AM 3

TUTORIAL 02

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D10 A 01

1) $\frac{s+2}{(s+3)(s+1)^3}$

now, $\frac{A}{(s+3)} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)^3} = \frac{s+2}{(s+3)(s+1)^3}$

$\therefore A(s+1)^3 + B(s+3)(s+1)^2 + C(s+3)(s+1) + D(s+3) = s+2$

$\therefore A(s^3+3s^2+3s+1)$
$+ B(s^3+5s^2+7s+3)$
$+ C(s^2+4s+3)$
$+ D(s+3) = s+2$

On solving the equations,

$A = 1/8, \quad B = -1/8, \quad C = 1/4, \quad D = 1/2$

$= \frac{1}{8(s+3)} - \frac{1}{8(s+1)} + \frac{1}{4(s+1)^2} + \frac{1}{2(s+1)^3}$

now, $L^{-1}\left\{\frac{1}{8(s+3)} - \frac{1}{8(s+1)} + \frac{1}{4(s+1)^2} + \frac{1}{2(s+1)^3}\right\}$

$= \frac{1}{8} \cdot e^{-3t} - \frac{1}{8} \cdot e^{-t} + \frac{1}{4} \cdot \frac{1}{1!} e^{-t} \cdot t + \frac{1}{2} \cdot \frac{1}{2!} \cdot e^{-t} \cdot t^2 \ldots$

$L^{-1}\left\{\frac{1}{(s+b)^{n+1}}\right\} = \frac{1e^{-bt} \cdot t^n}{n!}$

$= \frac{1}{8}\left[e^{-3t} - e^{-t} + 2e^{-t}t + 2e^{-t} \cdot t^2\right]$

$\therefore L^{-1}\left\{\frac{s+2}{(s+3)(s+1)^3}\right\} = \frac{1}{8}\left[e^{-3t} + e^{-t}(2t^2+2t-1)\right]$

2) $\dfrac{s^2+8s+8}{s^4+64} = F(s)$

$$= \frac{s^2+8s+8}{(s^2)^2+8^2}$$

$$= \frac{s^2+8s+8}{(s^2+8)^2-(4s)^2}$$

$$= \frac{s^2+8s+8}{(s^2+8+4s)(s^2+8-4s)}$$

let
$(s^2+8+4s)=X, \quad (s^2+8-4s)=Y$

$\therefore X-Y = 8s$
$X+Y = 2(s^2+8)$

$$= \frac{(s^2+8)+8s}{X\ Y}$$

$$= \frac{\left[\frac{1}{2}(X+Y) + (X-Y)\right]}{X\ Y}$$

$$= \frac{\left(\frac{3}{2}X\right) - \left(\frac{1}{2}Y\right)}{XY}$$

$$= \frac{3}{2}\cdot\frac{1}{Y} - \frac{1}{2}\left(\frac{1}{X}\right)$$

$$= \frac{1}{2}\left[\frac{3}{(s^2+8-4s} - \frac{1}{s^2+8+4s}\right]$$

$$= \frac{1}{2}\left[\frac{3}{(s-2)^2+4} - \frac{1}{(s+2)^2+4}\right]$$

$$L^{-1}\{F(s)\} = \frac{1}{2}\left[\cdot 3e^{2t}\cdot L^{-1}\left\{\frac{1}{s^2+4}\right\} - e^{-2t}\cdot L^{-1}\left\{\frac{1}{s^2+4}\right\}\right]$$

$$= \frac{1}{2}\left[\frac{3}{2}e^{2t}\sin 2t - \frac{e^{-2t}}{2}\sin 2t\right]$$

$$L^{-1}\{F(s)\} = \frac{1}{4}\sin 2t\left(3e^{2t} - e^{-2t}\right)$$

3) $F(s) = \frac{s+2}{(s^2+2s+3)^2}$

$= \frac{s+2}{(s^2+2s+1+2)^2}$

$= \frac{s+2}{((s+1)^2+2)^2}$

$\therefore L^{-1} F(s) = L^{-1}\left\{ \frac{(s+1)+1}{((s+1)^2+2)^2} \right\}$

$= e^{-t} . L^{-1}\left\{ \frac{s+1}{(s^2+2)^2} \right\}$

$= e^{-t} . L^{-1}\left\{ \frac{s+1}{(s^2+2)} \times \frac{1}{(s^2+2)} \right\}$

now, $L^{-1}\left\{ \frac{s+1}{(s^2+2)} \right\}$

$= L^{-1}\left\{ \frac{s}{s^2+2} + \frac{1}{s^2+2} \right\}$

$= \cos\sqrt{2}t + \frac{1}{\sqrt{2}} \sin\sqrt{2}t$

now, $L^{-1}\left\{ \frac{1}{s^2+2} \right\} = \frac{1}{\sqrt{2}} \sin\sqrt{2}t$

$\therefore L^{-1}\{F(s)\} = e^{-t} . \operatorname{conv}\left(\cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t , \frac{1}{\sqrt{2}} \sin\sqrt{2}t \right)$

$= e^{-t}\left[\int_0^t \left(\cos\sqrt{2}u + \frac{1}{\sqrt{2}}\sin\sqrt{2}u\right)\left(\frac{1}{\sqrt{2}}\sin\sqrt{2}(t-u)\right) du \right]$

$= e^{-t}\left[\int_0^t \frac{1}{\sqrt{2}} \sin(\sqrt{2}t - \sqrt{2}u) . \cos\sqrt{2}u + \frac{1}{2}\sin(\sqrt{2}t-\sqrt{2}u) . \sin\sqrt{2}u \, du \right]$

$$= e^{-t}\left[\int_0^t \frac{1}{2\sqrt{2}}\left(\sin(\sqrt{2}t) + \sin(\sqrt{2}t - 2\sqrt{2}u)\right) + \frac{1}{4}\left(\cos(\sqrt{2}t - 2\sqrt{2}u) - \cos(\sqrt{2}t)\right)\right] du$$

$$= \frac{e^{-t}}{4}\int_0^t \left[\sqrt{2}\sin(\sqrt{2}t) + \sqrt{2}\sin(\sqrt{2}t \cancel{+} - 2\sqrt{2}u) + \cos(\sqrt{2}t - 2\sqrt{2}u) - \cos(\sqrt{2}t)\right] du$$

$$= \frac{e^{-t}}{4}\left[\sqrt{2}\,t\sin\sqrt{2}t - t\cos\sqrt{2}t + \frac{1}{2}\cos\sqrt{2}t + \frac{1}{2\sqrt{2}}\sin\sqrt{2}t - \frac{1}{2}\cos\sqrt{2}t + \frac{1}{2\sqrt{2}}\sin\sqrt{2}t\right].$$

$$= \frac{e^{-t}}{4}\left\{\sqrt{2}\,t\sin\sqrt{2}t - t\cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t\right\}$$

$$\therefore L^{-1}\{F(s)\} = \frac{e^{-t}}{4}\left(\sqrt{2}\,t\sin\sqrt{2}t - t\cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t\right)$$

34) $\log\left(1+\frac{1}{s^2}\right) = F(s)$

$\therefore L^{-1}\left\{F(s)\right\} = L^{-1}\left\{\log\left(1+\frac{1}{s^2}\right)\right\}$

$= -\frac{1}{t} L^{-1}\left\{\frac{d}{ds}\left(1+\frac{1}{s^2}\right)\right\}$ .... $L^{-1}(\phi(s)) = -\frac{1}{t} L^{-1}(\phi'(s))$

$= -\frac{1}{t} \cdot L^{-1}\left\{\frac{-2}{s^3}\right\}$

$= \frac{2}{t} \cdot \left[\frac{t^2}{2!}\right]$

$= t$

$\therefore L^{-1}\left\{F(s)\right\} = t$

5) $y'' + 3y' + 2y = t\,\delta(t-1)$; $y(0)=0$, $y'(0)=0$

taking Laplace on both side

$$L\{y'' + 3y' + 2y\} = L\{t\,\delta(t-1)\}$$

$$\therefore L\{y''\} + 3L\{y'\} + 2L\{y\} = e^{-s}\cdot 1$$

$$\therefore \{s^2\bar{y} - s\,\cancel{y(0)}^{0} - \cancel{y'(0)}^{0} + 3(s\bar{y} - \cancel{y(0)}^{0})\} + 2(\bar{y}) = e^{-s} = e^{-s}$$

$$\therefore s^2\bar{y} + 3s\bar{y} + 2\bar{y} = e^{-s}$$

$$\bar{y} = \frac{e^{-s}}{s^2+3s+2}$$

Taking laplace inverse on both side

$$L^{-1}\left\{\frac{1}{s^2+3s+2}\right\} = L^{-1}\left\{\frac{1}{(s+\frac{3}{2})^2 - \frac{1}{4}}\right\}$$

$$= e^{-3t/2}\cdot L^{-1}\left\{\frac{1}{s^2 - 1/4}\right\}$$

$$= e^{-3t/2}\cdot 2\sinh t/2$$

$$\therefore L^{-1}\left\{\frac{e^{-s}}{s^2+3s+2}\right\} = L^{-1}\left\{\frac{1}{s^2+3s+2}\right\}\Bigg|_{s\Rightarrow t-1} \cdot H(t-1)$$

$$= 2e^{-3(t-1)/2}\cdot \sinh\left(\frac{t-1}{2}\right)\cdot H(t-1)$$