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Consensus control of stochastic multi-agent systems: a survey

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Abstract In this article, we provide a review of the consensus control problem for stochastic multi-agent systems (MASs). Recent advances are surveyed according to the method of occurrence of the stochasticity of the MASs. First, the consensus problem is discussed for MASs, wherein individual agents are corrupted by random noises, i.e., the dynamics of agents involve stochasticity in process and/or measurement equations. Both additive noises and multiplicative noises are surveyed in detail and special attention is paid to the MASs whose dynamics are governed by Itô differential equations. Moreover, particular effort is devoted to presenting the latest progress on the consensus problem for a special type of stochastic MAS with Markovian jump parameters. Subsequently, the relevant research is summarized for MASs with noisy communication environments and stochastic sampling. Further, we provide a systematic review of the consensus problems for MASs whose communication topology varies randomly in the process of data propagation among agents. Finally, conclusions are drawn and several potential future research directions are outlined.

Keywords stochastic multi-agent systems, consensus control, stochastic noises, Markovian jump systems, random topology

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1 Introduction

In the past decade, there has been a surge of research interest on multi-agent systems (MASs) primarily owing to their extensive application in various fields ranging from the chemistry manufacturing industry, geological exploration, and building automation, to military and aerospace industries [1–11]. MASs are comprised of numerous interaction/cooperation units called agents, which directly interact with their neighbors according to a given topology. The MASs often demonstrate rich yet complex behavior even when all the agents have tractable models and interact with their neighbors in a simple and predictable

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fashion [12]. Owing to its clear physical and engineering insights, the consensus problem of MASs has been garnering considerable research attention and many results have been reported in the literature, such as those on the cooperative control of unmanned air vehicles (UAVs) or unmanned underwater vehicles [13], formation control for multi-robot systems [14, 15], collective behaviors of flocks or swarms [16], and distributed sensor networks [17–22].

One of the major topics of research on MASs is the so-called consensus control, whose objective is to design a consensus algorithm (or protocol) using local neighboring information such that the states of a team of agents reach some common features. The common features are dependent on the states of all the agents and examples of such features include position, phase, velocity, and attitude. The past several years have witnessed a considerable surge of interest regarding MASs with close ties to consensus problems. These relevant research fields include but are not limited to 1) consensus, which indicates that a collective of agents (sub-systems) reaches a common value dependent on the states of all agents [14]; 2) collective behavior of flocks and swarms, such as fish, birds, and bees where a large number of agents interact with their neighbors with a predetermined common group goal [23]; 3) sensor fusion in which a distributed filtering algorithm is adopted, which allows the nodes of a sensor network to track the average of n sensor measurements using an average consensus-based distributed filter called consensus filter [24–26]; 4) random networks whose topology varies over time in a manner that the existence of an information channel among individual agents is random and obeys certain probabilistic distribution [27]; 5) synchronization of coupled oscillators, which is recognized as a popular phenomenon in physics, engineering, biology, and other relevant fields, where a large quantity of coupled agents oscillate at a common pulsation [28, 29]; 6) algebraic connectivity of complex networks, which is expressed as the second smallest eigenvalue of the Laplacian matrix of the network, characterizing the speed of convergence of the consensus algorithm [30, 31]; 7) asynchronous distributed algorithms by which each individual agent calculates and updates at its own pace rather than the conventional mechanism where the decisions of all the agents have to be synchronized to a common clock [32]; 8) formation control whose purpose is to drive a collection of mobile autonomous agents to accomplish predetermined tasks while moving together according to a certain required formation [15, 33]; and 9) cooperative control problem, which deals with the cooperation among a group of interacting agents to perform a shared task by exchanging the information of individual agents with corresponding neighbors via communication networks [14, 34].

Notably, the fundamental problem among the above problems is the consensus problem. Generally, the consensus problem aims to design a control scheme/protocol that characterizes the way in which the individual agent interacts with neighbors, thereby driving all the states of a network of agents to reach a certain common value of interest. The following provides an explicit definition of the consensus of a MAS.

Definition 1. Consider a MAS with N agents, where x_i $(i = 1, 2, 3, \ldots, N)$ denotes the state of the ith agent. The MAS is said to reach consensus if $\forall i \neq j$, $\|x_i - x_j\| \rightarrow 0$ when $t \rightarrow +\infty$.

The basic idea of a consensus algorithm/protocol is that each individual agent utilizes the communication network to exchange information and accordingly designs the appropriate distributed control algorithm such that the complicated system resulting from the combination of dynamics of agents and network topology reaches consensus or synchronization. If the interaction network is capable of continuous communication, or if the network bandwidth is sufficiently large, the updating of the state information of each individual agent can be modeled by a differential equation. However, if the information exchange is performed in discrete fashion, the difference equation is utilized to illustrate the updating of the state information of agents.

As is well known, in the past few decades, stochastic systems have gathered increasing research interest and have found wide applications in many fields because the stochastic phenomenon is inevitable and cannot be avoided in the real-world systems. Therefore, the analysis and synthesis problems concerning stochastic systems have attracted much attention, see [35–40] and the references therein. In the context of MASs, the randomness might occur in many ways. First, it could occur in the system dynamics. For example, the random noises originating from either an internal device or external environment impose stochasticity in both the process and measurement equations. Moreover, the abrupt change of working conditions or the aging or erosion of certain equipment also creates randomness in the system dynamics. Second, the random switching of the network topology, which is frequently observed in practical engineering, also renders the MAS a stochastic system. Third, the utilization of the communication network to transmit information encounters many issues such as time delay, which probably occurs in a random way. Furthermore, the topology according to which the agents share their information might vary stochastically.

All these aforementioned stochastic sources result in difficulties in both the analysis and synthesis of the consensus problem for MASs. This is mainly because the traditional consensus definition, which is proposed for deterministic dynamics and requires all the states of agents to converge to the same point, is no longer applicable in the context of stochastic MASs owing to the existence of randomness. Consequently, further development of consensus control for stochastic MASs is a major concern for practitioners and scientists in the fields of systems sciences and control engineering.

This survey aims to provide a thorough enumeration, classification, and analysis of recent contributions to the research on consensus control of stochastic MASs. Although previous reviews have investigated the progress on this topic for deterministic MASs, this work is the first deep overview of recent advances in the branches of stochastic MASs, additionally outlining some vital future challenges that should be addressed to ensure meaningful progress and development of novel methods.

The rest of the paper is organized as follows. In Section 2, the consensus control problem of MASs is surveyed, where the dynamics of the individual agents are stochastic. Some recent advances are reviewed from different aspects according to the method of occurrence of stochasticity. Section 3 reviews the consensus control for MASs whose communication topology varies in a random way. The conclusions and future work are provided in Section 4.

2 Consensus of MASs with stochastic dynamics

In this section, we first provide an overview of the consensus control for MASs whose dynamics are governed by a stochastic differential equation or stochastic difference equations. Subsequently, we review the consensus control problem in which the communication channels are affected by random noises, which render the MASs under investigation stochastic systems.

Owing to the existence of stochasticity, the conventional consensus definitions for deterministic MASs, which require the states of all the agents to converge to the same point, are not capable of characterizing the consensus process in the stochastic context. Therefore, by borrowing ideas from the stochastic analysis and control theory, researchers and engineers have proposed several consensus conceptions in different probabilistic senses to describe the consensus behaviors of stochastic MASs. These conceptions include but are not limited to mean square consensus, consensus in pth moment, consensus in probability, and almost sure consensus. In the following subsection, we review the recent progress from the perspective of different locations where the stochasticity occurs in the system dynamics.

2.1 Stochasticity in the process equations of agents

In contrast to most of the multi-agent consensus literature dealing with L_2 energy-bounded noises [41–44] or unknown-but-bounded noises [45], in this subsection, we review the recent progress on the consensus control problem for MASs whose process equations contain random noises. We mainly discuss the problem from three aspects: consensus subject to additive noises, consensus subject to multiplicative noises (also known as state-dependent noises), and consensus subject to Markovian jump parameters.

• Additive noises. The additive noises frequently observed in the real-world engineering practice include maneuvering target tracking [46], distributed sensing [47,48], flight control [49,50], and UAV [51, 52]. In a noisy environment, the dynamics of MASs can naturally be modeled by differential/difference equations subject to additive stochastic noises. The MAS with additive noises in the process equations can be represented by the following state-space equation:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Dw_i(t), \tag{1}$$

where $x_i(t)$ and $u_i(t)$, respectively, represent the state and control input, respectively, of the ith agent; $w_i(t)$ represents the additive process noises satisfying certain statistical properties and all the matrices have compatible dimensions.

Based on the model (1), some representative work can be summarized as follows. The optimal control problem has been discussed in [53] for continuous MASs in which each of the sub-systems is subject to additive stochastic noises. The objective is to drive the individual agent to reach certain predetermined target states. Owing to the existence of stochasticity in the system dynamics, an optimal control scheme has been developed aiming at minimizing, in the mean square sense, the so-called accumulated joint cost. Notably, the developed optimization control technique could be applied in the optimal consensus control of MASs with certain appropriate manipulations. However, in [54], the distributed tracking-type games have been considered for a class of coupled discrete MASs subject to additive random disturbances. By resorting to the Nash certainty equivalence principle, the authors have designed the controller by guaranteeing that the states of all the agents converge to a certain required function of the population state average. In [55], owing to the existence of Gaussian white noise in the state feedback input, the definition consensus with probability one has been proposed and utilized to handle the finite-time consensus control problem. By using probability theory, the consensus problem has been solved in the framework of Lyapunov theory.

• Multiplicative noises. In practical engineering, apart from additive noises, the MASs are often confronted with multiplicative noises (also known as state-dependent noises), which can be expressed by the following state-space equation:

$$x_i(k+1) = Ax_i(k) + Bu_i(k) + Mx_i(k)w_i(k), \tag{2}$$

where $w_i(k)$ denotes the stochastic multiplicative noises and all the matrices have compatible dimensions.

For discrete-time MASs described by (2), in [56], in order to deal with the multiplicative noises that enter both the process and measurement equations of agent dynamics, the authors have proposed a novel type of consensus, i.e., consensus in probability. Subsequently, by borrowing ideas from those publications concerning the input-to-state stability, sufficient conditions have been established for the addressed stochastic MASs to reach the desired consensus in probability. Note that, contrary to [55] where the consensus with probability one is discussed, the control strategy proposed in [56] has the ability to drive all the agents to reach consensus with any predetermined probability.

With regard to the continuous case of MASs subject to multiplicative noises, the Itô-type stochastic systems have played an important role and their dynamics are shown as follows:

$$\mathrm{d}x_i(t) = (Ax_i(t) + Bu_i(t))\mathrm{d}t + (Ex_i(t) + Hu_i(t))\mathrm{d}W(t), \tag{3}$$

where $W(t)$ is a scalar Brownian motion.

Recently, in [57], the exponential consensus problem has been considered for a type of nonlinear MAS whose dynamics are governed by Itô-type differential equations. The sufficient conditions have been derived for the existence of the desired control protocol using Lyapunov theory and a comparison principle, which is capable of guaranteeing the exponential leader-following consensus in the mean square sense. Similarly, a linear matrix inequality (LMI)-based framework has been established in [58] for a class of Itô stochastic MASs to solve the passivity-based mean square exponential consensus control problem.

By virtue of Lyapunov theory, for a second-order stochastic MAS governed by nonlinear Itô stochastic differential equations, the control protocol has been designed in [59] in order to drive all the agents to reach finite-time consensus in probability. Within a similar theoretical framework, the containment control in the mean square and mean square leader-following consensus problems have been discussed in [60] and [61], respectively, by using Lyapunov theory in combination with the LMI approach.

Recently, in [62], the exponential consensus in the mean square sense has been proposed to characterize the consensus process for a class of MASs with asynchronous switching topology. In this work, the time delays originating from the reception of confirmation of mode switching have also been considered before applying the matched controller. Using the so-called extended comparison principle, several easy-to-verify conditions for the existence of an asynchronously switched distributed controller are derived such that the addressed stochastic delayed MASs with asynchronous switching and nonlinear dynamics can achieve global exponential consensus in the mean square. Furthermore, in the technical note [63], the authors have investigated the stability problem of stochastic delayed systems by using a more general Halanay-type inequality with time-varying coefficients. A new comparison principle of the proposed Halanay-type inequality is developed to ensure the stability of stochastic delayed systems. The pth moment stability, pth moment asymptotic stability, and pth moment exponential stability are investigated using the proposed general Halanay-type inequality.

• Markovian jump dynamics. In recent years, the Markovian jump system has attracted tremendous research interest within the systems science and control communities primarily owing to its ability to model random variations, see [18, 19, 64–72] and the references therein. Accordingly, the corresponding analysis and design problems for MASs subject to Markovian parameters have started to gain increasing attention. Some recent representative results and the corresponding approaches can be summarized as follows.

In [73], an output feedback control scheme has been presented for the stochastic MASs whose dynamics equations and performance index are expressed in terms of Markovian jump parameters. By using the Markovian jump optimal filtering theory in combination with the mean field approach, the distributed feedback control algorithm has been designed to drive all the agents to approach the desired position. For heterogeneous MASs, in [74], a stochastic model has been introduced to characterize the switching between two types of agents, i.e., active agents and passive agents. The heterogeneous structure of the addressed MAS has been cast into an appropriate chosen Markov chain and a unified framework has thereafter been established within which a necessary and sufficient condition has been provided to ensure the requested consensus. In [41], the synchronization problem has been first solved for a class of complex dynamical networks with Markovian jump parameters that switch among a finite set. Subsequently, the obtained results have been extended to the relevant MAS with stochastic time delays. The broadcast control has been investigated in [75] for a group of Markovian MASs. A sufficient condition has been presented for the addressed Markovian MAS to stabilize the resulting broadcast control systems.

Furthermore, in addition to describing system dynamics, the Markov chain has also been extensively employed to characterize many other frequently observed stochastic phenomena in the consensus control problems for MASs. For instance, the data missing phenomenon that usually occurs during the information transmission among agents has been handled in [76], where the Markov chain has been adopted to illustrate the random nature of the data missing. Similarly, the Markov chain has been employed in [77] to characterize the random input time delay in a class of MASs. By using the Lyapunov theory and LMI approach, necessary and sufficient conditions have been derived to achieve the desired consensus.

Recently, with regard to the second-order MASs, the impulsive control methodology has been exploited in [78] where the sampled information used for generating the feedback control input is subject to random heterogeneous time delays governed by a Markov chain. Similarly, in [79], two mutually independent Markov chains have been employed to characterize the stochastic switching topology and random communication delays. The key idea behind the proposed approach is to convert the original systems into an expanded analogous error system with two Markovian parameters using model transformation.

Within the H_∞ framework, the control routing strategy has been proposed in [41] for mobile multi-agent networked systems. The Markovian jump linear system theory has been used to develop the required decentralized scheme with a prescribed H_∞ disturbance attenuation level. Such an H_∞ framework has also found applications in [43], where the fault detection and isolation problems have been handled for a class of discrete-time MASs with Markovian jump parameters. In this work, the imperfect communication channel and stochastic packet dropping effects have also been considered. The sufficient conditions have been established where the required fault detection and isolation algorithms have been proposed with an

H_∞ prescribed performance criterion.

• Other random phenomena. Apart from the aforementioned literature concerning the consensus control problem subject to stochastic noises, random data missing, and random communication time delays, some literature has been concerned with the corresponding problems with other random phenomena. For instance, the stochastic communication failure occurring in the data transmission channels has been considered in [80], whereas the randomly changing leader-following issue has been considered in [81] where the leader changes stochastically.

2.2 Stochasticity in the output/measurement equations of agents

Consider the following MASs:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t), \\ y_i(t) = Cx_i(t) + Dv_i(t), \\ u_i(t) = -\displaystyle\sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(y_j(t) - y_i(t)), \end{cases} \tag{4}$$

where $y_i(t)$ represents the measurement output of agent i; $v_i(t)$ denotes the additive random noises corrupting the measurements; the real-valued coefficient $a_{ij}(t)$ represents the weighting parameter imposed on the information sent to agent i from agent j. If at time t, agent j has not sent information to agent i, $a_{ij}(t) = 0$. All the matrices have compatible dimensions. From (4), we can observe that the stochasticity appearing in the measurement output influences the consensus performance via the propagation of information among the agents according to the communication topologies.

For discrete-time MAS with additive noises in measurements, several consensus notions in the probabilistic sense, such as weak consensus, strong consensus, mean square (m.s.) consensus, and almost sure (a.s.) consensus have been proposed in [82–84] to characterize the consensus behavior of the addressed stochastic MASs. A unified framework has been established within which the proposed consensus problems in the stochastic context can be discussed. By using the developed stochastic approximation-type algorithms with a decay factor with adjustable decreasing step size, all the individual states can be driven toward each other, thereby achieving the required consensus in the desired stochastic sense.

Similar notions have been proposed in [85, 86] for continuous stochastic MASs, where the definitions of average-consensus in the pth moment and almost sure consensus have been proposed to handle the stochastic consensus problem for a class of MASs whose measurements are affected by Gaussian white noise. In [87], a necessary and sufficient condition has been derived for the existence of an asymptotic unbiased protocol, ensuring the mean square average-consensus of the considered MASs with measurement noises.

With additive measurement noises, the work in [88] has been concerned with the stochastic consensus of MASs, where the information exchange among agents is described by a directed graph. By combining the algebraic graph theory, matrix theory, and stochastic analysis, the stochastic weak and strong consensus are examined. For the case with general digraphs, the authors have provided the necessary and sufficient conditions for the almost sure strong consensus and show that the mean square strong consensus and almost sure strong consensus are equivalent. Moreover, some necessary conditions and sufficient conditions have been obtained for the mean square and almost sure weak consensus. Especially, the necessary and sufficient conditions for the mean square weak consensus are provided for the case with undirected graphs.

For discrete MASs, in [89], the mean square consensus has been replaced by the so-called approximated consensus owing to the adoption of the local voting protocols with non-vanishing (e.g., constant) step size. Such an approximation methodology for consensus in a noisy environment has originated in [82, 83], where it has been demonstrated that, if the random noises have certain statistical properties (i.e., zero mean, bounded variances), the consensus can always be achieved in the almost sure sense by adopting a monotonically decreasing gain algorithm. The key idea of the proposed consensus schemes is to solve the nonlinear equations with measurements corrupted by noises, thereby performing the recursive stochastic approximation procedures [90–93].

Recently, by using the aforementioned stochastic approximation technique, a similar idea has been proposed in [8] to handle the consensus issues subject to a noisy environment by using the averaging approaches with stopping rules. Notably, the number of iterations in the stochastic approximation procedure has been provided explicitly in terms of coefficients relevant to the desired consensus performance (accuracy or precision) and probabilistic constraints. A similar framework has been established in [94] for a class of MASs with noisy communication, where a stopping-rule- oriented stochastic approximation method has been proposed in terms of characteristic values of communication network graphs. In comparison to the aforementioned literature concerning the steady state performance of the MASs, this paper has made the first attempt to investigate the transient performance during the consensus process.

The aforementioned literature is concerned with the case of additive noises. Notably, in [95], the corresponding consensus issue has been discussed while the measurement outputs are affected by multiplicative measurement noises. By using the algebraic graph theory and matrix theory, the consensus problem has been converted into a stochastic stability problem of stochastic differential equations driven by multiplicative noises. In this work, both almost sure consensus and mean square consensus have been considered and the corresponding sufficient condition has been established. It is shown that, for any bounded noise intensities, the desired mean square and the almost sure consensus can be achieved by carefully choosing the control gain. In [96], the conception of average consensus in pth has been first proposed and thereafter applied to solve the consensus problem for a class of continuous MASs subject to communication noises that are described as multiplicative noises dependent on the states in the output measurements. Based on the Lyapunov theory and stochastic analysis technique, the sufficient conditions for the addressed consensus problem have been established.

2.3 Stochasticity in the communication channels

In real-world applications, the measurement output information is often incomplete owing to various reasons, such as random communication delays, random network topologies, and stochastic Gaussian fading channels, which induces certain stochasticity in the analysis and synthesis issues in consensus problem for MASs. The MASs subject to stochastic disturbances in communication channels are expressed as follows:

$$\begin{cases} x_i(k+1) = Ax_i(k) + Bu_i(k), \\ y_i(k) = Cx_i(k), \\ u_i(k) = -\displaystyle\sum_{j\in\mathcal{N}_i(k)} a_{ij}(k)(y_j(k) - y_i(k) + \delta_i(k)), \end{cases} \tag{5}$$

where $\delta_i(k)$ represents the noises or disturbances during the information propagation via communication networks.

Some recent representative works can be summarized as follows. For instance, a distributed consensus-based protocol named Average TimeSync has been proposed in [97] where the case of random communication has been considered. In [98], the phenomenon of random time delay occurring in the communication channels has been modeled by introducing a Markov chain. By considering the transition probability of the time delay, the authors have obtained a less conservative design algorithm for the consensus of the addressed MASs. For heterogeneous MASs, a similar problem has been solved in [99] by using an event-triggering tracking control algorithm. Very recently, in [100], the authors have studied a general synchronization problem while the communication networks are subject to random delays. In this work, the communication delays are assumed to be bounded and obeying some independent identical distribution with certain known statistical properties. A much more general problem has been considered in [101] for distributed networked control systems where various communication constraints such as time delays, noises, and link failures are incorporated in a unified framework.

Apart from the random communication delays, the channels used for signal transmission always encounter the corruption of external stochastic noises or disturbances. Recently, for a class of MASs with noisy communication, a stopping-rule- oriented stochastic approximation method has been proposed in [94] in terms of the characteristic values of communication network graphs. In comparison to the aforementioned literature concerning the steady state of the MASs, this paper made the first attempt to handle the transient performance during the consensus process. Moreover, in [83], two algorithms have been designed to handle the average consensus problems subject to communication noises. In order to handle the so-called bias-variance dilemma (which indicates that running consensus for a longer period leads to more accurate consensus performance, but could probably result in larger variance), two consensus algorithms have been proposed to reach a satisfactory balance between the two essential performance indices by playing with the tradeoffs. In this paper, similar to the studies in [8, 89–94], the stochastic approximation arguments and Markov process theory have been utilized to derive the condition for the addressed MASs to reach the almost sure consensus. In [84], the average consensus control problem has been investigated for the first-order discrete-time MAS subject to uncertain communication environments. By using the probability limit theory and algebraic graph theory, the addressed problem has been solved despite the existence of stochastic communication noises. For the continuous case, the authors in [102] have proposed a stochastic version of the averaging approach originally proposed in [103,104] to solve the mean square leader-following consensus problem with additive communication noises.

2.4 Stochasticity in sampling

As is well known, in the real-world applications, the signal received by the sensor of each agent should be sampled before it is transferred or utilized, because in the context of wireless communication networks, the information is usually sent digitally. This leads to the control or filtering algorithms using sampling techniques. Accordingly, the consensus problems for MASs where various sampling approaches are utilized have been attracting increasing research interest recently, see, e.g., [21,105–118] . Notably, however, owing to the limitation of devices, sometimes, the signals might not be captured, which leads to an undesirable phenomenon where the sampled information is lost in probability. In this case, the randomness in the sampling inevitably impacts the MAS through the propagation of measurement output via communication networks.

Recently, such an issue has been attracting research attention. For example, in [119], the mean square node-to-node consensus tracking issue has been investigated under a stochastic sampling mechanism. The original problem has been converted to a control problem for systems with stochastic time delay, and therefore, sufficient conditions have been derived by adopting the routine techniques for the analysis and synthesis of stochastic time delay systems (e.g., Lyapunov functional approach, LMI algorithm, stochastic analysis).

In [120], the mean square exponential consensus problem with an H_∞ criterion has been solved by using the LMI approach within the framework of the Lyapunov theory. Notably, in this work, the sampling period of each agent switches between two possible values in a probabilistic manner. In a similar manner, the stochastic sampled-data leader-following consensus problem of nonlinear MASs has been addressed in [109]. By introducing certain random variables obeying the Bernoulli distribution, such a stochastic sampling phenomenon has been properly characterized. With regard to the continuous case, a similar problem has been considered in [121] where the sampling instants have been assumed to be stochastic with values from a finite set. Based on such a stochastic sampling mechanism, the event-triggering technique has been also employed by utilizing the sampled data to solve the corresponding consensus control problem.

3 Consensus subject to stochastic topologies

In this section, we review some recent results concerning the consensus problems for MASs with stochastic topologies.

So far, consensus control for MASs subject to stochastic topologies has mainly focused on Markovian switching communication topologies, see e.g., [42, 122, 123]. This is primarily due to the ability of

Markov chains to model the variation in the random changes of communication situations during the data transmission among agents. Some representative publications can be summarized as follows.

For a class of heterogeneous MASs comprised first- and second- order agents, the mean square consensus control problem has been investigated in [124] where the topology switches in a finite set according to a known Markov chain. The necessary and sufficient condition has been established for the requested consensus by using a stochastic irreducible aperiodic matrix method. The mean square containment control has been discussed in [125] under the same Markovian switching topology. A similar mean square consensus control problem has been solved in [126] for nonlinear continuous MASs.

In [127], the considered topology is assumed to not only switch according to a known Markov chain, but also be subject to time delays. The time delays under investigation enter the MAS dynamics from both the measurement transmission and state feedback. In the case of simultaneous occurrence of Markovian topologies, communication noises, and delays, an ergodicity approach has been proposed in [7] for backward products of degenerating stochastic matrices via a discrete-time dynamic system approach to analyze the consensus stabilization problem. The relevant problems have been solved in [128–131] where the addressed MASs have Markovian switching topologies, communication noises, and/or time delays.

In addition to the mean square consensus, several other consensus definitions and performance indices have been proposed to characterize different consensus processes/requirements for MASs subject to Markovian topologies. For instance, in [132], the necessary and sufficient condition has been derived for a class of MASs to achieve the expected consensus with the guarantee of minimum communication cost. An $L_2 - L_\infty$ performance criterion has been proposed in [44] for the studied MAS to reach the desired consensus with satisfactory steady-state performance. In [42], the H_∞ consensus problem for continuous-time MASs with Markovian network topologies and external disturbance has been investigated. In contrast to some existing results, global jumping modes of the Markovian network topologies are not required to be completely available for consensus protocol design. By developing a so-called network topology mode regulator, the unavailable global modes have been decomposed into several overlapping groups. Subsequently, a new group mode-dependent distributed consensus protocol on the basis of relative measurement outputs of neighboring agents has been proposed. The sufficient conditions for the existence of the desired distributed consensus protocols have also been derived in [42] to ensure the consensus of the MAS with a prescribed H_∞ performance level.

Notably, in contrast to the aforementioned literature regarding Markovian switching topologies, in [133], a series of mutually independent Bernoulli sequences have been introduced to illustrate the random switching topology. The resulting MAS is named a Bernoulli network and the considered mean square consensus problem can be converted into the mean square stabilization problem of $N - 1$ delay systems subject to a stochastic switching signal. The sufficient conditions for the desired consensus performance have subsequently been established within an LMI framework.

4 Conclusion and future work

In this paper, consensus control problems were reviewed along with the recent progress on stochastic MASs. The latest results on consensus analysis and protocol design issues for stochastic MASs were surveyed. Based on the literature review, potential relevant topics for future research are listed as follows.

In real-world engineering applications, there are still some complicated yet important categories of stochastic MASs that have not attracted adequate research attention. Consequently, the consensus control problems still remain open and challenging for MASs whose dynamics are more general and complex, such as those investigated in [134–144]. Moreover, in practice, as the energy that can be used by each agent is always limited, another future research direction is to further investigate the consensus control problem for stochastic MASs with full/better utilization of the limited energy. The event-based communication mechanism [17,20,45,56,145–153] might be a possible way to solve the energy-constrained consensus control problem. Furthermore, as the communication protocol plays an important role in the consensus of MASs, more efforts should be devoted to the investigation of the protocol design issues for MASs in the presence of stochasticity. Different protocols, such as those studied in [154, 155], could be adopted to enhance the consensus performance of stochastic MASs.

Furthermore, most of the existing literature regarding the addressed problems always assumes that the topologies are fixed and/or time-invariant, and the limited work concerning random topologies mainly focuses on the Markovian switching topology. However, in practical engineering, such a random topology does not always explicitly reflect the real situation of the time-varying and stochastic topology. Consequently, it would be interesting to study the consensus problem for MASs whose topology varies according to a different probabilistic distribution than a Markov chain.

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