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CATE

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Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
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Title:	Logic	**Assessment:**	Individual
FAO:	Craven, Robert (rac101)	**Submission:**	Electronic

Student Declaration - Version 1

- I acknowledge the following people for help through our original discussions:

Monika Jotautaite

Signed: (electronic signature) Date: 2020-11-03 02:42:50

For Markers only: (circle appropriate grade)

JOTAUTAITE, Monika (mj720)	01967465	t5	2020-11-03 02:42:50	**A* A B C D E F**

Question 1

Tuesday, November 3, 2020 2:56 AM

1 i)

p - Michael is fulfilled

q - Michael is rich

r - Michael will live for another 5 yrs

$$((\neg(p \vee q)) \rightarrow (\neg r))$$

(ii) p storm arrives

q Raheem will wear his boots

r I'm sure storm arrives

$$((\neg p \vee q) \wedge r)$$

(iii) p - Akira is on set,

q - Toshiro is on set,

r - filming will begin,

s - caterers clear out.

$((p \wedge q) \rightarrow r) \leftrightarrow s)$

(iv) p - Imad arrived,

q - Sarah arrived.

$(p \vee \neg q) \wedge (p \wedge \neg q)$

(v) p - Herbert heard performance

q - Anne-Sophie heard performace

r - Anne-Sophie answered phone call

$\neg r \rightarrow \neg(\neg(p \wedge q))$

Question 2

Tuesday, November 3, 2020 2:38 AM

(2) (i) A propositional formula A is satisfiable if there is some v such that $h_v(A) = t$.

(ii) Two propositional formulas A and B are logically equivalent if, for every v, $h_v(A) = h_v(B)$.

(iii) '$\Rightarrow$'

A propositional formula $\neg A$ is satisfiable, so $\exists v : h_v(\neg A) = t$ and so $\exists v : h_v(\neg\neg A) = f$.

From Def 1.5 we know that for any v $h_v(\top) = t$. Hence, $h_v(\neg\neg A) \neq h(\top)$ and so this doesn't satisfy the definition of equivalence. Hence, $\neg\neg A \not\equiv \top$.

($\Leftarrow$) If $\neg\neg A \not\equiv \top$, then $\neg\neg A \equiv \bot$.

From equivalence definition we know that for every v $h_v(\neg\neg A) = h_v(\bot)$.

But we know from definition 1.5

that $h_v(\perp) = f$ for all v. Hence, $h_v(\neg\neg A) = f$ and $h_v(\neg A) = t$ for every v. This satisfies the above definition of satisfiability and so $\neg\neg A \not\equiv \top$ implies that $\neg A$ is a satisfiable propositional formula.

Question 3

Tuesday, November 3, 2020 2:38 AM

(3) Since we have variables p, q, r we need 8 rows.

Notice that $q \leftrightarrow \neg\neg q$ as

and $\neg(\neg r \vee \neg p) \leftrightarrow r \wedge p$

				A	B	C		
p	q	r	$\neg\neg q$	$\neg\neg q \to r$	$\neg(\neg r \vee \neg p)$	$p \wedge \neg q$	$C \leftrightarrow B$	$(C \leftrightarrow B) \to A$
0	0	0	0	1	0	0	1	1
0	0	1	0	1	0	0	1	1
0	1	0	1	0	0	0	1	0
0	1	1	1	1	0	0	1	1
1	0	0	0	1	0	1	0	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	1	0
1	1	1	1	1	1	0	0	1

Since it is not true for $(p,q,r) = (0,1,0)$ and $(1,1,0)$ so the propositional formula is not valid.

Question 4

Tuesday, November 3, 2020 2:38 AM

4 a. $p \wedge (\neg q \vee r)$ is CNF with clauses p and $\neg q \vee r$.

b. $\neg p$ is both CNF and DNF.

c. $p \wedge (q \vee (p \wedge r))$ is neither.

d. $\top$ is a clause and so both CNF and DNF.

e. $(p \wedge q) \vee (p \wedge q)$ is DNF.

f. $\neg\neg p \wedge (q \vee p)$ is neither since $\neg\neg p$ is not a literal (a literal is atomic or atomic with single sign of negation)

g. $p \wedge q$ is both CNF and DNF

h. $p \vee q$ is both CNF and DNF.

(ii) The refutation-soundness and

(ii) The refutation-soundness and completeness property says that for S in CNF we have $S \vdash_{res(PL)} \emptyset$ if and only if $S \models \bot$.
It is important as its corollary says that S is satisfiable if no derivation of S results in $\emptyset$.

(iii)
a) $\{ \{p, s\}, \{q, r\}, \{-s, q\}, \{\neg p, \neg r, \neg s\} \}$

$\Rightarrow \{ \{p, s\}, \{\neg p, \neg r, \neg s\} \}$ [pure rule on q]

$\Rightarrow \{ \{p, s\} \}$ [pure rule on ¬r]

$\Rightarrow \{ \}$ [pure rule on p]

$\Rightarrow$ Satisfiable

b. $\{ \{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}$

$\Rightarrow \{ \{\neg p, r\}, \{p, r\}, \{\neg r\} \}$ [unit prop on ¬q]

$\Rightarrow \{ \{\neg p\}, \{p\} \}$ [unit prop on ¬r]

$\Rightarrow \{ \{\} \}$ [unit prop on p]

$\Rightarrow$ Unsatisfiable [since ϕ is in the set]

5. Using p: I am going, q: you are going
r: Tara is going,
We formalise it as follows

$p \to \neg q$, $\neg q \to \neg r$, $r \land \neg p$
$r \land p$, therefore q.

So we must check $p \to \neg q, \neg q \to \neg r, r \land \neg p,$
$r \lor p \models q$.

We know that $A_1, \dots, A_n \models B \iff A_1 \land \dots A_n \land \neg B$
is unsatisfiable. So we can check if
$(p \to \neg q) \land (\neg q \to \neg r) \land (r \lor \neg p) \land (r \lor p) \models q$
satisfiable by converting to CNF

$$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$$

applying DP as follows:

$\{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$ [unit prop on $\neg q$]

$\Rightarrow \{\{\neg p\}, \{p\}\}$ [unit prop on $\neg r$]

$\Rightarrow \{\{\}\}$ [unit prop on p]

$\Rightarrow$ Unsatisfiable [since contains $\emptyset$]

6.

i).

aunt (X) ('X's aunt')
transaction (X, Y, Z) (X gives Y to Z')

$C = \{$ Andrea, cupcake $\}$
$P_1 = \{$ aunt $\}$
$P_3 = \{$ transaction $\}$

transaction ($\forall X$ (aunt (aunt (Andrea)),

cupcake, $\exists Y$ ($Y \neq$ Andrea))

(ii) $\exists X \forall Y$ (connected (computer(X) , computer(Y))

$\wedge (X \neq Y) \wedge \neg$ connected (X, X))

$P_1 = \{$ computer $\}$ computer (X) X is a computer,

$P_2 = \{$ connected $\}$ connected (X, Y) X is connected to Y.

($\forall X$ (PaulKlee (X) $\wedge$ Britishgallery (X)) $\rightarrow$

$\forall Y$ (Kandinsky (Y) $\wedge$ Britishgallery (Y) $\wedge$ room (Y, Z) $\wedge$ room (X, Z))

$P_1 = \{$ Britishgallery, PaulKlee, Kandinsky $\}$

$P_2 = \{$ room $\}$

British gallery (X) ('X is in British gallery)

PaulKlee(X) ('Painting X is by Paul Klee')

Kandinsky(X) ('Painting X is by Kandinsky')

room(X, Y) ('Painting X is in room Y')

(iv) $\exists X \forall Y (\neg loves(X, Y)) \rightarrow \neg \forall X \exists Y (loves(X,Y))$

$\mathcal{P}_2 = \{ loves \}$

7. i) For this to be true, any X in the domain D such that k has arrow to X cannot be j. Clearly that's false since there is

$(k, j) \in \varphi(a)$.

(ii) The statement is saying that if l is circular, then there exists at least one black circular object that l points to. Indeed, $l \in \varphi(c)$ and the consequent is true since $j \in \varphi(b) \wedge \varphi(c)$ and $(l, j) \in \varphi(a)$.

(iii) The statement is saying that there is X that doesn't have arrow pointing at something else. But we have

, hence the statement is true.

(iv) The statement is saying that for any non-square object there is a black circular object that it points to. This is false, since j is not square, but its only arrow goes to l, which is not black.

(V) The statement is saying that for any object X pointing to something else, there is also an object Y that both X points to and Y points to X. This is false, since k points to j, but j is not pointing back to k nor k is pointing back to l.

(VI) The statement is false as for X=k and Y=k, k does not have an arrow pointed to itself.