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Student Declaration - Version 1

- I acknowledge the following people for help through our original discussions:

Marwan Mousa (mm7515)

Signed: (electronic signature) Date: 2020-10-28 22:46:09

For Markers only: (circle appropriate grade)

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Introduction to Symbolic AI: Logic Coursework

Course:	Introduction to Symbolic AI
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Date:	November 2, 2020

1 Question 1

i If Michel isn't either fulfilled or rich, he won't live another five years.

- p: Michel is fulfilled.
- q: Michel is rich.
- r: Michel will live another five years.

$$(\neg(p \vee q) \rightarrow (\neg r))$$

ii Unless the snowstorm doesn't arrive, Raheem will wear his boots; but I'm sure it will arrive.

- p: The snowstorm arrives.
- q: Raheem will wear his boots.
- r: I'm sure the snowstorm will arrive

$$(((\neg p) \vee q) \wedge r)$$

iii If Akira and Toshiro are on set, then filming will begin if and only if the caterers have cleared out.

- p: Akira is on set.
- q: Toshiro is on set.
- r: Filming will begin.
- s: The caterers have cleared out.

$$((p \wedge q) \rightarrow (r \leftrightarrow s))$$

iv Either Irad arrived, or Sarah didn't: but not both!

- p: Arad arrived.
- q: Sarah arrived.

$$((p \vee (\neg q)) \wedge (\neg(p \wedge (\neg q))))$$

v It's not the case both that Herbert heard the performance and Anne-Sophie did, if the latter didn't answer her phone calls.

- p: Herbert heard the performance.
- q: Anne-Sophie heard the performance.
- r: Anne-Sophie answered her phone calls.

$$((\neg r) \rightarrow (\neg(p \wedge q)))$$

2 Question 2

i A propositional formula A is *satisfiable* if there exists some atomic evaluation function v such that the corresponding propositional evaluation function $h_v(A) = \mathbf{t}$.

ii Two propositional formulas A and B are *logically equivalent* , $A \equiv B$, if for every atomic evaluation function v, $h_v(A) = h_v(B)$

iii If $\neg\neg A \not\equiv T$ then $\neg\neg A$ is not valid as validity requires $\neg\neg A \equiv \top$. This means that $\neg\neg\neg A$ is satisfiable. Since $\neg\neg\neg A \equiv \neg A$, then $\neg A$ is also satisfiable. The reverse can also be argued where, if $\neg A$ is satisfiable then $\neg\neg A$ is not valid and if $\neg\neg A$ is not valid then $\neg\neg A \not\equiv \top$.

3 Question 3

For an argument $A \vDash B$ to be valid, it requires $A \to B$ to hold for any valuation v. This can be evaluated using a truth table.
Let $A : (p \wedge \neg q \leftrightarrow (\neg r \vee \neg p))$ and $B : (\neg\neg q \to r)$ express $(p \wedge \neg q \leftrightarrow (\neg r \vee \neg p)) \to (\neg\neg q \to r)$ in the form $A \to B$.

p	q	r	$((p$	$\wedge$	$\neg q)$	$\leftrightarrow$	$\neg$	$(\neg r$	$\vee$	$\neg p))$	$\to$	$(\neg\neg q$	$\to$	$r)$
t	**t**	**t**	t	**f**	f	**f**	**t**	f	**f**	f	**t**	t	**t**	t
t	**t**	**f**	t	**f**	f	**t**	**f**	t	**t**	f	**f**	t	**f**	f
t	**f**	**t**	t	**t**	t	**t**	**t**	f	**f**	f	**t**	f	**t**	t
t	**f**	**f**	t	**t**	t	**f**	**f**	t	**t**	f	**t**	f	**t**	f
f	**t**	**t**	f	**f**	f	**t**	**f**	f	**t**	t	**t**	t	**t**	t
f	**t**	**f**	f	**f**	f	**t**	**f**	t	**t**	t	**f**	t	**f**	f
f	**f**	**t**	f	**f**	t	**t**	**f**	f	**t**	t	**t**	f	**t**	t
f	**f**	**f**	f	**f**	t	**t**	**f**	t	**t**	t	**t**	f	**t**	f

It can be seen from the table above when $h_v(p) = \mathbf{t}$, $h_v(q) = \mathbf{t}$, $h_v(r) = \mathbf{f}$ and when $h_v(p) = \mathbf{f}$, $h_v(q) = \mathbf{t}$, $h_v(r) = \mathbf{f}$ then $h_v(A) = \mathbf{t}$ while $h_v(B) = \mathbf{f}$, therefore the formula $(p \wedge \neg q \leftrightarrow (\neg r \vee \neg p)) \to (\neg\neg q \to r)$ is **NOT** valid.

4 Question 4

i (a) $p \wedge (\neg q \vee r)$ is in **CNF**.
  (b) $\neg p$ is in **CNF**.
  (c) $p \wedge (q \vee (p \wedge r))$ is neither **CNF** nor **DNF**.
  (d) $\top$ is in **CNF**.
  (e) $(p \wedge q) \vee (p \wedge q)$ is in **DNF**.
  (f) $\neg\neg p \wedge (q \vee p)$ is neither **CNF** nor **DNF**.
  (g) $p \wedge q$ is in **CNF**.
  (h) $p \vee q$ is in **CNF**.

ii A resolution is said to be sound if when $C = resolvent(C_1, C_2, p)$, then $\{C_1, C_2\} \vDash C$, that is two clauses logically imply their resolvent. This means that a derivation by propositional resolution of a clause C from the set of clauses S in CNF is logically implied by it. Now the theorem of *refutation-soundness and -completeness* states that $S \vdash_{res(PL)} \emptyset$ if and only if $S = \bot$. This property means that if $\emptyset$ can be derived from S then S is false and vice-versa. This is used to determine whether S is satisfiable or not, where S is satisfiable only if $S \nvdash \emptyset$. This property is important as it allows the more computationally efficient methods of derivation to be used to determine the satisfiablity of any formula rather than truth tables or direct argument.

iii (a) $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$
  - Applying pure rule on q: $\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$
  - Applying pure rule on $\neg r$: $\{\{p, s\}\}$

  Can't be reduced any further so formula is satisfiable.

  (b) $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$
  - Applying unit propagation on $\neg q$: $\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$
  - Applying unit propagation on $\neg r$: $\{\{\neg p\}, \{p\}\}$
  - Applying unit propagation on p: $\{\{\emptyset\}\}$

  Contains the empty set, therefore formula is unsatisfiable.

5 Question 5

Let

- p: I am going.
- q: You are going.
- r: Tara is going.

Then

- If I'm going, then you aren't. $(p \to \neg q) \equiv (\neg p \vee \neg q)$
- If you're not going, then neither is Tara. $(\neg q \to \neg r) \equiv (q \vee \neg r)$
- Either Tara's going or I'm not. $(r \vee \neg p)$
- Tara's going unless I am. $(r \vee p)$
- **So**, you're going. (q)

Let $(\bigwedge A_i)$: $(\neg p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (r \vee p)$ and B: (q). For the above argument to be valid i.e. $(\bigwedge A_i) \models B$ we need $(\bigwedge A_i) \wedge \neg B$ to be unsatisfiable.
This can be written in the form: $\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$

Using David-Putnam:

- Applying unit propagation on $\neg q$: $\{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$
- Applying unit propagation on $\neg r$: $\{\{\neg p\}, \{p\}\}$
- Applying unit propagation on p: $\{\{\emptyset\}\}$

Since the empty set was found, then $(\bigwedge A_i) \wedge \neg B$ is indeed unsatisfiable which means the original argument is **valid**.

6 Question 6

i Let $\mathcal{L}_1$ be a signature where:

- $\mathcal{C} = \{Andrea\}$
- $\mathcal{P}_1 = \{cupcake\}$ where $cupcake(X)$: ('X is a cupcake')
- $\mathcal{P}_2 = \{aunt\}$ where $aunt(X, Y)$: ('X is aunt of Y')
- $\mathcal{P}_3 = \{gave\}$ where $gave(X, Y, Z)$: (X gave Y to Z)

Then **All of Andrea's aunts' aunts gave a cupcake to someone other than Andrea** can be written as:

$$\forall X \forall W \exists Y \exists Z \Big(gave(aunt(X, aunt(W, Andrea)), cupcake(Y), Z) \\ \land \neg gave(aunt(X, aunt(W, Andrea)), cupcake(Y), Andrea) \Big)$$

ii Let $\mathcal{L}_1$ be a signature where:

- $\mathcal{P}_1 = \{computer\}$ where $computer(X)$: ('X is a computer')
- $\mathcal{P}_2 = \{connected\}$ where $connected(X, Y)$: ('X is connected to Y')

Then **There's a computer connected to every computer which isn't connected to itself** can be written as:

$$\exists Y \forall X \Big(connected(computer(Y), computer(X)) \land \neg connected(computer(X), computer(X)) \Big)$$

iii Let $\mathcal{L}_1$ be a signature where:

- $\mathcal{C} = \{PaulKlee, Kandinsky\}$
- $\mathcal{P}_1 = \{british\}$ where $british(X)$: ('X is British')
- $\mathcal{P}_2 = \{painting, hangs, gallery\}$ where $painting(X, Y)$: ('X is a painting by Y'), $hangs(X, Y)$: ('X hangs in Y'), $gallery(X, Y)$: ('X is in gallery Y')

Then **Any painting by Paul Klee in a British gallery hangs in a room where all Kandinsky paintings in that gallery hang** can be written as:

$$\forall X \exists Y \Big(british(gallery(painting(X, PaulKlee), Y)) \rightarrow \\ \forall W \exists Z \big(hangs(X, room(Z)) \land hangs(W, room(Z)) \land gallery(painting(W, Kandinsky), Y) \big) \Big)$$

iv Let $\mathcal{L}_1$ be a signature where:

- $\mathcal{P}_2 = \{loves\}$ where $loves(X, Y)$: ('X loves to Y')

Then **If there's somebody who loves nobody, then it's false that everybody loves somebody** can be written as:

$$\exists X \neg \exists Y loves(X, Y) \rightarrow \forall Z \exists W \neg loves(Z, W)$$

7 Question 7

i **False**: There exists an assignment σ of X where $a(k, X) = \mathbf{t}$ and $\neg(X = j) = \mathbf{f}$, thus making the formula false. This occurs when $\sigma(X) = \varphi(j)$.

ii **True**: When $c(l) = \mathbf{t}$, there exists an assignment σ such that $(b(X) \wedge c(X) \wedge a(l, X)) = \mathbf{t}$. This occurs when $\sigma(X) = \varphi(k)$.

iii **True**: There exists an assignment σ such that the formula is true. Consider the assignment $\sigma(X)$ in $\varphi(b)$ and $\varphi(s)$; in this case, there indeed does not exist an assignment $\sigma(Y)$ such that $\neg(X = Y) \wedge a(X, Y)$ is true.

iv **False**: There exists an assignment σ of X such that $\neg s(X) = \mathbf{t}$ and $\exists Y(c(Y) \wedge b(Y) \wedge a(X, Y)) = \mathbf{f}$ thus making the formula false. This occurs when $\sigma(X) = \varphi(j)$; in this case, there exists no assignment of Y such that $(c(Y) \wedge b(Y) \wedge a(X, Y)) = \mathbf{t}$.

v **False**: There exists an assignment, σ of X such that $\exists Y(\neg(X = Y) \wedge a(X, Y)) = \mathbf{t}$ and $\exists Y(a(X, Y) \wedge a(Y, X)) = \mathbf{f}$, thus making the formula false. This occurs when $\sigma(X) = \varphi(k)$, where there exists an assignment of Y such that $\neg(X = Y) \wedge a(X, Y)$ is true, but there exists no assignment of Y such that $a(X, Y) \wedge a(Y, X)$ is true.

vi **False**: There exists an assignment, σ of X and Y such that $a(X, j) \wedge a(Y, j) = \mathbf{t}$ and $a(X, Y) \vee a(Y, X) = \mathbf{f}$ thus making the formula false. Consider the assignments $\sigma(X) = \sigma(Y) = \varphi(k)$ and $\sigma(X) = \sigma(Y) = \varphi(l)$; these assignments are such that $a(X, j) \wedge a(Y, j)$ true while $a(X, Y) \vee a(Y, X)$ is false.