CHOI, Ling (lyc15)

Imperial College London

Department of Computing

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- I declare that this final submitted version is my unaided work.

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Coursework 1

1. (i) p = Michel is either fulfilled or rich
q = Michel will live another five years.

$((\neg p) \rightarrow (\neg q))$.

(ii) p = snowstorm arrives
q = Raheem wears his boots.
r = I'm sure it will arrive.

$(((\neg p) \vee q) \wedge r)$

(iii) p = Akira is on set
q = Toshiro is on set
r = filming will begin
s = caterers have cleared out

$((p \wedge q) \rightarrow (r \leftrightarrow s))$

(iv) p = Irad arrived
q = Sarah arrived

$((p \vee (\neg q)) \wedge (\neg (p \wedge q)))$

(v) p = Herbert heard the performance.
q = Anne-Sophie heard the performance
r = Anne-Sophie answers her phone calls

$((\neg r) \rightarrow (\neg (p \wedge q)))$

2. (i) A propositional formula A is satisfiable if there is some v such that $h_v(A) = t$

(ii) Two propositional formulas A, B are logically equivalent if, for every v, $h_v(A) = h_v(B)$.

(iii) Suppose $\neg A$ is satisfiable. Then there is some v such that $h_v(\neg A) = t$. From lectures, we then know that $h_v(\neg\neg A) = f$. Since $h_v(T) = t$, we have shown that $\neg\neg A \not\equiv T$.

Suppose $\neg\neg A \not\equiv T$. Then for some v, $h_v(\neg\neg A) \not\equiv h_v(T)$. Since $h_v(T) = t$, then $h_v(\neg\neg A)$ must equal f. From lectures, $h_v(\neg A) = t$ iff $h_v(A) = f$. So since $h_v(\neg\neg A) = f$, then $h_v(\neg A) = t$. Hence there is a v such that $h_v(\neg A) = t$. So $\neg A$ is satisfiable.

3.

p	q	r	$(p$	$\wedge$	$\neg$	q	$\leftrightarrow$	$\neg$	$(\neg$	r	$\vee$	$\neg$	$p))$	$\rightarrow$	$(\neg$	$\neg$	q	$\rightarrow$	$r)$
t	t	t	t	f	f	t	t	f	f	t	t	f	t	t	t	f	t	t	t
t	t	f	t	f	f	t	t	f	t	f	t	f	t	f	t	f	t	f	f
t	f	t	t	t	t	f	t	t	f	t	f	f	t	t	f	t	f	t	t
t	f	f	t	t	t	f	f	f	t	f	t	f	t	t	f	t	f	t	f
f	t	t	f	f	f	t	t	f	f	t	t	t	f	t	t	f	t	t	t
f	t	f	f	f	f	t	t	f	t	f	t	t	f	f	t	f	t	f	f
f	f	t	f	f	t	f	t	f	f	t	t	t	f	t	f	t	f	t	t
f	f	f	f	f	t	f	t	f	t	f	t	t	f	t	f	t	f	t	f

These two columns determine the final column

Since this isn't true for all v, this is not valid.

4. (i) (a) CNF (b) Both (c) Neither (d) Neither (e) DNF (f) Neither (g) Both (h) Both.

(ii) Let S be in CNF. $S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$.
This is an important property, as this allows us to determine whether a formula or a set of formula is satisfiable or not, and to conclude whether or not an argument is valid.

(iii) (a) $\{\{p,s\},\{q,r\},\{\neg s,q\},\{\neg p,\neg r,\neg s\}\}$.

$\Rightarrow \{\{p,s\},\{\neg p,\neg r,\neg s\}\}$ [q was pure].

$\Rightarrow \{\{p,s\}\}$ [$\neg r$ was pure]

$\Rightarrow \{\ \}$ [p was pure].

$\Rightarrow$ Satisfiable (since no conditions for further application of rules apply)

(b) $\{\{\neg p,q,r\},\{\neg q\},\{p,r,q\},\{\neg r,q\}\}$

$\Rightarrow \{\{\neg p,r\},\{p,r\},\{\neg r\}\}$ [Unit propagation by unit clause $\{\neg q\}$]

$\Rightarrow \{\{\neg p\},\{p\}\}$ [Unit propagation by unit clause $\{\neg r\}$]

$\Rightarrow \{\{\}\}$ [Unit propagation by unit clause $\{\neg p\}$].

$\Rightarrow$ Unsatisfiable (since $\emptyset$ is in the set)

5. p = I'm going. q = you're going r = Tara's going.

$$((p \to \neg q) \land (\neg q \to \neg r) \land (r \lor \neg p) \land (r \lor p)) \models q \quad (*)$$

We must check $(*)$. We know from lectures, $A_1, \ldots, A_n \models B$ iff $A_1 \land \ldots \land A_n \land \neg B$ is unsatisfiable. So we check

$$((p \to \neg q) \land (\neg q \to \neg r) \land (r \lor \neg p) \land (r \lor p)) \land \neg q$$

So we convert into clausal form CNF and apply unit propagation repeatedly.

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{\neg q\}\}$.

$\Rightarrow \{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$ [unit propagation by unit clause $\{\neg q\}$]

$\Rightarrow \{\{\neg p\}, \{p\}\}$ [Unit propagation by unit clause $\{\neg r\}$]

$\Rightarrow \{\{\}\}$ [Unit propagation by unit clause $\{\neg p\}$]

$\Rightarrow$ Unsatisfiable [since $\emptyset$ is in the set]

Hence the original argument is propositionally valid.

6. (i) $C = \{andrea\}$ $\mathcal{P}_1 = \{cupcake\}$ $\mathcal{P}_2 = \{aunt\}$ $\mathcal{P}_3 = \{give\}$

$$\forall X \forall Y \exists V \exists Z \,(aunt(X, andrea) \wedge aunt(Y, X) \wedge give(Y, V, Z) \wedge cupcake(Z) \wedge \neg(V = andrea))$$

cupcake(X) X is a cupcake.
aunt(X, Y) X is an aunt of Y.
give(X, Y, Z) X gave Y, Z.

(ii) $\mathcal{P}_1 = \{computer\}$ $\mathcal{P}_2 = \{connect\}$.

$$\exists X \forall Y \,(computer(X) \wedge computer(Y) \wedge connect(X, Y) \wedge \neg (X = Y))$$

computer(X) X is a computer
connect(X, Y) X is connected to Y.

(iii) $C = \{paulklee, kandinsky\}$
$\mathcal{P}_1 = \{painting, britishgallery, room\}$
$\mathcal{P}_2 = \{painted\}$

$$\forall X (\exists Y\ painting(X) \wedge painted(X, kandinsky) \wedge britishgallery(X) \wedge room(Y) \rightarrow \forall Z\ painting(Z) \wedge painted(Z, paulklee) \wedge britishgallery(Z) \wedge room(Y))$$

painting (X) X is a painting
painted (X, Y) X is painted by Y
room (X) X hangs in a room
britishgallery (X) X is in a British gallery.

(iv) $\mathcal{P}_2 = \{loves\}$

$$(\exists X \forall Y\ \neg loves(X, Y)) \rightarrow \neg(\forall X \exists Y\ loves(X, Y))$$

loves (X, Y) X loves Y.

7. Let M be the $\mathcal{L}$-structure (D, φ).

(i) False, the only object connected from $\varphi(k)$ is $\varphi(j)$, so the statement must be false.

(ii) True. If we assume $c(l)$, then we see that $\varphi(X) = j$, and $\varphi(X) = K$ are the only objects that satisfy $a(l, X)$. These two objects $(\varphi(j), \varphi(K))$ are also in $\varphi(c)$ and $\varphi(b)$.
(iii) False, this statement says if there is an X then there's no Y that is different from X and connects from X. If we consider $\varphi(X) = j$, then for $\varphi(Y) = l$, the condition $(\neg(X = Y) \wedge a(X, Y))$ holds as they are diffent objects, and there's a directed arrow from $\varphi(j)$ to $\varphi(l)$

(iv) False, if we consider the object $\varphi(j)$, then we see that it is not in $\varphi(S)$, as it is circular. Then for the object $\varphi(l)$, there is a directed arrow from $\varphi(j)$ to $\varphi(l)$, and $\varphi(l)$ is both filled and circular. $\varphi(l)$ is the only object that connects from $\varphi(j)$.

(v) False, this is not true if we consider $\varphi(X)=K$. Then the antecedent is true if we consider the object $\varphi(j)$, since $\varphi(j)$ and $\varphi(K)$ are different objects and there is a directed arrow from $\varphi(K)$ to $\varphi(j)$. However, for $\varphi(K)$, there is no connected object with directed arrows to $\varphi(K)$.*

(vi) False, if we take $\varphi(X)$ and $\varphi(Y)$ to be the same object, then if it is connected to $\varphi(j)$, [only $\varphi(K)$ and $\varphi(l)$ are connected to $\varphi(j)$], then there is not directed arrow to itself for objects. $\varphi(K)$, $\varphi(l)$.

* There is a directed arrow from K to j, but not the other way around, and the same is true for K and l as well.