Université Libre de Bruxelles

Summary

Turbomachinery
MECA-H-402

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Appel à contribution

Synthèse Open Source

Ce document est grandement inspiré de l'excellent cours donné par Patrick HENDRICK à l'EPB (École Polytechnique de Bruxelles), faculté de l'ULB (Université Libre de Bruxelles). Il est écrit par les auteurs susnommés avec l'aide de tous les autres étudiants et votre aide est la bienvenue ! En effet, il y a toujours moyen de l'améliorer surtout que si le cours change, la synthèse doit être changée en conséquence. On peut retrouver le code source à l'adresse suivante

`https://github.com/nenglebert/Syntheses`

Pour contribuer à cette synthèse, il vous suffira de créer un compte sur *Github.com*. De légères modifications (petites coquilles, orthographe, ...) peuvent directement être faites sur le site ! Vous avez vu une petite faute ? Si oui, la corriger de cette façon ne prendra que quelques secondes, une bonne raison de le faire !

Pour de plus longues modifications, il est intéressant de disposer des fichiers : il vous faudra pour cela installer LaTeX, mais aussi *git*. Si cela pose problème, nous sommes évidemment ouverts à des contributeurs envoyant leur changement par mail ou n'importe quel autre moyen.

Le lien donné ci-dessus contient aussi un `README` contenant de plus amples informations, vous êtes invités à le lire si vous voulez faire avancer ce projet !

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Merci !

Contents

Chapter 1

Fundamental equations of turbomachinery

1.1 Basics and principles

1.1.1 Introduction

About the exam, he likes the drawings and likes to give a sentence and asks if it is the reality or not. The questions are about the **lectures** and not the notes. This summary is thus mainly based on the lectures.

The course is organized as:

1. **Turbopumps** and the system involved: a pump is something applying pressure to a fluid $\Delta p > 0$, which will first be a liquid $\rho = cst$.
2. **Turbines:** in this case the fluid involves a pressure loss $\Delta p < 0$, so expansion (a valve, simple releaser). We will see the gas turbines and the hydraulic turbines. Some machines can be used in the two ways, both roles, same mechanical component is acting as a pump and as a turbine in the other direction (reversible).
3. **Volumetric compressors:** as in other courses we have polytropic or isentropic efficiency, we can define volumetric compressors.
4. **Compressors $\boldsymbol{\rho \neq cst}$:** we need to consider the axial and centrifugal systems separately because they are very different and complicated.

1.1.2 Classification of turbomachines

If the role of the machine is to extract energy from the fluid to the shaft we speak about **turboproducers** or **turbomotors** (ex: hydraulic turbines), and in the other case **turboabsorbers** or **turbogenerators**.

1.1.3 General organization of turbomachines

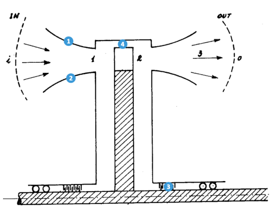

Figure 1.1

We always have a **shaft**, and on the shaft we have a **disk** or a **rotor** where the energy transformation takes place, and on this we put a **blade** (white rectangle on Figure 1.1), an element with a given geometry. The blade is situated at a distance r_H from the shaft and implies high tangential velocities. The device is closed by a **carter** and we have to be careful at the top of the blade since there can be leakage, this is why the clearance is very small (μm). We can also have active clearance control by blowing fluid on the carter. Remark that there is an **external carter** (blue 1) and an **internal carter** (blue 2) that can constitute a convergent distributor and a divergent diffuser. These can contain non rotating parts called **vane**, we speak of **vaned convergent**, **vaned divergent** or **vaneless** nozzles. The internal carter plays the role of support and is connected to the shaft via **bearings** and **seals** (to avoid air preferring this way to reach atmospheric pressure).

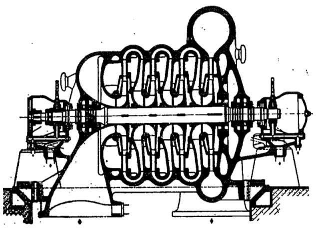

Figure 1.2

The major types of machine have an axial configuration so that the particles flows parallel to the shaft, radial configuration where flows enters horizontal and leaves vertical also exists. Most turbines or compressors are axial turbines. Turbopumps are most of the time radial, centrifugal. Several disks can be mounted on a same shaft (or several shafts), with fixed components changing the direction of the flow (multistage pump on Figure 1.2).

1.1.4 Notations

On Figure 1.1, 0 is the inlet of the distributor, 1 the inlet of the rotor, 2 outlet of the rotor, 3 outlet of the diffuser and o/s the upstream/downstream plenum. There are 3 velocity components: the absolute velocity $\bar{v}$, the frame tangential velocity $\bar{u}$ and the relative velocity $\bar{w}$ due to the rotation of fluid particles.

1.1.5 Velocity triangle

The absolute velocity respects:

$$\bar{v} = \bar{u} + \bar{w}. \tag{1.1}$$

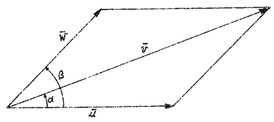

Figure 1.3

By defining α and β the angle between $\bar{u}$ and respectively $\bar{v}$ and $\bar{w}$, we can write:

$$v \cos\alpha = u + \cos\beta \qquad w^2 = u^2 + v^2 + 2uv\cos\alpha \tag{1.2}$$

The velocity triangle is the keypoint of a turbomachine, we have to play with the velocities by means of the mass flow rate $\dot{m}$ and the machine rpm N. It is important to see that these parameters are linked to the velocity triangle, for the mass flow for example $\dot{m} = \rho u A$.

1.2 Fundamental equations of the flow

1.2.1 Equations of the flow in a fixed frame

For all developments, only **steady state** regime is considered, so that $\omega = cst$ and the flow properties are not time dependent. Moreover, boundary layers are neglected and the flow propagates in parallel slices.

Energy equation

Let's consider sections A_0 and A_1 denoting the input and output of a non interrupted flow section, for instance 0 to 1 in previous figure. The total energy equation is:

$$q = h_1 - h_0 + \frac{v_1^2 - v_0^2}{2} + g(z_1 - z_0) \tag{1.3}$$

where q is the heat exchanged with the outside [J/kg], v is the absolute velocity [m/s] and h is the enthalpy [J/kg]. In adiabatic systems $q = 0$. We see that if there is thermal, kinetic or potential energy loss, it will create heat. Be careful that this equation is not applicable around the blade.

Equation of kinetic energy

We know that kinetic and potential energy can be transformed into work in a machine. We have:

$$\frac{v_1^2 - v_0^2}{2} + g(z_1 - z_0) = -\int_{p_0}^{p_1} \nu dp - w'_f \tag{1.4}$$

where $\nu = 1/\rho$ is the specific volume [m^3/kg] and w'_f the work resulting from all friction effects [J/kg]. We see that if velocity increases/decreases, pressure decreases/increases and the friction is a loss, so "-" signs.

Global thermodynamic equation

By considering (1.4) in (1.3):

$$q + w'_f = h_1 - h_0 - \int_{p_0}^{p_1} \nu dp \tag{1.5}$$

The physical content is different, we see that in fact the losses corresponds to pressure and temperature losses.

Mass flow rate equation

The mass flow is constant over a tube:

$$\dot{m} = \rho v A = \rho_0 v_0 A_0 = \rho_1 v_1 A_1 = \frac{v_1 A_1}{\nu_1} \tag{1.6}$$

where $\dot{m}$ [kg/s].

1.2.2 Equations of the flow in a moving frame

Now we can consider the indices 1 and 2 referring to the blade space.

Equation of kinetic energy in a relative space

He skipped the long demo page 8. The equations are the same as before, the only difference is that we replace the absolute velocity by the relative one and an additional kinetic energy term due to the rotation of the frame:

$$\frac{w_2^2 - w_1^2}{2} + g(z_2 - z_1) = -\int_{p_1}^{p_2} \nu dp - w''_f + \frac{u_2 - u_1}{2} \tag{1.7}$$

Remark that this term can play a huge role because in a centrifugal system the term is non zero while zero in axial system.

Global thermodynamic equation

The equation is exactly the same as before, except w'_f that becomes w''_f.

$$q + w''_f = h_2 - h_1 - \int_{p_1}^{p_2} \nu dp \tag{1.8}$$

Energy equation

If we combine the two previous equation we get:

$$q + \frac{u_2 - u_1}{2} = h_2 - h_1 + \frac{w_2^2 - w_1^2}{2} + g(z_2 - z_1) \tag{1.9}$$

Mass flow equation rate equation

For a section normal to the relative velocity:

$$\dot{m} = \rho w A = \rho_1 w_1 A_1 = \rho_2 w_2 A_2 \tag{1.10}$$

1.3 Fundamental equations of turbomachinery

1.3.1 Compressors

Internal losses

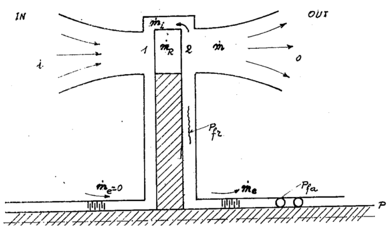

Figure 1.4

Several kind of losses are present in a compressor. First of all, friction in the distributor gives birth to losses noted w'_f. Then, the fluid go through the interblade channel where its energy increases, but a loss due to friction is present w''_f. In addition, the clearance between the blade tip and the carter is non zero and leads to back-flow leakage $\dot{m}_i$ as the pressure upstream is lower. If we denote the mass flow rate through the blades $\dot{m}_R$, we have:

$$\dot{m}_R = \dot{m} + \dot{m}_i \tag{1.11}$$

Big part of the flow goes then through the diffuser, but a small part escapes from the seal to the outside $\dot{m}_e$. The small gap between the rotor and the internal carter is filled with part of the fluid. Even if this fluid does not contribute to the energy exchanges (stagnation), part of the power supplied to the shaft will dissipate due to fluid friction P_{fr}.

External losses

As discussed, part of the flow escapes through the seals. The upstream one is not a problem since the pressure is close to the atmospheric one, but for the downstream $\dot{m}_e \neq 0$ since the pressure is higher. This will be neglected in the study. The last losses are due to bearings and other mechanical components such as the fuel pump that we all denote in a single absorbed power P_{fa}.

Energy equation

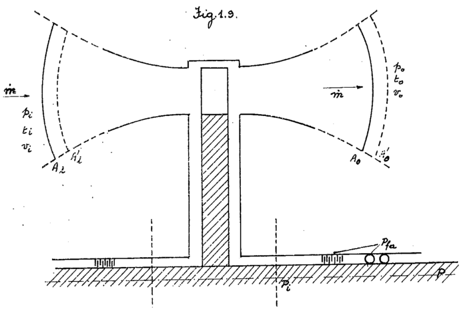

Figure 1.5

Before going through all the losses appearing in the engine, let's have a global view limiting the study region as on the figure. The total energy equation between input and output is:

$$q + W_{e \to s} = u_o - u_i + \frac{v_o^2 - v_i^2}{2} + g(z_o - z_i) \quad (1.12)$$

where we have the heat exchanged with the outside, the work from external forces to the system due to fluid pressure $p_i \nu_i - p_o \nu_o$ and work applied on the shaft $\frac{P - P_{fa}}{\dot{m}} = \frac{P_i}{\dot{m}}$ equal to the variation of internal energy, kinetic energy and potential energy. Heat exchanges and the height difference in such engine is negligible, by introducing the enthalpy instead of internal energy and fluid pressure, we have:

$$P_i = P - P_{fa} = \dot{m}\left(h_o - h_i + \frac{v_o^2 - v_i^2}{2}\right) = \dot{m}(h_{t_o} - h_{t_i}) = \dot{m}c_p(T_{t_o} - T_{t_i}) \quad (1.13)$$

where the index t denotes the total or stagnation quantities. We conclude that the work transferred through the shaft increases the total energy of the fluid.

Equation of kinetic energy

This adapted to the work $\frac{P_i}{\dot{m}}$ gives:

$$P_i = \dot{m}\underbrace{\left(\frac{v_o^2 - v_i^2}{2} + \int_i^o \nu dp + g(z_o - z_i)\right)}_{e} + P_f = \dot{m}e + P_f \quad (1.14)$$

P_f is the internal friction loss with the active as well as the non-active fluid and e is the energy transferred to the fluid.

A first distribution of the power - efficiencies

The figure summarizes all the above equations, for the efficiency we have $\dot{m}e$ at the end and we inject P so:

$$\eta_g = \frac{\dot{m}e}{P} = \frac{\dot{m}e}{P_i}\frac{P_i}{P} = \eta_i \eta_m \quad (1.15)$$

that we rewrite as internal efficiency and external mechanical efficiency.

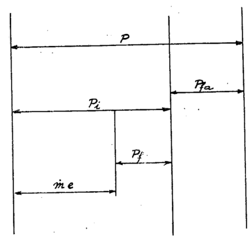

Figure 1.6

Energy transfer to the rotor

The only new equation we have is the **Euler-Rateau equation**. The energy given to the flow in a pump is linked to the energy provided to the shaft. We have to consider the kinetic moment (moment of the quantity of movement) related to the shaft:

$$\frac{d}{dt}\sum M_{axe}(m\bar{v}) = \sum M_{axe}\bar{F}_e \quad (1.16)$$

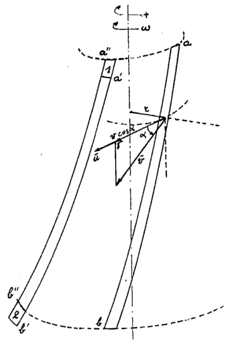

Figure 1.7

The variation of the kinetic moment of the rotor is zero since the rotation speed is constant. For a fluid element with mass m and at a distance r of the shaft, the velocity $\bar{v}$ is composed of a component // to the shaft, radial component and tangential component on the rotor. Only the tangential component delivers a moment:

$$M_{axe}(m\bar{v}) = mrv\cos\alpha \tag{1.17}$$

and since the flow is permanent, after a time dt very small, if the mass flow is constant:

$$\sum_{b'b"} mrv\cos\alpha - \sum_{a'a"} mrv\cos\alpha = m'(r_2v_2\cos\alpha_2 - r_1v_1\cos\alpha_1) = m'(r_2v_{2u} - r_1v_{1u}) \tag{1.18}$$

Accepting $r_2v_{2u} - r_1v_{1u} = cst$ for all channels we get:

$$\frac{d}{dt}\sum M_{axe}(m\bar{v}) = \frac{d}{dt}\left[(r_2v_{2u} - r_1v_{1u})\sum m'\right] = \dot{m}_R(r_2v_{2u} - r_1v_{1u}) \tag{1.19}$$

For the right hand side of (1.16), the weight of the wheel, the weight of the fluid, the pressure at inlet, at outlet and at side walls are null due to symmetry. Only the **driving torque** applied on the shaft $\mathbf{M_i}$ and the **resistive torque** $-\mathbf{M_{fr}}$ due to friction between the wheel and the non-active fluid are present, so that we finally get:

$$\dot{m}_R(r_2v_{2u} - r_1v_{1u}) = M_i - M_{fr} \tag{1.20}$$

And if we multiply by ω:

Equation of Euler-Rateau

$$P_i - P_{fr} = P - P_{fa} - P_{fr} = P_R = \dot{m}_R(u_2v_{2u} - u_1v_{1u}) = \dot{m}_R\Delta(uv_u) \tag{1.21}$$

This equation tells that in order to be transferred to the mass flow rate $\dot{m}_R$, the power at the rotor shaft P_R must show an increase of the quantity uv_u which is directly related to the velocity triangle.

As $w^2 = v^2 + u^2 - 2uv_u$, one can also write:

$$P_R = \dot{m}_R\left(\frac{v_2^2 - v_1^2}{2} + \frac{u_2^2 - u_1^2}{2} - \frac{w_2^2 - w_1^2}{2}\right) \tag{1.22}$$

Energy generated by the rotor

Remember (1.7) and replace this in (1.22), we get:

$$P_R = \underbrace{\dot{m}_R\left(\frac{v_2^2 - v_1^2}{2} + \int_{p_1}^{p_2}\nu dp + g(z_2 - z_1)\right)}_{e_R} + \dot{m}_R w_f^" \tag{1.23}$$

where e_R is the energy transferred to the fluid through the channels on the rotor between section 1 and 2 on the common figure. $\dot{m}_R w_f^" = P_f^"$ being the power absorbed by friction effects, we get:

$$\boxed{P_R = \dot{m}_R e_R + P_f^{"} \qquad \text{(and } P_R = \dot{m}(h_{t_2} - h_{t_1})\text{)}} \tag{1.24}$$

And we get thus 4 different expression for P_R, where the last expression was given in lecture.

1.3.2 Detailed power distribution

The different losses can be depicted thanks to the previous equation:

$$P_i = P - P_{fa} \qquad P_R = P_i - P_{fr} \qquad \dot{m}_R e_R = P_R - P_f^{"} \qquad \dot{m}_R = \dot{m} + \dot{m}_i \tag{1.25}$$

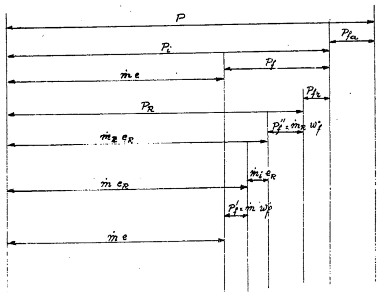

Figure 1.8

where we clearly see the mechanical losses, the loss due to stationary fluid, the loss at the rotor and the back flow. To this we have to add the loss at the diffuser and distributor which we wrote as:

$$P_f' = \dot{m} w_f' \tag{1.26}$$

On the figure, we first see the loss due to mechanical components giving P_i, then the fluid losses composed of stationary fluid losses giving P_R, then rotor channels loss giving $\dot{m}_R e_R$, but we have the back flow loss $\dot{m}_i e_R$ giving $\dot{m} e_R$ and the last venturi losses that gives the output power $\dot{m}e$ plus all the losses.

1.4 Turbines

Description

The equations are in fact the same except that we have to adapt the signs and indexes to the working principle since the energy is flowing from the fluid to the shaft. As before, there is a clearance flow $\dot{m}_i$ such that:

$$\dot{m} = \dot{m}_R + \dot{m}_i \tag{1.27}$$

The losses due to mechanical components P_{fa} and the leakage flows $\dot{m}_e$ (negligible), the energy losses in non rotating (w_f') and rotating ($w_f^{"}$) frame and the loss due to non-active fluid P_{fr} are present. The useful power P_u replaces the P we had before.

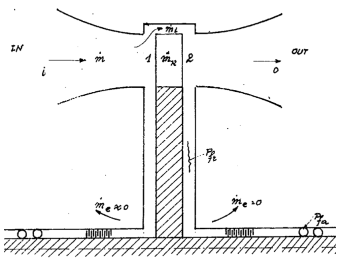

Figure 1.9

Energy equation

It is the same as before except that the indexes are inverted and the useful power is $P_u = P_i - P_{fa}$:

$$P_i = P_u + P_{fa} = \dot{m}(h_{t_i} - h_{t_o}) \tag{1.28}$$

Equation of kinetic energy

$$P_i = \dot{m} \underbrace{\left(\frac{v_i^2 - v_o^2}{2} + \int_{p_o}^{p_i} \nu dp + g(z_i - z_o) \right)}_{e} - P_f = \dot{m}e - P_f \tag{1.29}$$

First distribution of the power

The efficiencies are inverted:

$$\eta_g = \frac{P_u}{\dot{m}e} = \frac{P_u}{P_i}\frac{P_i}{\dot{m}e} = \eta_m \eta_i \tag{1.30}$$

On Figure 1.10 is represented a first plot of the different powers, remark that it is similar to what we had, only the fluid losses comes before the mechanical losses. The fluid losses are the same as before.

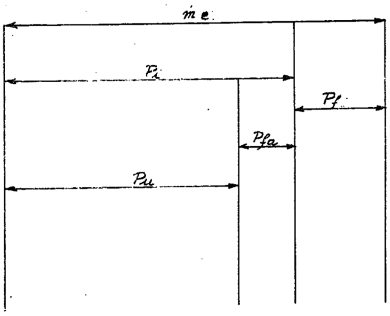

Figure 1.10

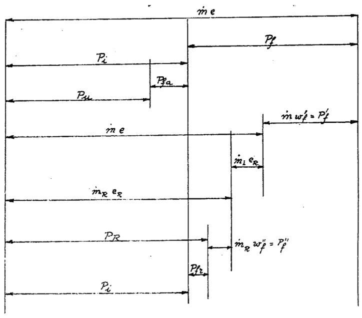

Figure 1.11

Equation of Euler-Rateau

We can rewrite P_R as the power transfered to the rotor from the fluid $\dot{m}_R$:

$$P_R = P_i + P_{fr} = \dot{m}_R(u_1 v_1 \cos\alpha_1 - u_2 v_2 \cos\alpha_2) \tag{1.31}$$

Energy absorbed by the rotor

As before, combing the above equation and the equation for rotating frame we get:

$$P_R = \dot{m}_R e_R - P_f^{"} \tag{1.32}$$

1.5 Axial thrust and disc friction

1.5.1 Definition

Since the turbomachinery is used to increase the energy of the fluid or extract energy, the pressure at the two sides of the rotor is different. This leads to an axial force and we need **thrust bearings** to avoid the shaft to slip.

1.5.2 Equation of the axial thrust

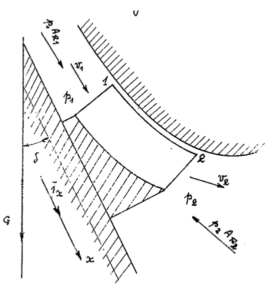

Figure 1.12

We apply the equation of quantity of movements to the wheel and fluid without the shaft between section 1 and 2:

$$\sum \frac{d}{dt}(m\bar{v}) = \sum \bar{F}_e \tag{1.33}$$

We will make a projection along x-axis. The external forces in presence are: the pressure forces $p_1 A_{R1}$ and $p_2 A_{R2}$, the weight of the rotor and the fluid projected is $G\cos\delta$ and the reaction force of the shaft on the rotor with projection F_{ax} so $-F_{ax}$ as seen by the rotor (projection of moments is null). The equation of quantity of movement becomes:

$$\begin{aligned} & \dot{m}_R\,(v_{2ax}-v_{1ax}) = p_1 A_{R1} - p_2 A_{R2} + F_{ax} + G\cos\delta \\ \Leftrightarrow \quad & F_{ax} = \dot{m}_R\, dt(v_{2ax}-v_{1ax}) + P_2 A_{R2} - P_1 A_{R1} - G\cos\delta \end{aligned} \tag{1.34}$$

1.5.3 Friction of the disc

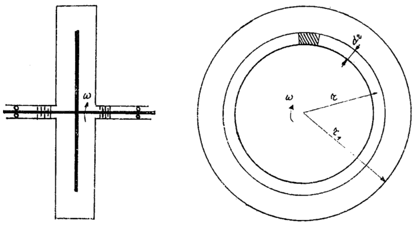

Figure 1.13

As mentioned before, there is friction between the disc and the non-active fluid. It is obvious that the friction torque will increase with the radius (more surface), the rpm (u) and the volumetric mass of the fluid. Considering an elementary surface $2\pi r\,dr$, we find experimentally for the elementary friction force and torque:

$$dF_{fr} = K\rho u^2 2\pi r\,dr \qquad dC_{fr} = 2K\rho 2\pi r^2\,dr\,u^2 \tag{1.35}$$

the 2 appears by considering the 2 sides of the disc. We integrate from internal radius r to the external one r_1 and multiply by ω to have powers (neglect r^5 compared to r_1^5):

$$C_{fr} = K\rho\omega^2 r_1^5 \qquad \Rightarrow P_{fr} = K\rho\omega^3 r_1^5 \tag{1.36}$$

Remark that one could think that we will have enormous values for ω^3, but be aware that if ω increases, the mechanical stresses on the device increases and force to reduce the r_1. This is why the friction is finite. If the side walls are very smooth, the coefficient K is related to the Reynolds number:

$$Re = \frac{\rho\omega r_1^2}{\mu} \tag{1.37}$$

Chapter 2

Centrifugal pumps

2.1 Generalities

2.1.1 Description - Type of turbopump

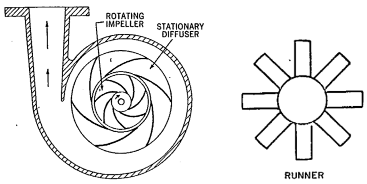

Figure 2.1

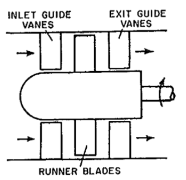

Figure 2.2

The task of the turbopump is to transfer energy to a liquid. Above we can see a centrifugal and an axial turbopump. Between these two extremes, we can have a variety of of types depending on the requirements. Each turbopump is composed of one or several wheels that can be mounted in parallel (increase mass flow rate) or in series (higher energy transfer), see Figure 1.2.

2.1.2 Installation of a turbopump

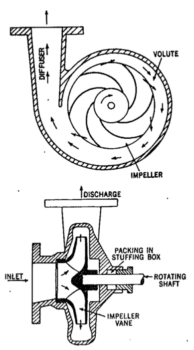

Figure 2.3

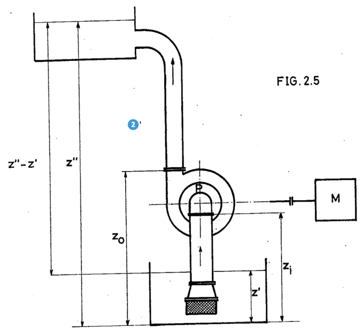

Figure 2.4

The general scheme is shown here, observe that the flow enters at the middle in the rotating blade and is projected into the volute. This last has a growing section from the beginning to the end as the mass flow increases. The turbopump is commonly used to transfer liquid from a downstream reservoir to an upstream reservoir situated higher. We have to be careful to avoid cavitation (evaporation of the fluid due to too low pressures) and we also have a control valve at the suction section to always have a contact blade-fluid.

2.1.3 Energy developed by the turbopump - flow rate

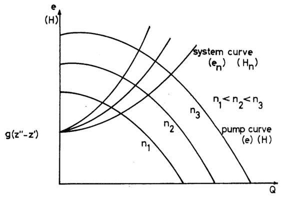

Figure 2.5

This is the type of curve we will have with the equations, where we see the characteristic curves of the pump and of the overall system (model of the resistance). These curves will be very similar to compressors. Depending on the rpm, we will consider different curves. The fundamental equations are simplified considering $\rho = cst \to \nu = cst$ for non compressible fluids:

$$e = \frac{v_o^2 - v_i^2}{2} + \int_{p_i}^{p_o} \nu dp + g(h_o - h_i) = \frac{v_o^2 - v_i^2}{2} + \frac{p_o - p_i}{\rho} + g(h_o - h_i) \tag{2.1}$$

The velocity is low in order to limit the head losses and thus the pressure term is the highest (height change in a compressor is low too). The energy delivered by the pump to the fluid can be rewritten in terms of the volumetric flow rate Q $[m^3/s]$:

$$e_p = \frac{p_a - p_i}{\rho} - \frac{p_a - p_o}{\rho} + \frac{A_i^2 - A_o^2}{2A_iA_o}Q^2 + g(z_o - z_i) \tag{2.2}$$

2.1.4 Useful power or hydraulic power

The power transfered from the input of the pump until the exit and the global efficiency of the pump are:

$$P_h = \dot{m}e = \rho Q e \quad [W] \qquad \eta = \frac{\rho Q e}{P_m} \tag{2.3}$$

where P_m is the mechanical power to drive the pump.

2.1.5 Working point of a turbopump

Consider Figure 2.4 and let's apply Bernouilli equation (kinetic energy equation) between z' and z_i then z_o and $z^"$:

$$\frac{v_i^2 - v'^2}{2} + g(z_i - z') = -\frac{p_i - p'}{\rho} - w'_{fa} \frac{v^{"2} - {v_o}^2}{2} + g(z^" - z_o) = -\frac{p^" - p_o}{\rho} - w'_{fr} \tag{2.4}$$

One can make the sum of the two expression and regroup the terms of the reservoirs in a new **energy requested by the circuit e_n**. If we consider large reservoirs $v^" \approx v'$ and $p^" \approx p' \approx p_a$, we have:

$$e_n = g(z^" - z') + \underbrace{w'_{fa} + w'_{fr}}_{w'_f} \qquad \Rightarrow e_p = e_n \tag{2.5}$$

This is always valid in **steady state**.

2.1.6 Characteristic of the hydraulic circuit

The system curves on Figure 2.5 plot e_n which depends on the height difference and the mass flow rate (because $w'_f \propto v^2$ of the flow) and depends thus on the square of the volumetric mass flow rate. This is why we have a parabolic shape, the slope depends on the head loss coefficient K. If we have a valve, the closer the valve, the higher the slope.

2.1.7 Performance curve of a pump

Similar curves can be established for the e, Q relations at different rpm. With a control valve at the exit, and by fixing the rpm of the engine, we can find them and are plotted on Figure 2.5.

2.1.8 Working regimes

Practical analysis shows that if two of the three parameters e, Q, n are fixed, the working point too: $f(e, Q, n) = 0$

2.1.9 Practical units

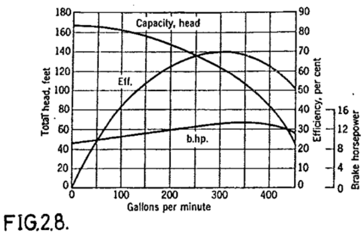

Figure 2.6

Here we express the energy in J/kg but we know that it is also $g\Delta z$ in [m]. Thus we will use instead of e, $H = e/g$ [m]. The energy transferred to the fluid is often called the **height** of the **head**. For example $H = 10$ m means that we transfer energy such that we increase z of 10 m. As last remark, be aware that efficiency curves are provided by the manufacturer and the pump has to be chosen specifically to the circuit where it should operate to get the maximum efficiency.

2.2 The centrifugal pump

2.2.1 Organization of a centrifugal pump

We have an inlet distributor D charged of guiding the fluid towards the entrance 1 of the rotor R or also called **impeller**. The rotor is made of one or two disks on which are mounted the blades beginning at a certain external radius r_1 and finishing at r_2. A fixed diffuser d composed of 2 parallel discs surrounding the rotor, connected with vanes surrounds the exit of the blades, sometimes it is not used. A **volute** or **collector** c with increasing volume directs the flow to the exit section of the machine.

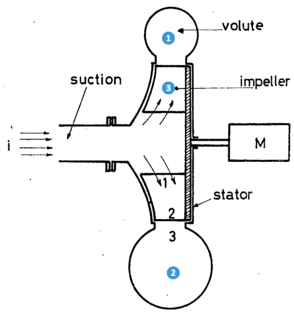

Figure 2.7

2.2.2 The distributor

If there is no vane in the distributor, the flow penetrates in the rotor axially since we assume no fluid particle to rotate before entering in the rotor, and becomes radial symmetrically at intrance 1. If there is vane, the direction of the flow is imposed by the vanes but we take the first case here. The equation of kinetic energy applied between i and 1 when neglecting the height difference is:

$$\frac{v_i^2 - v_1^2}{2} + \frac{p_i - p_1}{\rho} = w'_{fD} \tag{2.6}$$

where w'_{fD} represents the pressure losses in the distributor, proportional to the square of Q and thus to v_1^2: $w'_{fD} = K_D \frac{v_1^2}{2}$ where $K_D \approx 5.10^{-3}$

2.2.3 The rotor

The impeller starts at r_1 and finish at r_2, the section of the rotor at these levels are:

$$A_1 = 2\pi r_1 b_1 e_1 \qquad A_1 = 2\pi r_2 b_2 e_2 \tag{2.7}$$

where e_1, e_2 are blockage coefficients taking into account the decrease in area due to the thickness of the impeller.

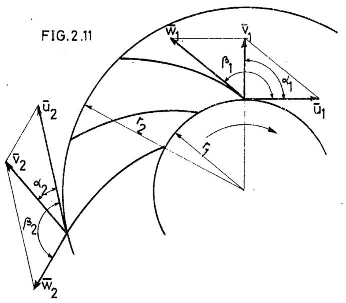

Figure 2.8

The rotor and impeller velocity triangles are represented on the figure. v_1 is known by the previous discussion and is purely radial and u_1 can be computed:

$$v_1 = \frac{Q_R}{2\pi r_1 b_1 e_1} \qquad u_1 = r_1\omega_1 = \frac{2\pi n}{60} r_1 \qquad \alpha_1 = 90^\circ \tag{2.8}$$

The missing components of the velocity triangles are w_1 and β_1 and can be retrieved by construction. In addition we make an assumption for β_1 (fluid angle) which must be equal to $\bar{\beta}_1$ (solid angle), this is imposed by the design to avoid collision or separation. Indeed, the pump is designed to work with a certain Q_R and a certain n, if this changes, shocks and separation can occur, leading to losses. Same considered for β_2, $u_2 = r_2\omega$ and this time the radial velocity is the projection of w_2:

$$u_2 = r_2\omega_2 \qquad w_2 \sin\beta_2 = \frac{Q_R}{2\pi r_2 b_2 e_2} \tag{2.9}$$

β_2 is chosen larger than 90°in order to make v_2 small and thus limit the diffuser size (limit the losses, β_2 between 145°and 165°). We are now able to retrieve v_2 and w_2.

2.2.4 Number and shape of the blades

The number of blades determine the volume available to the flow and the guidance. The more we have blades, the more the fluid is guided but the more we have pressure losses. The designer must make a trade-off, generally there are 6 to 12 blades. The profile of the blades must be so that the angles $\beta_1 = \bar{\beta}_1$ and $\beta_2 = \bar{\beta}_2$ are respected.

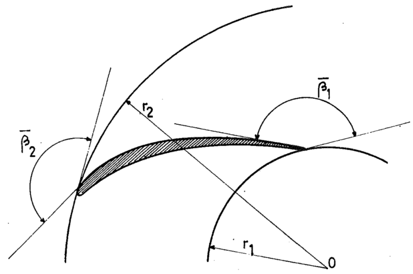

Figure 2.9

If the blades are made of two surfaces, one active and one non-active as represented on the figure, if the the two surfaces makes an angle too large at the entrance, it is impossible that β_1 is tangent to both and lead to shocks. At the exit, since there is a pressure gradient between active and non active sides, the β_2 is "sucked" by the non-active part where the pressure is lower.

2.3 Head of Euler of the rotor

Using Euler-Rateau and power distribution, we have:

$$P_R = \dot{m}_R(u_2v_{2u} - u_1v_{1u}) = \dot{m}_R e_R + \dot{m}_R w''_{fR} \qquad \Rightarrow e_R = u_2v_{2u} - u_1v_{1u} - w''_{fR} \tag{2.10}$$

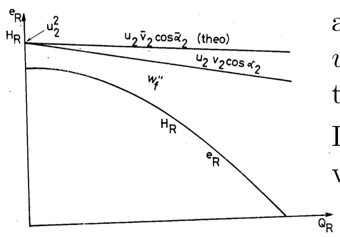

Figure 2.10

as we have seen, $\alpha_1 = 90^\circ \Rightarrow v_{1u} = 0$ and $e_R = u_2v_{2u} - w''_f$. The term u_2v_{2u} is called the **energy of Euler** and is the theoretical energy that the rotor transfers to the fluid. The **head of Euler** is $\frac{u_2v_{2u}}{g}$. Lets show that this energy is function of $Q_R, N, \bar{\beta}_1$ and $\bar{\beta}_2$ using the velocity triangle relation:

$$\boldsymbol{u_2} v_2 \cos\alpha 2 = \boldsymbol{u_2}(u_2 + w_2\cos\bar{\beta}_2) \Rightarrow e_r = u_2^2 + \frac{Q_R}{2\pi r_2 e_2 b_2 \tan\bar{\beta}_2} \tag{2.11}$$

where we used (2.9). We see that as $\bar{\beta}_2 > 90^\circ$ in practice, we have a linearly decreasing function. We still don't have the characteristics since we are underestimating the angles deviation, the number of blades and the fluid losses.

2.3.1 Losses in the rotor due to friction

He skipped the previous section. The term $w_f^{"}$ regroups the different losses that occurs in the rotor and can be separated in:

- the losses due to the development of the boundary layer in the channel sidewalls and $\propto Q_R^2$:

$$k_1 Q_R^2 = K_R \frac{w_1^2}{2} \qquad K_R \approx 0.025 \tag{2.12}$$

- a second loss due to the shocks and separation of the boundary layer each time w_2 is not tangent to the blade. Looking to the situation on Figure 2.11, we find experimentally that these losses are $\propto (\Delta w)^2$ that is $\propto v_1$ that is $\propto Q_R - Q_{RD}$ where Q_{RD} is the flow rate in design conditions $\beta_1 = \bar{\beta}_1$:

$$Q_R = 2\pi r_1 b_1 e_1 v_1 \qquad Q_{RD} = 2\pi r_1 b_1 e_1 v_{1D} \tag{2.13}$$

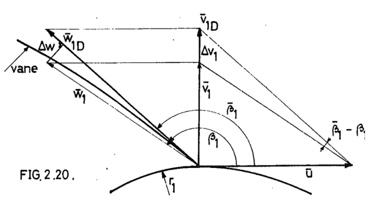

Figure 2.11

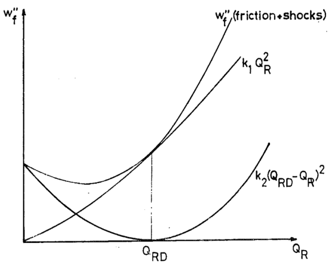

Figure 2.12

The sum is represented on Figure 2.12 and we see that even at low Q_R $w_f^{"}$ is high, this is due to the second loss.

2.3.2 Loss due to the internal leak flow

We already know what it is, it goes from 1 for large pumps to 10% of $\dot{m}_R$ for small pumps. This is due to the fact that the clearance cannot be reduced under an absolute size and the seals in large machines cannot be more efficient than in small machines. The power loss is:

$$\dot{m}_i e_R = (0.01 \text{ } to \text{ } 0.1)\dot{m}_R e_R \quad [W] \tag{2.14}$$

2.3.3 Friction of the disc on the non-active fluid

Per side of the disc, it can be estimated as:

$$P_{fr} = 1.21\,10^{-3} u_2^3 D_2^2 \quad [ch] \tag{2.15}$$

2.3.4 The diffuser

Energy transformation in the diffuser

The velocity v_2 at the exit of the pump is generally too high for some applications, the diffuser converts part of the kinetic energy into pressure energy. There exists 4 types of diffuser: straight parallel sidewalls or inclined, and for each we can have vaned or vaneless. The kinetic energy equation in a fixed frame with $z_3 - z_2 = 0$ is:

$$\frac{p_3 - p_2}{\rho} = -\frac{v_3^2 - v_2^2}{2} - w'_{fd} \tag{2.16}$$

where $w'_{fd} \approx 0.02 - 0.03 v_2^2/2$ [J/kg]. We see that kinetic energy gives pressure and loss.

The vaneless diffuser with flanges

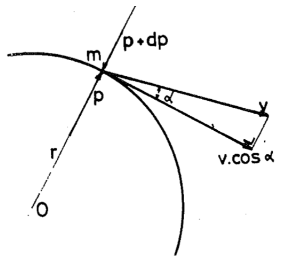

Figure 2.13

The common architecture of the diffuser is composed of two circular flanges put around and in the continuity of the exhaust of the rotor. Let's apply the equation of the kinetic moment to a fluid element of mass m and at a radius r:

$$\frac{d}{dt} M_{axis}(m\bar{v}) = M_{axis}\bar{F}_e \tag{2.17}$$

The situation is represented on Figure 2.13, the flow in the diffuser is axisymmetric. If one neglect the weight of the particles, the external forces moment is null since the pressure is radial. All particles are facing the same pressure for symmetrical reasons and have thus the same trajectory. We have:

$$\frac{d}{dt}(mvr\cos\alpha) = 0 \qquad \Leftrightarrow mvr\cos\alpha = rv_u = cst \tag{2.18}$$