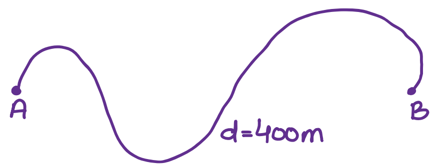
KINEMATICS

Distance :- (d)

the total length of path between two points.

SI units :- meter (m)

Nature :- scalar quantity

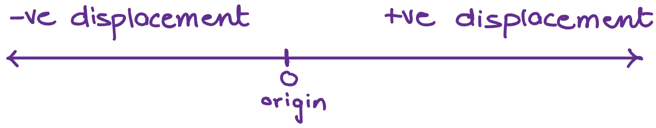


Displacement :- (s)

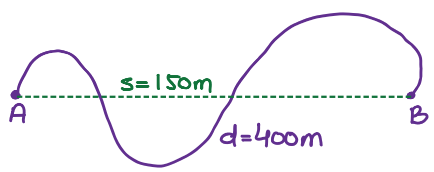
the shortest distance of an object from a fixed point in a specified direction.

SI units :- meter (m)

Nature :- vector quantity



∴ the sign of displacement is switched once it crosses the origin. [the sign of displacement is not switched as object change its direction]



distance (d) = 400m

displacement (s) = 150m

Speed :- (u, v, c)

the rate of change of distance of an object.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

SI units :- meter/second (ms^{-1})

Nature :- scalar quantity

∴ Average speed is always calculated using the formula :-

$$\text{avg speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

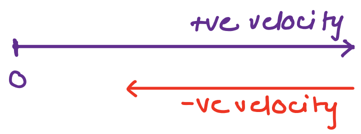
Velocity :- (u, v, c)

the rate of change of displacement of an object.

$$\text{velocity} = \frac{\text{change in displacement}}{\text{change in time}} = \frac{\Delta s}{\Delta t}$$

SI units :- meter/second (ms^{-1})

Nature :- vector quantity



∴ the sign of velocity is switched as soon as the object change its direction.

Acceleration :- (a)

the rate of change of velocity of an object.

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{change in time}} = \frac{\Delta v}{\Delta t}$$

$$\text{acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{change in time}} = \frac{v-u}{t}$$

SI units :- meter/second^2 (ms^{-2})

Nature :- vector quantity

∴ +ve acceleration :- velocity increases with time.

∴ -ve acceleration (deceleration) :- velocity decreases with time.

∴ sign of acceleration has nothing to do with the direction of motion.

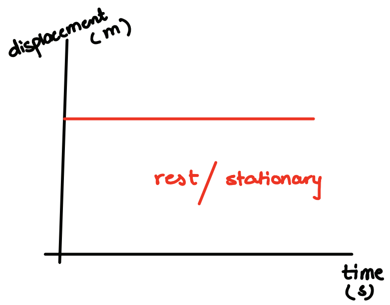
∴ sign of acceleration tells the direction of resultant force.

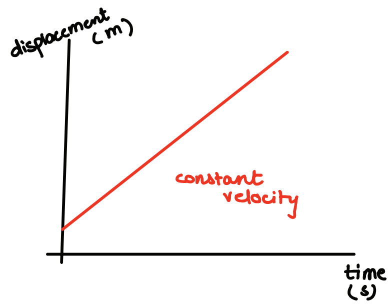
Motion Graphs :-

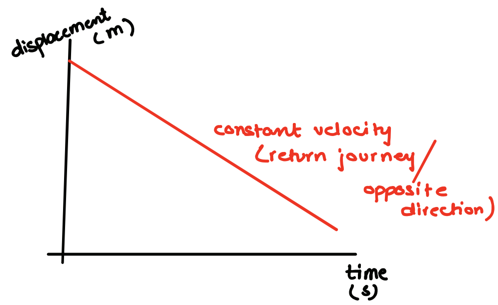
the motion of any object can be represented by the graph. The tree types of graph which can be used to represent motion are :-

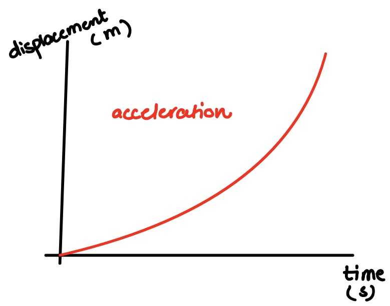
- Displacement - time graph
- Velocity - time graph
- Acceleration - time graph

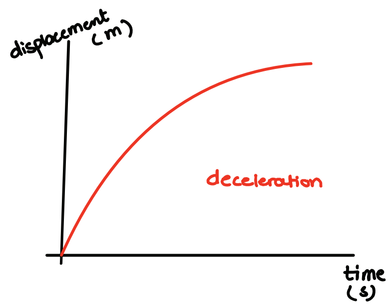
Displacement - time graph :-











∴ gradient / slope equals velocity.

∴ y-intercept equals initial displacement.

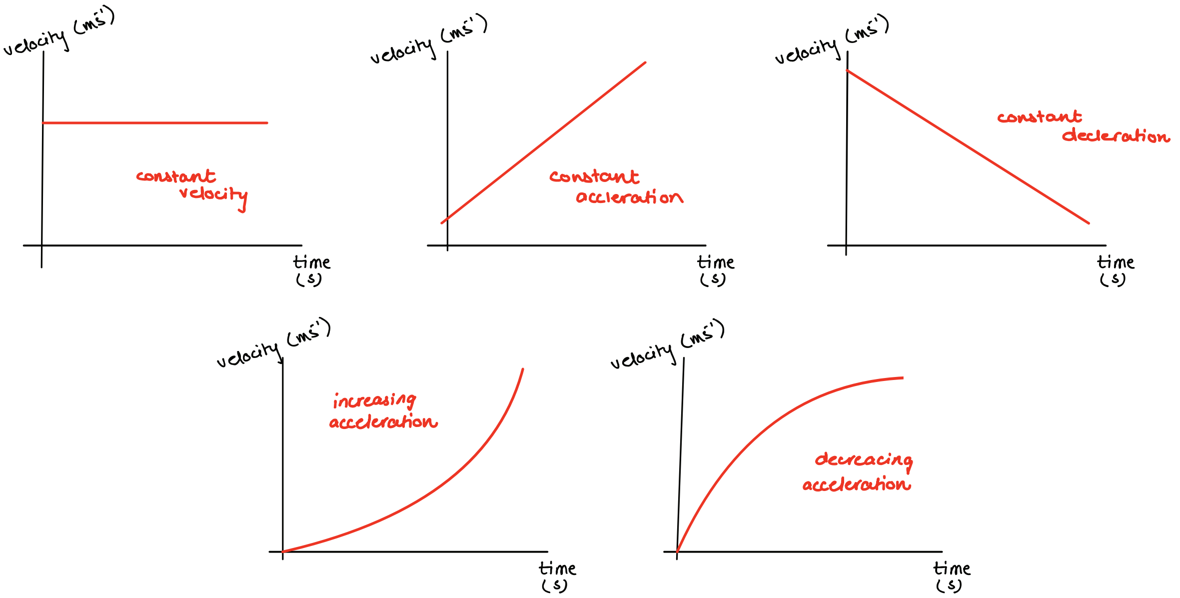
∴ the straight line represents constant velocity.

∴ +ve / -ve slope represents the direction of velocity.

∴ horizontal line represents a state of rest.

∴ the area under the curve is meaningless.

Velocity-time graph :-



∴ gradient / slope equals acceleration.

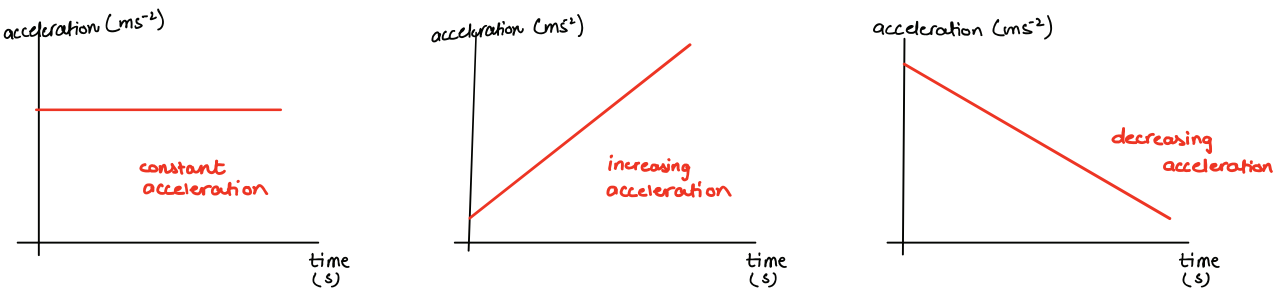
∴ y-intercept equals initial velocity.

∴ the straight line represents constant / uniform acceleration

∴ horizontal line represents constant velocity.

∴ the area under the curve equals the distance travelled.

Acceleration-time graph :-



∴ gradient / slope is meaningless.

∴ y-intercept equals initial acceleration.

∴ horizontal line represents constant acceleration.

∴ the area under the curve equals the change in velocity.

Equations of Motion :-

These are the set of equations which can describe the motion of any object with the constant acceleration

↳ Remember, equations of motion are only applicable when acceleration is constant.

∴ Key Variables :-

s = displacement

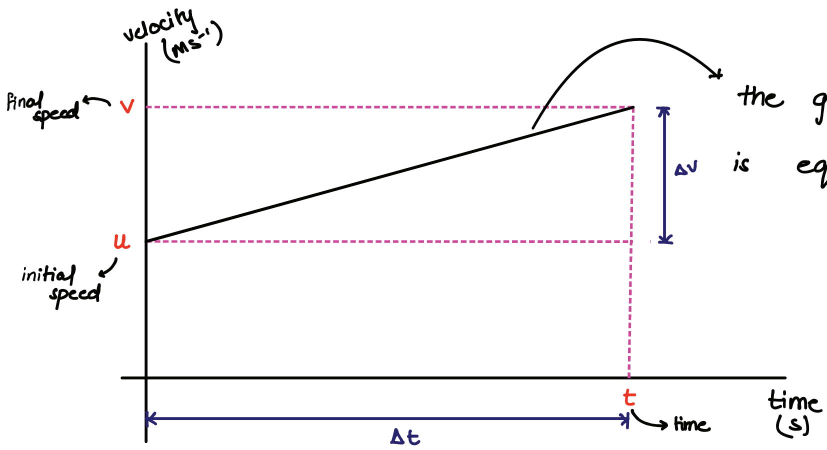
u = initial velocity

v = final velocity

a = acceleration

t = time

First equation :- ($v = u + at$)



the gradient of the line is equal to acceleration.

gradient = $\frac{\Delta y}{\Delta x}$ ⟶ acceleration

$$a = \frac{\Delta y}{\Delta x} = \frac{\Delta v}{\Delta t} = \frac{v-u}{t}$$

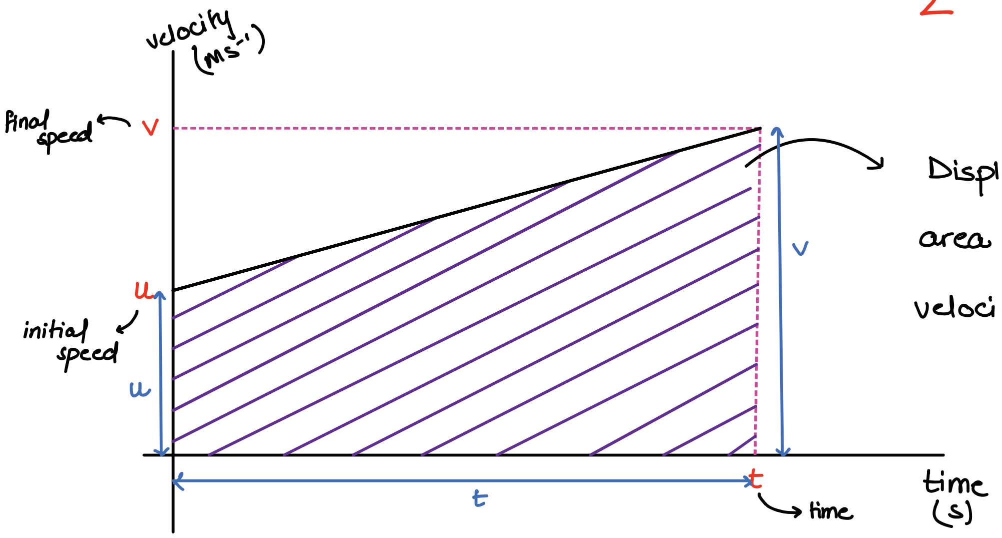
$$a = \frac{v-u}{t}$$

$$at = v - u$$

$$v = u + at$$

$$\boxed{v = u + at}$$

Second Equation :- ($s = \left(\frac{u+v}{2}\right) t$)



Displacement is the area under the velocity-time graph.

Area of trapezium = $\frac{1}{2}$ (Sum of parallel sides) height

displacement = $\frac{1}{2}(u+v)\,t$

$$\boxed{s = \frac{(u+v)}{2}\,t}$$

Third Equation :- ($s = ut + \frac{1}{2}at^2$)

∴ Using first and second equation:

1) $v = u + at$

2) $s = \left(\frac{u+v}{2}\right) t$

• Substitute eq 1 into eq 2 :

$$s = \frac{(u + (u+at))\,t}{2}$$

$$s = \frac{ut + (ut + at^2)}{2}$$

$$s = \frac{2ut + at^2}{2}$$

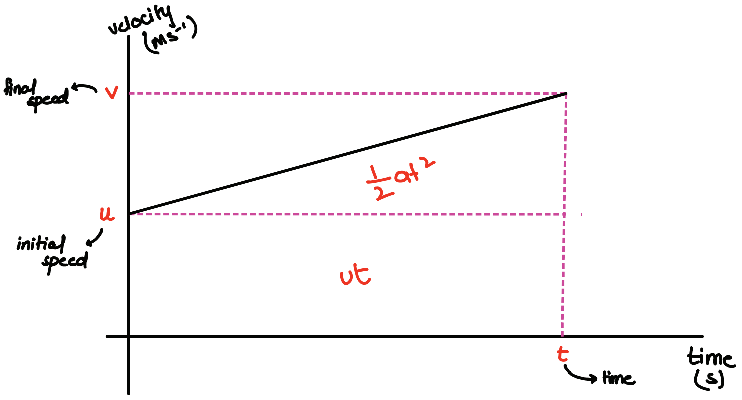
$$s = \frac{\cancel{2}ut}{\cancel{2}} + \frac{at^2}{2}$$

$$s = ut + \frac{1}{2}at^2$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

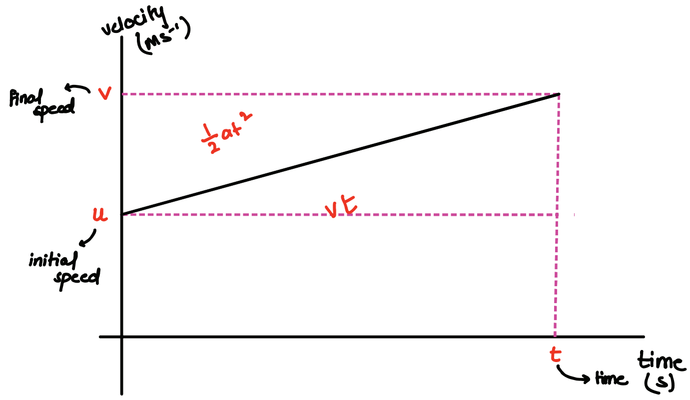
∴ This equation can be used as :-

$s = ut + \frac{1}{2}at^2 \Rightarrow s = vt - \frac{1}{2}at^2$



the two terms ut and $\frac{1}{2}at^2$ make up the area under graph

$s = ut + \frac{1}{2}at^2$



vt is the area of big rectangle and if we subtract $\frac{1}{2}at^2$ from it, it will result into area under graph.

$s = vt - \frac{1}{2}at^2$

Fourth Equation :- $(v^2 = u^2 - 2as)$

∴ Using first and second equation:

1) $v = u + at$ ⟶ make t subject

$t = \frac{v-u}{a}$

2) $s = \left(\frac{u+v}{2}\right)t$ ∴ Substitute t into equation 2

$s = \frac{(u+v)}{2}\left(\frac{v-u}{a}\right) \Rightarrow s = \frac{v^2-u^2}{2a} \Rightarrow 2as = v^2 - u^2$

$\boxed{v^2 = u^2 + 2as}$

Summary :-

1) $v = u + at$

2) $s = \left(\frac{v+u}{2}\right) t$

3) $s = ut + \frac{1}{2}at^2$ OR $s = vt - \frac{1}{2}at^2$

4) $v^2 = u^2 + 2as$

How to use these equations :-

• Step 1 :

Write out the variables that are given in the question, both known and unknown, and use the context of the question to deduce any quantities that aren't explicitly given:

e.g → starts from rest ⟶ $u = 0\,ms^{-1}$

→ eventually comes to rest ⟶ $v = 0\,ms^{-1}$

→ acceleration of freefall ⟶ $a = \pm 9.81\,ms^{-2}$

• Step 2 :

Choose the equation which contains the quantities you have listed

e.g equation which links s, u, a and t.

$$s = ut + \frac{1}{2}at^2$$

• Step 3 :

Convert any units to SI units and then insert the quantities into the equation and rearrange allgebrically to determine the unknown (answer).

EXAMPLE :-

The train coming from the left travel at the speed of $50ms^{-1}$. At marker one, the driver must apply the brakes so that the train decelerates uniformly in order to pass marker two at no more than $10ms^{-1}$. The train carries a detector that notes the time when the train passes each marker and will apply an emergency brake if the time between markers is less than 20 seconds.

(i) How far from marker 2 should marker 1 be placed ?

(ii) Calculate the deceleration of train for this motion ?

Solution

$u = 50ms^{-1}$

$v = 10ms^{-1}$

$t = 20 s$

$s = ?$

$a = ?$

(i) $s = \left(\frac{u+v}{2}\right) t$

$s = \left(\frac{50+10}{2}\right)(20)$

$s = (30)(20)$

$s = 600m$

(ii) $v = u + at$

$10 = 50 + a(20)$

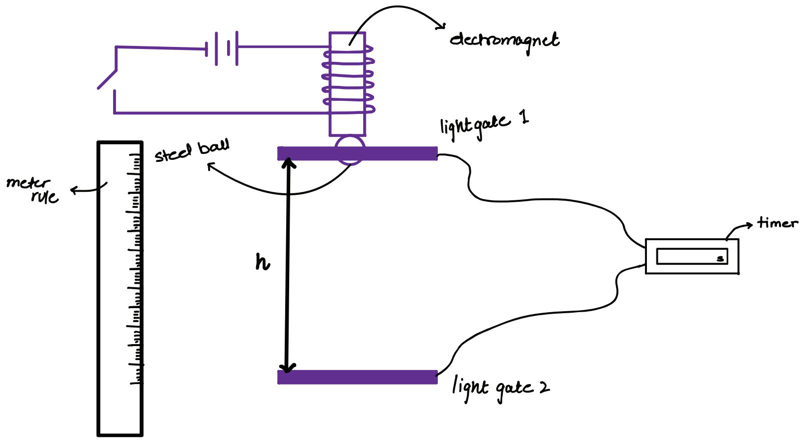
$-40 = 20a$

$a = -2ms^{-2}$

deceleration $= 2ms^{-2}$

Determine Acceleration of Freefall :-

A common experiment to determine acceleration of a falling object which can be carried out in the Lab.



➡ A steel ball is held on an electromagnet, when electromagnet is switched off, ball interrupts a beam of light and a timer starts.

As the ball falls, it interrupts a second beam of light and timer stops.

A meter rule is used to measure h.

Analysing Data

Displacement $= s = h$

Time taken $= t$

Initial velocity $= u = 0\,ms^{-1}$

Acceleration $= a = g$ (unknown)

$s = \cancel{ut}^{\,0} + \frac{1}{2}at^2$

$\boxed{h = \frac{1}{2}gt^2}$

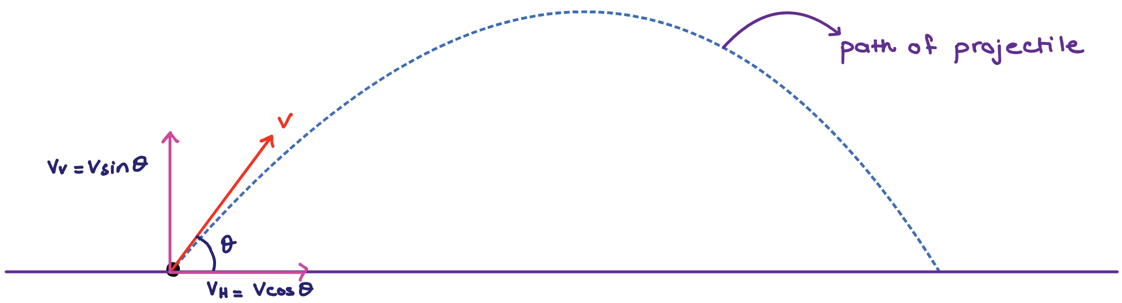
∴ Plot the graph of h against t^2

$\boxed{\text{gradient} = \frac{1}{2}g}$

Projectile Motion :-

It is also known as motion in two dimensions.

∴ The trajectory of an object undergoing projectile motion cosists of a vertical and horizontal component, these components need to evaluated separately.



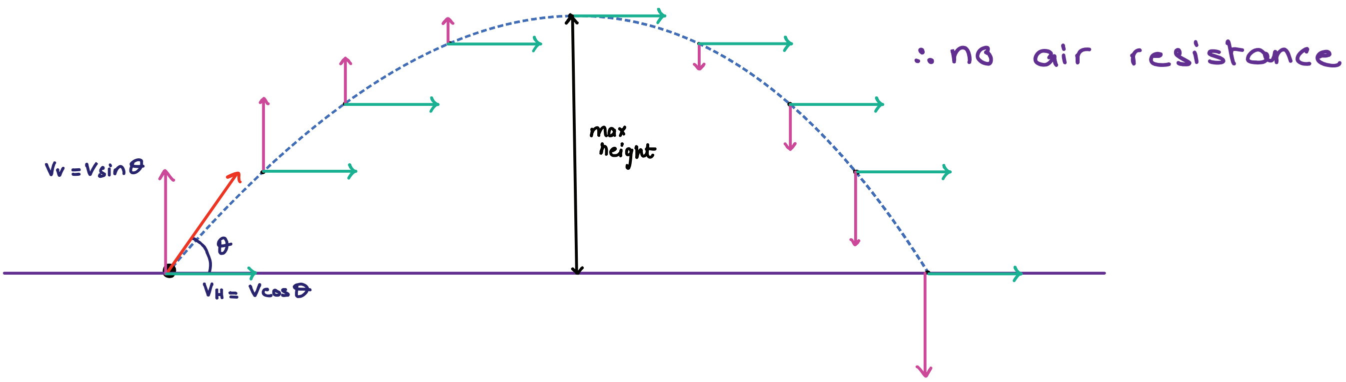
Vertical Component :-

The only force acting on the projectile, after it has been released, is gravity. Its a vertical force so it only affects the vertical component of the projectile.

Initially at launch, the vertical component is maximum as the projectile climbs the vertical component decreases and become zero at maximum height. As the object desends the vertical component starts to increase and become maximum again just before hitting the ground but in opposite direction.

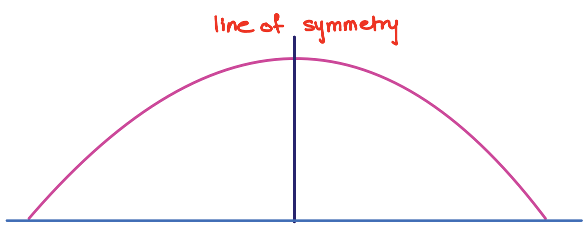
Horizontal Component :-

The force which acts in the horizontal direction is air resistance, it will decrease the horizontal component. While solving the numericals of projectile motion air resistance is ignored so the horizontal component stays constant through out the motion.

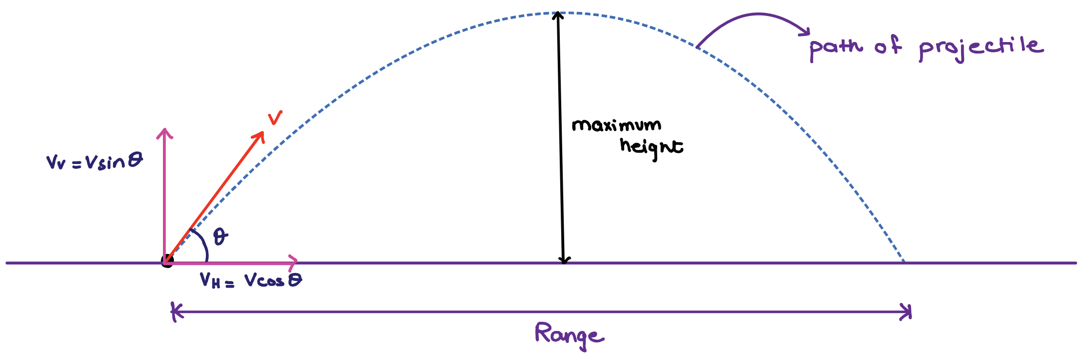


∴ The horizontal and vertical motions in a projectile are dealt seperately. Equations of motion will be used. Remember both motions are happening simultaneously so the time will be same.

∴ The path of projectile motion is symmetrical path along the line of maximum height.



Key Terms :-



Time of flight :-

how long the projectile is in the air.

∴ the time taken to reach the maximum height is half the time of flight as the path is symmetrical.

$$\text{Time of flight} = \text{Time to reach maximum height} \times 2$$

Maximum height attained :-

It is maximum height achieved by the projectile. At this point vertical component of the velocity is zero. (momentarily at rest).

Range :-

the horizontal distance travelled by the projectile.

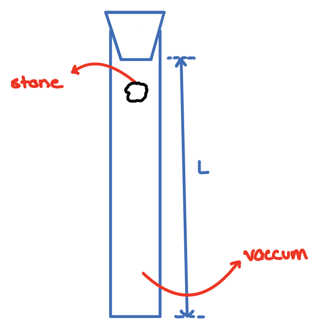
∴ Usually horizontal component of velocity is constant as air resistance is ignored. (acceleration is zero), so $v = \frac{d}{t}$ can be used to calculate range of projectile.

∴ There are three possible scenarios for projectile motion:

1) Vertical Projection
2) Horizontal Projection
3) Projection at an Angle

Vertical Projection :-

Q. The diagram shows a laboratory experiment in which a stone falls from rest in a long evacuated vertical tube of length L.



The time of fall from rest is 0.5 s.
What is the length of the tube, L?

Sol

∴ The feather is in vertical motion, so there is no horizontal component of velocity.

$u = 0 ms^{-1}$

$a = 9.81 ms^{-2}$

$t = 0.5 s$

$s = L = ?$

$s = ut + \frac{1}{2}at^2$

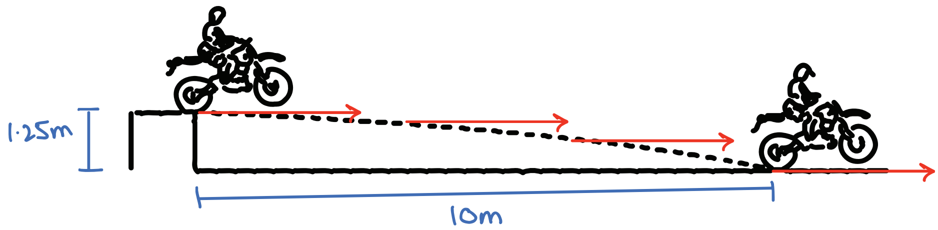
$L = (0)(0.5) + \frac{1}{2}(9.81)(0.5)^2$

$L = 1.226 m$

$\boxed{L \approx 1.23 m}$

Horizontal Projection :-

Q. A motorcyclist stund-rider moving horizontally takes off from a point 1·25 m above the ground, landing 10 m away as shown.
What was the speed of motorcyclist at take-off ?



Sol

∴ Range of motion is given (10m), we need the time of flight to calculate the horizontal component of the velocity. (horizontal component is our inital velocity)
To calculate time of flight we can consider vertical motion. Initially, the bike is moving horizontally which shows that inital vertical velocity is zero.

vertical motion

$u = 0ms^{-1}$ $a = 9.81\ ms^{-2}$ $s = 1.25m$ $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$1.25 = (0)(t) + \frac{1}{2}(9.81)(t^2)$$

$$1.25 = \frac{1}{2}(9.81)(t^2)$$

$$\sqrt{\frac{2.5}{9.81}} = t$$

$$t = 0.505s$$

horizontal motion

$$v = \frac{d}{t} = \frac{10}{0.505} = \boxed{19.8\ ms^{-1}}$$

Projection at an Angle :-

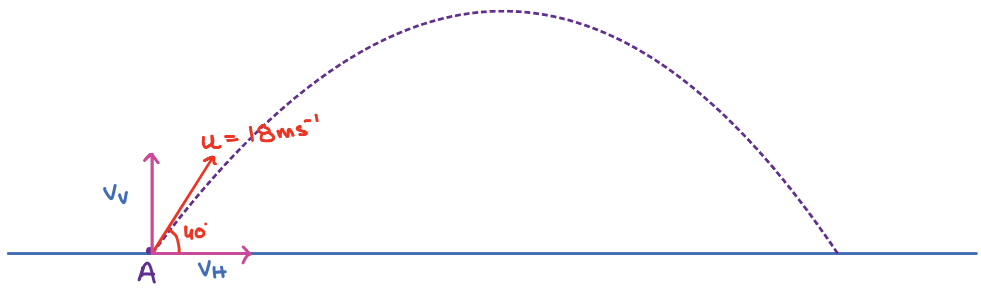
Q. The ball is thrown from a point A with an initial velocity of $18 ms^{-1}$ at $40°$ to the horizontal. What is the value of :-

(a) the maximum height ?

(b) time of flight ?

(c) range ?

Sol



$V_H = u\cos\theta = 18\cos(40) = 13.79\ ms^{-1}$

$V_V = u\sin\theta = 18\sin(40) = 11.57\ ms^{-1}$

(a)

∴ for max height vertical motion will be used.

at max height $V_V = 0 ms^{-1}$

$u = 11.57 ms^{-1}$

$v = 0 ms^{-1}$

$a = -9.81 ms^{-2}$

$s = H_{max} = ?$

$2as = v^2 - u^2$

$2(-9.81)(H_{max}) = 0^2 - (11.57)^2$

$H_{max} = \frac{-133.87}{-19.62}$

$\boxed{H_{max} = 6.82\ m}$

(b)

∴ for time of flight we will calculate the time taken to reach maximum height and will double it (symmetrical path)

vertical motion

$u = 11.57\ ms^{-1}$

$v = 0\ ms^{-1}$

$a = -9.81\ ms^{-2}$

$t = ?$

$v = u + at$

$0 = 11.57 + (-9.81)(t)$

$\frac{-11.57}{-9.81} = t$

$t = 1.179\ s$

$$\text{Time of flight} = 1.179 \times 2$$

$$= 2.358$$

$$\boxed{\approx 2.36\ s}$$

(c)

For range use $v = \frac{d}{t}$

horizontal motion

$V_H = 13.79\ ms^{-1}$

$t = 2.36\ s$

$$v = \frac{d}{t}$$

$$13.79 = \frac{d}{2.36}$$

$$d = (13.79)(2.36)$$

$$d = 32.54\ m$$

$$\text{Range} \Rightarrow \boxed{d \approx 32.5\ m}$$