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Reentry Trajectory Optimization under Atmospheric Uncertainty as a Differential Game*

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Dedicated to Professor Dr. Dr. h. c. Roland Bulirsch
on the occasion of his 60th birthday.

Abstract. If solutions of optimal control problems are to be realized in practical applications, one has to take into account the influence of unpredictable disturbances. If lower and upper bounds for the disturbances are known, one can investigate the so-called worst case. This worst case can be formulated as a two-person differential game and its solution in closed form provides the optimal feedback controller against all possible disturbances. Following this approach, realistically modelled optimal control problems lead in general to nonseparable, non zero-sum differential games where inequality constraints have to be taken into account. A maximum cross-range reentry of a space-shuttle in the presence of uncertain air density fluctuations serves as an example. For the treatment of terminal conditions and path constraints in which controls of both players are involved, the responsibility of the players for obeying these constraints is investigated, and a new transformation technique is used. The open-loop representation of the optimal feedback controller is computed along various saddle-point trajectories representing the worst case. The computational method is based on the numerical solution of the multipoint boundary-value problem which arises from the necessary conditions for a saddle-point solution of the game. Extensive numerical results are presented for a space-shuttle reentry under a dynamic pressure and an aerodynamic heating constraint. It is outlined, that the proposed method allows not only the analysis of the worst case, but is also the first step towards the construction of the optimal feedback controller via a successive computation of the open-loop representation or via a neural network training.

Key Words. Constrained differential games, unknown disturbances, space-shuttle reentry, air density fluctuations, optimal control problems.

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1 Introduction

The purpose of this paper is to develop the theoretical basis and a numerical method for the computation of the optimal feedback controller for the following rather general optimal control problem with unknown disturbances:
Let the dynamic system and the associated boundary conditions be given in the form

$$\dot{z}(t) = f(z(t), u(t), w(t)) \ , \tag{1}$$

$$z(t_0) = z_0 \ , \quad ||\psi(z(t_f))||_p \le \varepsilon \tag{2}$$

where $z(t)$ denotes the state vector, $u(t)$ the control vector and $w(t)$ the unknown disturbance vector. The derivative with respect to time t, $t \in [t_0, t_f]$, is denoted by a dot. The initial time is t_0 and the final time t_f which can be specified or unspecified. It is assumed that the disturbance vector $w(t)$ lies within a known bounded set $\Omega(t, z(t))$ for all $t \in [t_0, t_f]$. The initial state is prescribed by the vector z_0, and the terminal conditions are given by an inequality constraint. In order to have a differentiable norm, we require p to be an even integer. The accuracy by which the terminal conditions are to be satisfied can be regulated by the parameter ε. Furthermore, constraints might occur in the form of so-called control variable inequality or mixed constraints,

$$C(z(t), u(t), w(t)) \le 0 \ . \tag{3}$$

We now search for control functions and their associated solutions of the differential equations so that the boundary conditions as well as the inequality constraints are satisfied. If a higher accuracy for the terminal conditions than ε can be enforced against all possible disturbances w, a performance index

$$I[u] = \phi(t_f, z(t_f)) \tag{4}$$

to be minimized, can be additionally taken into account.

The optimal open-loop solution $\tilde{z}^*(t)$ of the related undisturbed optimal control problem is usually computed first. Next, the problem (1)–(4) is linearized around $\tilde{z}^*(t)$. Then, a feedback controller tracking $\tilde{z}^*(t)$ against the unkown disturbances must be constructed, for example by using Lyapunov theory [13], H^∞-control theory [1], or the theory of linear differential games [3]. On the other hand, the construction of a feedback controller using neighboring optimal trajectories is also possible; see [14] and [12]. However, all these feedback controllers are only suboptimal for the disturbed optimal control problem (1)–(4).

In the present paper, we show that the problem of constructing the optimal feedback controller for the disturbed optimal control problem, can be formulated as a two-person differential game. For this purpose, the theory of separable inequality-constrained zero-sum games [4] is generalized to nonseparable games. The application of the maximinimum principle allows the computation of the optimal open-loop strategies and yields a criterion to verify whether the game is zero-sum. In addition, a method for the treatment of terminal conditions and path constraints, in which even controls of both players are involved, is described. This method also includes an investigation of the responsibility of the players for those constraints. The computation of the optimal controls, or more precisely of the open-loop representations of the optimal strategies, is based on the numerical computation of a solution of the multipoint boundary-value problem which arises from the necessary conditions for optimality. By this method, the optimal feedback strategies against all possible disturbances are obtained along the so-called saddle-point trajectories. An extensive insight is gained into the structure of the optimal solution for the worst case. Moreover, a benchmark for the assessment of various suboptimal feedback controllers can be obtained.

A realistically modelled space-shuttle reentry problem where the cross-range is to be maximized and where a dynamic pressure and an aerodynamic heating constraint are taken into account, serves as a real-life example to apply the theory and the numerical method. The unpredictable fluctuations of the air density, for which lower and upper bounds are known, are the unknown disturbances. These disturbances are of great influence on the constraints and the performance index. The optimal feedback controller for the worst possible variations of the air density and the associated reentry trajectories are computed and presented for that space-shuttle reentry problem.

2 Optimal Control Problem for a Space-Shuttle Reentry

Assuming no planet rotation and oblateness, no winds and a stationary atmosphere, a point mass vehicle and a constant drag polar, the equations of motion for a space-shuttle-type vehicle in a flight path oriented coordinate system can be written as

$$\dot{h} = v \sin\gamma \ , \tag{5}$$

$$\dot{\Lambda} = \frac{v}{R+h} \cos\gamma \sin\chi \ , \tag{6}$$

$$\dot{\Theta} = \frac{v}{R+h} \frac{\cos\gamma \cos\chi}{\cos\Lambda} \ , \tag{7}$$

$$\dot{v} = -C_L^n \frac{S\, v^2 \rho}{2\, m} \sigma - C_{D_0} \frac{S\, v^2 \rho}{2m} \sigma - g \sin\gamma \ , \tag{8}$$

$$\dot{\chi} = C_L \sin\mu \frac{S\, v\, \rho}{2\, m \cos\gamma} \sigma - \frac{v}{R+h} \cos\gamma \cos\chi \tan\Lambda \ , \tag{9}$$

$$\dot{\gamma} \quad = \quad C_L \, \cos\mu \, \frac{S \, v \, \rho}{2 \, m} \, \sigma + \left(\frac{v}{R+h} - \frac{g}{v} \right) \cos\gamma \; . \qquad (10)$$

Here h denotes the altitude above the earth's surface, Λ the cross-range angle, Θ the down-range angle, v the velocity, χ the heading angle, and γ the flight path angle. Besides these six state variables there are two control variables, the lift coefficient C_L ($C_L \geq 0$) and the bank angle μ ($-90[\text{deg}] \leq \mu \leq 90[\text{deg}]$). Moreover, R is the earth's radius and $g(h)$ the gravity acceleration; S, m, C_{D_0}, and n are the aerodynamic reference area, the mass, the zero-drag coefficient, and the exponent of C_L in the induced drag of the shuttle, respectively. The air density $\rho \, \sigma$ consists of a prescribed air density model $\rho(h)$, e.g., $\rho = \rho_0 \, \exp(-\beta \, h)$, and a variable deviation factor σ to describe the density fluctuations. For more details of the space-shuttle model, see [12].

Table 1: Initial and final conditions.

	$t = 0$	$t = t_f$
h [km]	95.0	30.0
Λ [deg]	0.00	
Θ [deg]		3.0
v [$\text{km}\,\text{s}^{-1}$]	7.850	1.116
χ [deg]	0.00	
γ [deg]	-1.25	-2.70

The initial conditions at the reentry into the atmosphere and the final conditions at the unspecified terminal time t_f are given in Table 1 . The initial conditions correspond to a reentry out off the equatorial plane with 97% of the total energy for a stable circular orbit in an altitude of 242 km. The terminal conditions for h, v and γ enable the entry into the final quasi-steady glide for a landing at an appropriate landing site.

For a realistic description of a reentry maneuver, the limitation of the dynamic pressure and of the aerodynamic heating at a critical stagnation point is of utmost importance. Both state constraints are of the first order and are transformed into the single zeroth-order state constraint

$$C_L \leq C_{L,\max}(h, v \, \sqrt[6]{\sigma}; \vartheta_{\max}) \; ; \qquad (11)$$

see [7]. The complicated function $C_{L,\max}$ can be found in [12]. The parameter $\vartheta_{\max}$ indicates the maximum permitted skin temperature at the critical stagnation point. For the given data, only the heating constraint becomes active. The maximum dynamic pressure $q_{\max} = 40\,000$ [N/m^2] is not reached in the problems investigated here; see Figs. 6 and 10. Therefore, the term $v \, \sqrt[6]{\sigma}$ occurs in Eq. (11), because the temperature is proportional to $v^3 \, \sqrt{\rho \, \sigma}$.

Since the terminal conditions for h, v and γ can be met within a higher than the prescribed accuracy against all possible disturbances for technical reasons, it is possible to use the remaining degree of freedom to maximize the cross-range. This enables to reach landing sites in southern France. Thus, we have to minimize the performance index

$$I[C_L, \mu, \sigma] := -\Lambda(t_f) \ . \tag{12}$$

In order to define the preceding optimal control problem completely, σ must be known as a function of the time and/or the state variables. However, only lower and upper bounds depending on the altitude are available in reality,

$$\sigma_{\min}(h) \leq \sigma \leq \sigma_{\max}(h) \ . \tag{13}$$

Some values of ρ, $\sigma_{\min}$ and $\sigma_{\max}$ are given in Table 2. Note that the dependence of the air density on season, day time, and locus is assumed to be already considered in the density model $\rho(h)$. The density model used for the reentry problem is based on data of Refs. [18] and [16].

Table 2: Air density and bounds for the density deviation.

h [km]	ρ [g m^{-3}]	$\sigma_{\max}$ [1]	$\sigma_{\min}$ [1]
100	.000391	1.106	.894
90	.00347	1.133	.867
80	.0219	1.148	.852
70	.0974	1.148	.852
60	.343	1.133	.867
50	1.14	1.106	.894
40	4.33	1.078	.922
30	19.1	1.050	.950

3 Differential Game Approach for a Space-Shuttle Reentry under Uncertain Air Density

By means of the necessary conditions for optimality (see, e.g., [5]), a multipoint boundary-value problem is obtained that must be satisfied by the solution of the optimal control problem. This boundary-value problem can be solved by the multiple shooting method; see, e.g., [17] and [6]. Note that the numerical solution of the boundary-value problem is only a candidate for the optimal solution of the control problem. The numerical solution for the space-shuttle reentry problem of section 2 with $\sigma \equiv 1$ was first given in [8].

In this section we investigate first the optimal control problem with the control constraint (11) omitted. The optimal solution for the control unconstrained case is shown in Fig. 1 (thin line).

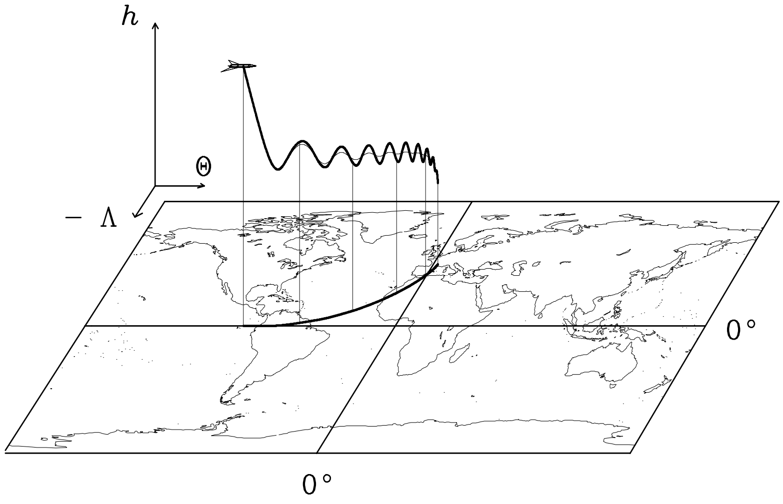

Figure 1: Optimal reentry trajectories with $\sigma \equiv 1$ (thin line) and $\sigma_{\min} \leq \sigma^* \leq \sigma_{\max}$ (thick line); control unconstrained cases.

In order to take into account fluctuations of the air density in the optimization process, the unknown disturbance factor σ is now considered as a control variable of an opposite player, say, the disturber D, in a two-person differential game. Here, complete information of the actual state variables and the dynamic system is available for both players, the disturber D and the optimizer O. Contrary to this, we assume that the disturber D has an information advantage with respect to the control variables, i. e. , he knows the actual value of the control variables choosen by the optimizer O. Recall, that we want to investigate the worst case here.

The optimizer O must be able to achieve the final conditions of Table 1 against all admissible disturbances at least approximately. This controllability proposition can be satisfied for the problem under investigation if, for example, the terminal conditions of Table 1 are replaced by the terminal inequality

$$F := \left\| \left(\frac{v(t_f) - v_f}{v_f}, \frac{\gamma(t_f) - \gamma_f}{\gamma_f}, \frac{h(t_f) - h_f}{h_f} \right)^\top \right\|_2 \leq \varepsilon \ . \tag{14}$$

We call the resulting problem the modified optimal control problem. If the optimizer O can enforce penetration into the terminal set defined by (14), the terminal time t_f of the game is reached when the optimizer O can no longer keep the state within this hyperset. Of course, this special situation is induced here by the monotonous increase of the cross-range angle Λ. We want to emphasize here that a primary objective of O is to reach the terminal set against all admissible disturbance strategies of D. We say, O is responsible for reaching the terminal set. The minimization of the performance index is only the secondary objective of the player O.

The first step for the treatment of the disturbed optimal control problem is a homotopy in the parameter ε from $\varepsilon = 0$ to, say, $\varepsilon = 0.01$. This value guarantees that the deviations from the prescribed terminal conditions given in Table 1 stay below 1%. The differential game is now mounted on this modified optimal control problem.

To compute the optimal strategies, we first consider the Hamiltonian of the problem,

$$H(z, \lambda, C_L, \mu, \sigma) := \lambda^\top f(z, C_L, \mu, \sigma) \tag{15}$$

with z denoting the state vector and f the right-hand sides of the dynamic equations (5)–(10). The vector λ denotes the gradient of the performance index I with respect to the state variables. For details see, e.g., [4]. The Hamiltonian can be rewritten in the form

$$H(z, \lambda, C_L, \mu, \sigma) = C_1(z)\, A(z, \lambda, C_L, \mu)\, \sigma + C_2(z, \lambda)\ . \tag{16}$$

Note that this Hamiltonian coincides with the Hamiltonian of the modified optimal control problem for $\sigma \equiv 1$. Since the present differential game is nonseparable, the optimal open-loop strategies for the two players can be computed by Isaacs' maximinimum principle (see [10] and [11]). To calculate the optimal strategies C_L^*, μ^* and σ^*, we first compute

$$\tilde{\sigma}(z, \lambda, C_L, \mu) := \arg \max_{\sigma} H(z, \lambda, C_L, \mu, \sigma)\ , \tag{17}$$

$$(C_L^*(z, \lambda), \mu^*(z, \lambda)) := \arg \min_{C_L,\, \mu} H(z, \lambda, C_L, \mu, \tilde{\sigma}(z, \lambda, C_L, \mu))\ , \tag{18}$$

and then

$$\sigma^*(z, \lambda) := \arg \max_{\sigma} H(z, \lambda, C_L^*(z, \lambda), \mu^*(z, \lambda), \sigma)\ . \tag{19}$$

Following this procedure, we obtain for the optimal open-loop strategies,

$$\sin \mu^* = -\frac{\lambda_\chi / \cos\gamma}{\sqrt{(\lambda_\chi / \cos\gamma)^2 + \lambda_\gamma^2}}\ , \tag{20}$$

$$\cos \mu^* = -\frac{\lambda_\gamma}{\sqrt{(\lambda_\chi / \cos\gamma)^2 + \lambda_\gamma^2}}\ , \tag{21}$$

$$C_L^* = \left(- \frac{\sqrt{(\lambda_\chi / \cos\gamma)^2 + \lambda_\gamma^2}}{v\,\lambda_v\, n} \right)^{\dfrac{1}{n-1}} , \tag{22}$$

$$\sigma^* = \begin{cases} \sigma_{\min} & \text{if} \quad A < 0 \ , \\ \text{undefined} & \text{if} \quad A = 0 \ , \\ \sigma_{\max} & \text{if} \quad A > 0 \ . \end{cases} \tag{23}$$

Since both players optimize the same performance index, the minimax condition

$$\min_{C_L,\,\mu} \max_{\sigma} H(z, \lambda, C_L, \mu, \sigma) = \max_{\sigma} \min_{C_L,\,\mu} H(z, \lambda, C_L, \mu, \sigma) = 0 \ , \tag{24}$$

if holding for all z and λ in a neighborhood of an optimal trajectory, is necessary and sufficient for the game to be zero-sum. This condition is always satisfied for the reentry problem in discussion. Moreover, the optimal open-loop strategies C_L^* and μ^* coincide here with the optimal control variables of the modified optimal control problem. A formulation of the game with an information advantage for O, i. e. with $\min\max H$ instead of $\max\min H$, leads to the same open-loop strategies for O and D. The calculations in this section are based on the assumption, which is reasonable, that σ^* is of bang-bang type. The case of singular subarcs, along which there holds $A \equiv 0$, is not considered here.

Since the performance index I is known on the terminal hypermanifold defined by the equality sign in (14), the terminal conditions for λ are obtained by differentiating the performance index with respect to the state variables. Hence, we have the following nonlinear terminal conditions for the adjoint variables,

$$H(z, \lambda, C_L^*, \mu^*, \sigma^*)\big|_{t=t_f} = 0 \ , \tag{25}$$

$$\lambda_\Lambda = -1 \ , \tag{26}$$

$$\lambda_v = \lambda_h \, \frac{h_f^2}{h(t_f) - h_f} \, \frac{v(t_f) - v_f}{v_f^2} \ , \tag{27}$$

$$\lambda_\chi = 0 \ , \tag{28}$$

$$\lambda_\gamma = \lambda_h \, \frac{h_f^2}{h(t_f) - h_f} \, \frac{\gamma(t_f) - \gamma_f}{\gamma_f^2} \ . \tag{29}$$

The boundary-value problem is completed by the differential equation for λ,

$$\dot\lambda = -\frac{\partial}{\partial z} H(z, \lambda, C_L^*(z, \lambda), \mu^*(z, \lambda), \sigma^*(z, \lambda)) \ . \tag{30}$$

For details see, e.g., Refs. [2] and [4]. Note that the differential equation for the down-range angle is decoupled.

For every saddle-point trajectory $z^*(t)$ related to the secondary objective minimization of I the compliance with the primary objectives of the optimizer O has to be verified a posteriori. In detail, for all $z \in z^*(t)$ the precondition

$$\max_{\sigma} \min_{C_L,\,\mu} \left(\min_{t \in [0,\infty[} F(t) \right) \leq \varepsilon \tag{31}$$

has to be checked with an auxiliary two-person differential game. However, in the reentry problem in consideration the precondition (31) is satisfied for technical reasons.

The optimal solution of the differential game can now be interpreted as the optimal solution of the modified optimal control problem if a most unfavourable course of the air density is assumed. Furthermore, the close relationship between these two problems enables the following procedure. Since $C_1 > 0$, the switching function A in Eq. (16) along the optimal solution of the modified control problem indicates the bang-bang switching structure of the optimal open-loop strategy σ^* for the disturber D to be expected if the bounds $\sigma_{\min}$ and $\sigma_{\max}$ are slightly freed from one. Therefore, the numerical solution of the multipoint boundary-value problem associated with the modified optimal control problem can serve as an appropriate starting trajectory for the homotopy in the two parameters $\sigma_{\min}$ and $\sigma_{\max}$. After the first homotopy step, the solution of the multipoint boundary-value problem, now associated with the differential game, is a candidate for the saddle-point trajectory of the game. At the end of the homotopy, $\sigma_{\min}(h)$ and $\sigma_{\max}(h)$ reach the functions the values of which are given in Table 2. The numerical solutions of the various multipoint boundary-value problems have been obtained by the multiple shooting algorithm given in [9].

The history of the function $C_1\,A$ is shown in Fig. 2. The magnitude of the function $C_1\,A$ is a measure for the influence of the disturbance on the performance index. The optimal open-loop strategies σ^*, C_L^*, and μ^* are shown in Figs. 3–5. In Fig. 1, the reentry trajectories for both the modified optimal control problem (thin line) and the disturbed optimal control problem (thick line) are compared. The flight oscillations are typical for maximum-range flights of gliders (pumping). The same trajectories are shown in an altitude-velocity diagram in Fig. 6. This diagram indicates that the dynamic pressure constraint is nowhere violated. In addition, the loss of the initial total energy E_0 during the reentry can be seen. The skin temperature ϑ due to the aerodynamic heating for both problems is given in Fig. 7. The discontinuities are caused by the discontinuities in σ^*; compare Eq. (11). Note that the maximum temperature of about 1522 [°C] occurs at the first peak in the undisturbed problem, but of the magnitude of about 1701 [°C] at the fourth peak in the disturbed problem. In the next section the maximum temperature $\vartheta_{\max}$ is decreased to 1400 [°C].

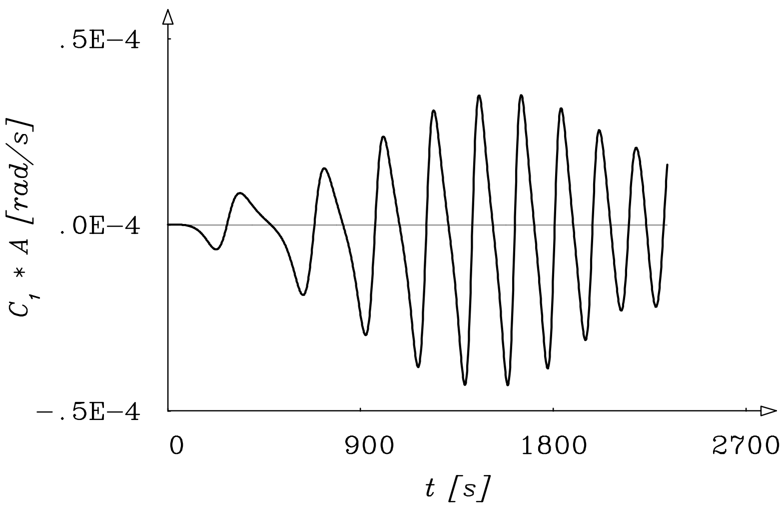

Figure 2: Switching function for σ^*; disturbed unconstrained optimal control problem.

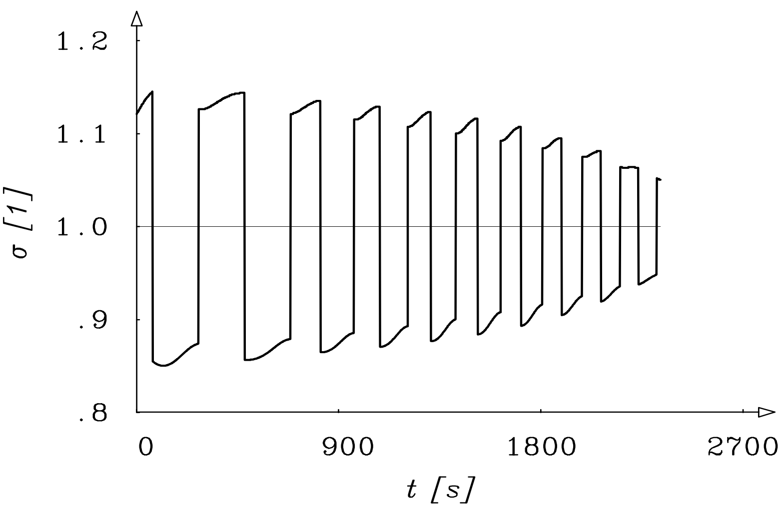

Figure 3: Optimal open-loop strategy σ^*; disturbed unconstrained optimal control problem.

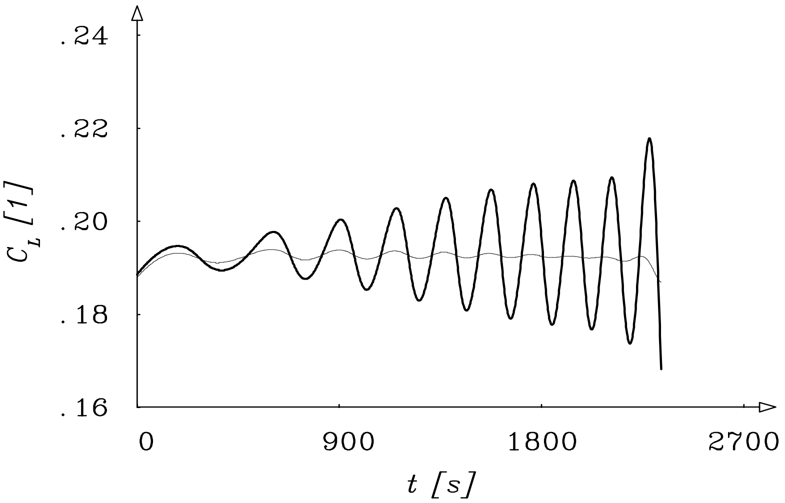

Figure 4: Optimal open-loop strategy C_L^* (thick line) and optimal control C_L^* (thin line); unconstrained problems.

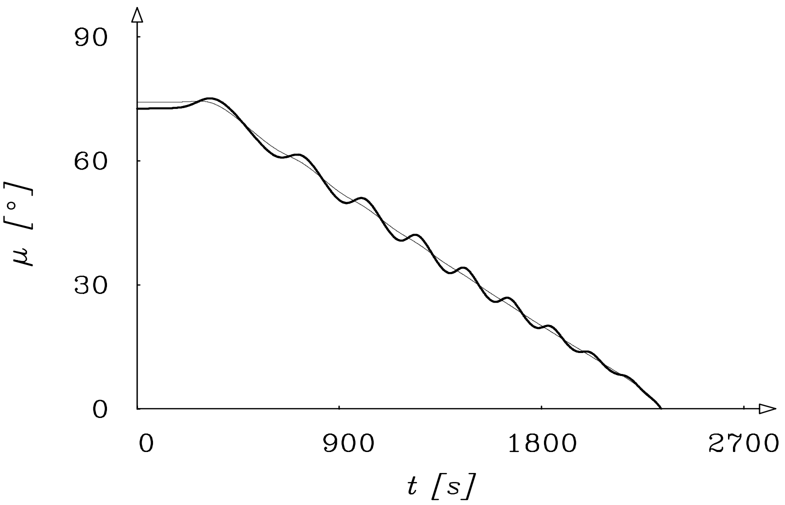

Figure 5: Optimal open-loop strategy μ^* (thick line) and optimal control μ^* (thin line); unconstrained problems.

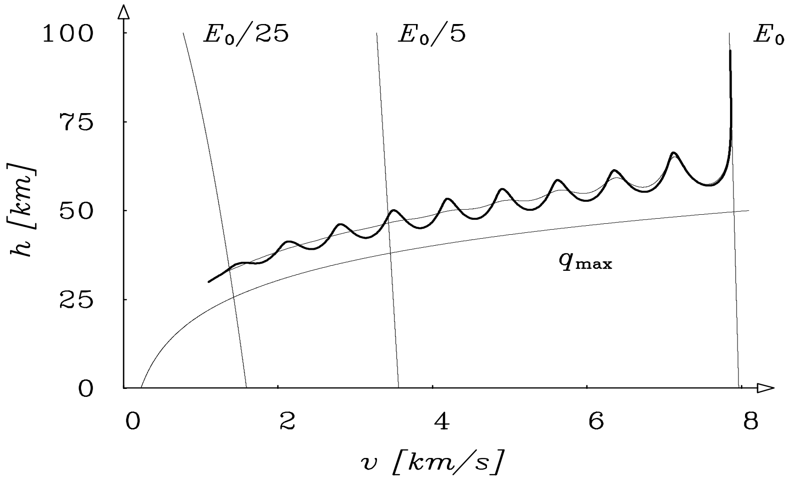

Figure 6: Optimal trajectories for the disturbed optimal control problem (thick line) and for the modified optimal control problem (thin line); unconstrained cases.

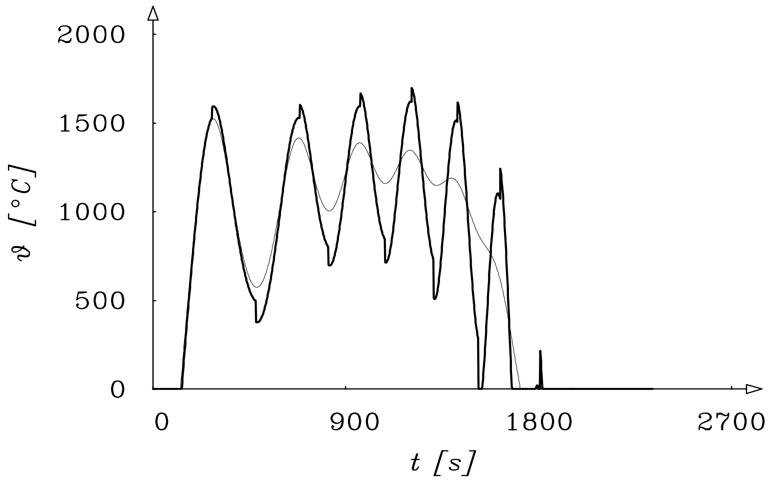

Figure 7: Temperature ϑ for the disturbed optimal control problem (thick line) and the modified optimal control problem (thin line); unconstrained cases.

4 Differential Game for the Temperature-Constrained Disturbed Optimal Control Problem

The compliance with the temperature constraint (11) is now the second primary objective of the optimizer O, i.e., O is also responsible for the strict observance of this constraint. Therefore, we must transform the inequality constraint (11) such that this constraint can be satisfied at any time against all admissible air density fluctuations σ. Hence, there always must hold

$$C_L \leq \min_{\sigma} C_{L,\max}(h, v\ \sqrt[6]{\sigma}; \vartheta_{\max}) \ . \tag{32}$$

Because of the special function $C_{L,\max}$, the minimum is always taken at $\sigma = \sigma_{\max}$. This modified constraint is called active if there exists an admissible disturbance σ such that the equality sign holds in Eq. (32). In this case, the subarcs are called constrained and the optimizer O must choose the optimal open-loop strategy C_L^* as

$$C_L^* = C_{L,\max}(h, v\ \sqrt[6]{\sigma_{\max}}; \vartheta_{\max}) \ . \tag{33}$$

Note that, on constrained subarcs, the term H_{C_L} no longer vanishes in the differential equations for the adjoint variables (30).

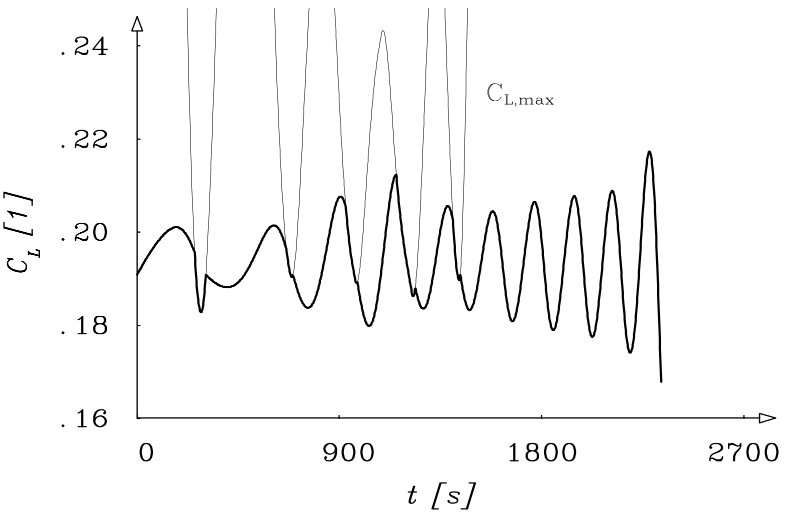

Figure 8: Optimal open-loop strategy C_L^* (thick line) and $C_{L,\max}$ for $\vartheta_{\max} = 1400$ [°C] (thin line); constrained case.

By a second homotopy, the temperature constraint is now introduced and tightened step by step until $\vartheta_{\max} = 1400$ [°C] is reached. As the first result, the history of the optimal lift coefficient is shown in Fig. 8. Five constrained subarcs occur.

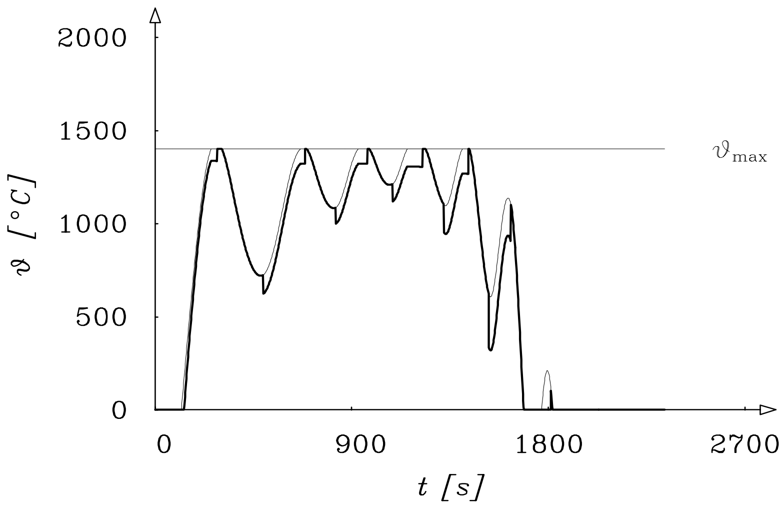

Figure 9: Temperature ϑ for the disturbed optimal control problem (thick line) and temperature for $\sigma \equiv \sigma_{\max}$ (thin line); constrained case.

The history of the temperature is shown in Fig. 9. The thin line indicates the history of the temperature which is the worst possible one for the optimizer O. Note that, along the constrained subarcs, i.e., where the transformed inequality (32) is active, the temperature ϑ is not everywhere equal to $\vartheta_{\max}$. The discontinuities in ϑ result from the bang-bang structure of the optimal open-loop strategy σ^*. Finally, the optimal trajectories for the disturbed optimal control problems are depicted in Fig. 10.

5 Conclusions

Since the worst case of the air density fluctuations is extremly unlikely, the actual trajectory will deviate from the precalculated saddle-point trajectory immediately after the start of the reentry maneuver. Nevertheless it is not only possible to analyze the worst case by the method presented in this paper. We can benefit from this method in two more ways. At first, many saddle-point trajectories filling up the relevant part of the state space can be computed. They can serve as a basis for the training of a neural network to approximate the optimal feedback controller. Secondly neighboring boundary-value problems can be solved successively. Their solutions are connected with the saddle-point trajectories for the remaining flight and enable tests with realistic air density distributions. These tests yield benchmarks for suboptimal feedback controllers. Moreover, the method is expected to become capable of real-time applications if one takes advantage of the inherent parallel structure of the multiple shooting method. For deterministic optimal control problems, first suggestions for a real-time-capable multiple shooting method have been already developed; see [15].

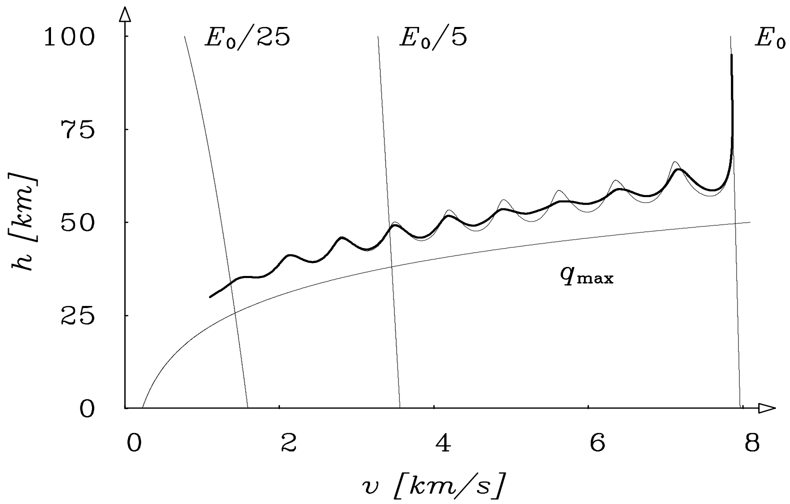

Figure 10: Optimal trajectories for the disturbed optimal control problems: constrained case (thick line) and unconstrained case (thin line).

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