IT-081 Pankhania Aanandi R.
DAA

Floyd Algorithm

i) Problem Analysis:-

i/p:- vertices : $V = \{1, 2, \ldots, n\}$
Assume that graph is represented by an $n \times n$ matrix, with weights of the edges.

$$w_{ij} = \begin{cases} 0 & ; \text{if } i=j \\ w(i,j) & ; \text{if } i \neq j \ \& \ (i,j) \in E \\ \infty & ; \text{if } i \neq j \ \& \ (i,j) \notin E \end{cases}$$

O/p:- $n \times n$ matrix $D = [d_{ij}]$, where d_{ij} is the length of the shortest path from vertex i to j.

(ii) Development Model:-

Smallest problem:- When there is no intermediate node between i^{th} node to j^{th} node.

So, $d_0[i,j]$.

When there is 1 ~~as~~ intermediate node

$d_1[i,j]$

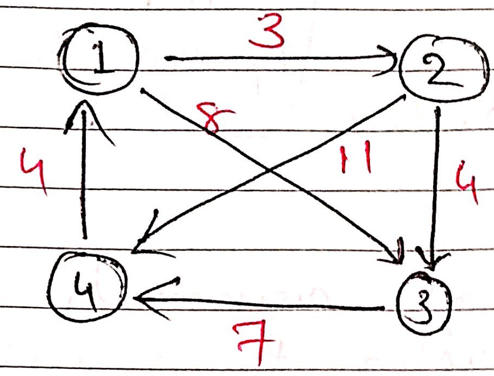
$d_0[i,j]$ $\quad$ $d_0[i,k] + d_0[k,j]$ $\quad$; $k=1$.

$$\text{So, } d_1[i,j] = \min\left(d_0[i,j],\ d_0[i,1] + d_0[1,j]\right)$$

When there is k intermediate nodes i.e $(1,2,\ldots,k)$

$$d_k[i][j] = \min\left(d_{k-1}[i,j];\ d_{k-1}[i,k] + d_{k-1}[k,j]\right)$$

example (Bottom-up calculation)



$$D_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 8 & \infty \\ 2 & \infty & 0 & 4 & 11 \\ 3 & \infty & \infty & 0 & 7 \\ 4 & 4 & \infty & \infty & 0 \end{array}$$

$D_1 \Rightarrow$ intermediate node $\boxed{1}$

$$D_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 8 & \infty \\ 2 & \infty & 0 & 4 & 11 \\ 3 & \infty & \infty & 0 & 7 \\ 4 & 4 & 7 & 12 & 0 \end{array}$$

4,3
$\Rightarrow$ 4,1 + 1,3
4 + 8
= 12 < ∞

$2,1 \Rightarrow (2,1)+(1,1) = (2,1)$

$2,3 \Rightarrow \min\left(\underbrace{((2,1)+(1,3))}_{\infty}, \underbrace{(2,3)}_{4}\right) = 4$

$2,4 \Rightarrow \min(\infty, 11) = 11$

$3,2 \Rightarrow (3,1) \Rightarrow \infty$ ∴

$3,4 \Rightarrow 4$ (∵ $(3,1) \Rightarrow \infty$)

$4,2 \Rightarrow \min\left((4,1)+(1,2), (4,2)\right) = \min(4+3, \infty)$
$= \boxed{7}$

$$D_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 7 & 14 \\ 2 & \infty & 0 & 4 & 11 \\ 3 & \infty & \infty & 0 & 7 \\ 4 & 4 & 7 & 11 & 0 \end{array}$$

$1,3 \rightarrow (1,2)+(2,3) = 3+4 = 7 < 8 = 7$

$1,4 \rightarrow 1,2 + 2,4 = 3+11 = 14 < \infty = 14$

$3,1 \rightarrow 3,2 + 2,1 = \infty$

$3,4 \rightarrow 3,2 + 2,4 = \infty + \ldots > 7 = 7$

$4,1 \rightarrow 4,2 + 2,1 = \quad > 4$

$4,3 \rightarrow 4,2 + 2,3 = 7 + 4 = 11 < 12$

$$D_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & \cancel{8}3 & 7 & 14 \\ 2 & \infty & 0 & 4 & 11 \\ 3 & \infty & \infty & 0 & 7 \\ 4 & 4 & 7 & 11 & 0 \end{array}$$

$1,2 \rightarrow 1,3 + 3,2 = 7 + \infty > 3$

$1,4 \rightarrow 1,3 + 3,4 = 7 + \cancel{4}\,7 = 14 = 14.$

$2,1 \rightarrow 2,3 + 3,1 = 4 + \infty \approx \infty$

$2,4 \rightarrow 2,3 + 3,4 = 4 + 7 = 11 = 11$

$4,1 \rightarrow 4,3 + 3,1 \sim \infty < 4$

$4,2 \rightarrow 4,3 + 3,2 \sim \infty < 7$

$4,$

$$D_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 7 & 14 \\ 2 & \infty & 0 & 4 & 11 \\ 3 & \infty & \infty & 0 & 7 \\ 4 & 4 & 7 & 11 & 0 \end{array}$$

$$D_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 7 & 14 \\ 2 & 15 & 0 & 4 & 11 \\ 3 & 11 & 14 & 0 & 7 \\ 4 & 4 & 7 & 11 & 0 \end{array}$$

$1 \to 2 : 1 \to 4 + 4 \to 2 = 14 + 7 > \cancel{\infty}\ 3$

$1 \to 3 : 1,4 + 4,3 = 14 + 11 > 7$

$2 \to 1 : 2,4 + 4,1 = 11 + 4 = 15 < \infty \quad 15$

$2 \to 3 : 2,4 + 4,3 = 11 + 11 = 22 > 4$

$3 \to 1 : 3,4 + 4,1 = 7 + 4 = 11 < \infty$

$3 \to 2 : 3,4 + 4,2 = 7 + 7 = 14 < \infty$

Algorithm :-

Floyd - Warshall (W)

$n \leftarrow rows[W]$
$D^{0} \leftarrow W$
for $k \leftarrow 1$ to n
  do for $i \leftarrow 1$ to n
    do for $j \leftarrow 1$ to n
      do
        $d_{ij}^{(k)} \leftarrow \min\left(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right)$

return $D^{(n)}$

Time complexity :- $O(n^3)$ for $\rightarrow$ 3 nested for loops

Space Complexity :- $O(|n|^2)$ $\rightarrow$ for matrix $(n \times n)$: n^2