Find the Inverse Laplace Transform of following:-

(1) $\dfrac{2s+4}{s^2+4}$

Soln

$$F(s) = \frac{2s+4}{s^2+4}$$

$$= 2\frac{s}{s^2+4} + 4\frac{1}{s^2+4}$$

$$= 2\frac{s}{s^2+2^2} + 4\frac{1}{s^2+2^2}$$

$$\therefore L^{-1}\{F(s)\} = 2\cos 2t + 4\cdot\frac{1}{2}\sin 2t$$

$$= 2[\cos 2t + \sin 2t]$$

(2) $F(s) = \dfrac{s^2-3s+4}{s^3}$

Soln

$$F(s) = \frac{1}{s} - \frac{3}{s^2} + \frac{4}{s^3}$$

$$L^{-1}\{F(s)\} = 1 - 3t + 4\cdot\frac{t^2}{2!}$$

$$= 1 - 3t + 2t^2$$

(3) $\left(\frac{1-\sqrt{s}}{s^2}\right)^2$

Sol^n $F(s) = \left(\frac{1-s^{1/2}}{s^2}\right)^2$

$= \frac{1}{s^4}(1-s^{1/2})^2$

$= \frac{1}{s^4}(1-2s^{1/2}+s)$

$= \frac{1}{s^4} - \frac{2}{s^{7/2}} + \frac{1}{s^3}$

$\cdot L^{-1}\{F(s)\} = \frac{t^3}{3!} - 2\frac{t^{5/2}}{\Gamma 7/2} + \frac{t^2}{2!}$

$= \frac{t^3}{6} - 2\frac{t^{5/2}}{5/2 \times 3/2 \times 1/2\sqrt{\pi}} + \frac{t^2}{2}$

$= \frac{t^3}{6} - \frac{16}{15\sqrt{\pi}}t^{5/2} + \frac{t^2}{2}$

(4) $\frac{2s+5}{s^2+4s+8}$

Sol^n $F(s) = \frac{2s+5}{s^2+4s+8}$

$= \frac{2s+5}{s^2+4s+2^2-2^2+8}$

$$F(s) = \frac{2s+5}{(s+2)^2+4}$$

$$= \frac{2(s+2-2)+5}{(s+2)^2+4}$$

$$= \frac{2(s+2)-4+5}{(s+2)^2+4}$$

$$= \frac{2(s+2)+1}{(s+2)^2+4}$$

$$\therefore L^{-1}\{F(s)\} = e^{-2t} L^{-1}\left\{\frac{2s+1}{s^2+4}\right\}$$

$$= e^{-2t} L^{-1}\left\{2\frac{s}{s^2+2^2} + \frac{1}{s^2+2^2}\right\}$$

$$= e^{-2t}\left[2\cos 2t + \frac{1}{2}\sin 2t\right]$$

5) $\dfrac{3s+7}{s^2-2s-3}$

Soln

$$F(s) = \frac{3s+7}{s^2-2s-3}$$

$$= \frac{3s+7}{s^2-2s+1-1-3}$$

$$F(s) = \frac{3s+7}{(s-1)^2 - 4}$$

$$= \frac{3(s-1+1)+7}{(s-1)^2 - 4}$$

$$= \frac{3(s-1)+10}{(s-1)^2 - 4}$$

$$L^{-1}\{F(s)\} = e^{t} L^{-1}\left\{ \frac{3s+10}{s^2-4} \right\}$$

$$= e^{t} L^{-1}\left\{ 3\frac{s}{s^2-2^2} + \frac{10}{s^2-2^2} \right\}$$

$$\cancel{= e^{t}\left[3 \cosh 2\right.}$$

$$= e^{t}\left[3\cosh 2t + 10\,\frac{1}{2}\sinh 2t\right]$$

$$= e^{t}\left[3\cosh 2t + 5\sinh 2t\right]$$