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The effect of hysteresis on equilibrium selection in coordination games☆

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ABSTRACT

One of the fundamental problems in both economics and organization is to understand how individuals coordinate. The widely used minimum-effort coordination game has served as a simplified model to better understand this problem. This paper first presents theoretical results that give conditions under which the minimum-effort coordination game exhibits hysteresis. It then uses these theoretical results to develop and confirm some experimental hypotheses using human subjects in the laboratory. The main insight is that play in a given game is heavily dependent on the history of parameters leading up to that game. For example, the experiments show that when cost $c = 0.5$ in the minimum-effort coordination game, there is significantly more high effort if the cost has increased to $c = 0.5$ compared to when the cost has decreased to $c = 0.5$. One implication of this is that a temporary change in parameters may be able move the economic system from a bad equilibrium to a good equilibrium.

1. Introduction

One common cause of sub-optimal economic outcomes is the inability of individuals to coordinate their actions. For example, there are many situations in which individuals could mutually benefit from coordinating their actions, but the environment makes coordination difficult. When faced with these situations, it is important to understand what aspects of the environment make coordination difficult and to try to determine what can be done to help individuals attain better outcomes. This paper focuses on hysteresis and its effect on equilibrium selection in coordination games. The paper starts with a simple model and develops theoretical results describing conditions under which hysteresis occurs in coordination games. Some experimental hypotheses, formulated from the theoretical results, are then confirmed using laboratory experiments with the minimum-effort coordination game. These results suggest that temporary changes in the environment can lead to a significant increase in the amount of coordination.

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The minimum-effort coordination game (or weak-link game) has multiple Pareto-Ranked equilibria and therefore has been studied extensively to try to better understand the equilibrium selection problem (Schelling, 1960; Ochs, 1995). The game is of interest to macroeconomists as a simplified example of an economy with a low-output and high-output equilibrium (Cooper and John, 1988). The game is also of interest as a simplified model of a firm to those studying organization (Weber, 2006). The game consists of N players, each of which selects an effort level. Players' payoffs depend on both the group outcome and their individual effort. The group outcome is determined by the minimum effort in the group. Effort is costly and determined by a parameter c. The higher the value of c, the more costly it is for players to exert effort. For example, consider a group of people working together to make a product. Each person can either exert high effort or low effort. If even one of the individuals exerts low effort, then the product is low quality, which is bad for everyone that worked on it. Therefore, if other players are exerting low effort, then the best response is to exert low effort as well. However, if all other players exert high effort, then the best response would be to exert high effort as well. In this example with two effort levels, there would be two pure-strategy Nash equilibria; one in which all players exert high effort and one in which all players exert low effort. Among the equilibria, the one in which all players exert high effort is more desirable because it leads to a higher payoff for all players.

Hysteresis is a specific form of path dependence in which the state of the system lags behind as a parameter in the system changes. In the context of the minimum effort coordination game with parameter c, players behavior will lag as the cost parameter is changed. Therefore, if a group starts with a low cost parameter, players usually exert high effort. As the cost is increased to an intermediate level of $c = 0.5$, this high effort behavior persists. Alternatively, if a group starts with a high cost parameter, players typically exert low effort. As the cost is decreased to an intermediate level of $c = 0.5$, the low effort behavior persists. Therefore, the behavior displayed at the intermediate value depends heavily on whether the group started with a high or low cost. To better understand hysteresis in the minimum-effort coordination game, this paper looks at a theoretical model and finds conditions under which this phenomenon is likely to occur. In particular, this paper focuses on s-shaped equilibrium correspondences[1] because these correspondences lead to hysteresis.

Using the theoretical results from this model, some experimental hypotheses are developed and then tested. The experiments involve subjects playing the minimum-effort coordination game repeatedly as the cost is varied. There are two treatments, one in which the cost is varied from low to high to low, and another in which it is varied from high to low to high. Varying the cost in this organized manner provides a test for determining whether hysteresis occurs in the minimum-effort coordination game. The experiments provide strong support for hysteresis in the minimum-effort coordination game, showing large differences in effort levels at $c = 0.5$, depending on whether cost has increased or decreased to $c = 0.5$. Specifically, groups chose high effort 85% of the time when c started low and increased to 0.5, compared to 13% of the time when c started high and decreased to 0.5. Another important implication of the experimental results is that a temporary decrease in the cost parameter may move the system from a bad equilibrium to a good equilibrium. In the experiments, 13% of groups played high effort at $c = 0.5$, but after the cost was temporarily decreased and then increased to $c = 0.5$, the percentage of groups playing high effort increased significantly, to 69%.

An important implication of the results is that when a system exhibits hysteresis, a temporary change in one parameter can move the system from a "bad" equilibrium to a "good" equilibrium. For example, if a manager that is overseeing an under-performing group project could offer a temporary incentive (to decrease the cost of effort), which would cause the group to move from the "bad" (under-performing) equilibrium to the "good" equilibrium. However, the results suggest that the converse is also true: a temporary increase in the cost of effort could move a group from the "good" equilibrium to a "bad" equilibrium.

The equilibrium selection problem has been studied extensively in the literature. One approach to better understanding the equilibrium selection problem is to impose additional equilibrium criteria, which refines the set of equilibria and makes predictions more clear. In their seminal work, Harsanyi and Selten (1988) suggest two different methods for selecting equilibria in games with multiple equilibria: payoff dominance and risk dominance. In certain situations, these two selection criteria may conflict, meaning that the payoff-dominant equilibrium is not the risk-dominant equilibrium. When there is conflict, Harsanyi and Selten (1988) suggest that payoff dominance should be used instead of risk dominance. Since this seminal study, however, a growing support for the risk-dominant equilibrium has emerged (Kandori et al., 1993; Young, 1993; Carlsson et al., 1993). Rather than suggesting that one equilibrium is played in a specific environment, this paper shows that there is hysteresis in coordination games, which means that different equilibria may be selected in a single environment, depending on the history.

Another approach to better understanding the equilibrium selection problem is to run experiments with human subjects in the laboratory. The game commonly used to model these situations in the laboratory is the minimum-effort coordination game. Experimental results suggest that coordination is more difficult with larger groups and higher costs of effort (Cooper et al., 1990; Van Huyck et al., 1990; Goeree and Holt, 2005). Building on these results, others have examined how changes in the environment can lead to higher levels of coordination. Some examples include adding communication (Cooper et al., 1992), competition between groups (Myung, 2013), and entrance fees (Cachon and Camerer, 1996). The results in this paper suggest that since there is hysteresis, changes in the environment need only be temporary to result in increased coordination.

[1] S-shaped equilibrium correspondences are correspondences that are shaped like either an S or a reflected S.

Systems exhibiting hysteresis are by no means rare. Hysteresis is present in a wide variety of physical settings, including magnetism and elasticity, but has also been observed in economics. Blanchard and Summers (1986) present a model in which the natural unemployment rate exhibits hysteresis in the presence of shocks. Employers make employment decisions in advance, with the goal of maintaining steady employment in expectation. Employment shocks change these expectations and lead to more-permanent changes in the natural unemployment rate. Baldwin (1988) shows that overvaluation of the dollar leads to hysteresis in United States import prices. Dixit (1989) examines entry of Japanese firms into the U.S. market based on exchange-rate fluctuations and finds that due to sunk costs, firms may remain in the U.S. even after the favorable exchange-rate fluctuation has subsided. Nyberg (1997) examines an evolutionary model of honesty and finds that once a society loses its honesty, hysteresis makes it difficult to reestablish. Wolpert et al. (2012) use the maximum entropy principle to examine the shape of the QRE equilibrium correspondence. They find that by imposing an individual specific tax rate, the system exhibits hysteresis as this rate varies. Finally, Göcke (2002) provides a detailed survey of some of the research on hysteresis in economics. This paper not only finds that there is hysteresis in coordination games, but also that hysteresis has important effects on equilibrium selection.

Some experimental papers have examined coordination games and found evidence of hysteresis. Weber (2006) examines the effect of changing group sizes on a group's ability to coordinate in a minimum-effort coordination game. He finds that coordination in large groups is possible if the group starts with a small number of subjects and gradually increases to a size of 12 subjects per group. In contrast, groups that start with 12 subjects per group are never able to coordinate on high-effort levels. Brandts and Cooper (2006) examine the effect of using payoff bonuses as a means of inducing cooperation in the minimum-effort coordination game. They find that adding bonuses helps bring groups from lower to higher effort levels. In addition, they find that when the payoffs are decreased back to initial levels after the temporary bonuses, effort levels are higher than they were before the bonuses. Hamman et al. (2007) examine different "all-or-none" incentives, and their effectiveness in moving the system from an inefficient equilibrium to an efficient one. Ruffle et al. (2010) examine a market in which a minimum number of buyers (critical mass) is needed to make the purchase worthwhile. They find that it is difficult to sustain coordination when the minimum number of buyers is increased immediately from a low number to high number. However, they also find that coordination is more likely to be sustained when that number is gradually increased from a low number to high number. Examining the effect of gradual increases in the stakes in weakest-link games under several information feedback structures, Ye (2011) finds that gradually increasing stakes does lead to more coordination. This paper adds to current literature by providing a simple model that suggests hysteresis occurs as the cost is varied in the MECG, and that this hysteresis has important implications on equilibrium selection. In addition, the model shows that hysteresis can be either good or bad; the model can move from a "bad" equilibrium to a "good" equilibrium as a parameter is varied, or vice-versa. Finally, the model is comprehensive, and suggests that hysteresis is not just a function of varied costs, but can also occur with a number of parameters (Section 5.2) and in other commonly studied coordination games (Section 5.3).

The paper proceeds as follows. Section 2 introduces the model. Section 3 gives the theoretical results and experimental hypotheses. Section 4 presents the experimental design and the experimental results. Section 5 provides further discussion of some technical extensions of the model. Finally, Section 6 concludes.

2. Model

Let $g(\gamma)$ be a game consisting of n players, $I=\{1, 2, \ldots, n\}$. Each player has m pure actions, $S_i=\{s_i^1, s_i^2, \ldots, s_i^m\}$. A joint-action profile is denoted by $\boldsymbol{s}=(s_1, \ldots, s_n)$. Each player faces a payoff function $u_i(\boldsymbol{s}, \gamma)$ that depends on the parameter γ from parameter space Γ.

The set of mixed strategies is denoted by Σ_i, which is the set of probability distributions over S_i. A mixed strategy is denoted by $\sigma_i \in \Sigma_i$, which is a mapping from S_i to $[0, 1]$, where $\sigma_i(s_j)$ is the probability that player i plays pure-action s_j, and $\Sigma=\Sigma_1 \times \cdots \Sigma_n$ is the set of mixed-strategy profiles. A joint mixed-strategy profile is denoted $\boldsymbol{\sigma}=\{\sigma_1, \ldots, \sigma_n\}$. Player i's expected payoff for mixed-strategy profile $\boldsymbol{\sigma}$ is $u_i(\boldsymbol{\sigma}, \gamma)=\sum_{\boldsymbol{s}\in S_1\times\cdots\times S_n} p(\boldsymbol{s})u_i(\boldsymbol{s}, \gamma)$, where $p(\boldsymbol{s})=\Pi_{i\in I}\sigma_i(s_i)$ is the probability of the pure-strategy profile $\boldsymbol{s}$ given mixed strategy profile $\boldsymbol{\sigma}$.

A joint-strategy profile $\boldsymbol{\sigma}$ is an equilibrium of the game $g(\gamma)$ if the equilibrium function $f:\Sigma\times\Gamma\times\Lambda\to\mathbb{R}$, dependent on parameter $\lambda\in\Lambda$, satisfies $f(\boldsymbol{\sigma}, \gamma, \lambda)=0$. For example, f could be the logit quantal response equilibrium function,

$$f(\boldsymbol{\sigma}, \gamma, \lambda)=\sum_{i=1}^{n}\sum_{j=1}^{m}\left|\frac{e^{\lambda u_i(s_j, \boldsymbol{\sigma}_{-\boldsymbol{i}}, \gamma)}}{\sum_{k=1}^{m} e^{\lambda u_i(s_k, \boldsymbol{\sigma}_{-\boldsymbol{i}}, \gamma)}}-\sigma_i(s_j)\right|=0. \tag{1}$$

Given γ and λ, any joint mixed-strategy profile $\boldsymbol{\sigma}$ is an equilibrium if (1) is satisfied. The game $g(\gamma)$ has multiple equilibria for parameter λ if $f(\boldsymbol{\sigma}, \gamma, \lambda)=0$ for more than one joint mixed-strategy profile $\boldsymbol{\sigma}$. Let $\boldsymbol{\Sigma}^*(\gamma, \lambda)=\{\boldsymbol{\sigma}|f(\boldsymbol{\sigma}, \gamma, \lambda)=0\}$ be the set of equilibria of game $g(\gamma)$ according to equilibrium function f with parameter λ.

Definition 2.1 *(Continuous variation).* The equilibrium correspondence $\boldsymbol{\Sigma}^*(\gamma, \lambda)$ varies continuously from $\underline{\gamma}$ to $\bar{\gamma}$ starting at $\underline{\boldsymbol{\sigma}}\in\boldsymbol{\Sigma}^*(\underline{\gamma}, \lambda)$ and ending at $\bar{\boldsymbol{\sigma}}\in\boldsymbol{\Sigma}^*(\bar{\gamma}, \lambda)$ if and only if there exists a continuous function $h:\Gamma\to\mathbb{R}$ such that,

1. $h(\underline{\gamma})=\underline{\boldsymbol{\sigma}}$ and $h(\bar{\gamma})=\bar{\boldsymbol{\sigma}}$,
2. for all $\gamma\in[\underline{\gamma}, \bar{\gamma}]$, $h(\gamma)\in\boldsymbol{\Sigma}^*(\gamma, \lambda)$, and

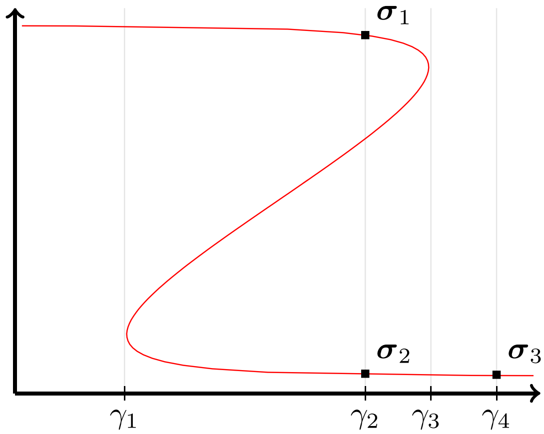


Fig. 1. Example of continuous variation of a correspondence between two parameter values.

3. there exists some $\bar{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \bar{\varepsilon})$, $(h(\gamma) - \varepsilon, h(\gamma) + \varepsilon) \cap \boldsymbol{\Sigma}^{*}(\gamma, \lambda)$ is a singleton.[2,3]

The equilibrium correspondence varies continuously if there is only one continuous path along the correspondence between $\underline{\boldsymbol{\sigma}}$ and $\bar{\boldsymbol{\sigma}}$ while remaining in the domain $\gamma \in [\underline{\gamma}, \bar{\gamma}]$. For example, in Fig. 1, the equilibrium correspondence varies continuously from γ_2 to γ_4 starting at $\boldsymbol{\sigma}_2$ and ending at $\boldsymbol{\sigma}_3$. However, the equilibrium correspondence *does not* vary continuously from γ_2 to γ_4 starting at $\boldsymbol{\sigma}_1$ and ending at $\boldsymbol{\sigma}_3$.

Assumption #1: *When faced with* $g(\gamma)$*, players will play one of the equilibria, call this* $\boldsymbol{\sigma}(g(\gamma)) \in \boldsymbol{\Sigma}^{*}(\gamma, \lambda)$.
Assumption #2: *If the equilibrium correspondence* $\boldsymbol{\Sigma}^{*}(\gamma, \lambda)$ *varies continuously from* γ_1 *to* γ_2 *starting at* $\boldsymbol{\sigma}_1$ *and ending at* $\boldsymbol{\sigma}_2$*, and players play equilibrium* $\boldsymbol{\sigma}_1$ *when game* $g(\gamma_1)$ *is played; then players will play* $\boldsymbol{\sigma}_2$ *when game* $g(\gamma_2)$ *is played.*

Assumption #2 says that if players have reached a certain equilibrium, and then a parameter of the game is changed slightly, they will move to a similar equilibrium as long as it exists at the new parameter. Several papers have examined the effect of spillovers between games. Cason et al. (2011) examine spillover between coordination games and find coordination spillover when moving from the median-effort game to the minimum-effort game. Huck et al. (2011) vary subjects information about previous play and observe feedback spillover between similar games. Devetag (2005) finds that historical precedents, such as playing efficient equilibria in the critical mass game transfer over to the minimum-effort coordination game. Cooper and Kagel (2003) find that players are able to learn across entry limit-pricing games as the cost of entry is changed. These results suggest that there are spillovers between games, which provides some support for Assumption #2.

Assumption #2 can also be related to the literature on catastrophe theory. Zeeman (1977) proposes several rules by which equilibria are selected as a parameter varies across a bifurcation point. The first is the delay rule, which, like assumption #2, suggests that the equilibrium does not jump between branches of the correspondence unless one of the equilibria ceases to exist. The second is the Maxwell rule, which, in opposition to assumption #2, suggests that the players jump to the payoff dominant equilibrium as long as it exists. In light of the experimental results in Section 4 and the brief examination of adaptive dynamics in Section 5.1, assumption #2, rather than an assumption similar to the Maxwell rule, seems the more natural assumption for this setting.

Based on the above definitions and assumptions, the definition of hysteresis is as follows:

Definition 2.2 *(Hysteresis).* The equilibrium correspondence $\boldsymbol{\Sigma}^{*}(\gamma, \lambda)$ exhibits hysteresis for equilibrium function f with parameter λ if there exist points $\gamma_1, \gamma_2, \gamma_3$ such that,

1. the correspondence varies continuously from γ_1 to γ_2, starting at $\boldsymbol{\sigma}_1 \in \boldsymbol{\Sigma}^{*}(\gamma_1, \lambda)$ and ending at $\boldsymbol{\sigma}_2 \in \boldsymbol{\Sigma}^{*}(\gamma_2, \lambda)$;
2. the correspondence varies continuously from γ_3 to γ_2, starting at $\boldsymbol{\sigma}_3 \in \boldsymbol{\Sigma}^{*}(\gamma_3, \lambda)$ and ending at $\boldsymbol{\sigma}'_2 \in \boldsymbol{\Sigma}^{*}(\gamma_2, \lambda)$; and
3. $\boldsymbol{\sigma}_2 \neq \boldsymbol{\sigma}'_2$.

The next section examines the minimum-effort coordination game, and shows that it exhibits hysteresis.

[2] This definition differs from the normal continuity definition of a correspondence using upper and lower hemicontinuity. Rather, this definition focuses on the properties of a specific part of the correspondence.

[3] The third condition ensures that there is a unique continuous path between the start and end point, and rules out regions of the correspondence that are thick.

Player i	$s_j = x_H$ for all $j \neq i$	$s_j = x_L$ for some $j \neq i$
x_H	$x_H(1-c)$	$x_L - cx_H$
x_L	$x_L(1-c)$	$x_L(1-c)$

Fig. 2. Minimum-effort coordination game.

2.1. Minimum-effort coordination game

A minimum-effort coordination game consists of n players, $I = \{1, \ldots, n\}$. Each player has two actions: They can choose to exert either high effort or low effort, $S_i = \{x_L, x_H\}$ for $x_L, x_H \in \mathbb{R}$ and $x_L < x_H$. The joint pure-action profile is denoted by $\boldsymbol{s} \in \{x_L, x_H\}^n$. Performing the high effort is more costly than performing the low effort. The high effort benefits players only if every player chooses the high-effort action. If any player chooses the low-effort action, then all players receive only the benefit from the low action. This yields payoffs,

$$u_i(\boldsymbol{s}) = \min_{j=1,\ldots,n} s_j - cs_i.$$

Fig. 2 displays the normal form of the minimum-effort coordination game. For cost $c \in \mathbb{R}$, the minimum-effort coordination game is denoted by $g^{\mathcal{CG}}(c)$. The class of all minimum-effort coordination games is $\mathcal{CG} = \{g^{\mathcal{CG}}(c) | c \in \mathbb{R}\}$.[4]

2.2. Nash equilibria

If $c > 1$, the cost of exerting the high effort outweighs the benefit, so action x_L strictly dominates x_H for all players. Therefore, all players playing x_L is the only pure-strategy Nash equilibrium when $c > 1$. Similarly, if $c < 0$, then the cost is negative, so action x_H strictly dominates x_L. Thus, all players playing x_H is the unique Nash equilibrium. When $c \in [0, 1]$, the game has two pure-strategy Nash Equilibria: one where everyone plays the high effort x_H, and one where everyone plays the low effort x_L. There is also one mixed-strategy equilibrium in which all players play x_H with probability $c^{1/(N-1)}$, which is clearly increasing in N for $c \in (0, 1)$.

For all values of $c \in (0, 1)$, the equilibrium in which all players play x_H with probability 1 is the *payoff-dominant* equilibrium. For levels of c close to 1, the difference between the high-effort and the low-effort equilibrium payoffs becomes small. However, a large loss is possible if the high-effort action is played, while no loss is possible if the low-effort action is played. Therefore, when c is close to 1, the gain from coordinating on the high-effort equilibrium is small compared to the loss from playing the high-effort action while others play the low-effort action, so the high-effort action is risky.

In the experiment, the players play the MECG repeatedly for a finite number of periods. There are a plethora of equilibria of the finitely repeated game which, among others, include all strategies where players play one of the pure-strategy Nash equilibria of the stage game in every period. For example, this would include the strategy where players all play x_H in even periods and all players play x_L in odd periods whenever $c \in [0, 1]$ and play the unique stage-game Nash-equilibrium otherwise.

2.3. Quantal-response equilibria

This section studies properties of the quantal-response equilibria (McKelvey and Palfrey, 1995) (QRE hereafter). In particular, this paper focuses on the logit QRE, which is commonly used in practice.[5] In the minimum-effort coordination game,

[4] Not all games in $\mathcal{CG}$ are minimum-effort coordination games (i.e., sufficiently high or low values of c).

[5] Haile et al. (2008) urge caution when using QRE as it can rationalize any distribution of behavior in any normal form game under certain assumptions. They emphasize that "Testing requires maintained hypotheses beyond those of the QRE notion itself." In this paper QRE is used as a theoretical tool to explain a certain trend in behavior as parameters change with well defined hypotheses. The hypotheses set forth in this paper are definitely falsifiable, and therefore the use of QRE should not be a concern here for the reasons pointed out in the paper.

since all payoffs are symmetric (in the sense that player's name doesn't affect their payoff), all QRE are symmetric. Let σ_H be the probability that a player plays the high-effort action x_H, and $\sigma_L = 1 - \sigma_H$ be the probability that a player plays the low-effort action. Using the logit QRE function, a QRE must satisfy the equation,

$$f(\sigma_H, c, \lambda) = \frac{1}{1 + e^{\lambda[(x_h - x_L)(\sigma_H{}^{N-1} - c)]}} - \sigma_H = 0. \tag{2}$$

When $\lambda = 0$, there is always a unique QRE, $\sigma_H = 0.5$. The intuition for this is that when $\lambda = 0$, players are completely unresponsive to payoffs, and therefore choose each action with equal probability. In addition, in the limit as $\lambda \to \infty$, there are always three solutions because the set of QREs approaches the set of Nash equilibria as $\lambda \to \infty$, and there are always three Nash equilibria for games with $c \in (0, 1)$. To sum up, low λ players are not very responsive to payoff, so the only equilibrium is for everyone to play essentially randomly. However, when λ increases, then players are more sensitive to payoffs, and the set of equilibria becomes closer to the set of Nash equilibria. The next section examines the properties of the QRE correspondence as the cost parameter c varies.

3. Results

This section determines properties of the logit QRE correspondence for the minimum-effort coordination game. The main result states conditions required for an s-shaped QRE correspondence, which leads to hysteresis. An s-shaped equilibrium correspondence has two critical values (parameter values where the number of equilibria changes as that parameter value is traversed). For example, the s-shaped curve in Fig. 1 has a critical value at γ_1 where the number of solutions changes from one to three as the parameter is increased, and another critical value at γ_3 where the number of solutions changes from three to one as the parameter is increased. The results are summarized in the following proposition:

Proposition 3.1. *For the class of coordination games $\mathcal{CG}$:*

1. *There exists a λ^* such that the logit QRE correspondence, $\boldsymbol{\Sigma}^*(c, \lambda)$, exhibits hysteresis for all $\lambda > \lambda^*$, where,*

$$\lambda^* = \left(\frac{N}{N-1}\right)^N \frac{1}{x_h - x_l}. \tag{3}$$

2. *The critical value λ^* is decreasing in N.*
3. *For $N = 2$, the probability of playing H at the critical values are given by,*

$$\sigma_H = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \frac{4}{\lambda(x_h - x_L)}}.$$

4. *For $N > 2$, probability of playing H at the critical values is not symmetric around $\sigma_H = (1/2)$.*

This proposition states that for any coordination game of the given form, if the players have a high enough payoff responsiveness (sufficiently high λ), then the QRE correspondence should exhibit hysteresis. This means that given a game and an equilibrium function, it is possible to vary one parameter slightly and then change it back, and, thus, the system could be at a completely different equilibrium. This is important if one of the equilibria is more desirable than the other and all that is required is a small perturbation of the system to go from the less desired to the more desired equilibrium. The second point in the proposition says that this critical value is decreasing as the size of the group gets larger. Assuming that the values of λ for the individuals are not dependent on group size, this means that hysteresis is more likely as the group size increases. The third part gives the analytical solution for the probability of playing H at the critical values for the $N = 2$ case. It is not possible to find the analytical solution for the $N > 2$ case, but with numerical analysis, it is clear that both critical values are decreasing as the group size gets larger (as shown in Fig. 3). Also, for group size larger than two, the probability of playing H at the critical value is not symmetric, meaning that the probabilities are not equidistant from 0.5. However, in the $N = 2$ case, the probability of playing H at the critical values is symmetric around (1/2). As $\lambda \to \infty$, the QRE correspondence approaches the Nash equilibrium correspondence, which is shown for several values of N in Fig. 4. Since the proposition shows that the QRE correspondence exhibits hysteresis for sufficiently high λ, the Nash equilibrium correspondence also exhibits hysteresis.

The proof of this proposition is given in the Appendix. The proof uses the implicit function theorem to solve for the equilibrium value of c as a function of σ_H, $c^*(\sigma_H)$. The equilibrium correspondence $\boldsymbol{\Sigma}^*(c, \lambda)$ is an s-shaped curve if $c^*(\sigma_H) \to \infty$ as $\sigma_H \to 0$, $c^*(\sigma_H) \to -\infty$ as $\sigma_H \to 1$, and $(\partial c^*(\sigma_H))/(\partial \sigma_H) > 0$ for some value of $\sigma_H \in (0, 1)$. This process ensures the existence of an s-shaped curve for sufficiently large values of λ. The rest of the proposition is obtained from comparative statics that are detailed in the Appendix.

This setup yields some testable implications, the most important being that the equilibrium correspondence of the minimum-effort coordination game exhibits hysteresis. To test this, it is necessary to run an experiment with multiple

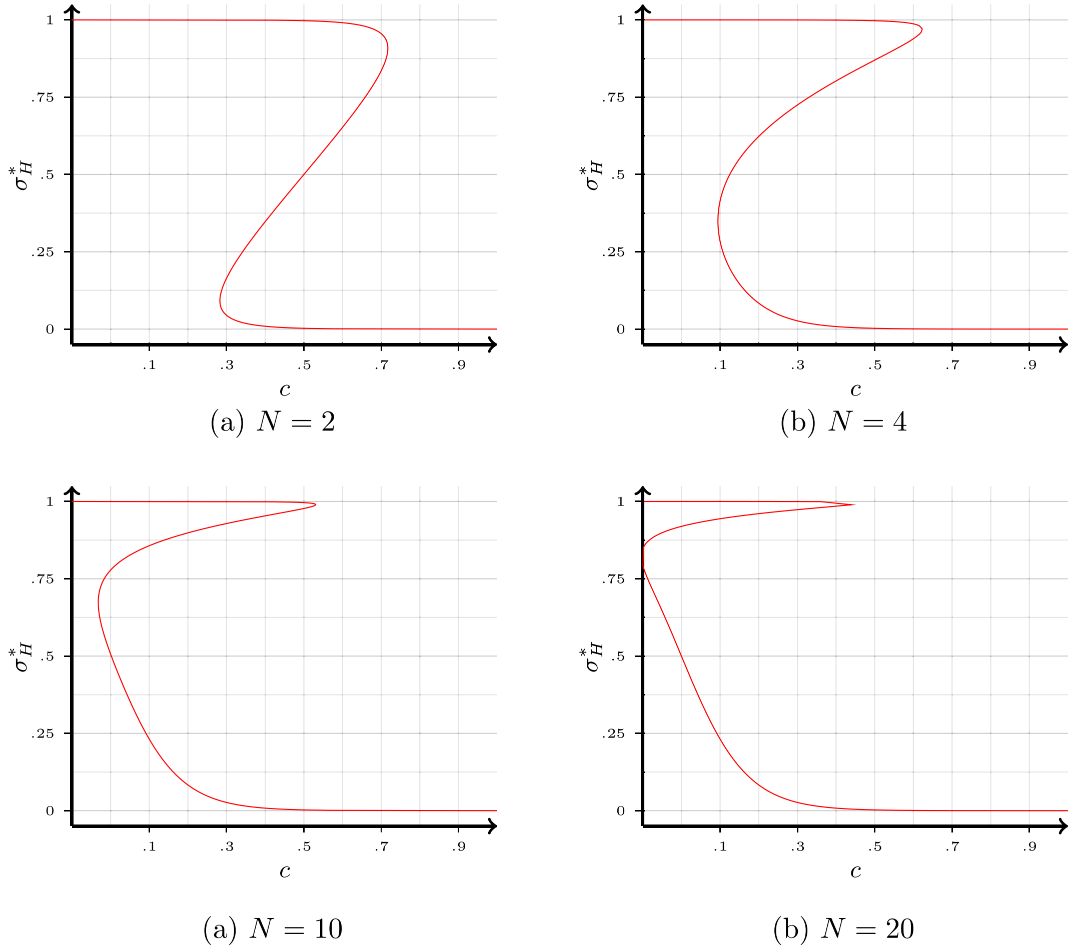


Fig. 3. QRE correspondences as c is varied for different values of N and $\lambda = 4$.

games, in which the games are varied in an organized manner to determine whether or not the experimental outcomes exhibit hysteresis. Based on the theoretical results, the experiment is used to test the following three hypotheses:

Hypothesis 1: In the minimum-effort coordination game, groups that start with a sufficiently[6] low cost that gradually increases to $c = 0.5$ should have significantly higher effort levels at $c = 0.5$ than groups that start with a sufficiently high cost that gradually decreases to $c = 0.5$.
Hypothesis 2: If the cost is varied gradually from a sufficiently low value to a sufficiently high value and back to a sufficiently low value, then, for an intermediate cost level c^*, there should be significantly **higher effort levels** the first time the group faces c^* (going from low to high) than there are the second time the group faces c^* (going from high to low).
Hypothesis 3: If the cost is gradually varied from a sufficiently high value to a sufficiently low value and back to a sufficiently high value, then, for an intermediate cost level c^*, there should be significantly **lower effort levels** the first time the group faces c^* (going from high to low) than there are the second time the group faces c^* (going from low to high).

These hypotheses predict that there is hysteresis in the minimum-effort coordination games. The next section details the experimental methods and results.[7]

[6] For all of the hypotheses sufficiently low means that cost is low enough such that the model predicts a unique equilibrium. Also, sufficiently high means that cost is high enough such that the model predicts a unique equilibrium. This ensures that the cost variations cross the saddle node points.
[7] Though the behavior predicted by the hypotheses are a subgame perfect Nash equilibrium of the finitely repeated game (for large λ), the prediction consists of a very specific type of behavior among the large possible set of behaviors that can be supported as a subgame perfect Nash equilibrium.

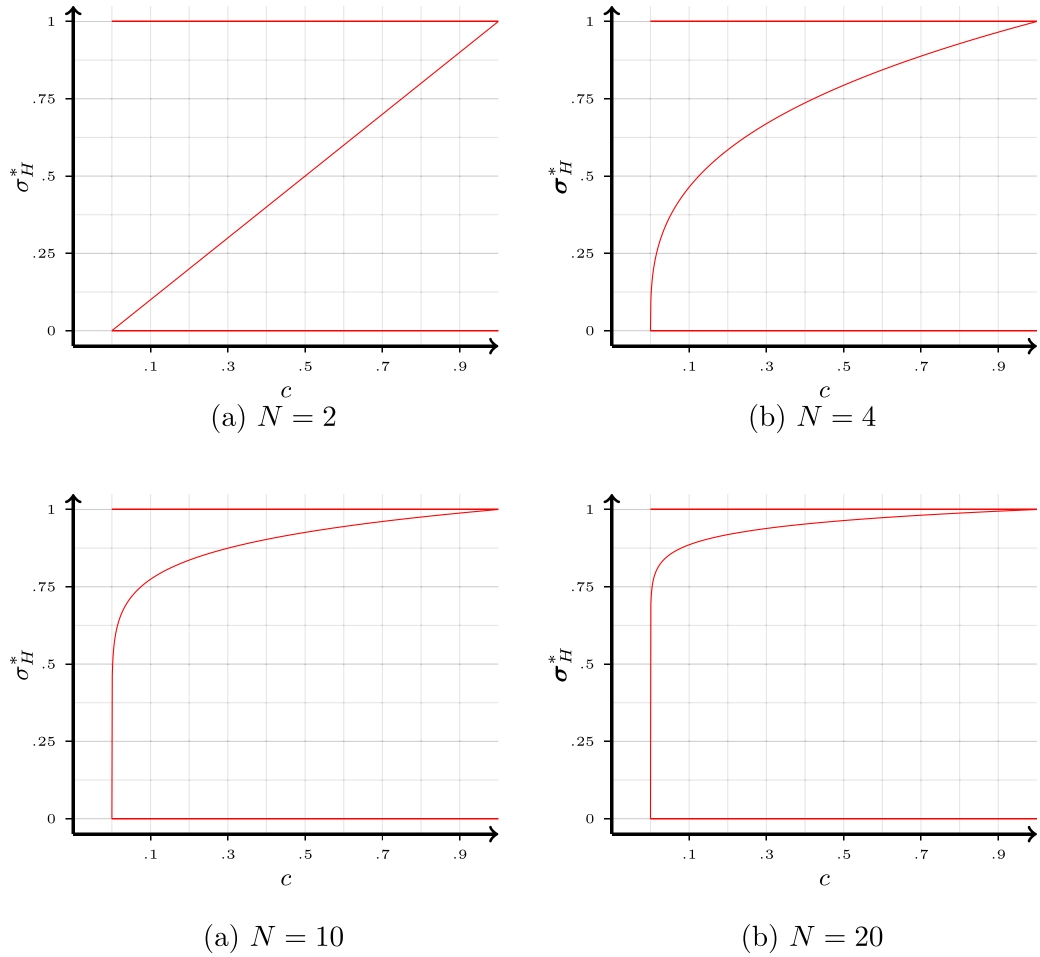


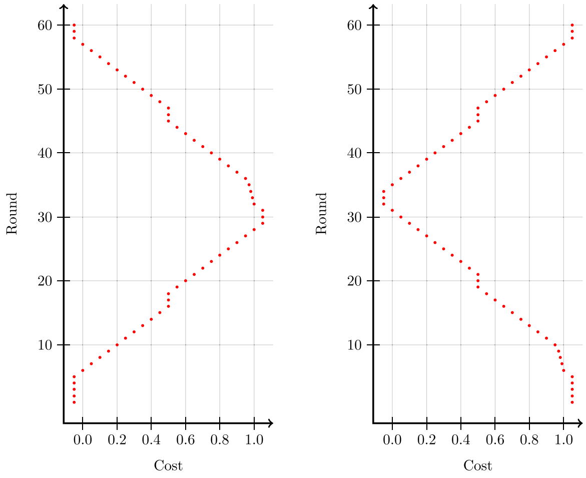
Fig. 4. Nash equilibrium correspondences as c is varied for different values of N.

4. Experiments

4.1. Experimental design

The subjects were drawn from a pool of Purdue University undergraduate students that were signed up to participate in experiments at the Vernon Smith Experimental Economics Laboratory. Upon entering the lab, the subjects were randomly assigned to a computer and given a handout containing the instructions. After all subjects had been seated, the instructions were read aloud. Then, the subjects completed a quiz to make sure that they understood the format of the game (see supplementary material for the instructions and quiz). The experiment did not start until all subjects had correctly answered all of the questions on the quiz. Each session, including the instructions portion, took about 45 minutes to complete. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007) (for a screenshot of the experimental interface, see supplementary material). During the experiments, all payoffs were displayed in U.S. dollars. Subjects' final payoff was the sum of earnings from three randomly selected rounds. After the experiment, subjects privately received their payments in cash, with the average payoff totaling $12.50.

The experiment was divided into six sessions, each of which consisted of 60 rounds. At the beginning of the session, subjects were divided into groups of four, and remained in the same group for all 60 rounds. In each round, the subjects played a minimum-effort coordination game in which they were asked to choose one of two options, high effort (labeled *X*) or low effort (labeled *Y*). If all the members of a group chose high effort, then the group choice was high effort (labeled *X*), and if anyone chose low effort, then the group choice was low effort (labeled *Y*). Their payoffs were displayed in a table similar to that in Fig. 2, with $x_H = 6$, $x_L = 1.5$, and varying cost



(a) LHL Treatment (b) HLH Treatment

Fig. 5. Progression of costs in LHL and HLH treatments.

parameters $c \in [-0.05,\ 1.05]$.[8] In the table, all payoffs were multiplied out, so the subjects saw just a single number in each box. After all subjects had selected their effort, they got to see their payoff for the round, as well as their group effort.[9] The players also were asked to record their choices and payoffs from each round on a record sheet.

The experiment consisted of two treatments – one in which cost was varied from low to high to low (LHL), and another in which cost was varied from high to low to high (HLH). Each treatment had three sessions with a total of 64 subjects divided into 16 groups. In the LHL treatment, the cost started at $c = -0.05$, gradually increased to $c = 1.05$, and then decreased back to $c = -0.05$. In the HLH treatment, the cost started at $c = 1.05$, gradually decreased to $c = -0.05$, and then increased back to $c = 1.05$. Fig. 5 shows the full progression of the costs for each treatment.[10,11] In the instructions, the subjects were told that the costs would change, but were not told the specific cost values or the manner in which the cost would be changed.[12]

4.2. *Experimental results*

Table 1 displays effort rates for different costs for both the LHL and the HLH treatments. The effort rate for the intermediate cost value $c = 0.5$ is displayed for each direction (whether cost is increasing to $c = 0.5$ or decreasing to $c = 0.5$).

4.2.1. *Hypothesis 1*

Hypothesis 1 predicted that there would be significantly more high effort at $c = 0.5$ in groups that started at a low cost and gradually increased to $c = 0.5$ than in groups that started at a high cost and gradually decreased to $c = 0.5$. In other words, different groups have different outcomes depending on the starting point. In LHL, the groups started with a cost of $c = -0.05$, and then the cost gradually increased until it reached $c = 0.5$ in rounds 16–18. In HLH, the groups started with a cost of

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[8] Different values of λ lead to different critical values on the s-shaped correspondence. For example, the s-shaped correspondence for $\lambda = 4$ and $N = 4$ in Fig. 3(b) has critical values $c = 0.1$ and $c = 0.63$. As λ increases, the critical values spread apart. The s-shaped correspondence for $\lambda = \infty$ and $N = 4$ in Fig. 4(b) has critical values $c = 0$ and $c = 1$. Since the subjects in the lab may have different rates of payoff responsiveness, the critical values are likely to be different from one group to the next. In order to ensure that the cost traverses the critical values of all groups, the value of c was set to vary below $c = 0$ and above $c = 1$.

[9] The group choice provides less information than the choice of each member of the group does.

[10] Note that $c = 0.5$ was played multiple times as this is the intermediate cost value that is used for comparison. The ideal experiment would consist of subjects playing many games at every cost level (giving them a chance to converge to equilibrium and satisfying assumption #1), and then looping from high to low cost many times to ensure the hysteresis loop. Unfortunately this would require keeping subjects in the lab for a very long time, and is therefore difficult to implement. As a compromise, I have the two different sessions which highlight that the hysteresis is present regardless of whether the initial cost is low or the initial cost is high.

[11] The cost increased and decreased at the same rate ($\Delta c = \pm 0.05$) except for when the cost was decreasing from $c = 1$ when the rate was decreased in magnitude to $\Delta c = -.01$. The delay and the Maxwell rules proposed by Zeeman (1977) are not in conflict for any value of λ as the cost increases from $c = 0$ (both predict high-effort equilibrium), but are in conflict for large values of λ as the cost decreases from $c = 1$ (delay rule predicts low-effort equilibrium while Maxwell rule predicts high-effort equilibrium). Because this region of the parameter space was seen as a potentially important region, the magnitude of the rate of change was decreased around $c = 1$.

[12] Alternatively, subjects could have been informed about the complete schedule of costs. This may complicate things as subjects would have to be more informed about the functional form of the payoffs, rather than just seeing payoff values in the table as they did in the current experiment. In addition subjects would have to have been given the entire schedule of costs which may also complicate things. The current choice was made for simplicity, but the hypotheses would remain the same in the alternate setting where subjects were given the entire cost schedule.

Table 1
Rates of high effort of individuals and groups for each treatment.

Treatment	Rounds	Direction	Cost	% Indv. = High	% Group = High
LHL	1–5		−0.05	99%(317/320)	96%(77/80)
	16–18	Increasing	0.50	88%(169/192)	85%(41/48)
	29–31		1.05	2%(4/192)	0%(0/48)
	45–47	Decreasing	0.50	70%(136/192)	56%%(27/48)
	58–60		−0.05	100%(192/192)	100%(48/48)
HLH	1–5		1.05	9%(30/320)	0%(0/80)
	19–21	Decreasing	0.50	34%(67/192)	12%(6/48)
	32–34		−0.05	98%(189/192)	93%(45/48)
	45–47	Increasing	0.50	71%(138/192)	68%(33/48)
	58–60		1.05	0%(1/192)	0%(0/48)

$c = 1.05$, and then the cost gradually decreased until it reached $c = 0.5$ in rounds 19–21. Table 1 shows that when players start with a low cost, 88% (169/192) of individuals play high effort, which leads to 85% (41/48) of groups attaining the high effort when $c = 0.5$ in rounds 16–18 of the LHL treatment. Alternatively, when players start with a high cost, only 35% (67/192) of individuals play high effort, which leads to 13% (6/48) of groups attaining the high effort when $c = 0.5$ in rounds 19–21 of the HLH treatment. So, when the group started at a low cost and increased to $c = 0.5$, it attained the high effort 85% of the time as compared to only 13% of the time when the group started at a high cost and decreased to $c = 0.5$. This difference is significant using a one-tailed Wilcoxon Rank-Sum Test with normal approximation ($p = 8.5 \times 10^{-5}$).[13] This provides strong support for Hypothesis 1, and suggests that there is hysteresis in the minimum-effort coordination game.

4.2.2. Hypothesis 2

While Hypothesis 1 predicted that different groups will have different outcomes at $c = 0.5$ depending on the starting point, Hypothesis 2 and Hypothesis 3 went even further by predicting that the same group has different outcomes at $c = 0.5$ depending on whether the cost is increasing or decreasing. Hypothesis 2 predicted that in LHL, there would be significantly more high effort when the subjects play the minimum-effort coordination game at $c = 0.5$ when the cost is changing from low to high than when the cost is going from high to low. In LHL, the cost starts at $c = -0.05$ and gradually increases to $c = 0.5$ in rounds 16–18. Then, the cost increases to $c = 1.05$, before it starts to decrease again, and reaches $c = 0.5$ again in rounds 45–47. Table 1 shows details of the level of high effort at the individual and group levels in the LHL treatment. As Hypothesis 2 predicted, a one-tailed Wilcoxon Rank-Sum Test with normal approximation ($p = 0.045$) confirms that there is significantly more high effort at $c = 0.5$ when the cost is increasing (85%) than when it is decreasing (56%).

Fig. 6 shows the full progression of the level of high effort for both individuals and groups. One striking observation from Fig. 6(a) is the spike that occurs when the cost is $c = 0.99$. The game with $c = 0.99$ is the first game played, as the cost is decreasing, where playing the high-effort action is not weakly dominated. The game with $c = 0.99$ is also very risky. Subjects who played the low-effort action were guaranteed \$0.02. Subjects who played the high-effort action got \$0.06 if everyone else played high effort and −\$4.44 if one other person chose low effort. Despite this risk, 66% (42/64) of the subjects played high effort in the game. The $c = 0.99$ game seems to be decisive for determining the level of coordination on the high-effort action in the remaining games. If all members of a group played the high-effort action in this game, then they were able to maintain the high-effort outcome as a group as the cost decreased back to $c = -0.05$. If at least one of the members of a group played the low-effort action in this game, then the group had a more difficult time attaining the high effort. Assuming that 66% is the rate at which subjects play the high effort at $c = 0.99$, then the probability that a group achieves the high-effort equilibrium is 0.66^N. This decreases quickly as N increases. Therefore, since it would be more difficult to get group coordination in the $c = 0.99$ game for larger N, one could speculate that the hysteresis would be even stronger for large N.

4.2.3. Hypothesis 3

Hypothesis 3 predicted that in HLH, subjects would exert significantly less high effort the first time they play at $c = 0.5$ (as cost is decreasing) compared to the second time they play at $c = 0.5$ (as cost is increasing). Table 1 shows that 13% of the groups attained the high effort as the cost is decreasing, while 69% attained the high effort as the cost is increasing. The difference in the levels of high-effort groups is significant using a one-tailed Wilcoxon Rank-Sum Test with normal approximation ($p = 1.6 \times 10^{-3}$). This shows that a temporary decrease in the cost can have a large impact on equilibrium selection. This temporary decrease in the cost helped increase the percentage of high-effort groups from 13% to 69%. These results strongly support Hypothesis 3. Fig. 7 shows the full progression of both group and individual choices in the HLH treatment.

[13] To test the significance of the difference in efforts between the two sets of groups, a Wilcoxon rank-sum test is used. Each group plays at $c = 0.5$ for three periods, and receives a score of 0–3, depending on how many times attained the high effort in those three periods. Rank-sums are then determined for these sets, and a normal approximation is used.

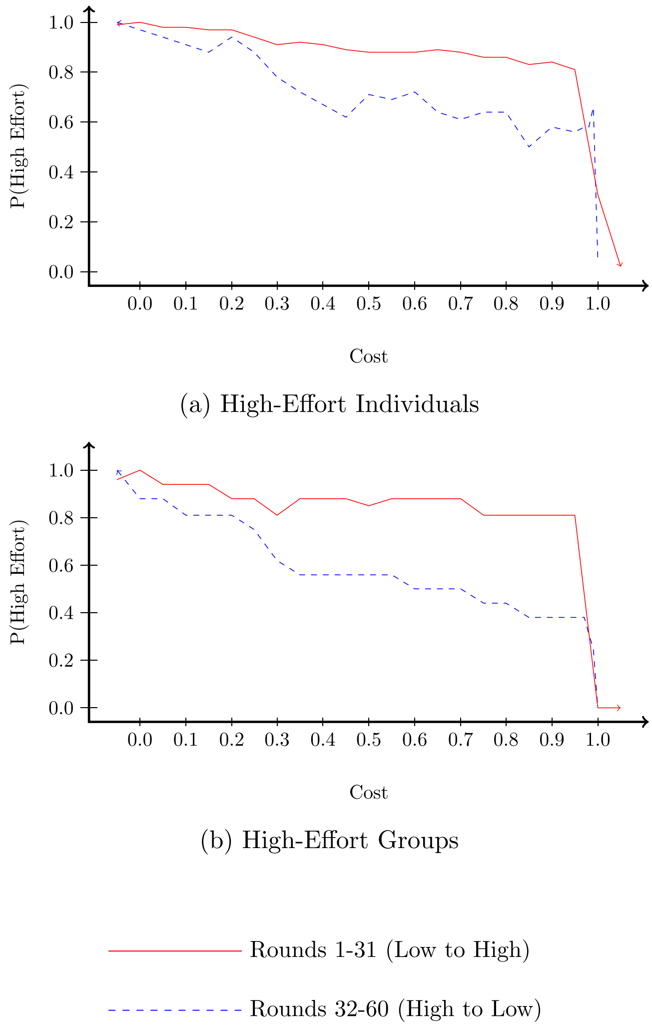


Fig. 6. Progression of high effort in LHL for individuals and groups.

5. Further discussion

Section 2 presented a relatively simple model of hysteresis. The benefit of the simplistic model is that it is clean and allows experimental predictions without getting caught up in technical details. This section elaborates on some of these technical details, including a quick examination of adding adaptive dynamics to the model.

5.1. Adaptive dynamics

This section considers two types of adaptive dynamics: best-response dynamics (Matsui, 1992) and smoothed best-response dynamics (Fudenberg and Levine, 1988). In best-response dynamics, a player best responds to the joint-strategy profile of the other players from the previous round.[14] The probability that all other players play H is $\prod_{j \neq i} \sigma_j$. For notational simplicity, let $\rho_i(\boldsymbol{\sigma}) = (\prod_{j \neq i} \sigma_j)^{N-1}$. For the game in Fig. 2 with N players, the best response is,

$$BR_i^{NE}(\boldsymbol{\sigma}) = \begin{cases} 1 & \rho_i(\boldsymbol{\sigma}) \geq c^{(1/(N+1))} \\ 0 & \rho_i(\boldsymbol{\sigma}) < c^{(1/(N+1))} \end{cases}.$$

[14] In this case it may be more appropriate to assume that players play the game for blocks of T rounds, and then best respond to the empirical distribution of play from the previous block. This would allow the players to converge on (possibly mixed) equilibrium before the parameter changes.

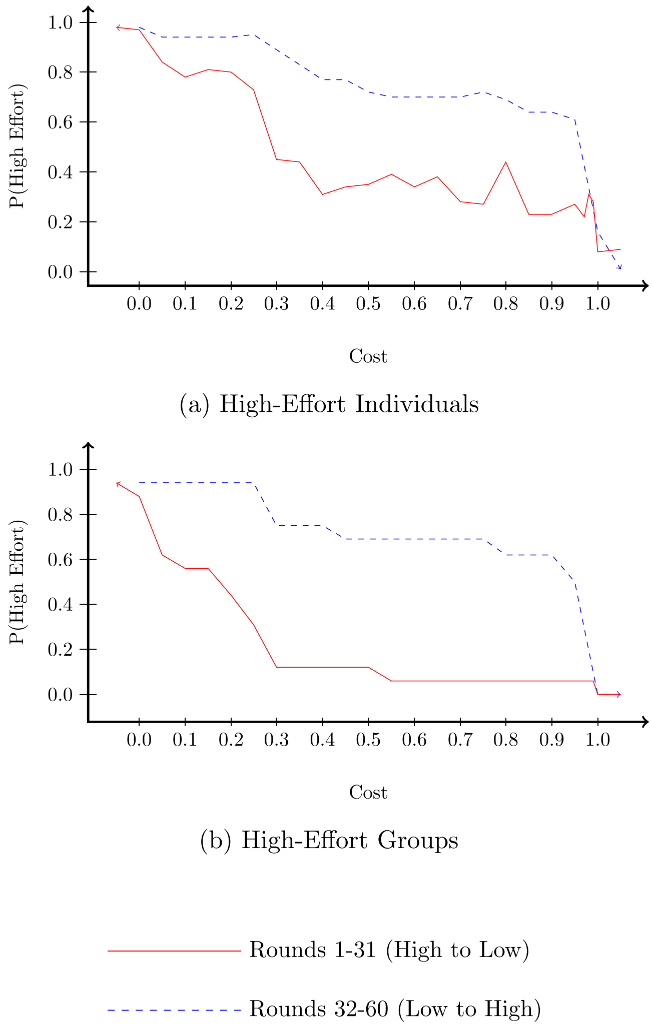


Fig. 7. Progression of high effort in HLH for individuals and groups.

The best-response dynamics are, therefore, specified as,

$$\dot{\boldsymbol{\sigma}} = BR^{NE}(\boldsymbol{\sigma}) - \boldsymbol{\sigma}$$

The second adaptive dynamic, smoothed best-response dynamics, uses a logistic function to determine the best response of player *i*. In this case, the best response is,

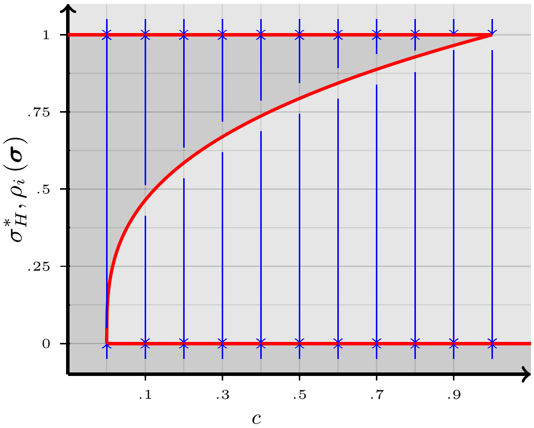
$$BR_i^L(\boldsymbol{\sigma}) = \frac{1}{1 + e^{\lambda(x_H - x_L)(c - \rho_i(\boldsymbol{\sigma})^{(1/(N-1))})}}.$$

The smoothed best-response dynamics are, therefore, specified as,

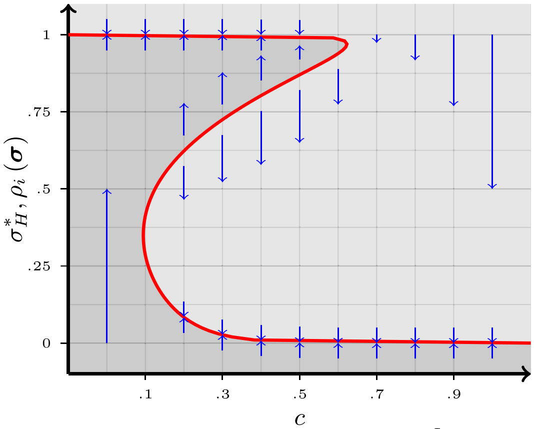
$$\dot{\boldsymbol{\sigma}} = BR^L(\boldsymbol{\sigma}) - \boldsymbol{\sigma}.$$

Fig. 8(a) shows the Nash equilibrium correspondence for $N = 4$ and how the system changes under best-response dynamics. Fig. 8(b) shows the QRE correspondence for $\lambda = 4$ and $N = 4$ and how the system changes if the players use smoothed best-response dynamics with $\lambda = 4$. The arrows in these figures show how the adaptive dynamics would change play. The arrows start at various values of $\rho_i(\boldsymbol{\sigma})$ and end at the corresponding best response σ_i^H. For example, Fig. 8(b) shows that if $c = 0.9$, $\lambda = 4$, and $\rho_i(\boldsymbol{\sigma}) = 1$, then the adaptive dynamic would lead to player *i* playing $\sigma_i^H = BR_i^L(\boldsymbol{\sigma}) \approx 0.77$.

The darker-shaded regions of the figures are where the adaptive dynamics would lead to an increase in the probability of playing *H*. That is, if $\rho_i(\boldsymbol{\sigma})$ falls in the darker-shaded region, then the resulting play σ_i^H must satisfy $\sigma_i^H > \rho_i(\boldsymbol{\sigma})$. Similarly, the lighter-shaded regions of the figures are where the adaptive dynamics lead to a decrease in the probability of playing *H*.



(a) NE Correspondence, BR_i^{NE} dynamics



(b) QRE Correspondence, $\lambda = 4$, BR_i^L dynamics

Fig. 8. Adaptive dynamics for $N = 4$.

This graph suggests that the middle branch of the equilibrium correspondence is unstable and that the two outside branches are stable.

Assumption #2, the main assumption from the model, states that if players play a certain equilibrium, and then the parameters of the game are changed slightly, players will move to the close equilibrium. A model of adaptive dynamics and changing parameters can provide additional support for this assumption. Fig. 9 shows the QRE equilibrium correspondence for $N = 4$ and $\lambda = 4$. Suppose that the players have been playing the game at $c = 0.1$ and have converged on the high-effort equilibrium. Assume that as c increases away from $c = 0.1$, players' beliefs about the other player's strategies fall into the shaded region. So, when the change in c is small, players' beliefs about others are quite similar to what they have been seeing. However, when c changes a large amount, the variance of beliefs becomes larger. Based on smoothed best-response dynamics, a small change in c would attract the players back to the similar equilibrium. For example, in the figure, if c increased to $c = 0.4$, then the adaptive dynamics would eventually settle on the similar equilibrium. However, if c increased to $c = 0.5$, then the variance in the beliefs is large enough that the adaptive dynamics could lead the system to the other equilibrium. A model of adaptive dynamics like this would suggest that Assumption #2 would hold only if the change is small.

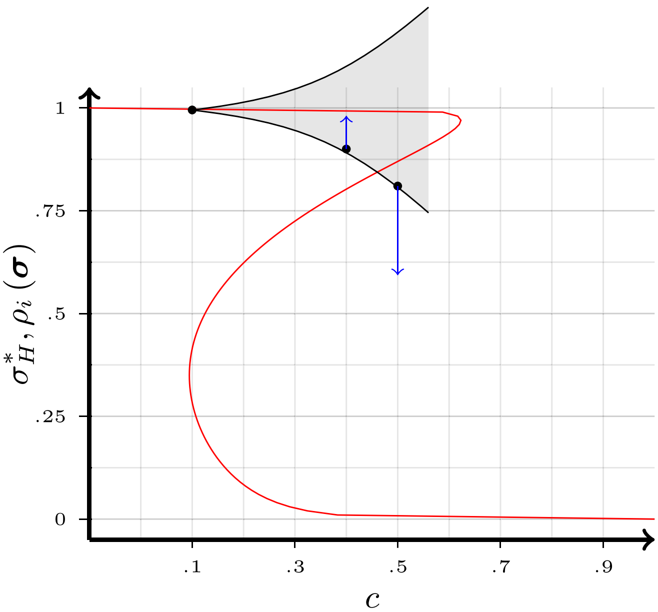
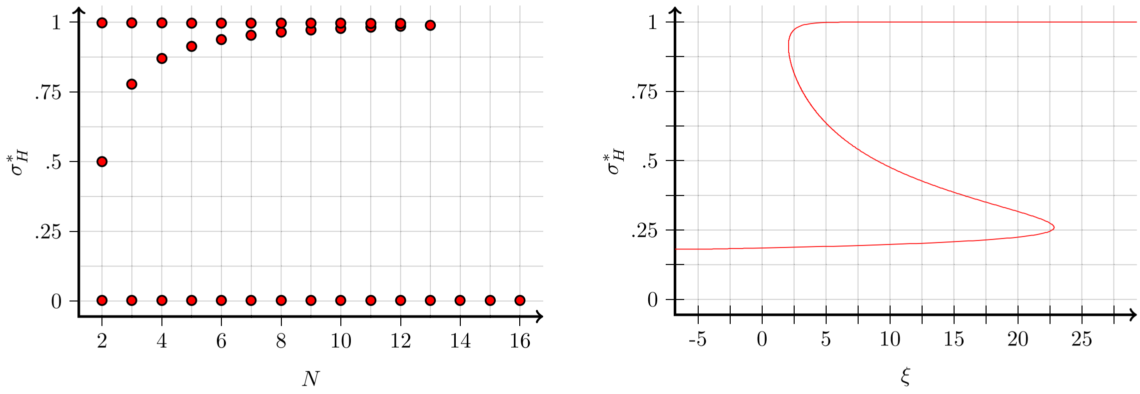


Fig. 9. Model of adaptive dynamics with changing parameters.



(a) Varying group size N for $\lambda = 4$. (b) Varying payoff parameter ξ for $\lambda = 1$.

Fig. 10. QRE correspondence as different parameters are varied for $c = 0.5$.

5.2. Varying other parameters

The results in this paper focus on the QRE equilibrium correspondence as the parameter c is varied. This section looks briefly at what happens as other parameters are varied. The goal of this section is to show that these s-shaped correspondences are not a knife-edge case, but actually occur for many different parameters.

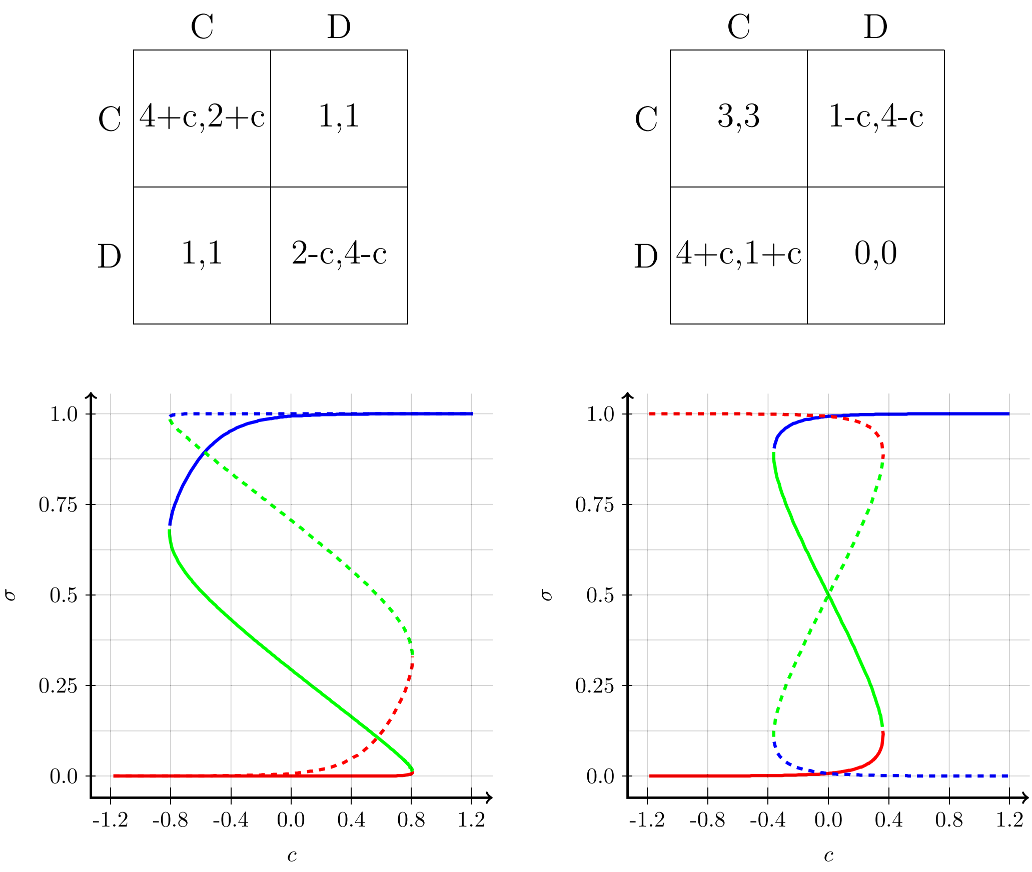
Fig. 10(a) shows the QRE correspondence for $\lambda = 4$ and $c = 0.5$ as N is varied. A model of adaptive dynamics similar to the one discussed in the previous section would predict that if a group of individuals starts at $N = 12$, they would converge to the low-effort equilibrium. However, if the group starts at $N = 2$, and then the group size N is gradually increased, the group can attain the high-effort equilibrium at $N = 12$. This is similar to the result found in Weber (2006). This result helps to show the robustness of the current model to different situations.

Consider a slight change to the minimum-effort coordination game presented in Fig. 2 where, now, if all players play x_H, the payoff is $x_H(1 - c) + \xi$ and all of the other payoffs are the same. Fig. 10(b) shows the QRE correspondence for $c = 0.5$, $\lambda = 1$, and $N = 4$ as payoff parameter ξ varies. The correspondence exhibits the s-shaped form, and, therefore, as long as Assumption #2 is satisfied, the correspondence exhibits hysteresis. These examples show that hysteresis does not just occur when the cost is varied, but also in other cases. In these other cases however, the same hypotheses hold, which suggest that a temporary change in the parameter can lead to a significant change in behavior.

5.3. Other games

In addition to the robustness to different parameters seen in the last section, similar properties can be seen in other commonly studied coordination games. Fig. 11 displays a parameterized Battle of the Sexes game as well as a parameterized Chicken game. In these games, the QRE correspondence is not symmetric, so for any value of c the players may play C with different probabilities. The Battle of the Sexes game, displayed in panel (a), has two pure strategy Nash equilibrium ((C, C) and (D, D)) for values of $c \in (-1, 1)$. The QRE correspondence is displayed below the normal form game table. The solid line represents the QRE correspondence for player 1 and the dashed line represent the QRE correspondence for player 2. When $c \leq -.8$, then the unique QRE is for both players to play D. Similarly, when $c \geq .8$, the unique QRE is for both players to play C. When $c \in (-.8, .8)$, there are three quantal-response equilibria. For example, when $c = 0$, one QRE has both players playing C with high probability (the blue portion of the solid and dashed lines), another QRE has both players playing C with low probability (the red portion of the solid and dashed lines), and a third QRE where both players mix by playing each action with non-negligible probabilities (the green portion of the solid and dashed lines). If players initially face a low value of c, then they both play C with low probability since that is the unique QRE. As c is increased, if assumption 2 holds, then they will remain playing that equilibrium even when moving into the range with multiple equilibria ($c \in (-.8, .8)$). Conversely, if the players initially face a high value of c, then they both play C with high probability since that is the unique QRE. As c is decreased, again assuming assumption 2 holds, then they will remain playing that equilibrium even when moving into the region with multiple equilibria.

Panel (b) of Fig. 11 shows a parameterized version of the commonly studied Chicken game. This game has two pure-strategy equilibria as long as $c \in (-1, 1)$. The QRE correspondences are shown below the normal form game table. In this case, we have a unique equilibrium as long as $c \leq -.4$ in which player 1 plays C with high probability and player 2 plays D with high probability. For values of $c \in (-.4, .4)$, there are multiple QREs. Finally, when $c \geq .4$, there is a unique QRE where player 1 plays D with high probability and player 2 plays C with high probability. Again, if assumption 2 holds, then the equilibrium played at $c = 0$ depends on whether the players started with a high or low value of c.



(a) Battle of the Sexes (b) Chicken

Fig. 11. Examples of hysteresis in other coordination games.

6. Conclusion

This paper started with the idea that hysteresis, a property that is common in a wide range of physical settings, is also present in an economic system. Proposition 3.1 shows that the minimum-effort coordination game exhibits hysteresis as long as several assumptions are satisfied. Based on this theoretical result, some experimental hypotheses were developed and then tested in the lab using human subjects. The experimental results, presented in Section 4, provide support for the theoretical results, suggesting that hysteresis occurs in the minimum-effort coordination game.

Probably the most important implication of this hysteresis is on equilibrium selection. The experimental results showed that in a minimum-effort coordination game with cost $c = 0.5$, 13% of groups played high effort when the cost was decreasing. However, when the *exact same groups* returned to $c = 0.5$ after a temporary decrease in cost, 69% of groups played the high effort. This suggests that a group of people can behave one way in a certain game, but then behave completely differently in the same game after a parameter has been temporarily changed. With regard to equilibrium selection, this suggests that we may be able to move from one equilibrium to a better equilibrium just by temporarily changing certain parameters.

Another interesting implication of the s-shaped curves that was not studied in this paper is the fact that they can lead to oscillations or cycles in the system. In the experiments presented here, the cost was changed exogenously. However, in many situations, the parameters may change endogenously. For example, if coordination on the high effort pushes the cost upward, while the groups that exert low effort have decreasing costs, then this would lead to a cycle. The effect of this cycling on equilibrium selection provides some interesting questions for future work.

Given these implications, it is important to determine how likely this hysteresis is in a framework more general than the minimum-effort coordination game. Section 5.2 briefly examined situations in which this s-shaped curve shows up when parameters other than c are varied. It is hypothesized that the s-shaped curve is not a knife-edge case, but is likely to occur in a large number of settings. A better understanding of when this s-shaped curve occurs may help clarify the equilibrium selection process in more-general settings. Further examination of the genericity of this hysteresis is saved for future work.

Finally, the model presented in this paper is quite simple, and does not factor in a lot of things that are likely going on as the parameters change in these games. However, even this simple model is able to provide clean predictions about what we see in the experimental lab, so for this paper it is sufficient.[15] Another interesting avenue for future work would be to try to develop a more-sophisticated model of learning in close games to see if that provides any additional insights into the phenomena studied in this paper.

Appendix A. Proofs

Proof of Proposition 3.1. First, calculate the quantal-response equilibrium of the game. Suppose that all players play x_H with probability σ_H and x_L with probability σ_L. So, the probabilities are

$$P(\text{All others play H}) = \sigma_H^{N-1}, \text{ and}$$

$$P(\text{At least one other L}) = 1 - \sigma_H^{N-1}. \tag{4}$$

The payoffs are as follows:

$$u_i(x_L, \sigma_{-i}) = (1-c)x_L$$
$$u_i(x_H, \sigma_{-i}) = x_L(1-\sigma_H^{N-1}) + x_H\sigma_H^{N-1} - cx_H = x_L - cx_H + \sigma_H^{N-1}(x_H - x_L).$$

Therefore, the logit quantal response equilibrium must satisfy the following equations:

$$\sigma_H = \frac{e^{\lambda u_i(x_H,\sigma_{-i})}}{e^{\lambda u_i(x_L,\sigma_{-i})} + e^{\lambda u_i(x_H,\sigma_{-i})}} = \frac{1}{1 + e^{\lambda[u_i(x_L,\sigma_{-i}) - u_i(x_H,\sigma_{-i})]}} = \frac{1}{1 + e^{\lambda(x_H - x_L)(c - \sigma_H^{N-1})}}. \tag{5}$$

In order to show that hysteresis is possible, it is necessary to show that the correspondence $\boldsymbol{\Sigma}^*(c, \lambda)$ is an s-shaped curve. To do this, we find $c^*(\sigma_H)$, which is a function. Next, we show that $\lim_{\sigma_H \to 1} c^*(\sigma_H) \to -\infty$, $\lim_{\sigma_H \to 0} c^*(\sigma_H) \to \infty$, and $(\partial c^*(\sigma_H))/(\partial \sigma_H) > 0$ for some $\sigma_H \in (0, 1)$. If these conditions hold, then the correspondence $\boldsymbol{\Sigma}^*(c, \lambda)$ is an s-shaped curve. Rearrange Eq. (5) to get

$$c^*(\sigma_H) = \sigma_H^{N-1} + \frac{\ln((1-\sigma_H)/\sigma_H)}{\lambda(x_H - x_L)}, \tag{6}$$

which has a unique value of c^* for each value of σ_H. From this, notice that,

$$\lim_{\sigma_H \to 0} c^*(\sigma_H) \to \frac{\ln \infty}{\lambda(x_H - x_L)} = \infty,$$

and

$$\lim_{\sigma_H \to 1} c^*(\sigma_H) \to 1 + \frac{\ln 0}{\lambda(x_H - x_L)} = -\infty. \tag{7}$$

Finally,

$$\frac{\partial c^*}{\partial \sigma_H}(\sigma_H) = (N-1)\sigma_H^{N-2} - \frac{1}{\lambda(x_H - x_L)}\left(\frac{1}{\sigma_H(1-\sigma_H)}\right). \tag{8}$$

Therefore,

$$\frac{\partial c^*}{\partial \sigma_H}(\sigma_H) > 0 \Leftrightarrow \lambda > \frac{1}{(N-1)\sigma_H^{N-1}(1-\sigma_H)(x_H - x_L)}. \tag{9}$$

In order to get the s-shaped curve, this needs to hold for some $\sigma_H \in (0, 1)$. Since the right side of the above equation is minimized when $\sigma_H = ((N-1)/N)$, it is clear that,

$$\frac{1}{(N-1)\sigma_H^{N-1}(1-\sigma_H)(x_H - x_L)} \geq \frac{1}{(N-1)((N-1)/N)^{N-1}(1/N)(x_H - x_L)} = \frac{1}{((N-1)/N)^N(x_H - x_L)} = ((N-1)/N)^N \frac{1}{x_H - x_L}.$$

Therefore, if

$$\lambda^* = \left(\frac{N}{N-1}\right)^N \frac{1}{x_H - x_L},$$

then for all $\lambda \geq \lambda^*$, the correspondence $\boldsymbol{\Sigma}^*(c, \lambda)(c)$ has the desired s-shaped form. Also, note that,

$$\left(\frac{N}{N-1}\right)^N \geq \left(\frac{N+1}{N}\right)^{N+1} \text{ for all } N \geq 2.$$

[15] Section 5.1 presented a brief explanation of some more-sophisticated models of adaptive dynamics.

This holds by the Bernoulli Inequality.[16] Therefore, λ^* is decreasing in N. This means that as the group size increases, holding everything else constant, the s-shaped curve is more likely.

Finally for any fixed value of λ, the critical values of the s-shaped curve are at the two points where,

$$\frac{\partial c}{\partial \sigma_H}(\sigma_H) = 0. \tag{10}$$

Then, set (8) to zero, and rearrange to get,

$$(1-\sigma_H)\sigma_H^{N-1} - \frac{1}{\lambda(x_H - x_L)(N-1)} = 0. \tag{11}$$

An explicit solution for this equation in not tractable unless $N = 2$. In the $N = 2$ case, solving this gives the solution

$$\sigma_H = \frac{\lambda(x_H - x_L) \pm \sqrt{\lambda^2(x_H - x_L)^2 - 4\lambda(x_H - x_L)}}{2\lambda(x_H - x_L)} = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \frac{4}{\lambda(x_H - x_L)}}.$$

Also see that if $\lambda \geq \lambda^* = (4/(x_H - x_L))$, then

$$1 \geq \frac{4}{\lambda(x_H - x_L)}.$$

So, if $\lambda \geq \lambda^*$, then the two roots are always real, and if $\lambda < \lambda^*$, then there are no real roots, which is what we would expect. These two critical values are symmetric around $\sigma_H = (1/2)$ for the $N = 2$ case. However, for the $N > 2$ case, we would not expect to see this symmetry due to the form of (11).

Appendix B. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jebo.2014.12.029.

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[16] The Bernoulli Inequality says that for $1 \geq \alpha > 0$ and $\delta \geq -1$,

$$(1+\delta)^\alpha \leq 1 + \alpha\delta.$$

Set $\alpha = (N/(N+1))$ and $\delta = -(1/N)$, then the inequality shows that,

$$\left(1 - \frac{1}{N}\right)^{(N/(N+1))} \leq 1 - \frac{N}{N+1} \times \frac{1}{N} = \frac{N}{N+1}.$$

Taking the reciprocal,

$$\left(\frac{N}{N-1}\right)^{(N/(N+1))} \geq \frac{N+1}{N}.$$

Or equivalently,

$$\left(\frac{N}{N-1}\right)^{N} \geq \left(\frac{N+1}{N}\right)^{N+1}.$$

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