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Exercise Information

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Title:	Logic	**Assessment:**	Individual
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Student Declaration - Version 1

- I acknowledge the following people for help through our original discussions:

Sneha Naik (sn914)

Signed: (electronic signature) Date: 2020-11-03 08:48:14

For Markers only: (circle appropriate grade)

NAIK, Sneha (sn914)	00927813	t5	2020-11-03 08:48:14	A* A B C D E F

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1. i) p = Michael is fulfilled
q = Michael is rich
r = Michael will live another five years

$((\neg(p \vee q)) \to (\neg r))$

ii) p = The snowstorm arrives
q = Raheem will wear his boots
r = I am sure the snowstorm will arrive

$(((\neg p) \vee q) \wedge r)$

iii) p = Akira is on set
q = Toshiro is on set
r = filming will begin
s = caterers have cleared out

$((p \wedge q) \to (r \leftrightarrow s))$

iv) p = Irad arrived
q = Sarah arrived

$((p \vee (\neg q)) \wedge (\neg(p \wedge (\neg q))))$

v) p = Herbert heard performance
q = Anne sophie heard performance
r = Anne sophie answered her phone calls

$((\neg r) \to (\neg(p \wedge q)))$

2. i) A propositional formula A is satisfiable if there exists some valuation, v, such that $h_v(A) = t$

ii) Two propositional formulas A, B are logically equivalent if for every valuation, v, $h_v(A) = h_v(B)$

~~iii)~~

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2.iii) $(\neg A)$ is satisfiable if for some valuation function v, $h_v(\neg A) = t$

* Suppose $\neg\neg A \equiv \top$: Since $h_v(\top)$ is true for all valuations, v, this means that $h_v(\neg\neg A) \equiv \top$ for all v.
  Since $\neg\neg A$ can be expressed $\neg(\neg A)$, if $h_v(\neg(\neg A)) \equiv t$ for all v, then $h_v(\neg A)$ would evaluate to false for all v ie $\neg A$ would be unsatisfiable if $\neg\neg A = \top$; satisfiable iff $\neg\neg A \not\equiv \top$

* Reasoning from other direction

  Suppose $\neg\neg A \not\equiv \top$, $h_v(\neg\neg A) \equiv \bot$ for all v

  ie. $h_v(\neg(\neg A)) \equiv \bot$, $h_v(\neg A) \equiv \top$ ie

  satisfiable.

3.

compare ④ ⑤

p	q	r	$(p$	$\wedge$ ①	$\neg q$	$\leftrightarrow$ ④	$\neg$ ③	$(\neg r$	$\vee$ ②	$\neg p))$	$\rightarrow$ ⑥	$(\neg\neg q$	$\rightarrow$ ⑤	$r)$
t	t	t	t	f	f	f	t	f	f	f	t	t	t	t
t	t	f	t	f	f	t	f	t	t	f	f	t	f	f
t	f	t	t	t	t	t	t	f	f	f	t	f	t	t
f	t	t	f	f	f	t	f	f	t	t	t	t	t	t
t	f	f	t	t	t	f	f	t	t	f	t	f	t	f
f	t	f	f	f	f	t	f	t	t	t	f	t	f	f
f	f	t	f	f	t	t	f	f	t	t	t	f	t	t
f	f	f	f	f	t	t	f	t	t	t	t	f	t	f
											☆			

☆ $(p \wedge \neg q \leftrightarrow \neg(\neg r \vee \neg p)) \rightarrow (\neg\neg q \rightarrow r)$

is NOT valid. For a formula to be valid, $h_v(A) = t$ for any valuation v. From the truth table, we see that the formula evaluates to false in two cases:

① $h_v(p) = t$, $h_v(q) = t$ and $h_v(r) = f$

② $h_v(p) = f$, $h_v(q) = t$ and $h_v(r) = f$

Therefore the formula is not valid

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4. i) a) $p \wedge (\neg q \vee r)$ is CNF $\{\{p\}, \{\neg q, r\}\}$

b) $\neg p$ is CNF as a single negated atomic is a clause
is also in DNF for the same reason
$\{\neg p\}$

[DNF = disjunction of ONE or more conjunctions of ONE or more literals
CNF = conjunction of ONE or more clauses (disjunction of one or more literal)]

c) $p \wedge (\underbrace{q \vee (p \wedge r)}_{\text{this isnt a clause}})$ Not CNF due to brackets
Not DNF

this isnt a clause
as a clause is of form $L_1 \vee L_2 \ldots \vee L_n$ L_i = literal

d) $\top$ is a clause because $\top$ is a literal
Therefore CNF and DNF for same reason as (b)

e) $(p \wedge q) \vee (p \wedge q)$ is DNF

f) $\neg\neg p \wedge (q \vee p)$ not CNF, not DNF because $\neg\neg p$ is not a literal (only atomic or negated atomic - can't have double negation)

g) $p \wedge q$ is CNF $\{\{p\}, \{q\}\}$ conjunction of 2 clauses
and is also DNF with $(p \wedge q)$ as a single conjunct in DNF

h) $p \vee q$ is CNF and DNF similar to above
$\downarrow$ CNF: $\underbrace{\{\{p, q\}\}}_{\text{single clause}}$
DNF: disjunction of literals p and q

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4ii) If S is in CNF $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$

This is key because it is how many SAT solvers operate. If it is impossible to derive $\emptyset$ from S by resolution derivation, then S is satisfiable.

SAT solvers try to construct the derivation of $\emptyset$ from S, if fails, we know original CNF is satisfiable, else S is not satisfiable

Can be used to show whether arguments of the form

$A_1 \ldots A_n \models B$ are valid by showing that

$(\bigwedge A_i) \wedge (\neg B)$ are unsatisfiable

4 iii) a) $\{\{p,s\},\{q,r\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\}$

q is pure $\Rightarrow \{\{p,s\},\{\neg p,\neg r,\neg s\}\}$

$\neg r$ is pure $\Rightarrow \{\{p,s\}\}$

p is pure $\Rightarrow \{\}$ Empty set is not in the set

↳ empty conjunction : $\top$

4 iii) b) $\{\{\neg p,q,r\},\{\neg q\},\{p,r,q\},\{\neg r,q\}\}$

Unit clause $\neg q \Rightarrow \{\{\neg p,r\},\{p,r\},\{\neg r\}\}$

Unit clause $\neg r \Rightarrow \{\{\neg p\},\{p\}\}$

Unit clause $\neg p \Rightarrow \{\{\emptyset\}\}$ Empty set is in the set

↳ empty disjunction : $\bot$

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5. p = I am going

q = You are going

r = Tara is going

1. If I'm going, then you aren't $(p \to (\neg q))$
2. If you're not going, then neither is Tara $((\neg q) \to (\neg r))$
3. Either Tara's going or I'm not $(r \vee (\neg p))$
4. Tara's going unless I am $((\neg r) \to p)$

~~Rewording:~~ 1

5. ~~Therefore~~ So, you're going "Therefore" q

1. $(\neg p \vee \neg q)$
2. $(q \vee \neg r)$
3. $(r \vee \neg p)$
4. $(r \vee p)$

Task: ~~Show~~ If valid, show $\underbrace{(\neg p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (r \vee p)} \models q$

ie show $(\wedge A_i) \wedge (\neg q)$ unsatisfiable

$\{ \{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\} \}$

Using DP: Unit clause on $\{\neg q\}$ $\Rightarrow \{ \{\neg r\}, \{r, \neg p\}, \{r, p\} \}$

Unit clause on $\{\neg r\} \Rightarrow \{ \{\neg p\}, \{p\} \}$

Unit clause on $\{\neg p\} \Rightarrow \{ \{\emptyset\} \} \Rightarrow$ CNF unsatisfiable

Therefore original argument $A_i \ldots A_n \models B$ is valid!

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all other P_i F_i are $\emptyset$

6.i) $\mathcal{L}$: $C = \{andrea\}$
$p_1 = \{cupcake\}$, $p_2 = \{aunt\}$, $p_3 = \{gave\}$

where cupcake(X) means "X is a cupcake"
aunt(X,Y) means "X is an aunt of Y"
gave(X,Y,Z) means "X gave Z to Y"

$$\forall X \forall Y[aunt(X,Y) \wedge aunt(Y, andrea) \rightarrow$$
$$\exists Z \exists W\,(gave(X,Z,W) \wedge cupcake(W) \wedge \neg(Z = andrea)]$$

ii) $\mathcal{L}$: $p_1 = \{computer\}$ $p_2 = \{connected\}$

where computer(X) means "X is a computer"
connected(X,Y) means "X is connected to Y"

$$\forall X \exists Y\,[\neg connected(X,X) \wedge computer(X) \rightarrow$$
$$connected(X,Y) \wedge computer(Y)]$$

iii) $\mathcal{L}$: $C = \{Paul\ Klee, Kandinsky\}$
$p_1 = \{british, gallery, room, painting\}$
$p_2 = \{belongs, hangs\}$
$f_1 = \{painter\}$

All other P_i, F_i are $\emptyset$

where british(X) means 'X is british', gallery(X) 'X is a gallery', room(X) "X is a room", painting(X) 'X is a painting'
where belongs(X,Y) means 'X belongs to Y' and hangs(X,Y) means "X hangs in Y"
where painter(X) maps to a constant {Paul Klee, Kandinsky} and means painter(X) is the painter of X.
↳ I used a function here because every painting has exactly ~~only~~ one painter ∴ f_1

$$\forall X \exists Y\,[painting(X) \wedge belongs(X,Y) \wedge gallery(Y) \wedge british(Y) \wedge (painter(X) = Paul\ Klee)$$
$$\rightarrow (\exists Z \forall W(room(Z) \wedge hangs(X,Z) \wedge hangs(W,Z) \wedge painting(W) \ldots \wedge (painter(W) = Kandinsky))]$$

All paul Klee paintings belonging to a british gallery
There is a room Z that X hangs in and all Ws hang in where W is a painting by Kandinsky

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6. iv) $\mathcal{L}$: $P_2 = \{loves\}$ where loves(x,y) means 'x loves y'

$\exists W \neg \exists V$ loves$(V,W) \rightarrow \neg(\forall x \exists y$ loves$(x,y))$

7. i) "for all x if there is a directed arrow from k to x, then x isn't equal to j"

False the only arrow from k is to j

ii) "If l is a circle, then there is some x for which x is black, circular and there is a directed arrow from l to x"

True e.g. k is black, circular and $a(l,k)$ is true
same for j

iii) "There is some x for which x isn't connected to anything that is not itself"

True object ■ is only connected to itself

iv) "for all x, if x is not a square, then there exists some y where y is a black circle and there's a directed arrow from $x \rightarrow y$"

consider all x which aren't square

$x = k$ $k \rightarrow j$ ✓
$x = j$ $j \rightarrow l$ ✗ l is a white circle ✗
$x = l$ $l \rightarrow j, \square$ ✓ for j

Not true $\forall x$ ∴ False

v) "for all x, if there exists some $y \neq x$ and for which $x \rightarrow y$ then there exists some y for which $x \rightarrow y$ AND $y \rightarrow x$

False eg. $x = k$ some y e.g j, $j \neq k$
$a(k,j) = $ True $a(j,k) = $ False ∴ False

vi) "for all x, for all y its true that if there is $x \rightarrow j$ and $y \rightarrow j$ then either $(x \rightarrow y)$ or $(y \rightarrow x)$"

Key no condition preventing $x = y$

$x\,y$ $k \rightarrow j$ x,y can be $\underbrace{k,l;\ k,k;\ l,l}_{x \rightarrow y \text{ or } y \rightarrow x \text{ false}}$ ∴ False
$l \rightarrow j$