Why Poisson arrival coincide the blocking and the loss probability.

question posted 18 days ago by **gmikawa**

Could you explain a little bit more coincidence of the blocking and the loss probability in Poisson arrival case?

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1 response

PatrickMaille (Staff)
18 days ago

That result comes from the PASTA property: the probability that a user sees the system full upon arrival (loss probability) equals the probability that the system is full (blocking probability).

I'm now trying to convince you of the PASTA property (Poisson Arrivals See Time Averages): with Poisson arrivals, whatever the state you are in (and how long you've been there), you have the same chance of having an arrival in the next Δ_t seconds, and on average you have λ arrivals per second. So over a long enough duration T seconds, we spend about $\pi_i T$ seconds (π_i: steady-state probability) in State $i$and there are about $\lambda\pi_i T$ arrivals occurring when in State i over T. Or, said differently, a proportion π_i *of the arrivals* over that period occur while in State i: that is the "probability seen by clients", i.e., an arriving client has a probability π_i of being among those who find the system in State i upon arrival. Hence the PASTA property.

Thank you, Patrick. I understand that the expected arrival No. to the state i is $\lambda.\pi i.T$, but I lost your argument in the last sentence containing "probability seen by the clients". It would be nice if you could expand your reasoning a little bit more. The probability of loss P(loss) occurs when the system is full with stationary probability πK in M/M/1/K. Can I simply apply the total probability theorem to P(loss), i.e. $P(loss)=\Sigma P(System\ is\ full|X=i)P(X=i)=\Sigma \pi K P(X=i)=\pi K$, where $P(X=i)$ is Poisson PMF, sum over from i=0 to infinite and assuming the customer arrival and system saturation is independent.

posted 18 days ago by**gmikawa**

I don't understand your reasoning, in particular I don't see what you mean with P(X=i). If X is the state of the Markov chain, then the system is full if and only if X=k, hence the loss probability is just P(X=k), which is given in steady state by the probability π_k.

Or, you mean something else with P(X=i)?

posted 18 days ago by**PatrickMaille** (Staff)

X is the number of arriving customers but rejected by the system. The system is already blocked with probability πk.

posted 18 days ago by**gmikawa**

I'm not sure I understand, if you can X the "number of arriving customers but rejected by the system", do you mean "over the period T" as I was suggesting before? So you would have X following a Poisson distribution with average $\lambda\pi_k T$, right? I.e.,

$$\mathbf{P}(X=i)=e^{-\lambda\pi_k T}\frac{(\lambda\pi_k T)^i}{i!}$$

?

(Just to be sure I understand your reasoning)

To go back to your

To go back to your previous question regarding the "probability seen by the client". Assume I'm an arriving client and I'm wondering with what probability I'll find the system in State i: well, that's the probability that I am among the (about) $\lambda\pi_i T$ clients that found it in that state. But in total over T, there are (about) λT arrivals, hence **my**probability (as an arriving client, i.e., the ***probability seen by the clients***) is the ratio $\frac{\lambda\pi_i T}{\lambda T}$, which is just π_i and establishes the PASTA property. In summary, *from the client point of view*, the probabilities of the states upon arrival equal the steady-state probabilities.

And you can directly apply that to get loss probabilities.

posted 17 days ago by**PatrickMaille** (Staff)

Good morning Patrick. Yes, I meant X~Pois(λπkT). I understand what you meant by the "probability seen by the client". Loss probability is the special case seen by the arriving client when the system is blocked.

posted 17 days ago by**gmikawa**