

# Mid Topics

## DFA

- ~~Finite Alphabet of its Output~~
- 1 F<sup>t</sup> input of each transition
- 1 f<sup>t</sup> | Multiple Transition Allowed

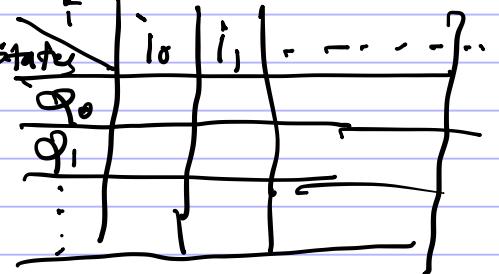
### Definition:

① States,  $Q = \{ \dots \}$

② Inputs,  $\Sigma = \{ \dots \}$

→ ③ Starting states,  $Q_0 = \boxed{\dots}$

\* ④ Finishing States,  $Q_f = \{ \dots \}$

⑤ Transition table: 

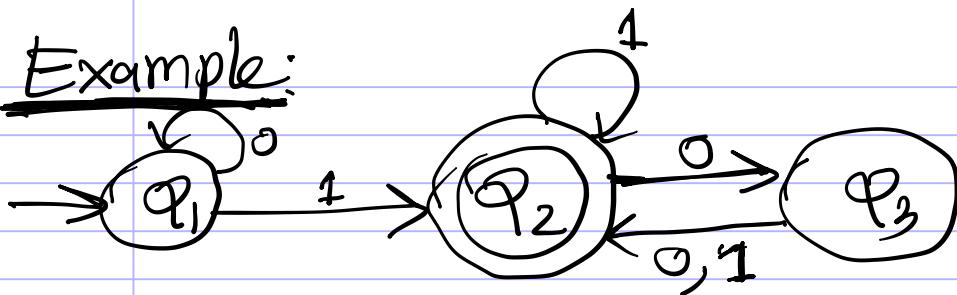
Design  
must be ~~good~~

⑥ Transition Equation:

$$g(\text{current state, input}) = \text{next state}$$

⑦ Language:

Example:



- ① States,  $Q = \{Q_1, Q_2, Q_3\}$
- ② inputs,  $\Sigma = \{0, 1\}$
- ③ Starting state,  $Q_0 = Q_1$
- \* ④ Finishing states =  $\{Q_2\}$

⑤ Transition table:

		0	1
states		0	1
→	$Q_1$	$Q_1$	$Q_2$
*	$Q_2$	$Q_3$	$Q_2$
	$Q_3$	$Q_1$	$Q_2$

⑥  $S(Q_1, 0) \rightarrow Q_1$

⑦  $L = \{w1w : \text{set of all strings that start with 1 and ends with even zero or 1}\}$

## Designing a DFA

\* যেসব টাইপের DFA এটা কৈ ?

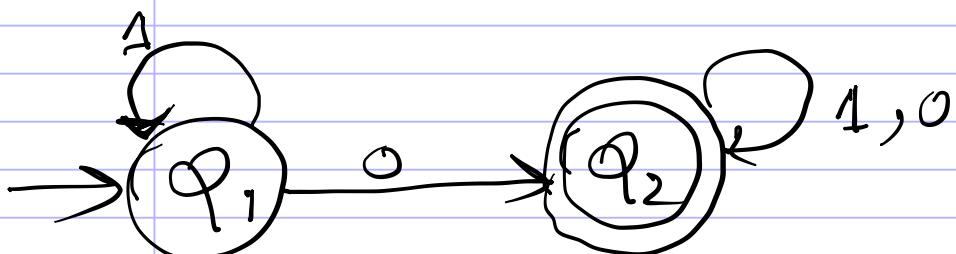
তাৰ প্ৰয়োজনীয়তা কৈমিক বিভিন্ন category-ক  
divide কৰে আৰু তাৰ মাধ্যমে সুলভ  
ৱায়ৰতা পাব।

### Contain Type

\* String that contains 0.  $\Sigma = \{0, 1\}$

Ans:

<u>Anv</u>	<u>N/A</u>
0	1
00	111
00	111.....
100 - ...	



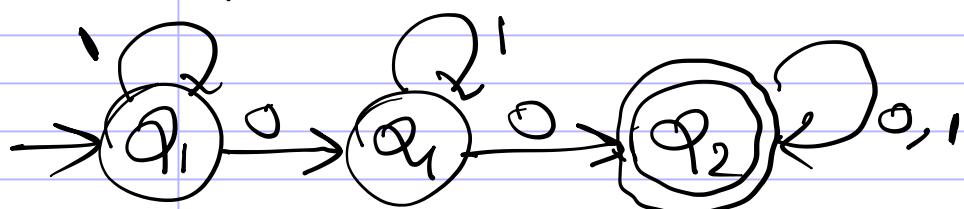
\* Contains atleast 2 zero  $\Sigma = \{0, 1\}$

A<sup>N</sup>

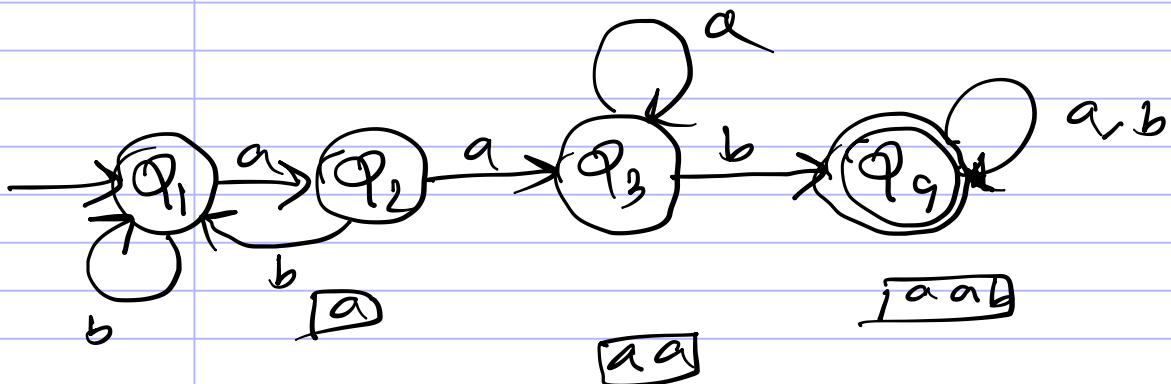
0011  
000  
100  
1010

NA

0  
101  
111 - - -

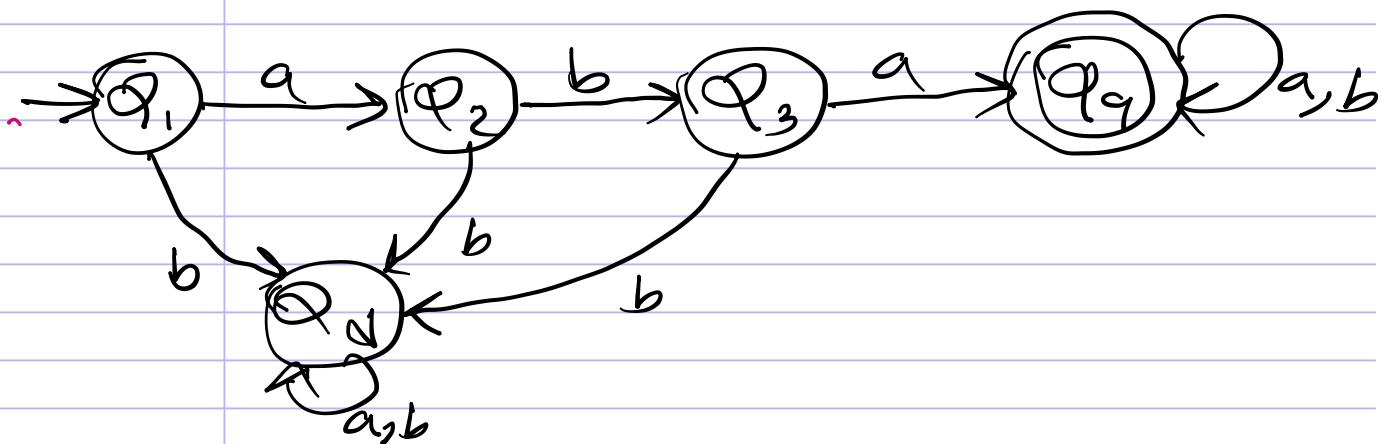


\* Draw a DFA with substring "aab"  
 $\Sigma = \{a, b\} \cup \{\text{ (contain type)}\}$



## Starts Type

\* Design a DFA that starts with "aba";  $\Sigma = \{a, b\}$



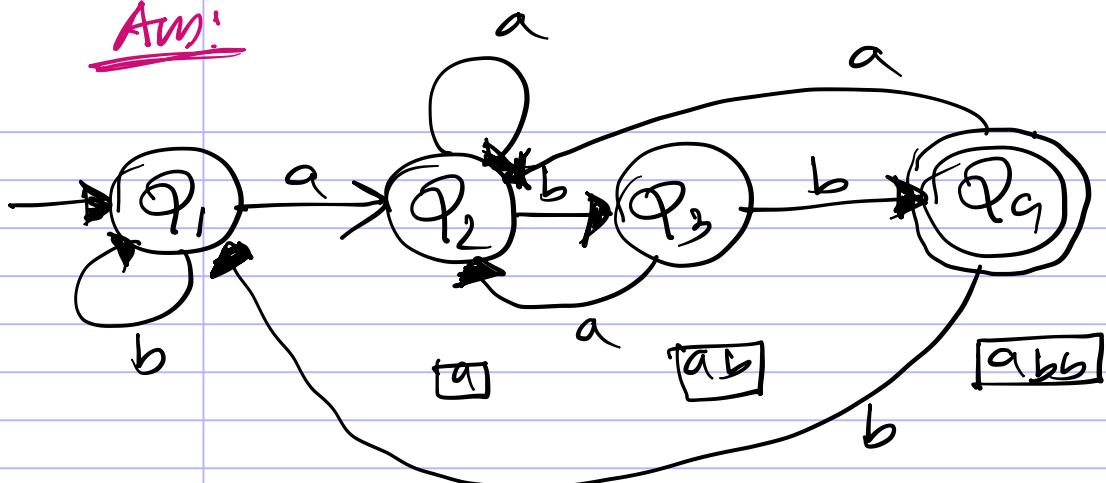
Note: Starts with ~~expected~~ with  
क्योंकि state  $\rightarrow$  expected input ताकि  
स्वयंस्थिर dead state  $\rightarrow$  इन छाड़ि।

## Ends with Type

[String फ़िर exactly word by word  
match करें end २०१७]

\* Design a DFA that ends with "abb";  $\Sigma = \{a, b\}$ .

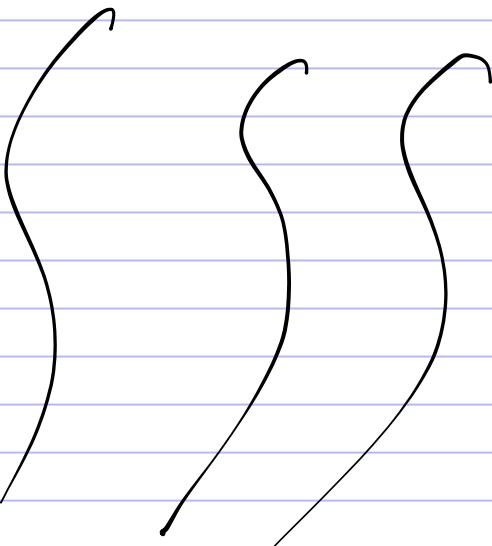
Ans:



ab  
—  
—ab

abb  
—  
—abb

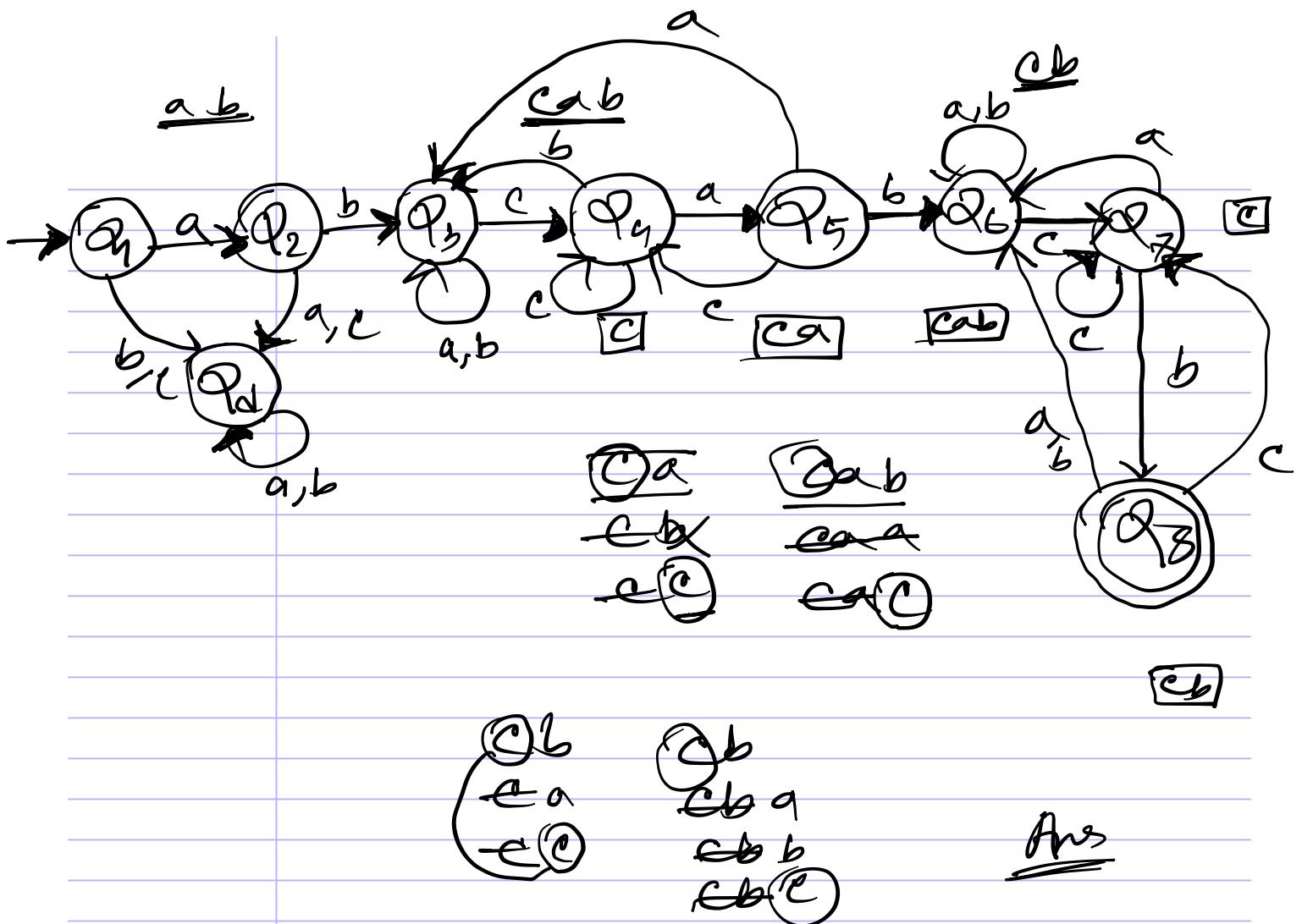
abb  
abba  
abbb



\* 3-type mixed strings \*

④ Design a DFA that starts with "ab"; contains "cab" and ends with "cb".

$\Sigma = \{a, b, c\}$



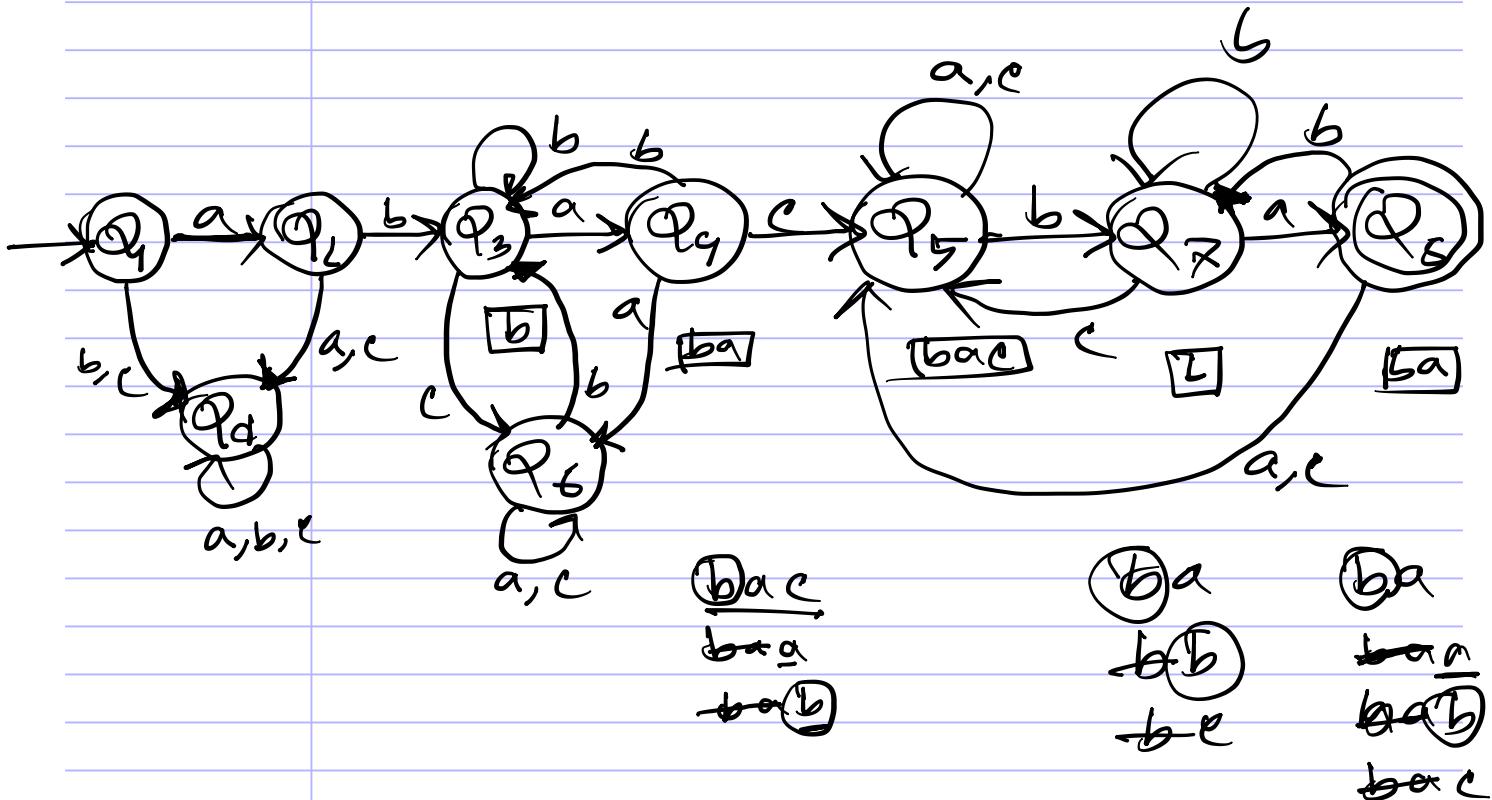
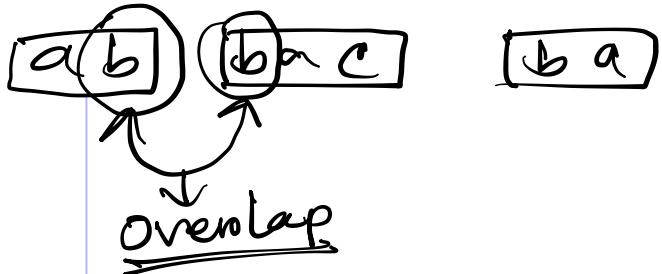
## Overlapping

[Overlapping  $\rightarrow$  start current state  
at  $\Sigma$  Always.]

Fall-29 (1-b)

Start with "ab", contains "bac", ends with "ba".  $\Sigma = \{a, b, c\}$ .

Sol'n:



## Trick for Unmatched:

→ check কোনো target এর প্রথম sequentially কোনো part এর সাথে match করে ফিনা।

( $a|b$ ) → target

( $a|a$ ) → aa এর small match কর

aa-contained state → return হার্ড।

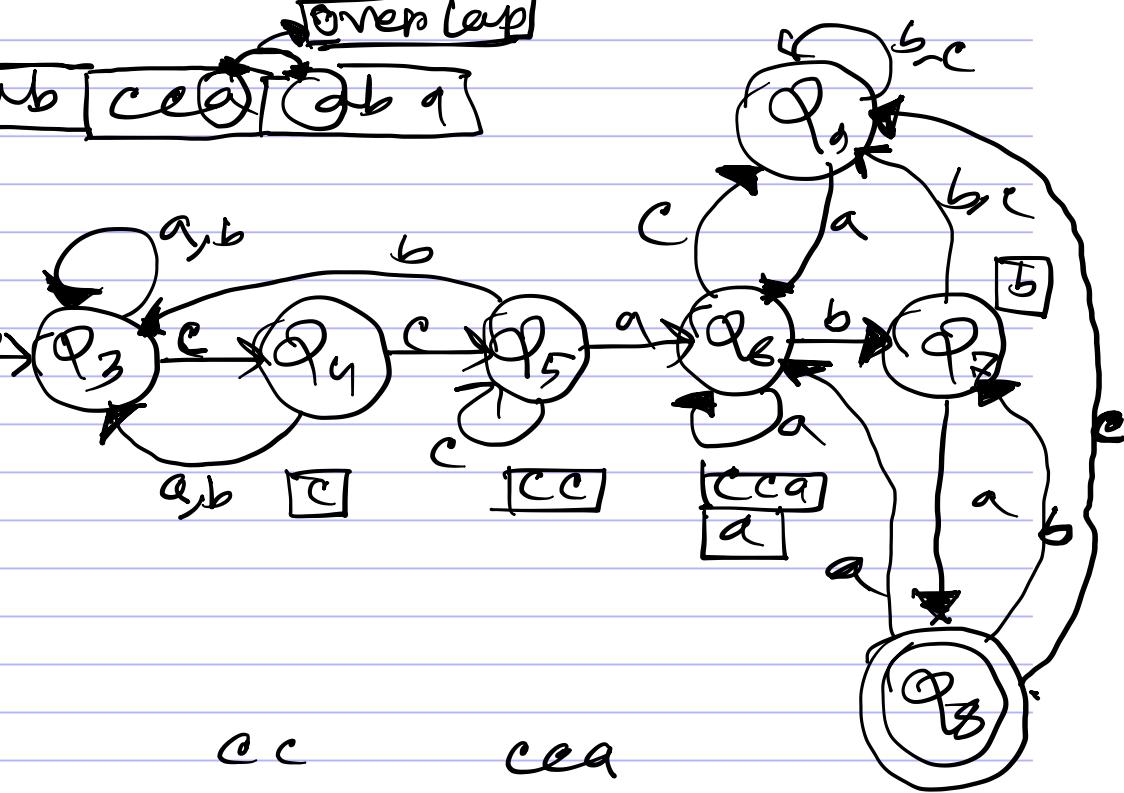
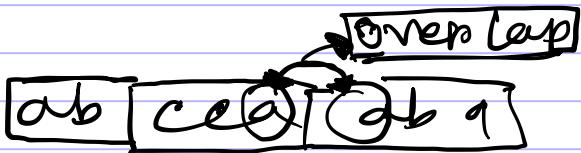
Unmatched string টির প্রথম ছেকে। If কোনো word কোনো এর target string এর small check করো এবং দ্যব্য কোনো অন্তর্ভুক্ত স্ট্রিং match করো ফিনা।

$\text{Q}_1|b$   
 $\begin{array}{c} a \\ \hline a \\ b \\ a \end{array}$  → no match  
 $\begin{array}{c} a \\ \hline a \\ b \\ \text{Q}_1 \end{array}$  → a-match

## Summer-29: 1-(b)

\* Starts with  $\boxed{ab}$ , contains  $\boxed{cca}$ , ends with  $\boxed{aba}$ ;  $\Sigma = \{a, b, c\}$

Sol'n:



~~cc~~  
~~ea~~  
~~eb~~  
cca  
cea  
ceb  
ccc

~~aba~~  
~~abb~~  
~~abc~~

~~aba~~  
~~aba~~  
~~aba~~  
abac

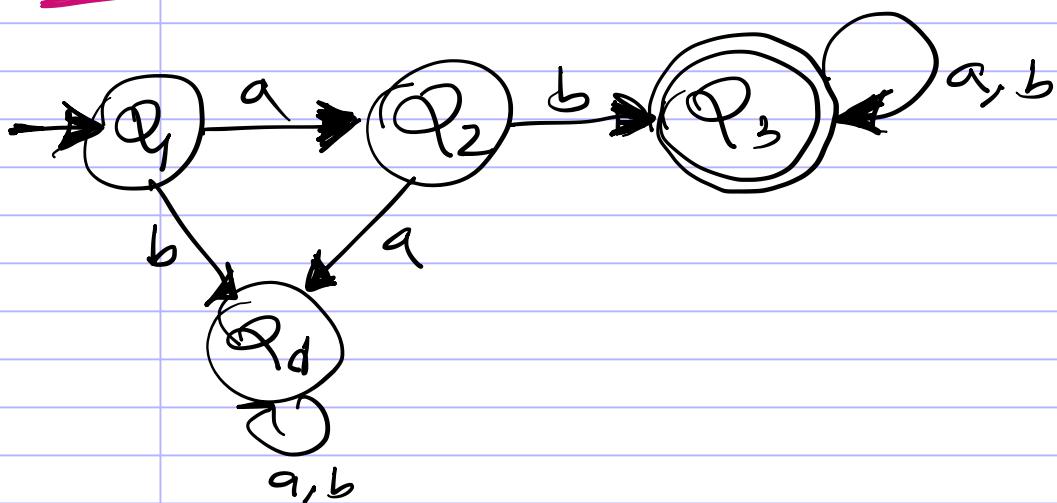
~~Ab~~

More Practice on  
till now completed types

~~(\*)~~ contains all strings that starts with a b

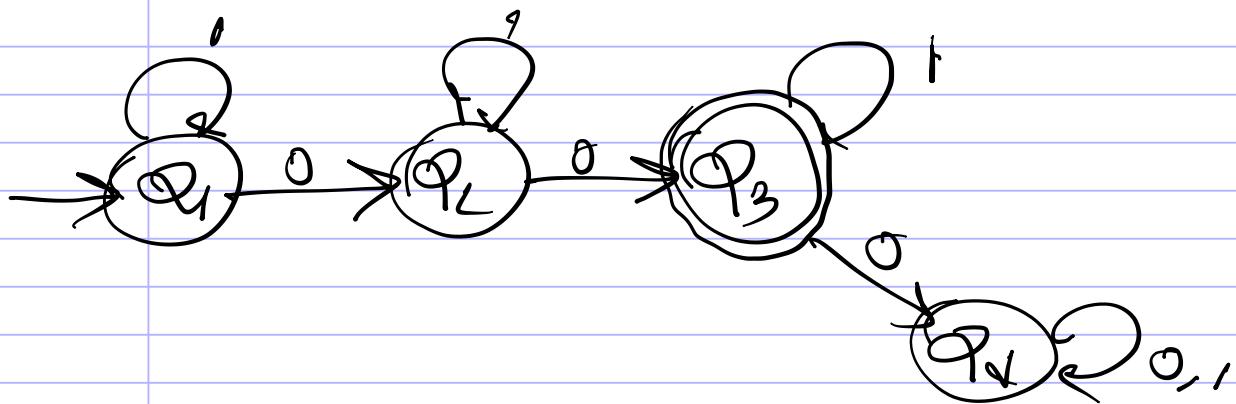
$$L \subseteq \{a, b\}^*$$

Ans:



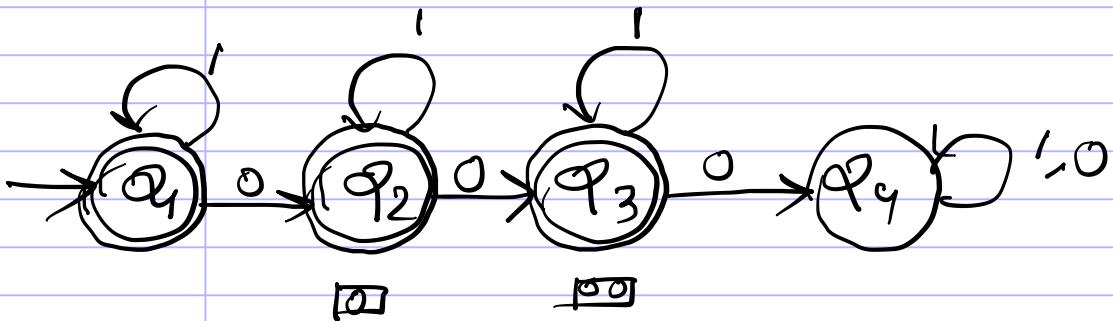
~~(\*)~~ contains exactly two zeroes.

$$\Sigma = \{0, 1\}^*$$



\* Contains at most 2 zeros  $\cdot \Sigma = 20, 14$

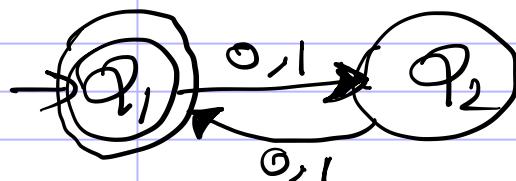
Ans:



Length-related type

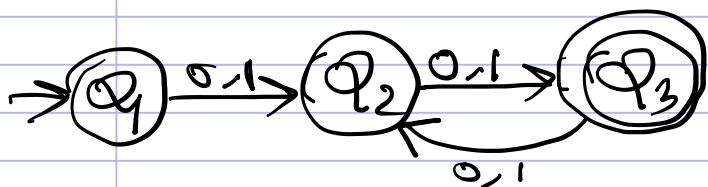
\* Length Even  $\cdot \Sigma = 20, 14$

Ans:

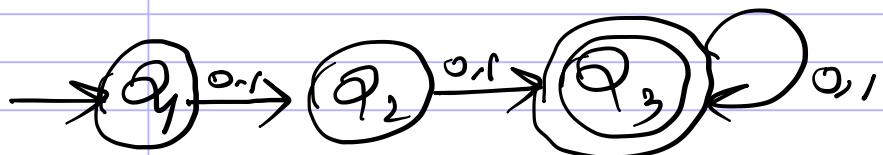


\* Length even where length is min 2.  $\Sigma = 20, 14$

Ans:

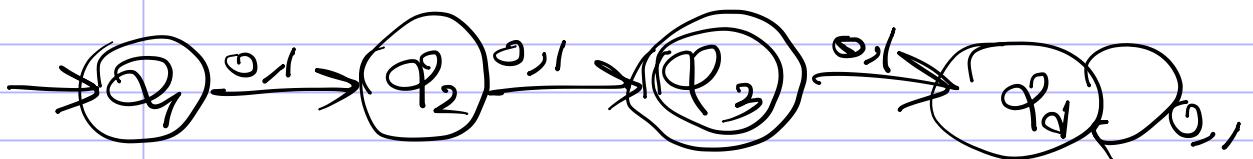


~~\*~~ Length at least two.  $\leq = 20, 14$



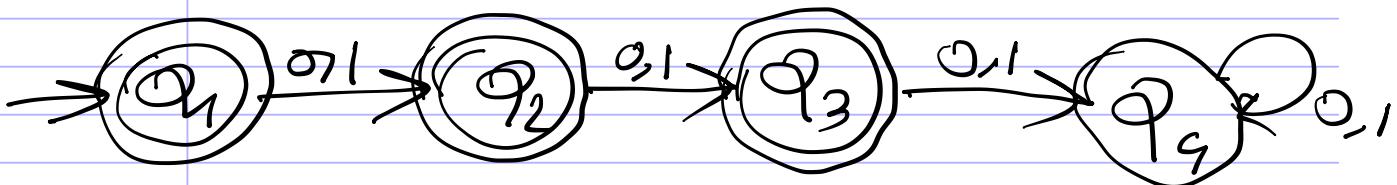
~~\*~~ Length is exactly  $2 \cdot 2 = 20, 14$

Ans:



~~\*~~ Length at most  $2 \cdot 2 = 20, 14$

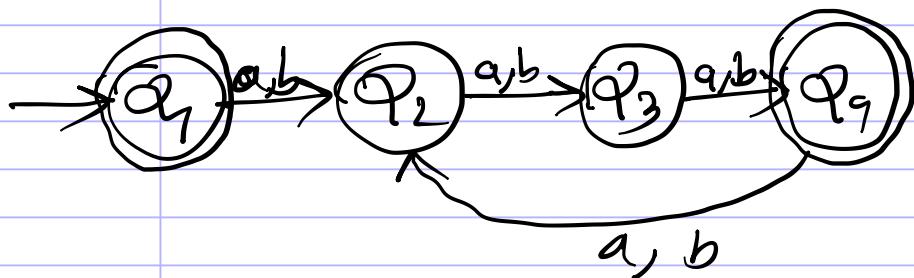
Ans:



## Divisible/Multiple-Type

\* Length is divisible by  $3 \cdot \Sigma = 2a, b$

Ans: AV: 0, 3, 6, 9, 12, ...

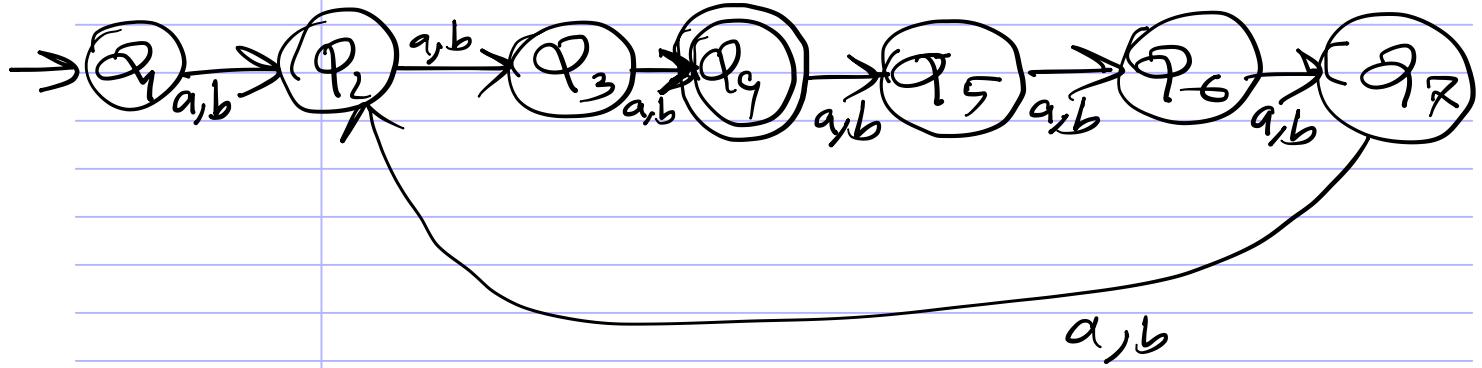


\* Length is divisible by 3 not 2 -  
 $\Sigma = 2a, b$ .

Ans:

AV: 3, 9, 15, 21, ...

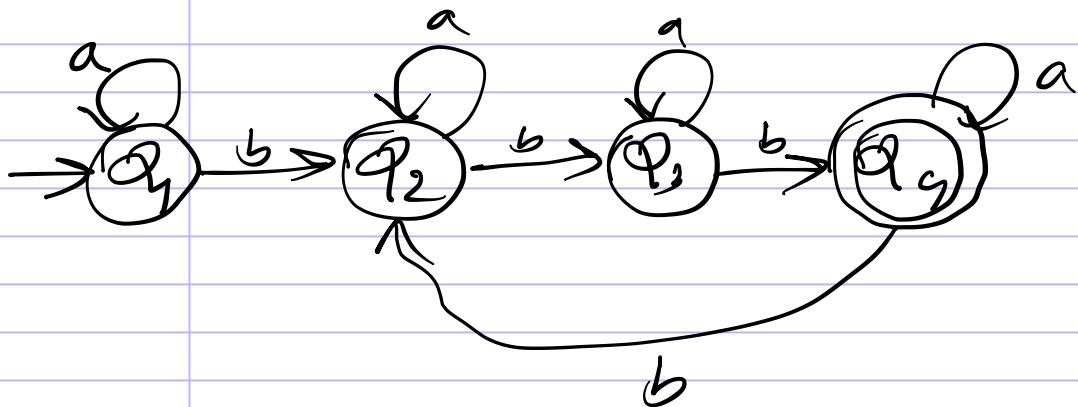
$\begin{matrix} \checkmark \\ 6 \end{matrix}$     $\begin{matrix} \checkmark \\ 6 \end{matrix}$     $\begin{matrix} \checkmark \\ 6 \end{matrix}$  }  $\rightarrow$  difference 6



\* Accept all strings where number of 'b' is divisible by 3 |  $\Sigma = \{a, b\}$ .

Ans:

AN:  $bbb, bbbb, \dots$



### \* Binary division/Multiplication-type \*

\* Accepts all strings that are divisible by three |  $\Sigma = \{0, 1\}$ .

⇒ എന്തെങ്കിലും ഒരു കമ്പ്യൂട്ടർ മുൻ്നായിരുന്ന് അതിനു സമാദായാക്കണമെന്ന് ഒരു binary string ദ്വേരുവായി എഴുതാൻ ശ്രമിച്ചു. അതിനു സഹായിക്കാൻ ഒരു ഗണിതജ്ഞന്റെ കാര്യാലയം ഒരു മാഷ്ട്രൈഡ് മൈഷ്യൂണ്ട് ആക്കുകയോജിപ്പിച്ചു. അതിനു സഹായിക്കാൻ ഒരു ഗണിതജ്ഞന്റെ കാര്യാലയം ഒരു മാഷ്ട്രൈഡ് മൈഷ്യൂണ്ട് ആക്കുകയോജിപ്പിച്ചു.

\* Every remainder = 0 state always start and accepted state.

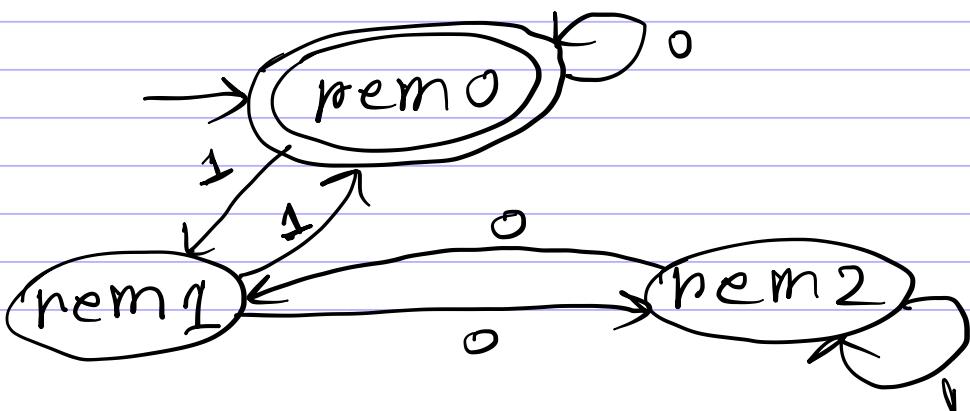
### Tips:

- ① প্রদত্ত ক্ষেত্রফল এবং যথেষ্ট remainder অঙ্কগুচ্ছ, তিনি তাঁর states নির্ণয়।
- ② প্রদত্ত এক ক্ষেত্রফল length এবং দ্বিমুক্ত binary অঙ্কগুচ্ছ নির্ণয়।
- ③ Binary অঙ্কগুচ্ছ প্রতিটির decimal দৈর্ঘ্য remainder হিসেব করবে, এবং state  $n \mod 2$  হিসেব করে remainder এর state নির্ণয় করবে।
- ④ Sample string অঙ্কগুচ্ছের input ফর্মে চিহ্ন করে প্রতিটি state এর অঙ্ক next state কেও করবে।

৩ টাকা ক্ষেত্রফল remainder mod 2 থেকে।

ট্রিভাস, remainders: 0, 1, 2

অঙ্কগুচ্ছ ৩টি states: rem0, rem1, rem2



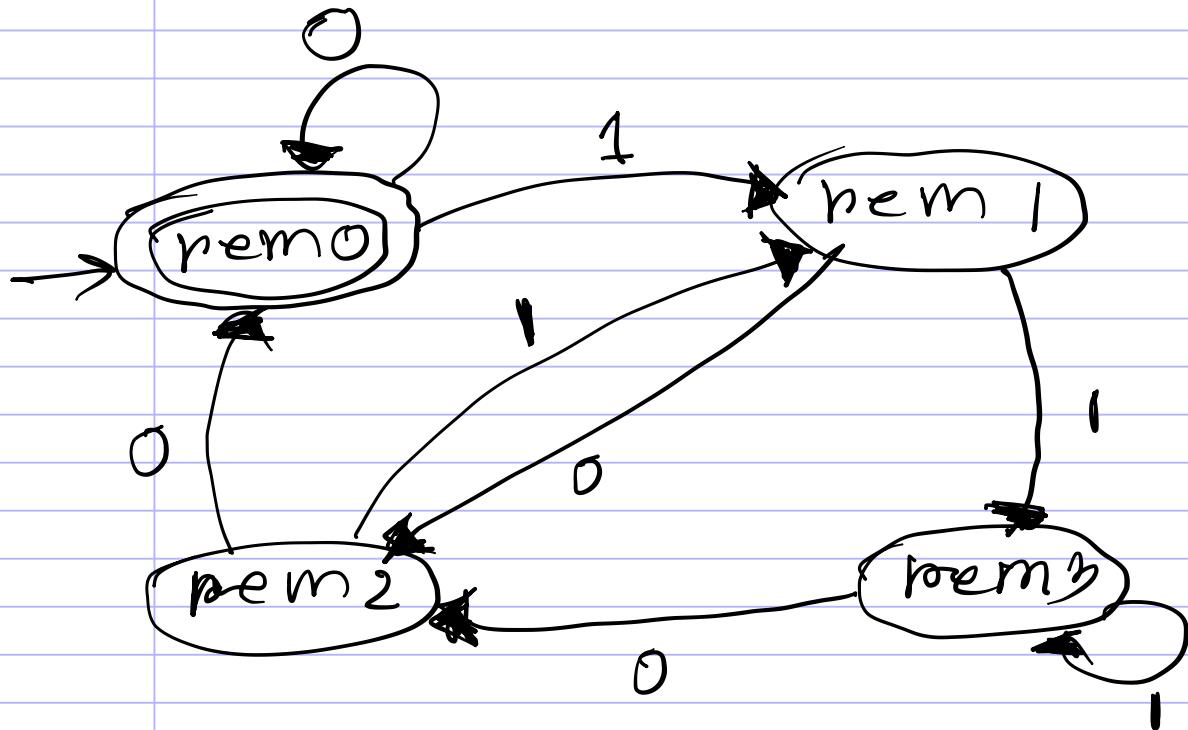
<u>decimal</u>	<u>binary</u>	<u>remainder</u>
0	000	000
1	001	001
2	010	010
3	011	000
4	100	001
5	101	010
6	110	000
7	111	001 <u>Ans</u>
- - - - - - - - - - -		

\* Binary String divisible by 4.  
 $\Leftrightarrow$  0, 14,

Ans: remainders: 0, 1, 2, 3

$\Rightarrow$  0000, 0001, 0010, 0011, - - - - -

<u>Decimal</u>	<u>Binary</u>	<u>remainder</u>
0	0000	0000(0)
1	0001	0001(1)
2	0010	0010(2)
3	0011	0011(3)
9	0100	0000(0)
5	0101	0001(1)
6	0110	0010(2)
7	0111	0011(3)



## And/Or type

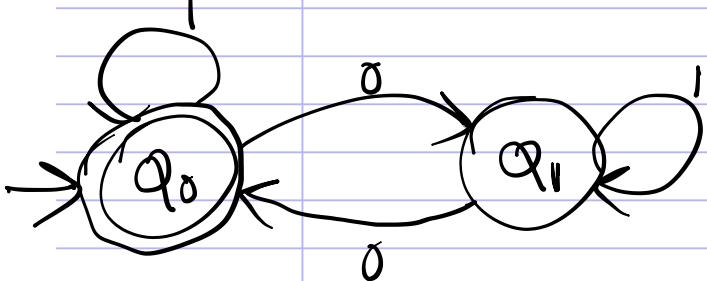
### Steps:

- ① প্রতিটি যোগসমূহ ক্ষেত্র individual DFA'র Automata হাতাবে শোনা দ্বা পাবে।  
এই গুদের states সুলভের মতোই common নামে থাকবে পার্শ্বের।
- ② প্রতিটি যোগ ক্ষেত্র transition table হাতাব।
- ③ Transition table সুলভের মতোই cartesian product করব, তাহুন  
ক্ষেত্র transition table এবং,
- ④ নতুন transition table হৃতে নতুন  
DFA - Diagram হাতাবে।
- ⑤ প্রথম 3 ২য় ক্ষেত্র transition table  
এই start state সুলভ করো  
নতুন start state পাবে।
- ⑥ ~~প্রথম~~ AND ক্ষেত্র : Both transition table  
→ final state, new table এই ২টি  
state জোগবে। ক্ষেত্র হৃতে।  
→ OR ক্ষেত্র : Both table এই final  
state সুলভ করোনা ক্ষেত্র state,

new table of के यहाँ state ड  
मार्किंग करें final state डॉ

\* Even numbers of "0" and Odd  
numbers of "1" alphabet  $q_0, q_1$ .

Ans:



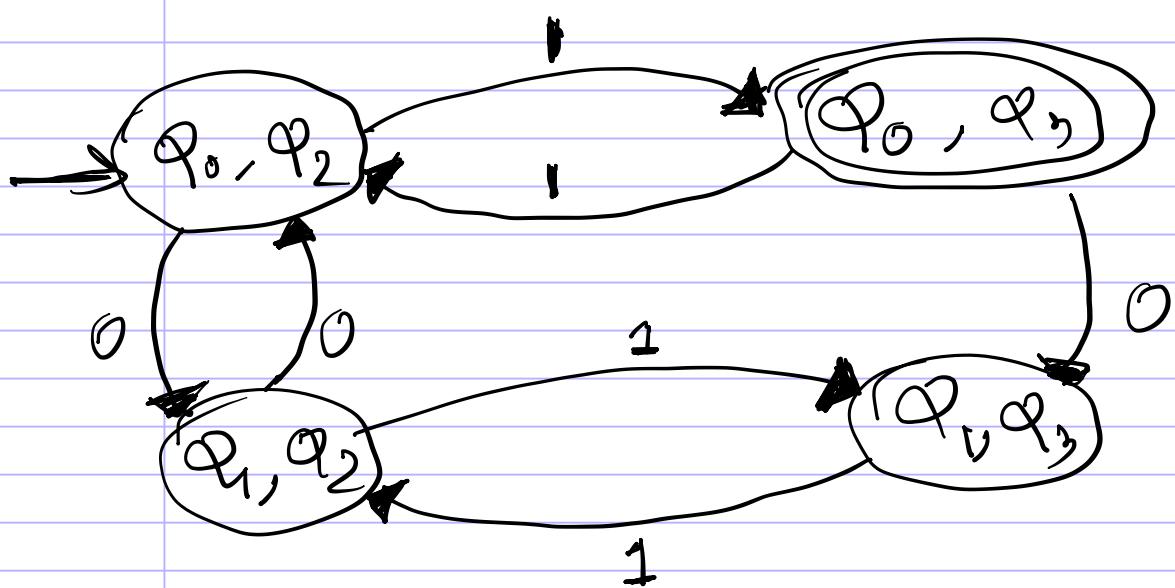
$T_1$ input states	0	1
$\rightarrow q_0$	$q_1$	$q_0$
$\rightarrow q_1$	$q_0$	$q_1$

$T_2$ input states	0	1
$\rightarrow q_2$	$q_2$	$q_3$
$\rightarrow q_3$	$q_3$	$q_2$

$T_1 \times T_2 \Rightarrow$  Cartesian Product

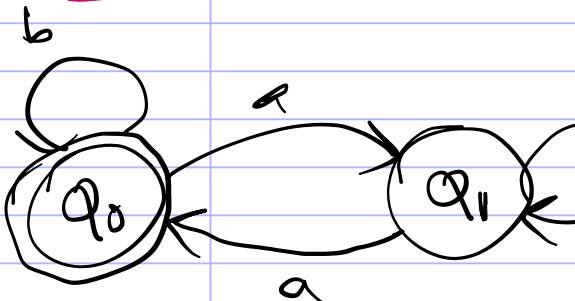
Input

States	0	1
$\rightarrow \{q_0, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_3\}$
$* \{q_0, q_3\}$	$\{q_1, q_3\}$	$\{q_0, q_2\}$
$\{q_1, q_2\}$	$\{q_0, q_2\}$	$\{q_1, q_3\}$
$\{q_1, q_3\}$	$\{q_0, q_3\}$	$\{q_1, q_2\}$

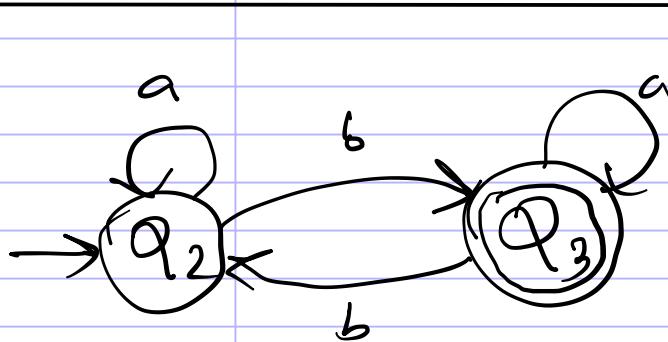


\* Even number of "a" OR Odd number of "b" alphabet  $\{a, b\}$ .

Ans:



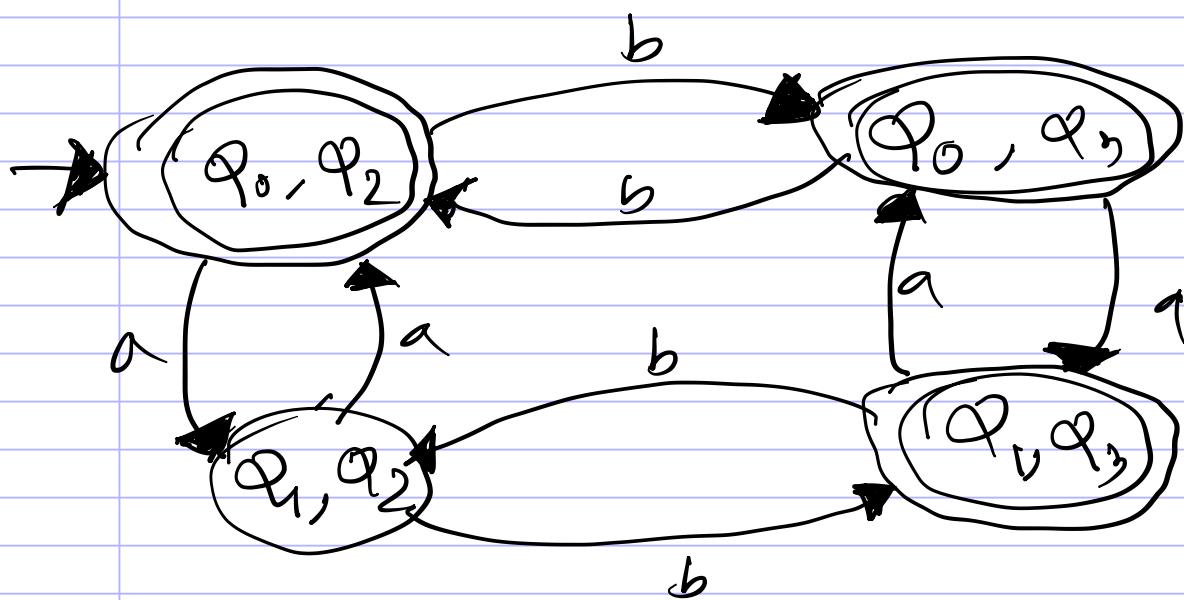
<del>T1 input states</del>	a	b
<del>*</del>	$q_0$	$q_1$
	$q_1$	$q_0$



<del>T2 input states</del>	a	b
<del>*</del>	$q_2$	$q_3$
	$q_3$	$q_2$

$T_1 \times T_2 \Rightarrow$  Cartesian Product

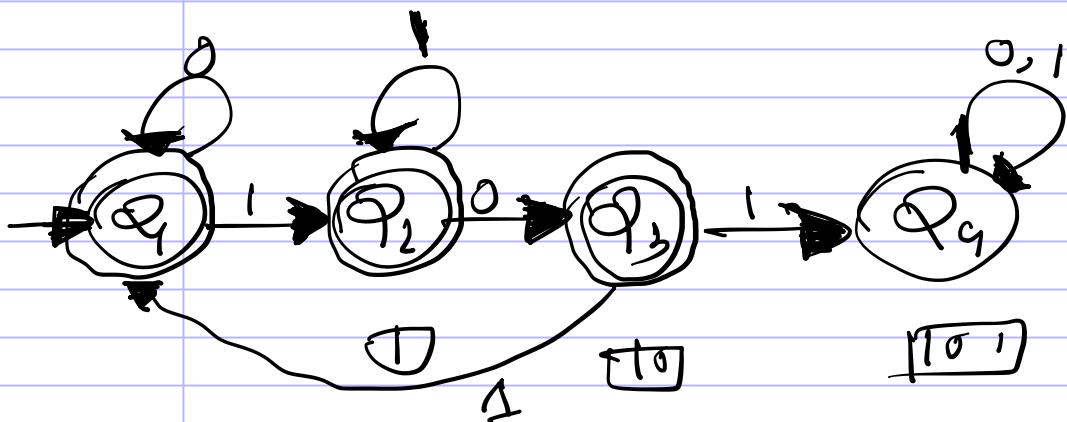
Input	a	b	
States			
*	$\{q_0, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_3\}$
*	$\{q_0, q_3\}$	$\{q_1, q_3\}$	$\{q_0, q_2\}$
	$\{q_1, q_2\}$	$\{q_0, q_2\}$	$\{q_1, q_3\}$
*	$\{q_1, q_3\}$	$\{q_0, q_3\}$	$\{q_1, q_2\}$



## Does not - Type

\* २ वा type एँ कोट्या प्रभावामध्ये Does एव्या  
स्टेट Automata design रश्यावर, एकूण प्रत्येक  
स्टेटला state रुपे toggle करावे दिला।

■ Doesn't contain  $101 \cdot \Sigma^{\neq 0, 1}$



$$\begin{matrix} 0 \\ + \\ 1 \end{matrix} \quad (01) \quad + \quad 00$$

Fall-29: 1(a)

\* Doesn't contain "001" but ends  
with 11 ·  $\Sigma^{\neq 0, 1}$

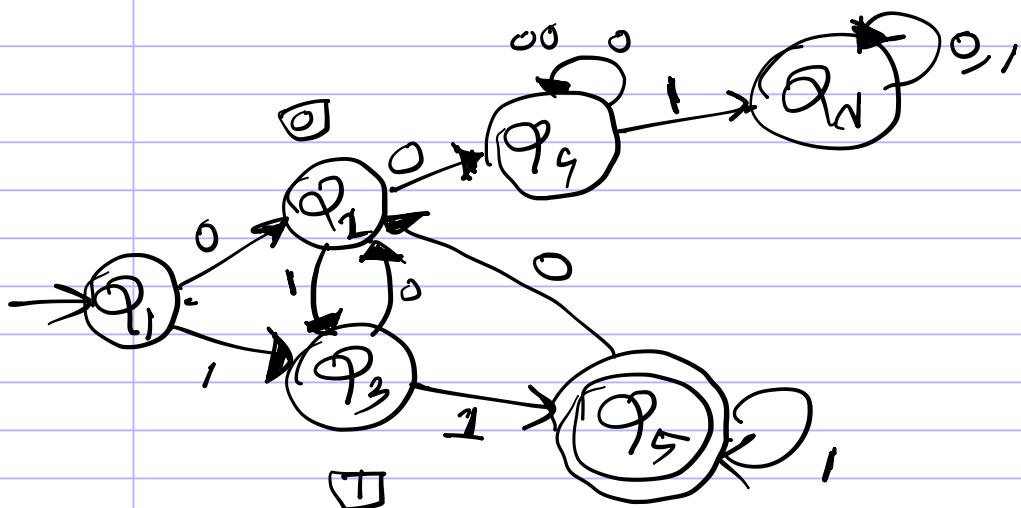
Ans:

A<sub>2</sub>

11	
1011	
011	...

N/A  
001

0011	
1100	



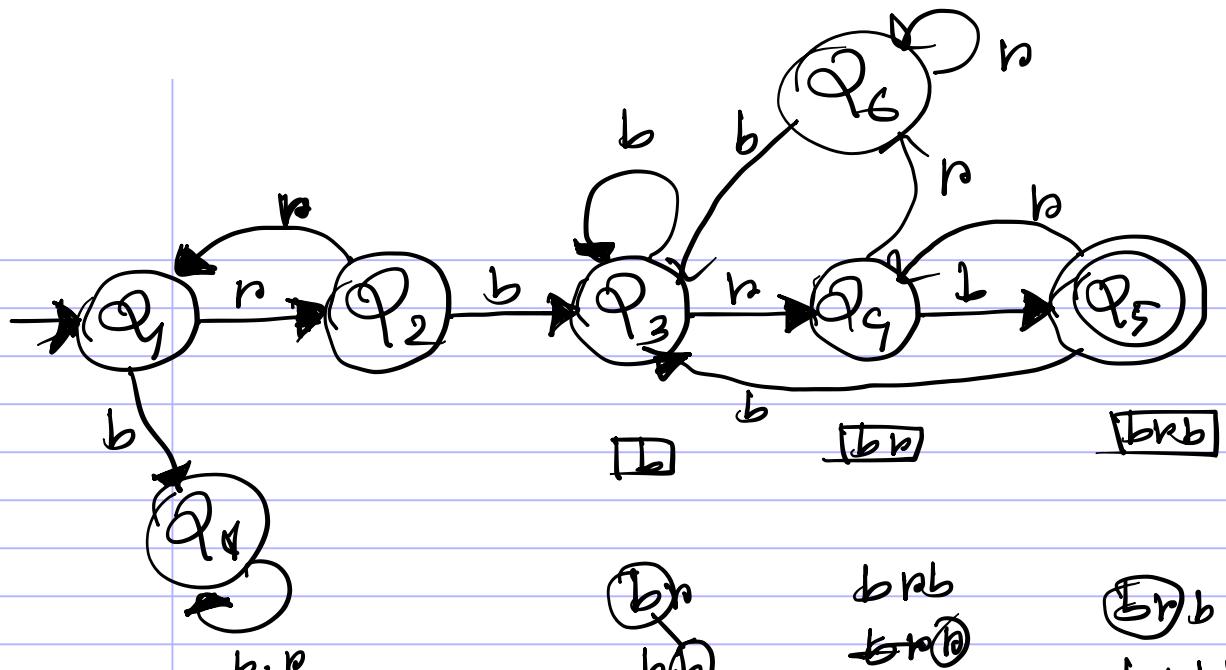
A<sub>2</sub>

\* Lets Practice Some General type \*

Fall - 23 (1 - b)

(\*) Starts with odd no. of "r", ends with "bbb",  $\Sigma = \{b, r\}$ ,

Ans:



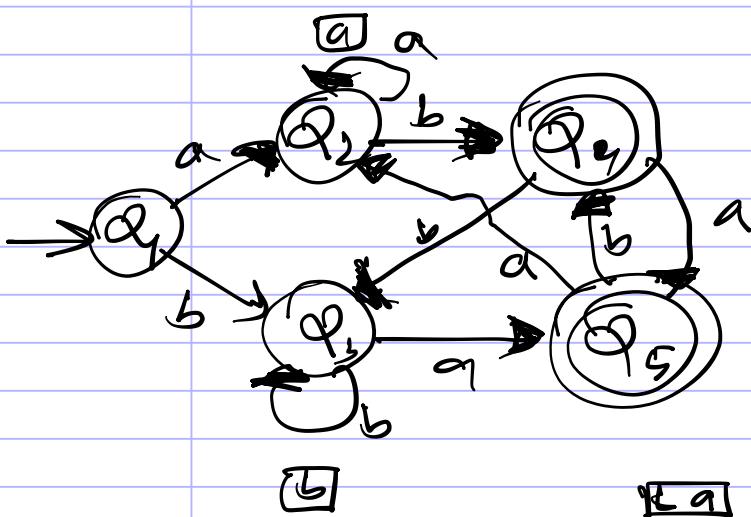
$brb$   
 $brb$   
 $brb$   
 $bpb$   
 $bpb$   
 $bpb$

Aus

### Path 1-23 (1-c)

\* Last two symbol are different.  
 $\leq = 2a, 1b$ .

Av: ...ab | ...ba | NA: ...aa, ...bb



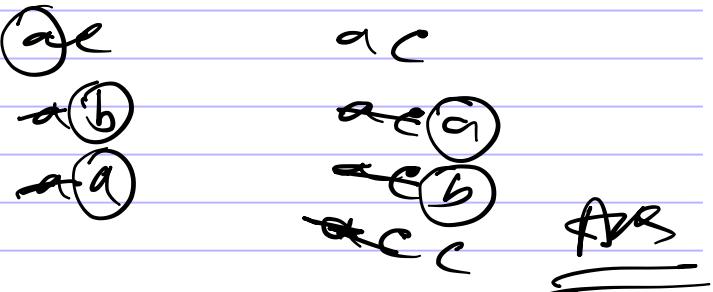
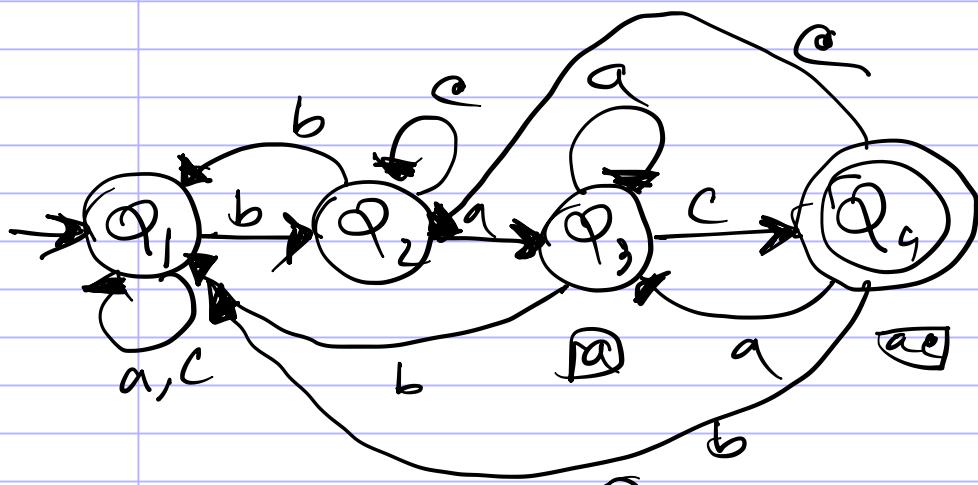
$ab$   
 $ba$   
 $a$   
 $b$

$ab$   
 $-ab$   
 $a$   
 $ab$   
 $ba$   
 $a$   
 $b$   
 $ab$

## Spring - 29 (1-c)

\* Contains odd no. of "b" and ends with "ac".  $\Sigma = 2a, b, c$

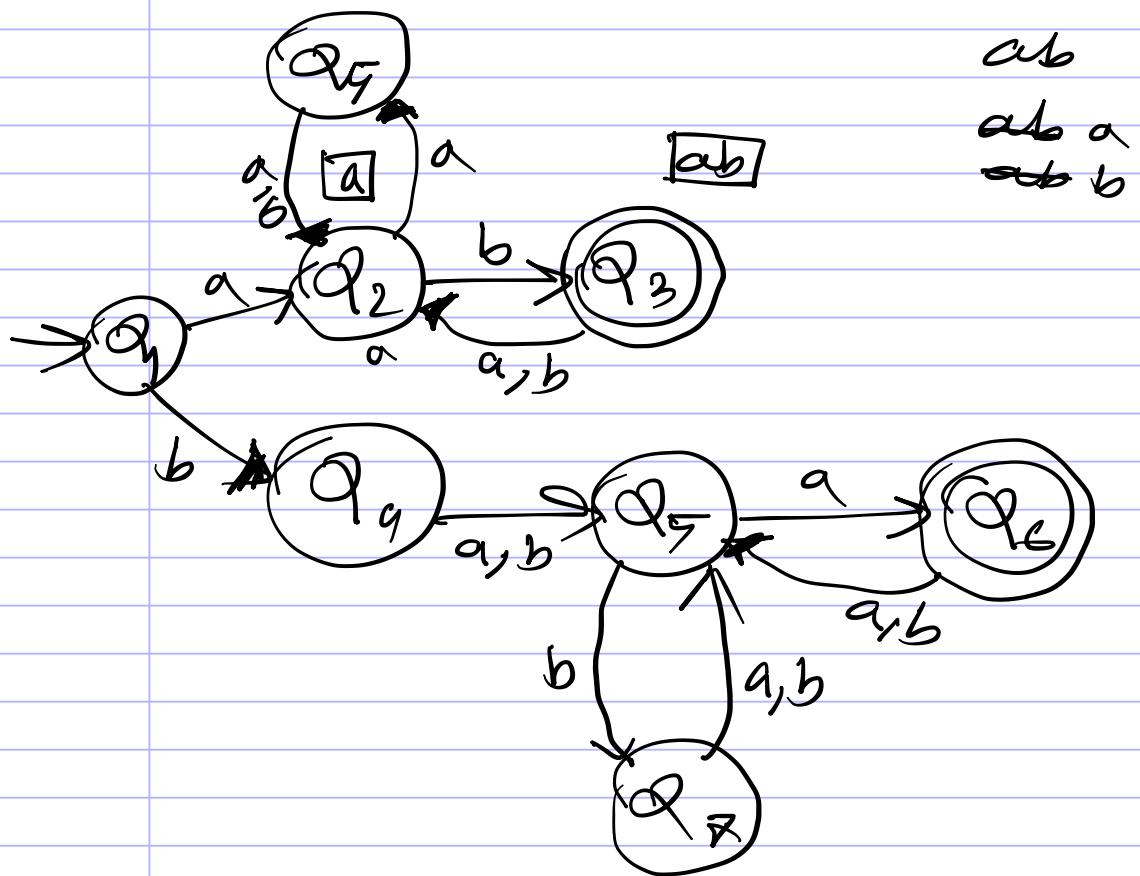
Sol:



## Summers - 29:

\* Starts and ends with different symbol, having even length strings when starts with "a" or odd length strings when starts with "b". Where  $\Sigma = \{a, b\}$ .

Ans:



Ans

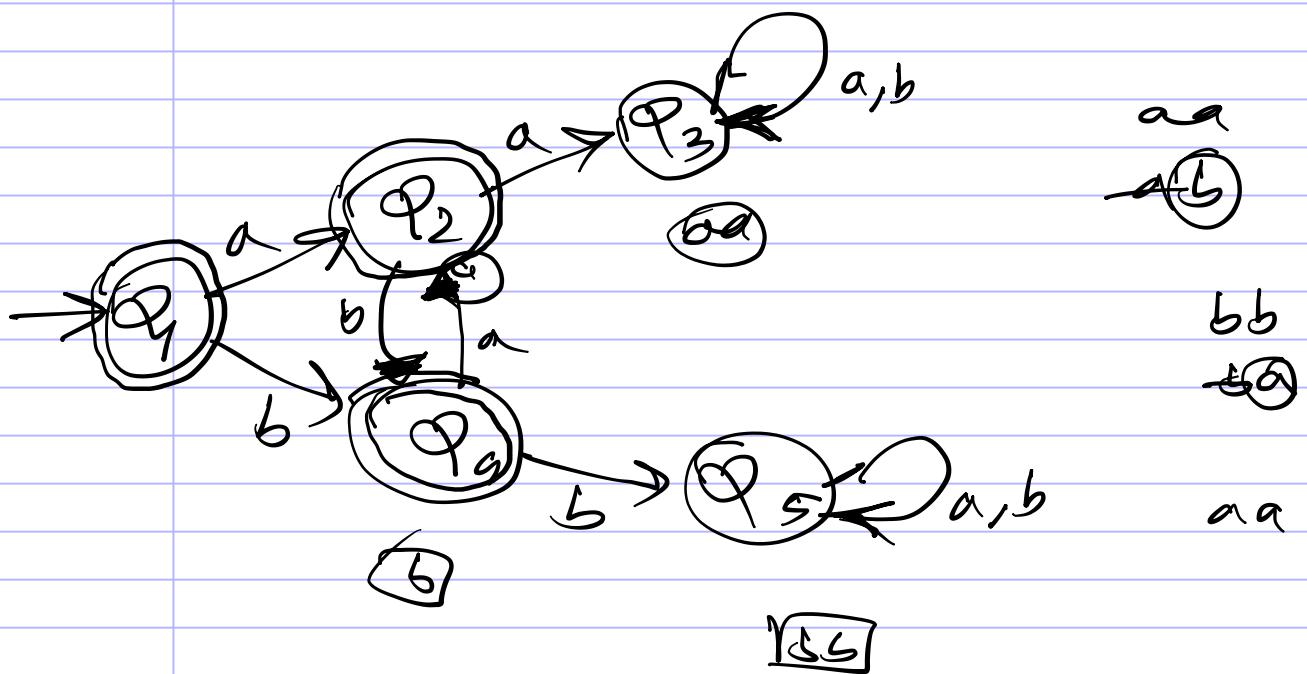
## Summer-29 (1+c)

\* Neither contains two consecutive a's nor two consecutive b's.  
 $\leq = \{a, b\}$ .

Ans:

NA ✓  
... aa - -  
-- bb - -

A ✓  
a  
b  
ab bba  
ba .. - -



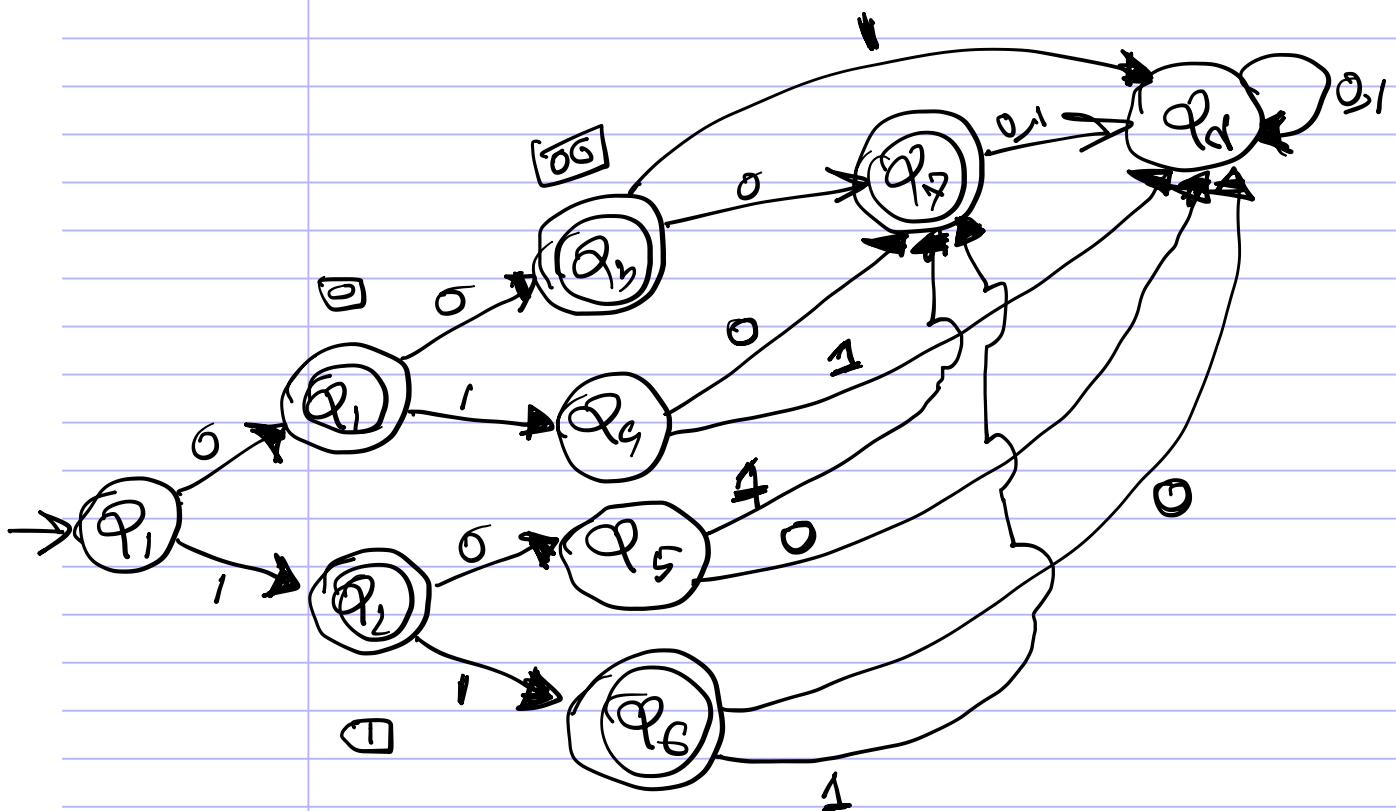
# Palindrom

Summer - 23 [1-C]

\*  $L = \{a_1 a_2 \dots a_n\}$  is a palindrom with max length of  $|a_i| \leq 2, 0, 1$

Ans:

AV: 0, 1, 00, 11, 000, 111, 010, 101



Ans

## Summer-23 (1-d)

$L = \{a^i b^j \mid i \geq 0, j \geq 0, i+j \text{ is an odd number}\}$   
 $| \Sigma = \{a, b\}.$

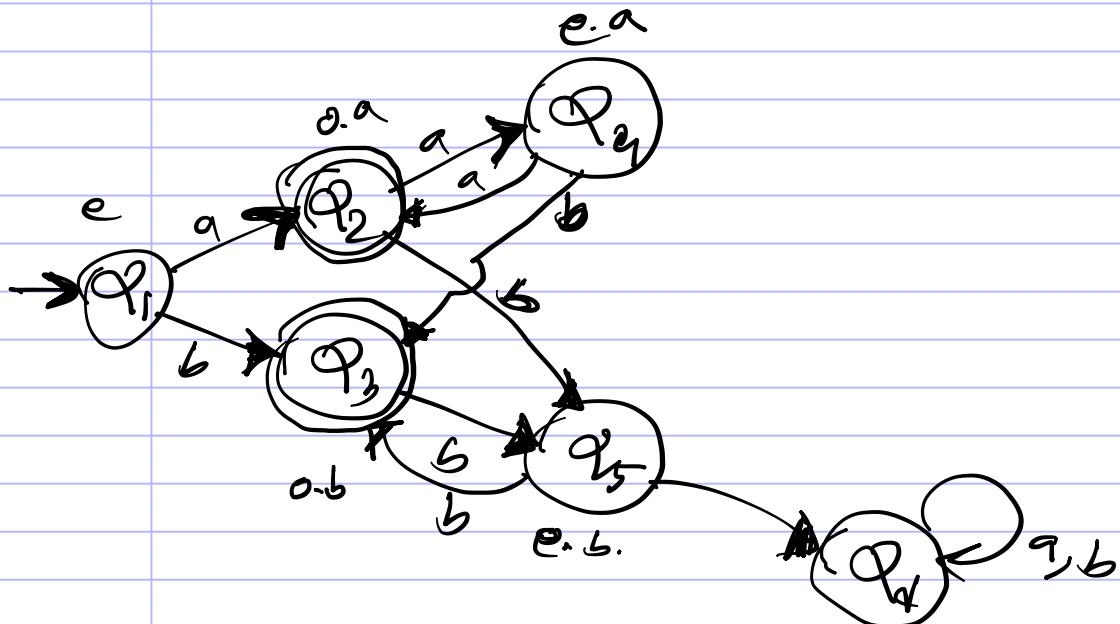
Ans:

odd + odd = even  $\times$

even-a + odd-b = odd  $\checkmark$

odd-a + even-b = odd  $\checkmark$

even+even = even  $\times$



Ans

Spring - 29: 1-(a)

⊗ L =  $q_w w q_1$  starts with "23", contains "943" as a substring and ends with "32"  $\Sigma = \{2, 3, 9\}$

solution like overlap

