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Exercise Information

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- I acknowledge the following people for help through our original discussions:

Ding Ke (kd120)

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For Markers only: (circle appropriate grade)

DING, Ke (kd120)	01328574	t5	2020-11-03 15:18:38	A* A B C D E F

Q1(i)

p: Michel is fulfilled

q: Michel is rich

r: Michel will live another five years

$$\left((\neg(p \vee q)) \longrightarrow (\neg r)\right)$$

Q1(ii)

p: The snowstorm arrives

q: Raheem will wear his boots

r: I am sure that the snowstorm will arrive

$$\left(\left((\neg(\neg p)) \longrightarrow q\right) \wedge r\right)$$

Q1(iii)

p: Akira is on set

q: Toshiro is on set

r: Filming will begin

s: Caterers have cleared out

$$((p \vee q) \longrightarrow (r \leftrightarrow s))$$

Q1(iv)

p: Irad arrived

q: Sarah arrived

$$\left((p \vee (\neg q)) \wedge \left(\neg(p \wedge q)\right)\right)$$

Q1(v)

p: Herbert heard the performance

q: Anne-Sophie heard the performance

r: Anne-Sophie answered her phone calls

$$\left((\neg r) \longrightarrow \left(\neg(p \wedge q)\right)\right)$$

Q2(i)

If there exist some propositional evaluation function v such that $h_v(A) = t$, propositional formula A is be said to be satisfiable.

Q2(ii)

If $h_v(A) = h_v(B)$ holds for every evaluation v, then A and B are logically equivalent.

Q2(iii)

The statement to prove contains the "if and only if" term which indicates that the statement should be proved in two-ways, to demonstrate the symmetry(i.e. vice-versa):

$$\begin{cases} (1) if\ \neg\neg A \not\equiv \top, then\ \neg A\ \ is\ satisfiable \\ (2) if\ \neg A\ is\ satisfiable, then\ \neg\neg A \not\equiv \top \end{cases}$$

To prove (1):

First assume that $\neg\neg A \not\equiv \top$, then find the evaluation v such that the evaluation allows the antecedent to be true: thus $h_v(A) = f$, since if $h_v(A) = f$, then $h_v(\neg\neg A) = f$, given it is always the case that $h_v(\top) = t$, thus we have a case in which $h_v(\neg\neg A) \neq h_v(\top)$, therefore $\neg\neg A \not\equiv \top$by definition of logical equivalence in Q2(ii).

Now for the v that $h_v(A) = f$, $h_v(\neg A) = t$, then this case also makes $\neg A$ satisfiable, by definition of satisfiability in Q2(i). So (1) is proved given all above.

To prove (2):

First assume that $\neg A$ is satisfiable, find the evaluation v such that $h_v(\neg A) = t$ (if $h_v(\neg A) = t$ then $\neg A$ is satisfiable). Thus we have the case where $h_v(A) = f$, and then $h_v(\neg\neg A) = f$. Now given that it is always the case that $h_v(\top) = t$, this means for this evaluation $h_v(\neg\neg A) \neq h_v(\top)$, which then leads to $\neg\neg A \not\equiv \top$. Thus (2) is also proved.

Now that the two ways have all been proved, the statement is proved.

Q3

Let the given formula be A. 3 unique atoms appear in A, thus 8 different valuation functions are required. Note in the below truth table for every single negated atom only one truth value is written down. The resulting truth-table is attached below as a picture:

p	q	r	$(p$	$\wedge$	$\neg q$	$\leftrightarrow$	$\neg($	$\neg r$	$\vee$	$\neg p))$	$\longrightarrow$	$(\neg$	$\neg q$	$\rightarrow$	$r)$
t	t	t	t	f	f	f	t	f	f	f	t	t	f	t	t
t	t	f	t	f	f	t	f	t	t	f	f	t	f	f	f
t	f	t	t	t	t	t	t	f	f	f	t	f	t	t	t
f	t	t	f	f	f	t	f	f	t	t	t	t	f	t	t
t	f	f	t	t	t	f	f	t	t	f	t	f	t	t	f
f	t	f	f	f	f	t	f	t	t	t	f	t	f	f	f
f	f	t	f	f	t	t	f	f	t	t	t	f	t	t	t
f	f	f	f	f	t	t	f	t	t	t	t	f	t	t	f

It can be observed that the pinpointed column is the final valuation result for the given formula A. There are two rows with false, which means not all valuation v leads to $h_v(A) = t$, thus the given formula is not valid.

Q4(i)

CNF: a, b, d, e, g, h

DNF: b, d, e, g

Q4(ii)

Let S be in CNF, there exists a derivation of $\emptyset$ from S if and only if $S \vDash \bot$(which means S is never satisfiable). This is important as it allows one to check the satisfiability: S is satisfiable when it is impossible to derive $\emptyset$ from S by a resolution derivation.

Q4(iii)(a)

$$\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$$

$$= \{\{p, s\}, \{\neg p, \neg r, \neg s\}\} \quad (pure\ rule\ on\ q)$$

$$= \{\{p, s\}\} \quad (pure\ rule\ on\ \neg r)$$

Q4(iii)(b)

$$\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$$

$$= \{\{\neg p, r\}, \{p, r\}, \{\neg r\}\} \quad (unit\ propagation\ by\ \neg q)$$

$$= \{\{\neg p\}, \{p\}\} \quad (unit\ propagation\ by\ \neg r)$$

$$= \{\{\emptyset\}\} \quad (unit\ propagation\ by\ p)$$

Q5

p: I am going

q: You are going

r: Tara is going

Then the argument can be written in PL forms:

$$p \longrightarrow \neg q, \neg q \longrightarrow \neg r, r \vee \neg p, r \vee p \ \vDash \ q$$

For this kind of argument $A_1 \dots \dots A_n \vDash B$ if and only if $A_1 \wedge \dots \dots \wedge A_n \wedge \neg B$ is unsatisfiable. Therefore rewrite the given argument into the below desired form and check if it is satisfiable:

$$(\neg p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (r \vee p) \wedge (\neg q)$$

Transform the above into clausal-form CNF:

$$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$$

Apply DP:

$$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$$

$$= \{\{\neg r\}, \{r, \neg p\}, \{r, p\}\} \quad (unit\ propagation\ by\ \neg q)$$

$$= \{\{\neg p\}, \{p\}\} \quad (unit\ propagation\ by\ \neg r)$$

$$= \{\{\emptyset\}\} \quad (unit\ propagation\ by\ p)$$

Since $\emptyset$ is in the set, which means it is unsatisfiable, thus the argument is valid.

Q6(i)

$$C = \{Andrea, cupcake\}$$

$$P_1 = \{human\}$$

$$P_3 = \{gave\} \quad (gave(X, Y, Z): X\ was\ given\ by\ Y\ to\ Z)$$

$$F_1 = \{aunt\}$$

Then in first order logic:

$$\forall X(human(X) \wedge (X = aunt(aunt(Andrea))) \longrightarrow \exists Y(human(Y) \wedge \neg(Y = Andrea) \wedge gave(cupcake, X, Y)))$$

Q6(ii)

$$P_1 = \{computer\}$$

$$P_2 = \{connect\} \quad (connect(X, Y): X\ is\ connected\ to\ Y)$$

Then in first order logic:

$$\forall X(computer(X) \wedge \neg connect(X, X) \longrightarrow \exists Y connect(Y, X))$$

Q6(iii)

$$C = \{Paul\ Klee, Kandinsky, room, British\ Gallery\}$$

$$P_1 = \{painting\}$$

$$P_2 = \{belong, hang\} \quad (belong(X, Y): X\ belongs\ to\ Y; hang(X, Y): X\ is\ hung\ in\ Y)$$

Then in first order logic:

$$\begin{aligned}\forall X \exists Y(&painting(X) \wedge belong(X, Kandinsky) \wedge hang(X, room) \wedge hang(X, British\ Gallery)\\ &\longrightarrow painting(Y) \wedge belong(Y, Paul\ Klee) \wedge hang(Y, room) \wedge hang(Y, British\ Gallery))\end{aligned}$$

Q6(iv)

$$P_2 = \{love\} \quad (love(X, Y): X\ loves\ Y)$$

Then in first order logic:

$$\forall Y \exists X(\neg love(X, Y) \wedge \neg(X = Y) \longrightarrow \neg love(Y, X) \wedge \neg(X = Y))$$

Q7(i)

Let σ be such that $(\varphi(k), \sigma(X)) \in \varphi(a)$, then according to the structure and the meaning of binary relation symbol a: $\sigma(X) = \varphi(j)$, which means X can only be j (as from the diagram k only connects to j). Since $\sigma(X) = \varphi(j)$, so $\neg(X = j)$ is false, and when the antecedent is true and the consequent is false, the material conditional is false. Thus the whole thing is false.

Q7(ii)

From the diagram, $c(l)$ is always true, so the antecedent is always true. Now let $\sigma(X) = \varphi(j)$, then in this case $b(X) \wedge c(X) \wedge a(l, X)$ is also true according to the structure, thus there exists an X to enable the $\exists X(\dots)$ to be true under the structure, thus the whole thing is true.

Q7(iii)

Let there be a σ such that $\sigma(X) = \varphi(l)$ and $\sigma(Y) = \varphi(k)$, then $\neg(X = Y) \wedge a(X, Y)$ is true under the structure and σ , thus the $\neg\exists Y(\dots)$ is false, the whole is false.

Q7(iv)

Let σ be such that $\sigma_1(X) = \varphi(k)$; $\sigma_2(X) = \varphi(j)$; $\sigma_3(X) = \varphi(l)$ to make the antecedent true. Among the three cases, consider $\sigma_2(X) = \varphi(j)$, for this $\sigma_2(X)$, it is impossible for σ_2 to assign Y to an object that makes $c(Y) \wedge b(Y) \wedge a(X, Y)$ true, i.e. there is no $\sigma_2(Y)$ that can refer to an object that itself is circular and black and to be connected to by object j which is what $\sigma_2(X)$ refers to. Thus the whole thing is false.

Q7(v)

List all possible $\sigma(X)$ that can make the antecedent true: $\sigma_1(X) = \varphi(k)$; $\sigma_2(X) = \varphi(j)$; $\sigma_3(X) = \varphi(l)$. Consider $\sigma_1(X) = \varphi(k)$: it is impossible for $\sigma_1(Y)$ to map Y to an object that makes $a(X, Y) \wedge a(Y, X)$ true, i.e. there is no object that connects to k while k connects back to itself. This makes the $\exists Y(\dots)$ in the consequent false, thus the whole is false.

Q7(vi)

Find all possible σ that must make the antecedent true: $\begin{cases}\sigma_1(X) = \varphi(k)\\ \sigma_1(Y) = \varphi(l)\end{cases}$, $\begin{cases}\sigma_2(X) = \varphi(l)\\ \sigma_2(Y) = \varphi(k)\end{cases}$, $\begin{cases}\sigma_3(X) = \varphi(k)\\ \sigma_3(Y) = \varphi(k)\end{cases}$, $\begin{cases}\sigma_4(X) = \varphi(l)\\ \sigma_4(Y) = \varphi(l)\end{cases}$. Now consider both the σ_3 case and the σ_4 case, in both cases the consequent $a(X, Y) \wedge a(Y, X)$ cannot be true as for both objects k and l, they do not connect to themselves. Thus the whole is false.