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# Department of Computing

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## Exercise Information

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| **Module:** | 70051 Introduction to Symbolic Artificial Intelligence (MSc AI) | **Issued:** | Tue - 20 Oct 2020 |
| **Exercise:** | 2 (CW) | **Due:** | Tue - 03 Nov 2020 |
| **Title:** | Logic | **Assessment:** | Individual |
| **FAO:** | Craven, Robert (rac101) | **Submission:** | Electronic |

## Student Declaration - Version 1

- I declare that this final submitted version is my unaided work.

Signed: (electronic signature) Date: 2020-11-01 13:34:23

**For Markers only:** (circle appropriate grade)

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# Introduction to Symbolic AI

## Coursework 1: Logic

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Q1

i. $((\neg(p \lor q)) \to (\neg r))$
p: Michel is fulfilled
q: Michel is rich
r: Michel lived another five years

ii. $((p \lor q) \land r)$
p: The snowstorm doesn't arrive
q: Raheem will wear his boots
r: I'm sure the snowstorm will arrive

iii. $((p \to q) \leftrightarrow r)$
p: Akira and Toshiro are on set
q: Filming will begin
r: The caterers have cleared out

iv. $((p \lor q) \land (\neg(p \land q)))$
p: Irad arrived
q: Sarah didn't arrive

v. $(r \to (\neg(p \land q)))$
p: Herbert heard the performance
q: Anne-Sophie heard the performance
r: Anne-Sophie didn't answer her phone calls

Q2

i. A propositional formula A is satisfiable if there is some valuation v such that, $h_v(A) = t$.

ii. Two propositional formulas A, B are logically equivalent if, for every valuation v, $h_v(A) = h_v(B)$.

iii. Consider the right-hand-side of the biconditional to be $\neg\neg A \equiv \top$.
For every v, $h_v(\neg\neg A) = h_v(\top) = t$, which implies $h_v(\neg A) = h_v(\bot) = f$.
Therefore $\neg A$ is not satisfiable in this case.
Hence, $\neg A$ is satisfiable iff $\neg\neg A \not\equiv \top$.

Q3

Since the second valuation of the truth table returned false, the formula is not valid.

| **p** | **q** | **r** | $(p \land \neg q \leftrightarrow \neg(\neg r \lor \neg p)) \to (\neg\neg q \to r)$ |
|---|---|---|---|
| t | t | t | t f f f t f f f t t t t |
| t | t | f | t f f t f t t f f t f f |
| t | f | t | |
| f | t | t | |
| t | f | f | |

| | | | |
|---|---|---|---|
| f | t | f | |
| f | f | t | |
| f | f | f | |

Q4

i. CNF: a b d
DNF: e

ii. Let S be in CNF. $S \vdash_{res(PL)} \emptyset \ iff \ S \vDash \bot$.

An immediate corollary is that,
Let S be in CNF. $S \text{ is satisfiable } iff \ S \nvdash_{res(PL)} \emptyset$, which means that if it is impossible to derive ∅ from S by a resolution derivation, then S is satisfiable. Together with the preservation of satisfiability, we are able to build all resolution-derivations from S to check the satisfiability of a formula.

iii. $\{\{p, s\}, \{q, r\}, \{\neg s, q\}, \{\neg p, \neg r, \neg s\}\}$
$\{\{p, s\}, \{\neg p, \neg r, \neg s\}\}$ [$q$ was pure]
$\{\{p, s\}\}$ [$\neg r$ was pure]
$\{\ \}$ [$p$ was pure]
Satisfiable

$\{\{\neg p, q, r\}, \{\neg q\}, \{p, r, q\}, \{\neg r, q\}\}$
$\{\{\neg p, r\}, \{p, r\}, \{\neg r\}\}$ [unit clause $\{\neg q\}$]
$\{\{\neg p\}, \{p\}\}$ [unit clause $\{\neg r\}$]
$\{\{\ \}\}$ [unit clause $\{\neg p\}$]
Unsatisfiable

Q5

Formalize the argument as: $(p \to \neg q), (\neg q \to \neg r), (r \vee \neg p), (p \vee r) \vDash q$
p: I’m going
q: You’re going
r: Tara is going

Since $(\bigwedge A_i) \vDash B \ iff \ (\bigwedge A_i) \wedge \neg B \text{ is unsatisfiable}$, the argument is valid if,

$(p \to \neg q) \wedge (\neg q \to \neg r) \wedge (r \vee \neg p) \wedge (p \vee r) \wedge (\neg q)$ is unsatisfiable.

Convert to clausal-form CNF

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{p, r\}, \{\neg q\}\}$

Apply DP

$\{\{\neg p, \neg q\}, \{q, \neg r\}, \{r, \neg p\}, \{p, r\}, \{\neg q\}\}$

$\{\{\neg r\}, \{r, \neg p\}, \{p, r\}\}$ [unit clause $\{\neg q\}$]

$\{\{\neg p\},\{p\}\}$ [unit clause $\{\neg r\}$]

$\{\{\ \}\}$ [unit clause $\{\neg p\}$]

Unsatisfiable since ∅ is in the set

Hence, the argument is valid.

Q6

i. $C = \{andrea\}$
   $P_2 = \{giveCupcake\}$
   $F_1 = \{aunt\}$

$\forall X((X = aunt(aunt(andrea))) \rightarrow \exists Y(giveCupcake(X,Y) \wedge \neg(Y = andrea)))$

ii. $P_1 = \{computer\}$
   $P_2 = \{connect\}$

$\exists X \forall Y(computer(X) \wedge computer(Y) \wedge connect(X,Y) \wedge \neg(connect(Y,Y)))$

iii. $P_1 = \{paulKlee, gallery, kandinsky, room\}$
   $P_2 = \{hang\}$

$$\forall X(paulKlee(X) \wedge gallery(X) \\ \rightarrow \exists Y \forall Z(room(Y) \wedge hang(X,Y) \wedge kandinsky(Z) \wedge gallery(Z) \wedge hang(Z,Y)))$$

iv. $P_2 = \{loves\}$

$\exists X \forall Y(\neg loves(X,Y) \rightarrow \neg(loves(Y,X)))$

Q7

Let $M = (D, \varphi)$

i. False
   For let $\sigma$ be such that $(\varphi(k), \sigma(X)) \in \varphi(a)$, then $\sigma(X) = \varphi(j)$ only, which is negated by $\neg(X = j)$

ii. True
   The antecedent states that "$l$ is circular" which is true.
   The consequent states that "there exists something which is black, circular and is pointed by the arrow from $l$". Both j and k fulfill these conditions.
   Truth of the consequent determines the whole statement is true.

iii. True
   The formula states that "something points to nothing, and that something is not nothing". The white square is such an object.

iv. False

The formula states that “every non-squared object, in this case j k $l$, points to something which is circular and black, in this case j k.” Since j neither points to k nor itself. The whole is false.

v. False

The formula states that “for everything that points to something other than itself, in this case j k $l$, each of them points to something, and that something points back.” This is not true since k points to j only, but j does not point back.

vi. True

The formula states that “for everything that points to j, in this case $l$ and k, one points to another (including themselves).” This is true since $l$ points to k.