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Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
Exercise:	2 (CW)	**Due:**	Tue - 03 Nov 2020
Title:	Logic	**Assessment:**	Individual
FAO:	Craven, Robert (rac101)	**Submission:**	Electronic

Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-10-27 20:49:39

For Markers only: (circle appropriate grade)

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Intro to Symbolic AI

CW 1: Logic

1\. i. Michel is fulfilled $= p$

Michel is rich $= q$

Michel will live another five years $= r$

$$(((\neg p) \vee (\neg q)) \rightarrow (\neg r))$$

ii\. The snowstorm will arrive $= p$

Raheem will wear his boots $= q$

I am sure the snowstorm will arrive $= r$

$$((\neg p) \vee q \wedge r)$$

iii) Akira and Toshino are on set $= p$

The filming will begin $= q$

The caterers have cleared out $= r$

$$(p \rightarrow (q \leftrightarrow r))$$

iv) ~~Either~~ Irad arrived $= p$ Sarah arrived $= q$

$$((p \vee (\neg q)) \wedge (\neg (p \wedge q)))$$

v) Herbert heard the performance $= p$

Anne-Sophie heard the performance $= q$

Anne-Sophie ~~didn't~~ answered her phone calls $= r$

$$((\neg r) \rightarrow \neg (p \wedge q))$$

2. i) A propositional formula A is satisfiable if there is an propositional evaluation function such that $h_v(A) = T$

ii) Two propositional formulas are equivalent if $h_v(A) = h_v(B)$

iii) ~~If $\nvdash \neg A \not\equiv T$, then there exists a v such that $h_v(\neg\neg A) = \bot$. Therefore $\neg h_v(\neg\neg A) = T$~~

Consider an evaluation function such that $h_v(\neg A) = T$
Therefore $h_v(\neg(\neg A)) = \bot \not\equiv T$. Therefore $\neg A$ is satisfiable

3. $(p \wedge \neg q \leftrightarrow (\neg r \vee \neg p)) \rightarrow (\neg\neg q \rightarrow r)$ is valid

p	q	r											
T	T	T	T	F	F	T	F	F	F	T	T	T	T
T	F	T	T	T	T	F	F	F	F	T	F	T	T
F	T	T	F	F	F	F	F	T	T	T	T	T	T
F	F	F	F	F	T	F	T	T	T	T	F	T	F
T	T	F	T	F	F	F	T	T	F	T	T	F	F
F	F	T	F	F	T	F	F	T	T	T	F	T	T
T	F	F	T	T	T	T	T	T	F	T	F	T	F
F	T	F	F	F	F	F	T	T	T	T	T	F	F

4\. i) a) $p \wedge (\neg q \vee r)$ CNF

b) $\neg p$ either CNF or DNF

c) $p \wedge (q \vee (p \wedge r))$ neither CNF nor DNF

d) $\top$ neither CNF nor DNF

e) $(p \wedge q) \vee (p \wedge q)$ DNF

f) $\neg\neg p \wedge (q \vee p)$ CNF

g) $p \wedge q$ DNF

h) $p \vee q$ CNF

ii) Let S be in CNF. $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$

This means that if S does not resolve in $\emptyset$, then it is satisfiable. This is the basis of SAT-solving.

iii) a. $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

Unit propagation $\{p\} \Rightarrow \{\{q, r\}, \{\neg s, q\}, \{\neg r, \neg s\}\}$

Unit propagation $\{r\} \Rightarrow \{\{\neg s, q\}, \{\neg s\}\}$

Pure rule $\{q\} \Rightarrow$ ~~$\{\{q\}\}$~~ $\{\{\neg s\}\}$

Unit propagation $\{\neg s\} \Rightarrow \{\}$ The CNF is satisfiable

b. $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

Unit propagation $\{p\} \Rightarrow \{\{q, r\}, \{\neg q\}, \{\neg r, q\}\}$

Unit propagation $\{q\} \Rightarrow \{\{\}\}$

The CNF is unsatisfiable

5. I am going $= p$

You are ~~not~~ going $= q$

Tara is going $= r$

$$[(p \rightarrow \neg q) \wedge (\neg q \rightarrow \neg r) \wedge (r \vee \neg p) \wedge (r \vee p)] \models q$$

Equivalent to

$$((\neg p \vee q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (r \vee p)) \models q$$

$$\{\{\neg p, q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}\}$$

Unit propagation $\{p\} \Rightarrow \{\{q\}, \{q, \neg r\}, \{r\}\}$

Unit propagation $\{r\} \Rightarrow \{\{q\}, \{q\}\}$

~~Pure rule $\Rightarrow$ [illegible]~~ $(q \wedge q) \models q$ Argument is valid

6. i) L - signature : $C = \{andrea, \cancel{cupcake}\}$ $P = \{givescupcake\}$ $F = \{aunt\}$

~~$\forall X \forall Y ((X = aunt(aunt(andrea))) \rightarrow$ [illegible] $(gives(X, cupcake) \rightarrow \neg(Y = andrea)))$~~

$$\forall X (X = aunt(aunt(andrea)) \rightarrow \exists Y (givescupcake(X, Y) \wedge \neg(Y = andrea)))$$

ii) $C = \{computer\}$ $P = \{connected\}$

$$\forall X \forall Y (computer(X) \wedge computer(Y) \wedge connected(X, Y) \wedge \neg(X = Y))$$

iii) $C = \{$Paul Klee, Kandinsky$\}$ $P = \{painting, author, room\}$

$$\forall X (painting(X) \wedge (author(X) = \text{Paul Klee}) \rightarrow \forall Y (painting(Y) \wedge (author(Y) = \text{Kandinsky}) \wedge (room(X) = room(Y))))$$

iv) $\exists X \forall Y \forall Z (\neg loves(X, Y) \rightarrow \neg loves(Y, Z))$

~~7.~~ 7. i) $\forall X (a(K, X) \to \neg(X = j))$

False as there is a black arrow from K to j : $a(K, j)$ in domain D.

ii) $\underbrace{c(l)}_{\text{true as } l \text{ is a circle}} \to \underbrace{\exists X (b(X) \wedge c(X) \wedge a(l, X))}_{\text{true as K is a black circle with a line coming from } l}$

$T \to T$ is T

iii) $\exists X \neg \exists Y (\neg(X = Y) \wedge a(X, Y))$

equivalent to $\exists X \forall Y ((X = Y) \wedge \neg a(X, Y))$

True eg. node K

iv) $\forall X (\neg s(X) \to \exists Y (c(Y) \wedge b(Y) \wedge a(X, Y))$

False if $X = j$, there is no arrow pointing from j towards a black circle. This means $T \to \perp$ which is false.

v) $\forall X (\underbrace{\exists Y (\neg(X = Y) \wedge a(X, Y))}_{\text{false for white square}} \to \underbrace{\exists Y (a(X, Y) \wedge a(Y, X))}_{\text{true for black square top right pointing to itself}})$

$\perp \to T$ is true

vi) $\forall X \forall Y (a(X, j) \wedge a(Y, j)) \to (a(X, Y) \vee a(Y, X)))$

False as there is no direct arrow from X= black square and Y= white square to node j or between each other.