ISSN 0005-1179, Automation and Remote Control, 2017, Vol. 78, No. 2, pp. 276–288. © Pleiades Publishing, Ltd., 2017.
Original Russian Text © S.I. Tomashevich, 2017, published in Avtomatika i Telemekhanika, 2017, No. 2, pp. 99–114.

Control for a System of Linear Agents Based on a High Order Adaptation Algorithm

S. I. Tomashevich

St. Petersburg National Research University of Information Technologies, Mechanics, and Optics, St. Petersburg, Russia
Institute of Problems of Mechanical Engineering, St. Petersburg, Russia
e-mail: tomashevich.stanislav@gmail.com

Received February 13, 2015

Abstract—We solve the problem of synchronizing a network of linear agents with unknown parameters and unknown network topology given that the Laplacian that defines it has no complex eigenvalues. To solve this problem, we use a modified high order adaptation algorithm. We obtain conditions for reaching consensus with the proposed algorithm. We show modeling results that demonstrate the efficiency of the proposed approach.

Keywords: multiagent systems, Laplacian, decentralized control, matrix of connections, adaptation algorithm.

DOI: 10.1134/S0005117917020072

1. INTRODUCTION

Lately, multiagent systems have become one of the most comprehensively studied fields of control theory. For multiagent systems the problem of decentralized control becomes important, where agents are controlled not from a single center but work out their controls with local controllers based on the information available to them. For example, in case of mobile robots exploring a territory the use of decentralized control lets them perform their tasks even when it is impossible to use centralized control due to organizational and technical conditions. Information on dangerous zones can be passed from agent to agent, allowing them to correct their routes and increasing reliability and security. However, there is a problem in studying such systems that lies in their mutual interconnection: the behavior of each agent depends on the others, which complicates the dynamics of such systems in general [1].

Multiagent systems can be used in different fields: they help to design order distribution systems, control systems for groups of intelligent robots, groups of unmanned flying vehicles, and in many other domains. They can solve the following problems: transitioning a group of agents into a common state [2] (the rendezvous problem), formation control [3] (when agents depict certain forms in space), load balancing for the nodes of a decentralized network [4] or flocking [5]. A more detailed survey can be found in [6].

Multiagent systems usually employ the notion of consensus (see more details in [1, 7–10]). This notion corresponds to one of the forms of synchronization [11]. One example of consensus behavior is the problem of ensuring car traffic moving with the same speed [12–14]. We should note that similar problems had been posed before this direction of studies gained wide popularity: traffic models had been studied since the 1950s. Generally speaking, “approach” can occur both with respect to system output and with respect to individual components of the state vector or the whole vector. Problems of consensus have been the subject of many works in both Russian and foreign literature. We note the works of Agaev and Chebotarev [10, 15] and especially [16], which shows a fundamental theorem. Frequency-domain consensus criteria for various types of systems are shown, for example, in [17–22].

There are many control algorithms and laws for isolated individual systems, while in multiagent systems agents are connected to each other and exchange information between each other. This information may include measurable outputs, disagreements, or state vectors. All of this makes it impossible to use regular methods that do not account for these interactions. The behavior of such systems significantly depends on the system structure, i.e., of the form of the graph of connections that defines network structure and its properties [23]. It has been shown in [24] how to construct controllers for agents with respect to eigenvalues of the Laplacian that defines network structure.

One can also consider multiagent systems with leaders (real or virtual), and this example has been considered in [25]. The leader's motion is the reference influence, and the system's motion in general occurs according to this reference. The system is non-autonomous and controlled by an external system that can know about outside disturbances. A similar approach can be found in works that perform absolute planning (e.g., [26]). Another known approach (see, e.g., [27]) presumes that control is decentralized, each agent influences the motion up to exactly the degree to which is participates in network topology, but there is no single center that would control the motion. Decentralized control also includes motion with a leader that does not transmit commands immediately to every other agent but does it through the network's mediation. An example of a multiagent system with two such leaders located at the ends of the chain is shown in [28].

Similar to the single-agent system, agents in a multiagent system can also be subject to methods of adaptive control. In many works (see, e.g., [27, 29–34]) the design of adaptation algorithms uses prior knowledge of the eigenvalues of the Laplacian that characterizes the network topology. These and other works construct controllers whose synthesis primarily uses the eigenvalues of the Laplacian or at least its smallest positive eigenvalue, i.e., one has to know the specific form of the graph of connections. These conditions are hard to satisfy in systems with a large number of agents, e.g., in systems with switching connections. Thus, the problem of constructing adaptive control laws for multiagent systems under lack of information regarding the structure of connections is indeed highly relevant.

In this work, we propose a modified high order adaptation algorithm based on [25]. Unlike [25], it is tuned not only for the deviation of agents' parameters but also to different network topologies. For the first time, the high order adaptation algorithm (high order tuner) was published in [35]. Later, many modifications of this algorithm were proposed, and the most close to the one used in this work are algorithms [36–38]. This algorithm was chosen because it is possible to introduce Laplacian eigenvalues to the vector of unknown parameters and not account for it directly, but knowing only the boundaries of the admissible set. Moreover, this algorithm presumes the use of a reference model (leading subsystem for each agent), which lets us work with non-identical agents.

2. PROBLEM SETTING

Consider a multiagent system M that consists of N agents M_i and a digraph $\Gamma = (V, E)$ associated with the multiagent system M, where each vertex of digraph Γ corresponds to an agent M_i, $i \in \overline{1, N}$, $V = v_1, \ldots, v_N$ is the set of vertices, $E \subseteq V \times V$ is the set of edges. We assume that the digraph has no loops. We also denote by $J_i \subset [1, N] \backslash \{i\}$ the set of adjacent vertices for a node v_i for which v_i is the final vertex, and by $d_o(v_i)$ the number of arcs outgoing from vertex v_i. We assume that this set is nonzero. We also assume that digraph Γ must have an incoming spanning tree.

According to [39], the Laplacian is defined as $L = I - G$, where I is a unit matrix of size $N \times N$, and matrix G consists of elements $G_{ij} = 1/d_o(v_i)$ if the arc $(v_i, v_j) \in \Gamma$, and $G_{ij} = 0$ otherwise.

Theorem 1 (Agaev, Chebotarev) [16]. *The rank of the Laplacian of graph Γ equals $N - v$, where v is the forest dimension of the graph by incoming trees. In particular, rank $L = N - 1$, i.e., the zero eigenvalue of matrix L has unit multiplicity if and only if the digraph Γ has an incoming spanning tree.*

At every time moment t, the value of $y_i(t)$ defines the state of agent i $\{i = 1, \ldots, N\}$. Suppose that for each agent the dynamics of its state changes has the form

$$Q_i(p)y_i(t) = k_i R_i(p)u_i(t), \quad i \in \overline{1, N}, \tag{1}$$

where $y_i(t)$ and $u_i(t)$ are the scalar output and input signals of the ith agent respectively, $p = d/dt$ is the differentiation operator, $R_i(p)$ and $Q_i(p)$ are linear differential operators with unknown coefficients, $\deg R_i(p) = m$ and $\deg Q_i(p) = n$, $R_i(\cdot)$ are Hurwitz polynomials, $n - m \geqslant 1$, $k_i > 0$ are unknown coefficients. The coefficients of operators $R_i(p)$, $Q_i(p)$ and coefficients $k_i > 0$ belong to a known bounded set Ξ.

Suppose that each agent corresponds to a leading subsystem that defines the reference motion to which the agent's motion must converge. We define local leading subsystems for each agent as

$$Q_m(p)y_{mi}(t) = k_m R_m(p)z_i(t), \quad i \in \overline{1, N}, \tag{2}$$

where $y_{mi}(t)$ is the output of the leading subsystem, $z_i(t)$ is the reference leading influence.

According to [17], we define the consensus control protocol $u_i(t)$ for the ith agent as

$$u_i(t) = -\left(y_i(t) - \frac{1}{|J_i|}\sum_{j\in J_i} y_j(t)\right), \tag{3}$$

where y_i is the output of the ith agent, $|J_i|$ is the power of set J_i. Thus, we have constructed a multiagent system M that consists of agents whose dynamics is defined by expression (1), and the leading subsystem (2) will be turned on later. The communication channel along which the information is transmitted is assumed to be perfect, disregarding its bounded throughput, time discretization, loss and delays of data packets, and other distortions characteristic for real world communication channels.

The purpose of the control is to synthesize a continuous control law that would ensure the objective condition

$$\overline{\lim_{t\to\infty}} |y_i(t) - y_{mi}(t)| < \delta, \quad i \in \overline{1, N} \tag{4}$$

and the output consensus condition

$$\overline{\lim_{t\to\infty}} |y_i(t) - y_j(t)| < 2\delta, \quad i, j \in \overline{1, N}, \tag{5}$$

and also the boundedness of all signals in the closed system where $\delta > 0$ is the control accuracy in the established mode. The control law must be synthesized without knowledge of specific values of the Laplacian's eigenvalues.

3. SYNTHESIS OF THE CONTROL SYSTEM

Due to Theorem 1, the assumption about the incoming spanning tree is equivalent to the fact that zero is a simple eigenvalue of the Laplacian, and in case of an undirected graph Γ it holds that $0 = \lambda_1 < \lambda_2 \leqslant \ldots \leqslant \lambda_N$. This assumption would ensure consensus in the multiagent system M if the dynamics of agent (1) are defined by an integrator. For the general case this lets us use the Fax–Murray theorem [17] that can be reformulated as follows [40].

Theorem 2. *If the transition function of agent $W_a(s) = \frac{R(s)}{Q(s)}$ is stable, and the frequency-domain transition function $W_a(j\omega) = U(\omega) + jV(\omega)$, as ω ranges from 0 to ∞, does not encircle any of the points $-\frac{1}{\lambda_i}$, where λ_i are all nonzero eigenvalues of the Laplacian, then the multiagent system M reaches consensus. If the transition function $W_a(s)$ is not stable, then in order for multiagent system M to reach consensus it is necessary and sufficient that the hodograph of this function encircles every point $-\frac{1}{\lambda_i}$, and as the frequency changes from 0 to ∞ circles around it counterclockwise by angle $\varphi = n\frac{\pi}{2}$, where n is the order of differential equations that define the agent's dynamics.*

Theorem 2 deals with stabilization, and the result for consensus is not formulated explicitly but is understood. We can judge about the fulfillment of objective (4) for the original multiagent system M by system $\overline{M}$ composed as follows: i separate agents with dynamics (1) are encircled by inverse connections with coefficients λ_i, where λ_i are nonzero eigenvalues of the Laplacian L, and $i \in \overline{1, N-1}$. If system $\overline{M}$ is stable, then the consensus condition holds for the original system M. Consequently, for a further study of the multiagent system M we can consider its simplified model represented by equation

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_{N-1}(t) \end{bmatrix} = -W_a(s) \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{N-1} \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_{N-1}(t) \end{bmatrix}. \tag{6}$$

The choice of a consensus criterion is based on the convenience of representation for the entire multiagent system as separate subsystems, which will further let us construct a control that would ensure this criterion.

We should note that by the problem setting the graph associated with the multiagent system M has an incoming spanning tree, i.e., there is one zero eigenvalue $\lambda_N = 0$. Therefore, the system in expression (6) has smaller dimension than the multiagent system M. Theorem 2 itself can be reformulated as follows: the convergence of a multiagent system M to consensus is equivalent to the stability of system (6).

It follows from the problem setting that each agent's parameters are unknown, and the hodograph of each agent's transition function can encircle points $-\frac{1}{\lambda_i}$, $i \in \overline{1, N-1}$. Then, in order to ensure control objective (4) under uncertainty we use an algorithm proposed in [41].

Let us consider expression (6) in more detail: eigenvalues of the Laplacian can be either real or imaginary. In what follows we assume that the Laplacian has only real eigenvalues. This assumption can hold when the Laplacian is symmetrical and positive semidefinite (the case of an undirected graph or a tree-like structure without loops).

Remark 1. The assumption that Laplacian eigenvalues do not have imaginary parts is in reality rather restrictive. It is important to note that complex eigenvalues are admissible and do not contradict the proof of Statement 1. But it is incorrect to consider real systems with complex amplification coefficients in the feedback. Considering this case is a subject for further study.

We should note that in real life systems information transmission from agent to agent is done through data channels that have a delay. The presence of identical delays in the input signal of all agents simply introduces an additional negative phase shift in the agent's hodograph by a corresponding angle, still leaving it possible to apply Theorem [17]. Delays certainly influence the system's stability, shifting the hodograph to critical points $-\lambda_i^{-1}$. The stability of a system in this case has to be proven separately with the proposed algorithm, which is a subject for subsequent works.

Figure 1 shows the structural scheme for the proposed control system and introduces the following notation: CO is the control object, RM is the reference model, AA is the adaptation algorithm.

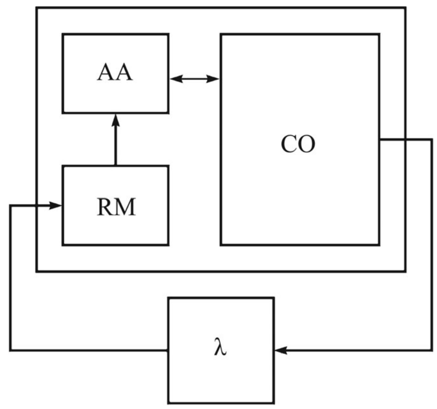

Fig. 1. Structural scheme of the control system.

The figure shows that the collection of CO, RM, and AA can be represented by a new agent, but this agent is nonlinear. The new agent's input is the input of the leading subsystem $z_i(t)$. Therefore, we now define for it the consensus control (3), and the new input $u_i(t)$ will be constructed later, due to AA. In order for Theorem 2 to hold at every time moment we assume that after adaptation time every system of this form will be defined by RM equations. Then we can decide its stability with a system similar to (6), where instead of $W_a(s)$ we have the collection of CO, RM, and AA. Therefore, by Theorem 2 this system is similar to a system with control $z_i(t) = \lambda_i y_i(t)$, where λ_i is an unknown feedback coefficient. If all $N-1$ systems constructed in this way are stable, then the multiagent system M will satisfy objective (5).

Let us rewrite operators $Q_i(p)$ and $R_i(p)$ as the following sums:

$$\begin{cases} Q_i(p) = Q_m(p) + \Delta Q_i(p) \\ R_i(p) = R_m(p) + \Delta R_i(p), \end{cases} \tag{7}$$

where $\deg \Delta Q_i(p) \leqslant n-1$, $\deg \Delta R_i(p) \leqslant m-1$. Substituting (7) into (1), we get

$$Q_m(p)y_i(t) = k_i R_m(p)u_i(t) + k_i \Delta R_i(p)u_i(t) - \Delta Q_i(p)y_i(t).$$

According to [41], we define the control law for each agent as

$$u_i(t) = T(p)\hat{v}_i(t), \tag{8}$$

where $T(p)$ is chosen in such a way that the transition function $\frac{R_m(s)T(s)}{Q_m(s)}$ is a strictly positive real function, $\hat{v}_i(t)$ is an estimate for the auxiliary controlling influence $v_i(t)$ that will be constructed below.

Taking into account (7), (8), and (2), we construct the equation for the tracking error $e_i(t) = y_i(t) - y_{mi}(t)$ as

$$e_i(t) = \frac{k_i R_m(p)T(p)}{Q_m(p)} \left(\hat{v}_i(t) + \frac{\Delta R_i(p)}{R_m(p)}\hat{v}_i(t) - \frac{\Delta Q_i(p)}{k_i R_m(p)T(p)}y_i(t) - \frac{k_m \lambda}{k_i T(p)}y_i(t) \right). \tag{9}$$

Let us take the whole part of the third term in brackets in (9):

$$\frac{\Delta Q_i(p)}{k_i R_m(p)T(p)} = K_{yi} + \frac{\Delta \overline{Q}_i(p)}{k_i R_m(p)T(p)},$$

where $\deg \Delta\overline{Q}_i(p) \leqslant n-2$, and transform Eq. (9):

$$e_i(t) = \frac{k_i R_m(p) T(p)}{Q_m(p)} \left(\hat{v}_i(t) + \frac{\Delta R_i(p)}{R_m(p)} \hat{v}_i(t) - K_{yi} y_i(t) - \frac{\Delta \overline{Q}_i(p)}{k_i R_m(p) T(p)} y_i(t) - \frac{k_m \lambda}{k_i T(p)} y_i(t) \right). \quad (10)$$

We construct state filters:

$$\begin{cases} \dot{V}_{1i}(t) = F_1 V_{1i}(t) + b\hat{v}_i(t) \\ \dot{V}_{2i}(t) = F_2 V_{2i}(t) + b y_i(t), \end{cases} \quad (11)$$

where $F_1,\ F_2$ is a matrix in Frobenius form with characteristic polynomials $R_m(p)$ and $R_m(p)T(p)$ respectively, $b = \begin{bmatrix} 0 & \ldots & 0 & 1 \end{bmatrix}^{\mathrm{T}}$.

Taking into account (11), we rewrite Eq. (9) as

$$e_i(t) = \frac{k_i R_m(p) T(p)}{Q_m(p)} \Big(\hat{v}_i(t) - c_{0i}^{\mathrm{T}} \omega_i(t) \Big), \quad (12)$$

where c_{0i} is the vector of unknown parameters for the ith agent that depends on the coefficient K_{yi}, coefficients of operators $\Delta R_i(p)$ and $\Delta\overline{Q}_i(p)$, and also includes λ in itself, $\omega_i(t) = \begin{bmatrix} V_{1i}^{\mathrm{T}}(t) & y_i(t) & V_{2i}^{\mathrm{T}}(t) & \frac{k_m}{T(p)} y_i(t) \end{bmatrix}^{\mathrm{T}}$ is the regression vector.

We define the auxiliary controlling influence $v_i(t)$ as

$$v_i(t) = c_i^{\mathrm{T}} \omega_i(t), \quad (13)$$

where $c_i(t)$ is the vector of unknown parameters.

To implement the control law (8) we use an observer [42] defined in the form

$$\begin{cases} \dot{\zeta}_i(t) = G_0 \zeta_i(t) + B(\hat{v}_i(t) - v_i(t)) \\ \hat{v}_i(t) = \mathcal{L} \zeta_i(t), \end{cases} \quad (14)$$

where $\zeta_i(t) \in \mathbb{R}^\gamma$, $G_0 = \begin{bmatrix} 0 & I_{\gamma-1} \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -\frac{d_1}{\mu} & \ldots & -\frac{d_\gamma}{\mu^\gamma} \end{bmatrix}^{\mathrm{T}}$, $\mathcal{L} = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}$, $d_1, \ldots, d_\gamma$ are chosen in such a way that the matrix $G = G_0 + \begin{bmatrix} d_1 & \ldots & d_\gamma \end{bmatrix}^{\mathrm{T}} \mathcal{L}$ is Hurwitz, and $\mu > 0$ is a sufficiently small number.

Next we formulate the main result of this work.

Statement 1. *There exists a number $\mu_0 > 0$ such that for $\mu < \mu_0$ the control system* (8), (11), (13), (14) *together with the adaptation algorithm*

$$\dot{c}_i(t) = -\alpha e_i(t) \omega_i(t), \quad (15)$$

where $\alpha > 0$, ensures the objective condition (4).

Proof of Statement 1 is given in the Appendix.

Note that the difference from [36, 41] is that the vector of unknown parameters contains not only parametric deviations but also feedback coefficients λ_i, and the system adapts not only to the reference model but also to the structure of connections between agents which is characterized by these possible unknown eigenvalues λ_i. The eigenvalues themselves may be unknown, but we do know the range that contains them. In the general case, for the Laplacian this range is a circle on the complex plane of unit radius centered at the point $(1, 0j)$, and for the proposed constraint of real eigenvalues it is the interval $[0, 2]$. More details regarding the location of eigenvalues can be found in [43]. This algorithm is also applicable to discrete systems, which means that it can be used with most modern digital systems.

4. EXAMPLE

Consider a multiagent system with four agents with the graph of connections shown on Fig. 2. Suppose that each agent corresponds to one of the Eqs. (16)–(19):

$$\left(p^3+2p+2\right)y_1(t)=\left(5p^2+p+20\right)u_1(t), \tag{16}$$

$$\left(p^3+8p+9\right)y_2(t)=\left(6p^2+p+10\right)u_2(t), \tag{17}$$

$$\left(p^3+2p+1\right)y_3(t)=\left(p^2+2p+3\right)u_3(t), \tag{18}$$

$$\left(p^3+10p+16\right)y_4(t)=\left(6p^2+2p+4\right)u_4(t). \tag{19}$$

Such dynamics equations can describe individual agents that are not connected into a formation, for example, quadrocopters [44] or electrical cars [45, 46].

Let us now define the control system. According to Fig. 2, we construct the Laplacian

$$L=\begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & 1 & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 1 \end{bmatrix}. \tag{20}$$

For the Laplacian that defines this network structure one eigenvalue is zero, and the other $N-1$ are $1+\frac{1}{N-1}$ [17], i.e., for $N=4$ the eigenvalues are $\lambda_{2,3,4}=\frac{4}{3}$.

If the multiagent system consisted of identical agents with dynamics described by one of the Eqs. (16)–(19), then by Theorem 2 we would not be able to reach consensus. We can say that

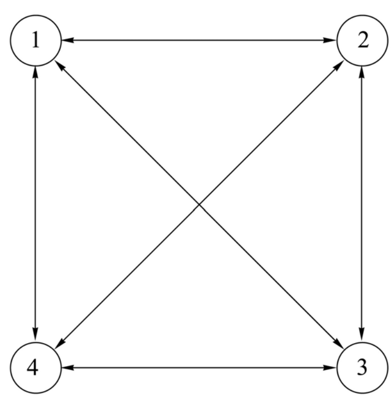

Fig. 2. The graph of connections for a multiagent system.

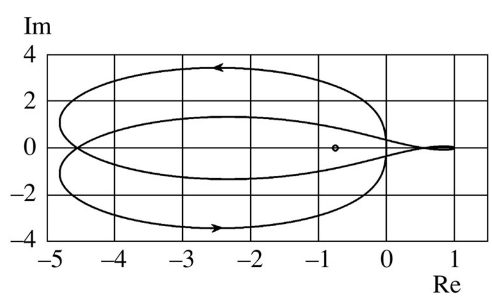

Fig. 3. Hodograph of the frequency-domain transition function $W(j\omega)$ agent (16).

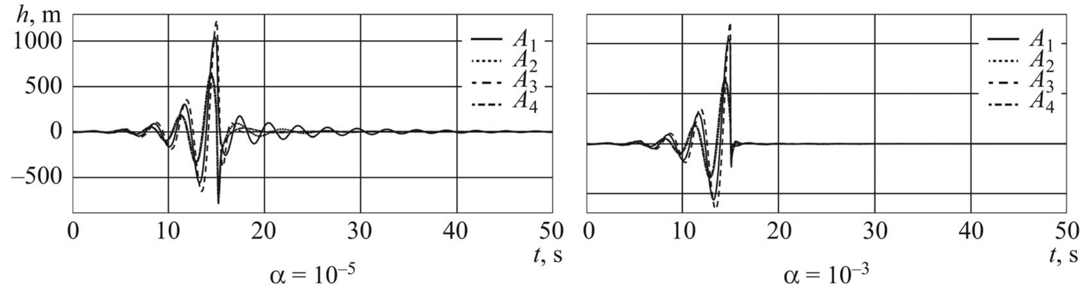

Fig. 4. Graphs of processes $y_i(t)$ on the agents' outputs.

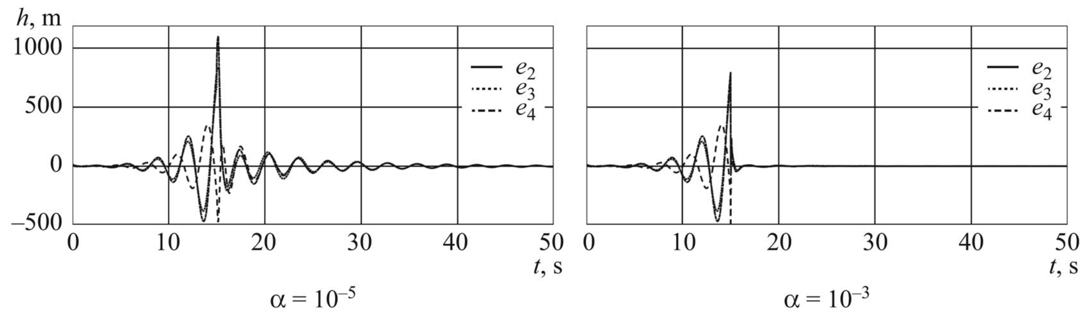

Fig. 5. Graphs of desynchronization processes between outputs $y_2(t), y_3(t), y_4(t)$ and output $y_1(t)$.

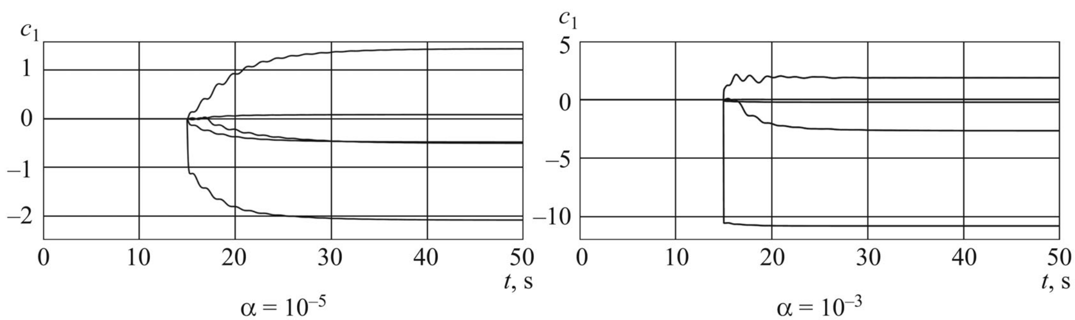

Fig. 6. Tuning controller coefficients for the agent (16).

because the hodographs of frequency characteristics of the agents (16)–(19) encircle the point $\sigma_{2,3,4} = -\lambda_{2,3,4}^{-1}$, i.e., $(-\frac{3}{4}, 0)$. For example, on Fig. 3 we show the hodograph of the frequency-domain transition function $W(j\omega)$ for agent (16), and hodographs of frequency characteristics of agents (17)–(19) are similar.

We define the dynamics of reference models with equations

$$\left(p^3 + 3p^2 + 3p + 1\right) y_{mi}(t) = \left(p^2 + 2p + 1\right) z_i(t), \quad i \in \overline{1,4}. \tag{21}$$

Judging by the properties of the reference model (21), $T(p)$ can be chosen as $T(p) = 1$. Performing the modeling, we will consider different values of the coefficient α in the adaptation law. We show the results for each case on two graphs: motion of all agents simultaneously and their desynchronization with one of the agents.

In the system's operation we use, for clarity, different initial conditions and show the motion of the entire formation without the algorithm adaptation and with it. To do that, we first switch the adaptation algorithm off and then switch it on at time moment $t = 15$. This gives us a possibility to see that the system indeed does not converge to consensus, and it does so with the adaptation algorithm.

Figures 4–6 show that changing the coefficient α influences the control's rigidity: an increase in α lets us reach objectives (4) and (5) as soon as possible for all agents.

Remark 2. It is important to note that the system will reach consensus even if the reference model's hodograph encircles eigenvalues σ_i, $i \in \overline{1, N}$. If λ_i were not introduced in the vector of unknown parameters, and the control algorithm simply reduced the dynamics of agent (1) to the dynamics of the leading subsystem (2), then the multiagent system would not reach consensus by Theorem 2.

5. CONCLUSION

In this work, we have proposed an adaptive control algorithm for multiagent systems with linear agents for the case when the eigenvalues of the Laplacian that defines system structure are real numbers. We find the conditions for the agents reaching consensus with the proposed algorithm. One advantage of this algorithm is that it can adapt to unknown eigenvalues of the matrix of connections, i.e., to different network structures, and lets one work with non-identical agents. The network structure can change, for instance, if some agent will not be able to receive information from another due to flaws in the operation of the communication channel. This approach has an obvious advantage compared to other algorithms that require one to know the minimal nonzero eigenvalue of the Laplacian. With an example we have shown that this algorithm works for a system that cannot reach consensus. This approach lets one, with a simple variation of one parameter of the adaptation algorithm, to change quantitative characteristics of the agents' convergence, e.g., the transition process duration.

ACKNOWLEDGMENTS

This work was supported by the Government of the Russian Federation, project no. 074-U01, and Russian Foundation for Basic Research, project no. 14-08-01015. Design of the adaptive network control algorithm (Section 3) was performed at the IPME RAS and supported exclusively by the Russian Science Foundation, project no. 14-29-00142.

The author is grateful to Prof. I.B. Furtat for his help in the preparation of this paper.

APPENDIX

Consider the vector $\overline{\eta}_i(t) = \overline{D}^{-1}\delta_i(t)$ where $\overline{D} = \mathrm{diag}\{\mu^{\gamma-1}, \mu^{\gamma-2}, \ldots, \mu, 1\}$. Let us find the full derivative with respect to time of $\overline{\eta}_i(t)$ due to (14):

$$\dot{\overline{\eta}}_i(t) = \mu^{-1}G\overline{\eta}_i(t) + \overline{b}\zeta_i^{(\gamma)}(t), \quad \overline{\Delta}_i(t) = \mu^{\gamma-1}\mathcal{L}\overline{\eta}_i(t). \tag{A.1}$$

We transform the penultimate equation to an equivalent equation on the output $\overline{\Delta}_i(t)$:

$$\dot{\eta}_i(t) = \mu^{-1}G\eta_i(t) + b\dot{v}_i(t), \quad \overline{\Delta}_i(t) = \mu^{\gamma-1}\mathcal{L}\eta_i(t). \tag{A.2}$$

Here $\eta_i(t) \in \mathbb{R}^\gamma$, $\eta_i^1(t) = \overline{\eta}_i^1(t)$ are first components of vectors $\eta_i(t)$ and $\overline{\eta}_i(t)$, $b = \begin{bmatrix}1 & 0 & \ldots & 0\end{bmatrix}^{\mathrm{T}}$. Let

$$\frac{R_m(s)T(s)}{Q_m(s)} = \frac{1}{s+a}, \quad \text{where} \quad a > 0.$$

Taking into account (A.2), we transform (12) to

$$\dot{e}_i(t) = -ae_i(t) + k_i\left(c_i(t) - c_{0i}\right)^{\mathrm{T}}\omega_i(t) + \mu^{\gamma-1}\mathcal{L}\eta_i(t). \tag{A.3}$$

Let us rewrite Eqs. (A.2) and (A.3) as

$$\begin{cases} \dot{e}_i(t) = -ae_i(t) + k_i\left(c_i(t) - c_{0i}\right)^{\mathrm{T}}\omega_i(t) + \mu_2^{\gamma-1}\mathcal{L}\eta_i(t) \\ \mu_1\dot{\eta}_i(t) = G\eta_i(t) + \mu_2\overline{b}\dot{v}_i(t), \end{cases} \tag{A.4}$$

where $\mu_1 = \mu_2 = \mu_0$, and use lemma [47].

Lemma. *Consider a dynamical system*

$$\dot{x} = f(x,\ \mu_1,\ \mu_2,\ t), \tag{A.5}$$

where $x \in R^{s_1}$, $\mu = col(\mu_1, \mu_2) \in R^{s_2}$, $f(x, \mu_1, \mu_2, t)$ is a Lipschitz function in x. Let system (A.5) *has a closed bounded attraction set $\Omega = \{x \mid P(x) \leqslant C\}$ when $\mu_2 = 0$, where $P(x)$ is a piecewise smooth positive definite function on R^{s_1}. We also assume that there exist certain numbers $C_1 > 0$ and $\overline{\mu}_1$ such that the following condition holds:*

$$\sup_{|\mu_1|\leqslant\overline{\mu}_1}\left[\left[\frac{\partial P(x)}{\partial x}\right]^{\mathrm{T}} f(x,\ \mu_1,\ 0,\ t)\Big| P(x) = C\right] \leqslant -C_1. \tag{A.6}$$

Then there exists $\mu_0 > 0$ such that system (A.5) *has the same attraction set Ω for $\mu_2 \leqslant \mu_0$.*

This lemma is a generalization of the first lemma in [48] for studying the behavior of a system of non-autonomous differential equations.

Let us check the lemma's conditions. Suppose that in (A.4) $\mu_2 = 0$. We choose the Lyapunov function for Eq. (A.4) as

$$V = \sum_{i=1}^{N}\left(\frac{1}{2}e_i^2(t) + \eta_i^{\mathrm{T}}(t)P\eta_i(t) + \frac{1}{2}(c_i(t) - c_{0i})^{\mathrm{T}}(c_i(t) - c_{0i})\right), \tag{A.7}$$

where $P = P^{\mathrm{T}} > 0$ is a solution of equations $G^{\mathrm{T}}P + PG = -Q$, $Q = Q^{\mathrm{T}} > 0$. We take from function V its full derivative with respect to time along the trajectories (A.4):

$$\dot{V} = -\sum_{i=1}^{k}\left[ae_i^2(t) + \frac{1}{\mu_1}\eta_i^{\mathrm{T}}(t)Q\eta_i(t)\right]. \tag{A.8}$$

Now (A.8) implies that $e_i(t)$ and $\eta_i(t)$ are bounded. We can prove that the other variables are bounded for $\mu_2 = 0$ similar to the proof in [49]. Thus, for $\mu_2 = 0$ we get an asymptotically stable system of Eqs. (A.4). Consequently, conditions of the lemma hold for system (A.4). But the asymptotic stability of the reduced model does not imply asymptotic stability of the original equation.

Suppose that in (A.4) $\mu_1 = \mu_2 = \mu_0$. We take again the Lyapunov function (A.7) and compute now its full derivative with respect to time along the trajectories (15) and (A.4) due to the result (A.8):

$$\dot{V} = \sum_{i=1}^{k}\left[-ae_i^2(t) + \mu_0^{\gamma-1}e_i(t)\mathcal{L}\eta_i - \frac{1}{\mu_0}\eta_i^{\mathrm{T}}(t)Q\eta_i(t) + 2\eta_i^{\mathrm{T}}(t)P\overline{b}\dot{v}_i(t)\right]. \tag{A.9}$$

Consider the bounds

$$\begin{cases} \mu_0^{\gamma-1} e_i(t)\mathcal{L}\eta_i(t) \leqslant \dfrac{1}{2}\mu_0^{\gamma-1} e_i^2(t) + \dfrac{1}{2}\mu_0^{\gamma-1}\eta_i^{\mathrm{T}}(t)\mathcal{L}\eta_i(t) \\ 2\eta_i^{\mathrm{T}}(t)P\bar{b}\dot{v}_i(t) \leqslant \eta_i^{\mathrm{T}}(t)P\overline{b}\overline{b}^{\mathrm{T}}P\eta_i(t) + \dot{v}_i^2(t). \end{cases} \tag{A.10}$$

Then due to these bounds we rewrite (A.9) as

$$\dot{V} = -\sum_{i=1}^{N}\left[\widetilde{a}e_i^2(t) + \frac{1}{\mu_0}\eta_i^{\mathrm{T}}(t)\widetilde{Q}\eta_i(t)\right] + N\psi, \tag{A.11}$$

where $\psi = \sup\{\dot{v}_i^2(t)\}$, $\widetilde{a} = a - \frac{1}{2}\mu_0^{\gamma-1}$, $\widetilde{Q} = Q - \mu_0 P\overline{b}\overline{b}^{\mathrm{T}}P - \frac{1}{2}\mu_0^{\gamma-1}\mathcal{L}^{\mathrm{T}}\mathcal{L}$. Obviously, there exists a number μ_0 such that $\widetilde{a} > 0$ and $\widetilde{Q} > 0$, consequently, the objective condition (4) will hold. This completes the proof of the statement.

REFERENCES

1. Amelina, N.O., Anan'evskii, M.S., Andrievskii, B.R., et al., *Problemy setevogo upravleniya* (Network Control Problems), Fradkov, A.L., Ed., Moscow–Izhevsk: Izhevsk. Inst. Comp. Issled., 2015.

2. Cheng, Z., Zhang, H.-T., and Fan, M.-C., Consensus and Rendezvous Predictive Control for Multi-agent Systems with Input Constraints, *33rd Chinese Control Conf. (CCC)*, 2014, pp. 1438–1443.

3. Wu, Z., Guan, Z., Wu, X., and Li, T., Consensus Based Formation Control and Trajectory Tracing of Multi-Agent Robot Systems, *J. Intelligent Robot. Syst.*, 2007, vol. 48, no. 3, pp. 397–410.

4. Amelina, N., Fradkov, A., Jiang, Y., and Vergados, D.J., Approximate Consesus in Stochastic Networks with Application to Load Balancing, *IEEE Trans. Inform. Theory*, 2015, vol. 61, no. 4, pp. 1739–1752.

5. Leonard, N.E. and Fiorelli, E., Virtual Leaders, Artificial Potentials, and Coordinated Control of Groups, *Proc. 40 IEEE Conf. Decision Control*, 2001, pp. 2968–2973.

6. Amelina, N.O., Multiagent Technologies, Adaptation, Self-Organization, and Reaching Consensus, *Stokhast. Optim. Informat.*, 2011, vol. 7, no. 1, pp. 149–185.

7. Olfati-Saber, R. and Murray, R., Consensus Problems in Networks of Agents with Switching Topology and Time-Delays, *IEEE Trans. Autom. Control*, 2004, vol. 49, no. 9, pp. 1520–1533.

8. Ren, W. and Beard, R.W., *Distributed Consensus in Multi-vehicle Cooperative Control*, London: Springer-Verlag, 2008.

9. Olfati-Saber, R., Fax, A., and Murray, R., Consensus and Cooperation in Networked Multi-agent Systems, *Proc. IEEE*, 2007, vol. 95, no. 1, pp. 215–233.

10. Agaev, R.P. and Chebotarev, P.Yu., Coordination in Multiagent Systems and Laplacian Spectra of Digraphs, *Autom. Remote Control*, 2009, vol. 70, no. 3, pp. 469–483.

11. Fradkov, A.L., *Kiberneticheskaya fizika* (Cybernetic Physics), St. Petersburg: Nauka, 2003.

12. Gazis, D.C., Herman, R., and Rothery, R.W., Nonlinear Follow-the-Leader Models of Traffic Flow Operations Research, *INFORMS*, 1961, no. 9, pp. 545–567.

13. Chandler, R.E., Herman, R., and Montroll, E.W., Traffic Dynamics: Studies in Car Following, *Oper. Res. Informs.*, 1958, no. 6, pp. 165–184.

14. Newell, G.F., Nonlinear Effects in the Dynamics of Car Following, *Oper. Res.*, 1961, vol. 9, no. 2, pp. 209–229.

15. Agaev, R.P. and Chebotarev, P.Yu., The Matrix of Maximal Outgoing Forests of a Digraph and Its Applications, *Autom. Remote Control*, 2000, vol. 61, no. 9, pp. 1424–1450.

16. Agaev, R.P. and Chebotarev, P.Yu., Spanning Forests of a Digraph and Their Applications, *Autom. Remote Control*, 2001, vol. 62, no. 3, pp. 443–466.

17. Fax, J.A. and Murray, R.M., Information Flow and Cooperative Control of Vehicle Formations, *IEEE Trans. Autom. Control*, 2004, no. 8, pp. 1465–1476.

18. Proskurnikov, A.V., The Popov Criterion for Consensus Between Delayed Agents, *Proc. 9 IFAC Nonlinear Control Syst. Sympos. NOLCOS-2013*, Toulouse, France, pp. 693–698.

19. Li, Z., Duan, Z., and Chen, G., Consensus of Multiagent Systems and Synchronization of Complex Networks: A Unified Viewpoint, *IEEE Transact. Circuits Systems I*, Regular Papers, 2010, no. 1, pp. 213–224.

20. Polyak, B.T. and Tsypkin, Ya.Z., Stability and Robust Stability of Uniform System, *Autom. Remote Control*, 1996, vol. 57, no. 11, pp. 1606–1617.

21. Hara, S., Shimizu, H., and Kim, T.-H., Consensus in Hierarchical Multi-agent Dynamical Systems with Low-rank Interconnections: Analysis of Stability and Convergence Rates, *Am. Control Conf.*, 2009, pp. 5192–5197.

22. Proskurnikov, A.V., Frequency-Domain Criteria for Consensus in Multiagent Systems with Nonlinear Sector-shaped Couplings, *Autom. Remote Control*, 2014, vol. 75, no. 11, pp. 1982–1995.

23. Godsil, C. and Royle, G., *Algebraic Graph Theory*, New York: Springer-Verlag, 2001.

24. Tomashevich, S. and Andrievsky, B., Stability and Performance of Networked Control of Quadrocopters Formation Flight, *Proc. 6 Int. Congr. Ultra Modern Telecommun. Control Syst. Workshops (ICUMT 2014)*, St. Petersburg, Russia, 2014, pp. 331–336.

25. Furtat, I.B., Adaptive Control for a Dynamic Network with Linear Subsystems, *Vestn. Astrakhan. Gos. Tekh. Univ., Ser. Upravlen., Vychisl. Tekh. Informatika*, 2012, no. 1, pp. 69–78.

26. Chen, Y. and Yuping Tian, Y., Cooperative Control of Multi-agent Moving Along a Set of Given Curves, *J. Syst. Sci. Complex.*, 2011, vol. 24, pp. 631–646.

27. Selivanov, A., Fradkov, A.L., and Junessov, I., Robust and Adaptive Passification Based Consensus Control of Dynamical Networks, *IFAC Int. Workshop Adapt. Learn. Control Signal Proc.*, 2013, pp. 707–711.

28. Parsegov, S.E., Joining Coordinates and Hierarchical Algorithms in the Problem of Equidistant Location of Agents on a Segment, *Upravlen. Bol'shimi Sist.*, 2012, no. 39, pp. 264–287.

29. Dzhunusov, I.A. and Fradkov, A.L., Synchronization in Networks of Linear Agents with Output Feedbacks, *Autom. Remote Control*, 2011, vol. 72, no. 8, pp. 1615–1626.

30. Furtat, I.B., Consensus Output Control for a Linear Dynamical Network with Disturbance Compensation, *Mekhatronika, Avtomatiz., Upravlen.*, 2011, no. 4, pp. 12–18.

31. Li, Z., Ren, W., Liu, X., and Fu, M., Consensus of Multi-Agent Systems With General Linear and Lipschitz Nonlinear Dynamics Using Distributed Adaptive Protocols, *IEEE Transact. Automat. Control*, 2012, vol. 58, no. 7, pp. 1786–1791.

32. Li, Z., Liu, X., Ren, W., and Xie, L., Distributed Consensus of Linear Multi-Agent Systems with Adaptive Dynamic Protocols, *Automatica*, 2013, vol. 49, no. 7, pp. 1986–1995.

33. Selivanov, A.A., Control over Synchronization of Networks with Nonlinearities and Delayed Connections, *Vestn. Nizhegorod. Univ. im. N.I. Lobachevskogo*, 2014, no. 1 (3), pp. 265–271.

34. Hara, S. and Tsubakino, D., Eigenvector-Based Intergroup Connection of Low Rank for Hierarchical Multi-agent Dynamical Systems, *Syst. Control Lett.*, 2012, vol. 61, no. 2, pp. 354–361.

35. Morse, A.S., High-order Parameter Tuners for the Adaptive Control of Nonlinear Systems, in *Systems, Models and Feedback: Theory Appl.*, Isidori, A. and Tarn, T.J., Eds., Basel: Birkhauser, 1992, pp. 339–364.

36. Tsykunov, A.M., A Modified High-order Adaptive Output Feedback Control Algorithm for Linear Plants, *Autom. Remote Control*, 2006, vol. 67, no. 8, pp. 1311–1321.

37. Miroshnik, I.V., Nikiforov, V.O., and Fradkov, A.L., *Nelineinoe i adaptivnoe upravlenie slozhnymi dinamicheskimi sistemami* (Nonlinear and Adaptive Control over Complex Dynamical Systems), St. Petersburg: Nauka, 2000.

38. Nikiforov, V.O. and Fradkov, A.L., Adaptive Control Schemes with Extended Error, *Autom. Remote Control*, 1994, vol. 55, no. 9, pp. 1239–1255.

39. Chung, F.R.K., *Spectral Graph Theory*, Ser. *Regional Conference Series in Mathematics*, Providence: Am. Math. Soc., 1997, vol. 92.

40. Tomashevich, S.I., Stability of Multiagent Linear Scalar Systems and Its Dependence on the Graph of Connections, *Nauch.-tekhn. Vestn. Inform. Tekhnol., Mekh. Optiki*, 2014, vol. 14, no. 2, pp. 72–78.

41. Furtat, I.B. and Tsykunov, A.M., A Modified Algorithm of High Order Adaptation for Systems with State Delay, *Vestn. Astrakhan. Gos. Tekh. Univ.*, 2006, no. 1, pp. 24–33.

42. Atassi, A.N. and Khalil, H.K., A Separation Principle for the Stabilization of Class of Nonlinear Systems, *IEEE Trans. Autom. Control*, 1999, vol. 44, no. 9, pp. 1672–1687.

43. Fax, J.A., Optimal and Cooperative Control of Vehicle Formations, *PhD Dissertation*, California Inst. Technol., Pasadena, CA, 2002, pp. 55–58.

44. Amelin, K., Tomashevich, S., and Andrievsky, B., Recursive Identification of Motion Model Parameters for Ultralight UAV, *IFAC-PapersOnLine 1st IFAC Conf. Modell., Identificat. Control Nonlin. Syst. (MICNON 2015)*, Russia, St. Petersburg, 2015, vol. 48, no. 11, pp. 233–237.

45. Fradkov, A.L. and Furtat, I.B., Robust Control for a Network of Electric Power Generators, *Autom. Remote Control*, 2013, vol. 74, no. 11, pp. 18851–1862.

46. Furtat, I.B. and Fradkov, A.L., Robust Control of Multi-Machine Power Systems with Compensation of Disturbances, *Electr. Power Energy Syst.*, 2015, no. 73, pp. 584–590.

47. Furtat, I.B., Robust Control for a Certain Class of Nonminimally–Phase Dynamical Networks, *Izv. Ross. Akad. Nauk, Teor. Sist. Upravlen.*, 2014, no. 1, pp. 35–48.

48. Brusin, V.A., On a Class of Singular Perturbed Adaptive Systems, *Autom. Remote Control*, 1995, vol. 56, no. 4, pp. 552–559.

49. Furtat, I.B. and Tsykunov, A.M., Adaptive Control of Plants of Unknown Relative Degree, *Autom. Remote Control*, 2010, vol. 71, no. 6, pp. 1076–1084.

This paper was recommended for publication by A.A. Fradkov, a member of the Editorial Board