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Academic Year **2020-2021**

Page created Tue Nov 3 23:15:04 GMT 2020

CATE

70051 rac101 2

t5 ah820 v1

0 0 1 2 0 0 0 1 4 3 2 5

Electronic submission

Mon - 02 Nov 2020
21:51:44

ah820

Exercise Information

Module: 70051 Introduction to Symbolic Artificial Intelligence (MSc AI)
Exercise: 2 (CW)
Title: Logic
FAO: Craven, Robert (rac101)

Issued: Tue - 20 Oct 2020
Due: Tue - 03 Nov 2020
Assessment: Individual
Submission: Electronic

Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-10-30 16:14:17

For Markers only: (circle appropriate grade)

HLEDIKOVA, Anna (ah820)	01932146	t5	2020-10-30 16:14:17	**A* A B C D E F**

SYMBOLIC AI coursework #1

1 (i) $(\neg(p \vee q) \rightarrow (\neg r))$

p: Michael is fullfilled
q: Michael is rich
r: Michael will live another 5 years

(ii) $(((\neg p) \vee q) \wedge r)$

p: Snowstorm arrives
q: Raheem will wear his boots
r: I'm sure the storm arrives

(iii) $((p \wedge q) \rightarrow (r \leftrightarrow s))$

p: Akira is on set
q: Toshiro is on set
r: filming will begin
S: caterers have cleared out

(iv) $((p \vee (\neg q)) \wedge \neg(p \wedge (\neg q)))$

p: Irad arrived
q: Sara arrived

(v) $((\neg r) \rightarrow \neg(p \wedge q))$

p: Herbert heard the performance
q: Anne-Sophie heard the performance
r: Anne-Sophie answered her phone calls

2 (i) A propositional formula A is satisfiable if there is some v such that $h_v(A) = t$.

(ii) Two propositional formulas A, B are logically equivalent if, for every v, $h_v(A) = h_v(B)$

(iii) First, let's look at direction $\boxed{\Rightarrow}$, i.e. if $\neg A$ is satisfiable then $\neg\neg A \not\equiv T$.
$\neg A$ is satisfiable, so there exists v such that $h_v(\neg A) = t$.
Then, under the same v we have $h_v(\neg\neg A) = f$, which means that $\neg\neg A \not\equiv T$.

$\boxed{\Leftarrow}$, i.e. if $\neg\neg A \not\equiv T$ then $\neg A$ is satisfiable.
We have $\neg\neg A \not\equiv T$, which means that there exists v such that $h_v(\neg\neg A) = f$.
Then, under the same v we have $h_v(\neg A) = t$, and so by definition $\neg A$ is satisfiable. ■

3

p	q	r	(p	∧	¬	q	↔	¬	(¬	r	∨	¬	p))	**→**	(¬	¬	q	→	r)
t	t	t	t	f	f	t	f	t	f	t	f	f	t	**t**	t	f	t	t	t
t	t	f	t	f	f	t	t	f	t	f	t	f	t	**f**	t	f	t	f	f
t	f	t	t	t	t	f	t	t	f	t	f	f	t	**t**	f	t	f	t	t
t	f	f	t	t	t	f	f	f	t	f	t	f	t	**t**	f	t	f	t	f
f	t	t	f	f	f	t	t	f	f	t	t	t	f	**t**	t	f	t	t	t
f	f	t	f	f	t	f	t	f	f	t	t	t	f	**t**	f	t	f	t	t
f	t	f	f	f	f	t	t	f	t	f	t	t	f	**f**	t	f	t	f	f
f	f	f	f	f	t	f	t	f	t	f	t	t	f	**t**	f	t	f	t	f

In order for a formula to be valid, it must be true for any possible evaluation function.
That is clearly not the case based on the truth table,
So the formula is not valid.

4 a. CNF - is a conjunction of clauses
b. satisfies conditions for both CNF & DNF
c. neither, $(p \wedge r)$ is not a clause
d. ~~satisfies conditions for both CNF & DNF~~ neither (based on definition given in lectures)
e. DNF - is a disjunction of conjunctions
f. neither, as $\neg\neg p$ is not a literal
g. DNF
h. CNF

4 continued

(ii) Theorem 2.7: Let S be in CNF. $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$.
Corollary 2.8: Let S be in CNF. S is satisfiable iff $S \nvdash_{res(PL)} \emptyset$

Thm 2.5: If $C = \text{resolvent}(C_1, C_2, P)$ then $\{C_1, C_2\} \models C$.

The above tells us that the resolvent is implied by $\{C_1, C_2\}$ and also that a sentence S is not satisfiable iff we get $\emptyset$ as a resolvent.
This is useful, because any sentence can be transformed into a logically equivalent one in CNF, and thus we can use ~~resolution~~ resolution to determine the satisfiability of any PL sentence.
The same technique can be also used to determine validity and equivalence.

(iii) a. $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$
1. ~~unit~~ pure rule on q: $\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$
2. pure rule on $\neg r$: $\{\{p, s\}\}$
3. pure rule on p: $\{\}$ (satisfiable)

b. $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$
1. unit propagation on $\neg q$: $\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$
2. unit propagation on $\neg r$: $\{\{\neg p\}, \{p\}\}$
3. unit propagation on p: $\{\{\}\}$ (not satisfiable)

5 p: I'm going
q: you're going
r: Tara is going

The statements can be represented in PL as follows: $p \to \neg q$, $\neg q \to r$, $r \vee \neg p$, $\neg r \to p$, q

From the lectures we know that $A_1, \dots, A_n \models B$ is valid iff $(\wedge A_i) \wedge \neg B$ is unsatisfiable.
Thus, we rewrite the above as

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$

1. unit propagation by $\{\neg q\}$ gives $\{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$
2. unit propagation by $\{\neg r\}$ gives $\{\{p\}, \{\neg p\}\}$
3. unit propagation by $\{p\}$ gives $\{\{\}\}$ $\to$ unsatisfiable $\to$ valid

6 (i) $C = \{andrea, cupcake\}$
$P_3 = \{give\}$ → give (X, Y, Z) reads 'X gave Z to Y'
$F_1 = \{aunt\}$ → aunt (X) reads 'X's aunt'

$\exists Y \forall X \; ((X = aunt(aunt(andrea))) \to (give(X, Y, cupcake) \wedge \neg (Y = andrea)))$

(ii) $P_1 = \{computer\}$ → computer (X) reads 'X is a computer'
$P_2 = \{connected\}$ → connected (X, Y) reads 'X and Y are connected'

$\forall X \exists Y \; ((\neg connected(X, X) \wedge computer(X)) \to (connected(X, Y) \wedge computer(Y)))$

(iii) $C = \{paul klee, kandinski\}$
$P_1 = \{gallery, british, room\}$ → gallery(X) reads 'X is a gallery'
british(X) reads 'X is british'
room (X) reads 'X is a room'

$P_2 = \{painting, hangs, in\}$ → painting (X, Y) reads 'X is a painting by Y'
hangs (X, Y) reads 'X hangs in Y'
in (X, Y) reads 'X is in Y'

$\forall X \; \forall Y \; \exists Z ((painting(X, PaulKlee) \wedge in(X, british(gallery(Y))) \to (hang(X, room(Z))$
$\wedge \; \forall W ((painting(W, Kandinski) \wedge in(W, british(gallery(Y))))$
$\to hang(W, room(Z))))$

structure: $A \to (B \to C)$

(iv) $P_2 = \{love\}$ → love (X, Y) reads 'X loves Y'

$\exists X (\forall Y \; \neg loves(X, Y)) \to \neg (\forall W \; \exists Z \; loves(W, Z))$

7 (i) FALSE, since $a(k, X)$ where $X = j$ is true

(ii) TRUE, $c(e)$ is true and for $X = k$ $(b(X) \wedge c(X) \wedge a(e, X))$ is also true

(iii) TRUE, the statement says "there is an X s.t. there is no Y for which $X \neq Y$ and there's an edge from X to Y
■ and □ satisfy this.

(iv) FALSE, for j there is no arrow (edge) from j to a black circle.

(v) FALSE, for example for k this doesn't hold - there is no Y such that $a(k, Y) \wedge \neg a(Y, k)$

(vi) FALSE, if $X = Y = k$ this does not hold.