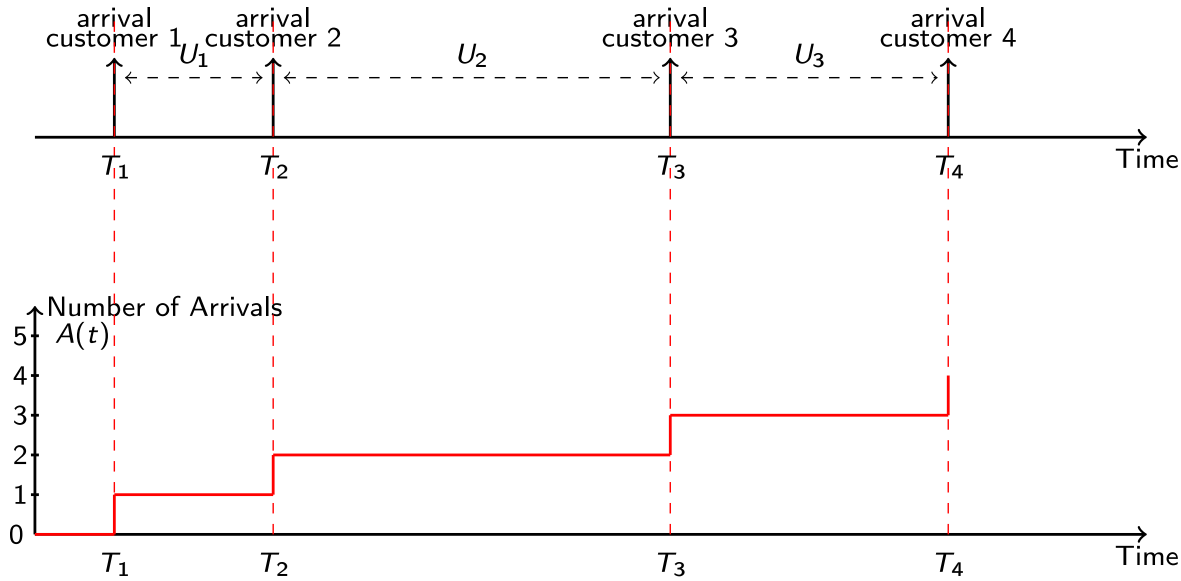
What is a queue?

Poisson process

Characterizing Arrivals with the Counting Process



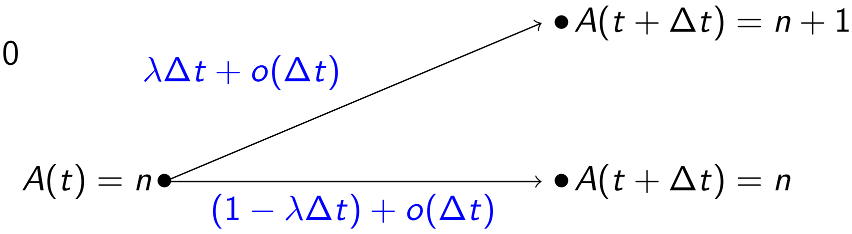
$$\boxed{A(t) = \sum_{i=0}^{\infty} \mathbb{I}_{T_i \leq t}}$$

Now let's go back to the characterization of customers' arrivals. T_i denotes the arrival time of customer i and U_i is the interarrival time between customer i and customer $(i+1)$. The counting process $A(t)$ counts the number of customers that have arrived up to time t.

For example, if 3 customers have already arrived at time t, then $A(t)$ equals 3. Equivalently, $A(t)$ is the sum over i of the indicator functions of "T_i lower than t", so, the number of indices i such that T_i is lower than t.

Poisson process

$\Delta t << 1$
$\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$



$$\mathbf{P}(A(t+\Delta t) = n+1 \mid A(t) = n) = \lambda \Delta t + o(\Delta t)$$

$$\mathbf{P}(A(t+\Delta t) = n \mid A(t) = n) = 1 - \lambda \Delta t + o(\Delta t)$$

Let's assume that the arrival process is a Poisson process and suppose that at time t the counting process $A(t)$ is equal to n. Let's look at a very small time interval Δt. Between time t and time $t+\Delta t$, there can be zero customer arrivals or one customer arrival. The probability that one client arrives between t and $t+\Delta t$ is equal to $\lambda\Delta t$, where λ is the rate of the Poisson process.
Reciprocally, the probability that no customer arrives in the meantime is equal to 1 minus $\lambda\Delta t$. The probability that more than one customer arrives is negligible (with respect to Δt). In these probabilities we omit all terms that are negligible with respect to Δt. They are denoted as $o(\Delta t)$.