the original and the propositional atoms):

   ? i. If Michel isn't either fulfilled or rich, he won't live another five years.

   ii. Unless the snowstorm doesn't arrive, Raheem will wear his boots; but I'm sure it *will* arrive.

   ? iii. If Akira and Toshiro are on set, then filming will begin if and only if the caterers have cleared out.

   ? iv. Either Irad arrived, or Sarah didn't: but not both!

   v. It's not the case both that Herbert heard the performance and Anne-Sophie did, if the latter didn't answer her phone calls.

i) p = Michel is fulfilled
q = Michel is rich
r = Michel will live another five years

$$\boxed{((\neg(p \lor q)) \to (\neg r))}$$

ii) p = the snowstorm arrives
q = Raheem will wear his boots
r = I'm sure it will arrive

$$\boxed{(((\neg p) \lor q) \land r)}$$

$\neg p \lor q$

$\parallel$

$p \to q$

$((p \to q) \land r)$

iii) p = Akira is on set q = Toshiro is on set
r = filming will begin s = the caterers have cleared out.

$((p \wedge q) \rightarrow (r \leftrightarrow s))$

iv) p = Fred answered   q = Sarah did (answer)

$((p \vee (\neg q)) \wedge (\neg(p \wedge q)))$

v) p = Herbert heard the performance
q = Anne-Sophie did (hear the performance)
r = The latter did answer her phone calls

$((\neg r) \rightarrow (\neg(p \wedge q)))$

2. [**8 marks**]

i. What is the definition of the *satisfiability* of a propositional formula, A?
ii. What is the definition of the *logical equivalence* of two propositional formulas A and B?
iii. Prove that a propositional formula $\neg A$ is satisfiable iff $\neg\neg A \not\equiv \top$ (i.e., iff it's not the case that $\neg\neg A \equiv \top$).

i) A is satisfiable if there exists a valuation v for which the propositional evaluation $h_v(A) = t$

ii) two propositional formulas A and B are equivalent if for every valuation v $h_v(A) = h_v(B)$

iii) Assume $\neg A$ is satisfiable then there exists a valuation v^* s.t. $h_{v^*}(\neg A) = t$ $\rightarrow$ $h_{v^*}(\neg\neg A) = f$ thus $\neg\neg A$ can be false and $\neg\neg A \not\equiv \top$

assume $\neg\neg \not\equiv T$ then there exists at least 1 valuation v^+ s.t $h_{v^+}(\neg\neg A) = f$ $\Rightarrow$ $h_{v^+}(\neg A) = t$ thus $\neg A$ is satisfiable

3. [**8 marks**] Use truth-tables to determine whether the following is valid or not: $(p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \to (\neg\neg q \to r)$.

- $(\neg\neg q \to r) \equiv (q \to r)$
- $\neg(\neg r \vee \neg p) \equiv (r \wedge p)$

$\therefore (p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \to (\neg\neg q \to r)$

$\equiv (p \wedge \neg q \leftrightarrow r \wedge p) \to (q \to r)$

p	q	r	$(p \wedge \neg q \leftrightarrow r \wedge p) \to (q \to r)$
t	t	t	t
t	t	f	f
t	f	t	t
t	f	f	t
f	t	t	t
f	t	f	f
f	f	t	t
f	f	f	t

Not a valid argument as the proposition is false when $p = f$, $q = t$, $r = f$

4. [**14 marks**]

i. Which of the following are in CNF? Which are in DNF?

a. $p \wedge (\neg q \vee r)$
b. $\neg p$
c. $p \wedge (q \vee (p \wedge r))$
d. $\top$
e. $(p \wedge q) \vee (p \wedge q)$
f. $\neg\neg p \wedge (q \vee p)$
g. $p \wedge q$
h. $p \vee q$

ii. Define the property of the *refutation-soundness and -completeness* of a resolution derivation. Why is this property important?

iii. Apply unit propagation and the pure rule repeatedly, in order to reduce the following to their simplist forms (stating which rule you're applying, and indicate the literal involved):

a. $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$
b. $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

i) a) CNF b) CNF & DNF c) neither

d) CNF & DNF e) DNF f) neither

g) CNF & DNF h) CNF & DNF

ii) the refutation-soundness and completeness of a resolution states that if S is a CNF $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$
It is important because it implies that S is satisfiable iff its resolved formula is satisfiable. This is at the basis of many SAT solvers.

iii) a) $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

q is pure $\downarrow$

$\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$

$\neg r$ is pure

$\{\{p, s\}\}$

p and s are pure

$\{\}$

b) $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

$\neg q$ unit propagation

$\{\{\neg p, r\}, \{p, r\}, \{p, r\}, \{\neg r\}\}$

$\neg r$ unit propagation

$\{\{\neg p\}, \{p\}, \{p\}\}$

p unit propagation

$\{\{\}\}$

5. [**8 marks**] Use DP to determine whether the following argument is valid or not:

> If I'm going, then you aren't.
> If you're not going, then neither is Tara.
> Either Tara's going or I'm not.
> Tara's going unless I am.
> *So*, you're going.

The argument is in the form

p = I'm going q = you are going

r = Tara is going

$(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (r \vee p) \to q$

We transform it in a CDF by using $A \to B \equiv \neg A \vee B$

$(\neg p \vee q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (r \vee p) \to q$

to prove that the argument is valid we need to prove that the following is unsatisfiable

$\{\{\neg p, q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$

$\neg q$ unit propagation

$\{\{\neg p\}, \{\neg r\}, \{r, \neg p\}, \{r, p\}\}$

$\neg p$ unit propagation

$\{\{\neg r\}, \{r\}\}$

we resolve on r

$\{\{\}\}$

It is valid

we have proved that the argument is unsatisfiable.

6. [**12 marks**] Translate into first-order logic, giving as much logical structure as possible. Be sure to specify the signature for each part.

   i. All of Andrea's aunts' aunts gave a cupcake to someone other than Andrea.
   ii. There's a computer connected to every computer which isn't connected to itself.
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   iv. If there's somebody who loves nobody, then it's false that everybody loves somebody.

i) $C = \{andrea\}$

$P_2 = \{givescupcake, isaunt\}$

$givescupcake(X, Y)$ is read X gives a cupcake to Y

$isaunt(X, Y)$ is read X is Y's aunt

$\forall X \forall Y (isaunt(X, andrea) \wedge isaunt(Y,X) \rightarrow \exists Z\ gueuscupcake(Y,Z) \wedge \neg(Z = andrea))$

ii)

$C = \{\ \} \leftarrow$ computers

$P_2 = \{ areconnected \}$ $\quad$ areconnected(X,Y) reads X is connected to Y

$\exists X\ \forall Y (\neg areconnected(Y,Y) \rightarrow areconnected(X,Y))$

iii)

$C = \{\ \} \leftarrow$ paintings

$P_1 = \{ paulklee, kandinsky, britishgallery \}$

$P_2 = \{ hangs \}$

- paulklee(x) reads x is made by paul klee
- kandinsky(x) reads x is made by kandinsky
- british gallery(x) reads x is in a british gallery
- hangs(x,y) reads x and y are hanged in the same room

$\forall x \forall y (paulklee(x) \wedge britishgallery(x) \wedge kandinsky(y) \wedge hangs(x,y))$

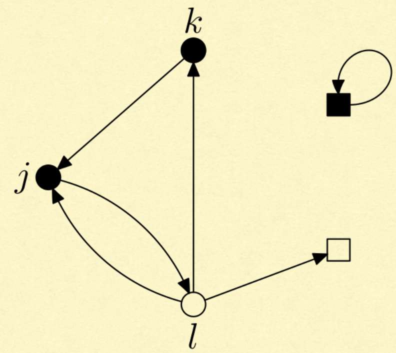
iv)

$C = \{\ \} \leftarrow$ people

$P_2 = \{ loves \}$

$\exists x (\forall y (\neg loves(x,y))) \rightarrow \neg \forall W (\exists Z (loves(W,Z)))$

7. [**12 marks**] Let $\mathcal{L}$ be a signature containing just four unary predicate symbols b, w, s and c, and a single binary relation symbol a; and three constants j, k and l. Consider the following $\mathcal{L}$-structure (D, φ), containing five objects:



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v. $\forall X(\exists Y(\neg(X = Y) \land a(X, Y)) \to \exists Y(a(X, Y) \land a(Y, X)))$
vi. $\forall X \forall Y(a(X, j) \land a(Y, j) \to (a(X, Y) \lor a(Y, X)))$

$C = \{j, k, l\}$

$P_1 = \{b, w, s, c\}$

$P_2 = \{a\}$

i) False: The only object that k points toward is j

thus $a(k, j) \to \neg(j = j)$

$t \to f$

f

ii) True: $c(l)$ is always true so we ignore it

so we are asking if there is an object that l points toward, that is black and circular. There are two j and k

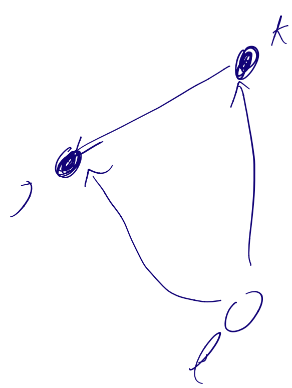
iii) True: it is saying that there exists an object that only points to itself. Yes 

iv) False: it is saying that for all circular objects x there exists another object y that x points towards and it is black and circular.

No: take j. It only points toward l and l is not black

v) False: it is saying that for all objects pointing to something other than itself, that object points to something that points back at it.

vi) True:



for each pair of objects that are both pointing at j at least one of them points at the other