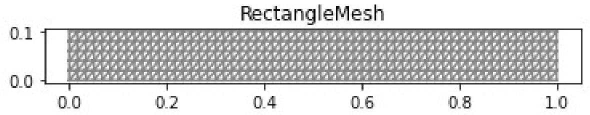
General introduction:

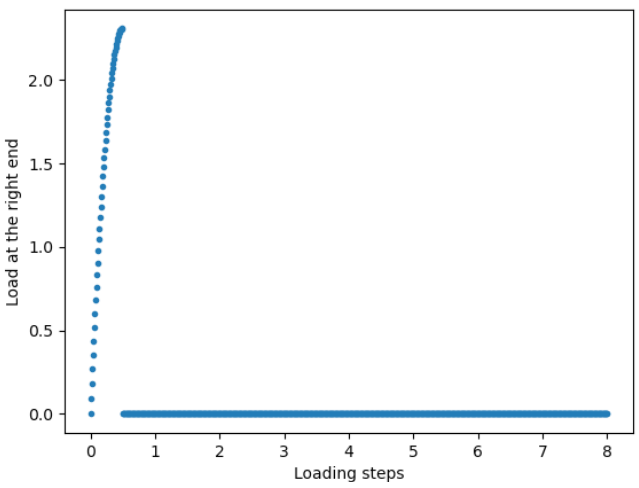
The scripts used for simulation are modified from two examples provided by FEniCS:
- Elasto-plastic analysis of a 2D von Mises material:
https://comet-fenics.readthedocs.io/en/latest/demo/2D_plasticity/vonMises_plasticity.py.html
- Time-integration of elastodynamics equation:
https://fenicsproject.org/olddocs/dolfin/latest/python/demos/elastodynamics/demo_elastodynamics.py.html

1 2D von Mises plasto-elastic strain-hardening material

The static elasto-plastic code was first extended to account for dynamic effects. Newmark-beta algorithm is applied for time discetization.

The simulation is conducted on a square domain, where the left side is fixed while the rest boundaries are set free. The wave is generated by applying a horizontal uniformly distributed load on the right surface.





The stress-strain curve is shown in Figure XX. It is observed that both loading and unloading behave as expected.

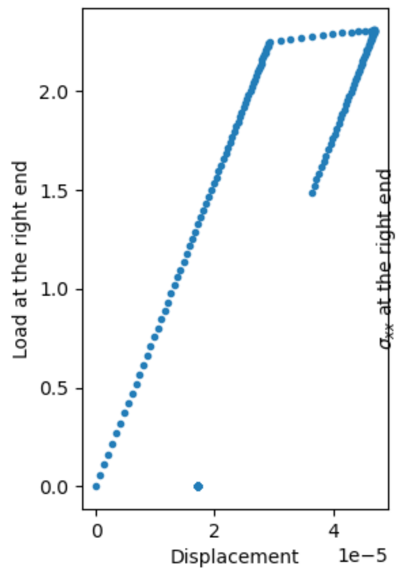
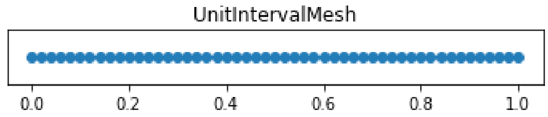


Figure XX shows the developed plastic zone after wave propagation:

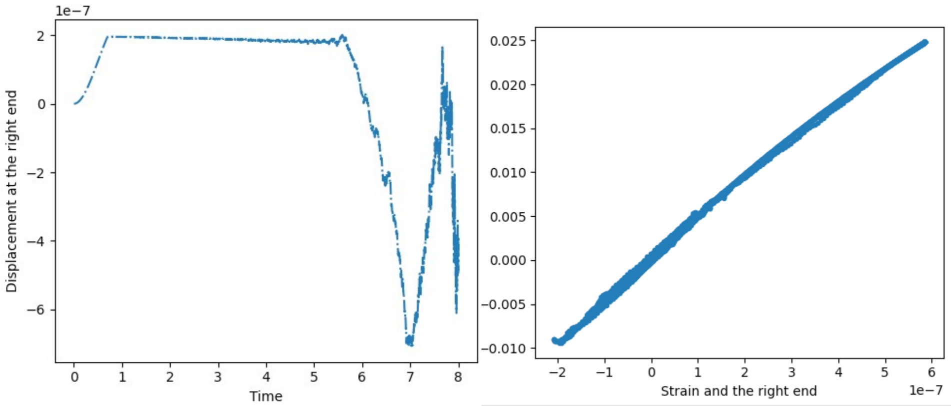
2 1D non-linear elasticity

2D simulate can provide some beautiful videos. But it is hard to check the accuracy of a nonlinear system. I degenerated the code to 1D version and to prepare for the more complex MPII model, pure nonlinear elasticity rheology is tested, which is the bone curve of the MPII model. Only SH wave is simulated. The tangent shear modulus of the stress-strain curve is determined by $\frac{G}{G_0}=1/(1+\frac{\gamma}{\gamma_{ref}})$. Same load is applied as the 2D case.

The geometry is quite simple.



Probably due to nonlinearity, the results become very inaccurate at the later stage of simulation (requires 1/100 times smaller time stepping). The displacement and stress-strain curve at the right end is shown in Figure XX.

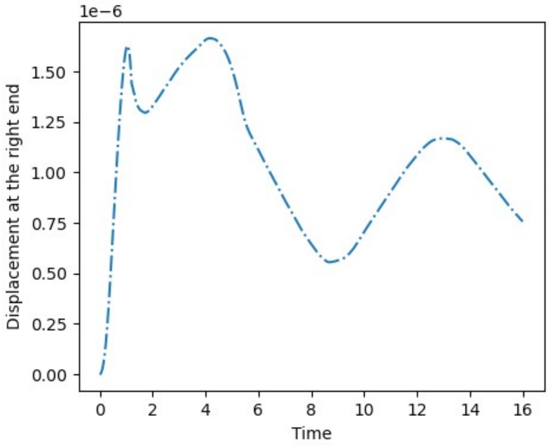


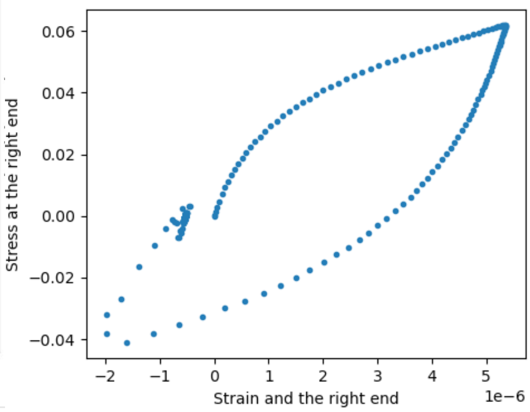
I will need to figure out a way to make the simulation more accurate by considering adaptive time stepping.

3 1D MPII model

The above 1D simulation code contains most of the components of the code for 1D MPII model, but the more complex rheology. It is kind of a combination of non-linear elasticity and von-Mises plasticity with strain hardening. The bone-curve is discretized with yielding surfaces with strain-hardening behaviors so that the first loading will follow the bone-curve. The details are explained by Hardin and Drnevich (1972).

The displacement and stress-strain curve at the right end is shown in Figure XX. This more complex models requires much bigger time steps, probably due to the piece-wise linear nature of the multiple yielding surface.





Next steps:

- Study the possibility of having adaptive time stepping
- Find a benchmark problem to verify the simulation