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Exercise Information

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Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

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SPOONER, Jordan (js4416)	01201572	c4	2020-02-17 18:32:23	**A* A B C D E F**

Q1. (a) For a model M, path π and LTL formulas φ and ψ:

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$(M,\pi) \models \varphi R \psi$ iff there is some $i \geq 0$ such that $(M, \pi[i..\infty]) \models \varphi$ and for all $0 \leq j \leq i$ we have $(M, \pi[j..\infty]) \models \psi$, or for all $k \geq 0$, $(M, \pi[k..\infty]) \models \psi$.

(b) $$\varphi R \psi \equiv \underbrace{(\neg(\top U \neg\psi))}_{\psi \text{ holds forever}} \vee \underbrace{(\psi U (\psi \wedge \varphi))}_{\psi \text{ until and including when } \varphi \text{ holds}}$$

ψ holds forever or ψ until and including when φ holds

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Solution could have been simplified futher. However, the explanation given is very strong

(c) $(M,\pi) \models (\neg(\top U \neg\psi)) \vee (\psi U (\psi \wedge \varphi))$

(by defn 1.4) iff $(M,\pi) \models \neg(\top U \neg\psi)$ or $(M,\pi) \models \psi U (\psi \wedge \varphi)$

(by 1.4) iff $(M,\pi) \not\models \top U \neg\psi$ or $\underbrace{(M,\pi) \models \psi U (\psi \wedge \varphi)}_{*}$

(by 1.4) iff $\neg[(M, \pi[k..\infty]) \models \neg\psi$ for some $k \geq 0$, and $(M, \pi[\ell..\infty]) \models \top$ for all $0 \leq \ell < k]$, or $*$

(by 1.4) iff $\neg[(M, \pi[k..\infty]) \models \neg\psi$ for some $k \geq 0]$, or $*$

(by 1.4) iff $\neg[(M, \pi[k..\infty]) \not\models \psi$ for some $k \geq 0]$, or $*$

($\neg\exists\neg X \equiv \neg\neg\forall X \equiv \forall X$) iff $\underbrace{\text{for all } k \geq 0, (M,\pi) \models \psi}_{\dagger}$ or $*$

(by rewriting) iff $\oplus$ or $(M,\pi) \models \psi \,U\, (\psi \wedge \varphi)$

(1.4) iff $\oplus$ or $(M,\pi[i..\infty]) \models (\psi \wedge \varphi)$ for some $i \geq 0$
and $(M,\pi[j..\infty]) \models \psi$ for all $0 \leq j < i$

(1.4) iff $\oplus$ or $(M,\pi[i..\infty]) \models \psi$ and
$(M,\pi[i..\infty]) \models \varphi$ for some $i \geq 0$
and $(M,\pi[j..\infty]) \models \psi$ for all $0 \leq j < i$.

(rewrite) iff $(M,\pi[i..\infty]) \models \varphi$ for some $i \geq 0$ and
$(M,\pi[j..\infty]) \models \psi$ for all $0 \leq j \leq i$,
or for all $k \geq 0$ $(M,\pi[k..\infty]) \models \psi$.

which is the condition provided in part (a).

Solution correct and extremely well explained 3

(d) $\perp R \psi \equiv (\neg(\top U \neg\psi)) \vee (\psi \,U\, (\psi \wedge \perp))$

(from parts (a) to (c)) $\equiv (\neg F \neg\psi) \vee (\psi \,U\, (\psi \wedge \perp))$

(by defn F) $\equiv G\psi \vee (\psi \,U\, (\psi \wedge \perp))$

(by defn G) $\equiv G\psi \vee (\psi \,U\, \perp)$

(since $X \wedge \top \equiv X$) $\equiv G\psi \vee \perp$

(since $X \vee \perp \equiv X$) $\equiv G\psi$

by defn 1.4, this would require $\lambda[i..\infty] \models \perp$ for some $i \geq 0$ to hold for some path λ. Clearly no such λ exists, so it is equivalent to $\perp$

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solution correct and well explained, though the explanation for steps could have been presented in a stronger manner. However, since the main explanation is the truth condition, the solution is adequate

Q2.(i) $(M,q) \models EF\phi$

iff $(M,q) \models E(T \cup \phi)$ (given)

iff for some path λ starting from q,
$(M,\lambda) \models T \cup \phi$ (defn 1.7)

iff for some path λ starting from q,
$(M,\lambda[j]) \models \phi$ for some $j \geq 0$
and $(M,\lambda[k]) \models T$ for all $0 \leq k < j$ (defn 1.8)

iff for some path λ starting from q,
for some $j \geq 0$ $(M,\lambda[j]) \models \phi$.

(since $(M,s) \models T$) $\circledast$

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(ii) $(M,q) \models AF\phi$

iff $(M,q) \models A(T \cup \phi)$ (given)

iff for every path λ starting from q,
$(M,\lambda) \models T \cup \phi$ (1.7)

iff for every path λ starting from q,
for some $j \geq 0$ $(M,\lambda[j]) \models \phi$.

(by the same reasoning as in $\circledast$)

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(iii) $(M,q) \models EG\phi$

iff $(M,q) \models \neg AF \neg \phi$ (given)

iff $(M,q) \models \neg A(\top U \neg \phi)$ (given)

iff $(M,q) \not\models A(\top U \neg \phi)$ (defn 1.7)

iff it is not the case that for all paths λ starting from q, $(M,\lambda) \models \top U \neg \phi$ (1.7)

iff for some path λ starting from q, $(M,\lambda) \not\models \top U \neg \phi$ $(\neg \exists X \equiv \forall \neg X)$

iff for some path λ starting from q, it is not the case that $(M,\lambda[j]) \models \neg \phi$ for some $j \geq 0$ and $(M,\lambda[k]) \models \top$ for all $0 \leq k < j$. (1.8)

(†) iff for some path λ starting from q, it is not the case that $(M,\lambda[j]) \models \neg\phi$ for some $j \geq 0$ $((M,s) \models \top)$

iff for some path λ starting from q, for all $j \geq 0$ $(M,\lambda[j]) \not\models \neg\phi$ $(\neg \exists X \equiv \forall \neg X)$

iff for some path λ starting from q for all $j \geq 0$ $(M,\lambda[j]) \models \phi$ (1.7 and $\neg\neg X \equiv X$)

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(iv) $(M,q) \models AG\phi$

iff $(M,q) \models \neg EF \neg \phi$ (given)

iff $(M,q) \models \neg E(\top U \neg \phi)$ (given)

iff $(M,q) \not\models E(\top U \neg \phi)$ (1.7)

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iff there is some path λ starting from q s.t. $(M,\lambda) \models \top U \neg \phi$

iff for all paths λ starting from q, $(M,\lambda) \not\models \top U \neg \phi$ (1.7) (switch quantifiers)

iff for all paths λ starting from q, for all $j \geq 0$ $(M,\lambda[j]) \models \phi$. (by the same reasoning as in (†))

Q3. (a)

Consider an arbitrary CTL formula ϕ
According to the definition in lecture 5, ϕ
can take seven forms:

$$\phi ::= a \mid \neg\phi_1 \mid \phi_1 \wedge \phi_2 \mid EX\phi_1 \mid AX\phi_1 \mid E(\phi_1 \cup \phi_2) \mid A(\phi_1 \cup \phi_2)$$

where ϕ_1 and ϕ_2 are CTL formulas.

We show by induction that ϕ is a formula CTL*.

Specifically, our inductive hypothesis is that any CTL formula is a CTL* (state) formula.

① Base case : ϕ is an atom a

By definition 1, an atom a is a (state) formula of CTL* as required.

② Inductive case 1 : ϕ is $\neg\phi_1$

By our I.H., assume ϕ_1 is a (state) formula CTL*.

By defn 1 (line 1), $\neg\phi_1$ is therefore also a (state) formula in CTL*.

③ Inductive case 2 : Φ is $\phi_1 \wedge \phi_2$

By the IH, assume Φ_1 and ϕ_2 are (state) formulas

By defn 1, $\Phi_1 \wedge \Phi_2$ is then a (state) formula in CTL* as required.

④ Inductive case 3 : Φ is $EX\Phi_1$

By the IH, assume Φ_1 is a (state) formula of CTL*.

(*) { Then by defn 1 (line 2) Φ_1 is a path formula also.
By defn 1 (line 2), $X\Phi_1$ is a path formula also.
By defn 1 (line 1), $EX\Phi_1$ is a (state) formula of CTL* as required.

⑤ Inductive case 4 : Φ is $AX\phi_1$

By the IH, assume Φ_1 is a (state) formula of CTL*.

By (*), $X\Phi_1$ is a path formula of CTL*

By defn 1 (line 1), $AX\Phi_1$ is a (state) formula of CTL*.

⑥ Inductive case 5 : ϕ is $E(\Phi_1 \cup \Phi_2)$

(†) { By the IH, assume Φ_1 and Φ_2 are (state) formulas of CTL*
By defn 1, they are also path formulas of CTL*.
By defn 1, $\Phi_1 \cup \Phi_2$ is a path formula of CTL*.
By defn 1, $E(\Phi_1 \cup \Phi_2)$ is a (state) formula of CTL*.

(7) Inductive case 6 : ϕ is $A(\phi_1 \cup \phi_2)$

By (+), we get $\phi_1 \cup \phi_2$ is a path formula of CTL*.

By defn 1, $A(\phi_1 \cup \phi_2)$ is a (state) formula of CTL* as required.

Hence every formula ϕ of CTL is a (state) formula in CTL*.

(b) Consider the CTL* formula

$$Ea$$

This is a formula of CTL*, derived as follows:

$$\phi \rightarrow E\psi \rightarrow E\phi \rightarrow Ea$$

It is not a formula of CTL since $E\psi$ can only be accepted if ψ is of the form $X\phi$ or $\phi \cup \phi$; and 'a' is clearly neither.

Hence there exists a formula of CTL* that does not belong to CTL.

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Q4. We denote entailment in CTL as $\models$ and entailment in CTL* as $\models^*$.

We must show that every construction in CTL (i.e. those seen in the previous question) have the same semantics in CTL*.

Once again, we will show this by induction.

Let M be an arbitrary model, q be an arbitrary state, and ϕ be an arbitrary CTL* (state) formula which is also a (state) formula in CTL.

Our inductive hypothesis is that $(M, q) \models^* \phi$ iff $(M, q) \models \phi$.

We will use ϕ_i to denote CTL* (state) formulas which are also (state) formulas of CTL.

Inductive case 3 $\quad \phi$ takes the form $EX\phi_1$

$(M, q) \models^* EX\phi_1$

iff for some path λ starting from q,
$(M, \lambda) \models^* X\phi_1$ (defn 2)

iff for some path λ starting from q,
$(M, \lambda[1..\infty]) \models^* \phi_1$ (defn 2)

iff for some path λ starting from q,
$(M, \lambda[1]) \models^* \phi_1$ (defn 2, since ϕ_1 is a state formula)

iff for some path λ starting from q,
$(M, \lambda[1]) \models \phi_1$ (I.H., since $\lambda[1]$ is a state)

iff for some path λ starting from q,
$(M, \lambda) \models X\phi_1$ (defn 1.8)

(*)

iff $(M, \lambda) \models EX\phi_1$ (defn 1.7)

as required.

Inductive case 4 $\quad \phi$ takes the form $AX\phi_1$

$(M, q) \models^* AX\phi_1$

iff for all paths λ starting from q,
$(M, \lambda) \models^* X\phi_1$ (defn 2)

iff for all paths λ starting from q,
$(M, \lambda) \models X\phi_1$ (same as (*))

iff $(M, \lambda) \models AX\phi_1$ (defn 1.7)

as required.

Base case ϕ is an atom a

$$(M, q) \models^* a \text{ iff } q \in V(a) \qquad \text{(defn 2)}$$

$$\text{iff } (M, q) \models a \qquad \text{(defn 1.7)}$$

as required.

Inductive case 1 ϕ is of the form $\neg \phi_1$

$$(M, q) \models^* \neg \phi_1 \text{ iff } (M, q) \not\models^* \phi_1 \qquad \text{(defn 2)}$$

$$\text{iff } (M, q) \not\models \phi_1 \qquad \text{(by I.H.)}$$

$$\text{iff } (M, q) \models \neg \phi_1 \qquad \text{(by defn 1.7)}$$

as required.

Inductive case 2 ϕ takes the form $\phi_1 \wedge \phi_2$

$$(M, q) \models^* \phi_1 \wedge \phi_2$$

$$\text{iff } (M, q) \models^* \phi_1 \text{ and } (M, q) \models^* \phi_2 \qquad \text{(defn 2)}$$

$$\text{iff } (M, q) \models \phi_1 \text{ and } (M, q) \models \phi_2 \qquad \text{(by I.H.)}$$

$$\text{iff } (M, q) \models \phi_1 \wedge \phi_2 \qquad \text{(by defn 1.7)}$$

as required.

Inductive case 5 ϕ takes the form $E(\phi_1 \cup \phi_2)$

$(M, q) \models^* E(\phi_1 \cup \phi_2)$

$\oplus$

iff for some path λ starting from q,
$(M, \lambda) \models^* \phi_1 \cup \phi_2$ (defn 2)

iff for some path λ starting from q,
$(M, \lambda[i..\infty]) \models^* \phi_1$ for some $i \geq 0$, and
$(M, \lambda[j..\infty]) \models^* \phi_2$ for all $0 \leq j < i$. (defn 2)

iff for some path λ starting from q
$(M, \lambda[i]) \models^* \phi_1$ for some $i \geq 0$ and
$(M, \lambda[j]) \models^* \phi_2$ for all $0 \leq j < i$ (defn 2, since ϕ_1 and ϕ_2 are state formulas)

iff for some path λ starting from q
$(M, \lambda[i]) \models \phi_1$ for some $i \geq 0$ and
$(M, \lambda[j]) \models \phi_2$ for all $0 \leq j < i$ (I.H., since $\lambda[i]$ and $\lambda[j]$ are states)

iff for some path λ starting from q
$(M, \lambda) \models \phi_1 \cup \phi_2$ (defn 1.8)

iff $(M, q) \models E(\phi_1 \cup \phi_2)$ as required. (defn 1.7)

Inductive case 6 ϕ takes the form $A(\phi_1 \cup \phi_2)$

$(M, q) \models^* A(\phi_1 \cup \phi_2)$

iff for all paths λ starting from q,
$(M, \lambda) \models^* \phi_1 \cup \phi_2$ (defn 2)

iff for all paths λ starting from q,
$(M, \lambda) \models \phi_1 \cup \phi_2$ (same as $\oplus$)

iff $(M, q) \models A(\phi_1 \cup \phi_2)$ as required. (defn 1.7)

Note that we have shown the I.H. holds over all formulas of CTL, as per the definition on slide 142. Also this is a strict subset of the formulas of CTL*, by the previous question. Hence the formulas of CTL* that are also formulas of CTL have the same semantics in both logics, as required.

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Q5. (a)

Consider an arbitrary formula ϕ of CTL, an arbitrary model M and an arbitrary state s.

By question 3(a), $\phi' = \phi$ a formula of CTL*.
By question 4, $(M, s) \models \phi$ iff $(M, s) \models^* \phi'$, as required.
Hence CTL* is more expressive than CTL.

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(b) Consider the formula of CTL*:

$$\phi = A(\top U (a \wedge Xa))$$
$$\equiv AF(a \wedge Xa) \quad (\text{by defn of } F, \text{ slide } 142)$$

ϕ is equivalent to the LTL formula $F(a \wedge Xa)$ (slide 126)

ϕ is not expressible in CTL (slide 208)

Hence we have that CTL* is strictly more expressive than CTL, as required.

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Q6. We do induction over defn 2.

Given models $M = (St, \rightarrow, V)$ and $M' = (St', \rightarrow', V')$, arbitrary states $t \in St$ and $t' \in St'$, arbitrary paths $\pi \in (St, \rightarrow)$ and $\pi' \in (St', \rightarrow')$, arbitrary state formula ϕ and arbitrary path formula ψ, such that $\underbrace{(M, t) \approx (M', t')}_{\dagger}$ and $\underbrace{(M, \pi) \approx (M', \pi')}_{\ddagger}$, our I.H. is that:

$(M, t) \models \phi$ iff $(M', t') \models \phi$

and $(M, \pi) \models \psi$ iff $(M', \pi') \models \psi$

Base case ϕ takes the form p.

$$
\begin{aligned}
(M, t) \models p \quad & \text{iff } t \in V(p) && \text{(defn 2)} \\
& \text{iff } t' \in V'(p) && (\dagger \text{ and defn 3a}) \\
& \text{iff } (M', t') \models p && \text{(defn 2)}
\end{aligned}
$$

as required

Inductive case 1 ϕ takes the form $\neg\phi_1$

$$
\begin{aligned}
(M, t) \models \neg\phi_1 \quad & \text{iff } (M, t) \not\models \phi_1 && \text{(defn 2)} \\
& \text{iff } (M', t') \not\models \phi_1 && \text{(I.H.)} \\
& \text{iff } (M', t') \models \neg\phi_1 && \text{(defn 2)}
\end{aligned}
$$

as required

Inductive case 2 ϕ takes the form $\phi_1 \wedge \phi_2$

$$(M, t) \models \phi_1 \wedge \phi_2$$

iff $(M, t) \models \phi_1$ and $(M, t) \models \phi_2$ (defn 2)

iff $(M', t') \models \phi_1$ and $(M', t') \models \phi_2$ (I. H.)

iff $(M', t') \models \phi_1 \wedge \phi_2$ (defn 2)

as required.

For the next inductive cases, we first prove the following:

Lemma 1 Given a path λ in M, starting at some state $\lambda[0]$, and a model M' such that $(M, \lambda[0]) \approx (M', \lambda'[0])$, ⊛ $(M, \lambda) \approx (M', \lambda')$, and vice versa.

Proof ① Forwards:

We must show that there exists a λ' s.t. $(M, \lambda[i]) \approx (M', \lambda'[i])$ for all $i \geq 0$.

We must show that there exists a bisimulation B between M and M' s.t. $B(\lambda[i], \lambda'[i])$ for all $i \geq 0$.

By ⊛, we know there exists a bisimulation B between M and M' s.t. $B(\lambda[0], \lambda'[0])$.

We now show by induction over $\mathbb{N}$ that $B(\lambda[i], \lambda'[i])$ for all $i \geq 0$.

Our I.H. is that $B(\lambda[k], \lambda'[k])$.

Base case $B(\lambda[0], \lambda'[0])$ holds as stated earlier.

Inductive case Assume $B(\lambda[k], \lambda'[k])$
we know that $\lambda[k] \in St$ and $\lambda[k] \to \lambda[k+1]$
By defn 3(b) (forth), there exists some state, call it $\lambda'[k+1] \in St'$ s.t. $B(\lambda[k], \lambda'[k+1])$, as required.

② Backwards: the proof is symmetrical, it relies on defn 3(c) (back) instead of 3(b) (forth).

Inductive case 3 ϕ takes the form $E\psi_1$

$(M, t) \models E\psi_1$

iff for some path λ starting from t,

$(M, \lambda) \models \psi_1$ (defn 2)

Now we note that $(M, t) \approx (M', t')$ by (+), and that $\lambda[0] = t$. If we have $\lambda'[0] = t'$, we can therefore apply Lemma 1 to get that $(M, \lambda) \approx (M', \lambda')$ and vice versa. Hence, this is equivalent to the condition that:

for some path λ' starting from t',

$(M', \lambda') \models \psi_1$ (by the I.H.)

iff $(M', \lambda') \models E\psi_1$ (defn 2)

as required

Inductive case 4 ϕ takes the form $A\psi_1$

$(M, t) \models A\psi_1$

iff for all paths λ starting from t,

$(M, \lambda) \models \psi_1$ (defn 2)

iff for all paths λ' starting from t'

$(M', \lambda') \models \psi_1$ (same reasoning as above)

iff $(M', t') \models A\psi_1$ (defn 2).

Inductive case 5 ψ takes the form ϕ_1

$$(M, \pi) \models \phi_1$$
$$\text{iff} \quad (M, \pi[0]) \models \phi_1 \qquad (\text{defn } 2)$$

Note that $(M, \pi) \approx (M', \pi')$ by ⊕. Hence by the definition of bisimilar paths, $(M, \pi[0]) \approx (M', \pi'[0])$, so this condition is equivalent to:

$$(M', \pi'[0]) \models \phi_1 \qquad (\text{by the I.H.})$$
$$\text{iff} \quad (M', \pi') \models \phi_1 \qquad (\text{defn } 2)$$

Inductive case 6 ψ takes the form $\neg\psi_1$

$$(M, \pi) \models \neg\psi_1$$
$$\text{iff} \quad (M, \pi) \not\models \psi_1 \qquad (\text{defn } 2)$$
$$\text{iff} \quad (M', \pi') \not\models \psi_1 \qquad (\text{I.H.})$$
$$\text{iff} \quad (M', \pi') \models \neg\psi_1 \qquad (\text{defn } 2)$$

as required.

Inductive case 7 ψ takes the form $\psi_1 \wedge \psi_2$

$(M, \pi) \models \psi_1 \wedge \psi_2$

iff $(M, \pi) \models \psi_1$ and $(M, \pi) \models \psi_2$ (defn 2)

iff $(M', \pi') \models \psi_1$ and $(M', \pi') \models \psi_2$ (I. H.)

iff $(M', \pi') \models \psi_1 \wedge \psi_2$ (defn 2)

as required.

Inductive case 8 ψ takes the form $X \psi_1$

$(M, \pi) \models X \psi_1$

iff $(M, \pi[1..\infty]) \models \psi_1$ (defn 2)

Note that $(M, \pi) \approx (M', \pi')$ by (‡). Hence by the definition of a bisimilar path, $(M, \pi[i]) \approx (M', \pi'[i])$ for all $i \geq 0$. Again, by defn, we therefore have that $(M, \pi[1..\infty]) \approx (M', \pi'[1..\infty])$. Hence this condition is equivalent to

$(M', \pi'[1..\infty]) \models \psi_1$ (I. H.)

iff $(M', \pi') \models X \psi_1$ as required (defn 2)

Inductive case 9 ψ takes the form $\psi_1 \cup \psi_2$

$(M, \pi) \models \psi_1 \cup \psi_2$

iff $(M, \pi[i..\infty]) \models \psi_1$ for some $i \geq 0$

and $(M, \pi[j..\infty]) \models \psi_2$ for all $0 \leq j < i$

(defn 2)

By a similar argument as before

$(M, \pi[k..\infty]) \approx (M', \pi'[k..\infty])$ for any $k \geq 0$

Hence this is equivalent to the condition

$(M', \pi'[i..\infty]) \models \psi_1$ for some $i \geq 0$

and $(M', \pi'[j..\infty]) \models \psi_2$ for all $0 \leq j < i$

(I.H.)

iff $(M', \pi') \models \psi_1 \cup \psi_2$ (defn 2)

as required.

Hence we have shown that for all CTL* formulas, if $(M, t) \approx (M', t')$:

$(M, t) \models \phi$ iff $(M', t') \models \phi$

That is, the truth of CTL* formulas is preserved by bisimulations.

Assume $t \in M$ and $t' \in M'$ are CTL-equivalent.

Q7. We must show that (M, t) and (M', t') are bisimilar.

We must show that there exists a bisimulation B from M to M' and that $B(t, t')$.

Specifically, we show that the CTL-equivalence relation (which we denote a $\leftrightsquigarrow$) is such a bisimulation.

1. $\leftrightsquigarrow$ is a bisimulation from M to M'

Consider any states $u \in St$ and $u' \in St'$ such that $u \leftrightsquigarrow u'$. (*)

a. For all atoms p, we must show $u \in V(p)$ iff $u' \in V'(p)$

$u \in V(p)$
iff $(M, u) \models p$ (defn 2)
iff $(M', u') \models p$ (*)
iff $u' \in V'(p)$ (defn 2) as required

b. We must show that if $v \in St$ and $u \to v$, then there is $v' \in St'$ such that $u' \to' v'$ and $v \leftrightsquigarrow v'$.

- Assume there exists a $v \in St$ s.t. $u \to v$. (1)
- Assume for the purpose of contradiction that there is no $v' \in St'$ s.t. $u' \to' v'$ and $v \leftrightsquigarrow v'$. (2)
- Let $S' = \{w' \in St' \mid u' \to' w'\}$.
  Note that $(M, u) \models EXT$ (since $u \to v$ (1)) and $u \leftrightsquigarrow u'$ so $(M', u') \models EXT$. So S' is non-empty and finite.

- By (2), for every $w_i \in S'$ there exists a CTL formula ϕ_i such that $(M, v) \models \phi_i$ but $(M', w_i') \not\models \phi_i$.

- It follows that $(M, u) \models EX(\phi_1 \wedge \dots \wedge \phi_n)$ and that $(M', u') \not\models EX(\phi_1 \wedge \dots \wedge \phi_n)$.
- But that contradicts that $u \leftrightarrow u'$ (*).
- Hence the forth condition holds.

(c) The back condition can be shown similarly.

(2) We must show that $B(t, t')$

This holds directly from the definition of B and that t and t' are CTL-equivalent.

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Correct methodology but no actual attempt is seen to prove the back relation

Q8. We show that CTL and CTL* have the same distinguishing power.

Forwards :
- Assume (M, t) and (M', t') satisfy the same formulas of CTL.
- That is, for any CTL formula ϕ, $(M, t) \models \phi$ iff $(M', t') \models \phi$.
- That is, $t \in M$ and $t' \in M'$ are CTL-equivalent.
- By Q7, $(M, t) \simeq (M', t')$.
- By Q6, truth of CTL* formulas is preserved by bisimulations. That is, $(M, t) \models^* \phi^*$ iff $(M', t') \models^* \phi^*$ for any CTL* formula ϕ^*.
- Hence (M, t) and (M', t') satisfy the same formulas of CTL*, as required.

Backwards :
- Assume (M, t) and (M', t') satisfy the same formulas of CTL*.
- That is, for any CTL* formula ϕ^* $(M, t) \models^* \phi^*$ iff $(M', t') \models^* \phi^*$.
- By Q5a, for every CTL formula ϕ there exists a CTL* formula ϕ' such that ϕ and ϕ' are equivalent : $(M, t) \models \phi$ iff $(M, t) \models^* \phi'$ and $(M', t') \models \phi$ iff $(M', t') \models^* \phi'$.
- By the previous two points, we conclude that $(M, t) \models \phi$ iff $(M', t') \models \phi$ for every CTL formula ϕ.
- That is (M, t) and (M', t') satisfy the same CTL formulas, as required.

Distinguishing power is determined by a logic's ability to discern between particular models, whilst expressiveness refers to the definability of certain properties by formulas of a logic.

Since CTL* is strictly more expressive than CTL, but has the same distinguishing power, this means that although CTL* can express properties which CTL cannot (such as AF $(a \wedge X a)$ – in all paths, there are two consecutive nodes satisfying a), it is always possible to write a CTL formula which is satisfied in the same finite models as those satisfied by a CTL* formula.

Specifically, any non-bisimilar models (as per defn 3) can be distinguished, and so any set of non-bisimilar models could simply be distinguished by a disjunction of CTL formulas.

4

All correct but no attempt to resolve the contradiction

46 **Out of** **49**

1			
a/**2**	b/**2**	c/**3**	d/**3**
	Solution could have been simplified futher. However, the explanation given is very strong	Solution correct and extremely well explained	Solution correct and well explained, though the explanation for steps could have been presented in a stronger manner. However, since the main explanation is the truth condition, the solution is adequate
2	1	3	3

2			
a/**2**	b/**2**	c/**2**	d/**2**
2	2	2	2

3	
a/**3**	b/**2**
3	2

4
/5
5

5	
a/**2**	b/**2**
	Example well justified
2	2

6	7	8
/6	**/6**	**/5**
	Correct methodology but no actual attempt is seen to prove the back relation	All correct but no attempt to resolve the contradiction
6	5	4