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Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

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Introduction to Symbolic AI
Coursework 1: Logic

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Total possible: 72 marks

1 [10 marks]

Formalize each of the following in propositional logic, including all brackets required by the strict definition of a propositional formula (remember to give the correspondence between the basic sentences of the original and the propositional atoms)

i. If Michel isn't either fulfilled or rich, he won't live another five years.

Answer: Let p : Michael is fulfilled, q : Michael is rich, r : Michael will live another five years. Then we get:

$$((\neg(p \vee q)) \rightarrow (\neg r))$$

ii. Unless the snowstorm doesn't arrive, Raheem will wear his boots; but I'm sure it *will* arrive.

Answer: Let p : The snowstorm will arrive, q : Raheem will wear his boots. Then we get:

$$(((\neg p) \vee q) \wedge p))$$

iii. If Akira and Toshiro are on set, then filming will begin if and only if the caterers have cleared out.

Answer: Let p : Akira is on set, q : Toshiro is on set, r : filming will begin, s : Caterers have cleared out. Then we get:

$$((p \wedge q) \rightarrow (r \leftrightarrow s))$$

iv. Either Irad arrived, or Sarah didn't: but not both!

Answer: Let p : Irad arrived, q : Sarah arrived. Then we get:

$$((p \vee (\neg q)) \wedge (\neg(p \wedge (\neg q))))$$

v. It's not the case both that Herbert heard the performance and Anne-Sophie did, if the latter didn't answer her phone calls.

Answer: Let p : Herbert heard the performance, q : Anne-Sophie heard the performance, r : Anne-Sophie answered her phone calls. Then we get

$$((\neg r) \rightarrow (\neg(p \wedge q)))$$

2 [8 marks]

i. What is the definition of the *satisfiability* of a propositional formula, A?
Answer:
A propositional formula A is satisfiable if there exists some valuation v such that $h_v(A) = t$, where h_v is the propositional valuation function based on v.

ii. What is the definition of the logical equivalence of two propositional formulas A and B?
Answer:
Two propositional formulas A,B are logically equivalent if, for every valuation v, $h_v(A) = h_v(B)$.

iii. Prove that a propositional formula $\neg A$ is satisfiable iff $\neg\neg A \not\equiv \top$ (i.e., iff it's not the case that $\neg\neg A \equiv \top$).
Answer:
We need to prove this in both directions.
($\neg A$ is satisfiable $\rightarrow \neg\neg A \not\equiv \top$): First, let's assume that $\neg A$ is satisfiable. That means that there exists some valuation v such that $h_v(\neg A) = t$. Using the definition of the propositional valuation formula with regards to $\neg$, we get that $h_v(\neg\neg A) = f$. Therefore there exists such a v that makes $h_v(\neg\neg A) = f$, whereas $h_v(\top) = t$. Thus it is not the case that $\neg\neg A \equiv \top$
($\neg\neg A \not\equiv \top \rightarrow \neg A$ is satisfiable): Now, let's assume that $\neg\neg A \not\equiv \top$. That means that there exists some valuation v such that $h_v(\neg\neg A) \neq h_v(\top) = t$, so $h_v(\neg\neg A) = f$. By the definition of the propositional valuation formula with regards to $\neg$, we get $h_v(\neg A) = t$. That means, by the definition of satisfiability, that $\neg A$ is satisfiable.
Thus we have proved that $\neg A$ is satisfiable $\leftrightarrow \neg\neg A \not\equiv \top$

3 [8 marks]

Use truth-tables to determine whether the following is valid or not: $(p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \rightarrow (\neg\neg q \rightarrow r)$

Table 1: Truth table for $(p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \rightarrow (\neg\neg q \rightarrow r)$

case	p	q	r	$(p$	$\wedge$	$\neg$	q	$\leftrightarrow$	$\neg$	$(\neg$	r	$\vee$	$\neg$	$p))$	$\rightarrow$	$(\neg$	$\neg$	q	$\rightarrow$	$r)$
1	t	t	t	t	f	f	t	f	t	f	t	f	f	t	t	t	f	t	t	t
2	t	t	f	t	f	f	t	t	f	t	f	t	f	t	**f**	t	f	t	f	f
3	t	f	t	t	t	t	f	t	t	f	t	f	f	t	t	f	t	f	t	t
4	t	f	f	t	t	t	f	f	f	t	f	t	f	t	t	f	t	f	t	f
5	f	t	t	f	f	f	t	t	f	f	t	t	t	f	t	t	f	t	t	t
6	f	t	f	f	f	f	t	t	f	t	f	t	t	f	**f**	t	f	t	f	f
7	f	f	t	f	f	t	f	t	f	f	t	t	t	f	t	f	t	f	t	t
8	f	f	f	f	f	t	f	t	f	t	f	t	t	f	t	f	t	f	t	f

As can be seen in cases 2 and 6, this not valid.

4 [14 marks]

i. Which of the following are in CNF? Which are in DNF?
Answers:

a. $p \wedge (\neg q \vee r)$ is CNF.

b. $\neg p$ is both CNF and DNF, since it is both a conjunction of clauses (a single unit clause, to be exact) and a disjunction of conjuncts (disjunction of a single literal)

c. $p \wedge (q \vee (p \wedge r))$ is neither, since it is neither a conjunction of clauses nor a disjunction of conjuncts.

d. $\top$ is both a CNF and DNF consisting of a single literal, $\top$.

e. $(p \wedge q) \vee (p \wedge q)$ is a DNF.

f. $\neg\neg p \wedge (q \vee p)$ is neither, since $\neg\neg p$ is not a literal.

g. $p \wedge q$ is both a CNF and DNF, since it is either a conjunction of unit clauses, or a disjunct of a single conjunct.

h. $p \vee q$ is both a CNF and DNF, since it is a conjunction of a single clause, or a conjunction of single literal disjuncts.

ii. Define the property of the refutation-soundness and -completeness of a resolution derivation. Why is this property important?
Answer:
Refutation-soundness and -completeness property: Let a formula S be in CNF. Then $S \vdash_{res(PL)} \emptyset \iff S \vDash \bot$.
This means that any formula in CNF can be checked for satisfiability using propositional resolution. Since any propositional logic formula has a logically equivalent formula in CNF, this means that any formula can be checked for satisfiability by finding the logically equivalent CNF formula and applying propositional resolution.

iii. Apply unit propagation and the pure rule repeatedly, in order to reduce the following to their simplist forms (stating which rule you're applying, and indicate the literal involved):

a. $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$
Answer:

$$\begin{aligned}
&\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\} && \\
\Rightarrow&\{\{p, s\}, \{\neg p, \neg r, \neg s\}\} && \text{[pure rule on } q\text{]} \\
\Rightarrow&\{\{p, s\}\} && \text{[pure rule on } \neg r\text{]}
\end{aligned}$$

b. $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$
Answer:

$$\begin{aligned}
&\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\} && \\
\Rightarrow&\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\} && \text{[unit propogation on } \neg q\text{]} \\
\Rightarrow&\{\{\neg p\}, \{p\}\} && \text{[unit propogation on } \neg r\text{]} \\
\Rightarrow&\{\{\}\} && \text{[unit propogation on } p\text{]}
\end{aligned}$$

5 [8 marks]

Use DP to determine whether the following argument is valid or not:
If I'm going, then you aren't.
If you're not going, then neither is Tara.
Either Tara's going or I'm not.
Tara's going unless I am.
So, you're going.
Answer:
Let p : I'm going, q : You're going, r : Tara's going.
The sentences above can be translated as such:
$p \to \neg q$ is the translation of "If I'm going, then you aren't."
$\neg q \to \neg r$ is the translation of "If you're not going, then neither is Tara."
$r \vee \neg p$ is the translation of "Either Tara's going or I'm not."
$p \vee r$ is the translation of "Tara's going unless I am.", as the term "unless" is usually translated as "or".
q is the translation of "You're going."

To show that the statement is valid, we use the fact that $A_1 \wedge ... \wedge A_n \models B$ iff $A_1 \wedge ... \wedge A_n \wedge \neg B$ is unsatisfiable. Let's use Davis-Putnam algorithm to determine satisfiability.

Let's convert the formula into CNF

$$\text{The formula } (p \rightarrow \neg q) \wedge (\neg q \rightarrow \neg r) \wedge (r \vee \neg p) \wedge (p \vee r) \wedge (\neg q)$$
$$\text{is equivalent to } (\neg p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee r) \wedge (\neg q)$$

Using set notation, and applying DP, we get:

$$\begin{aligned}
&\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{p, r\}, \{\neg q\}\} \\
\Rightarrow &\{\{\neg r\}, \{r, \neg p\}, \{p, r\}\} && \text{[unit propogation on } \neg q] \\
\Rightarrow &\{\{\neg p\}, \{p\}\} && \text{[unit propogation on } \neg r] \\
\Rightarrow &\{\{\}\} && \text{[unit propogation on } p] \\
\Rightarrow &\text{Unsatisfiable, since } \emptyset \text{ is in the set.}
\end{aligned}$$

Therefore, we have shown that the argument is valid ($A_1 \wedge ... \wedge A_n \models B$).

6 [12 marks]

Translate into first-order logic, giving as much logical structure as possible. Be sure to specify the signature for each part.

i. All of Andrea's aunts' aunts gave a cupcake to someone other than Andrea.
Answer:
For the structure

$$\mathcal{C} = \{andrea\}, \mathcal{P}_1 = \{cupcake\}, \mathcal{P}_2 = \{aunt\}, \mathcal{P}_3 = \{gives\}$$

where $cupcake(X)$ means X is cupcake, $aunt(X, Y)$ means X is an aunt of Y, and $gives(X, Y, Z)$ means X gives Y to Z.
We get:

$$\forall X \forall Y (aunt(X, Y) \wedge aunt(Y, andrea) \rightarrow \exists Z \exists Q (\neg(Z = andrea) \wedge cupcake(Q) \wedge gives(X, Q, Z)))$$

ii. There's a computer connected to every computer which isn't connected to itself.
Answer:
For the structure

$$\mathcal{P}_1 = \{computer\}, \mathcal{P}_2 = \{connected\}$$

where $computer(X)$ means X is a computer, $connected(X, Y)$ means X is connected to Y.
We get

$$\forall X (computer(X) \wedge \neg connected(X, X) \rightarrow \exists Y (computer(Y) \wedge connected(Y, X)))$$

iii. Any painting by Paul Klee in a British gallery hangs in a room where all Kandinsky paintings in that gallery hang.
Answer:
For the structure

$$\mathcal{C} = \{paulKlee, kandinsky\}, \mathcal{P}_1 = \{room, painting, gallery\}, \mathcal{P}_2 = \{painted, in\}$$

where $paulKlee$ and $kandinsky$ are constants, $room(X)$ means X is a room, $painting(X)$ means X is a painting, $gallery(X)$ means X is a gallery, $painted(X, Y)$ means X painted Y, $in(X, Y)$ means X is in Y.
We get

$$\begin{aligned}
\forall X \forall Y (painting(X) \wedge painted(paulKlee, X) \wedge gallery(Y) \wedge british(Y) \wedge in(X, Y) \\
\rightarrow \exists Z (room(Z) \wedge \forall Q (painting(Q) \wedge painted(kandinsky, Q) \wedge in(Q, Y) \rightarrow in(Q, Z)) \wedge in(X, Z)))
\end{aligned}$$

iv. If there's somebody who loves nobody, then it's false that everybody loves somebody.
Answer:
For the structure

$$\mathcal{P}_2 = \{loves\}$$

where $loves(X, Y)$ means X loves Y.
We get:

$$\exists X \neg \exists Y (loves(X, Y)) \to \neg(\forall X \exists Y (loves(X, Y))$$

7 [12 marks]

Let $\mathcal{L}$ be a signature containing just four unary predicate symbols b, w, s and c, and a single binary relation symbol a; and three constants j, k and l. Consider the following $\mathcal{L}$-structure (D, φ), containing five objects: The objects $\varphi(j), \varphi(k) and \varphi(l)$ are shown (indicated by the relevant letters). Further:

- $\varphi(b)$ is the set of filled ('black') objects
- $\varphi(w)$ is the set of unfilled ('white') objects
- $\varphi(s)$ is the set of square objects
- $\varphi(c)$ is the set of circular objects.
- $\varphi(a)$ is the set of pairs (x, y) such that there is a directed arrow from x to y

For example, the object $\varphi(k)$, to the top in the diagram, is in $\varphi(b)$ and $\varphi(c)$, since it is drawn filled and circular. Determine, for each of the following, whether it is true or false, and provide a justification in each case.

i. $\forall X(a(k, X) \to \neg(X = j))$
Answer:
This is false. It states that everything that k points to (i.e. there is an arrow from k to it) is not j. If we take the assignment $\sigma(X) = j$, then $(k, \sigma(X)) \in \varphi(a)$. The the left-hand side of the formula is true, but the right-hand side is false, so it does not hold for all X in our structure.

ii. $c(l) \to \exists X(b(X) \wedge c(X) \wedge a(l, X))$
Answer:
This is true. Note that for this structure, $l \in \varphi(c)$ so the left-hand side of the formula is always true. We need to show that there exists an assignment $\sigma(X)$ such that the right-hand side $(\exists X(b(X) \wedge c(X) \wedge a(l, X)))$ is also true. We need to find an black circle that l points to.
Let's take $\sigma(X) = j$. Then $\sigma(X) \in \varphi(b)$,$\sigma(X) \in \varphi(c)$, $(l, \sigma(X)) \in \varphi(a)$. Therefore the formula is true in our structure.

iii. $\exists X \neg \exists Y(\neg(X = Y) \wedge a(X, Y))$
Answer:
This is true. This means that there exists something that doesn't point to anything other than itself. The black square satisfies this condition, since the only element of $\varphi(a)$ that contains the black square represents the arrow that points to itself.

iv. $\forall X(\neg s(X) \to \exists Y(c(Y) \wedge b(Y) \wedge a(X, Y)))$
Answer:
This is false. This states that all non-square elements have an arrow from itself to a black circle. Let's take $\sigma(X) = j$, which is not a square. The only element of $\varphi(a)$ which starts with a j is (j, l), but l is not a black circle. Therefore the formula is false in this structure.

v. $\forall X(\exists Y(\neg(X = Y) \wedge a(X, Y)) \rightarrow \exists Y(a(X, Y) \wedge a(Y, X)))$
Answer:
This is false. It states that for all elements which point to something other than itself, there exists an arrow in the opposite direction. A counterexample is $\sigma(X) = k$, which has an arrow going from k to j, but there is no arrow from j to k.

vi. $\forall X \forall Y(a(X, j) \wedge a(Y, j) \rightarrow (a(X, Y) \vee a(Y, X)))$
Answer:
This is false. It states that for all two elements (not necessarily distinct!) that point to j, there exists an arrow between them (in either direction).
A counterexample is $\sigma(X) = \sigma(Y) = k$. It satisfies that $(\sigma(X), j) \in \varphi(a)$ and $(\sigma(Y), j) \in \varphi(a)$, but the element $(\sigma(X), \sigma(X)) = (k, k) \notin \varphi(a)$. Therefore the formula is false in our structure.