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Journal of Control and Decision

ISSN: 2330-7706 (Print) 2330-7714 (Online) Journal homepage: https://www.tandfonline.com/loi/tjcd20

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Journal of Control and Decision, 2014
Vol. 1, No. 2, 115–148, http://dx.doi.org/10.1080/23307706.2014.913837

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Robust model predictive control: reflections and opportunities

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(*Received 4 April 2014; accepted 8 April 2014*)

The past three decades have witnessed important developments in the theory and practice of model predictive control (MPC). In particular, considerable effort has been devoted to robust MPC theory. There have also been many successful applications. This paper will give a brief overview of existing results and summarise experience gained in two real-world applications. We also present some reflections on issues which, in the authors' opinion, deserve further attention.

Keywords: robust MPC; application of MPC; power electronics; induction heating

1. Introduction

MPC, together with the Internal Model Principle, have arguably had more impact in practical control than any other control theory development over the past four decades. The success of MPC can be mainly attributed to its ability to account for multi-variable interactions, and its capability to take constraints and nonlinearities into consideration. The last couple of decades have witnessed many advances in the theory and practice of the method. (See, e.g. Goodwin, Seron, & Doná, 2005; Rawlings & Mayne, 2009, and the references therein.) Recent literature on MPC has focused on the robustness issue (Kouvaritakis et al., 2000; Løvaas, Seron, & Goodwin, 2008a; Mayne, Rawlings, Rao, & Scokaert, 2000; Mayne, Seron, & Raković, 2005; Raković, Findeisen, & Allgöwer, 2006), the problem of reference tracking (Løvaas, Seron, & Goodwin, 2010; Maeder & Morari, 2010), networked control (Kong, Goodwin, & Seron, in press; Liu, Xia, Chen, & Rees, 2007; Sun, Wu, & Liu, 2014), the extension to the stochastic setting (Bernardini & Bemporad, 2012; Goodwin, Østergaard, & Quevedo, 2009) and solutions in a distributed framework (Scattolini, 2009). In addition, there has been recent interest in developing sparse MPC. (See, e.g. Annergren, Hansson, & Wahlberg, 2012; Gallieri & Maciejowski, 2012; Kong et al., in press, and the references therein.)

Whilst it might be reasonable to consider MPC as a relatively mature technique both in theory and practice, much of the supporting theory focuses on analysis rather than design. Also, the theory emphasises the nominal case with known model and where the states are measured. On the other hand, in practice, a key issue is the role of imperfect models and the need to estimate unmeasured states. There has also been interest in aligning MPC controllers with an existing unconstrained controller designed by linear methods (see, e.g. Di Cairano & Bemporad, 2010; Hartley & Maciejowski, 2013; Kong, Goodwin, & Seron, 2012, 2013, and the references therein). In particular, Hartley and Maciejowski

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(2013), Di Cairano and Bemporad (2010) and Kong et al. (2012) show how to tune the weighting matrices in MPC so that the MPC controller performs the same as an existing linear controller, when the constraints are inactive. By doing so, the MPC controller inherits the small-signal properties of the linear control design and can still deal with constraints during transients. A recent survey of this class of tuning methods can be found in Garriga and Soroush (2010). A related idea is discussed in Kong et al. (2013), where a general tuning method called predictive metamorphic control (PMC) has been proposed allowing one to move smoothly from an existing controller to a new MPC strategy.

An important issue which has received little attention in the MPC literature is how to choose an observer in the presence of disturbances and/or uncertainties (DU). In classical and modern control theory (De Oliveira, Bernussou, & Geromel, 1999; De Oliveira, Geromel, & Bernussou, 2002; Duan, 2010; Goodwin, Graebe, & Salgado, 2001), for the *unconstrained* case, the feedback control law and the observer play equally important roles in determining the performance of the system. However, the joint design of feedback laws and observers in the presence of constraints is a difficult problem. While the results in Mayne et al. (2000, 2005, 2006) and Løvaas et al. (2008a, 2010), have provided much insight into robust issues for MPC, the observer gains in these papers are typically chosen with the only additional requirement that they render a stable closed-loop system. To the authors' knowledge, Pannocchia and Bemporad (2007) is one of the few works that has considered designing an observer in a robust MPC problem.

In this paper, we will give a very brief overview of the current state of the theory of robust MPC. More detailed reviews can be found in other recent papers, e.g. Mayne (in press). We will also reflect on this theory in the light of two practical case studies with which we have been involved. This leads us to formulate a set of opportunities and challenges for future research.

The layout of the remainder of the paper is as follows:

Section 2 gives a brief introduction to MPC. Section 3 reviews recent results related to the theory of robust MPC. Section 4 outlines practical experience gained from two applications. Section 5 describes opportunities and challenges in robust MPC. Conclusions are drawn in Section 6.

Notation: We use A^{T} to denote the transpose of matrix A. $\mathbf{R}^n$ stands for the n-dimensional Euclidean space. The notation $M > 0$ (≥ 0) means that M is real symmetric and positive definite (semi-definite). I_n stands for identity matrices of n dimensions. 0_n denotes matrices of n dimensions whose elements are all 0. Given two sets $\mathcal{U}$ and $\mathcal{V}$, such that $\mathcal{U} \subset \mathbf{R}^n$ and $\mathcal{V} \subset \mathbf{R}^n$, the Minkowski set addition is defined as $\mathcal{U} \oplus \mathcal{V} \triangleq \{u + v \mid u \in \mathcal{U}, v \in \mathcal{V}\}$.

2. Preliminaries on MPC

To set the scene, consider the simplest possible case of a discrete-time linear time-invariant (LTI) state space model:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k \end{cases}, \tag{1}$$

where $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times r}$ and $C \in \mathbf{R}^{m\times n}$ are constant matrices, x, u and y are the state vector, input vector and output vector, respectively. Assume that when MPC is used as the control strategy for the system (1), at kth sampling instant, the desired performance is measured via a quadratic cost function of the following form:

$$J_N(x_k, \mathbf{u}) = \sum_{t=0}^{N-1} \left[x_{k+t|k}^{\mathrm{T}} Q x_{k+t|k} + u_{k+t|k}^{\mathrm{T}} R u_{k+t|k} \right] + x_{k+N|k}^{\mathrm{T}} P x_{k+N|k}, \tag{2}$$

where

$$\begin{cases} P > 0, \quad Q \geq 0, \quad R > 0, \quad x_{k|k} = x_0 \\ u_{k+t|k} \in \mathbf{U}, \qquad \text{for } t = 0, \dots N-1 \\ y_{k+t|k} \in \mathbf{Y}, \qquad \text{for } t = 1, \dots N \\ x_{k+t|k} \in \mathbf{X}, \qquad \text{for } t = 0, \dots N \\ x_{k+N|k} \in \mathbf{X}_f \end{cases}, \tag{3}$$

and $\mathbf{u}$ contains the control moves u_{k+t} defined over the finite horizon $t \in [0, N-1]$ with $N \in \mathbb{Z}^+$ and $N \geq 1$, that is,

$$\mathbf{u} = \left[u_{k|k}^{\mathrm{T}}, u_{k+1|k}^{\mathrm{T}}, \cdots, u_{k+N-1|k}^{\mathrm{T}} \right]^{\mathrm{T}}. \tag{4}$$

The vector $x_{k+t|k}$ in (2) represents the prediction of the state vector of the system at time $k+t$ given the information available at time k and based on a prediction form of the model in (1), i.e.

$$\begin{cases} x_{k+t+1|k} = Ax_{k+t|k} + Bu_{k+t|k}, & k = 0, 1, \dots, \\ y_{k+t|k} = Cx_{k+t|k}, & x_{k|k} = x_k \end{cases} \tag{5}$$

Remark 1 The above notation is simplistic and may lead to confusion in some sophisticated treatments of MPC. If there is potential for misunderstanding, then it is best to be explicit i.e. to use a notation, such as, $\bar{x}^{\mathbf{u},\boldsymbol{\omega}}(k; \bar{x}(j))$ which should be interpreted as the solution at time step k to the prediction model (which may differ from the true system) when the input sequence is chosen to be $\mathbf{u}$, the disturbance sequence is chosen to be $\boldsymbol{\omega}$ and when the initial state at time step j is selected to be $\bar{x}(j)$.

The basic idea of MPC is to start with a fixed optimisation horizon, N, as in (2), using the current measured (or estimated) state of the plant as the initial state. We then optimise the objective function (2) over the fixed interval accounting for constraints to obtain a sequence of N control moves. Only the first control (action) in the sequence $\mathbf{u}$ is applied to the plant. Time then advances one step and the same N-step optimisation problem is repeated using the new state of the plant as the initial state. Thus, one continuously revises the current control action based on the current state and accounting for the constraints over an optimisation horizon of length N. The matrix P, terminal constraint set $\mathbf{X}_f$ and a local stabilising controller in $\mathbf{X}_f$ are typically selected to ensure closed-loop stability (Rawlings & Mayne, 2009).

3. Overview of robust MPC theory

The simple MPC formulation described in the previous section implicitly assumes that the nominal model (1) is an exact description of the plant so that the prediction model used for computation of the optimising sequence is the same as the real plant. However, the above assumption is never satisfied in practice. This has motivated the development of classical robust control theory (see, e.g. Petersen, Ugrinovskii, & Savkin, 2000; Zhou, Doyle, & Glover, 1996) and other control strategies such as adaptive control (Middleton, Goodwin, Hill, & Mayne, 1988). We refer to the problem of MPC for the ideal plant (1) as the nominal MPC question. In general, the presence of disturbances and uncertainty (DU) has important consequences in the theory of MPC since it affects both the issues of stability and constraint satisfaction. Therefore, the impact of DU has to be examined so that the stability and feasibility results for the nominal MPC problem are reasonably revised so that robust stability and robust constraint fulfillment are obtained.

The issue of how best to model a system's DU depends on a large number of factors such as the physical model of the system, the complexity of the model, and the necessary effort needed to perform model identification and validation techniques. Note that the latter issues are not the core focus of the current paper. Instead, we shall concentrate our attention on available robust MPC methods for certain given classes of DU based on the assumption that these DU models adequately capture the uncertainties in the plant. Moreover, it is expected that different types of DU may have nonidentical effects on the system. Thus, they need dissimilar treatments. Hence, it becomes natural for us to divide our discussion based on the characteristics of the DU. In the remainder of this section, we will present the most studied cases of DU and make some observations on the existing methodologies to tackle them. In the following, we will categorise the DU into several categories: bounded additive disturbances, parametric uncertainties (including polytopic uncertainties, and norm-bounded model uncertainties), and unstructured uncertainties/unmodelled dynamics.

Most recent results on robust MPC are based on state-space models. We remark that our discussion does not cover some earlier results on robust MPC for systems with uncertainties in coefficients of the step-response and impulse-response. (Interested readers can refer to Bemporad and Morari (1999) and the references therein.) We also remark that MPC for systems with stochastic DU is becoming an active area of research (see, e.g. Bernardini & Bemporad, 2012; Goodwin et al., 2009 and the references therein). We do not include a discussion of this area, for the sake of space. For a relatively recent and more comprehensive survey of MPC, we refer to Mayne (in press).

3.1. Robust MPC for systems with bounded additive disturbances

In general, this problem setup differs from the nominal one in that the real system is assumed to be subject to additive bounded process/measurement disturbances. In this case, the plant is modelled as follows:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + \omega_k \\ y_k = Cx_k + \gamma_k \end{cases}, \tag{6}$$

where A, B and C are defined as in (1), x_k, u_k and y_k are the state vector, input vector and output vector, respectively. $\omega_k \in \mathbf{R}^n$ is an unknown state disturbance and $\gamma_k \in \mathbf{R}^m$ is an unknown output disturbance. The process and measurement disturbances ω_k and γ_k are known only to the extent that they lie, in the C-sets $\mathscr{W} \subset \mathbf{R}^n$ and $\mathscr{R} \subset \mathbf{R}^m$, respectively. The system (1) is referred to as the nominal system for (6). There are several existing results in the robust MPC literature dedicated to the above DU framework. There are three main approaches, namely the tube-based approach, the input-to-state stability approach and the min-max approach. In the following, we will discuss only the tube-based approach. Readers interested in the other two approaches are referred to the recent survey papers (Limon et al., 2009; Raimondo, Limon, Lazar, & Magni, 2009).

To the best of the authors' knowledge, tube-based robust MPC for linear systems was first conceived in a series of papers (Chisci, Rossiter, & Zappa, 2001; Langson, Chryssochoos, Raković, & Mayne, 2004; Mayne & Langson, 2001; Mayne et al., 2005, 2006). The tube-based robust output feedback MPC controller deploys a stable Luenberger observer:

$$\begin{cases} \widehat{x}_{k+1} = A\widehat{x}_k + Bu_k + L(y_k - \widehat{y}_k) \\ \widehat{y}_k = C\widehat{x}_k \end{cases}, \tag{7}$$

to estimate the state of the real plant (6). L is chosen such that $\rho(A_L) < 1$, where $A_L = A - LC$ and $\rho(\cdot)$ denotes the largest among the eigenvalue magnitudes. It can be verified that the estimated state $\widehat{x}$ satisfies the following uncertain difference equation:

$$\widehat{x}_{k+1} = A\widehat{x}_k + Bu_k + \delta_k$$

where

$$\delta_k = L(C\widetilde{x}_k + \gamma_k)$$

and $\widetilde{x}_k \triangleq x_k - \widehat{x}_k$ satisfies

$$\widetilde{x}_{k+1} = A_L\widetilde{x}_k + \widetilde{\omega}_k,$$

where

$$\widetilde{\omega}_k = \omega_k - L\gamma_k.$$

Moreover, $\widetilde{\omega}_k$ takes values in the C-set $\widetilde{\mathscr{W}}$ defined by

$$\widetilde{\mathscr{W}} := \mathscr{W} \oplus (-L\mathscr{R}).$$

The state estimation error $\widetilde{x}_k$ is guaranteed to be bounded by an invariant set. In tube-based MPC, the observer (7) is controlled such that the real state

$$x_k = \widehat{x}_k + \widetilde{x}_k$$

satisfies the state constraint and the associated control satisfies the given control constraint. To achieve this, a nominal system

$$\overline{x}_{k+1} = A\overline{x}_k + B\overline{u}_k,$$

is introduced by neglecting the disturbances ω_k and γ_k. In order to counteract the disturbances, the tube-based controller takes the form

$$u_k = \overline{u}_k + Ke_k,$$

where $\overline{u}_k$ is a general MPC controller for the nominal system, and Ke_k is a feedback part with K satisfying $\rho(A + BK) < 1$. The tracking error, e_k, is defined as follows

$$e_k \triangleq \widehat{x}_k - \overline{x}_k.$$

The analysis proceeds by showing that, the tracking error, e, is bounded by an invariant set. This ensures that all possible realisations of the state trajectory lie in a simple uncertainty tube the "centre" of which is the solution of a nominal system and the "cross-section" of which is invariant, if the observer starts in "steady-state". Satisfaction of the state and input constraints for the original system is guaranteed by employing tighter constraint sets for the nominal system. The complexity of the resultant controller is similar to that required for nominal model predictive control. We remark that tube-based robust MPC relies on set theory (see e.g. Blanchini, 1999; Blanchini & Miani, 2008 and the references therein) and is facilitated by recently proposed methods aimed at computing robust positive invariant sets. (See e.g. Kolmanovsky & Gilbert, 1998; Raković, Kerrigan, Kouramas, & Mayne, 2005; Raković, Kerrigana, Mayne, & Kouramas, 2007 and the references therein).

The tube-based method has been generalised in several directions. For example, for discrete-time LTI systems, it has been enhanced in Raković, Kouvaritakis, Findeisen, and Cannon (2012), Raković, Kouvaritakis, Cannon, Panos, and Findeisen (2012) and Raković, Kouvaritakis, Cannon, and Panos (2012), through more general parameterisations of the state and control tubes based on set theory and recent developments in the computation of the robust invariant sets. (See e.g. Raković & Baric, 2010 and the references therein.) In

Kong et al. (2013), the PMC method was used to control the nominal system in tube-based robust MPC. Robust stability of the PMC method and constraint satisfaction have been guaranteed by the tube-based MPC mechanism. In Mayne, Raković, Findeisen, and Allgöwer (2009), the tube-based robust MPC method has been extended by allowing the estimator to be time varying so as to cover the problem of output feedback MPC for discrete time systems in the presence of additive but bounded state and output disturbances. Tube-based MPC has also been generalised to the control of nonlinear systems (Cannon, Buerger, Kouvaritakis, & Raković, 2011; Mayne, Kerrigan, van Wyk, & Falugi, 2011; Yu, Maier, Chen, & Allgöwer, 2013).

Of course, assuming bounded disturbances does not allow the theory to cover unmodelled dynamics since, in the latter case, the disturbance ω in (6) is input dependent.

3.2. *Robust MPC for systems with parametric uncertainties*

In this section, we review existing results on robust MPC for systems with parametric uncertainties. We first review the problem setup. A linear discrete-time system

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k \end{cases}, \tag{8}$$

is said to have polytopic uncertainty, if $[A, B] \in \Omega \triangleq Co\{[A_1, B_1], \cdots, [A_M, B_M]\}$ with Co denoting convex hull. The parameter variations are described by

$$[A, B] = \sum_{i=1}^{M} \lambda^i \left[A^i, B^i\right] \tag{9}$$

where λ^i is nonnegative for $i = 1, \ldots, M$ and $\sum_{i=1}^{M} \lambda^i = 1$.

A linear discrete-time system with structured norm-bounded model uncertainty typically refers to a system with perturbations appearing in the feedback loop in the following form:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + B_p p_k \\ y_k = Cx_k \\ q_k = C_p x_k + D_p u_k \\ p_k = \triangle \cdot q_k \end{cases}, \tag{10}$$

where q_k and p_k are variables accounting for the uncertainty and $\triangle$ is usually a norm-bounded time-varying matrix in a set, defined as

$$\triangle = \begin{bmatrix} \triangle_1 & & \\ & \ddots & \\ & & \triangle_r \end{bmatrix}$$

with $\|\triangle_i\|_2 \leq 1$, for $i = 1, \cdots, r$. It has been shown in the robust control literature that a number of control systems with uncertainties can be recast in these two frameworks. (See e.g. Boyd, El Ghaoui, Feron, & Balakrishnan, 1994; Petersen et al., 2000; Zhou et al., 1996 and the references therein.) Depending on the problem setup, the constraint set may be different. Therefore, we do not explicitly formulate the constraint sets here but adopt the constraint expression in Section 2.

The literature on the above two robust MPC setups, especially the former one, is large. To the best of the authors' knowledge, Kothare, Balakrishnan, and Morari (1996) is the earliest general work on robust MPC for systems with polytopic uncertainties, or structured norm-bounded model uncertainties. Moreover, the framework in Kothare et al. (1996) has been adopted by many other researchers and generalised to many other problems. We therefore review the basic idea of Kothare et al. (1996) and then discuss other related works.

The method in Kothare et al. (1996) is developed in three steps: (i) Formulate an infinite horizon robust unconstrained problem. Then, by making appropriate assumptions, it is recognised that the cost has an upper bound. (ii) A constant state-feedback law is then constructed via convex optimisation which can satisfy the aforementioned assumptions. (iii) By imposing additional requirements on some matrices in the previous convex optimisation problem, an invariant ellipsoid is obtained for the system, thereby allowing constraint satisfaction and robust stability to be achieved. We proceed to give further details of these three steps. The cost considered in Kothare et al. (1996) takes the following form

$$\begin{cases} J_\infty(x) = \min_u \max_\lambda \sum_{i=0}^{\infty} \left[|x_{k+i}|_Q^2 + |u_{k+i-1}|_R^2 \right] \\ \text{s.t. (8) holds with } x \text{ the current state at time } k, (A, B) \text{ given by equation (9),} \\ \quad u_{k+i} \in U, y_{k+i} \in Y, \lambda_{k+i} \in \Lambda \triangleq \left\{ \lambda | \lambda^j > 0, \Sigma\lambda^j = 1 \right\} \end{cases} \tag{11}$$

It has been assumed that there is no control action after the prediction horizon. It can be concluded that the above min-max optimisation problem is convex. However, the problem (11) is not computationally tractable. Therefore, deriving an upper bound on the robust performance objective in (11) becomes an alternative. Consider a quadratic Lyapunov function $V(x) = x^{\mathrm{T}}Px$ with $P > 0$. At kth sampling instant, suppose that V satisfies the following inequality for all predictions $u_{k+t|k}, x_{k+t|k}, t \geq 0$ based on (8),

$$V(x_{k+t+1|k}) - V(x_{k+t|k}) \leq -x_{k+t|k}^{\mathrm{T}} Q x_{k+t|k} - u_{k+t|k}^{\mathrm{T}} R u_{k+t|k}. \tag{12}$$

For the robust performance objective in (11) to be finite, one must have $x_{\infty|k} = 0$, therefore, $V(x_{\infty|k}) = 0$. Summing (12) from $t = 0$ to $t = \infty$, we obtain

$$-V(x_{k|k}) \leq -J_\infty(k),$$

thereby establishing $\max J_\infty(k) \leq V(x_{k|k})$. This is an upper bound on the robust performance objective. After that a constant state-feedback law

$$u_{k+t|k} = F x_{k+t|k}$$

minimising the upper bound $V(x_{k|k})$ is investigated and the appropriate F and $P > 0$ are found by formulating a set of LMIs. The LMIs are solved at each sampling instant and only the first move $u_{k|k} = F x_{k|k}$ is applied. After this, by adding more LMI constraints on the matrix P in the convex programming problem, an invariant ellipsoid for the predicted states of the uncertain system is constructed. It is shown in Kothare et al. (1996) how input and output constraints can be incorporated as additional LMI constraints in the robust MPC problem. The proposed method possesses desirable properties such as recursive feasibility and robust stability, as proven in Kothare et al. (1996). Several extensions such as constant set-point tracking, reference trajectory tracking, disturbance rejection and application to delay systems have been presented. The methodology has been adopted in many other papers and the results of Kothare et al. (1996) have been extended and generalised in many directions (see e.g. Angeli, Casavola, Franze, & Mosca, 2008; Cao & Lin, 2005; Cuzzola, Geromel, & Morari, 2002; Ding, Xi, & Li, 2004; Gautam, Chu, & Soh, 2012; Lee, Cannon, & Kouvaritakis, 2005; Kouvaritakis, Rossiter, & Schuurmans, 2000;

Yu, Böhm, Chen, & Allgöwer, 2012; Zheng, Li, Xi, & Zhang, 2013 and the references therein).

Other methods have also been proposed in the literature. For example, in Lee and Yu (1997), it is assumed that the uncertainties lie in the system matrices and are characterised by an uncertain and bounded (in a certain sense) vector that parameterises the system matrices which can be constant or time-varying. Two different predictive control formulations, one assuming open-loop control and the other considering closed-loop control, are developed based on minimisation of the worst-case quadratic cost for systems with the bounded parameters. The associated closed-loop properties, such as asymptotic stability, are examined. Reference Badgwell (1997) presents a robust MPC algorithm for stable linear plants, subject to hard input and soft-state constraints. Model uncertainty is parameterised by a list of possible plants. Robust stability is achieved through the addition of constraints that prevent the sequence of optimal costs from increasing for the true plant. Asymptotic stability is demonstrated through a Lyapunov argument. In Lee and Cooley (2000), min-max MPC of a linear state-space system with uncertainties having the same form as those in Lee and Yu (1997) is studied based on a quadratic performance criterion. Both stable and integrating system dynamics as well as time-varying and time-invariant parameter cases are treated. Reference Bemporad, Borrelli, and Morari (2003) develops an approach to determine state feedback robust MPC controllers based on a min–max control formulation for constrained discrete-time uncertain linear systems with additive norm-bounded input disturbances and/or polyhedral parametric uncertainties in the state-space matrices. It has been shown that the finite-horizon robust optimal control law is a continuous piecewise affine function of the state vector and can be calculated by solving a sequence of multiparametric linear programs.

3.3. Robust MPC for systems with unstructured uncertainties

3.3.1. Linear systems

Compared to the uncertainty formulations discussed above, the case of constrained systems with unstructured uncertainties is much less studied in the literature (see Canale, Fagiano, & Signorile, 2013; Canale & Milanese, 2003; Falugi & Mayne, 2014; Hovland, Løvaas, Gravdahl, & Goodwin, 2008; Løvaas et al., 2008a; Løvaas, Seron, & Goodwin, 2008b; Løvaas et al., 2010). In Løvaas et al. (2008a), a class of robust output-feedback MPC policies for discrete-time systems with constraints and unstructured model uncertainty is presented. The discrete-time uncertain system is described by the feedback interconnection shown in Figure 1,

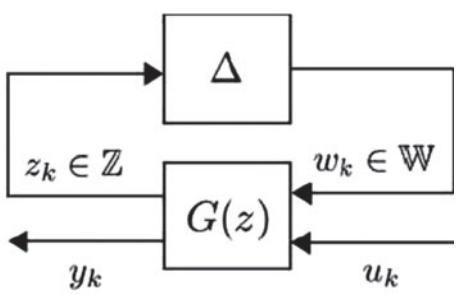

Figure 1. Uncertain system (reproduced from Løvaas et al. (2008a)).

where

$$G(z) = \begin{bmatrix} G_{11}(z) & G_{12}(z) \\ G_{21}(z) & G_{22}(z) \end{bmatrix} \tag{13}$$

is a known rational transfer function, and where $\triangle : \ell_{2e}(\mathbf{Z}) \rightarrow \ell_{2e}(\mathbf{W})$ is an operator satisfying $\|\triangle\|_\infty < 1$. For the differences between the uncertainty model in this section and the above two sections, we refer to Løvaas et al. (2008a) or Zhou et al. (1996), Petersen et al. (2000), Boyd et al. (1994) and the references therein. To impose strict causality from the control input $u_k \in \mathbf{R}^{n_u}$ to the system output $y_k \in \mathbf{R}^{n_y}$ for any causal $\triangle : \ell_{2e}(\mathbf{Z}) \rightarrow \ell_{2e}(\mathbf{W})$, it is required that $G_{21}(z)$ and $G_{22}(z)$ be strictly causal. To ensure that the system is well defined, $G_{11}(z)$ is assumed to be strictly causal. The Input–output map of the system in Figure 1 is given by the LFT

$$H = G_{22} + G_{21}\triangle(I - G_{11}\triangle)^{-1}G_{12}.$$

Therefore, G_{22} can be considered as the nominal model of the system. The norm-bound $\|\triangle\|_\infty < 1$, the sets $\mathbf{Z}$, $\mathbf{W}$,and G_{21}, G_{11}, G_{12} describe the model uncertainty associated with G_{22}. A state-space representation of the uncertain system has the form

$$\begin{cases} x_{k+1} = Ax_k + B_\omega \omega_k + Bu_k \\ z_k = C_z x_k + D_z u_k \\ y_k = Cx_k \end{cases}, \tag{14}$$

where $x_k \in \mathbf{R}^{n_x}$, $u_k \in \mathbf{R}^{n_u}$ and $y_k \in \mathbf{R}^{n_y}$ are the state vector, the input vector and the output vector, respectively; the auxiliary system input $\omega_k \in \mathbf{W} \subseteq \mathbf{R}^{n_\omega}$ is the response of $\triangle : \ell_{2e}(\mathbf{Z}) \rightarrow \ell_{2e}(\mathbf{W})$ to the auxiliary system output $z_k \in \mathbf{Z} \subseteq \mathbf{R}^{n_z}$. The system is subject to constraints which can be expressed in state-space coordinates as follows:

$$q_k \in \mathbf{Q}, \quad \forall k \geq N^+, \quad q_k = C_q x_k + D_q u_k \tag{15}$$

where C_q and D_q are matrices of appropriate dimension and $\mathbf{Q} \subseteq \mathbf{R}^{n_q}$ is some given (polyhedral) set. It is assumed that $\triangle$ describes *unstructured* uncertainty in the sense that it may comprise nonlinear, time-varying and infinite-dimensional dynamics, provided only that it is causal and that its ℓ_{2e}-gain is bounded. The MPC policies proposed in Løvaas et al. (2008a) employ a linear state estimator to estimate the state of the plant (14). Moreover, it is assumed that an existing feedback gain has been designed to satisfy a small-gain lemma. Also, a robust invariant set for the closed-loop system is assumed to be available. Then the pre-specified feedback gain is designed to be perturbed by the available degrees of design freedom including the quadratic cost function and the estimated state. At each time step k, the MPC policy minimises the cost function subject to a set of tighter constraints which guarantee that the predicted N-step perturbation of the pre-specified policy is constraint admissible and results in a "terminal state" that belongs to the existing robust invariant set for the closed-loop system. The unifying aspect is that all MPC policies within the class satisfy a robust stability test expressed in the form of an LMI condition which involves the parameters of the cost function. This is shown to always be feasible under an appropriate small-gain condition on the pre-determined feedback gain and the state estimator.

Moreover, by means of both theoretical and numerical results, it has been shown that choosing the cost function parameters subject to the proposed condition often leads to good nominal performance, whilst at the same time, guaranteeing robust stability. The results in Løvaas et al. (2008a) have been extended in Løvaas et al. (2010) to a class of systems, having nonvanishing constant output disturbances, hard constraints and unstructured uncertainty. Integral action is included in the proposed design by applying a similar procedure to that in

Maeder and Morari (2010). In Løvaas et al. (2008b) and Hovland et al. (2008), the method has been utilised to cover uncertain constrained systems with soft-state constraints, and stability analysis of MPC based on reduced-order models, respectively.

3.3.2. Nonlinear systems

Nonlinear constrained systems with unstructured uncertainty have been considered in Falugi and Mayne (2014). The structure of the system and uncertainty is the same as in Figure 1 except the system is nonlinear. The results in Falugi and Mayne (2014) extend tube-based model predictive control to the control of nonlinear systems with unmodelled dynamics. The problem of obtaining robustness against unstructured uncertainty is converted into the easier problem of achieving robustness against an additional bounded disturbance while satisfying an additional output constraint which depends on the maximum l_∞ gain of the unstructured dynamics. Reference Canale and Milanese (2003) presents a robust MPC strategy for systems with constraints and modelling errors. It is assumed that the system is described by an uncertainty model set made up of a nominal approximate model and a bound on the unmodelled dynamics. Assuming that the plant is open-loop stable, the proposed robust MPC law is shown to guarantee satisfaction of hard input constraints and zero regulation properties despite the presence of unmodelled dynamics. In Canale et al. (2013), the robust design of nonlinear MPC laws that employ approximate models, derived directly from input–output data by means of a nonlinear set membership identification technique, has been studied. The nonlinear identification technique is also able to provide an estimate of the uncertainty associated with the model, which is employed to design a robust predictive controller by using a min-max formulation of the finite horizon optimal control problem.

4. Some practical experience with MPC

In this section, we briefly summarise two practical problems for which the authors have investigated the use of MPC. The goal in presenting these case studies is to reveal future opportunities in MPC research.

4.1. Induction heating furnace control

4.1.1. Background

Further discussion of the work presented here can be found in Goodwin, Middleton, Seron, and Campos (2013). Induction Heating Furnaces are frequently used in industry. These systems consume extremely large amounts of energy. Indeed, the annual cost of electrical energy for a typical furnace can exceed US\$1M. Hence, there is substantial financial motivation to reduce energy consumption.

A schematic diagram of a typical furnace is given in Figure 2.

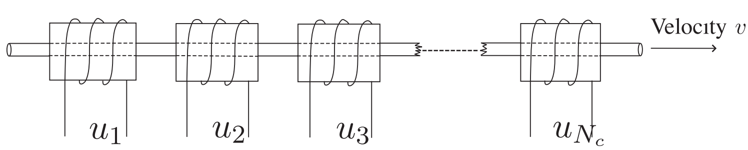

Figure 2. Schematic diagram of induction heating furnace $\{u_i \equiv$ (Voltage, Current, Frequency); $i = 1, \ldots, N_c\}$.

As illustrated in Figure 2, the key features of the apparatus are that a cylindrical rod is passed through a succession of coils. These coils are capable of being operated at different voltages, currents and frequencies. Moreover, the rod can be moved at different speeds through the furnace. The coils induce currents in the rod which, due to hysteresis effects, heat the rod. Most of the current in the rod is initially confined to the surface due to skin effect. Once the temperature passes the Curie point (Rudnex, Loveless, Cook, & Black, 2003), then the relative permeability drops from about 200 to near 1 (i.e. that of air).

This problem has a range of associated challenges (Scattolini, 2009; Zinn & Semiatin, 1987) including:

(i) Whilst one can reasonably assume radial symmetry of temperatures within the rod, the temperatures are not constant in the longitudinal direction. This complicates model development and can lead to excessive model complexity. For example, if one were to divide the radial and longitudinal spatial dimension into 1000 segments each, then one would have a model containing a million state variables!
(ii) Due to skin effect phenomena (Rudnex et al., 2003), much of the heating takes place in a narrow surface region. This region can be of the order of 1/2 mm or less in thickness and is associated with rapid thermal heating. By way of contrast, the core heats slowly. Hence, the model is inherently "stiff" with a wide range of associated time constants. Moreover, the skin effect region progressively moves into the core as the outer temperature passes the Curie point.
(iii) All of the parameters appearing in the model are highly temperature dependent. Moreover, the temperatures within the rod range from ambient to above the Curie point. Some parameters, e.g. relative permeability, change by several orders of magnitude as the rod traverses a given coil. Hence, the system is highly nonlinear.
(iv) The available measurements are very limited being restricted, in essence, to surface temperature measurements (made by pyrometers) and the external currents and voltages.
(v) The system has many degrees-of-freedom in the input including line speed, voltages, currents and frequencies in the induction coils (of which there may be 10 or 20 in a given set-up).
(vi) The system needs to satisfy a large number of constraints. These include:
    (1) A lower bound on minimum exit temperature.
    (2) An upper bound on maximum exit temperature.
    (3) Constraints on temperature gradients in both time and space to avoid metallurgical stress on the material.
    (4) Bounds on operating voltages, currents, frequencies and switching times dictated by the driving power electronics.

4.1.2. Model

The system is, in principal, described by a set of partial differential equations which include Fourier's Law (for heat conduction), Newton's Law (for thermal heating) and Maxwell's equations (for electromagnetic phenomena). Detailed models for this problem can be found in Bay, Labbe, Favennec, and Chenot (2003), Skoczkowski and Kalus (1989), Egan and Furlani (1991), Pascal, Conraux, and Bergheau (2003), Jang, Cho, Lee, Cho, and Park (2003), Yang, Wang, and Yan (2008), Qianzhe, Yibing, Yanping, and Weisong (2011), Chaboudez et al. (1997), Fisk (2008), ter Maten and Melissen (1992) and Chaboudez et al. (1994). Our approach was first to develop a *control-relevant* model containing approximately 1000 states. The model development is described in Goodwin et al. (2013). A brief outline is given below.

To develop the model, a thin cylindrical strip is tracked as it passes through each coil. Let the cylinder strip width be W and the length of each coil be L. Time is discretised so that the cylindrical strip moves forward a distance W in each time step. It then takes $N = L/W$ time steps to transit a coil. During transition between coils, the input is zero and hence the rod cools due to thermal radiation.

The model takes the following generic form:

$$\bar{x}^j_{k+1} = f\left(\bar{x}^j_k, u^j\right); \quad j = 1, \ldots, N_c; \quad k = 1, \ldots, N \tag{16}$$

where the superscript j refers to the coil and the subscript k denotes the time step (i.e. progress through the coil).

The model parameters were initially chosen based on published empirical data. The model was then calibrated using typical values found in industry for the manipulated and process variables.

The available control signals are the current into each coil and the frequency used in each coil. Also, the performance can be evaluated based on different rod velocities. Note that the control is constant in each coil. Between coils, the same model is used save that the externally applied input is zero and the number of time steps, N', will depend on the distance between coils.

4.1.3. Nonlinear MPC

The basic principle behind the development of the control law is to use finite horizon optimisation in a nonlinear MPC control law.

We use a cost function which captures energy utilisation in the N_c coils. This can be expressed as

$$J = \sum_{j=1}^{N_c} \ell\left(u^j\right) \tag{17}$$

We optimise (17) subject to (16) and the constraints listed earlier. Results presented in Goodwin et al. (2013) suggest that energy savings of the order of 10% can be achieved via this MPC control law.

4.1.4. Observer

The model used to develop the MPC controller depends on many simplifying assumptions. Hence, it cannot exactly describe the system behaviour. Consequently it is desirable that the model be first calibrated by analysing offline data from the system. In addition, the model should be “adapted” online by comparing the model response with online data. A nonlinear observer driven by the available measurements is proposed for the latter purpose.

Since the model is highly nonlinear, then design of the observer represents a challenge in its own right. One possible approach to designing the appropriate observer is to use optimisation to estimate the state using a cost function of the form

$$J_0 = \sum_{j=1}^{N_c} \sum_{k=1}^{N} \left(y^j_k - \hat{y}^j_k\right)^T \left(y^j_k - \hat{y}^j_k\right) + \left(v^j_k\right)^T Q v^j_k \tag{18}$$

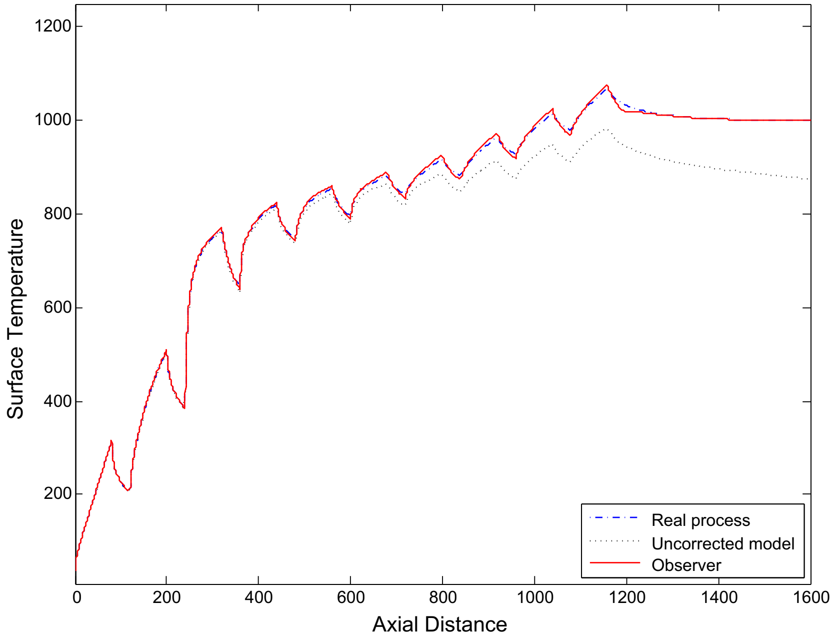

Figure 3. Surface temperature comparisons.

where y_k^j $\left(\text{normally modelled as } h\left(x_k^j\right)\right)$ is the sequence of available measurements along the furnace, $Q \geq 0$ is a weighting matrix and v_k^j is an estimate of the process noise which has been disregarded in (16). The observer takes the following form:

$$\hat{x}_{k+1}^j = f\left(\hat{x}_k^j, u^j\right) + L\, v_k^j \quad ; \quad \hat{y}_k^j = h\left(\hat{x}_k^j\right) \tag{19}$$

for some appropriate matrix L. The basic idea of the observer, is to adjust $\{v_k^j\}$ to minimise J_0. However, depending on the choice of the matrix L, there may be a major complexity issue since the dimension of v_k^j may be of the order of a thousand! Hence, some "parametrisation" of the process noise is desirable to regularise the state estimation problem. In our studies, we have reduced the complexity of the process noise by restricting it to errors in the heat radiated from the surface. This architectural change is important in obtaining successful results using the observer. This can be viewed as adding a sparsity constraint to the observer.

To illustrate the idea, we simulated a hypothetical "real" process by setting the exponent in the heat radiation loss from the surface to be 3 instead of the nominal value 4 used in the observer and controller model. (Note that this is a more dramatic change than considered in the earlier paper Goodwin et al., 2013).

Figure 3 shows the "real" surface temperature (thick dash-dotted line), the response of the uncorrected model (thin dotted line) and the observer output (red solid line). Figure 4 shows the corresponding results for the core temperature. We see that the observer correctly predicts the core temperature. Note that, since this is a simulation study, the "real" core temperature is available. This allows us to validate the observer performance. Of course, in practice, the core temperature is not available. Results using experimental data are described in Goodwin et al. (2013).

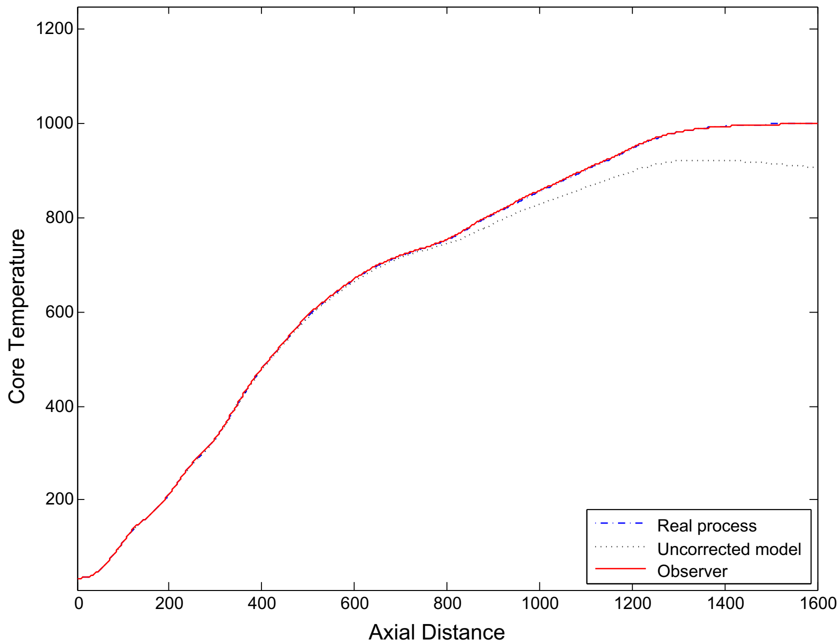

Figure 4. Core temperature comparisons.

4.1.5. *Distinctive features*

The distinctive features of this problem are:

(i) The real system is hugely complex so a much simplified (control relevant) model is needed.
(ii) The simple model needs to be calibrated using offline data and adapted to online data.
(iii) A nonlinear observer is necessary to provide state estimates for use in the MPC calculations.
(iv) Local minima problems are a source of difficulty since the problem is inherently nonconvex.

4.2. *Application of MPC to harmonic suppression in power electronics*

In this section, we describe the application of MPC to a problem in power electronics. In recent years, MPC has become extremely popular in this general area – see Cortes et al. (2010), Kouro et al. (2009), Rodriguez et al. (2004), Rodriguez et al. (2013), Goodwin et al. (2010) and Romero, Seron, and Goodwin (2011). Due to computational constraints it is common to use a one-step prediction horizon. We will adopt this restriction here but note that the extension to longer horizons is straightforward provided the issue of computational complexity is addressed.

4.2.1. *Background*

We will study the specific problem of harmonic suppression in the presence of nonlinear switching delays. An alternative view (based on a Feedback Quantizer) of the work presented here is described in Mirzaeva and Goodwin (in press).

An inverter is a switching electronics device that generates sinusoidal voltage waveforms, of variable amplitude and frequency, using a DC voltage source as its input (Holmes, Lipo, McGrath, & Kong, 2009). Inverters play a central role in modern society including integration of wind and solar energy into the grid, high voltage DC power transmission, electric motor speed control, power conditioning, uninterruptible power supplies, etc. An important requirement is that the waveform generated by an inverter should be as free as possible from certain types of harmonic distortion for reasons of efficiency, maintaining the integrity and stability of power infrastructure avoiding resonance in mechanical loads and mitigating electromagnetic interference with other devices. Hence, there has been substantial ongoing interest in harmonic suppression. This problem is made difficult by several core factors, especially the inherent switched nature of the waveforms. Also, the switching patterns are perturbed by unavoidable switching delays which depend, inter alia, on direction of current flow, charge, temperature, etc. Thus, the problem is beset by fundamental difficulties. Indeed, nonlinear switching delays (known as inverter "dead time delays") are a classical open problem mentioned in essentially every power electronics reference source, see, for example, Holmes and Lipo (2003). Here we show how nonlinear MPC can be used to address the combined problems of harmonic suppression and delay compensation.

4.2.2. Harmonic suppression

Harmonics are generated by the switched nature of the voltage waveform. These can be viewed as periodic disturbances of known frequency which appear on the inverter output voltage. In this context, we note that a core consequence of the Internal Model Principle (Francis & Wonham, 1975) is that one needs to include a model for the disturbance to achieve zero tracking error. See Maeder and Morari (2010) for a discussion of this concept within the MPC framework.

In the remainder of this subsection we ignore switching delays. The latter are considered in the next subsection. In the absence of delays, the input is of zero-order-hold type and a discrete model describing the at-sample response is easy to formulate by applying the matrix exponential to an underlying continuous-time model.

Remark 2 A similar idea to that used here for harmonic suppression also applies to achieving zero-state error in the face of constant off-sets and model gain mismatch. In the latter case, one should include a model for a constant off-set in the model. We will not pursue this aspect further here since it can be viewed as a special case of harmonic suppression (at zero frequency).

Returning to the problem of harmonic suppression, we note that a state-space model for a sinusoidal signal $\{d_k^h\}$ of nominal frequency ω^h can be written as

$$x_{k+1}^h = A^h x_k^h + \eta_k^h \tag{20}$$

$$d_k^h = C^h x_k^h \tag{21}$$

where $A^h = \begin{bmatrix} C & S \\ -S & C \end{bmatrix}$, $C = \cos\omega^h$, $S = \sin\omega^h$, and where $\{\eta_k^h\}$ is a white noise sequence.

We describe the other relevant dynamics related to the inverter operation by a linear state-space model of the form:

$$x_{k+1}^p = A^p x_k^p + B^p u_k + B^p d_k^h + \eta_k^p \tag{22}$$

$$y_k^p = C^p x_k + \upsilon_k^p \tag{23}$$

where $\{\eta_k^p\}$, $\{\upsilon_k^p\}$ are white noise sequences and where y_k^p denotes the available measurements at time k. Note that, for convenience, we have lumped the harmonic disturbance at the input.

Combining (20), (22), (23) leads to a composite model

$$\begin{bmatrix} x_{k+1}^h \\ x_{k+1}^p \end{bmatrix} = \begin{bmatrix} A^h & 0 \\ B^p C^h & A^p \end{bmatrix} \begin{bmatrix} x_k^h \\ x_k^p \end{bmatrix} + \begin{bmatrix} 0 \\ B^p \end{bmatrix} u_k + \begin{bmatrix} \eta_k^h \\ \eta_k^p \end{bmatrix} \tag{24}$$

$$y_k^p = \begin{bmatrix} 0 & C^p \end{bmatrix} \begin{bmatrix} x_k^h \\ x_k^p \end{bmatrix} + \upsilon_k^p \tag{25}$$

An observer may then be used to estimate the states. An appropriate observer takes the form:

$$\begin{aligned} \begin{bmatrix} \hat{x}_{k+1}^h \\ \hat{x}_{k+1}^p \end{bmatrix} &= \begin{bmatrix} A^h & 0 \\ B^p C^h & A^p \end{bmatrix} \begin{bmatrix} \hat{x}_k^h \\ \hat{x}_k^p \end{bmatrix} + \begin{bmatrix} 0 \\ B^p \end{bmatrix} u_k \\ &\quad + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} y_k - \begin{bmatrix} 0 & C^p \end{bmatrix} \begin{bmatrix} \hat{x}_k^h \\ \hat{x}_k^p \end{bmatrix} \Bigg\} \end{aligned} \tag{26}$$

Turning to the control problem, say that we wish to track a constant reference y^*. A suitable (one-step-ahead) cost function is

$$J = |E(u_k)|^2 + \lambda \left| u_k + C^h \hat{x}_k^h - u^* \right|^2 \tag{27}$$

where $E(u_k) = y'_{k+1}(u_k) - y^*$ and where $y'_{k+1}(u_k)$ denotes the predicted output given the estimates $\hat{x}_k^p$, $\hat{x}_k^h$ when u_k is used, i.e.

$$\hat{y}'_{k+1} = C^p A^p \hat{x}_k^p + C^p B^p C^h \hat{x}_k^h + C^p B^p u_k \tag{28}$$

$$u^* = \left[C^p \left(I - A^p \right)^{-1} B^p \right]^{-1} y^* \tag{29}$$

We note that the above cost function has been arranged so that the estimated input disturbance is cancelled and hence the problem is turned into a regulation problem around (y^*, u^*). Moreover, perfect harmonic disturbance rejection occurs even if the model is incorrect since the only possible steady state is when the output contains no component at frequency ω^h – see Goodwin et al. (2005), Francis and Wonham (1975) and Maeder and Morari (2010).

The optimal input which minimises the cost function (27), subject the constraint that u_k must lie in the allowable switching levels, is readily seen to be

$$u_k = [NN] \left\{ \left(g^2 + \lambda \right)^{-1} \left(-C^p A^p \hat{x}_k^p + y^* + \lambda u^* \right) - B^p C^h \hat{x}_k^h \right\} \tag{30}$$

where $g = (C^p B^p)$ and where the operator $[NN]$ selects the nearest neighbour from the allowable finite set of switching states.

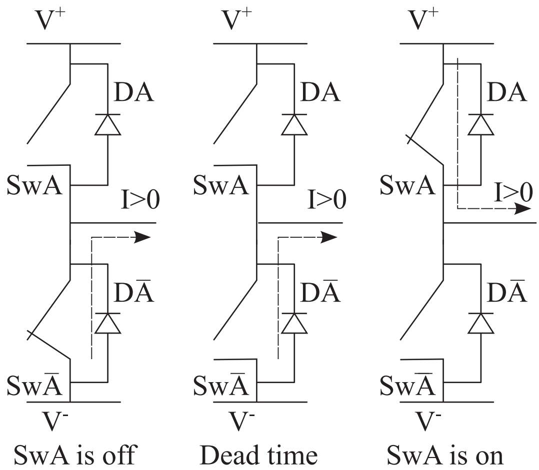

Figure 5. Dead time effect in one inverter leg (switch turns on).

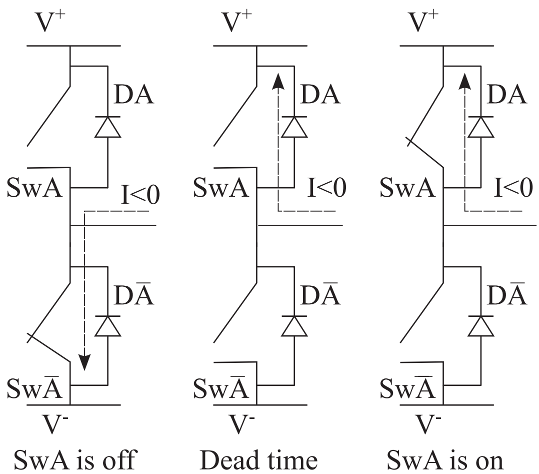

Figure 6. Dead time effect in one inverter leg (switch turns off).

4.2.3. Switching dead time

We next turn to the problem of nonlinear dead time. We explain the core concept of dead time delays with the aid of Figures 5 and 6. One inverter leg (repeatedly shown in Figures 5 and 6) includes two switches whose states are the inverse of each other at all times except during transitions. To perform a transition, a short time interval is introduced so that complementary switches are not conducting simultaneously. This is done to avoid short circuits across the inverter DC source. The associated interval is called the inverter dead time. Its typical duration is between 1 μsec and 4 μsec.

An inherent difficulty is that the effect of dead time on the voltage waveform is nonlinear since it depends upon the direction of current in the corresponding inverter leg. In the case illustrated by Figure 5, the current has positive direction $I > 0$ and its path is shown by an arrow. Switch SWA in Figure 5 is initially off (left-hand side circuit) and then it switches on (right-hand side circuit) via a dead time state (middle circuit). During the dead time, current is conducted via diode $D\overline{A}$, which clamps the output voltage to V^-. The transition from

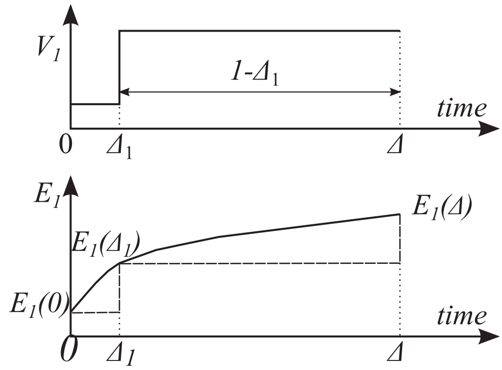

Figure 7. Delay Δ_1.

V^- to V^+ thus becomes delayed by the dead time. A similar situation, but with negative current $I < 0$, is shown in Figure 6. During the dead time current is conducted by diode DA, which clamps the output voltage to V^+. As a result, the transition from V^- to V^+ has no delay. The opposite is true for a transition from V^+ to V^-: it is delayed by the dead time if $I < 0$ and has no delay if $I > 0$. Clearly, dead time is both a nonlinear and complex issue.

There have been many previously suggested approaches to the dead time problem – see for example Leggate and Kerkman (1995), Holmes and Lipo (2003), Hwang and Kim (2010), Urasaki, Senjyu, Uezato, and Funabashi (2005), Deng, Oruganti, and Srinivasan (2005), Zhang, Zhao, and Zhu (2011), Cortes, Rodriguez, Silva, and Flores (2012) and Lee, Kim, and Ahn (2014). However, none is deemed to be a comprehensive solution.

4.2.4. *Variable prediction horizon nonlinear MPC for delay compensation*

The essential idea is to use the same cost function as in (27). However, the input is now not of zero-order-hold type. Thus, the observer is appropriately modified so that the input is not assumed constant over each sample period but can switch at some point within the sample period. The same observer gain can be used since output observations occur at the sample time. Similarly, to obtain predictions over the full sample period Δ, the prediction is divided into two steps, namely a prediction over horizon Δ_i (where the previous input applies) and then over horizon $\Delta - \Delta_i$ (where the new input applies). This motivates the name “variable prediction horizon nonlinear MPC”.

The system is nonlinear since the switching delay, Δ_i, will take different values depending on the current direction, charge, temperature, etc. Say that the allowable input set U contains choices $v^1, \ldots, v^m$ and that, at the present state, these choices come with a delay Δ_i, $i = 1, \ldots, m$. Then we can predict the value, $E(v^i)$, of the error E at the end of the sampling interval for each value of v^i. This is repeated for $i = 1, \ldots, m$ and we choose that v^i which minimises the cost function (27).

To illustrate, say there are only two possible choices of the inverter output voltage, namely, V_1 and V_2. This is indeed the case for a 2-level single-phase inverter. Say that, at the current state, the choice V_1 is associated with delay Δ_1 and the choice V_2 is associated with delay Δ_2 as shown in Figures 7 and 8, respectively. For each of the voltage choices, the error at the end of the sampling interval Δ can be predicted by using a discrete model with

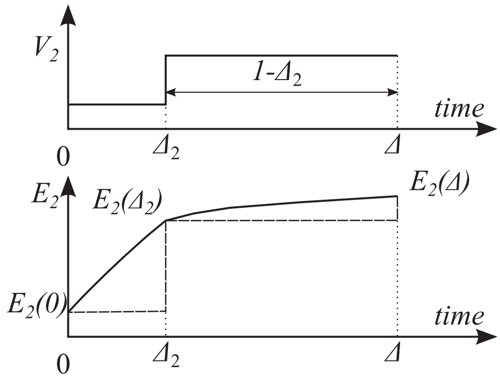

Figure 8. Delay Δ_2.

period Δ_i followed by a second discrete model with period $(\Delta - \Delta_i)$. Each of the errors is estimated at two time instants, i.e. at $t = \Delta_1$ and $t = \Delta$ for the choice V_1, and at $t = \Delta_2$ and $t = \Delta$ for the choice V_2. Evolution of the predicted errors E_1 and E_2 is illustrated in Figures 7 and 8, respectively, by dashed lines. At the end of the sampling interval, the predicted errors E_1 and E_2 are available. The best input is then chosen to minimise the cost function (27) with $E(u_k)$ replaced by E_1 when v_1 is chosen and E_2 when v_2 is chosen.

For the two voltage choices implied by a 2-level single-phase inverter, the error prediction model needs to be invoked four times, with variable horizon, over the sampling interval. The most common 2-level three-phase inverter would have three voltage choices at each interval, therefore, the error prediction model will be invoked six times over the sampling interval. One can see that this is only marginally more complex than the usual fixed horizon MPC algorithm, since the number of possible voltages is the same as in the nondelayed case.

4.2.5. Results

To validate the proposed concepts, we performed detailed simulations in the Matlab© environment. The ideas were implemented on a single-phase inverter operating at 50 Hz. To illustrate the core idea, we consider the suppression of the 3rd harmonic at 150 Hz. (Note that it is straightforward to extend the idea to multiple harmonics, see Mirzaeva & Goodwin, in press.) A realistic load model was assumed, namely, a series connection of a resistor $R = .037\Omega$ and an inductor $L = 3$ mH. We considered three scenarios:

- No switching delays in the system and using a design which assumes no delays; fixed horizon MPC;
- Switching delays in system but using a design which assumes no delays; fixed horizon MPC;
- Switching delays in system and a design which assumes the presence of delays; variable horizon nonlinear MPC.

The results are shown in Figures 9–11, respectively. Each scenario is illustrated by a plot showing the inverter voltage spectrum expressed in dB relative to the first harmonic. Note that, in the absence of delays, standard MPC (as described in Section 4.2.2) gives excellent harmonic suppression. In particular, note the notch at 150 Hz in the voltage spectrum shown in Figure 9. Unfortunately, these results are negated when nonlinear switching delays are introduced into the plant. In particular, we note that there is actually now a peak in the

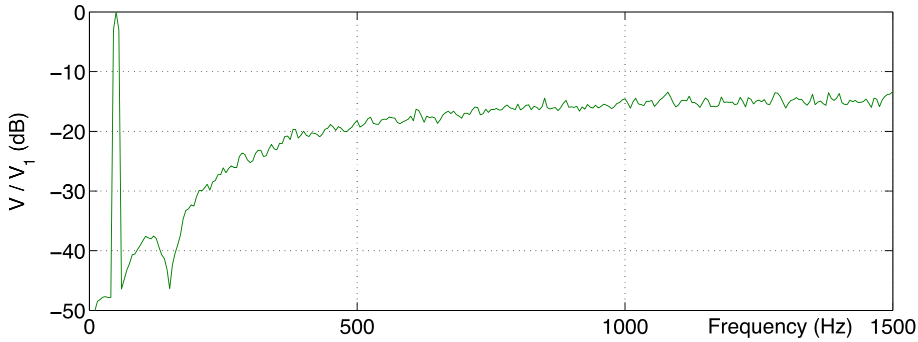

Figure 9. Voltage spectrum: no dead time in system, no compensation.

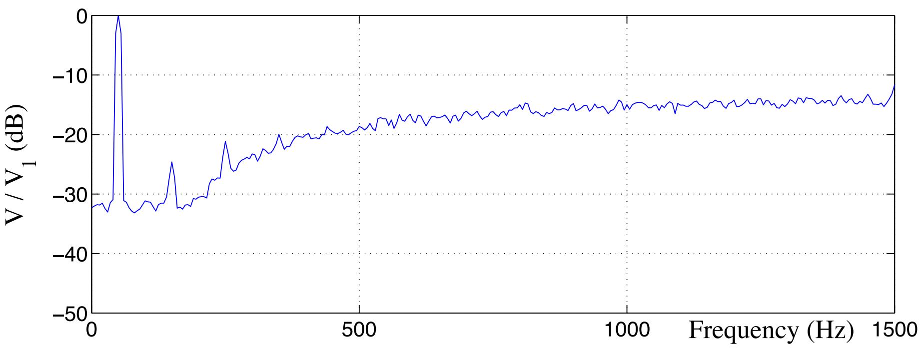

Figure 10. Voltage spectrum: dead time in system, no compensation.

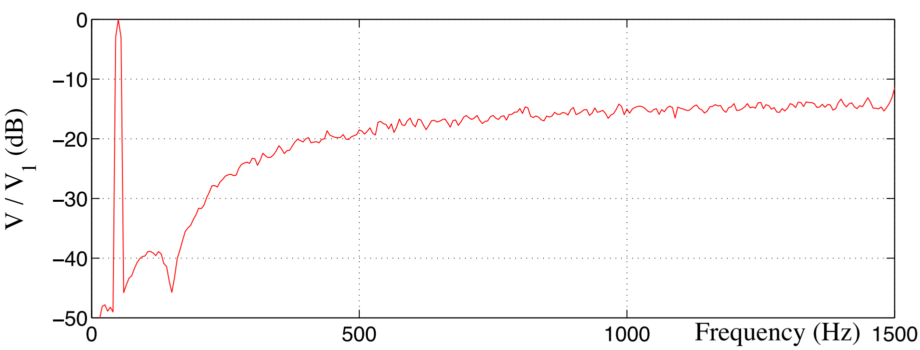

Figure 11. Voltage spectrum: dead time in system, compensation.

spectrum at 150 Hz as shown in Figure 10. The results in Figure 11 are obtained using the algorithm described in Section 4.2.4. Notice that the notch at 150 Hz has been reintroduced. Indeed the results in Figure 11 are very close (almost identical) to those in Figure 9, which demonstrates that joint harmonic suppression and delay compensation have been achieved by the application of the variable prediction horizon nonlinear MPC algorithm.

4.2.6. Distinctive features

The distinctive features of this problem are

- The model is inherently nonlinear.
- The model needs to contain a description of the harmonics (plus d.c. off-sets).
- An observer is needed to estimate the plant states and the harmonics (plus d.c. off-sets).
- The input space is finite and hence the problem is inherently nonconvex.
- Computational time is severely restricted (typical sample periods are of the order of .1 msec)
- Numerical issues can arise unless one uses appropriate techniques to implement observers and predictors – see discussion in Mirzaeva and Goodwin (in press), Goodwin, Aguero, Cea Garridos, Salgado, and Yuz (2013) and Goodall and Donoghue (1993).

5. Opportunities and challenges in robust MPC

Based on the review of theory as presented in Section 3 and the case studies presented in Section 4, it seems that there remain a number of issues which may warrant further study. Some of these issues are:

(a) Alternative approaches to robust MPC theory.
(b) Co-design of observer and MPC law.
(c) Treating nonconstant disturbances and reference signals.
(d) Control relevant models.
(e) Calibration and adaptation of control relevant models.
(f) Feedforward design.
(g) Fault detection diagnosis and compensation.

We briefly discuss these issues below.

5.1. Alternative approaches to robust MPC theory

It will be noticed that only a few ideas from robust MPC theory (as discussed in Section 3) are actually used in the particular applications described in Section 4. There are three main reasons for this:

5.1.1. Computational

There is a heavy computational load associated with techniques such as tube-based robustness, invariant sets, tightening of constraints and LMI conditions. For example, these would certainly be too complex for the heating furnace (due to the high state dimension).

5.1.2. Disturbance format

In many applications (including those discussed here), a key ingredient is being able to capture the distinctive disturbance features. Thus, one needs to go beyond saying that disturbances are “bounded”.

5.1.3. Conservativeness

Min-max type designs place emphasis on the “worst case scenario” which is often not well defined. Even in those cases where the “worst case’ is well defined, it usually corresponds to a low probability event. Thus, a better design goal might be to optimise performance for systems that lie near the nominal model whilst avoiding extremely poor, or unacceptable, performance for those (rare) systems that lie on the boundary of the set of conceivable models.

Some of the above points are taken up below. Note that these are not intended as criticisms of existing theory (which addresses issues of importance in many applications) but represent opportunities for alternative perspectives in future research.

5.2. Co-design of observer and MPC law

The co-design of observer and control law seems to be a neglected problem in the robust MPC literature.

5.2.1. Illustrative example

We first present a simple example to show that the choice of observer plays an important role in robust MPC. We adopt the system considered in Løvaas et al. (2008a), namely,

$$H = G_{22}(z) + W(z)z^{-1}\triangle(z),$$

where the nominal model $G_{22}(z)$ is a double integrator and the additive term $W(z)z^{-1}\triangle(z)$ represents the uncertainty. We assume that the system is subject to input constraint

$$u_k \in \mathbf{U} \triangleq \{u \mid -2 \leqslant u \leqslant 2\}.$$

We assume that $\|\triangle\|_{\ell_\infty} \leqslant 1$ and choose $\mathbf{Z} = \mathbf{Q} = \mathbf{W} = \mathbf{U}$ to satisfy the constraint consistency assumption (15). The system matrices are

$$A = \begin{bmatrix} .06 & .25 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_\omega = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ .5 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} -.0323 & .0207 & 1 & 0 \end{bmatrix},$$

$C_z = C_q = 0$ and $D_z = D_q = 1$. Our aim is to investigate the impact of an observer of the form (7) in robust MPC. For this purpose, we choose to compare two choices of the observer, namely,

$$L_1 = \begin{bmatrix} .005 & 0 & 1.99 & .99 \end{bmatrix}, \tag{31}$$

$$L_2 = \begin{bmatrix} 2 & 0 & .001 & .0001 \end{bmatrix}. \tag{32}$$

It can be easily verified that these two observer gains render both $A_1 = A - L_1C$ and $A_2 = A - L_2C$ stable. Moreover, it can be easily checked that the poles of A_1 and A_2

are $\{.0826, -.0062 + .0438i, -.0062 - .0438i, 0\}$ and $\{.1246, .9995 + .0103i, .9995 - .0103i, 0\}$, respectively.

For the robust MPC design, we use the ideas in Løvaas et al. (2008a) with an extra embellishment on the LMI condition based on the Extended Bounded Real Lemma, see de Souza and Xie (1992) and Kong (2014). The sets $\mathbf{E}_1$ and $\mathbf{E}_2$ (see Løvaas et al., 2008a) are computed as the maximal robustly invariant set (for the system with input $\omega_k \in \mathbf{W}$, and observer gains L_1 and L_2, respectively) satisfying $\mathbf{E}_1 \subseteq \mathbf{E}_0$ and $\mathbf{E}_2 \subseteq \mathbf{E}_0$ where

$$\mathbf{E}_0 \triangleq \{e|\ \|e\|_\infty \leq 1\}.$$

The resulting sets $\mathbf{E}_1$ and $\mathbf{E}_2$ are nonempty and contain the initial estimation errors used in the simulations below. A pre-stabilising feedback gain was chosen as follows:

$$F = \begin{bmatrix} 0 & 0 & .009 & .137 \end{bmatrix},$$

derived as the LQR gains for the nominal model using $R = 10^4$ in combination with $Q = I$. The prediction horizon is chosen to be $N = 8$. We set $\gamma = 10$ to satisfy the LMI conditions necessary for robust stability (see Løvaas et al., 2008a). The responses were evaluated when

$$\widehat{x}_0 = \begin{bmatrix} 5 & 0 & 10 & -5 \end{bmatrix}$$

with an initial estimation error of

$$e_0 = \begin{bmatrix} .5 & .5 & .5 & .5 \end{bmatrix}$$

and an admissible realisation of $\triangle$ given by

$$\triangle = \frac{.218z - .013}{z^2 + .390z - .378}.$$

This design is an instance of the optimisation problem formulated in Løvaas et al. (2008a) and Kong (2014) with weighting matrices $Q = C^{\mathrm{T}}C$ and $R = .001$.

In a first design, the observer gain was set to L_1 as in (31). The response is shown in the lower plots in Figure 12.

In a second design, we fix the cost function obtained from the first design, however, we change the observer to L_2. We re-run the optimisation problem. It is still feasible since its feasibility has been achieved through the addition of a slack variable, after the system matrix $\overline{A}$ has been adjusted due to the change of observer. The upper plot in Figure 12 represents the results for the second design using L_2. It can be easily seen that the second observer offers much worse performance than the first one. Note that the time scale of the two simulations have been chosen to be different to better show the results.

5.2.2. Beyond Luenberger observers

Given the results in Section 5.2.1, it seems reasonable to ask if there is ever motivation to go beyond the class of Luenberger observers. This question can be thought of as an extended form of the well-known Internal Model Principle, Francis and Wonham (1975) and Maeder and Morari (2010). For example, to deal with a constant off-set, one needs to incorporate a model for the offset, i.e. an integrator which then appears in the observer design; to cancel a harmonic disturbance one needs to incorporate a model for a sinusoidal signal – as discussed in Section 4.2.2. Also, if one believes that the process noise is coloured, then the corresponding model should be expanded to include a description of the noise colouring. Specifically, if the system is described by

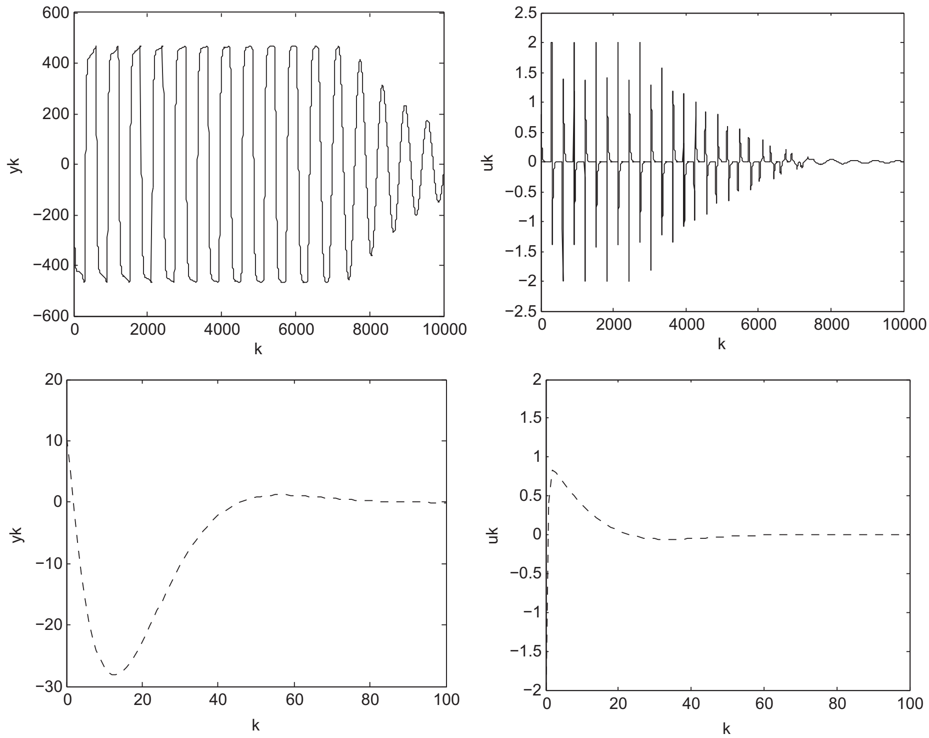

Figure 12. Comparisons of the system performance with different choices of the observers.

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + \eta_k \\ x'_{k+1} = A'x'_k + \omega'_k, \qquad\qquad \eta_k = C'x'_k \\ y_k = Cx_k + \upsilon_k \end{cases},$$

where ω'_k and υ_k represent white noise. Then an appropriate observer becomes

$$\widehat{x}''_{k+1} = A''\widehat{x}''_k + B''u_k + L''(y_k - C''\widehat{x}''_k),$$

where

$$\widehat{x}''_k = \begin{bmatrix} \widehat{x}_k \\ \widehat{x}'_k \end{bmatrix}, \qquad A'' = \begin{bmatrix} A & C' \\ 0 & A' \end{bmatrix},$$
$$B'' = \begin{bmatrix} B \\ 0 \end{bmatrix}, \qquad C'' = \begin{bmatrix} C & 0 \end{bmatrix}.$$

It is readily checked that this observer is an unbiased observer (Goodwin, Mayne, & Shim, 1995; Goodwin & Middleton, 1989; Seron, Braslavsky, & Goodwin, 1997). An unbiased observer has the property that (see Goodwin et al., 1995; Goodwin & Middleton, 1989; Seron et al., 1997) the transfer function from u_k to $\widehat{x}_k$ is the same as that from u_k to x_k. More generally, one can use other observers from the general class of unbiased observers (Goodwin & Middleton, 1989; Seron et al., 1997). The extra dynamics in these observers can be used to reduce the gain from unmodelled dynamics to the observer error (Carrasco & Goodwin, 2014). This is achieved whilst leaving the nominal dynamics unaffected. See also the comments made in Section 5.1.3.

A suitable observer from the general class of unbiased observers takes the form

$$\begin{cases} \widehat{\widehat{x}}_{k+1} = \overline{A}\widehat{\widehat{x}}_k + \overline{B}u_k + \overline{L}y_k, \\ \widehat{x}_k = \overline{C}\widehat{\widehat{x}}_k \end{cases}. \tag{33}$$

The condition for unbiasedness is

$$T_{\widehat{x}u} + T_{\widehat{x}y}C(zI - A)^{-1}B = (zI - A)^{-1}B$$

where A, B, C are as in the nominal part of the model (14) and

$$T_{\widehat{x}u} = \overline{C}(zI - \overline{A})^{-1}\overline{B},$$
$$T_{\widehat{x}y} = \overline{C}(zI - \overline{A})^{-1}\overline{L}.$$

Note that the dimension of $\widehat{\widehat{x}}_k$ will normally be greater than that of $\widehat{x}_k$.

Remark 3 Note that Luenberger and Kalman observers, including those which capture coloured noise, are a special case of the broad class of the unbiased observers. The key point is that the additional flexibility inherent in (33) relative to (7) allows one to influence the gain from both noise and unmodelled dynamics to the state estimate. Hence there is potential to achieve robust stability of the MPC law by adjusting the observer only whilst leaving the MPC cost function unaltered.

5.2.3. *General issues*

At one extreme, the co-design of observer and control law could be considered within the framework of Dual Control (see Bar-Shalon & Tse, 1974; Feldbaum, 1961). However, this is usually viewed as being impractical. On the other hand, even if a more pragmatic approach was adopted, it seems unclear what one could say, at a theoretical level, if a nonlinear observer (such as the one in (18), (19)) was interconnected with a nonlinear control law in the presence of model uncertainty.

5.3. *Treating nonconstant disturbances and reference signals*

Much of the existing MPC literature discusses regulation i.e. convergence to a fixed point. However, in practice, one is often more interested in transient behaviour and the "bandwidth" associated with disturbance rejection. These concepts lie outside of current robust MPC thinking. Similar remarks hold for reference tracking.

Within the literature on robust MPC, there has been some developments in MPC for references tracking (see e.g. Bemporad, Casavola, & Mosca, 1997; Chisci et al., 2001; Di Cairano & Borrelli, 2013; Gossner, Kouvaritakis, & Rossiter, 1997; Falugi & Mayne, 2012, 2013; Løvaas et al., 2010; Maeder & Morari, 2010 and the references therein). However, the case of general time-varying references tracking for constrained systems with unstructured uncertainties is still largely an open problem. The results in Abedor et al. (1994) and Abedor, Nagpal, Khargonekar, and Poolla (1995) for the unconstrained case may be helpful for the consideration of this topic. Apart from that, robust control of unconstrained nonlinear systems with unstructured uncertainties has gained some attention during the last few years

(Anderson, James, & Limebeer, 1998; Chen & Han, 1998; James & Petersen, 2005; Lin & Byrnes, 1995; Lin & Xie, 1998). Extension of these results to the constrained case pose hard, but interesting, questions. The development of reference tracking for nonlinear systems will rely on the advancement of nonlinear robust MPC theory.

5.4. Control relevant models

The single most important challenge in the applications discussed above has been the development of a control relevant model that captures key features of the system relevant to the design question. This model ignores other issues which could be highly relevant to other questions. Also, the architecture of the control law (e.g. what to measure, how to utilise feedforward etc) is usually a design choice. Alas, at the present time, these key aspects appear to be more of an "art" than a "science", yet they are arguably more important than determining what the controller gain should be.

5.5. Calibration and adaptation of low-order models for MPC

We have seen that in the case of the induction heating furnace, the "true" system description is hugely complex. This raises issues associated with; (i) the use of low-order models, (ii) calibration of simple models using offline data, and (iii) adaptation of simple models using online data. These issues are briefly discussed below.

5.5.1. Reduced-order MPC

Although MPC has been widely applied in practice, its applicability has been confined to low-dimensional systems or large-scale systems with slow dynamics, this is largely due to the heavy online computation burden. This has motivated the recent development of decentralised MPC and distributed MPC. In practice, smaller order controllers are normally preferred over complex higher order ones, because the former are easier to understand, computationally less demanding, and easier to implement and maintain. Therefore, a lower order controller should be sought as long as stability and control performance are guaranteed. As known in robust control (Zhou et al., 1996), certain forms of uncertainties are usually introduced to capture the characteristics of the reduction procedure, thereby rendering it a robust control problem. Reduced-order MPC has been considered in Hovland et al. (2008), Hovland, Willcox, and Gravdahl (2006), Hovland and Gravdahl (2008), Bonis and Theodoropoulos (2010), Hara and Kojima (2012), Unger, Kozek, and Jakubek (2012) and Sopasakis, Bernardini, and Bemporad (2013). However, it seems that more work could be carried out. Moreover, reduced-order MPC can be considered as an alternative to decentralised and distributed MPC for large-scale systems.

5.5.2. Model calibration

An issue related to reduced order models is how best to "tune" or "calibrate" a simple control-relevant model using offline data. One may feel that this is a simple question in system identification but that is far from the truth. Indeed, how to estimate parameters in a low-complexity model is a "hot topic" in the field of system identification. The key difficulty is how to deal with the fact that the system does not lie in the model set (as is usually assumed in system identification theory). The final goal would be to obtain confidence intervals for the parameters in the normal model as well as information about the residual, unstructured, model uncertainty.

5.5.3. Model adaptation

Another issue is that, even if one initially tunes (or calibrates) the model, the model will almost certainly vary over time. This is more difficult than initial model calibration since some form of online (or closed loop) estimation is needed.

5.6. Feedforward

In classical control, it is known that one can benefit from doing high gain feedforward based entirely on the nominal model. This is quite different to the concept of preview which simply informs a feedback controller of future values of the reference and disturbance (if known). Feedforward can yield significantly improved performance for systems "near" the nominal model but may yield slightly worse performance for systems "far from" the nominal model. This raises interesting design questions about how best to design the feedback and feedforward elements so as to trade-off improved performance for *most* systems at the expense of slightly worse performance for some systems whilst guaranteeing robust stability for all systems. (See the comments made in Section 5.1.1.) Note that this is quite different from the usual notion of worst-case design that is prevalent in modern robust control theory. For a recent discussion of feedforward MPC, see Carrasco and Goodwin (2011) and for an application to the development of an artificial pancreas, see Carrasco, Fu, Goodwin, King, and Medioli (2014). An extra ingredient in the latter problem is that appropriately designed feedforward can avoid some of the inherent lags and uncertainties that affect the feedback part of the design.

5.7. Fault detection, diagnosis and compensation

In practice one would only consider a high-performance controller, such as MPC, if there was considerable gain to be achieved by an increase in performance. However, such performance critical systems are also likely to be sensitive to control law failure. Some work has been done on actuator fault detection and compensation in the context of MPC (Yetendje, Seron, & De Dona, 2013). However, it seems that more work is desirable.

6. Conclusion

The paper has presented a brief review of robust MPC theory and has summarised two real-world applications. The paper has also pointed to several questions which seem to warrant further study.

Acknowledgements

The authors gratefully acknowledge collaboration with many people on the topic of MPC and its application, especially: Professor David Q. Mayne, Professor Richard H. Middleton and A/Professor José De Doná (Robust MPC); Dr Katrina Lau, Dr Torbjörn Wigren, Dr Stephen Craig and Dr Linda Brus (Mobile Telecommunications); Professor Richard H. Middleton and Dr Bernado Campos (Induction Heating Furnaces); Dr Adrian M. Medioli, Dr Diego S. Carrasco, Dr Yongi Fu and Dr Bruce King (Artificial Pancreas); Dr Adrian M. Medioli (Ambulance Scheduling); Dr Monica Romero, Dr Daniel E. Quevedo and Dr Colin Coates (Power Electronics).

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