# APPLIED MATHEMATICS - III

## Mini Project Report

## Topic:

Applications of Laplace to solve initial and boundary value problems and engineering applications involving ordinary differential equations. (Problems from Engineering Subjects)

## Theory:

Laplace transforms when applied to any single or a system of linear ordinary differential equations, converts it into mere algebraic manipulations. In case of partial differential equations involving two independent variables, laplace transform is applied to one of the variables and the resulting differential equation in the second variable is then solved by the usual method of ordinary differential equations. Thereafter, inverse Laplace transform of the resulting equation gives the solution of the given p.d.e.

Finding Laplace Transform of derivative of a function g(t),

$$\mathcal{L}\{g'(t)\} = \int_0^\infty e^{-st} g'(t)dt = \left[e^{-st} g(t)\right]_{t=0}^{\infty} - \int_0^\infty (-s)e^{-st} g(t)dt = -g(0) + s\mathcal{L}\{g(t)\}.$$

Thus for any function f(t),

$$L\{f'(t)\} = sL(f(t)\} - f(0)$$

Similarly,

$$L\{f''(t)\} = s^2L\{f(t)\} - sf(0) - f'(0)$$

This process will work for higher order derivatives also. With a quick induction method, it can be shown that,

$$L\{f^{(n)}(t)\} = s^nL\{f(t)\} - s^{n-1}f(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

## Examples:

**Question 1.** Solve using Laplace Transforms:

$$3\frac{dy}{dt}\,2y = e^{3t} \text{ at y = 1 and t = 0}$$

**Solution:**

Taking Laplace on both sides,

$$3\,L'(y) + 2\,L(y) = L(e^{3t})$$

$$3\,[s\bar{y} - y(0)] + 2\bar{y} = \frac{1}{s-3}$$

$$(3s + 2)\,\bar{y} = \frac{1}{s-3} + 3 = \frac{3s-8}{s-3}$$

$$\bar{y} = \frac{3s-8}{(s-3)(3s+2)}$$

$$\overline{y} = \frac{30}{11} \cdot \frac{1}{3s+2} + \frac{1}{11} \cdot \frac{1}{s-3}$$

$$\overline{y} = \frac{10}{11} \cdot \frac{1}{s+0.6667} + \frac{1}{11} \cdot \frac{1}{s-3}$$

Taking inverse Laplace transforms, the solution is:

$$y = \frac{10}{11} L^{-1}\left[\frac{1}{s+0.6667}\right] + \frac{1}{11} L^{-1}\left[\frac{1}{s-3}\right] = \frac{10}{11} e^{(-2/3)t} + \frac{1}{11} e^{3t}$$

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**Question 2.** Solve the following:

$$D^2 - 3D + 2 = 4e^{2t}, \text{ with } y(0) = -3 \text{ and } y'(0) = 5$$

**Solution:**

Let $L(y) = \overline{y}$. Then, taking Laplace Transform,

$$L(y'') - 3L(y') + 2L(y) = 4L(e^{2t})$$

But $\quad L(y') = s\,\overline{y} - y(0) = s\overline{y} + 3$

and $\quad L(y'') = s^2\overline{y} - sy(0) - y'(0) = s^2\overline{y} + 3s - 5$

The Equation becomes

$$(s^2\overline{y} + 3s - 5) - 3(s\overline{y} + 3) + 2\overline{y} = 4\frac{1}{s-2}$$

$$(s^2 - 3s + 2)\overline{y} = \frac{4}{s-2} + 14 - 3s = \frac{-3s^2 + 20s - 24}{s-2}$$

$$\overline{y} = \frac{-3s^2 + 20s - 24}{(s-2)(s^2 - 3s + 2)} = \frac{-3s^2 + 20s - 24}{(s-1)(s-2)^2}$$

By partial fractions, $\overline{y} = \frac{7}{1-s} + \frac{4}{s-2} + \frac{4}{(s-2)^2}$

Taking Inverse Laplace transform,

$$\overline{y} = -7L^{-1}\left[\frac{1}{s-1}\right] + 4L^{-1}\frac{1}{s-2} + 4L^{-1}\frac{1}{(s-2)^2}$$

$$\overline{y} = -7e^{t}L^{-1}\frac{1}{s} + 4e^{2t}L^{-1}\frac{1}{s^2} = -7e^{t} + 4e^{2t} + 4te^{2t}$$

Hence, the solution is:

$$y = -7e^{t} + 4e^{2t} + 4t\,e^{2t}$$

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**Question 3.** Using Laplace Transform solve:

$$(D^2 + 3D + 2)y = e^{-2t}$$

**Solution:**
Let L(y) = $\bar{y}$

Taking Laplace on both sides,

$$L(y'') + 3\,L(y') + 2\,L(y) = L\,(e^{-2t}\,sin\,t)$$

But, $L(y') = s\,\bar{y} - y(0) = s\,\bar{y}$

And $L(y'') = s^2\,\bar{y} - y(0) - y'(0) = s^2\,\bar{y}$

And $L(e^{-2t}\,sin\,t) = \frac{1}{(s+2)^2+1}$

The equation hence becomes,

$$s^2\,\bar{y} + 3s\,\bar{y} + 2\,\bar{y} = \frac{1}{s^2+4s+5}$$

$$(s^2 + 3s + 2)\,\bar{y} = \frac{1}{s^2+4s+5}$$

$$\bar{y} = \frac{1}{(s^2+3s+2)\,(s^2+4s+5)}$$

Let $\bar{y} = \frac{a}{s+1} + \frac{b}{s+2} + \frac{cs+d}{s^2+4s+5}$

Solving this, we get a = 0.5, b = -1, c = 0.5, d = 0.5

$$\bar{y} = \frac{0.5}{s+1} - \frac{1}{s+2} + \frac{1}{2}\left(\frac{s+1}{s^2+4s+5}\right)$$

Taking Inverse Laplace Transform,

$$y = \frac{1}{2}\,L^{-1}\left[\frac{1}{s+1}\right] - L^{-1}\left[\frac{1}{s+2}\right] + \frac{1}{2}\,L^{-1}\left[\frac{s+2-1}{(s+2)^2+1}\right]$$

$$y = \frac{1}{2}\,L^{-1}\left[\frac{1}{s+1}\right] - L^{-1}\left[\frac{1}{s+2}\right] + \frac{1}{2}\,L^{-1}\left[\frac{s+2}{(s+2)^2+1}\right] - \frac{1}{2}\,L^{-1}\left[\frac{1}{(s+2)^2+1}\right]$$

$$y = \frac{1}{2} L^{-1}\left[\frac{1}{s+1}\right] - L^{-1}\left[\frac{1}{s+2}\right] + \frac{1}{2} e^{-2t} L^{-1}\left[\frac{s+1}{s^2+1}\right] - \frac{1}{2} e^{-2t} L^{-1}\left[\frac{1}{s^2+1}\right]$$

Hence, the solution is:

$$y = \frac{1}{2} e^{-t} - e^{-2t} + \frac{1}{2} e^{-2t} \cos t - \frac{1}{2} e^{-2t} \sin t$$

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**Question 4.** Using Laplace transform, solve

$$\frac{d^2y}{dt^2} + y = t, \; y(0) = 1, \; y^{|}(0) = 0$$

**Solution:**

Let $\overline{y}$ be the Laplace transform of y i.e let L(y) = $\overline{y}$

Taking laplace on both sides

$$L(y^{||}) + L(y) = L(t)$$

Now,

$$L(y^{||}) = s^2\,\overline{y} - sy(0) - y^{|}(0) = s^2\,\overline{y} - s \qquad \text{and } L(t) = \frac{1}{s^2}$$

$$s^2\,\overline{y} - sy(0) = \frac{1}{s^2} \qquad \therefore\ s^2\,\overline{y} + y = s + \frac{1}{s^2} = \frac{s^3+1}{s^2}$$

$$\therefore\ (s^2+1)\,\overline{y} = \frac{s^3+1}{s^2} \qquad \therefore\ \overline{y} = \frac{s^3+1}{s^2(s^2+1)}$$

Let $\frac{s^3+1}{s^2(s^2+1)} = \frac{a}{b} + \frac{b}{s^2} + \frac{cs+d}{s^2+1}$

$$\therefore\ (s^3+1) = a\,s(s^2+1) + b(s^2+1) + (cs+d)s^2$$

$$= (a+c)s^3 + (b+d)s^2 + as + b$$

Equating like powers of s,

$$a + c = 1,\ b + d = 0,\ a = 0,\ b = 1$$

$$\therefore\ a = 0,\ b = 1,\ c = 1,\ d = -1$$

$$\therefore\ \overline{y} = \frac{1}{s^2} + \frac{s-1}{s^2+1} = \frac{1}{s^2} + \frac{s}{s^2+1} - \frac{1}{s^2+1}$$

Taking laplace inverse, the solution is:

$$y = L^{-1}\left(\frac{1}{s^2}\right) + L^{-1}\left(\frac{s}{s^2+1}\right) - L^{2}\left(\frac{1}{s^2+1}\right) = t + \cos(t) - \sin(t)$$

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**Question 5.** Solve $(D^3-2D^2+5D)y = 0$, with $y(0) = 0$, $y^{'}(0)=0$, $y^{''}(0)=1$.

**Solution:**

Let $L(y) = \overline{y}$.

Taking Laplace transform on both the sides ,

$$L(y''') - 2L(y'') + 5L(y') = 0$$

$$L(y') = s(\overline{y}) - y(0)\,,\ L(y'') = s2\overline{y} - s\,y(0) - y'(0)$$

$$L(y''') = s3\overline{y} - s2\,y(0) - sy'(0) - y''(0)$$

∴ From given conditions, the equation becomes

$$s3\overline{y} - 1 - 2s2\overline{y} + 5s\overline{y} = 0$$

$$\therefore \overline{y} = \frac{1}{s^3-2s^2+5s}$$

Taking inverse Laplace transform,

$$y = L^{-1}\left[\frac{1}{s^3-2s^2+5s}\right]$$

$$= L^{-1}\left[\frac{1}{s(s^2-2s+5)}\right]$$

$$= L^{-1}\left[\frac{1}{s[(s-1)^2+2^2]}\right]$$

We obtain the inverse by convolution theorem,

Let $\Phi_1(s) = \frac{1}{(s-1)^2+2^2}$ *and* $\Phi_2(s) = \frac{1}{s}$

$$\therefore \quad \Phi(s) = \Phi_1(s)\,.\,\Phi_2(s)$$

$$\therefore f1(t) = L^{-1}\,\Phi_1(s) = L^{-1}\left[\frac{1}{s^3-2s^2+5s}\right]$$

$$= e^t\,.\,L^{-1}\left[\frac{1}{s^2+2^2}\right]$$

$$= \frac{1}{2}\ e^t \sin 2t$$

$$\therefore f_2(t) = L^{-1}\,\Phi_2(s) = L^{-1}\left[\frac{1}{s}\right] = 1$$

$$\therefore f1(u) = \frac{1}{2}\,e^u \sin 2u$$

$$L^{-1}\Phi(s) = \int_0^t \frac{1}{2}e^u \sin 2u\,du$$

$$= \frac{1}{2}\,.\,\frac{1}{5}\,.\,[e^u(\sin 2u - 2\cos 2u)]_0^t$$

$$y = \frac{1}{10}[e^t(\sin 2t - 2\cos 2t) + 2]$$

∴ The solution is:

$$y = \frac{1}{5} - \frac{1}{5}e^t\cos 2t + \frac{1}{10}e^t \sin 2t$$

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**Question 6.** Solve the following using Laplace transform

$$\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t \text{ ,given that } y(0)=1.$$

**Solution:**
Let L(y)= $\bar{y}$ .

Taking Laplace transform on both the sides ,

$$L(y') + 2L(y) + L\left[\int_0^t y\,dt\right] = L(\sin t)$$

But $\quad L(y') = sL(y) - y(0) = s\bar{y} - 1$

$$L\left[\int_0^t y\,dt\right] = \tfrac{1}{s}L(y) = \tfrac{1}{s}\bar{y} \text{ , } L(\sin t) = \frac{1}{s^2+1}$$

The equation becomes,

$$s\bar{y} - 1 + 2\bar{y} + \tfrac{1}{s}\bar{y} = \frac{1}{s^2+1}$$

$$\therefore \quad (s + 2 + \tfrac{1}{s})\bar{y} = \frac{1}{s^2+1} + 1 = \frac{s^2+1+1}{s^2+1}$$

$$\therefore \quad \frac{(s^2+2s+1)}{s}\bar{y} = \frac{(s^2+2)}{s^2+1}$$

$$\therefore \quad \bar{y} = \frac{s(s^2+2)}{(s^2+1)(s+1)^2}$$

Let $\dfrac{s(s^2+2)}{(s^2+1)(s+1)^2} = \dfrac{a}{s+1} + \dfrac{b}{(s+1)^2} + \dfrac{cs+d}{s^2+1}$

$$\therefore s(s^2+2) = a(s+1)(s^2+1) + b(s^2+1) + (cs+d)(s+1)^2$$

Putting $s = -1, \ -3 = 2b \ \therefore b = -3/2$
Putting s = 0 , 0 = a+b+d
Equating coefficients of $s^2$ and $s^3$

$0 = a + b + 2c + d$ and $1 = a + c$

$\because b = -3/2 , \quad a + d = 3/2, a + 2c + d = 3/2$

But, $a + d = 3/2 \quad \therefore 2c = 0 \quad \therefore c = 0$
$\because 1 = a + c$ and $c = 0 \quad \therefore a = 1$
$\because a + d = 3/2$ and $a = 1 \quad \therefore d = 1/2$
$\therefore a = 1, \; b = 3/2, \; c = 0, \; d = 1/2$

$$\therefore \overline{y} = \frac{1}{s+1} - \frac{3}{2}.\frac{1}{(s+1)^2} + \frac{1}{2}.\frac{1}{s^2+1}$$

$$\therefore y = L^{-1}(\tfrac{1}{s+1}) - \tfrac{3}{2}e^{t}L^{-1}(\tfrac{1}{s^2}) + \tfrac{1}{2}L^{-1}(\tfrac{1}{s^2+1})$$

Hence, the solution is:

$$\therefore y = e^{-t} - \tfrac{3}{2}e^{-t}.t + \tfrac{1}{2}sin\; t$$

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## Applications:

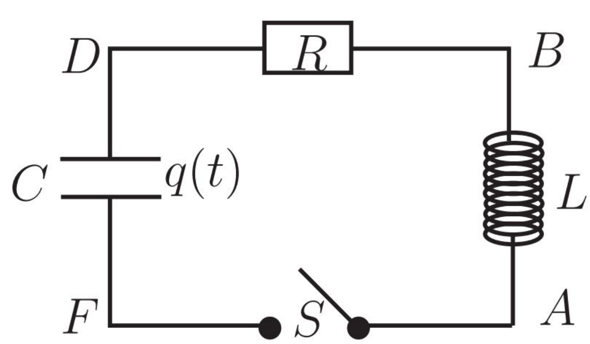
The Laplace Transform is widely used in the following science and engineering fields:

1. Analysis of electronic circuits
2. Mechanical Systems
3. Digital signal processing
4. Nuclear Physics

**Laplace Transform in Simple Electric Circuits:** Consider an electric circuit consisting of a resistance R, inductance L, a condenser of capacity C and electromotive power of voltage E in a series. A switch is also connected in the circuit. Then by Kirchhoff's law, we get the differential equation:

$$L(d^2q/dt^2) + R(dq/dt) + q/C = 0$$

<u>Example:</u>
In the circuit shown in figure, the switch S is closed at t = 0 with a capacitor charge $q(0) = q_0 =$ constant and $d_q / d_t(0) = 0$. Show that $q(t) = q_0(t)e^{-at}$ [cos ωt +α/w sin ωt] where $\alpha = R/2L$ and $\omega^2 = (1/LC) - \alpha^2$




Show that $q(t) = q_0(t)e^{-at}$ [cos ωt +α/w sin ωt] where $\alpha = R/2L$ and $\omega^2 = (1/LC) - \alpha^2$

When the switch S is closed, the inductance L, capacitance C and resistance R give rise to a.c. Voltages related by

$$VA - VB = L\, di/dt\ ,\ VB - VD = R\, i,\ VD - VF = q/C \text{ respectively.}$$

So since $VA - VF = (VA - VB) + (VB - VD) + (VD - VF\ ) = 0$ and $i = dq/dt$ we have

$$L(d^2q/dt^2) + R(dq/dt) + q/C = 0$$

Since the Laplace transform is linear, the transform of the differential equation is

$$L\{\ L(d^2q/dt^2) + R(dq/dt) + q/C\ \} = L\, L\{d^2q/dt^2\} + R\, L\{\ dq/dt\} +\ L\{q/C\} = 0.$$

$$\{d^2q/dt^2\} = s^2\ \mathcal{L}\{q(t)\} - dq/dt\ (0) = 0$$

So, using the initial conditions $q(0) = q0$ and $dq/dt\ (0) = 0$

$$\{d^2q/dt^2\} = s^2\mathcal{L}\{q(t)\} - s\ q(0).$$

$$\{dq/dt\} = s\mathcal{L}\{q(t)\} -\ q(0)$$

$$L[s^2\mathcal{L}\{q(t)\} - s\ q(0)] + R[s\mathcal{L}\{q(t)\}\text{-}q(0)] + 1/C\ \mathcal{L}\{q(t) = 0$$

$$\{q(t)\}[Ls^2 + Rs + 1/C] = L_{sq0} + R_{sq0}$$

$$\{q(t)\} = (Ls + R)\ /\ (Ls^2 + Rs + 1/C)\ q_0$$

Using the definitions $\alpha = R/2L$ and $\omega^2 = (1/LC) - \alpha^2$ enables the denominator to be expressed as the sum of two squares,

$$L\ s^2 + R\ s + 1/C = L[s^2 + Rs/L + 1/LC\ ] = L[s^2 + 2\alpha s + 1/LC\ ]$$

$$= L\ [s^2 + 2\alpha s + \alpha^2 + \omega^2] = L[\{s + \alpha\}^2 + \omega^2].$$

Consequently, with the new expression for the denominator, Equation becomes

$$L\{q(t)\} = q0\ [\ [s/(s + \alpha)^2 + \omega^2] + [R/L\ 1/(s + \alpha)^2 + \omega^2\ ]\ ]$$

The inverse Laplace transform is used to find q(t).

Taking the inverse Laplace transform of Equation and using the linearity properties

$$L^{-1}\{\{q(t)\}\} = q_0L^{-1}\{[s/(s + \alpha)^2 + \omega^2] + [R/L\ 1/(s + \alpha)^2 + \omega^2]\}$$

Using property this can be written as

$$q(t) = q_0\,\mathcal{L}^{-1}\ \{[(s + \alpha)/(s + \alpha)^2 + \omega^2] + [-\alpha/(s + \alpha)^2 + \omega^2] + (R/L\omega)\ [\omega/(s + \alpha)^2 + \omega^2\}$$

Using the linearity of the Laplace transform again

$$q(t) = q_0\,\mathcal{L}^{-1}\ \{[(s + \alpha)/(s + \alpha)^2 + \omega^2]\} + \mathcal{L}^{-1}\ \{[-\alpha/(s + \alpha)^2 + \omega^2]\} + \mathcal{L}^{-1}\ \{(R/L\omega)\ [\omega/(s + \alpha)^2 + \omega^2\}$$

Using properties,

$$\mathcal{L}^{-1}\ \{[(s + \alpha)/(s + \alpha)^2 + \omega^2]\} = e^{-at}\cos\omega t$$

Similarly,

$\mathcal{L}^{-1}$ {[- α/(s + α)$^2$+ ω$^2$]}= − (α/ω){$e^{-at}$ sin ωt}

and

$\mathcal{L}^{-1}$ {(R/Lω) [ω/(s + α)$^2$ + ω$^2$} = ( (R/Lω) $e^{-at}$ sin ωt

Substituting the equations, it gives

q(t) = $q_0$ $e^{-at}$ cos ωt +{(−α/ω + R/Lω)$e^{-at}$sin ωt

Substituting α = R/2L, it gives,

q(t) = q0 $e^{-at}$ [cos ωt + [−α/ω + 2α/ω] $e^{-at}$sin ωt]

= $q_0 e^{-at}$ [cos ωt +(α/ω)sin ωt ]

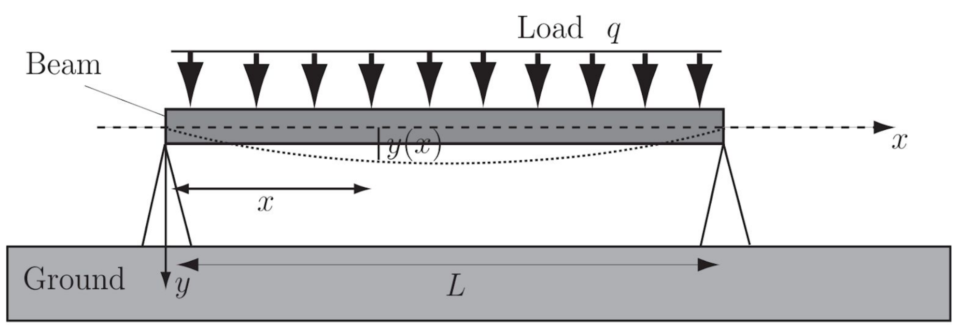
**Laplace Transform in Mechanical Engineering:** In the Mechanical engineering field Laplace Transform is widely used to solve differential equations occurring in mathematical modeling of mechanical systems to find the transfer function of that particular system.

Example:

A uniformly loaded beam of length L is supported at both ends. The deflection y(x) is a function of horizontal position x and obeys the differential equation:

$$\frac{d^4y}{dx^4}(x) - \frac{q}{EI} = 0 \qquad \text{.....(1)}$$

where E is Young's modulus, I is the moment of inertia and q(x) is the load per unit length at point x.




We assume in this problem that q(x) = q (a constant). The boundary conditions are (i) no deflection at x = 0 and x = L (ii) no curvature of the beam at x = 0 and x = L.

In addition to being subject to a uniformly distributed load, a beam is supported so that there is no deflection and no curvature of the beam at its ends. Applying a Laplace Transform to the differential equation (1), find the deflection of the beam as a function of horizontal position along the beam.

Using the linearity properties of the Laplace transform, (1) becomes

$$L\left\{\frac{d^4y}{dx^4}(x)\right\} - L\left\{\frac{q}{EI}\right\} = 0$$

Using property of laplace of a derivative of a function,

$$s^4 \, L\,\{y(x)\} - \sum_{k=1}^{4} s^{k-1} \, \frac{d^{4-k} f}{dx^{4-k}}\Big|_{x=0} = \frac{q}{EI}\,\frac{1}{s} = 0 \qquad (2)$$

The four terms of the sum are

$$\sum_{k=1}^{4} s^{k-1} \, \frac{d^{4-k} f}{dx^{4-k}} = \frac{d^3 y}{dx^3}\Big|_{x=0} + s\,\frac{d^2 y}{dx^2}\Big|_{x=0} + s^2\,\frac{d\,y}{dx}\Big|_{x=0} + s^2\, y(0)$$

The boundary conditions give y(0) = 0 and $\frac{d^2 y}{dx^2}\Big|_{x=0} = 0.$ so (2) becomes

$$s^4 \, L\{\, y(x) \,\} - \frac{d^3 y}{dx^3}\Big|_{x=0} - s^2\,\frac{d\,y}{dx}\Big|_{x=0} - \frac{q}{EI}\,\frac{1}{s} = 0 \qquad (3)$$

Here $\frac{d^3 y}{dx^3}\Big|_{x=0}$ and $\frac{d\,y}{dx}\Big|_{x=0}$ are unknown constants, but they can be determined by using the remaining two boundary conditions y(L) = 0 and $\frac{d^2 y}{dx^2}\Big|_{x=L} = 0$

Solving for L{ y(x) }, (3) leads to,

$$L\{\, y(x) \,\} = \frac{1}{s^4}\,\frac{d^3 y}{dx^3}\Big|_{x=0} + \frac{1}{s^2}\,\frac{d\,y}{dx}\Big|_{x=0} + \frac{q}{EI}\,\frac{1}{s^5}$$

Taking Laplace inverse on both sides of this equation,

$$L^{-1}\,\{\, L\,\{\, y(x) \,\}\,\} = \frac{d^3 y}{dx^3}\Big|_{x=0} \times L^{-1}\,\{\, \frac{1}{s^4} \,\} + \frac{d\,y}{dx}\Big|_{x=0} \times L^{-1}\,\{\, \frac{1}{s^2} \,\} + \frac{q}{EI}\, L^{-1}\{\, \frac{1}{s^5} \,\}$$

Hence,

$$y(x) = \frac{d^3 y}{dx^3}\Big|_{x=0} \times L^{-1}\,\{3!\,\frac{1}{s^4}\,\}\,/\,3! + \frac{d\,y}{dx}\Big|_{x=0} \times L^{-1}\,\{\, \frac{1}{s^2} \,\} + \frac{q}{EI}\, L^{-1}\{4!\,\frac{1}{s^5}\,\}\,/\,4!$$

$$y(x) = \frac{d^3 y}{dx^3}\Big|_{x=0} \times L^{-1}\{L\,\{x^3\}\}/6 + \frac{d\,y}{dx}\Big|_{x=0} \times L^{-1}\{\, L\{x^1\}\} + \frac{q}{EI}\, L^{-1}\{L\,\{x^4\}\}/\,24$$

Simplifying,

$$y(x) = \frac{d^3 y}{dx^3}\Big|_{x=0} \times x^3/6 + \frac{d\,y}{dx}\Big|_{x=0} \times x + \frac{q}{EI}\; x^4/24 \qquad (4)$$

To use the boundary condition $\frac{d^2 y}{dx^2}\Big|_{x=L} = 0,$ take the second derivative of (4), to obtain

$$\frac{d^2 y}{dx^2}(x) = \frac{d^3 y}{dx^3}\Big|_{x=0} \times x + \frac{q}{2EI}\; x^2$$

The boundary condition $\frac{d^2 y}{dx^2}\Big|_{x=L} = 0$ implies

$$\frac{d^3 y}{dx^3}\Big|_{x=0} = -\,\frac{q}{2EI}\,L \qquad (5)$$

Using the last boundary condition y(L) = 0 with (5) in (4)

$$\frac{d\,y}{dx}\Big|_{x=0} = \frac{q}{24EI}\,L^3 \qquad (6)$$

$$y(x) = \frac{q}{24EI}x^4 - \frac{qL}{12EI}x^3 + \frac{qL^3}{24EI}x.$$

Interpretation: The predicted deflection is zero at both ends as required.

## Conclusion:

The applications of Laplace Transform in different engineering fields, like Electronics, Mechanical, Physics etc. Laplace Transform is a very effective tool to simplify very complex problems in the area of stability and control. It goes without saying that Laplace Transform is put to tremendous use in various engineering fields.