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Introduction to Symbolic AI CW 1 Zhaojiang Liu

1\. i. $(\neg p \wedge \neg q) \rightarrow \neg r$

p: Michel is fulfilled.

q: Michel is rich.

r: Michel will live another five years.

ii. $(\neg p \vee q) \wedge r$

p: The snow storm arrives

q: Raheem will wear his boots

r: I'm sure ~~雪~~ snow storm will arrive

iii. $p \rightarrow (q \leftrightarrow r)$

p: Arika and Toshiro are on set

q: filming will begin

r: caterers have cleared out.

iv. $\cancel{(p \vee \neg q) \wedge p \wedge q}$ $(p \vee \neg q) \wedge (\neg(p \wedge q))$

p: Irad arrived

q: Sarah arrived

v. $\neg(\neg r \rightarrow (p \wedge q))$

p: Herbert heard the performance

q: Anne-Sophie heard the performance

r: Anne-Sophie answered her phone calls

2.

i. Satisfiability of a propositional formula A is that whether A is satisfiable. A propositional formula A is satisfiable if there is v such that $h_v(A)=t$.

ii. Logic equivalence of propositional formulas A and B is that whether A and B are logically equivalent.

Two propositional formula A, B are logically equivalent if, for every v, $h_v(A)=h_v(B)$.

iii.

① Proof of ($\neg A$ is satisfiable) $\rightarrow$ ($\neg\neg A \not\equiv T$).

If $\neg A$ is satisfiable, then there is some v s.t. ~~$h_v(A)=t$~~ $h_v(\neg A)=t$.

Also, for this v, $h_v(\neg\neg A)=$ ~~$h_v(\neg f)$~~ $h_v(\neg T)=f$.

Thus, there exists such a v, ~~$h_v(\neg\neg A)=f$~~ $h_v(\neg\neg A)=f$. So $\neg\neg A \not\equiv T$.

② Proof of ($\neg\neg A \not\equiv T$) $\rightarrow$ ($\neg A$ is satisfiable).

If $\neg\neg A \not\equiv T$, then there exists some v. s.t. $h_v(\neg\neg A)=f$.

Then for this v. $h_v(\neg A)=t$. Thus, ~~A~~ $\neg A$ is satisfiable.

3.

p	q	r	$(p$	$\wedge$	$\neg q$	$\leftrightarrow$	$\neg$	$(\neg r$	$\vee$	$\neg p))$	$\rightarrow$	$(\neg\neg q$	$\rightarrow$	$r)$
t	t	t	t	f	f	f	t	f	f	f	t	t	t	t
t	t	f	t	f	f	t	f	t	t	f	f	t	f	f
t	f	t	t	t	t	t	t	f	f	f	t	f	t	t
t	f	f	t	t	t	f	f	t	t	f	t	f	t	f
f	t	t	f	f	f	t	f	f	t	t	t	t	t	t
f	t	f	f	f	f	t	f	t	t	t	f	t	f	f
f	f	t	f	f	t	t	f	f	t	t	t	f	t	t
f	f	f	f	f	t	t	f	t	t	t	t	f	t	f

4. i.

a. $p \wedge (\neg q \vee r)$ is in CNF

b. $\neg p$ is in CNF and DNF

c. $p \wedge (q \vee (p \wedge r))$ is not in CNF or DNF

d. T is not in CNF or DNF.

e. $(p \wedge q) \vee (p \wedge q)$ is in DNF

f. $\neg\neg p \wedge (q \vee p)$ is not in CNF or DNF

g. $p \wedge q$ is in ~~DNF~~ CNF

h. $p \vee q$ is in DNF

ii.

Property: Let S be in CNF. $S \vdash_{res(PL)} \phi$ iff $S \models \bot$.

It's important since it's corollary: Let S be in CNF. S is satisfiable iff $S \nvdash_{res(PL)} \phi$.

By this corollary, we can check whether S is satisfiable.

We can build all resolution-derivations from S. If finally, $S \nvdash_{res(PL)} \phi$, then S is satisfiable.

iii.

a. $\{\{p,s\},\{q,r\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\}$

$\Rightarrow \{\{p,s\},\{\neg p,\neg r,\neg s\}\}$ [pure rule, q is pure]

$\Rightarrow \{\{p,s\}\}$ [pure rule, $\neg r$ is pure]

$\Rightarrow \{\}$ [pure rule, p is pure] $\Rightarrow$ satisfiable

b. $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

~~$\Rightarrow \{\{\}\}$ [unit propagation by unit clause~~

$\Rightarrow \{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$ [unit propagation by unit clause $\{\neg q\}$]

$\Rightarrow \{\{\neg p\}, \{p\}\}$ [unit propagation by unit clause $\{\neg r\}$]

$\Rightarrow \{\{\}\}$ [unit propagation by unit clause $\{p\}$]

$\Rightarrow$ unsatisfiable

5.

We formalize it as: $p \to \neg q$, $\neg q \to \neg r$, $r \vee \neg p$, $r \vee p$, therefore q

p: I'm going.

q: you're going.

r: Tara's going.

So we must check whether $p \to \neg q, \neg q \to \neg r, r \vee \neg p, r \vee p \models q$

We know that in general, $A_1, \cdots A_n \models B$ iff $A_1 \wedge A_2 \wedge A_3 \cdots \wedge A_n \wedge \neg B$ is unsatisfiable.

So we can check whether $(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (r \vee p) \wedge \neg q$ is satisfiable.

We convert it to CNF form: $\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$.

Now, apply DP:

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$

~~$\Rightarrow \{\{p, \neg q\}, \{q, \neg p\}, \{q, p\}, \{\neg q\}\}$~~

$\Rightarrow \{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$ [unit propagation by unit clause $\{\neg q\}$]

$\Rightarrow \{\{\neg p\}, \{p\}\}$ [unit propagation by unit clause $\{\neg r\}$].

$\Rightarrow \{\{\}\}$ [unit propagation by unit clause $\{p\}$]

$\Rightarrow$ unsatisfiable [since ϕ is in the set]

Since the CNF is unsatisfiable, the original argument is propositionally valid.

6. i.

$C = \{Andrea\}$

$P_1 = \{cupcake\}$ $P_2 = \{aunt\}$ $P_3 = \{give\}$ where $aunt(X,Y)$ is read as 'X is Y's aunt', $give(X,Y,Z)$ is read as 'X gives Y to Z'.

~~$\forall X \forall Y (aunt(X, Andrea) \land aunt(Y, X) \rightarrow \exists Z (give(Y, Z,) \land cupcake(Z) \land \neg(= Andrea)))$~~

Translation:

$\forall X \forall Y \left(aunt(X, Andrea) \land aunt(Y, X) \rightarrow \exists Z \exists Q (give(Y, Z, Q) \land cupcake(Z) \land \neg(Q = Andrea)) \right)$

ii. $P_1 = \{computer\}$

$P_2 = \{connect\}$

where $computer(X)$ is read as 'X is computer',

$connect(X,Y)$ is read as 'X connect to Y'.

Translation:

~~$\exists X \forall Y (computer(X) \land computer(Y) \land \neg connect(Y,Y) \rightarrow connect(X,Y))$~~

$\forall X \left(computer(X) \land \neg connect(X,X) \rightarrow \exists Y (computer(Y) \land connect(Y,X)) \right)$

iii. $C = \{Paul\ Klee, Kandinsky\}$

$P_1 = \{British\ gallery, room\}$

$P_2 = \{painting, hang\}$

where $British\ gallery(X)$ is read as 'X is in British gallery'.

$room(X)$ is 'X is ~~a~~ room'.

$painting(X, Y)$ is read as 'X is a painting by Y'.

$hang(X,Y)$ is read as 'X ~~is~~ hangs in Y'.

~~$\exists X \forall Y(painting(Y, Kandinsky) \wedge British\ gallery(Y) \rightarrow$~~

$$\forall X(painting(X, Kandinsky) \wedge British\ gallery(X) \rightarrow \exists Y(room(Y) \wedge hang(X,Y) \rightarrow \forall Z(painting(Z, Paul\ Klee) \wedge British\ gallery(Z) \rightarrow hang(Z,Y))))$$

iv.

~~$\exists X \neg \exists Y\ love(X,Y) \rightarrow \forall X \exists Y \neg love(X,Y)$~~

$P_2 = \{love\}$

where $love(X,Y)$ is read as 'X love Y'.

Translation:

$$\exists X \neg \exists Y\ love(X,Y) \rightarrow \forall X \exists Y \neg love(X,Y)$$

7.

i. False.

When $\sigma(X)=j$, $a(k,j)$ is ~~true~~ true. $\neg(X=j)$ is false.

This make $a(k,X) \to \neg(X=j)$ is false. Thus, the original formula is false.

ii. True

$c(l)$ is true. When X is k, $b(k)$ is true, $c(k)$ is true, $a(l,k)$ is true.

Then $b(k) \wedge c(k) \wedge a(l,k)$ is true. Thus $\exists X(b(X) \wedge c(X) \wedge a(l,X))$ is true.

Thus, the origin is true.

iii. False

When ~~X is j, is~~ X is j, Y is l, $\neg(X=Y)$ is true, $a(X,Y)$ is true.

This ~~means~~ makes $\neg(X=Y) \wedge a(X,Y)$ is true. Thus, $\exists X \neg \exists Y(\neg(X=Y) \wedge a(X,Y))$ is false.

iv. False.

k, j, l makes $\neg s(X)$ is true. For X is j, only $(g(j), g(l)) \in g(a)$.

But $g(l) \notin g(b)$. Thus, there isn't any Y ~~makes~~ s.t. $c(Y) \wedge b(Y) \wedge a(X,Y)$ is true

when X is j. Thus, ~~for~~ when X is j, $\neg s(X) \to \exists Y(c(Y) \wedge b(Y) \wedge a(X,Y))$ is false.

v. False.

For X is ~~j, k, l~~ j, k or l, $\exists Y(\neg(X=Y) \wedge a(X,Y))$ is true.

For all j, k, l, there isn't a Y s.t. $a(X,Y) \wedge a(Y,X)$. Thus, $\exists Y(a(X,Y) \wedge a(Y,X))$ is false. Thus, the origin is false

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vi. False.

Suppose that X is k and Y is k. Then $a(X,j) \wedge a(Y,j)$ is true, $a(X,Y) \vee a(Y,X)$ is false. Then $a(X,j) \wedge a(Y,j) \rightarrow (a(X,Y) \vee a(Y,X))$ is false. Then the origin is false.