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Department of Computing
Academic Year **2020-2021**
Page created Tue Nov 3 23:15:07 GMT 2020

70051 rac101 2 t5 sh2620 v1 0 0 1 2 0 0 0 1 3 5 0 3	Electronic submission **Wed - 28 Oct 2020 07:03:57** sh2620

Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
Exercise:	2 (CW)	**Due:**	Tue - 03 Nov 2020
Title:	Logic	**Assessment:**	Individual
FAO:	Craven, Robert (rac101)	**Submission:**	Electronic

Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-10-28 07:03:00

For Markers only: (circle appropriate grade)

HU, Shunlong (sh2620)	01921937	t5	2020-10-28 07:03:00	**A* A B C D E F**

1.

i.

p: Michel being fullfiled

q: Michel being rich

r: Michelliving for another five years

$((\neg(p \vee q)) \rightarrow (\neg r))$

ii.

p: storm will arrive

q: Raheem wears his boots

$(((\neg p) \vee q) \wedge p)$

iii.

p: Akira being on set

q: Toshiro being on set

r: filming will begin

s: the caterers have cleared out

$((p \wedge q) \rightarrow (r \leftrightarrow s))$

iv.

p: Irad arrived

q: Sarah arrived

$((p \vee (\neg q)) \wedge (\neg(p \wedge q)))$

v.

p: Herbert heard the performance

q: Anne-Sophie heard the performance

r: Anne-Sophie answered her phone calls

$((\neg r) \rightarrow (\neg(p \wedge q)))$

2.

i.

A propositional formula A is satisfiable if there is some v such that $h_v(A) = \mathbf{t}$.

ii.

Two propositional formulas A,B are logically equivalent if, for every v, $h_v(A) = h_v(B)$.

iii.

if $\neg A$ is satisfiable, then there is some v such that $h_v(\neg A) = \mathbf{t}$. Therefore, there is some v such that $h_v(\neg\neg A) \neq \mathbf{t}$, i.e. $h_v(\neg\neg A) = \mathbf{f}$. Thus, the condition such that for every v, $h_v(\neg\neg A) = \mathbf{t}$ is not satisfied. Hence, $\neg\neg A \not\equiv \top$ if $\neg A$ is satisfiable.

if $\neg\neg A \not\equiv \top$, then not for every v, $h_v(\neg\neg A) = \mathbf{t}$. Thus there is some v such that $h_v(\neg\neg A) \neq \mathbf{t}$, i.e. there is some v such that $h_v(\neg A) \neq \mathbf{f}$. Therefore, there is some v such that $h_v(\neg A) = \mathbf{t}$. Hence, $\neg A$ is satisfiable if $\neg\neg A \not\equiv \top$.

Since $\neg\neg A \not\equiv \top$ if $\neg A$ is satisfiable and $\neg A$ is satisfiable if $\neg\neg A \not\equiv \top$, $\neg A$ is satisfiable iff $\neg\neg A \not\equiv \top$.

3.

p	q	r	$(p$	$\wedge$	$\neg q$	$\leftrightarrow$	$\neg$	$(\neg r$	$\vee$	$\neg p))$	$\rightarrow$	$(\neg\neg q$	$\rightarrow$	$r)$
t	**t**	**t**	t	f	f	f	t	f	f	f	**t**	t	t	t
t	**t**	**f**	t	f	f	t	f	t	t	f	**f**	t	f	f

We can stop deriving the truth table here as when $p = \mathbf{t}, p = \mathbf{t}, p = \mathbf{f}$, the formula is **f**, so the formula is not valid.

4.
i.
In CNF: a, b, f, g
In DNF: b, e, h
ii.
The definition is: Let S be in CNF. $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$. This property is important because it implies that if it is impossible to derive $\emptyset$ from S by a resolution derivation, then S is satisfiable.
iii.
a. Apply pure rule to q:
$\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$
apply pure rule to $\neg r$:
$\{\{p, s\}\}$
apply pure rule to p:
$\{\}$

b. Apply unit propogation to $\neg q$:
$\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$
Apply unit propogation to $\neg r$:
$\{\{\neg p\}, \{p\}\}$
Apply unit propogation to p:
$\{\{\}\}$

5.
a: I'm going
b: You are going
c: Tara is going
The argument is valid if for all any atomic evaluation function, if value of the previous four clauses in conjunction is true, then the value of you are not going is true.
Evaluate the satisfiability of: you are not going $\wedge$ the previous four clauses in conjunction:
$(a \rightarrow \neg b) \wedge (\neg b \rightarrow \neg c) \wedge (c \vee \neg a) \wedge (c \vee a) \wedge (\neg b))$
Translate to CNF:
$\{\{\neg a, \neg b\}, \{b, \neg c\}, \{c, \neg a\}, \{c, a\}, \{\neg b\}\}$
$\Rightarrow$ $\{\{\neg c\}, \{c, \neg a\}, \{c, a\}\}$
$\Rightarrow$ $\{\{\neg a\}, \{a\}\}$

$\Rightarrow \quad \{\{\}\}$
$\Rightarrow \quad$ unsatisfiable

Therefore, the original argument is valid.

6.

i.

$\forall X(X = aunt(aunt(andrea)) \rightarrow \exists Y(\neg(Y = andrea) \wedge give\_a\_cupcake(X, Y)))$

The signature $\mathcal{L}$ is:
$\mathcal{C}_1 = \{andrea\}$
$\mathcal{P}_2 = \{give\_a\_cupcake\}$
$\mathcal{F}_1 = \{aunt\}$
$\mathcal{L} = \{\mathcal{C}_1, \mathcal{P}_2, \mathcal{F}_1\}$

ii.

$\exists X \forall Y(computer(X) \wedge computer(Y) \wedge \neg connect(Y, Y) \wedge connect(X, Y)))$

The signature $\mathcal{L}$ is:
$\mathcal{P}_2 = \{connect\}$
$\mathcal{P}_1 = \{computer\}$
$\mathcal{L} = \{\mathcal{P}_1, \mathcal{P}_2\}$

iii.

$\forall X \forall Y \forall Z \forall A \forall B(british\_gallery(X) \wedge is\_room\_in(Y, X) \wedge is\_room\_in(Z, X) \wedge hang(Y, A) \wedge hang(Z, B) \wedge paint(paul\_klee, A) \wedge paint(kandinsky, B)) \rightarrow Y = Z$

The signature $\mathcal{L}$ is:
$\mathcal{C}_1 = \{paul\_klee, kandinsky\}$
$\mathcal{P}_1 = \{british\_gallery\}$
$\mathcal{P}_2 = \{paint, is\_room\_in, hang\}$
$\mathcal{L} = \{\mathcal{C}_1, \mathcal{P}_1, \mathcal{P}_2\}$

iv.

$\exists X \forall Y(\neg love(X, Y)) \rightarrow \neg \forall X \exists Y(love(X, Y))$

The signature $\mathcal{L}$ is:
$\mathcal{P}_2 = \{love\}$
$\mathcal{L} = \{\mathcal{P}_2\}$

7.

i.

False.

Let σ_1 be the M assignment such that $X = j$.

In this case, $a(k, X) = \top$ but $\neg(X = j) = \bot$. Therefore, $M, \sigma_1 \models \bot$ and $M, \sigma \not\models \top$. Hence the original argument is false.

ii.

True.

Let σ_1 be the M assignment such that $X = j$.

In this case, $c(l) = \top$, $b(X) = \top$, $c(X) = \top$, $a(l, X) = \top$. Therefore, $c(l) \rightarrow b(X) \wedge c(x) \wedge a(l, X) = \top$. Thus $M, \sigma_1 \models \top$ and $M, \sigma \models \top$. Hence the original argument is True.

iii.

True.

Let σ_1 be the M assignment such that $X =$ the only white square and $Y = l$.

Let σ_2 be the M assignment such that $X =$ the only white square and $Y = j$.

Let σ_3 be the M assignment such that $X =$ the only white square and $Y = k$.

Let σ_4 be the M assignment such that $X =$ the only white square and $Y =$ the only black square.

Let σ_5 be the M assignment such that $X =$ the only white square and $Y =$ the only white square.

σ_1,σ_2,σ_3,σ_4,σ_5 are all X varient for $X =$ the only white square and $a(X,Y) = \perp$ for all X varient for $X =$ the only white square. Therefore, the original argument is true.

iv.

False.

Let σ_1 be the M assignment such that $X = j$ and $Y = l$.

Let σ_2 be the M assignment such that $X = j$ and $Y = j$.

Let σ_3 be the M assignment such that $X = j$ and $Y = k$.

Let σ_4 be the M assignment such that $X = j$ and $Y =$ the only black square.

Let σ_5 be the M assignment such that $X = j$ and $Y =$ the only white square.

Y satisfies $c(Y) \wedge b(Y)$, so we only need to consider σ_3. However, $a(X,Y) = \perp$, thus for X=j, there is no Y that satisfies $c(Y) \wedge b(Y) \wedge a(X,Y)$. Therefore, the original argument is false.

v.

False.

For $X = k$, $\neg(X = Y) \wedge a(X,Y) = \top$ when $Y = j$, but there is no Y such that $(a(X,Y) \wedge a(Y,X))$ is satisfied. Therefore, the original argument is false.

vi.

False.

Let σ_1 be the M assignment such that $X = k$ and $Y = k$.

$a(X,j) \wedge a(Y,j) = \top$, but $a(X,Y) \vee a(Y,X) = \perp$, so $a(X,j) \wedge a(Y,j) \rightarrow a(X,Y) \vee a(Y,X) = \perp$ and $M, \sigma_1 \models \perp$ and $M, \sigma \not\models \top$. Hence the original argument is false.