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Exercise Information

Module: 70051 Introduction to Symbolic Artificial Intelligence (MSc AI)

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Exercise: 2 (CW)

Due: Tue - 03 Nov 2020

Title: Logic

Assessment: Individual

FAO: Craven, Robert (rac101)

Submission: Electronic

Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-11-02 00:41:23

For Markers only: (circle appropriate grade)

SUN, Yipengjing (ys1220)	01905476	t5	2020-11-02 00:41:23	A*	A	B	C	D	E	F
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1.

i) p: Michel is fulfilled, q: Michel is rich
r: Michel will live another five years

$$((\neg(p \vee q)) \rightarrow (\neg r))$$

ii) p: the snowstorm arrives, q: Raheem will wear his boots.
r: I'm sure the snowstorm will arrive.

$$((p \vee q) \wedge r)$$

iii) p: Akira is on set, q: Toshio is on set
r: the film will begin, h: caterers have cleared out

$$(p \wedge q) \rightarrow (r \leftrightarrow h)$$

iv) p: Iraq arrived, q: Sarah arrived

$$((p \vee \neg q) \wedge (\neg(p \wedge \neg q)))$$

v) p: Herbert heard the performance

q: Anne-Sophie heard the performance

r: Anne-Sophie answered the phone calls

$$((\neg r) \rightarrow (\neg(p \wedge q)))$$

2. i) A propositional formula A is satisfiable if there is some v such that $h_v(A) = t$

ii) A, B are logically equivalent if for every v , $h_v(A) = h_v(B)$

iii) Suppose $(\neg A)$ is satisfiable, then $h_v(\neg A) = t$ for some v

If $h_v(A) = f$, then $h_v(\neg \neg A) = f$, but $h_v(\top) = t$
So $\neg \neg A \not\models \top$

On the other hand, if $\neg \neg A \not\models \top$, which means $h_v(\neg \neg A) = f$ for some v . It implies $h_v(\neg A) = t$ for some v . Therefore, $(\neg A)$ is satisfiable

3.

p	q	r	$(p \wedge \neg q \leftrightarrow \neg(\neg r \vee p)) \rightarrow (\neg \neg q \rightarrow r)$
t	t	t	<u>t</u> <u>f</u> <u>f</u> <u>t</u> <u>f</u> <u>f</u> <u>t</u> <u>t</u> <u>f</u> <u>t</u>
t	t	f	<u>t</u> <u>f</u> <u>f</u> <u>t</u> <u>f</u> <u>t</u> <u>f</u> <u>t</u> <u>f</u> <u>t</u>
t	f	t	<u>t</u> <u>f</u> <u>t</u>
t	f	f	<u>t</u> <u>f</u>
f	t	t	<u>f</u> <u>t</u>
f	t	f	<u>f</u> <u>t</u>
f	f	t	<u>f</u> <u>f</u>

$\vdash \vdash f \vdash$

It's not valid, since the function h not always equals to t for any r .

4. i)

CNF: $a \cdot b \cdot g \cdot h$

DNF: $b \cdot c \cdot g \cdot h$

ii)

Let S be in CNF. S is satisfiable iff
 $S \not\models \text{rep}_S \phi$

It's important since it's the heart of many SAT-solvers

iii) a. $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

$\Rightarrow q$ is pure, apply pure rule

$\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$

$\Rightarrow \neg r$ is pure, apply pure rule

$\{\{p, s\}\}$

h. Scenario $\neg q \wedge \neg r \wedge q \wedge \neg r$

• $\neg p \vee q, \neg q \vee r, \neg r \vee p$

$\Rightarrow \neg q$ is a unit clause, apply unit propagation

$$\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$$

$\Rightarrow \neg r$ is a unit clause, apply unit propagation

$$\{\{\neg p\}, \{p\}\}$$

$\Rightarrow p$ is a unit clause, apply unit propagation

$$\{\{\}\}$$

5. p: I'm going $\rightarrow q$: you're going, r: Tara is going

$$(p \rightarrow \neg q), (\neg q \rightarrow \neg r), r \vee \neg p, r \vee p$$

$$(\neg p \vee \neg q) \quad (q \vee \neg r)$$

$$S = \{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}\}$$

\Rightarrow resolvent $\{\neg r, \neg p\}, \{\neg r, p\}$ on p

$$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{\neg r\}\}$$

$\Rightarrow (\neg p)$ is pure

$$\{\{q, \neg r\}, \{r\}\}$$

\Rightarrow resolvent $\{q, \neg r\}, \{r\}$ on $\{p\}$

$$\{\{q\}\}$$

Since $S \vdash_{\text{recipL}} \{ q_0 \}$, $S \models q$

therefore, it's valid

6.

$$\text{i) } C = \{ \text{Andrea} \} \quad P_1 = \{ \text{cupcake} \}$$

$$P_2 = \{ \text{aunts} \} \quad P_3 = \{ \text{gave} \}$$

$$F_1 = \{ \text{aunts} \}$$

$$\forall X \exists Y \exists Z ((\text{aunts}(X, \text{aunts}(\text{Andrea})) \wedge \text{cupcake}(Y)) \wedge \text{gave}(X, Y, Z) \wedge \neg \text{gave}(X, Y, \text{Andrea}))$$

$$\text{ii) } P_1 = \{ \text{computer} \}$$

$$P_2 = \{ \text{connected} \}$$

$$\exists X \forall Y ((\text{computer}(X) \wedge \text{computer}(Y)) \wedge \text{connected}(X, Y) \wedge \neg \text{connected}(X, X))$$

$$\text{iii) } C = \{ \text{Paul Klee, Kandinsky} \}$$

$$P_1 = \{ \text{British Gallery, room} \}$$

$$P_2 = \{ \text{hangs, painting, in} \}$$

$\forall X \exists Y \exists Z \forall M (\text{Painting}(X, \text{paul klee}) \wedge \text{British Gallery}(Y) \wedge \text{Room}(Z)$
 $\wedge \text{in}(Z, Y) \wedge \text{hangs}(X, Z) \wedge \text{Painting}(M, \text{kandinsky})$
 $\wedge \text{hangs}(M, Z))$

iV) $P_2 = \{ \text{love} \}$

$\exists X \forall Y (\text{love}(X, Y) \rightarrow \text{love}(Y, X))$

T.

i) $\forall X (\text{a}(k, X) \rightarrow \neg(X=j))$

it will be true for any X such that $\text{a}(k, X)$ is false , since $\text{a}(k, l)$ is false , so $\text{a}(k, X)$ is false \Rightarrow it is true

ii) $c(l) \rightarrow \exists X (b(X) \wedge c(X) \wedge a(l, X))$

it will be true if $c(l)$ is false or

$\exists X (b(X) \wedge c(X) \wedge a(l, X))$ is true

since l is drawn circular , $c(l)$ is true

both j and k satisfy filled and circular and have a directed arrow from l , so $\exists X (b(X) \wedge c(X) \wedge a(l, X))$

is true

Therefore, it is true.

$$\text{iii) } \exists X \neg \exists Y (\top(x=Y) \wedge a(x, Y))$$

There exists X but does not exist Y such that $X \neq Y$, which means that there exists X such that for all Y , $x=Y$. Cannot find such X . So it is false

$$\text{iv) } \forall X (\top s(X) \rightarrow \exists Y (c(Y) \wedge b(Y) \wedge a(X, Y)))$$

For all X , X is not square objects.

it is false, since there are \heartsuit and



Therefore, it is true.

$$\text{v) } \forall X (\exists Y (\top(x=Y) \wedge a(x, Y)) \rightarrow \exists Y (a(x, Y) \wedge a(Y, x)))$$

It will be true if $\exists Y (\top(x=Y) \wedge a(x, Y))$ is false or

$\exists Y (a(x, Y) \wedge a(Y, x))$ is true

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For all X , such that there is Y , $X \neq Y$
and there is a directed arrow from X to Y

It's false since X contains Y , so there
must be one $X = Y$

Therefore it's true.

$$\forall i) \forall X \forall Y (a(x,j) \wedge a(Y,j) \rightarrow (a(x,Y) \vee a(Y,x)))$$

It's true since there are objects
that have no arrow to j

So $a(x,j) \wedge a(Y,j)$ is false.

then the whole formula is true