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Introduction to Symbolic AI
Coursework 1: Logic

Mathilde Outters

November 2, 2020

1 First Exercise

(i) If Michel isn't either fulfilled or rich, he won't live another five years.

p: Michael is fullfield

q: Michael is rich

r: Michael will live another five years

$((\neg(p \vee q)) \to (\neg s))$

(ii) Unless the snowstorm doesn't arrive, Raheem will wear his boots; but I'm sure it will arrive.

p: The snowstorm arrives

q: Raheem will wear his boots

r: I'm sure it will arrive

$(((\neg p) \vee q) \wedge r)$

(iii) If Akira and Toshiro are on set, then filming will begin if and only if the caterers have cleared out.

p: Akira is on set

q: Toshiro is on set

r: filming will begin

s: the caterers have cleared out

$((p \wedge q) \to (r \leftrightarrow s))$

(iv) Either Irad arrived, or Sarah didn't: but not both!

p: Irad arrived

q: Sarah arrived

$((p \wedge q) \vee ((\neg p) \wedge (\neg q)))$

(v) It's not the case both that Herbert heard the performance and Anne-Sophie did, if the latter didn't answer her phone calls.

p: Herbert heard the performance

q: Anne-Sophie heard the performance

r: Anne-Sophie answered her phone calls

$((\neg s) \to (\neg(p \wedge q)))$

2 Second Exercise

(i) A propositional formula A is *satisfiable* if there exists a combination of values for its propositional atoms (i.e. an atomic evaluation function v) that make A true (with propositional evaluation function notation: $h_v(A) = \mathbf{t}$). Equivalently, there is at least one true result in its truth-tables.

(ii) Two propositional formulas A, B are *logically equivalent* if, for every atomic evaluation function v, their propositional evaluation is the same: $h_v(A) = h_v(B)$.

(iii) $\neg A$ is satisfiable iif there exists an atomic evaluation function v such that $h_v(\neg A) = \mathbf{t}$. This is equivalent to $h_v(\neg(\neg A)) = h_v(\neg\neg A) = \mathbf{f}$ by definition of propositional evaluation. For this v, $h_v(\neg\neg A) \neq \mathbf{t} = h_v(\top)$ [by definition].

We have found a v such that $h_v(\neg\neg A) \neq h_v(\top)$, this is equivalent to $\neg\neg A \not\equiv \top$.

This proves that $\neg A$ is satisfiable iif $\neg\neg A \not\equiv \top$.

3 Third Exercise

$(p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \to (\neg\neg q \to r)$

p	q	r	(p	∧	¬	q	↔	¬	(¬	r	∨	¬	p))	→	(¬	¬	q	→	r)
t	t	t	t	f	f	t	f	t	f	t	f	f	t	t	t	f	t	t	t
t	t	f	t	f	f	t	t	f	t	f	t	f	t	f	t	f	t	f	f
t	f	t	t	t	t	f	t	t	f	t	f	f	t	t	f	t	f	t	t
t	f	f	t	t	t	f	f	f	t	f	t	f	t	t	f	t	f	t	f
f	t	t	f	f	f	t	t	f	f	t	t	t	f	t	t	f	t	t	t
f	t	f	f	f	f	t	t	f	t	f	t	t	f	f	t	f	t	f	f
f	f	t	f	f	t	f	t	f	f	t	t	f	t	t	f	t	f	t	t
f	f	f	f	f	t	f	t	f	t	f	t	f	f	t	f	t	f	t	f

The principal connective of this formula is $\to$ (overall logical form $A \to B$). We can see there exists atomic evaluations such that the formula evaluates to $\mathbf{f}$ (e.g. second line) hence it is NOT valid.

4 Fourth Exercise

(i) a) $p \wedge (\neg q \vee r)$ is in CNF, not in DNF.

b) $\neg p$ is in CNF and DNF.

c) $p \wedge (q \vee (p \wedge r))$ is not in CNF nor in DNF.

d) $\top$ is in CNF and DNF.

e) $(p \wedge q) \vee (p \wedge q)$ is in DNF, not in CNF.

f) $\neg\neg p \wedge (q \vee p)$ is not in DNF nor in CNF.

g) $p \wedge q$ is in CNF and DNF.

h) $p \vee q$ is in DNF and CNF.

(ii) Let S be a formula in Conjunctive Normal Form.

Refutation-soundness and -completeness of a resolution derivation states that if we can derive the empty set from a resolution of S , then S is not satisfiable.

This allows us to prove whether a complex S is satisfiable or not by applying a finite sequence of resolution ('resolution derivation').

(iii) a) $\{\{p,s\},\{q,r\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\}$
$\{\{p,s\},\{\neg p,\neg r,\neg s\}\}$ [literal q is pure]
$\{\{p,s\}\}$ [literal $\neg r$ is pure]

b) $\{\{\neg p,q,r\},\{\neg q\},\{p,r,q\},\{\neg r,q\}\}$
$\{\{\neg p,r\},\{p,r\},\{\neg r\}\}$ [unit propagation of unit clause $\{\neg q\}$]
$\{\{\neg p\},\{p\}\}$ [unit propagation of unit clause $\{\neg r\}$]
$\{\{\}\}$ [unit propagation of unit clause $\{p\}$]
(Hence the original set of clauses is not satisfiable.)

5 Fifth Exercise

p: I'm going
q: You are going
r: Tara is going
We must check whether $p \to \neg q, \neg q \to \neg r, r \vee \neg p, r \vee p \models q$.
So we can check whether $(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (r \vee p) \wedge (\neg q)$ is satisfiable.
We first convert it to clausal-form CNF: $\{\{\neg p,\neg q\},\{q,\neg r\},\{r,\neg p\},\{r,p\},\{\neg q\}\}$.
Now, applying DLL:
[p is true branch]
$\Rightarrow \{\{\neg q\},\{q,\neg r\},\{r\}\}$
$\Rightarrow \{\{\neg q\},\{q\}\}$ [unit propagation of unit clause {r}]
$\Rightarrow \{\{\}\}$ [unit propagation of unit clause {q}]
$\Rightarrow$ UNSATISFIABLE [since $\emptyset$ is in the set]
[p is false branch]
$\Rightarrow \{\{q,\neg r\},\{r\},\{\neg q\}\}$
$\Rightarrow \{\{q\},\{\neg q\}\}$
$\Rightarrow \{\{\}\}$
$\Rightarrow$ UNSATISFIABLE [since $\emptyset$ is in the set]
Since the second branch on p returned UNSATISFIABLE, the CNF is unsatisfiable and so the original argument is propositionally valid.

6 Sixth Exercise

(i) **All of Andrea's aunts' aunts gave a cupcake to someone other than Andrea.**

$\forall Z \forall Y(aunt(Z,Y) \wedge aunt(Y,andrea) \to \exists W \exists X(gave(Z,X,W) \wedge cupcake(W) \wedge \neg(X = andrea)))$

Where we used the following signature:

$$C = \{andrea\}$$
$$P_1 = \{cupcake\}$$
$$P_2 = \{aunt\}$$
$$P_3 = \{gave\}$$

Read: $gave(X,Y,Z)$ as 'X gave Y Z'.

Read: $aunt(X,Y)$ as 'X is an aunt of Y'.

(ii) **There's a computer connected to every computer which isn't connected to itself.**

$\exists X \forall Y (computer(X) \wedge connected(X, Y) \wedge computer(Y) \wedge \neg connected(X, X))$

Where we used the following signature:

$$P_1 = \{computer\}$$
$$P_2 = \{connected\}$$

Read: $computer(X)$ as 'X is a computer'

Read: $connected(X, Y)$ as 'X is connected to Y'.

(iii) **Any painting by Paul Klee in a British gallery hangs in a room where all Kandinsky paintings in that gallery hang.**

$\forall X (\exists Z (painted(paulKlee, X) \wedge in(X, Z) \wedge gallery(Z) \wedge british(Z)) \rightarrow \exists W \forall Y (hangs(X, W) \wedge room(W) \wedge painted(kandinsky, Y) \wedge in(Y, Z) \wedge hangs(Y, W)))$

Where we used the following signature:

$$C = \{paulKlee, kandinsky\}$$
$$P_1 = \{british, gallery, room\}$$
$$P_2 = \{painted, hangs, in\}$$

Read: $british(X)$ as 'X is British'.

Read: $gallery(X)$ as 'X is in a gallery'.

Read: $room(X)$ as 'X is a room'.

Read: $painted(X, Y)$ as 'X painted Y'.

Read: $hangs(X, Y)$ as 'X hangs in Y'.

Read: $in(X, Y)$ as 'X is in Y'.

(iv) **If there's somebody who loves nobody, then it's false that everybody loves somebody.**

$\exists X \forall Y \neg loves(X, Y) \rightarrow \neg(\forall U \exists V loves(U, V))$

Or, equivalently: $\exists X \neg \exists Y loves(X, Y) \rightarrow \neg(\forall U \exists V loves(U, V))$

Where we used the following signature:

$$P_2 = \{loves\}$$

Read: $loves(X, Y)$ as 'X loves Y'.

7 Seventh Exercise

Throughout the exercise, we will use the word 'linked'[by a directed arrow] as a shorthand for 'related according to the relation represented by a (i.e. $\varphi(a)$).

(i) FALSE

'Every object such that $\varphi(k)$ is linked to it is different from $\varphi(j)$.'

$\varphi(k)$ is only linked to one object in the structure: $\varphi(j)$.

(ii) TRUE

$c(l)$ is true in the structure as the object $\varphi(l)$ is a circle in the structure. There is a black-circle-object that $\varphi(l)$ is linked to (e.g $\varphi(j)$).

(iii) TRUE

'There exists an object that is linked to nothing else than itself.'

Example: the object depicted by the black square is only linked by $\varphi(a)$ to itself in the structure.

(iv) FALSE

'Everything that is not a square is linked by $\varphi(a)$ to at least one black circle object.'

This is not the case: the non-square object $\varphi(j)$ is not linked to a black-circle object via the relation $\varphi(a)$.

(v) FALSE

'For all objects linked to something different that itself, they have a symmetrical relation with an object.'

We can take the object $\varphi(k)$ as a counterexample k, it is linked to j (different than itself) but it has no symmetrical relation (j is not linked to k).

(vi) FALSE

If we take $\varphi_\sigma(X) = \varphi_\sigma(Y) = \varphi(k)$ then this object is indeed linked to $\varphi(j)$ but it is not linked to itself. We have found an object such that the premises are satisfied in the structure but the conclusion is false.