Assignment - I

Q1 Construct CFG for the following languages.

① Balanced parantheis.

$L = \{ w \in \{(,)\}^* \}$

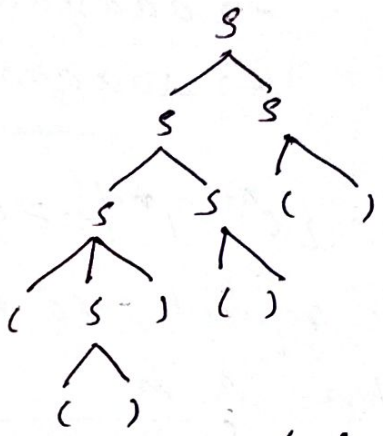
The required CFG in $\{\{S\}, \{(,)\}, R, S\}$

where R is

$$S \longrightarrow SS \mid (S)()$$

let $w = (())()()$

$S \rightarrow \underline{S}S$
$\rightarrow SS\underline{S}$
$\rightarrow (S)S\underline{S}$
$\rightarrow ((\,))S\underline{S}$
$\rightarrow ((\,))()()$



② $L = \{ w \in \{a,b\}^* \mid w$ contains substring $ab \}$

$L = \{ ab, aab, abb, bab \ldots\ldots \}$

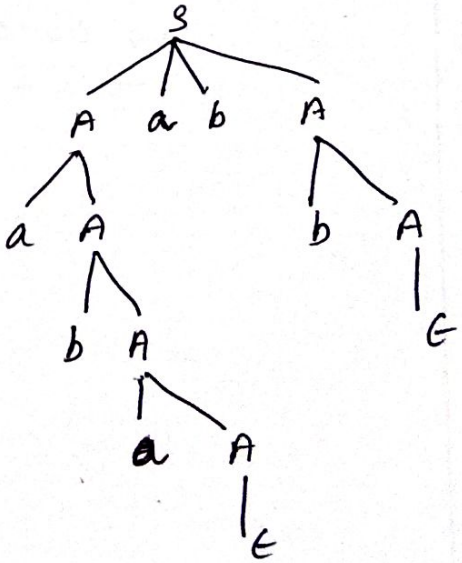
The required CFG in $\{\{A,S\}, \{a,b\}, R, S\}$

where R is

$$S \longrightarrow AabA$$
$$A \longrightarrow aA \mid bA \mid \epsilon$$

Let $w = abadbb$

$S \rightarrow \underline{A}abA$
$\rightarrow a\underline{A}abA$
$\rightarrow ab\underline{A}abA$
$\rightarrow aba\underline{A}abA$
$\rightarrow abaabb\underline{A}$
$\rightarrow abaabb$

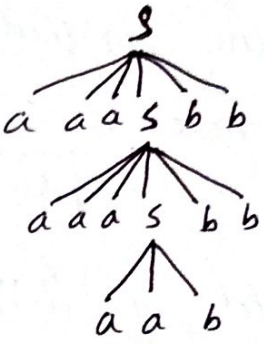


(iii) $L = \{a^i b^j \mid 2i = 3j+1\}$

The required CFG is $\{\{S\}, \{a, b\}, R, S\}$

where R is

$S \rightarrow aaaSbb \mid aab$



Let $w = aaaaaaaabbbbbb$

$S \rightarrow aaa\underline{S}bb$

$\rightarrow aaaaaaa\underline{S}bbbbbb$

$\rightarrow aaaaaaaabbbbbb$

(iv) $L = \{0^{n+2} 1^n \mid n \geq 1\}$

The required CFG is $\{\{S\}, \{0, 1\}, R, S\}$

where R is

$S \rightarrow 0S1 \mid 0001$

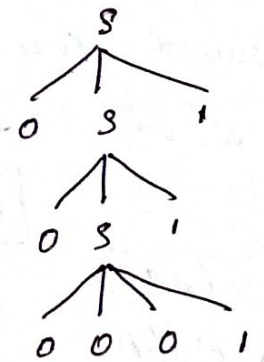
$L = \{0001, 000011, 00000111 \ldots\}$

Let $w = 00000111$

$S \rightarrow 0\underline{S}1$

$\rightarrow 00\underline{S}11$

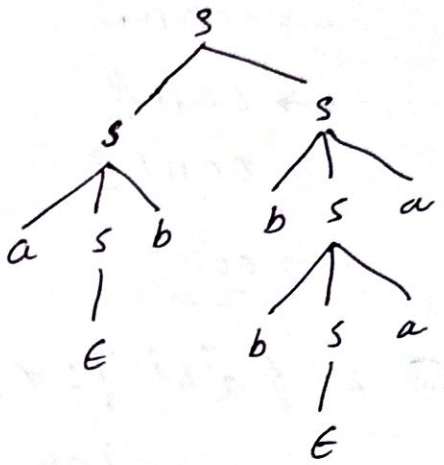
$\rightarrow 00000111$



(v) $L = \{ w \in \{a,b\}^* \mid n_a(w) = n_b(w) \}$

$L = \{abab,\ baba,\ abbbaa \ldots\ldots \}$

$\boxed{S \rightarrow SS \mid aSb \mid bSa \mid \epsilon}$



e.g: abbbaa

$S \rightarrow \underline{SS}$

$\rightarrow a\underline{S}bS$

$\rightarrow abb\underline{S}$

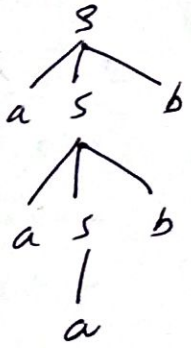
$\rightarrow abbb\underline{S}aa$

$\rightarrow abbbaa$

(vi) $L = \{a^i b^j \mid i = j+1 \}$

$L = \{a,\ aab,\ aaaab \ldots \}$

$\boxed{S \rightarrow a \mid aSb}$



e.g: aaabb

$S \rightarrow a\underline{S}b$

$\rightarrow aa\underline{S}bb$

$\rightarrow aaabb$

(vii) $L = \{0^m 1^m 2^n \mid m \geq 0,\ n \geq 0 \}$

$S \rightarrow AB$

$A \rightarrow 0A1 \mid \epsilon$

$B \rightarrow 2B \mid \epsilon$

e.g: 00112

$S \rightarrow \underline{A}B$

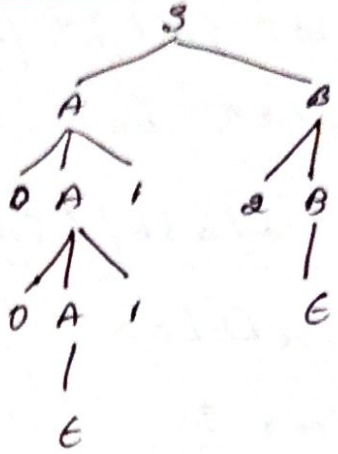
$\rightarrow 0\underline{A}1B$

$\rightarrow 00\underline{A}11B$

$\rightarrow 0011\underline{B}$

$\rightarrow 00112\underline{B}$

$\rightarrow 00112$



(viii) $L = \{ a^i b^j \mid i \neq j, i \geq 0, j \geq 0 \}$

$S \rightarrow BA \mid Ax$

$A \rightarrow aAb \mid \epsilon$

$B \rightarrow aB \mid a$

$X \rightarrow bX \mid b$

e.g: aaabb

$S \rightarrow \underline{A}X$

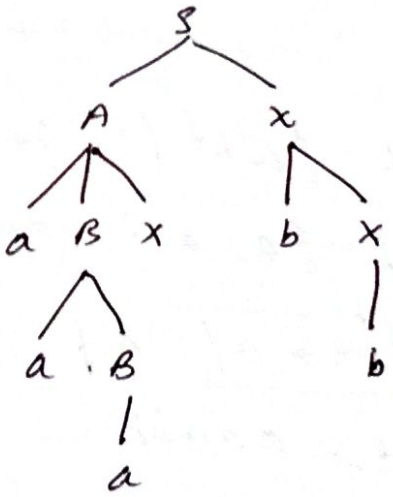
$\rightarrow a\underline{B}X$

$\rightarrow aa\underline{B}X$

$\rightarrow aaa\underline{X}$

$\rightarrow aaab\underline{X}$

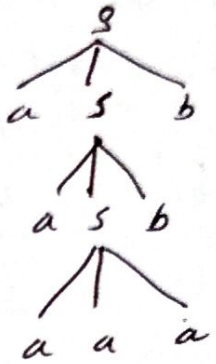
$\rightarrow aaabb$



(IX) $L = \{a^n b^{n-3} \mid n \geq 3\}$

$L = \{a^3, a^4b, a^5b^2, a^6b^3 \ldots\ldots\}$

$S \longrightarrow aSb \mid aaa$



eg: $w = a^5b^2$

$S \longrightarrow aSb$

$\longrightarrow aaSbb$

$\longrightarrow aaaaabb$

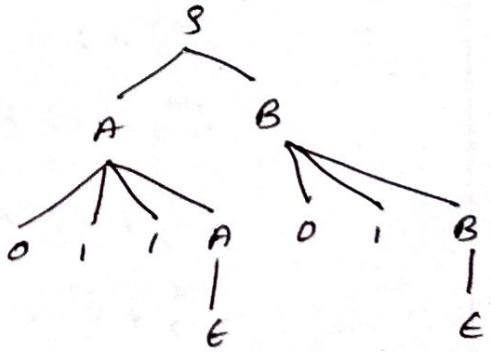
(X) $(011+1)^* (01)^*$

$L = \{\epsilon, 010101, 01111, 101, 01101 \ldots\ldots\}$

$S \longrightarrow AB$

$A \longrightarrow 011A \mid 1A \mid \epsilon$

$B \longrightarrow 01B \mid \epsilon$



e.g: 01101

$S \longrightarrow AB$

$\longrightarrow 011AB$

$\longrightarrow 011B$

$\longrightarrow 01101B$

$\longrightarrow 01101$

2) Consider the grammar.

$E \rightarrow +EE \mid *EE \mid -EE \mid x \mid y$

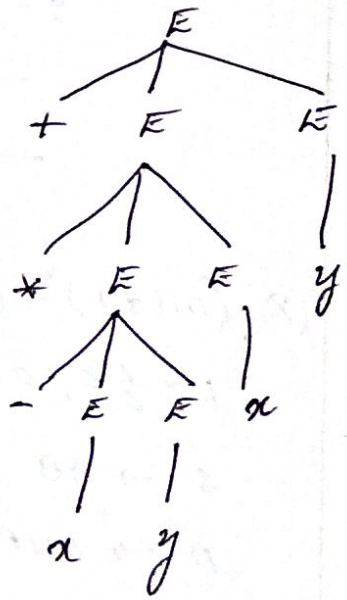
Find the LMD, RMD & parse tree for the string

$+*-xyxy$

⇒ LMD:

$$S \xrightarrow{lm} +EE$$
$$\xrightarrow{lm} +*EEE$$
$$\xrightarrow{lm} +*-EEEE$$
$$\xrightarrow{lm} +*-xEEE$$
$$\xrightarrow{lm} +*-xyEE$$
$$\xrightarrow{lm} +*-xyxE$$
$$\xrightarrow{lm} +*-xyxy$$

Parse Tree:



RMD:

$$S \xrightarrow{rm} +EE$$
$$\xrightarrow{rm} +Ey$$
$$\xrightarrow{rm} +*EEy$$
$$\xrightarrow{rm} +*Exy$$
$$\xrightarrow{rm} +*-EExy$$
$$\xrightarrow{rm} +*-Eyxy$$
$$\xrightarrow{rm} +*-xyxy$$

3] Consider the following grammar with production

$S \longrightarrow AbB$

$A \longrightarrow aA \mid \epsilon$

$B \longrightarrow aB \mid bB \mid \epsilon$

Find the LMD, RMD & parse tree for the string aaabab

LMD :

$S \xRightarrow{lm} \underline{A}bB$

$\xRightarrow{lm} a\underline{A}bB$

$\xRightarrow{lm} aa\underline{A}bB$

$\xRightarrow{lm} aaa\underline{A}bB$

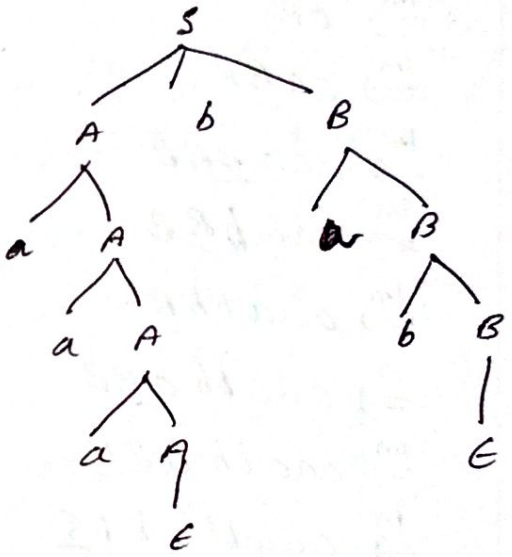
$\xRightarrow{lm} aaab\underline{B}$

$\xRightarrow{lm} aaaba\underline{B}$

$\xRightarrow{lm} aaababB$

$\Rightarrow aaabab$

Parse Tree :



RMD :

$S \xRightarrow{rm} Ab\underline{B}$

$\xRightarrow{rm} Aba\underline{B}$

$\xRightarrow{rm} Ababb\underline{B}$

$\xRightarrow{rm} Abab$

$\xRightarrow{rm} aAbab$

$\xRightarrow{rm} aa\underline{A}bab$

$\xRightarrow{rm} aaaAbab$

$\xRightarrow{rm} aaabab$

Q1 Let G be the following grammar.

$S \longrightarrow aB \mid bA$

$A \longrightarrow a \mid aS \mid bAA$

$B \longrightarrow b \mid bS \mid aBB$; string: aaabbabbba

Find LMD, RMD & parse Tree.

LMD:

$S \overset{lm}{\Rightarrow} aB$

$\overset{lm}{\Rightarrow} aa\underline{B}B$

$\overset{lm}{\Rightarrow} aaa\underline{B}BB$

$\overset{lm}{\Rightarrow} aaab\underline{B}B$

$\overset{lm}{\Rightarrow} aaabb\underline{B}$

$\overset{lm}{\Rightarrow} aaabba\underline{B}B$

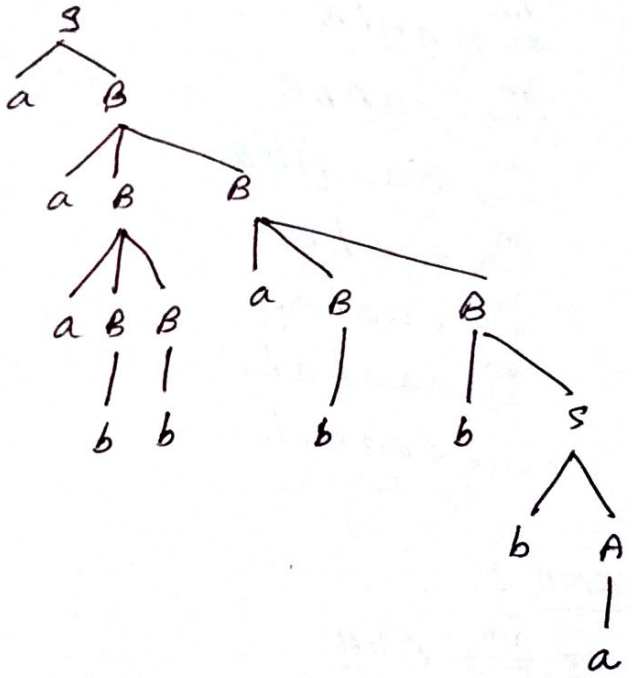
$\overset{lm}{\Rightarrow} aaabbab\underline{B}$

$\overset{lm}{\Rightarrow} aaabbabb\underline{S}$

$\overset{lm}{\Rightarrow} aaabbabbb\underline{A}$

$\overset{lm}{\Rightarrow} aaabbabbba$

Parse Tree:



RMD:

$S \overset{rm}{\Rightarrow} a\underline{B}$

$\overset{rm}{\Rightarrow} aa\underline{B}$

$\overset{rm}{\Rightarrow} aaB\underline{B}$

$\overset{rm}{\Rightarrow} aaBaB\underline{B}$

$\overset{rm}{\Rightarrow} aaBaBb\underline{S}$

$\overset{rm}{\Rightarrow} aaBaBbb\underline{A}$

$\overset{rm}{\Rightarrow} aaBa\underline{B}bba$

$\overset{rm}{\Rightarrow} aa\underline{B}abbba$

$\xRightarrow{rm}$ aaaBBabbba

$\xRightarrow{rm}$ aaaBbabbba

$\xRightarrow{rm}$ aaabbabbba

5] Show that the following grammar is ambiguous.

$S \longrightarrow aB/bA$

$A \longrightarrow aS/bAA/a$

$B \longrightarrow bS/aBB/b$

A grammar $G = (V, \Sigma, R, S)$ is said to be ambiguous grammar if for any string $w \in L(G)$ there exists more than one left most derivation or more than one Right most derivation or more than one distinct parse tree.

$w = aabbab$

LMD's: $S \xRightarrow{lm} aB$

$\xRightarrow{lm} aaBB$

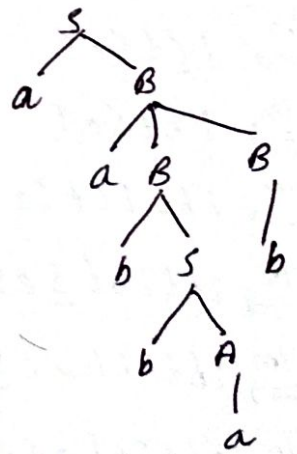
$\xRightarrow{lm} aabSB$

$\xRightarrow{lm} aabbAB$

$\xRightarrow{lm} aabbaB$

$\xRightarrow{lm} aabbab$

parse tree:



$S \xRightarrow{lm} aB$

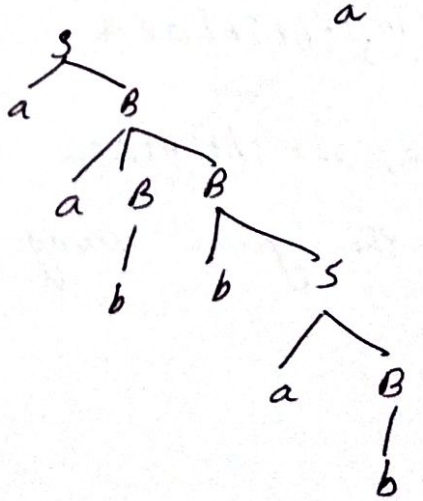
$\xRightarrow{lm} aaBB$

$\xRightarrow{lm} aabB$

$\xRightarrow{lm} aabbS$

$\xRightarrow{lm} aabbaB$

$\xRightarrow{lm} aabbab$



6) Is the following grammar ambiguous.

$S \longrightarrow iCtS / iCtSeS / a$

$C \longrightarrow b$

& $w = ibtibtaea$

⇒ LMD's:

$S \xRightarrow{lm} iCtS$

$\xRightarrow{lm} ibtS$

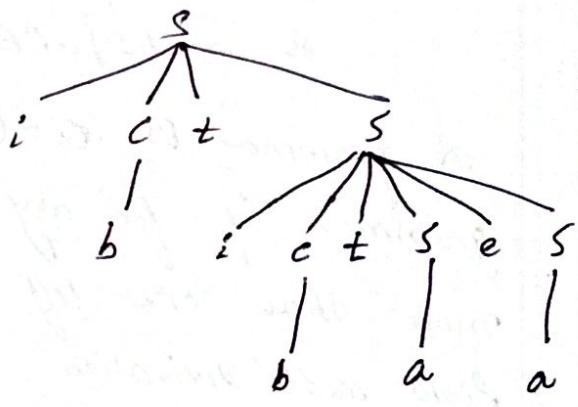
$\xRightarrow{lm} ibtiCtSeS$

$\xRightarrow{lm} ibtibtSeS$

$\xRightarrow{lm} ibtibtaeS$

$\xRightarrow{lm} ibtibtaea$

Parse Tree's



$S \xRightarrow{lm} iCtSeS$

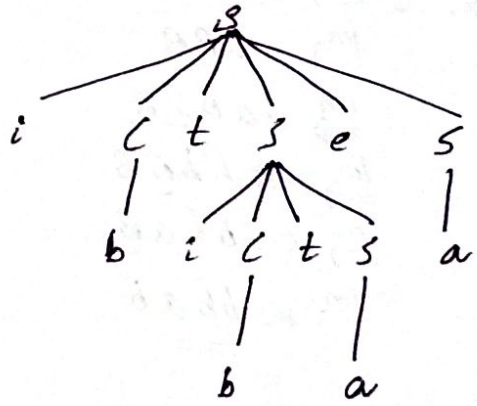
$\xRightarrow{lm} ibtSeS$

$\xRightarrow{lm} ibtiCtSeS$

$\xRightarrow{lm} ibtibtSeS$

$\xRightarrow{lm} ibtibtaeS$

$\xRightarrow{lm} ibtibtaea$



Since, $w = ibtibtaea$ has 2 LMD's & 2 parse tree's in the given string, hence it is ambiguous.