BEER, Anna (aab120)

Imperial College London

Department of Computing

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Exercise Information

Module:	70051 Introduction to Symbolic Artificial Intelligence (MSc AI)	**Issued:**	Tue - 20 Oct 2020
Exercise:	2 (CW)	**Due:**	Tue - 03 Nov 2020
Title:	Logic	**Assessment:**	Individual
FAO:	Craven, Robert (rac101)	**Submission:**	Electronic

Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-10-29 15:55:20

For Markers only: (circle appropriate grade)

BEER, Anna (aab120)	01832033	t5	2020-10-29 15:55:20	**A* A B C D E F**

INTRODUCTION TO SYMBOLIC AI - COURSE WORK I

1 i) If Michael isn't either fullfilled or rich, he won't live another five years.

p – Michael is fullfilled
q – Michael is rich
r – Michael will live another 5 years

$((\neg p \vee \neg q) \rightarrow \neg r)$

ii) Unless the snowstorm doesn't arrive, Raheem will wear his boots; but I'm sure it will arrive

p – snowstorm does arrive
q – Raheem will wear his boots
r – I'm sure it will arrive

$(((\neg p) \vee q) \wedge r)$

iii) If Aikira and Toshiro are on set, then filming will begin If and only if the caterers have cleared out

p – Aikira is on set
q – Toshiro is on set
r – filming will begin
s – caterers have cleared out

$(((p \wedge q) \rightarrow r) \leftrightarrow s)$

iv) Either Irad arrived or Sarah didn't: but not both!

p – Irad arrived
q – Sarah arrived

$((p \vee \neg q) \wedge (\neg p \vee q))$

v) It is not the case both that Herbert heard the performance and Anne-Sophie did, if the latter didn't answer her phone calls

p – Herbert heard the performance
q – Anne-Sophie heard the performance
r – Anne-Sophie did answer her phone call

$(\neg r \rightarrow \neg (p \wedge q))$

2 i) A propositional formula A is satisfiable if there is some v such that $h_v(A) = t$.

ii) Two propositional formulas A, B are logically equivalent if for every v $h_v(A) = h_v(B)$.

iii) Assume $h_v(\neg A) = t$ then $h_v(\neg\neg A) = f$

If $h_v(\neg\neg A) = t$ then $h_v(\neg A) = f$

Thus if $h_v(\neg A) = t$ which means that it is satisfiable then it has to hold that $h_v(\neg\neg A) = f$. If $h_v(\neg\neg A) = t$ then the propositional formula $\neg A$ would not be satisfiable as $h_v(\neg A) = f$ for all v

3)

p	q	r	$((p$	$\land$	$\neg q)$	$\Leftrightarrow$	$\neg$	$(\neg r$	$\lor$	$\neg p))$	$\rightarrow$	$(\neg$	$\neg q$	$\rightarrow$	$r)$
t	t	t	t	f	f	f	t	f	f	f	t	t	f	t	t
t	f	t	t	t	t	t	t	f	f	f	t	f	t	t	t
f	t	t	f	f	f	t	f	f	t	t	t	t	f	t	t
f	f	t	f	f	t	t	f	f	t	t	t	f	t	t	t
t	t	f	t	f	f	t	f	t	t	f	f	t	f	f	f
t	f	f	t	t	t	f	f	t	t	f	t	f	t	t	f
f	t	f	f	f	f	t	f	t	t	t	f	t	f	f	f
f	f	f	f	f	t	t	f	t	t	t	t	f	t	t	f

Not valid if $h_v(p) = t$, $h_v(q) = t$ $h_v(r) = f$

4 i) a) $p \land (\neg q \lor r)$ is in CNF

b) $\neg p$ is in CNF and DNF

c) $p \land (q \lor (p \land r))$ neither because it's a nested and and or

d) $\top$ is in CNF and DNF

e) $(p \land q) \lor (p \land q)$ is in DNF

f) $\neg\neg p \land (q \lor p)$ neither because $\neg\neg p$ is not a literal

g) $p \land q$ is in DNF and CNF

h) $p \lor q$ is in CNF and DNF

ii) Refutation - soundness and completeness: let S be a propositional formula in CNF then you can derive the empty clause if and only if S symantically entails false. ($S \vdash_{res(PL)} \emptyset$ iff $S \models \bot$)

This is important because if we apply resolution derivation then it is impossible to derive $\emptyset$ from S if S is satisfiable. ($S \nvdash_{res(PL)} \emptyset$

iii) a) $\{\{p,s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$

↳ $\{\{p,s\}, \{\neg p, \neg r, \neg s\}\}$ since q was pure

↳ $\{\{p,s\}\}$ since $\neg r$ was pure

↳ $\{\ \}$ since p was pure => satisfiable

b) $\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$

↳ $\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$ unit propagation on $\{\neg q\}$

↳ $\{\{\neg p\}, \{p\}\}$ unit propagation on $\{\neg r\}$

↳ $\{\{\ \}\}$ unit propagation of $\{p\}$ -> $\emptyset$ thus not satifiable

5) If I am going, then you are not.
If you are not going, then neither is Tara
Either Tara is going or I'm not.
Tara is going unless I am
So, you are going

$(p \to \neg q) \equiv (\neg p \lor \neg q)$
$(\neg q \to \neg r) \equiv (q \lor \neg r)$
$(r \lor \neg p)$
$(\neg p \to r) \equiv (p \lor r)$

p - I am going
q - you are going
r - Tara is going

Check satisfiable if $\neg q$ is true

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{r, p\}, \{q\}\}$

↳ $\{\{\neg r\}, \{r, \neg p\}, \{r, p\}\}$ unit propegation on $\{\neg q\}$

↳ $\{\{\neg p\}, \{p\}\}$ unit propagation $\{\neg r\}$

↳ $\{\{\}\}$ unit propegation on $\{\neg p\}$

↳ unstasifiable so the original is valid as $(\bigwedge_i A_i) \to B$ is valid iff $(\bigwedge A_i) \land \neg B$ is unsatifiable

6 i) All of Andreas aunts' aunts gave a cupcake to someone other than Andrea

$C = \{andrea\}$
$P_1 = \{aunt\}$
$P_2 = \{cupcake\}$
$P_3 = \{give\}$

$$\forall X((X = aunt(aunt(andrea))) \rightarrow (\forall Y\ cupcake(Y) \wedge give(X,Y)$$
$$\wedge \neg (Y = andrea)))$$

ii) There's a computer connected to every computer which isn't connected to itself.

$P_1 = \{computer\}$
$P_2 = \{connects\}$

$$\forall X\ \exists Y\ ((\neg connects(Y,Y) \wedge computer(Y)) \rightarrow (connects(X,Y)$$
$$\wedge\ Computer(X)))$$

iii) Any painting by Paul Klee in a british gallery hangs in a room where all Kandinsky paintings in that gallery hang

$C = \{paul\ klee,\ kandinsky\}$
$P_1 = \{painting\}$
$P_2 = \{painter\}$
$P_3 = \{room\}$
$P_4 = \{british\_gallery\}$

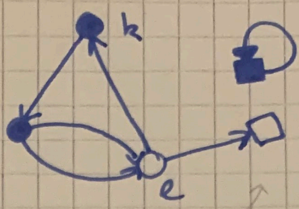
$$\forall Y \forall Z\ \forall X\ ((painter(paul\ klee) \wedge painting(X) \wedge british\_gallery(room(Y)))$$
$$\rightarrow (painter(kadinsky) \wedge painting(Z) \wedge british\_gallery(room(Y))))$$

iv) If there's somebody who loves nobody, then it's false that everybody loves somebody

$P = \{loves\}$

$$\exists Z, \forall W\ (\neg loves(Z,W) \rightarrow \neg(\exists Y\ loves(W,Y)))$$

7) $C = \{j, k, \ell\}$



$P_1 = \{b, w, s, c\}$

$P_2 = \{a\}$

$\varphi(j), \varphi(k), \varphi(\ell) \Rightarrow$ shown above

$\varphi(b)$ set of filled ('black') objects
$\varphi(w)$ set of unfilled ('white') objects
$\varphi(s)$ set of square objects
$\varphi(c)$ set of circular objects
$\varphi(a)$ set of pairs (x, y) s.t. there is a directed arrow from x to y

i) $\forall X(a(k,X) \to \neg(X=j)) \Rightarrow$ false because there is an arrow going k to j

ii) $c(\ell) \to \exists X(b(X) \wedge c(X) \wedge a(\ell,X)) \Rightarrow$ true because for $X=k$; ℓ is a circular object and when $X=k$ it holds that an arrow exists and that it is 'black' and circular

iii) $\exists X \neg \exists Y(\neg(X=Y) \wedge a(X,Y)) \Rightarrow$ true for ■ because if there exists an X for which there doesn't exist a Y where it doesn't hold that $X=Y$ and X and Y are connected. Then this actually means that $X=Y$ with a connecting arrow holds which is the case for ■

iv) $\forall X(\neg s(X) \to \exists Y(c(Y) \wedge b(Y) \wedge a(X,Y))) \Rightarrow$ false lets take $X=j$ then it holds that $s(j)$ is not a square object and lets take $Y=k$ then it holds that it circular and black but it doesn't hold that there is an arrow connecting from j to k

v) $\forall X(\exists Y(\neg(X=Y) \wedge a(X,Y)) \to \exists Y(a(X,Y) \wedge a(Y,X))) \Rightarrow$ false let's take $X=k$ then $Y=j$ because it is the only connecting arrow from k. Then it doesn't hold that there is a Y which is incoming and outgoing to k. because the incoming would be $\ell=Y$ and outgoing $Y=j$ and they are not the same

vi) $\forall X \forall Y(a(X,j) \wedge a(Y,j) \to (a(X,Y) \vee a(Y,X))) \Rightarrow$ false because X and Y have to be k for the arrow to exist. However, there doesn't exist an arrow going from k to k. Thus it is false