Game Theory has a rich history when considering an understanding of multi-agent systems. These begin in economics but have found a strong application in computation due to the rising need for distributed systems. Game Theory, therefore, branches across all of the categories in this section (although its synergy with swarms is requires significant development) since Dec-POMDP and MARL methods have both used game theory to support their frameworks. In fact, Dec-POMDP is a subset of Partially Observed Stochastic Games (POSG), in which all agents use the same payoff.

0.1 Market Based Methods

Garapati et al. [?] define a market based method as the setting where agents "follow their own interests and establish the mechanism of a market for distributing the tasks". Auctioning is the most widely used sub-field of market approaches and so I will use them interchangeably.

Whilst there are different variants to auctioning, the general procedure is that an auctioneer who has knowledge of a task (or multiple tasks) will set up an auction for said task. Agents can then make bids on these tasks and, once the auction is complete, the highest bid will win the task. In the specific application to robotics, a robot's bid will often reflect the costs, suitability or utility their undertaking the task [?]. This immediately highlights a few points. The first is that the method is not too heavily reliant upon a single processor to determine some joint policy. Tasks are allocated on a case-by-case basis and the utilities are calculated by the agent themselves. The only centralised process is the auctioneer's assessment of the winner which then relays this information back to them. The downside of this is that the system is heavily reliant upon strong communication channels, without which tasks may not be assigned, incorrect utilities may be communicated and, in general, sub-optimal solutions reached. Furthermore, the requirement that the agents themselves determine the cost of their actions assumes that they have the computational capability to do so. Furthermore, the bids placed by each agent need to be a strong representation of their capability to perform a task which may be hard to estimate without expert knowledge. However, market based methods are well suited to explanation through argumentation (similar to [?].

With well chosen payoffs, market based approaches work extremely well. For instance, in [?], the authors show that a free market approach (where agents try to maximise their own profits) can lead to a strong collaborative effort across teams. Similarly, in [?], Thomas et al. apply the auctioning scheme presented in [?] towards a robot construction team. However, it is important to note that these are both passive settings; tasks were assigned before the team were in the field and, in the case of [?], the system would repeat the bidding process if a robot failed. While both show strong performance, it cannot be said that either would be applicable in dangerous environments in which dynamic reassignment must happen within strict time constraints. Stancliff et al. [?] suggest that a more robust method to planning would be to account for failures a priori, a philosophy which is exemplified in [?] who consider the robot's reliability and relevance to a task as well as 'history relevance' which considers the relationship between pairs of robots with the aim of producing more effective teams.

There has also been some interesting work in probabilistic verification of market based approaches. Most notable to me is [?] which considers the case of conflict avoidance. Though their method focuses on collision avoidance, it highlights the need for verification of conflict resolution and goal achievement in market based approaches with different payoff structures. Sirigineedi et al. [?] make a step in this direction by considering the verification of cooperative surveillance along a route network. From my understanding, this means that they were able to verify that their agents were able to traverse along the network without interference.

0.2 Stochastic Games

TODO: For the love of God do this already

0.3 Game Theoretic Control

Game theory can often be applied to problems of control theory (particularly where there are multiple agents) to develop robust controllers which guarantee properties of stability and constraint satisfaction.

This idea is explored in [?]. Here, a zero-sum game is considered in which the players are a controller and an adversarial environment. The design of the controller must be such that it is able to drive the system to zero error. To illustrate, consider the problem of designing a controller for a re-entry vehicle, as in [**?**], in which vortices seek to destabilise the agent. This will allow us to build stable agents in a much more efficient manner since we can simulate the adversarial environment and hypothetical scenarios the agent may encounter without actually encountering them. The same notion is explored by Bardi et al [**?**].

Mylvaganam et al, in [**?**], consider the N-robot collision avoidance problem, similarly from the point of view of differential game theory. They develop a robust feedback system for the robots which they show to be able to drive the system towards predefined targets whilst providing guarantees of interference from other agents (or lack thereof). In [**?**], Mylvaganam also considers a game theoretic control of multi-agent systems in a distributed manner. Here, agents only consider their own payoff structure and have limited communication with one another. The author shows that an approximate equilibrium can be found using algebraic methods and illustrate the capabilities of the technique using a collision avoidance example. For the sake of brevity, I will not include all of the numerous strides that Mylvaganam has introduced to the area. However, I must conclude with those presented in [**?**]. Here, the author presents approximate solutions to a number of differential games, including linear-quadratic differential games (in which system dynamics are linear functions whilst payoff functions are quadratic), Stackelberg differential games, where a hierarchy is induced across the players (a notion was suggested in the research proposal) and mean-field games, which is discussed in 'MARL'. The importance of the linear-quadratic differential game is the stability of the solution; solutions for the Nash equilibria are admissable iff they are locally exponentially stable (which the author often shows with the aid of Lyapunov functions). Approximate solutions to the NE are developed which are more feasible to calculate online. The author then shows that this is not simply a theoretical exercise by applying the novel methods towards multi-agent collision problems and designs dynamic control laws which guarantee that each agent will reach their desired state whilst avoiding collision with the other agents. Similarly, the Stackelberg game is applied to the problem of optimal monitoring by a multi robot system.