

* Decision Tree

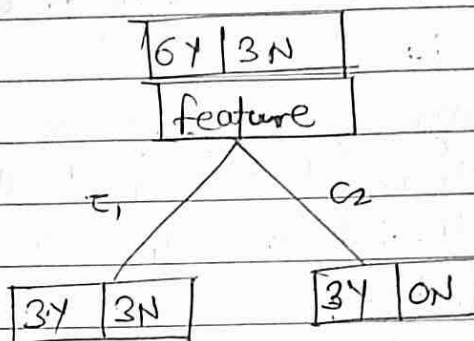
classmate

Date _____

Page _____

* Entropy Calculation →

feature	o/p
c ₁	Y
c ₂	Y
c ₁	Y
c ₂	Y
c ₁	Y
c ₁	N
c ₂	Y
c ₁	N
c ₁	N



$$\text{Entropy} = - \sum_{i=1}^N P_i \times \log_2(P_i)$$

N → Number of class. (2 → Y/N)

$$H(S) = -P_Y \log_2(P_Y) - P_N \log_2(P_N)$$

∴ Calculating for class 'c₁'

$$H(S) = -\frac{3}{6} \log_2\left(\frac{3}{6}\right) - \frac{3}{6} \log_2\left(\frac{3}{6}\right)$$

$$= -\frac{1}{2} (-1) \log_2 2 - \frac{1}{2} (-1) \log_2 2$$

Highest entropy that we can get

= 1

∴ This split is most impure split.

∴ Calculating for class c₂.

$$H(S) = -\frac{3}{3} \log_2\left(\frac{3}{3}\right) - \left(\frac{0}{3} \log_2\left(\frac{0}{3}\right)\right)$$

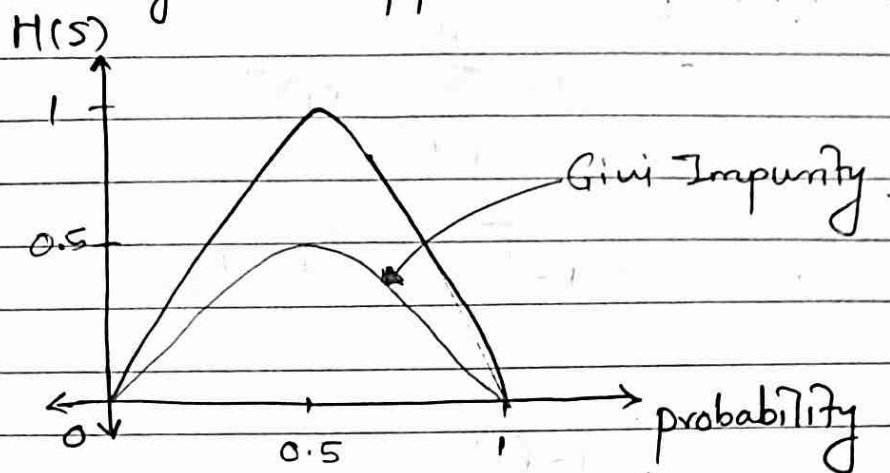
$$= -1 \log_2(1) = 0$$

lowest entropy that we can get

= 0

→ pure split.

* Graph probability vs entropy.



* Gini Impurity Calculation :-

- we are going to same above problem :-

$\boxed{3Y \mid 3N}$

$$G.I = 1 - [P_Y^2 + P_N^2]$$

$$G.I = 1 - \sum_{j=1}^N (P_i)^2 \quad \text{General}$$

$N \rightarrow$ Number of classes [Going to predict]

$$G.I = 1 - \left[\left(\frac{3}{6}\right)^2 + \left(\frac{3}{6}\right)^2 \right]$$

$$= 1 - \left[\frac{1}{4} + \frac{1}{4} \right]$$

Highest
G.I that we
can get

$$= 1 - 0.5$$

$$G.I = 0.5$$

most impure split

[can't draw any conclusion]

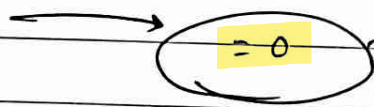
$$[3Y | 0N]$$

$$G.I = 1 - \sum_{i=1}^N (p_i)^2$$

$$= 1 - \left[\left(\frac{3}{3}\right)^2 + \left(\frac{0}{3}\right)^2 \right]$$

$$= 1 - (1)$$

least G.I
That we can
get



pure split we can conclude
as yes here.

* Calculation of information gain!

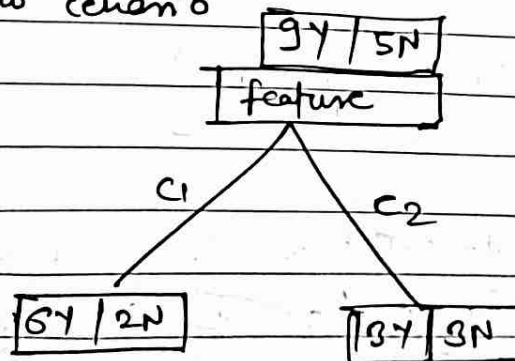
$$\text{Gain} = H(S) - \sum_{v \in \text{val}} \frac{|S_v|}{|S|} H(S_v)$$

Root node Entropy

child node entropy

$\Sigma \rightarrow$ of all child node

with below scenario



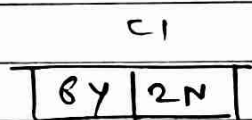
- entropy for root feature

$$H(S) = -P_Y \log_2(P_Y) - P_N \log_2(P_N)$$

$$= -\frac{9}{14} \log_2\left(\frac{9}{14}\right) - \frac{5}{14} \log_2\left(\frac{5}{14}\right)$$

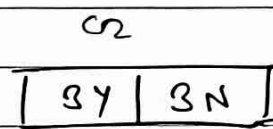
$$H(S) = 0.9402$$

- entropy for child Node



$$H(S) = -\frac{8}{2} \log_2\left(\frac{8}{2}\right) - \frac{2}{8} \log_2\left(\frac{2}{8}\right)$$

$$= 0.81$$



most impure split
= 1

$$\text{gain} = H(S) \sum_{\text{split value}} \frac{|S_v|}{|S|} H(S_v)$$

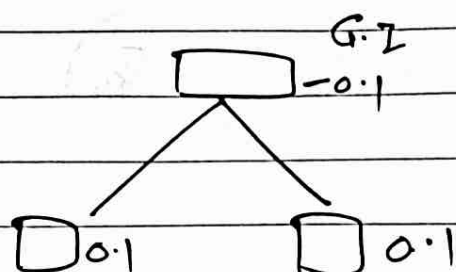
split value

$$= 0.94 - \left[\frac{8}{14} \times 0.81 + \frac{6}{14} \times 1 \right]$$

$$\text{gain} = 0.049$$

- ∴ The feature have more information gain that will be the root node selected by the Tree algorithms.
- The above process will be done on each level of node

- ✓ Information Gain Calculated in ID3 approach only
- ✓ In CART → Sum of all Gini Impurity is calculated to select the root node.



$$\text{Sum of G.I} = 0.1 + 0.1 + 0.1 = 0.3$$

The less the G.I sum is
This feature will be our root feature.

* pre-pruning

Before building a decision tree I can prune out the branches.

- if we going to build dT till complete depth?
 - ① overfitting [Train accuracy will be good test accuracy will be bad.]
 - ② Computational will be expensive.

* preprunning hyperparameter :-

- 1) Max depth
- 2) minimum Sample leaf
- 3) minimum Sample split.

1) Max depth :-

The depth of decision tree will decided by this parameter.

Ex: Max depth → 2 → Result

→ The minimum number of samples required to be at leaf node.

② Minimum-Samples leaf.

In the leaf node we can set minimum sample amount if leaf node contains

- The minimum number of samples required to be at leaf node.
- if we have value at certain split ~~more~~ than that remove it. less

③ minimum Samples

- The minimum number of samples required to split an internal node.

④ Max-depth

- The maximum depth of the trees.
- if none, then nodes are expanded until all leaves are pure or until all leaves contain less than min-samples-split samples.

* Decision tree regressor \Rightarrow

weight Heart-disease

220 Y

180 Y

225 Y

150 N

155 N

Q) How to solve the above problem coz in weight we have Continuous values

\rightarrow 1) Sort the values

2) Take the average of adjacent value

3) w.r.t every average value find out

[Gini impurity / average gain]

\rightarrow as like we did for categorical values.

① Sorted data

weight HD

167.5 { 155 N

185 { 180 Y

205 { 190 N

222.5 { 220 Y

225 Y

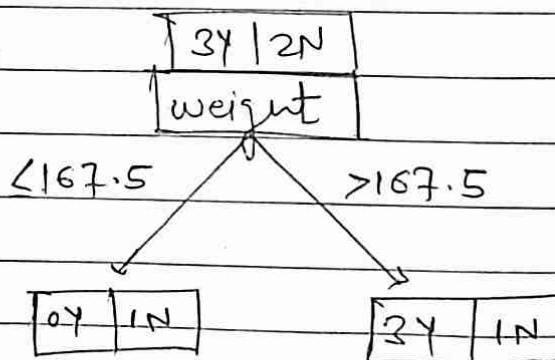
\downarrow
average of two
consecutive values.

\downarrow
This is like a various classes

\downarrow
for which classes we get
less entropy and more gain

That class will consider as
Root class [value].

③



✓ calculate the gini impurity for
above scenario

for left node

Gini impurity = 0

for Right node

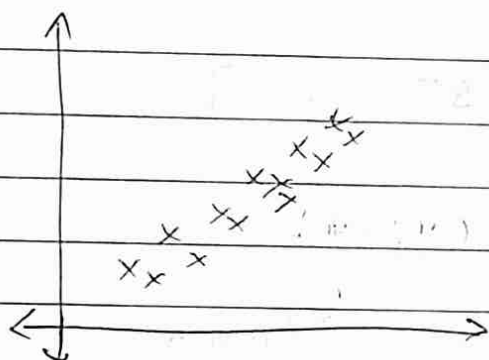
$$G.I = 1 - \left[\left(\frac{3}{4} \right)^2 + \left(\frac{1}{4} \right)^2 \right]$$

$$G.I = 0.37$$

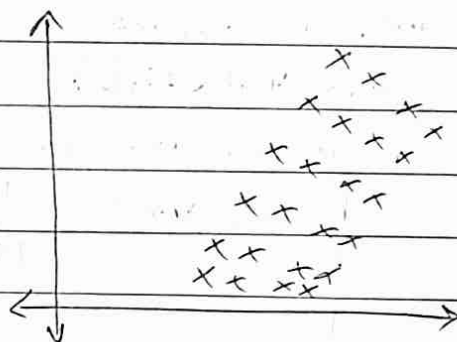
dependent feature

Date _____
Page _____

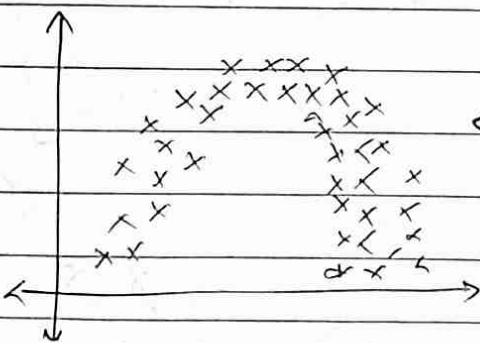
Height	weight	Gender	weight
165	66	M	66
160	50	F	50
180	90	M	90
170	85	F	85
175	70	M	70



problem Suitable for linear Regression



problem Suitable for SVM.



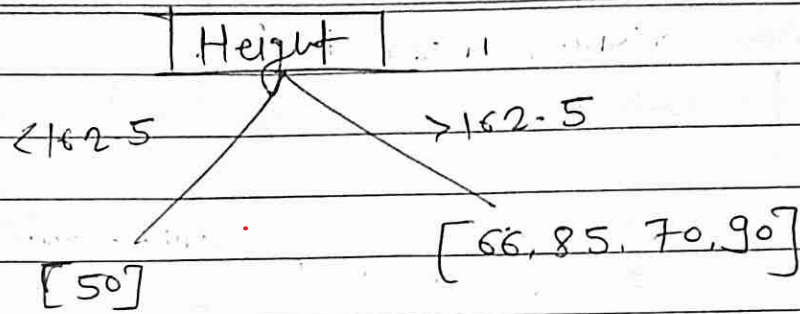
Not having linear Relation In features.

problem Suitable for DT

Height	weight
160	50
162.5	66
165	86
167.5	70
172.5	90
177.5	70
180	90

* Next page with this Example.

[50, 66, 85, 70, 90]



✓ for regression problem we found variance and redutance
In variance. [mse]

var(root) = [50, 65, 85, 70, 90]

$$\text{var} = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y})^2$$

↘ mean

$$\mu = 72$$

$$\text{var}(\text{root}) = \frac{1}{5} \left[(72-50)^2 + (72-65)^2 + (72-85)^2 + (72-70)^2 + (72-90)^2 \right]$$

$$\text{var}(\text{root}) = 206$$

$$\text{var}(\text{left child node}) = 0$$

$$\text{var}(\text{right child node}) = [66, 85, 70, 90]$$

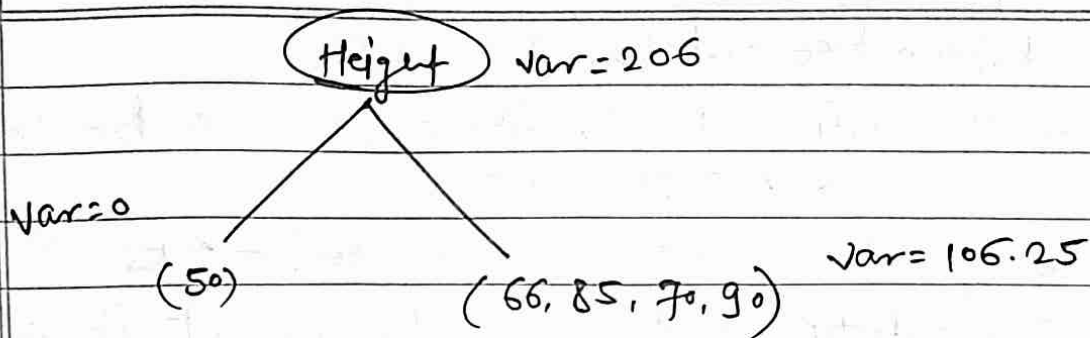
$$\text{var} = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y})^2$$

$$\mu = 77.5$$

$$= \frac{1}{4} \left[(77.5-66)^2 + (77.5-85)^2 + (77.5-70)^2 + (77.5-90)^2 \right]$$

$$= 106.25$$

✓ ~~wrong~~
calculate the
variance of child
Node with mean
of the root Node
 $\mu = 72.2$



$$\text{Reduction in var} = \text{var}(\text{root}) - \sum_{i=1}^N w_i \times \text{var}(\text{child})$$

~~$$= 206 - (106.25)$$~~

$$= 206 \left[\frac{1}{5} \times 0 + \frac{4}{5} \times 106.25 \right]$$

$$= 206 - [0 + 85]$$

$$\boxed{= 121}$$

for threshold value \rightarrow Height = 162.5 the Reduction in variance = 121.

- like that we have to calculate for every ^{Threshold value} value

weight threshold/value	Reduction in VAR
162.5	121
167.5	116
172.5	105 105
177.5	130 \rightarrow Root Node

* The threshold which have ^{greatest} ~~lowest~~ Reduction in VAR this Threshold we are selecting for splitting.

* The final output for test data calculated by using the mean of the sample in the leaf node where my test data is falling.