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Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
Exercise:	2 (CW)	**Due:**	Tue - 03 Nov 2020
Title:	Logic	**Assessment:**	Individual
FAO:	Craven, Robert (rac101)	**Submission:**	Electronic

Student Declaration - Version 1

- I acknowledge the following people for help through our original discussions:

Sicong Chen (sc5816)

Signed: (electronic signature) Date: 2020-10-28 20:16:18

For Markers only: (circle appropriate grade)

CHEN, SICONG (sc5816)	01190445	t5	2020-10-28 20:16:18	**A* A B C D E F**

Introduction to Symbolic AI Coursework 1: Logic

1. i p: Michel is fullfilled q: Michel is riched
r: He will live another five years

$$(\neg p \land \neg q) \rightarrow \neg r$$

ii p: Snowstorm does arrive q: Raheem will wear his boots
r: I'm sure snowstorm will arrive

$$(\neg p \lor q) \land r$$

iii p: Akira and Toshiro are on set. q: Filming will begin.
r: The ~~chan~~ caterers have cleared out.

$$(p \rightarrow q) \Leftrightarrow r$$

iv. p: Erad arrived q: Sarah arrived

$$(p \lor \neg q) \land \neg (p \land \neg q)$$

v. p: Herbert heard the performance q: Anne-Sophie did
r: Anne-Sophie answer her phone calls

$$\neg r \rightarrow \neg (p \land q)$$

2.

i. A propositional formula A is satisfiable if there is some v such that $h_v(A) = t$

ii. Two propositional formulas A, B are logically equivalent if, for every v, $h_v(A) = h_v(B)$

iii.

$\Rightarrow$: If $\neg A$ is satisfiable, then there is some v_1 such that $h_{v_1}(\neg A) = t$. Hence, $h_{v_1}(\neg\neg A) = f$. Therefore $\neg\neg A$ is not always true which is $\neg\neg A \not\equiv \top$.

$\Leftarrow$: If $\neg\neg A \not\equiv \top$, then there is some v_2 such that $h_{v_2}(\neg\neg A) = f$. Hence, $h_{v_2}(\neg A) = t$. Hence $\neg A$ is satisfiable. $\square$

3.

p	q	r	$(p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p))$	$\to$	$(\neg\neg q \to r)$
t	t	t	tff f t fff	T	ttt
t	t	f	tff t f ttf	F	tff
t	f	t	ttt t tfff	T	ftt
t	f	f	ttt f fttf	T	ftf
f	t	t	fff t fftt	T	ttt
f	t	f	fff t fttt	F	tff
f	f	t	fft t fftt	T	ftt
f	f	f	fft t fttt	T	ftf

Hence, this is not valid.

4.

i a. CNF b. both CNF and DNF

c. neither CNF nor DNF d. both CNF and DNF

e. DNF f. CNF

g. CNF h. DNF

ii Let S be in CNF. $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$.

This property is very important because we can use it to check the satisfiability of S. ~~which~~ In other words, S is satisfiable iff $S \nvdash_{res(PL)} \emptyset$.

iii a. Firstly, by applying the pure rule ~~on~~ with the literal q, we get

$\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$.

Then, by applying the pure rule ~~on~~ with the literal $\neg r$, we get

$\{\{p, s\}\}$.

b. Firstly, by applying the unit propagation with the unit clause $\neg q$, we get

$\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$.

Then, by applying the unit propagation with the unit clause $\neg r$, ~~we~~ we have

$\{\{\neg p\}, \{p\}\}$.

Finally, by applying the unit propagation with the unit clause p, we have

$\{\{\}\}$.

5. p: I am going q: You are going r: Tara is going

These 5 arguments could be written as

1. $p \to \neg q$
2. $\neg p \to \neg r$
3. $r \lor \neg p$
4. $r \lor p$
5. q

Since $p \to \neg q$ and $\neg p \lor \neg q$ are logically equivalent, and $\neg q \to \neg r$ and $q \lor \neg r$ are logically equivalent,

the CNF is

$$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{q\}\}$$

By applying the unit propagation on q,

$$\{\{\neg p\}, \{r, \neg p\}, \{r, p\}\}$$

By applying the unit propagation on $\neg p$,

$$\{\{r\}\}.$$

Hence, this is satistiable.

6.

i $C = \{Andrea\}$ $P_1 = \{human\}$ $P_2 = \{aunt, gave\}$

human(X) means X is a human.

aunt(X, Y) means X is Y's aunt.

gave(X, Y) means X gave Y a cupcake.

$$\forall X \exists Z\ (\forall Y\ aunt(X,Y) \wedge aunt(Y, Andrea)) \wedge gave(X, \cancel{human(Z)}\ Z) \wedge \neg(Z = Andrea) \wedge human(Z))$$

ii

$P_1 = \{comp\}$ $P_2 = \{connect\}$

comp(X) means X is a computer

connect(X, Y) means X connected to Y.

$$\exists X \forall Y\ \cancel{connect(comp(X), comp(Y))}$$
$$connect(X, Y) \wedge \neg(X = Y) \wedge comp(X) \wedge comp(Y)$$

iii $C = \{Paul\ Klee, Kandinsky\}$

$P_1 = \{BG\}$ $P_2 = \{Painting, room\}$

BG(X) means X is in British gallery.

painting(X, Y) means X is a painting of Y.

room(X, Y) means X is in room Y.

$$\exists Z \forall X \forall Y\ \cancel{\exists Z}\ painting(X, Paul\ Klee) \wedge BG(X) \wedge Painting(Y, Kandinsky) \wedge BG(Y) \wedge Room(X, Z) \wedge Room(Y, Z)$$

~~iv $P_1 = \{human\}$ $P_2 = \{love\}$~~

$$\cancel{\forall Y (\exists X\ love(human(X), human(Y)))}$$

$$\rightarrow \cancel{(\exists Z \neg love(human}$$

iv $P_1 = \{human\}$ $P_2 = \{love\}$

human(X) means X is a human.

love(X,Y) means X loves Y.

$$\left(\exists X \neg \exists Y \; love(X,Y) \wedge human(X) \wedge human(Y)\right)$$

$$\rightarrow \left(\forall Y \exists X \; \neg love(Y,X) \wedge human(X) \wedge human(Y)\right)$$

7.

i. False. If we take X to be j, then $a(k,X)$ is true. However, $\neg(X=j)$ is false. Hence, this statement is false.

ii. True. Since $c(l)$ is true, we need to find ~~X~~ such that it is a black circle and there is a directed arrow from l to it to make the statement true. If we take X to be j, ~~[illegible]~~ $b(X) \wedge c(X) \wedge a(l,X)$ is true. Hence, this statement is true.

iii. False. If we take X to be k, we can find a Y (i.e. Y to be j) such that $\neg(X=Y) \wedge a(X,Y)$ is true. Hence, this statement is false.

iv. False. If we take X to be j, then $\neg s(X)$ is true. However, we could not find Y such that it is a black circle and there is a directed arrow from X to it. ~~Hence~~ Therefore, $\exists Y(c(Y) \wedge b(Y) \wedge a(X,Y))$ is false. Hence, this statement is false.

v. False. ~~If we take X to be l and Y to be k.~~ If we take X to be k, then ~~$\exists X$~~ $\exists Y(\neg(X=Y) \wedge a(X,Y))$ is true since we can take Y to be j. However, $\exists Y(a(X,Y) \wedge a(Y,X))$ is false since we could not find Y such that both $a(X,Y)$ and $a(Y,X)$ is true. Hence, this statement is false.

vi. False. If we take ~~$X=Y$~~ X to be k and Y to be k as well, then $a(X,j) \wedge a(Y,j)$ is true. However, $a(X,Y) \vee a(Y,X)$ is false. Hence, this statement is false.