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Agri-food supply chain optimization through a decentralized production process in the olive oil industry

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ABSTRACT

A tactical/operational model is proposed to assist decision-making in harvest planning and olive oil production planning of the different producers in the rural sector, considering allocation and transportation decisions of the kilograms of olives to the production plant with temporary relocation decisions of the mobile olive oil mills through decentralized production activities in the agri-food supply chain. A Mixed Integer Linear Programming model is presented that maximizes the profit of small farmers in the production process of single-varietal extra virgin olive oil. The decisions are based on the geographical dispersion of the different olive growers and the oil concentration of the olive varieties in a certain period, which depends on the climatic conditions of each sector where the olives are cultivated. The mathematical programming model is applied in Coquimbo, Chile, using actual data collected from previous harvest seasons. The computational results of the optimization model indicate that transportation decisions impact olive harvesting and oil production decisions. It is concluded that cooperatives of small producers in the rural sector increase overall profits and reduce transportation costs and the number of kilometers traveled by at least 41% through a decentralized production process with mobile production plants.

1. Introduction

In recent decades, the olive oil industry has positioned itself worldwide as one of the most important in the food industry. According to data published by the International Olive Council, the area under olive cultivation worldwide is almost 12 million hectares. Over the last five years, consumption has exceeded 3 million tons of olive oil annually (IOC, 2023).

Mili and Bouhaddane (2021) forecasts an annual growth rate of between 2% and 10% in global olive oil demand until 2025, driven by the growing interest of consumers in healthier and more natural products, along with the diffusion of the properties of olive oil. The authors use the Delphi method to estimate oil production and consumption worldwide, considering production technology, the geographical dispersion of supply and demand, globalization, price competition, and consumer behavior as forecasting factors.

To take advantage of market opportunities, it is necessary to improve the overall performance of the Agri-Food Supply Chain (AFSC) by integrating tactical decisions (e.g., harvest planning, labor requirements, resource allocation) and operational decisions (e.g., production planning, machinery selection, inventory, and transportation) (Ahumada and Villalobos, 2009). In the olive oil sector, field and production facility decision-makers must synchronize harvest and transformation phases for specific days, facing the problem that the olives must be pressed within the first 24 h after harvest to maximize the organoleptic, nutritional and sensory properties of the olive oil (Rotondi et al., 2021).

An efficient harvesting plan must consider the climatic factors and the properties of the different olive varieties that affect the accumulation and quality of the olive oil obtained (Herrera-Cáceres et al., 2017). Consequently, a suitable production plan is required to yield a balanced harvest and coordination among olive growers to use the olive oil mills to increase the quality and quantity of produced olive oil. This process may be integrated inside the company (i.e., harvesting and production in the same field) or outsourced to producers who are not part of the work team of the production plants' owners or managers, or both.

Small communities or large cooperatives should plan harvesting, storage, and processing activities together, adapting to the production capacities of the olive oil mills. Better coordination along the

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agricultural supply chain in industries with perishable raw materials, such as olive oil, may help reduce losses and costs and increase the quality and quantity of their final products (Vlah Jerić and Šorić, 2010). Moreover, with the high production costs and low profits, small firms must integrate into a more extensive system to be more competitive in the market. In this context, one of the observed strategies in literature and practice is the decentralization of production activities through a dynamic relocation of the manufacturing units (Kusumastuti et al., 2016).

This research aims to integrate an olive oil mill with a dynamic location into the supply chain planning process. In our application case in the Province of Elqui, Coquimbo, Chile, we consider two mobile olive oil mills that can be moved between agricultural fields that comply with the corresponding sanitary resolution. The modularization, miniaturization, and mobility of production units allow solving the location problem at the tactical and operational level, unlike fixed manufacturing sites that usually solve the location problem at the strategic level (Alarcon-Gerbier and Buscher, 2022).

Coordination decisions are not straightforward, as they involve planning harvesting activities, fruit transportation, relocation of production facilities, and the olive transformation phase while considering various production limitations. Decision-makers must optimize the operating times of the production plant by ensuring timely supply and low-cost transportation of the small farmers' various harvests while also ensuring the quality and safety standards of the resulting olive oil.

According to the previously stated problem, a natural research question arises related to determining the impacts of transporting mobile olive oil mills over the integrating planning of olive harvesting and oil production. Moreover, it is relevant to investigate how integrating mobile olive oil mills affects the system's performance.

In concordance with the aforementioned problem and research questions, the main contribution of this research is to support decision-making through a mathematical programming model that seeks to provide producers with a plan for olive harvesting and olive oil production through a decentralized production process with mobile olive oil mills that can be relocated to different agricultural fields. We propose a Mixed Integer Linear Programming (MILP) model that integrates harvesting, transformation, and transportation decisions to maximize the overall profits of the members of the cooperative, considering the geographical dispersion of the different producers and the concentration of olive oil, that depends on the properties of each olive variety and the conditions of each region/sector where the olive growers' agricultural fields are located. Particularly, the proposed formulation explicitly models the setup process and downtime related to mill transportation, following the recommendations in Alarcon-Gerbier and Buscher (2022).

The remainder of this paper is organized as follows. Section 2 presents a summary of previous work-related studies. Section 3 briefly explains the problem statement and associated assumptions. The proposed MILP formulation is presented in Section 4. Section 5 subsequently presents the implementation of the optimization solution with all computational results and its sensitivity analysis. Section 6 discusses several research and managerial insights. Finally, Section 7 concludes our work and discusses further research directions.

2. Literature review

In their respective studies, Taşkıner and Bilgen (2021) and Nguyen et al. (2021) conduct literature reviews on optimization models for harvesting and production planning in AFSCs and the application of mathematical programming models in optimizing fresh fruit supply chains, respectively. Taşkıner and Bilgen (2021) reviews 74 papers published between January 2000 and October 2020, while Nguyen et al. (2021) reviews 70 articles published from 1976 to 2021. Both studies underscored the importance of developing mathematical models that incorporate integrated decisions to coordinate different stages of the food industry.

In the last decades, significant advances have been developed with optimization tools based on mathematical models to support harvest and production planning for perishable products. Ferrer et al. (2008) develops an MILP model that integrates harvest planning and routing of equipment and labor teams on a given day, which must store the grapes harvested in each warehouse. The optimization model aims to improve the quality of harvested grapes by balancing the costs of moving harvesting operations from one field area to another, including the operational cost of the harvesting process and the penalty costs of loss of quality. The case studies show that when routing decisions are not integrated into the objective function, the solution's total cost may increase by 20%.

Through a robust optimization model and a heuristic approach, Bohle et al. (2010) extends the work of Ferrer et al. (2008) considering the variation in labor productivity at harvest. Based on Bohle et al. (2010), Varas et al. (2020) proposes a novel multi-objective MILP model and a negotiation protocol that helps coordinate the harvesting of wine grapes, according to the competing objectives of the oenologist and the field manager. Similarly, Avanzini et al. (2021) extends the study Ferrer et al. (2008) by proposing two multi-stage stochastic optimization models that include product quality degradation and weather conditions uncertainty. The authors conclude that grape quality, rain probability, worker ability, and flexibility in decision-making may significantly affect harvest planning.

To support operational decision-making, Ahumada and Villalobos (2011) proposes a mathematical model that integrates the planning of harvesting, packing, and distribution of crops. The research considers labor management, operational limitations, post-harvest decomposition, transportation time, shipping costs, price dynamics, weather variable effects, and plant biology. The proposal's application is carried out in a case study based on peppers and tomatoes. In the tomato processing industry itself, mathematical programming models have been developed and implemented to support decision-making in agricultural activities (Rocco and Morabito, 2016; Alemany et al., 2021; and Esteso et al., 2023).

The sugar sector and its derivatives have been widely examined from the operations research perspective. Recent research has focused on incorporating strategic and tactical decision-making into mathematical models in order to redesign the AFSCs configuration (Carvajal et al., 2019; Jonkman et al., 2019; Fikry et al., 2021; Poltroniere et al., 2021; Filho et al., 2021, 2023; and Teixeira et al., 2023). Specifically, based on the case study of the sugar beet processing chain, Jonkman et al. (2019) highlights opportunities to enhance AFSCs via decentralized sugar beet processing/preprocessing.

Kusumastuti et al. (2016) states that from the existing literature on agricultural harvesting and processing models, the focus is predominantly on a centralized processing approach. This means crops from various farms or fields are transported to a fixed location. However, innovative supply chains may be more appropriate for rural sector development by involving a decentralized processing approach based on mobile units.

A recent literature review on facilities location problem of modular and mobile (Alarcon-Gerbier and Buscher, 2022) notes that the current market demands more flexible supply chain networks that may provide short-term solutions jointly with strategic ones. For a decentralized processing approach, the study recommends integrating operational production and distribution activities with planning location and transportation routes for mobile production units. Moreover, the study concludes that few articles consider mobile facilities' transportation, downtime, and setup times.

Particularly in the olive oil sector, decision support systems based on mathematical programming have focused mainly on harvesting, storage, production, and distribution. Vlah Jerić and Šorić (2010) proposes an MILP model to maximize the profit of small olive suppliers and small olive oil producers considering revenues, production, storage, and olive supply costs, together with a penalty of supplying in non-preferential periods due to the oversupply of olives. Vlah Jerić and Šorić (2019) extends its previous study to a case study in Croatia, using a multi-objective programming model that relates the producer and suppliers' objectives.

Kazaz (2004) presents a two-stage stochastic programming model to study olive oil production planning. The optimization model aims to maximize producers' profits, considering purchase costs and selling prices that depend on yield uncertainty and random demand. Kazaz and Webster (2011) extends its previous study and investigates the influence of yield-dependent trade costs on pricing, initial investment, expected profitability, and planning of olive oil production under supply uncertainty.

Cano Marchal et al. (2018) proposes a stochastic model to obtain a production plan for extra virgin olive oil that maximizes the company's profits. The optimization model determines the amount of olive oil obtained as a function of the properties of the processed olives and evaluates the appropriateness of including fixed costs and uncertainty in some parameters. Similarly, Herrera-Cáceres et al. (2017) estimates olive oil percentages depending on the olive lot and harvest period. Their study proposed an MILP model for planning harvests of different varieties of olives, aiming to maximize the quantity of olive oil while considering climatological phenomena. Given the closeness of the problem, our proposed model contains some constraints related to this study. Recently, Montenegro-Dos Santos et al. (2023) proposes a non-myopic Rolling Horizon methodology to face the uncertain changes produced in agricultural harvest planning. The authors proposed a bi-objective MILP formulation, minimizing the discrepancies between a base plan and the new one requested by some uncertain conditions that have changed and maximizing the total production. They tested this methodology using the olive oil production case presented in Herrera-Cáceres et al. (2017), using centralized production management, i.e., without taking into account mobile olive oil mills.

Yurt et al. (2019) investigates AFSC operations with a focus on olive oil bottling activities and its distribution network. They present a deterministic mathematical model for the profit maximization of a cooperative, differentiating the perceived profit between each type of bottled glass bottle. Their integer programming model considers the distances between suppliers, manufacturers, and demand points in some regions of Turkey.

According to the reviewed literature and to address the identified research gaps in Tables 1 and 2, the optimization model proposed in this research integrates olives harvesting, transformation, and transportation decisions for planning the usage of olive mills commonly owned by producers, considering a feasible harvesting time window and the olive oil yield differentiated by variety and region. In addition, the sequence of olive mills' relocation, downtime times, and their respective transport costs are considered. Thus, the proposed mobile and modular structure leads to an alternative design for the AFSCs by decentralizing the transformation process.

The tactical/operational model aims at optimizing the trade-off between a higher concentration of olive oil and the reduction of transportation costs, maximizing the profit of the small farmers of the agricultural cooperative in the olive oil industry, whose field has been little explored in the literature from an operations research point of view.

3. Problem statement

This section briefly describes the olive oil production process, portraying the different variables and the nature of the main parameters that affect harvesting, transformation, and transportation decisions. To this purpose, we describe the assumptions and constraints considered to develop the proposed mathematical programming model.

The main stages of the olive oil supply chain are farming, oil extraction from olives, and packaging. The first stage includes olive cultivation and harvesting, which can be manual, mechanized, or mechanical. The raw materials are then transported to the olive oil mills to be processed through defoliation, washing, pressing, malaxing, and extraction. The liquid phase (must oil) is separated from the solid one (pomace). The third stage consists of bottling and the treatment of olive oil production waste (Rapa and Ciano, 2022). More details on the olive oil production process can be found in Di Giovacchino et al. (2002).

Sizeable olive oil producers have the resources to carry out their products in their facilities, and they are accompanied by industrial machinery to harvest their olives. However, small farmers are conditioned by their harvesting capacity, mainly hand harvesting. They must adapt to the availability of the olive oil mills in the cooperatives or subcontract the owners' service. These communities are generally organized through a decision maker in charge of production planning at the olive oil mill, coordinating the days each olive grower must transport his harvest to the mill's location.

The cooperative's producers usually grow different olive varieties, whose natural properties and the weather conditions of each sector where the agricultural field is located affect the optimal harvesting period. Fig. 1 shows the supply of kilograms of olives to the production plant, which is limited by the milling and manual harvesting capacity. After the olive oil milling and storage process, producers decide whether their products will be for personal consumption only or bottled to generate additional profit from selling each type of monovarietal olive oil bottle on the local market.

Fig. 1 also shows that, unlike other AFSCs, production plants can be moved and temporarily installed in different agricultural fields to reduce transportation costs for producers further away from their current location during the harvest season. Along with all the information collected, decision-makers must respect the olive oil mills' limitations and the producers' daily harvesting capacity, guaranteeing the oil's quality and safety standards.

Decisions are complex since, depending on the defined production/processing plan, the costs and time associated with olive oil production may be less beneficial than selling olives in bulk for some producers. Currently, tactical and operational decisions depend mainly on the decision-maker's experience/intuition and reasonable criteria. We formulate an optimization model for small olive growers as a profit maximization tool to solve this problem. To do so, we make the following assumptions:

- All small farmers produce monovarietal extra virgin olive oil in different proportions, and the total profit per producer is calculated depending on whether they decide to produce olive oil for sale or only for their consumption, which is given by a retail price per liter of olive oil (i.e., it is an opportunity cost for buying in the market) that can generate an additional profit if they decide to bottle their product for commercialization.
- The producer anticipates which monovarietal olive oil to be bottled, considering that the entire production of oil of a given variety can be bottled in a single type of glass bottle. It is assumed that all products are sold at an additional profit depending on the type of bottle and variety of olive oil the producer chooses. If the olive oil is insufficient for the volume of a particular bottle, the olive growers will obtain a commercial price associated with the bulk format of the product, which is lower than the bottle price.
- Producers must harvest the olives one day before being assigned to milling at the production facility. Therefore, the day after harvesting, olive growers transport the daily harvest in their vehicles to the olive oil mill set to them to produce olive oil. Since they are small producers, the vehicle's capacity is assumed high enough to transport the kilograms of olives harvested.

Table 1
Papers on optimization problems types in the agri-food supply chain.

Author(s)	Number of products	Commodity type	Harvest	Production process	Mathematical modeling	Data type
Kazaz (2004)	S	Olive	*	*	SP	H
Ferrer et al. (2008)	S	Grape	Centralized	MC - MA	MILP	RC
Bohle et al. (2010)	M	Grape	Centralized	MC - MA	RO	RC
Vlah Jerić and Šorić (2010)	M	Olive	Centralized	*	MILP	H
Ahumada and Villalobos (2011)	M	Pepper and tomato	Centralized	MA	MILP	H
Kazaz and Webster (2011)	S	Olive	*	*	SP	H
Rocco and Morabito (2016)	M	Tomato	Centralized	*	LP	RC
Herrera-Cáceres et al. (2017)	M	Olive - Olive Oil	Centralized	MC - MA	MILP	RC
Cano Marchal et al. (2018)	M	Olive Oil	*	*	SP (Multi-stage)	RC
Carvajal et al. (2019)	S	Sugarcane	Centralized	MC	SP	RC
Jonkman et al. (2019)	M	Sugar beet	Centralized	*	MILP	H
Vlah Jerić and Šorić (2019)	M	Olive	Centralized	*	MILP	H
Yurt et al. (2019)	M	Olive Oil	*	*	IP	RC
Varas et al. (2020)	S	Grape	Centralized	MC - MA	MILP	H
Alemany et al. (2021)	M	Tomato	Centralized	*	MILP	RC
Avanzini et al. (2021)	S	Grape	Centralized	MC - MA	SP (Multi-stage)	H
Fikry et al. (2021)	M	Sugar beet	Centralized	*	MILP	H
Filho et al. (2021)	S	Sugarcane	Centralized	MC	MINLP	RC
Poltroniere et al. (2021)	M	Sugarcane	Centralized	*	MILP	RC
Esteso et al. (2023)	M	Tomato	Centralized	MA	MILP	RC
Filho et al. (2023)	M	Sugarcane	Centralized	MC	MINLP	RC
Montenegro-Dos Santos et al. (2023)	M	Olive - Olive Oil	Centralized	MC - MA	MILP	RC
Contribution	M	Olive - Olive Oil	Decentralized	MA - MZ	MILP	RC

Notes: S: Single product, M: Multiple products, MA: Manual, MZ: Mechanized, MC: Mechanical, C: Centralized, D: Decentralized, RC: Real case, H: Hypothetical, LP: Linear programming, IP: Integer programming, MILP: Mixed-integer linear programming, MINLP: Mixed-integer no linear programming, SP: Stochastic programming, RO: Robust optimization, *: Undefined.

Table 2
Papers on optimization models in the agri-food supply chain.

Author(s)	Decision variables								Decision levels	Time horizon	Objective function		
	CP	HP	PR	DT	IN	PC	FR	FT			CM	PM	OT
Kazaz (2004)			X						S	1P		X	
Ferrer et al. (2008)		X					X		T-O	MP	X		
Bohle et al. (2010)		X					X		T-O	MP		X	
Vlah Jerić and Šorić (2010)		X	X		X				T-O	MP		X	
Ahumada and Villalobos (2011)		X	X	X	X			X	O	MP		X	
Kazaz and Webster (2011)			X						S	1P		X	
Rocco and Morabito (2016)		X	X	X	X			X	T	MP	X		
Herrera-Cáceres et al. (2017)		X	X						O	MP			X
Cano Marchal et al. (2018)			X						O	MP		X	
Carvajal et al. (2019)	X	X	X		X				S-T	MP		X	
Jonkman et al. (2019)	X	X	X	X	X			X	S-T	MP		X	X
Vlah Jerić and Šorić (2019)		X	X		X				T-O	MP	X	X	
Yurt et al. (2019)			X	X				X	T	1P		X	
Varas et al. (2020)		X					X		T-O	MP	X		X
Alemany et al. (2021)	X	X	X	X	X			X	O	MP		X	X
Avanzini et al. (2021)		X					X		O	MP		X	
Fikry et al. (2021)	X	X			X			X	S-T	MP	X		
Filho et al. (2021)		X						X	T	MP	X		
Poltroniere et al. (2021)	X	X			X				T-O	MP		X	
Esteso et al. (2023)	X	X	X	X	X			X	T	MP	X	X	X
Filho et al. (2023)	X	X			X			X	T	MP	X	X	
Montenegro-Dos Santos et al. (2023)		X	X						O	MP			X
Contribution		X	X			X		X	T-O	MP		X	

Note: CP: Cultivation planning, HP: Harvest planning, PR: Production planning, DT: Distribution planning, IN: Inventory planning, PC: Mobile plant scheduling, FR: Freight routing/harvesting, FT: Freight transport, S: Strategic, T: Tactical, O: Operational, 1P: Single period, MP: Multiple periods, CM: Cost minimization, PM: Profit maximization, OT: Other objectives.

- If it is decided to relocate and temporarily install the mobile olive oil mills, variable costs will be incurred depending on the traveled distance by trailer. In addition, the production plants will not be available during the transfer day; thus, one production day is lost.
- The mill operator is paid a daily salary per production day and is responsible for receiving olives from growers to be processed on the same day they are received. Although the production facility can mill different varieties of olives separately daily, its capacity is limited by the kilograms processed and does not depend on the processing time.
- The harvest window considers the preferences and availability of producers to obtain olive oil with the desired quality and properties within a tolerance period defined according to maturity and estimated olive oil concentration.
- The concentration of olive oil follows a maturity curve for each type of olive variety and the geographical location of the producers, whose processing yield and quality distribution over time are estimated from the natural properties of each crop, the characteristics of the sector, and historical data available from the agricultural fields.

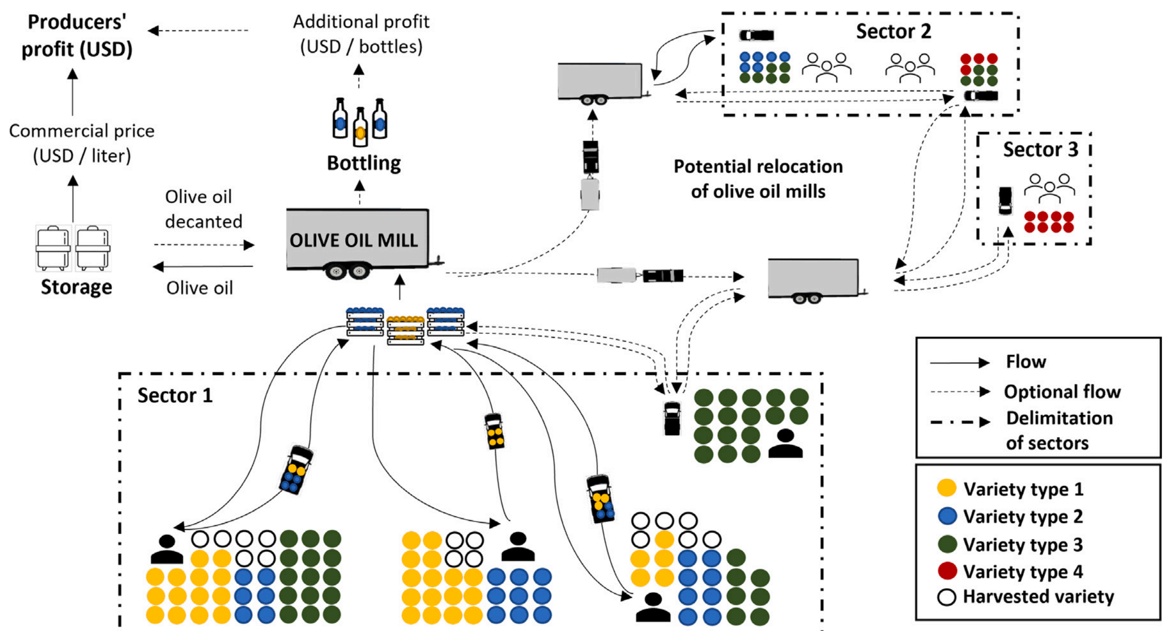

Fig. 1. Olive harvesting and olive oil processing in mobile olive oil mills.

- Each producer incurs a variable transportation cost that depends on the distance they travel to the assigned olive oil mill, considering a round trip as they store the olive oil in their fields. The bottling/labeling process of the olive oil decanted does not influence the milling capacity during production planning, as they are located in separate compartments in each mobile olive oil mill.
- The producers of the cooperative coordinate to use the production plants in common. However, all olive growers sell their products independently in the nearest locations, so the mathematical modeling does not consider a transportation cost to the customers to avoid overestimating or underestimating the overall costs.
- In terms of perishability or loss of productivity associated with the harvesting process, the proposed formulations assume very short periods between olive harvesting and processing, and thus, olive perishability is discarded. In addition, perishability associated with produced olive oil is not considered since it is assumed that they are rapidly bottled and sold.

4. Optimization model

This section presents an MILP model to help olive growers manage and plan to use olive oil mills. The optimization procedure consists of two steps. The first step is preprocessing, in which the olive oil bottling conditions are parameterized, and the most relevant harvesting and production/processing parameters are estimated. The second step is the data optimization model, which maximizes the producers' profit through the efficient utilization of the mobile olive oil mill and the timely supply of low-cost transportation of the different olive growers' productions based on the previously calculated parameters.

A multi-objective optimization model could consider profit maximization objectives through increased olive oil concentration and minimizing overall costs. Still, we have decided to combine them into a single objective function in light of the selling price and retail price of olive oil produced versus a series of costs. The MILP model with its sets, indices, parameters, variables, formulation, and explanations of the constraints are described below:

Sets

T	Sets of days within the planning horizon (indexed by $t \mid t \in \mathbb{Z}_0^+$).
V	Sets of olive varieties (indexed by v).
P	Sets of producers (indexed by p).
B	Sets of olive oil glass bottles (indexed by b).
A	Sets of olive oil mills (indexed by a).
L	Sets of potential olive oil mills locations (indexed by $\ell, r \mid \ell \neq r$).

Parameters

AO_b	Amount of olive oil (in liters) needed to fill the glass bottle b.
BO_{pvb}	Olive oil bottling. It takes the value 1 when the producer p decides to package the olive oil with variety v in glass bottle b; 0 otherwise.
CO_{pvt}	Olive oil concentration (in liters of oil per kilogram of olives) for variety v of producer p on day t.
H_{pvt}	Harvest window. It takes value 1 when producer p can harvest variety v on day t; 0 otherwise.
O_{pv}	Estimated quantity of olives at the beginning of the harvest season (in kilograms), of variety v of producer p.
Q_{pt}	Maximum quantity of olives (in kilograms) with which the producer p arrives at an olive oil mill on the day t, given his daily harvesting capacity.
K_a	Daily production capacity (in kilograms) at the olive oil mill a.
CP_v	Commercial price (in USD/liter) of extra virgin olive oil of the variety v.
UP_{vb}	Additional profit (in USD) from selling glass bottle b with extra virgin olive oil of variety v.

KC_b	Cost to package (in USD) the glass bottle b.
LC	Cost daily labor (in USD) to operate an olive oil mill on a given day.
PC	Cost of production (in USD) per kilogram processed in any olive oil mill.
TC	Cost of transportation (in USD) per kilometer of traveled distance.
$RC_{a\ell r}$	Cost of relocating (in USD) the olive oil mill a per day to location ℓ from location r.
$D_{p\ell}$	Traveled distance (in kilometers) for producer p to use an olive oil mill at the location ℓ.
M	Number of olive varieties to be harvested in the planning horizon ($M \in \mathbb{Z}^+$)
N	Number of producers in the study ($N \in \mathbb{Z}^+$)

Variables

u_{pvb}: Quantity of glass bottles b with olive oil of variety v of producer p.

x_{pvt}: Quantity of olives (in kilograms) of variety v harvested by producer p to be processed in an olive oil mill on day t.

o_{pvt}: Quantity of olives (in kilograms) remaining of variety v to be harvested by producer p, at the end of day t.

f_{pat}: Quantity of olives (in kilograms) that producer p transported to the olive oil mill a on day t.

$$y_{pvt} = \begin{cases} 1, \text{if producer } p \text{ harvests variety } v \text{ at the end of day } t-1 \text{ for production on day } t \\ 0, \text{otherwise.} \end{cases}$$

$$w_{pat} = \begin{cases} 1, \text{if producer } p \text{ is assigned to use olive oil mill } a \text{ on day } t \\ 0, \text{otherwise.} \end{cases}$$

$$z_{a\ell t} = \begin{cases} 1, \text{if an olive oil mill } a \text{ is located at site } \ell \text{ on day } t \\ 0, \text{otherwise.} \end{cases}$$

$$e_{a\ell t} = \begin{cases} 1, \text{if an olive oil mill } a \text{ is employed at the location } \ell \text{ on day } t \\ 0, \text{otherwise.} \end{cases}$$

$$\phi_{a\ell rt} = \begin{cases} 1, \text{if olive oil mill } a \text{ moves to location } \ell \text{ from location } r \text{ during day } t \\ 0, \text{otherwise.} \end{cases}$$

$$s_{pt} = \begin{cases} 1, \text{if producer } p \text{ harvests any variety of olive the day } t-1 \text{ for production on day } t \\ 0, \text{otherwise.} \end{cases}$$

$$\psi_{at} = \begin{cases} 1, \text{if olive oil mill } a \text{ is used on day } t \\ 0, \text{otherwise.} \end{cases}$$

$$g_{p\ell t} = \begin{cases} 1, \text{if producer } p \text{ is moved to the location } \ell \text{ on day } t \\ 0, \text{otherwise.} \end{cases}$$

Objective function

$$\begin{aligned} \max \sum_{t\in T}\sum_{v\in V}\sum_{p\in P} CP_v \cdot CO_{pvt} \cdot x_{pvt} + \sum_{b\in B}\sum_{v\in V}\sum_{p\in P}\left(U_{vb} - KC_b\right) \cdot BO_{pvb} \cdot u_{pvb} \\ - \sum_{t\in T}\sum_{a\in A} LC \cdot \psi_{at} \\ - \sum_{t\in T}\sum_{v\in V}\sum_{p\in P} PC \cdot x_{pvt} - \sum_{t\in T-\{0\}}\sum_{\ell,r\in L:\ell\neq r}\sum_{a\in A} RC_{a\ell r}\cdot\phi_{a\ell rt} \\ - \sum_{t\in T}\sum_{\ell\in L}\sum_{p\in P} 2\cdot TC\cdot D_{p\ell}\cdot g_{p\ell t} \end{aligned} \tag{1}$$

subject to:

$$AO_b \cdot u_{pvb} \leq BO_{pvb} \cdot \sum_{t\in T}\left(CO_{pvt}\cdot x_{pvt}\right) \quad \forall p\in P, v\in V, b\in B \tag{2}$$

$$o_{pv0} = O_{pv} \quad \forall p\in P, v\in V \tag{3}$$

$$o_{pvt} = o_{pv(t-1)} - x_{pvt} \quad \forall p\in P, v\in V, t\in T\setminus\{0\} \tag{4}$$

$$x_{pvt} \leq o_{pv(t-1)} \quad \forall p\in P, v\in V, t\in T\setminus\{0\} \tag{5}$$

$$\sum_{t\in T} x_{pvt} \leq O_{pv} \quad \forall p\in P, v\in V \tag{6}$$

$$x_{pvt} \leq Q_{pt}\cdot y_{pvt} \quad \forall p\in P, v\in V, t\in T \tag{7}$$

$$y_{pvt} \leq H_{pvt} \quad \forall p\in P, v\in V, t\in T \tag{8}$$

$$\sum_{v\in V} x_{pvt} \leq Q_{pt} \quad \forall p\in P, t\in T \tag{9}$$

$$\sum_{p\in P} f_{pat} \leq K_a \cdot \sum_{\ell\in L} e_{a\ell t} \quad \forall a\in A, t\in T \tag{10}$$

$$f_{pat} \geq \sum_{v\in V} x_{pvt} - Q_{pt}\cdot(1-w_{pat}) \quad \forall p\in P, a\in A, t\in T \tag{11}$$

$$f_{pat} \leq Q_{pt}\cdot w_{pat} \quad \forall p\in P, a\in A, t\in T \tag{12}$$

$$\sum_{v\in V} y_{pvt} \leq M\cdot s_{pt} \quad \forall p\in P, t\in T \tag{13}$$

$$s_{pt} \leq \sum_{a\in A} w_{pat} \quad \forall p\in P, t\in T \tag{14}$$

$$\sum_{p\in P} w_{pat} \leq N\cdot\psi_{at} \quad \forall a\in A, t\in T \tag{15}$$

$$\psi_{at} \geq \sum_{\ell\in L} e_{a\ell t} \quad \forall a\in A, t\in T \tag{16}$$

$$w_{pat} \leq \sum_{\ell\in L} e_{a\ell t} \quad \forall p\in P, a\in A, t\in T \tag{17}$$

$$\sum_{a\in A} w_{pat} \leq 1 \quad \forall p\in P, t\in T \tag{18}$$

$$e_{a\ell t} \leq z_{a\ell t} \quad \forall a\in A, \ell\in L, t\in T \tag{19}$$

$$\sum_{\ell\in L} z_{a\ell t} = 1 \quad \forall a\in A, t\in T \tag{20}$$

$$\sum_{\ell\in L} e_{a\ell t} \leq 1 \quad \forall a\in A, t\in T \tag{21}$$

$$\sum_{a\in A} e_{a\ell t} \leq 1 \quad \forall \ell\in L, t\in T \tag{22}$$

$$g_{p\ell t} \geq w_{pat} + e_{a\ell t} - 1 \quad \forall p\in P, a\in A, \ell\in L, t\in T \tag{23}$$

$$\phi_{a\ell rt} \geq z_{a\ell t} + z_{ar(t-1)} - 1 \quad \forall a\in A, \ell, r\in L : r\neq\ell, t\in T\setminus\{0\} \tag{24}$$

$$1 - \sum_{r\in L:\ell\neq r}\phi_{a\ell rt} \geq e_{a\ell t} \quad \forall a\in A, \ell\in L, t\in T \tag{25}$$

$$u_{pvb} \in \mathbb{Z}_0^+ \quad \forall p\in P, v\in V, b\in B \tag{26}$$

$$x_{pvt}, o_{pvt}, f_{pat} \geq 0 \quad \forall p\in P, a\in A, v\in V, t\in T \tag{27}$$

$$y_{pvt}, w_{pat}, z_{a\ell t}, e_{a\ell t} \in \{0,1\} \quad \forall p\in P, a\in A, v\in V, \ell\in L, t\in T \tag{28}$$

$$\phi_{a\ell rt}, g_{p\ell t}, s_{pt}, \psi_{at} \in \{0,1\} \quad \forall p\in P, a\in A, \ell, r\in L : r\neq\ell, t\in T \tag{29}$$

The objective function (1) maximizes olive growers' profits from producing extra virgin olive oil. The first term defines a retail price for the liters of monovarietal olive oil produced, which can generate an additional profit from selling the different types of bottles, according to the second term. The third term refers to the operator's salary for the production plants, while the fourth term is associated with the operating costs of olive milling. The last terms are the cost of towing the olive oil mills and the cost of raw material transportation by producers to the current location of the mills, respectively.

Constraints (2) provide an upper limit for bottling monovarietal olive oil, depending on the liters each olive grower produces if they decide to bottle previously. The availability to harvest olives at the beginning of the time horizon is given by (3). Constraints (4) establish for each producer the kilograms of olives remaining of the different olive varieties whose harvest for milling does not exceed the number of olives available at the end of the day $t-1$ according to constraints (5). Constraints (6) limit the olive oil production to the producer's estimated total harvest.

If, on certain days, it is decided to carry out the milling of olives from the producers, (7) activates the harvest variable, which is conditioned to a particular feasible time window according to the constraints (8). In (9), olive processing is limited to the daily harvesting capacity of each producer. Similarly, in (10), daily milling is limited to the olive oil mills' capacity, conditioning the kilograms of olives that producers transport to each available production plant. If it is decided to assign a producer to a specific olive oil mill, the constraints (11) and (12) input to variable f_{pat} the kilograms of the different varieties of olives that the grower can harvest during a given period.

Logical constraints (13) and (14) activate the assignment variable w_{pat} if the producer harvests at the end of the previous day to produce olive oil on the current day. In contrast, logical constraints (15) and (16) limit the employment of the olive oil mill as long as it is not decided to assign at least one producer to a particular production plant to carry out the production process. In addition, Eq. (17) activates the availability of these mills in a certain location if it is decided to assign producers to mills, considering that each producer can be assigned to only an olive oil mill per period, according to (18).

The employment of the olive oil mills defines their current location (19), which must be located in a place for each day according to the constraints (20). On certain days, the olive oil mills may not be available; however, they must be placed in single and different locations per period, according to the constraints (21) and (22). According to the restrictions (23), the cost of raw material transportation by producers is conditioned to their allocation to the current location of olive oil mills and the availability/employment of the mills in that period.

Constraints (24) activate the relocation and temporary installation of mobile olive oil mills in another agricultural field if the current location differs from the previous one. Consequently, constraints (25) limit the availability of the production plants when it is decided to move them (i.e., if it is determined to transport the olive oil mill to the ℓ position from the r position on the day t, then the production facility must be inactive at the ℓ destination on the day t but active on the day $t+1$). Finally, constraints (26), (27), (28), and (29) indicate the nature of decision variables.

5. Results and discussion of the application case

The optimization model proposed in this paper is applied to solve the problem of dozens of small olive growers in the Elqui Province, Coquimbo Region, Chile. Section 5.1 details the setup, parameter preprocessing, and model application using practical information from the previous study by the Instituto de Investigaciones Agropecuarias (INIA) and data provided by the olive grower associations. In Section 5.2, the computational package's capabilities and the model outputs are exemplified to obtain management insights. A sensitivity analysis of producers' harvesting capacities, the estimated quantity of olives at the beginning of the season, and olive oil concentration yield are performed in Section 5.3. The computational performance of the model, details on the optimization implementation, and the size of the instances are discussed in Section 5.4.

5.1. Case of application in Chile

Through the Chilean Ministry of Agriculture, INIA built two mobile olive oil mills to help small olive growers produce and package extra virgin olive oil. Each production plant has a surface area of 13 square meters, which houses the extraction, bottling/filtering, and packaging/labeling machinery, separated into three independent interior compartments. The structure is mounted on a double axle towing trolley that allows relocation in rural areas (see Fig. A.11). Tapia et al. (2020) detail the technical and economic aspects of the production, bottling, storage, and waste treatment process of olive oil mills.

The study area is situated within a zone with different microclimates between localities, influencing the optimum harvest date of the different varieties of olives. The olive growers are located and distributed geographically among the communes of La Higuera, La Serena, Andacollo, and Coquimbo, whose maximum distance between their locations slightly exceeds 170 kilometers. To cluster the homogeneous olive oil production zones, Tapia et al. (2020) conducts a multivariate statistical analysis considering the physical characteristics of the ground, climatology, irrigation water, chemical and sensory components of the olive oil. Based on the results, they defined three sectors with similar characteristics.

Depending on the climatology of each sector under study, which affects the olive oil's physicochemical properties and phenolic compounds, the olive oil yield will be classified into homogeneous territories according to the olive variety type and each producer's geographical location. Fig. 2 shows that for a given olive variety, the behavior of the olive oil concentration changes depending on the geographical area and the seasonality of the fruit over time, evaluated in the months that the olive harvesting season usually lasts. It can be seen that the olive oil yield is different in each sector and reaches its upper limit in different periods, maintaining an olive oil accumulation from 11% to 20% for that olive variety.

In the preprocessing stage, the percentage of olive oil concentration of the different olive varieties is estimated through historical data available about the yields and quality distributions of the crops planted according to the results of the previous study of Tapia et al. (2020). In Fig. 3, it can be visualized that when comparing the average olive oil concentration between different olive varieties, variety type 1 reaches the optimum of its growth curve in the first weeks, while for variety type 4, the maturity period is near the last weeks of the harvest season, being its average oil concentration lower than all varieties due to the physical characteristics of the fruit.

Olive oil concentration is a relevant parameter to define the optimal harvest date; however, harvested fruits must be kept within a tolerance period given each grower's daily harvesting capacities and the olive oil mill's processing capacities. To define the feasible harvest window and the total availability of olives for the season, the specialists test the agricultural fields according to the olives' maturity, considering the growers' production preferences and capacities. In addition, the bottling/labeling planning of monovarietal olive oil is parameterized according to the commercial strategy of each producer.

5.2. Computational results

To exemplify the capabilities of the computational package, we consider only one harvest season interval with a planning horizon of 20 working days plus the day before the start of olive processing. Each producer $P = \{p_1, p_2, \ldots, p_{30}\}$ must transport the harvest of his olive varieties $V = \{v_1, v_2\}$ to the olive oil mill $A = \{a_1, a_2\}$ assigned and available in a certain time period. Fig. 4 shows the potential locations $L = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5\}$ for the relocation of the production plants, together with the location of producers grouped by sector according to the properties of the olives. Glass bottles $B = \{b_1, b_2\}$ of 250 ml and 500 ml are available. The analysis of the optimization model considers an optimality gap of 0.1% for instances, the details of which are discussed in Section 5.4.

The international olive oil market does not have a universal benchmark price, as they vary according to supply, demand, and stock levels at each market (Mili and Bouhaddane, 2021). In particular, we consider the retail prices of Chilean supermarkets. The baseline profit of the producers is estimated according to the liters of monovarietal extra

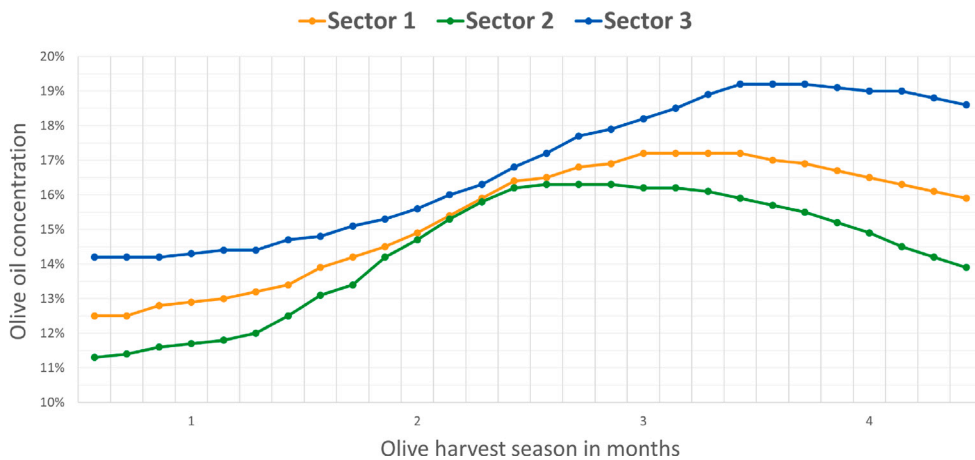

Fig. 2. Monovarietal olive oil concentration by homogeneous territories.

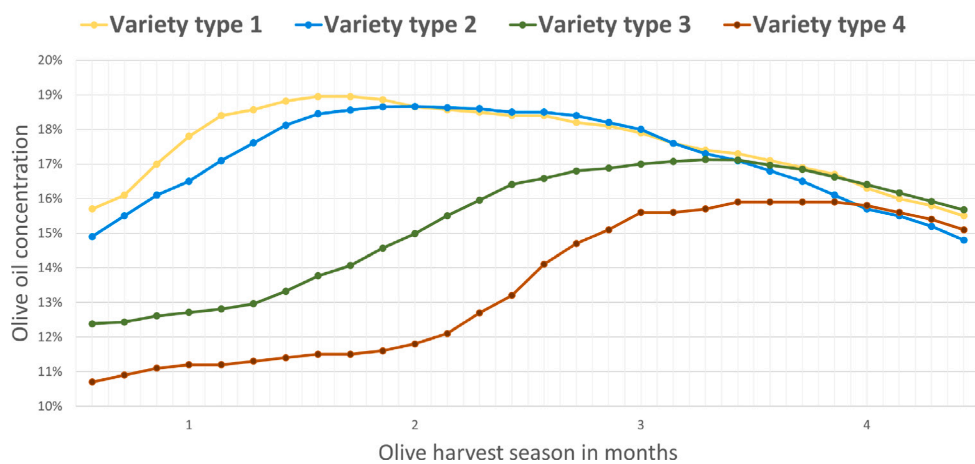

Fig. 3. Comparison of average olive oil concentration between varieties of olives.

virgin olive oil produced, which can generate an additional profit if they decide to bottle and sell a specific type of glass bottle (e.g., if a 1000 ml glass bottle of olive oil is priced at US$ 9 and a 250 ml glass bottle of olive oil of the same variety is priced at US$ 4, the additional profit for producing these four bottles is US$ 7 minus the respective bottling costs).

Fig. 5 shows that for olive growers who have two varieties of olives cultivated in a particular sector, it is preferable to harvest both during the same time interval to reduce transportation costs, as long as the level of olive oil concentration is in an acceptable range. Given the climatic characteristics of each sector, some producers have a greater volume of olives and, therefore, require more harvesting days, considering the harvesting capacity of small farmers. Similarly, it is normal for some producers not to cultivate a specific type of olive variety due to their preferences or unfavorable conditions for olive growth in a sector.

Producers may be assigned to different sites according to the location and availability of mobile olive oil mills, which depends on the variety and maturity of olives in each sector. Fig. 6 in the days that the production plants were used, olive oil mill a_2 has a higher average utilization of processed olives with a value of 72.80%, whereas olive oil mill a_1 reaches an average utilization of 53.96%. Specifically, the maximum utilization of olive oil mill a_1 does not exceed 75% over the planning horizon, while the maximum utilization of mill a_2 is 93.2%. Similarly, mill a_1 reaches a utilization of 30% in period one, while mill a_2 has a minimum daily utilization of 54%. Although overall profits are not affected by the low utilization of the production plants, their utilization should be in a range where profits exceed the olive oil processing costs.

The olive mills are available only if a relocation process is not being carried out between two fields starting at the end of the previous period and are only used if it is decided to assign producers to the mills, which is visualized in Fig. 7 with the olive oil mill a_2. In particular, in period 3, the production plant is moved from location ℓ_3 to location ℓ_1 to reduce the transportation costs of the assigned olive growers, whose action causes the loss of one day of production. Therefore, the olive oil mill will be available from period 4. However, even if the mill is available, producers may not necessarily employ it, as observed in periods 11, 15, 18, and 20.

To compare the solution obtained by using the mobile olive oil mills, the model is modified to restrict the relocation of the mills, fixing them in the set of available locations according to the solution previously found (see Figs. 7 and A.12). In this way, six instances have been evaluated considering the variations of the possible locations, considering them as fixed during the study horizon, as shown in Table 3. In addition, the economic impact of the solution obtained using mobile and fixed olive oil mills is presented.

As observed, the total revenues and costs generated by the olive oil production process maintain a similar economic margin for all the instances generated. The results indicate that using a decentralized production process generates at least 4.5% higher overall profit, achieving a decrease in transportation costs of 41.2% concerning the best solution obtained using fixed olive oil mills. Transportation costs

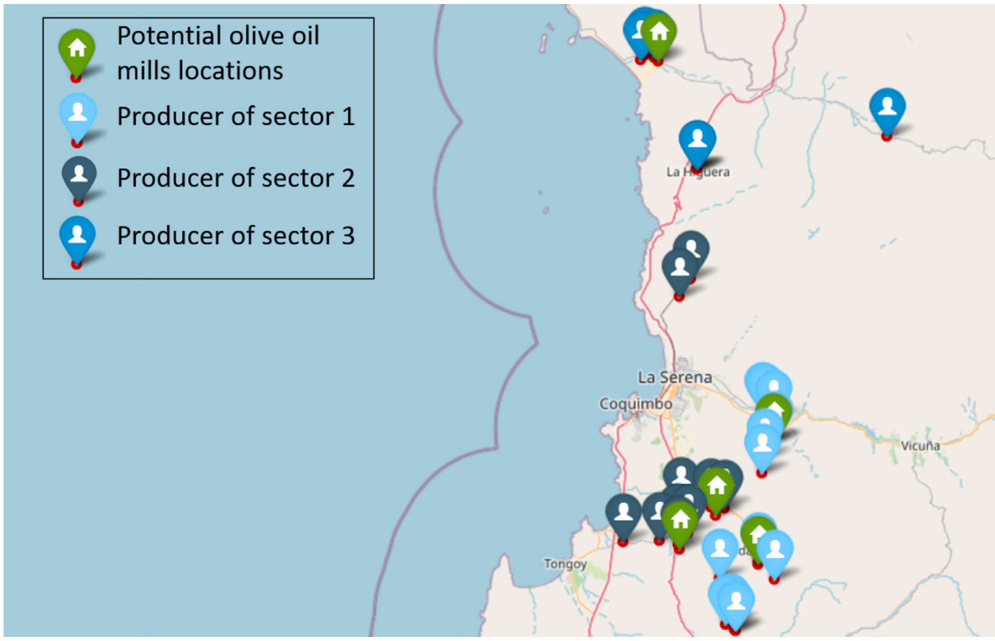

Fig. 4. Olive oil mill locations and producers by geographic.

Sector	Olive variety	Planning horizon in days																					Harvest days
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Sector 1	v_1	0	7	7	0	2	2	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	**20**
	v_2	0	0	0	0	0	0	0	0	0	0	2	0	5	5	6	0	6	5	0	6	0	**35**
Sector 2	v_1	0	0	0	0	6	6	6	6	6	0	3	0	0	0	0	0	0	0	0	0	0	**33**
	v_2	0	0	0	0	5	4	4	5	5	0	1	1	0	0	0	0	0	0	0	0	0	**25**
Sector 3	v_1	0	2	3	5	5	4	3	4	5	0	4	3	3	3	0	0	0	0	0	0	0	**44**
	v_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**

Fig. 5. Harvest planning of olives by variety and sector.

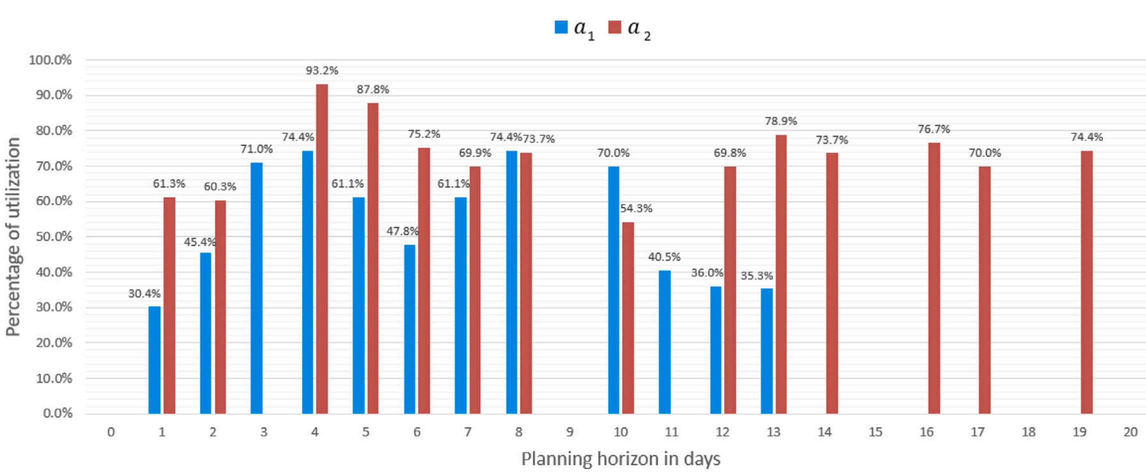

Fig. 6. Producers assigned to the mobile olive oil mills.

are based on the average price of gasoline in Chile during the year 2023 (ENAP, 2023), considering the average gasoline yield per mixed kilometer traveled by a private vehicle.

The agricultural fields in this study are located in rural areas far from each other. For this reason, in furtherance of calculating a realistic distance to be traveled by the small farmers and mobile olive oil mills, Open Source Routing Machine (OSRM) and OpenStreetMap were used to calculate this distance from the latitude and longitude data of the location of the agricultural fields (Huber and Rust, 2016). Thus, Table 4 presents the distance incurred for olive transportation (i.e., producers) and mill relocation and the total number of days the mills are employed. It is observed that the total kilometers traveled by producers is reduced by at least 42.4%, saving travel time in their vehicles, which could also be monetized in person-hours. It is important to note that

Location of the olive oil mill a_2

Potential locations	Planning horizon in days																				
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ℓ_1	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_2	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
ℓ_3	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Availability/employment of the olive oil mill a_2

Potential locations	Planning horizon in days																				
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ℓ_1	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_2	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	1	1	0	1	0
ℓ_3	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Fig. 7. Location and availability/employment of olive oil mill a_2.

Table 3
Profit and costs comparison between mobile and fixed olive oil mills.

Metrics (US$)	Mobile olive oil mill	Fixed olive oil mill					
		$a_1-\ell_4$, $a_2-\ell_3$	$a_1-\ell_4$, $a_2-\ell_2$	$a_1-\ell_4$, $a_2-\ell_1$	$a_1-\ell_5$, $a_2-\ell_3$	$a_1-\ell_5$, $a_2-\ell_2$	$a_1-\ell_5$, $a_2-\ell_1$
Commercial price	13,140	13,071	13,145	13,145	13,111	13,149	13,148
Additional profit	4,339	4,338	4,338	4,338	4,341	4,341	4,341
Cost of production	1,036	1,030	1,036	1,036	1,033	1,036	1,036
Cost of transportation	1,089	2,599	2,107	2,051	2,466	1,986	1,890
Cost of relocating	23	0	0	0	0	0	0
Cost labor	599	437	437	437	460	460	460
Objective function	14,733	13,342	13,903	13,959	13,492	14,006	14,102

* Exchange rate of the US dollar: US$ 868.77 Chilean pesos, taken on December 21, 2023, according to data from www.bcentral.cl.

Table 4
Comparison of days and traveled distance between mobile and fixed olive oil mills.

Metrics	Mobile olive oil mill	Fixed olive oil mill					
		$a_1-\ell_4$, $a_2-\ell_3$	$a_1-\ell_4$, $a_2-\ell_2$	$a_1-\ell_4$, $a_2-\ell_1$	$a_1-\ell_5$, $a_2-\ell_3$	$a_1-\ell_5$, $a_2-\ell_2$	$a_1-\ell_5$, $a_2-\ell_1$
Traveled distance by producers (in kilometers)	4,979	11,885	9,633	9,377	11,274	9,083	8,643
Traveled distance by olive oil mills (in kilometers)	105	0	0	0	0	0	0
Number of days the olive oil mills are used	26	19	19	19	20	20	20

such planning increases the number of days the mills must be used, thus proportionally increasing the costs of the operator's salaries.

5.3. Sensitivity analysis

Realistic instances were considered in the optimization model, along with historical information from the agricultural fields and feedback from the olive growers and agronomists of the application case. Therefore, the utilization of the olive oil mills for the entire study horizon is 39.7%, while considering only the days in which the mills are used, the average utilization is 64.1%. To evaluate the model's performance, the estimated quantity of olives of the producers, the harvesting capacity in each period, and the percentage of oil concentration of the olives for each variety and sector are sensitized.

5.3.1. Harvesting capacity

Considering that in the computational experiments, the mobile olive oil mills are used 37% more days than the fixed mills, the estimated harvest capacity and kilograms of olives calculated for each producer are doubled to evaluate the solution economically in the generated instances. Subsequently, the use of mills is stressed to their maximum capacity to assess the decision to relocate the mobile mills.

Accordingly, Table 5 shows system performance indicators for the solutions obtained with the increased harvesting capacities, considering fixed and mobile mills (similarly as shown in Tables 1 and 2). It can be observed that when using mobile mills, total profits are higher by at least 2.72%, whose value in the objective function is mainly affected by total costs, which are reduced by at least 17.36% when using the decentralized production process. The total kilometers traveled decreased by at least 43.97 compared to the solution obtained by setting the olive oil mills in specific locations throughout the planning horizon.

Sensitizing the values of the harvesting capacity parameters and the number of estimated kilograms of olives per producer increased the number of days of relocation of the olive oil mills and, in turn, reduced the number of days to use the mobile olive oil mills compared to the fixed olive oil mills. Therefore, the average utilization of mobile mills per day reached an average of 95.2%, considering the 20 working days for each one. Thus, it can be inferred that even if the number of kilograms of olives harvested increases, it is preferable to reduce transport costs than to increase the utilization of the olive oil mills.

To evaluate the relocation behavior of the mobile production mills, the harvesting capacity for each producer is proportionally increased up to the maximum milling capacity of the production plants. Table 6, similar to Table 3, shows system performance indicators for the solutions obtained with the new increased harvesting capacities, considering both fixed and mobile mills. In this case, the mobile mill scenario yields that only one mill is relocated once, and then only two

Table 5
Comparison between mobile and fixed olive oil mills with an increased harvesting capacity.

Metrics	Mobile olive oil mill	Fixed olive oil mill							
		$a_1-\ell_4$, $a_2-\ell_3$	$a_1-\ell_4$, $a_2-\ell_2$	$a_1-\ell_4$, $a_2-\ell_1$	$a_1-\ell_5$, $a_2-\ell_3$	$a_1-\ell_5$, $a_2-\ell_2$	$a_1-\ell_5$, $a_2-\ell_1$	$a_1-\ell_1$, $a_2-\ell_3$	$a_1-\ell_1$, $a_2-\ell_2$
Total profits (US$)	30,987	29,185	29,765	29,818	29,307	29,857	29,919	28,450	28,282
Total revenues (US$)	34,960	34,971	34,971	34,971	34,965	34,974	34,972	34,979	34,962
Total costs (US$)	3,972	5,542	4,962	4,908	5,414	4,872	4,808	6,284	6,435
Total traveled distance (in kilometers)	5,004	12,288	9,634	9,391	11,705	9,227	8,931	15,682	16,372
Number of days used	35	34	34	34	34	34	34	34	34

* Exchange rate of the US dollar: US$ 868.77 Chilean pesos, taken on December 21, 2023, according to data from www.bcentral.cl.

Table 6
Comparison between mobile and fixed olive oil mills at maximum milling capacity.

Metrics	Mobile olive oil mill	Fixed olive oil mill	
		$a_1-\ell_1$, $a_2-\ell_5$	$a_1-\ell_1$, $a_2-\ell_2$
Total profits (US$)	36,323	35,871	34,809
Total revenues (US$)	41,266	41,952	41,949
Total costs (US$)	4,943	6,081	7,141
Total traveled distance (in kilometers)	7,261	12,226	17,072
Number of days the olive oil mills are used	39	40	40

* Exchange rate of the US dollar: US$ 868.77 Chilean pesos, taken on December 21, 2023, according to data from www.bcentral.cl.

fixed mill location scenarios are considered. This way, the optimization model loses one production day to relocate the mobile olive oil mill to another sector to reduce the kilometers traveled. Such planning forces a decrease in overall revenues. Still, it simultaneously decreases the overall costs, thus increasing the total profits by 1.26% compared to the best solution obtained by fixing the mills at specific locations over the study horizon.

Fig. 8 illustrates the planning of olive milling in both mobile production plants provided by the model, achieving 97.5% of harvested kilograms compared to 100% of the harvested kilograms when setting olive oil mill a_1 at location ℓ_1 and olive oil mill a_2 at location ℓ_5 (see the planning of the fixed olive oil mills in Fig. A.13). It can be observed that during period 16, the maximum capacity of both mills is not used since olive oil mill a_1 is moved from location ℓ_5 to site ℓ_2 to get closer to the producers in Sector 1, where olive oil mill a_2 is located, thus losing one day of production in olive oil mill a_1.

In the last stage of the planning horizon, olive growers reduce the number of kilometers traveled to transport the kilograms of olives harvested; however, given the condition of losing a day of production by relocating the mobile olive oil mills, producers 19 and 20 will not be able to use the production plants given their milling capacity. Similarly, some producers will not be able to carry out the milling process for all their estimated olive production, considering that in terms of transportation costs, they are the furthest away from using the available olive oil mills.

5.3.2. *Olive oil concentration*

In agricultural planning, olive oil concentration is a relevant parameter for defining the harvest date, whose estimation depends on the properties of each olive variety and the characteristics of the area where it is harvested. By using quantitative analysis, Diamantakos et al. (2020) demonstrated that the degree of ripening of the olives at harvest date has a significant effect on the total phenolic content of olive oil, which decreases as it approaches higher degrees of ripening. To evaluate the robustness of the model's solution, the olive oil concentration percentage is sensitized for each variety and producer in each area, considering an interval of ± 2%.

Table 7 indicates the selected location for mill a_2 for each operational day and for each level of variation in olive oil concentration (i.e., −2%, −1%, 0%, 1%, 2%). Independent of the olive oil concentration variation, mill a_2 is relocated twice over the same locations (i.e., 1, 2, and 3). For all the simulated instances, olive mill a_2 relocates from location ℓ_3 to location ℓ_1 and then to place ℓ_2, considering the same mill relocation days for the different olive oil concentration percentages, except when it is decreased by 2% because certain olive varieties of the producers are mostly affected by this decrease. Regarding the olive oil mill a_1, it maintains its optimal configuration for all instances, relocating only once in period 9 from location ℓ_5 to site ℓ_4 (see Table 8).

Furthermore, Table 9 presents several system economic performance indicators for each level of variation in oil concentration. It shows that these changes significantly affect the solution economically, with revenues increasing or decreasing proportionally to the variation in that parameter. In particular, when the olive oil concentration was decreased by 2%, transportation costs were reduced, and labor costs increased compared to the initial solution. While the other costs remained constant, they were indifferent to the changes. While the variation in olive oil concentration influences the date of olive harvesting and its respective profits, the olive oil mill relocation, and olive grower allocation planning maintain a robust configuration in the different instances evaluated.

The computational results lead us to the conclusion that depending on the geographical location of the olive growers, the actual location of the mobile olive oil mills, and the properties of the olives in each homogeneous territory, the harvested olives do not necessarily reach the maximum olive oil yield because the model prefers to decrease the overall costs by harvesting the fruits earlier or later respecting the harvesting window established by each producer. Overall, the model prioritizes savings in transportation costs rather than greater utilization of olive oil mills, reaching a utilization rate of 30.4%, which still generates benefits over processing and labor costs.

This research demonstrates that decentralizing the production process in the agri-food supply chain through mobile olive oil mills increases overall profits and reduces costs for small farmers in agricultural cooperatives. From the results obtained in the application case, it is justified that by integrating harvesting, transportation, and production decisions in the mobile olive oil mills, transportation costs, and kilometers traveled by producers are reduced by at least 41%, considering the instances generated. Thus, we believe this work would

Producers	Planning horizon in days																					Kilograms/ producers	Percentage of kilograms harvested
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
p_1	0	124.1	0	0	150	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	274.1	100%
p_2	0	0	0	0	0	0	0	0	0	0	0	200	200	195.6	0	0	0	180	112.8	200	200	1,288.4	100%
p_3	0	0	0	0	0	0	0	0	0	0	0	0.0	0.0	0.0	0	0	0	72	0	48	72	192	100%
p_4	0	0	0	0	0	0	0	0	0	0	0	0.0	0.0	0	170	170	170	164.3	139.3	170	156	1,139.6	100%
p_5	0	0	0	0	0	0	0	0	0	0	0	190	0	0	0	190	0.0	176.5	141.6	190	190	1,078.1	100%
p_6	0	0	0	0	0	0	0	0	0	0	0	0	72.0	0	0	0	72	48	72	72	72	408	100%
p_7	0	0	0	0	0	0	0	0	0	0	0	0	0.0	0	150	0	150	150	125.9	130	150	855.9	100%
p_8	0	23.7	100	0	0	0	0	100	0	0	0	60	58	0	0	90	58	0	0	0	60	549.7	100%
p_9	0	0	0.0	51.9	0	0	72	0	0	0	0	0.0	0.0	0	0	0	0	0	0	0	0	123.9	100%
p_{10}	0	0	0	0	105	118.5	0.0	112.7	0	120	0	0.0	120	0	0	0	0	0	108.4	0	0	684.6	100%
p_{11}	0	0	0	0	0.0	0	0	0	0	0.0	0	0	0	110	0	0	0	109.2	0.0	90	0	309.2	100%
p_{12}	0	0	0	0	0.0	0	78.0	63.3	0	80	0	0	0	0	0	0	0	0	0	0	0	221.3	100%
p_{13}	0	0	176.8	0	0.0	0	200	0	200	0	200	0	0.0	0	0	0	0	0	200.0	0	0	976.8	100%
p_{14}	0	0	0	0	0	0	0.0	0	150	150	150	0	0	144.4	130	0	0	0.0	0	0	0	724.4	100%
p_{15}	0	0	0	0	0.0	48.7	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	548.7	100%
p_{16}	0	0	0	120	120	99.2	0.0	0.0	0.0	0.0	0	0	0	0	0	0	0	0	0	0	0	339.2	100%
p_{17}	0	75	43.2	74.7	75	75	0.0	0.0	0	0	0	0	0	0	0	0	0	0	0	0	0	342.9	100%
p_{18}	0	127.2	0	93.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	220.6	98.4%
p_{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0%
p_{20}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0%
p_{21}	0	0	130	0	0	108.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	238.6	100%
p_{22}	0	0	0	110	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	110	55.2%
p_{23}	0	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	59.0%
p_{24}	0	35	66	45	25.5	51.9	110	110	57.4	110	110	30	30	48	0	30	0	0	0	0	0	858.8	100%
p_{25}	0	270	270	0	0	0.0	195	195	247.6	0	195	270	270	0	155	270	0	0	0	0	0	2,337.6	100%
p_{26}	0	0	0	145	0	145	145	145	145	112.1	145	0	0	0	0	0	0	0	0	0	0	982.1	100%
p_{27}	0	0	114	0	136.8	0	0	0	0	0	0	150	150	150	150	150	0	0	0	0	0	1,000.8	100%
p_{28}	0	0	0	150	142.7	150	0	0	0	118.5	0	0	0	0.0	0	0	0	0	0	0	0	561.2	100%
p_{29}	0	0	0	110	0	0.0	0	74	0	109.4	0	0	0	107	0	0	0	0	0	0	0	400.4	100%
p_{30}	0	145	0	0	145	103.1	0	0	0	0	0	0	0	145	145	0	0	0	0	0	0	683.1	100%
Kilograms / day	0	900	900	900	900	900	900	900	900	900	900	900	900	900	900	900	450	900	900	900	900	17,550	97.5%

Fig. 8. Planning of olive milling in the two mobile olive oil mills with maximum capacity.

Table 7
Location of olive oil mill a_2 in the harvesting interval by olive oil concentration.

Metrics	Planning horizon in days																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Decrease of 2%	3	3	3	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
Decrease of 1%	3	3	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
Current olive oil concentration	3	3	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
Increase of 1%	3	3	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
Increase of 2%	3	3	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2

Table 8
Location of olive oil mill a_1 in the harvesting interval by olive oil concentration.

Metrics	Planning horizon in days																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Decrease of 2%	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5
Decrease of 1%	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5
Current olive oil concentration	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5
Increase of 1%	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5
Increase of 2%	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5

Table 9
Percentage change in income according to changes in olive oil concentration (OOC).

Metrics (%)	Decrease of 2%	Decrease of 1%	Current OOC	Increase of 1%	Increase of 1%
Commercial price	−10.31%	−5.16%	0%	5.16%	10.32%
Additional profit	−10.35%	−5.11%	0%	5.20%	10.39%
Cost of production	0%	0%	0%	0%	0%
Cost of transportation	−3.82%	0%	0%	0%	0%
Cost of relocating	0%	0%	0%	0%	0%
Cost labor	7.69%	0%	0%	0%	0%
Objective function	−12.27%	−6.11%	0%	6.13%	12.27%

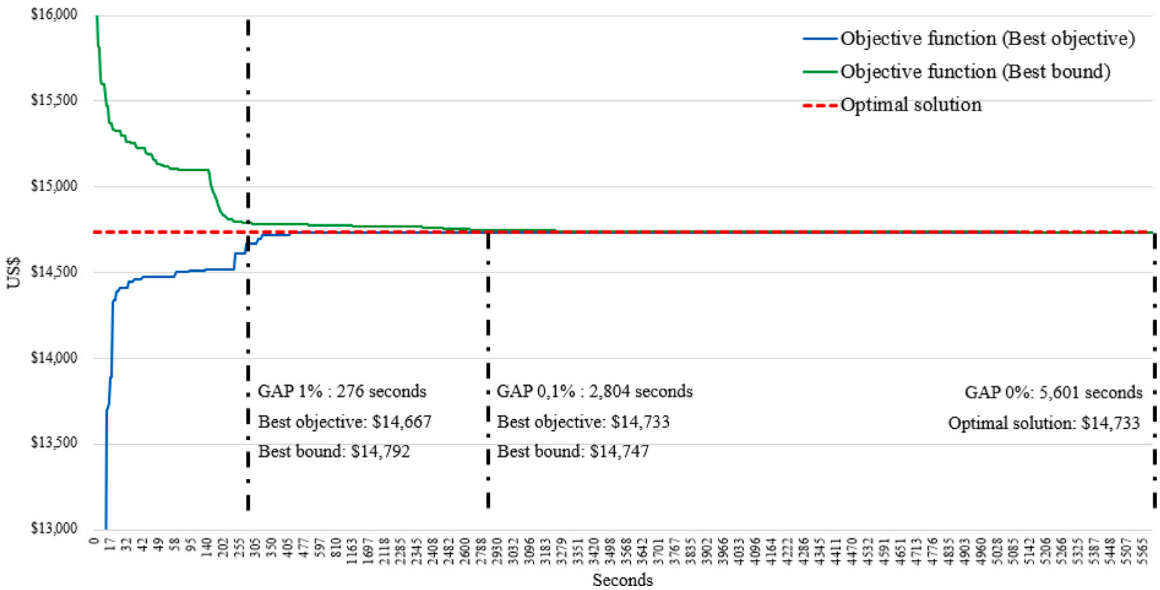

Fig. 9. Optimality gap in the application case.

benefit agricultural cooperatives with small producers in the rural sector who share common mills.

5.4. Computational performance

The instances were generated from actual data collected from previous harvest seasons and in communication with olive growers and agronomists experts in olive oil, who provided feedback and validation to the proposed model. The optimization model was solved on Gurobi v9.1.2 for 64-bit Windows 10 using Python 3.8. The experiments were carried out on a laptop computer with an Intel Core processor i7-8550U CPU 1.8 GHz, with 8.0 GB of RAM for computational consistency.

The optimization model for the application has 19,295 constraints, and 11,502 variables subdivided into 3,780 continuous variables and 7,722 integer variables. Fig. 9 presents the best objective function and the respective lower bound behavior along the algorithm execution (i.e., Gurobi Solver). At 276 s, the optimality gap of 1.0% with a percentage difference of 0.4% concerning the optimal result is observed. However, at 2,804 s, an optimality gap of 0.1% is reached, which in this case has no difference from the optimal result obtained at 5,601 s.

When evaluating the six instances of the base case with fixed oil mills, the average computational time to find a solution with a maximum optimality gap at 1% is less than 1 s. In contrast, by setting the optimality gap at 0.1%, the average computational time is no more than 7,500 s. However, finding the optimal solution in certain instances can take up to 55 h. Therefore, the optimality gap was set at 0.1%, while other solver parameters remained the default. Thus, in the sensitivity analysis, by requiring the maximum milling capacity of the mills with a proportional increase in the harvesting capacity and amount of raw material of the producers, the computational time reaches 21,418 s, equivalent to 357 min.

It can be concluded that the mathematical model can solve realistic instances of the problem in acceptable computational times for tactical/operational decision-making, considering an adequate range of optimality gap.

6. Managerial and research insights

Once the research has been done and based on the proposed MILP formulation and its application, several relevant recommendations for practitioners and the academic community may be emphasized.

For the practitioners within the industry of olive oil production, where usually a set of medium and small producers coexists in specific regions, it is worth highlighting the significant benefits that may arise from implementing a collaborative supply chain planning associated with the proposed formulation, involving harvesting, producing and some related transport decisions. In this context, the existence of tailored mobile olive oil mills that may be shared among producers is essential; instead of each one investing in its own olive oil mill, giving a set of shared static or fixed mills. This flexibility provides a better balance between the different system cost components and reduces the required investments compared to considering dedicated mills for each producer, thus providing more attractive investment projects for the producers. Moreover, this strategy allows for reasonable exploitation of the olive maturity curve for harvesting while balancing system costs, yielding better aggregate economic results for the whole producer association.

Naturally, the proposed strategy relies on an active collaboration among producers. In this context, the creation or the existence of an association of olive oil producers is indispensable, as is observed in the case study reported in Section 5. Under this collaborative scheme, planning the dynamic mill locations over suitable places along the time horizon must be addressed in an integrated manner for the whole set of producers. It should be noticed that, in the case study of Section 5, they already collaboratively plan the location and use of mills based on the decision variables and the performance indicators considered in our study, but without employing any decision support tool or systematic approach. Thus, the proposed model significantly aids in providing a systematic and efficient integrated planning process for the olive oil producer association.

Some future analysis may be performed based on the proposed formulation, in terms of alternative strategies and models, to allocate and share the benefits obtained with the proposed integrated plan for

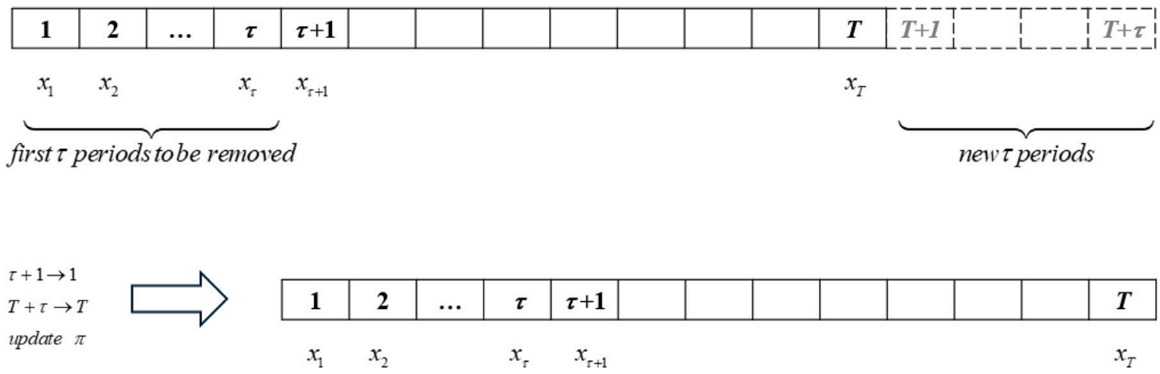

Fig. 10. Future research with rolling horizon method.

each producer belonging to the association, especially in comparison with potential isolated solutions.

Regarding uncertainty issues related to the studied problem, some elements, such as harvest yield, oil concentration, prices, and costs, are more significant. A natural reactive approach to deal with these uncertainties based on the proposed formulation may be considered following a rolling horizon-based implementation (see Montenegro-Dos Santos et al., 2023), where after each running period within the planning horizon (e.g., after each day or week), some model parameters may be updated or corrected according to changes are observed new data is received.

For example, decisions provided by the model related to the first period are assumed and implemented, parameters associated with the second and subsequent periods are updated, and the first period of the model is removed from the model and replaced by a new model period (i.e., $T+1$). Afterward, the model is run again, and the process is repeated after each running period, considering that data associated with the first period do not change significantly. This approach relies on employing the proposed deterministic, multi-period formulation to deal with uncertainty over future system settings and data. Naturally, if model parameters do not change significantly after more than one period, the model may be run after each τ period. In this case, the first τ periods are removed from the model and replaced by new τ periods, as shown in Fig. 10, where x is the vector of model decision variables, and π is the set of model parameters.

A second more proactive alternative relies on improving the proposed formulation based on a stochastic programming modeling structure, such as two-stage stochastic programming (Borodin et al., 2016), Sample average approximation (Kleywegt et al., 2002; Hu et al., 2012), a priori optimization (Bertsimas et al., 1990), or even robust optimization (Bertsimas and Sim, 2004; Bohle et al., 2010), which remain as interesting future research to be performed. For example, as can be observed in Tables 1 and 2, Kazaz (2004), Kazaz and Webster (2011), Cano Marchal et al. (2018), Carvajal et al. (2019), and Avanzini et al. (2021) developed specific stochastic programming approaches to deal with similar problems (focusing on olives, sugarcane and grapes, among other industries), but not addressing simultaneously decentralized production and harvest processes.

One significant element that may be studied is the quality or oil concentration of the olives while they are kept in the trees, as shown in Fig. 3. Thus, the parameters associated with this concentration (CO_{pvt}) may be reformulated by integrating a stochastic behavior. Other interesting model parameters are the available olives at the beginning of the planning horizon and selling prices, which may be indeed modeled including uncertainties (Avanzini et al., 2021).

In terms of perishability or loss of productivity associated with the harvesting process, it is worth highlighting that the proposed formulations assume very short periods between olive harvesting and processing, and thus, perishability is irrelevant. On the contrary, if longer periods are considered, the formulation should be modified by integrating olive perishability, remaining as future research.

7. Conclusions

This paper presents a Mixed Integer Linear Programming model to support small growers in decision-making regarding harvesting and processing planning in production plants that can be relocated to different strategic locations. In this context, we model the connections between harvesting and transport operations at each field by decentralized processing of mobile olive oil mills, considering the olive oil yield depending on the olive variety and region of the agricultural fields. This research contributes by providing a novel multi-period and multi-product MILP model that helps to solve a tactical/operational problem for agricultural communities or cooperatives, particularly in the growing olive oil industry. One distinctive feature of the proposed formulation is the inclusion of an innovative decentralized production strategy, which consists of employing mobile productive plants, i.e., olive oil mills, considering downtime and set-up times.

This research provides managerial insights concerning the impacts of olive and mill transport decisions on harvest and production planning. The study highlights the need to relocate mobile production units to different agricultural fields to minimize transportation costs for farmers. To do so, certain olive varieties are often required to be harvested before or after their optimum olive oil concentration time, which may have significant implications for production and harvest planning. In addition, the optimization model prioritizes reducing olive transportation costs over the utilization of the olive oil mills, even observing mills' productive days lost since farmers' transportation costs may be higher than the overall profits from olive oil production.

The proposed optimization model maximizes growers' total profits in olive oil production through integrated harvesting planning, production, and transportation of olives and mills. In future research, metrics that penalize decisions that lead to yield losses in olive oil concentration for particular olive growers or exclude some producers from benefiting overall revenues could be incorporated. In particular, individual growers' utilities may be addressed through multi-criteria, game theory, or bi-level programming approaches.

Currently, realistic instances are based on the historical data and theoretical information obtained in the past harvesting seasons, so in the short term, we wish to validate the performance of the proposed

Fig. A.11. Mobile olive oil mill in the province of Elqui, Coquimbo.
Source: Tapia et al. (2020)

Location of the olive oil mill a_1

Locations	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	Planning horizon in days																				
ℓ_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_4	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
ℓ_5	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0

Availability/employment of the olive oil mill a_1

Locations	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	Planning horizon in days																				
ℓ_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ℓ_4	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
ℓ_5	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0

Fig. A.12. Location and availability/employed of olive oil mills a_1.

Producers	Planning horizon in days																					Kilograms/ producers	Percentage of kilograms harvested
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
p_1	0	150	0	0	124.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	274.1	100%
p_2	0	0	0	0	0	0	0	0	0	0	0	0	186.7	0	200	0	160	170	171.7	200	200	1,288.4	100%
p_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	72	0	48	72	192.0	100%
p_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	170	168.6	170	170	170	165.4	125.6	1,139.6	100%
p_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	190	190	172.1	190	180	156	1,078.1	100%
p_6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	72	0	72	72	48	72	72	408.0	100%
p_7	0	0	0	0	0	0	0	0	0	0	0	0	143.3	128.7	0	0	0	133.9	150	150	150	855.9	100%
p_8	0	0	0	0	0	0	65	0	46.5	79.7	0	100	0	0	8	91.4	97.1	0	62	0	0	549.7	100%
p_9	0	55.4	0	0	0	0	0	0	68.5	0	0	0	0	0	0	0	0	0	0	0	0	123.9	100%
p_{10}	0	0	120	120	0	0	0	0	0	0	0	0	120	120	0	0	120	0	0	84.6	0	684.6	100%
p_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	90.9	110	108.3	0	0	309.2	100%
p_{12}	0	0	0	0	0	0	85	0	85	0	0	0	0	51.3	0	0	0	0	0	0	0	221.3	100%
p_{13}	0	0	0	0	0	0	200	200	0	176.8	200	200	0	0	0	0	0	0	0	0	0	976.8	100%
p_{14}	0	0	0	0	0	0	0	0	150	0	150	150	0	150	0	0	0	0	0	0	124.4	724.4	100%
p_{15}	0	0	0	0	0	48.7	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	548.7	100%
p_{16}	0	0	0	120	99.4	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	339.4	100%
p_{17}	0	72.9	75	58.1	75	62.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	343.7	100%
p_{18}	0	99.2	125	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	224.2	100.0%
p_{19}	0	0	0	64	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	144.0	100%
p_{20}	0	72.5	0	0	71.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	144.0	100%
p_{21}	0	0	130	0	0	108.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	238.6	100%
p_{22}	0	0	0	88	0	110	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	198.0	100.0%
p_{23}	0	0	0	0	0	0	0	0	0	93.5	0	76	0	0	0	0	0	0	0	0	0	169.5	100.0%
p_{24}	0	40.7	35.7	35	84.6	45	76.7	45	45	79.2	5	110	110	76.9	35	35	0	0	0	0	0	858.8	100%
p_{25}	0	264.3	264.3	270	0	0	0	0	0	270	0	264	195	270	270	270	0	0	0	0	0	2,337.6	100%
p_{26}	0	0	0	145	145	145	112.1	145	145	0	145	0	0	0	0	0	0	0	0	0	0	982.1	100%
p_{27}	0	0	150	0	150	150	150	150	0	100.8	150	0	0	0	0	0	0	0	0	0	0	1,000.8	100%
p_{28}	0	0	0	0	0	0	111.2	150	150	0	150	0	0	0	0	0	0	0	0	0	0	561.2	100%
p_{29}	0	0	0	0	70.4	110	0	110	110	0	0	0	0	0	0	0	0	0	0	0	0	400.4	100%
p_{30}	0	145	0	0	0	0	0	0	0	0	0	0	145	103.1	145	145	0	0	0	0	0	683.1	100%
Kilograms / day	0	900	900	900	900	900	900	900	900	900	900	900	900	900	900	900	900	900	900	900	900	18,000	100.0%

Fig. A.13. Planning of olive milling in fixed olive oil mills at location ℓ_1 and ℓ_5.

model with actual data on the physical–chemical composition of olive varieties in each sector, using sensors, remote sensing techniques and other information-technology (IT) sources that may provide real-time data to improve the estimation of olive oil concentration over the study horizon. Villalobos et al. (2019) presents interesting guidelines in their literature review on IT in the fresh produce supply chain for estimating the dynamic parameters of optimization models.

It is worth noting that the proposed methodology relies on a deterministic multi-period formulation, where data uncertainty cannot be explicitly addressed, such as olive oil concentration and harvest yield, among others, given weather or biological uncertainties. However, this limitation may be partially dealt with by rolling horizon-based implementation or considering a more proactive approach based on stochastic programming or related approaches.

Future research may consist of incorporating additional factors such as investment costs and sustainability into the proposed mathematical model through the valorization of olive pomace as a raw material (e.g., activated carbon, animal feed) or processed product (e.g., olive pomace oil biodiesel, exhausted olive pomace) along with environmental and economic objectives, as addressed by Gómez-Lagos et al. (2024) in their multi-objective linear programming model with the minimization of fruits losses and other objectives. In addition, in the same line of research, other future work may focus on waste treatment, whose main by-products of oil production are the olive pomace (35%–45%) and olive oil mill wastewater (38%–48%) (Rapa and Ciano, 2022).

CRediT authorship contribution statement

Bryan A. Urra-Calfuñir: Writing – original draft, Visualization, Software, Data curation. **Carlos A. Monardes-Concha:** Writing – review & editing, Validation, Supervision, Project administration, Methodology, Investigation, Formal analysis. **Pablo A. Miranda-González:** Writing – review & editing, Validation, Supervision, Methodology, Investigation, Formal analysis, Conceptualization.

Data availability

Data will be made available on request.

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Appendix A. Application case results

See Figs. A.11, A.12 and A.13.

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