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Exercise Information

Module:	499 Modal Logic for Strategic Reasoning in AI	**Issued:**	Wed - 05 Feb 2020
Exercise:	6 (CW)	**Due:**	Wed - 19 Feb 2020
Title:	Coursework2	**Assessment:**	Individual
FAO:	Belardinelli, Francesco (fbelard)	**Submission:**	Electronic

Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-02-11 15:58:26

For Markers only: (circle appropriate grade)

BARATH, Boris (bb816)	01187289	c4	2020-02-11 15:58:26	**A* A B C D E F**

1) a) $(M, \pi) \models \varphi R \psi$

φ releases ψ

iff $\left[\left(\pi[i..j] \models \psi \text{ and } \pi[j] \models \varphi \text{ for some } i \geq 0 \text{ for } j \geq i\right) \right.$ (1)

OR $\left.\left(\pi[i..\infty] \models \psi \text{ for all } i \geq 0\right)\right]$ (2)

ψ true forever

b) $\varphi R \psi \equiv (\varphi \wedge \psi) \vee (\psi \wedge X(\psi U(\varphi \wedge \psi)))$

c) the truth conditions match the LTL formula as:

1st part of the disjunction in the LTL formula satisfies the case if $i = j$ in (1)

2nd part of the disjunction says that either ψ will remain true indefinitely, which satisfies (2) or that ψ will remain true until $(\psi \wedge \varphi)$ is true which satisfies the $j > i$ case from (1)

d) $G\psi$ says that ψ will always be true.

since pRq says q will hold until and once p becomes true, using the fact that $\perp$ will never be true, we get that q will always be true

$\equiv Gq$ where $p = \perp$, $q = \psi$

2) for conciseness, I will use $\exists$ instead 'for some' and $\forall$ instead 'for all'

$(M,q) \models EF\phi$ iff $\exists \lambda$ from q, $\exists j \geq 0, (M, \lambda[j]) \models \phi$

iff $\exists \lambda$ from q, $\exists j \geq 0, (M, \lambda[j]) \models \phi$

always true $\rightarrow$ and $\forall i, 0 \leq i < j \ (M, \lambda[i]) \models true$

iff $(M,q) \models E(true \, U \phi)$

$(M,q) \models AF\phi$ iff $\forall \lambda$ from q, $\exists j \geq 0, (M, \lambda[j]) \models \phi$

iff $\forall \lambda$ from q, $\exists j \geq 0, (M, \lambda[j]) \models \phi$

always true $\rightarrow$ and $\forall i, 0 \leq i < j, (M, \lambda[i]) \models true$

iff $(M,q) \models A(true \, U \phi)$

$(M,q) \models EG\phi$ iff $\exists \lambda$ from q, $\forall j \geq 0, (M, \lambda[j]) \models \phi$

iff $\neg\neg\exists \lambda$ from q, $\forall j \geq 0, (M, \lambda[j]) \models \phi$

iff $\neg\forall \lambda$ from q, $\neg\forall j \geq 0, (M, \lambda[j]) \models \phi$

iff $\neg\forall \lambda$ from q, $\exists j \geq 0, (M, \lambda[j]) \models \neg\phi$

iff $(M,q) \models \neg AF \neg\phi$

$(M,q) \models AG\phi$ iff $\forall \lambda$ from q, $\forall j \geq 0, (M, \lambda[j]) \models \phi$

iff $\neg\neg\forall \lambda$ from q, $\forall j \geq 0, (M, \lambda[j]) \models \phi$

iff $\neg\exists \lambda$ from q, $\neg\forall j \geq 0, (M, \lambda[j]) \models \phi$

iff $\neg\exists \lambda$ from q, $\exists j \geq 0, (M, \lambda[j]) \models \neg\phi$

iff $(M,q) \models \neg EF \neg\phi$

3) Show that CTL is synt. frag. of CTL*

a) proof by comparing syntax of CTL and CTL*

let ϕ, ψ be arbitrary state formulas, σ, π arbitrary path form.

state formulas:

CTL	CTL*
p	p
$\neg\phi$	$\neg\phi$
$\phi \wedge \psi$	$\phi \wedge \psi$
$E\varphi$	$E\pi$
$A\pi$	$A\pi$

path formulas:

CTL	CTL*
	ϕ (path form.)
	$\neg\pi$
	$\pi \wedge \sigma$
$X\phi$	$X\pi$
$\phi U \psi$	$\pi U \sigma$

from this comparison, it becomes apparent that CTL syntax is a subset of CTL*, since CTL only supports state formulas after X and in U, while CTL* supports both state and path formulas after X and in U.

b) can be expressed in CTL* but not CTL: $AFG\phi$
$EXX\phi$

4) Let M be a model, s a state, π a path, ϕ, ϕ' state formulas and ψ, ψ' path formulas

if we restrict semantics of CTL* to CTL, we get:

$(M, s) \models p$ iff $s \in V(p)$
$(M, s) \models \neg \phi$ iff $(M, s) \not\models \phi$
$(M, s) \models \phi \wedge \phi$ iff $(M, s) \models \phi$ and $(M, s) \models \phi'$
$(M, s) \models E\psi$ iff for some path π from s, $(M, \pi) \models \psi$
$(M, s) \models A\psi$ iff for all paths π from s, $(M, \pi) \models \psi$

these semantic rules make up definition 1.7

To show definition 1.8 take:

$(M, \pi) \models X\phi$ iff $(M, \pi[1..\infty]) \models \phi$ which we get from CTL* $(M, \pi) \models X\psi$ and the fact that in CTL* syntax (in def. 1), a path formula ψ can consist of a state formula ϕ (*)

$(M, \pi) \models \phi \cup \phi$ can be derived similarly from CTL* $(M, \pi) \models \psi U \psi'$ and the same (*) from def. 1

these two rules make definition 1.8

5) a) in exercise 3 we showed that all expressions of CTL can be expressed in CTL* as CTL is a syntactic fragment of CTL*

furthermore, we showed that semantics of CTL are a subset of the semantics of CTL*.

from these facts, it follows that, given an arbitrary formula ϕ in CTL, we must have a syntactically and semantically equivalent formula ϕ' in CTL*

b) example formulas which can be expressed in CTL* but not CTL are formulas of the kind $X\psi$ or $\varphi U \pi$ where ψ, φ and π are path formulas. In CTL, only state formulas can occur before U and after X and U.

eg: $EXXp$

6) assume (M,t) and (M',t') are bisimilar

assume (M,π) and (M',π') are bisimilar

proof of $(M,t) \models \phi$ iff $(M',t') \models \phi$

by induction on structure of ϕ

i) $\phi = p$ trivial, follows from definition 3a) and definition of bisimulation

ii) $\neg\phi$ follows from the negation of 3a), def. bisim. and the semantic rule $(M,s) \models \phi \wedge \phi'$ iff $(M,s) \not\models \phi$

iii) $\phi \wedge \phi'$ follows from defintion of 3a) and the semantic rule $(M,s) \models \phi \wedge \phi'$ iff $(M,s) \models \phi \wedge (M,s) \models \phi'$

iv) $E\psi$ says there is a path where ψ holds.

from the semantic rule, we have that $(M,t) \models E\psi$ iff there exists a path π starting from t, s.t. $(M,\pi) \models \psi$

- since (M,t) and (M',t') and also (M,π) and (M',π') are bisimilar, there must also be a path π' from t' such that $(M',\pi') \models \psi$. proof on structure of ψ is below.

6) v) $A\psi$ says ψ holds on all paths.

from the semantic rule for $A\psi$ we have

$(M,t) \models A\psi$ iff for all paths π starting from t,

$(M,\pi) \models \psi$

since (M,t) and (M',t') and also (M,π) and (M',π')

are bisimilar, for every path π from t in M,

there exists a path π' from t' in M' s.t.

$(M',t') \models \psi$. (induction on ψ below)

proof of $(M,\pi) \models \psi$ iff $(M',\pi') \models \psi'$

by induction on the structure of ψ

i) $(M,\pi) \models \phi$ iff $(M,\pi[0]) \models \phi$

by the definition of bisimulation on (M,π)

and (M',π'), we have that $(M,\pi[0])$ and $(M',\pi'[0])$

are bisimilar. Furthermore, by the induction on ϕ

above, we have $(M,\pi) \models \phi$ iff $(M',\pi') \models \phi$

ii) $(M,\pi) \models \neg\psi$ iff $(M,\pi) \not\models \psi$ iff $(M',\pi') \models \psi$

follows from def. 3a)

iii) $(M,\pi) \models \psi \wedge \psi'$ iff $(M,\pi) \models \psi$ and $(M,\pi) \models \psi'$

iff $(M',\pi') \models \psi$ and $(M',\pi') \models \psi'$

follows from i) ii) and def. bisimulation

6) iv) $(M,\pi) \models X\psi$ iff $(M, \pi[1..\infty]) \models \psi$

by the definition of bisimulation we have that $(M,\pi[i])$ and $(M',\pi'[i])$ are bisimilar $\forall i \geq 0$

from def. bisimulation we have that $(M,\pi) \models \psi$ iff $(M',\pi') \models \psi$

Since ψ can be ϕ, which we have proven by induction. we have $(M,\pi) \models X\psi$ iff $(M',\pi') \models X\psi$

v) $(M,\pi) \models \psi \cup \psi'$ iff $(M,\pi[i..\infty]) \models \psi'$ for some $i \geq 0$ and $(M,\pi[j..\infty]) \models \psi$ for all $0 \leq j < i$

similarly to iv) above, $\forall i \geq 0$, $(M,\pi[i])$ and $(M',\pi'[i])$ are bisimilar.

we now have $(M,\pi[i..\infty]) \models \psi'$ iff $(M',\pi'[i..\infty]) \models \psi'$ and $(M,\pi[j..\infty]) \models \psi$ iff $(M',\pi'[j..\infty]) \models \psi$

which can be any of the cases proven above

7) assume St, St' are finite states of models M, M' respectively

Proof of Hennessy-Milner:

$\Rightarrow$: $\Rightarrow$ follows from the fact that since (M,t) and (M',t') are bisimilar, they satisfy the same formulas and hence are modally equivalent (by def. of modal equivalence)

$\Leftarrow$: assume (M,t) and (M',t') are modally equivalent. then, by def. of modal equivalence we know that they have the same theories (satisfy the same set of formulas.)

assume for a contradiction that there is a path π from t and there is no modally equivalent path π' from t'. But then, (M',t') and (M,t) do not necessarily satisfy the same set of formulas – contradiction to the modal equivalence of t, t'

8) proof:

$\Rightarrow$ assume (M,t) and (M',t') satisfy the same formulas of CTL. By exercise 7, (M,t) and (M',t') are bisimilar. From exercise 6 it follows that bisimulations preserve truth of modal formulas, so clearly (M,t) and (M',t') must satisfy the same formulas in CTL*

$\Leftarrow$ assume (M,t) and (M',t') satisfy the same formulas of CTL*

also, assume towards a contradiction that they do not satisfy the same formulas in CTL

- since CTL is a subset of CTL*, then they cannot satisfy the same formulas in CTL* which is a contradiction

40 **Out of** **49**

1			
a/**2**	b/**2**	c/**3**	d/**3**
	Solution could have been simplified futher	Solution is correct and explanation provided but no proof is given	Solution given but not well justified and no proof is attempted
2	1	2	1

2			
a/**2**	b/**2**	c/**2**	d/**2**
			Would have preferred if you had used the given abbreviations to arrive at the equivalences rather than the other way
2	2	2	2

3	
a/**3**	b/**2**
	By the argument you have presented in a, the example holds. However, it would be better to make the reasoning explicit (even if has already been stated) in every question
3	2

4
/5
Everything is correct, but the proof for the final statement (though it follows a similar form to before) is not given.
4

5	
a/**2**	b/**2**
2	2

6	7	8
/6	**/6**	**/5**
	Idea is correct but proof is lacking	Inductions are correct, but no comment is made resolving the apparent contradiction.

6	3	5