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Department of Computing
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Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
Exercise:	2 (CW)	**Due:**	Tue - 03 Nov 2020
Title:	Logic	**Assessment:**	Individual
FAO:	Craven, Robert (rac101)	**Submission:**	Electronic

Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-11-03 11:37:29

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Symbolic AI Coursework

1)

i) ~~p: [illegible]~~ P: michael is fulfilled

~~q: [illegible]~~ q: michael is rich

~~r: [illegible] 5 year~~ r: michael will live another 5 years

$$((\neg(p \lor q)) \rightarrow (\neg r))$$

ii) unless: treat as 'or'

~~p: [illegible]~~ P: the snowstorm does arrive

~~q: [illegible]~~ q: raheem will wear his boots

~~r: [illegible] will arrive~~

r: I am sure the snowstorm will arrive

$$(((\neg p) \lor q) \land r)$$

iii) ~~p: [illegible] set~~ ~~r: [illegible]~~

~~q: [illegible] set~~ ~~s: [illegible]~~

P: Akira is on set

q: Toshiro is on set

r: filming will begin

s: the caterers have cleared out

$$((p \land q) \rightarrow (r \leftrightarrow s))$$

iv) p: Irod arrived

q: Sarah arrived

$$((p \vee (\neg q)) \wedge (\neg(p \wedge (\neg q))))$$

v)

$$((\neg r) \rightarrow (\neg(p \wedge q)))$$

p: it is the case that Herbert heard the performance

q: it is the case that Anne-Sophie heard the performance

r: Anne-Sophie did answer her phone calls

2) i) a propositional formula A is satisfiable if there is some V such that

$h_V(A) = t$

ii) two propositional formulas A, B are logically equivalent if for every V,

$$h_V(A) = h_V(B)$$

iii) must show that $\neg A$ is satisfiable if and only if $\neg\neg A \equiv T$

assume $\neg A$ is satisfiable $\therefore$ take a certain propositional evaluation function V

then $h_V(\neg A) = t$ therefore by the meaning of $\neg$

$$\text{then } h_V(\neg\neg A) = f \neq h_V(T)$$

We have shown that this ~~is~~ leads to $\neg\neg A \not\equiv T$ as ~~they must~~

$h_V(\neg\neg A)$ and $h_V(T)$ must have the same value under h_V in order to be logically equivalent and wherever $h_V(\neg A) = t$ we have shown that

$\neg\neg A \not\equiv T$

• So we have shown under the satisfiability assumption that if $\neg A$ is true then $\neg\neg A$ must be false and this defines $\neg\neg A \not\equiv T$

3) a propositional formula is valid if $h_v(A) = t$ for any valuation V

~~Check all the valuations and see whether the formula is true in each~~

Check all the valuations and see whether the formula is true in each

3 atoms so 2^3 valuations of formula to check $\therefore 8$

$$(p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p)) \rightarrow (\neg\neg q \rightarrow r)$$

p	q	r	p	∧	¬	q	↔	¬	¬	r	∨	¬	p	→	¬	¬	q	→	r
t	t	t	t	f	f	t	f	t	f	t	f	f	t	t	t	f	t	t	t
t	t	f	t	f	f	t	t	f	t	f	t	f	t	f	t	f	t	f	f
t	f	t	t	t	t	f	t	t	f	t	f	f	t	t	f	t	f	t	t
f	t	t	f	f	f	t	t	f	f	t	t	t	f	t	t	f	t	t	t
f	t	f	f	f	f	t	t	f	t	f	t	t	f	f	t	f	t	f	f
f	f	t	f	f	t	f	t	f	f	t	t	t	f	t	f	t	f	t	t
t	f	f																	
f	f	f																	

Since when $h_v(p) = f$, $h_v(q) = t$ and $h_v(r) = f$

we get that $h_v((p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p)) \rightarrow (\neg\neg q \rightarrow r))$

$$= f$$

we can conclude that our formula is not valid

9) yes as $p \land q$ can be considered as a

a disjunction of literals

4) i) a formula is in conjunctive normal form if it is a conjunction of clauses

eg: $(P \vee \neg q) \wedge (r \vee \neg D)$

CNF

a) Yes

b) Yes

c) No this is not a conjunction of clauses

d) Yes T is a literal $\therefore$ T is a clause & T is an CNF

e) No

f) No as $\neg\neg P$ is not a literal due to the double negation sign

g) Yes

h) Yes as $P \vee q$ is itself a clause

if it is a disjunction of conjunctions

DNF a) No

b) Yes - both are conjunctions of P and truthity

c) No

d) Yes as T is logically equivalent to $(T \wedge T) \vee (T \wedge T)$

e) Yes $\therefore$ it is a disjunction of conjunctions

f) No

h) Yes as $P \vee q$ can be considered as $(P \wedge T) \vee (q \wedge T)$

g) Yes as $P \wedge q$ can be considered as $(A \wedge B) \vee (A \wedge \neg B) \vee (\neg A \wedge B)$

ii) if $C = \text{resolvent}(C_1, C_2, P)$, then $\{C_1, C_2\} \models C$

which leads to the refutation - soundness and completeness :

Let S be in CNF. $S \vdash_{res(PL)} \emptyset$ if and only if $S \models \bot$

This property is important as if S is in CNF, then S is satisfiable if and only if $S \nvdash_{res(PL)} \emptyset$

meaning we have a method now to prove satisfiability ~~[illegible]~~ of S

4) iii) a) $\{\{p,s\},\{q,r\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\}$

[q was pure]

$$\{\{p,s\},\{\neg p,\neg r,\neg s\}\}$$

[¬r was pure]

$$\{\{p,s\}\}$$

[p was pure] $\{\}$

b) $\{\{\neg p,q,r\},\{\neg q\},\{p,r,q\},\{\neg r,q\}\}$

[unit propagation by unit clause $\{\neg q\}$]

$$\{\{\neg p,r\},\{p,r\},\{\neg r\}\}$$

[unit propagation by unit clause $\{\neg r\}$] $\{\{\neg p\},\{p\}\}$

[unit propagation by unit clause $\{p\}$] $\{\{\}\} = \{\emptyset\}$ contains the null set

5) p: I am going
q: You are going
r: Tara is going

$$p \to \neg q$$

$$\neg q \to \neg r$$

$$r \vee \neg p$$

$$r \vee p \quad \models q$$

formalized as $p \to \neg q,\ \neg q \to \neg r,\ r \vee \neg p,\ r \vee p \models q$

we know $A_1, \ldots, A_n \models B$ iff $A_1 \wedge \ldots \wedge A_n \wedge \neg B$ is unsatisfiable

~~get convert to CN~~F

$$(\neg p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (r \vee p) \wedge (\neg q)$$

to CNF

$$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$$

apply DP

unit propagation by unit clause $\{\neg q\}$

$$\{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$$

rewritten on the next side

next Page →

$$\{\{\neg r\}, \{r, \neg P\}, \{r, P\}\}$$

unit propagation by unit clause $\{\neg r\}$

$$\{\{\neg P\}, \{P\}\}$$

unit propagation by unit clause $\{P\}$

$$\{\{\}\} = \{\emptyset\}$$ we have found the empty set

This means that the CNF is unsatisfiable which means that the original argument is propositionally valid

6) i) Let L be the Signature $(C, \{P_i\}_{i \in \mathbb{N}}, \{F_i\}_{i \in \mathbb{N}^+})$, where:

$$C = \{\text{Andrea}\}$$

$$P_2 = \{\text{auntsaunts}_{(X,Y)} : (X \text{ is the aunt's aunt of } Y),$$

$$\text{gaveacupcake}_{(X,Y)} : (X \text{ gave a cupcake to } Y)\}$$

$F_i = \emptyset$ ~~for all~~

~~for~~ all other component of the Signature are $\emptyset$

$\cancel{\forall X(\text{auntsaunts}(X, \text{Andrea}) \to \exists Y(\text{gaveacupcake}(X,Y) \land$}

for all X if X is an auntsaunt of Andrea, then they gave a cupcake to someone who is not Andrea

$$\forall X(\text{auntsaunts}(X, \text{Andrea}) \longrightarrow \exists Y(\text{gaveacupcake}(X, Y) \land \neg(Y = \text{Andrea})))$$

ii) Let L be the Signature $(C, \{P_i\}_{i \in \mathbb{N}}, \{F_i\}_{i \in \mathbb{N}^+})$ where:

$$P_1 = \{\text{Computer}(X) : (X \text{ is a Computer})\}$$

$$P_2 = \{\text{Connected}_{(X,Y)} : (X \text{ is connected to } Y)\}$$

There exists an X such that X is connected to every Y where Y is not connected to itself

$$\exists X(\text{computer}(X) \land \forall Y(\text{Computer}(Y) \land \neg\text{connected}(Y, Y) \longrightarrow \text{Connected}(X, Y)))$$

6) iii) for all X if X is a painting by Paul Klee and X hangs in a British gallery then X hangs in a room where if there exists a Y such that Y is a Kandinsky painting, then for all Y, Y hangs in this room

~~Signature~~

Let L be the Signature $(C, \{P_i\}_{i\in\mathbb{N}^+}, \{F_i\}_{i\in\mathbb{N}^+})$ where:

$C = \{$British Gallery, Room$\}$

$P_1 = \{$ Painting $(X) : (X$ is a painting$)$,

Paul Klee $(X) : (X$ is by Paul Klee$)$,

Kandinsky $(X) : (X$ is by Kandinsky$)$, $\}$

$P_2 = \{$ hangs $(X, Y) : (X$ hangs in $Y) \}$

$$\forall(X)\Big(\text{Painting}(X) \wedge \text{PaulKlee}(X) \wedge \text{hangs}(X, \text{British Gallery}) \longrightarrow \text{hangs}(X, \text{room})$$
$$\wedge\ \forall Y\big(\text{Painting}(Y) \wedge \text{Kandinsky}(Y) \longrightarrow \text{hangs}(Y, \text{room})\big)\Big)$$

6) iv) let L be the signature $(C, \{P_i\}_{i \in \mathbb{N}^+}, \{F_i\}_{i \in \mathbb{N}^+})$ where:

$$P_2 = \{ \text{loves}(x, y) : (x \text{ loves } y) \}$$

If there is someone X such that there is everyone Y, such that X does not love Y, then it is not the case that for everyone X there is someone Y such that X loves Y

$$\exists x \forall y \neg \text{loves}(x, y) \rightarrow \neg \forall x \exists y \text{ loves}(x, y)$$

$C : \{ j, k, l$

7) i) $P_1 : \{ b, w, s, c$

$P_2 : \{ a$

translation:

i) for all X if there is a ~~directed~~ directed ~~arrow~~ arrow from k to X then

X does not equal j

justification:

· looking at the object $\varphi(k)$ we can clearly see the only directed arrow from object $\varphi(k)$ goes to object $\varphi(j)$

therefore this formula is false

$\cdot \ (\varphi(k), \varphi(j)) \in \varphi(a) \quad \therefore$ ~~false~~

false

translation:

ii) if L is in the set of circular objects then there exists an X such that X is in the set of filled black objects and X is in the ~~the~~ set of circular objects and there is a directed arrow from L to X

justification:

- L is in the ~~circular~~ set of circular objects which makes the antecedent of the material conditional true, therefore for the entire formula to be true the consequent must also be true

the object ~~k~~ $\varphi(k)$ is ~~part of the set of circle~~ . an example

$(\varphi(k)) \in \varphi(c)$

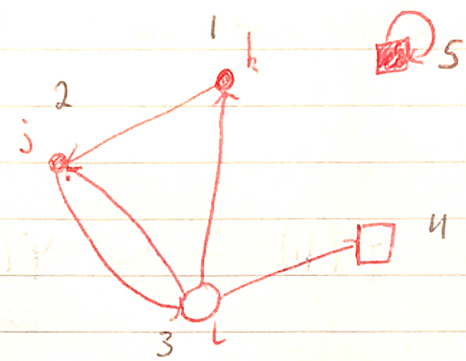
$(\varphi(k)) \in \varphi(b)$

$(\varphi(L), \varphi(k)) \in \varphi(a)$ So the formula is $\underline{\underline{\text{True}}}$

7) iii) translation:

there exists an X such that a Y does not exist such that X does not equal Y and there is a directed arrow from X to Y

~~there exists the~~



Justification:

$\exists y(\neg(x=y)\wedge a(x,y))$

is false for object 5 ~~object 5~~

therefore the negated version is true therefore the overall formula is True translation

iv) for all X if X is not a square then there exist a Y such that

Y is circle and Y is black and there is an arrow from X to Y

Justification

all ~~x~~ X that are not a square : is $\varphi(j)$

$\varphi(k)$

$\varphi(l)$

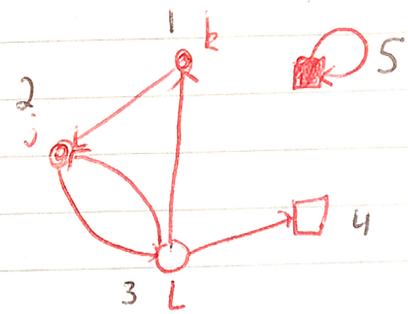
$\therefore$ for $\varphi(k)$ the object $\varphi(j)$ satisfies all 3 constraints of being black, a circle and there is a line from ~~k to~~ $\varphi(k)$ to $\varphi(l)$

for $\varphi(j)$ there is no object that is black a circle and there is a line from $\varphi(j)$ to this object

therefore this formula is <u>false</u>

Translation:

v) for all X if there exists a Y such that X does not equal Y and there is a directed arrow from X to Y then there exists a Y such that that there is ~~...~~ a directed arrow from X to Y and a directed arrow from Y to X



justification:

Looking at the antecedent of the formula:

examples ~~are~~ that make the antecedent true are:

$X = 3 \quad Y = 4$

$X = 3 \quad Y = 1$

$X = 3 \quad Y = 2$

$X = 2 \quad Y = 3$

$X = 1 \quad Y = 2$

then the consequent of the formula must be true for each X at least one for there to exist a Y such that $a(X, Y)$ and $a(Y, X)$

This is true for $X = 3$ and for $X = 2$, however for $X = 1$ there is no Y such that there is a directed arrow to X from Y and ~~from~~ to Y from X so this formula is False

translation:

vi) for all X and for all Y ~~if~~ ~~this~~ then is a directed arrow from X to j

and from Y to j then ~~this~~ there is a directed arrow from X to Y or from ~~Y to~~ ~~Y to~~

Y to X

Justification:

~~if ...~~

for the antecedent to be true

$X = l$	$Y = k$
$X = k$	$Y = l$
$X = k$	$Y = k$
$X = l$	$Y = l$

for each of these: the consequent then is a directed arrow from l to k

~~this~~ therefore for $X = l$ $Y = k$ $a(X, Y)$ is true

and for $X = k$ $Y = l$ $a(Y, X)$ is true so the consequent is always

true when the antecedent is true making formula True