

## Derivation of sum of output flow.

question posted 21 days ago by [gmikawa](#)

At 3:09 video clip in Lesson 3 week 4. May I ask you how you reduced the sum of output flow to  $\pi(i)q(ii)$ ? Many thanks.

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1 response

**laurent\_decreusefond**(Staff)

21 days ago



The flow from state  $i$  to state  $j$  is  $\pi_i q_{ij}$ . The output flow from state  $i$  is the sum over all over state  $j$  of the flow from  $i$  to  $j$ , that is to say  $\sum_{j \neq i} \pi_i q_{ij} = \pi_i \sum_{j \neq i} q_{ij}$  but the very definition of  $q_{ii}$  is exactly  $q_{ii} = -\sum_{j \neq i} q_{ij}$  so the output flow emanating from state  $i$  is  $-\pi_i q_{ii}$  or  $\pi_i |q_{ii}|$

hope this helps

That helps, Laurent.  
Thank you. This is going to be a tautologic question, but why the sum of row  $i$  of transition rate matrix need to be zero?  
Was it explained somewhere?



posted 21 days ago by [gmikawa](#)

The sum is zero by construction: since we decided to set

$$q_{ii} = -\sum_{j \neq i} q_{ij}$$

There is no intuitive signification to be searched for. The intuitive part is given by the  $p_{ij}$

(probability to jump from  $I$  to  $j$  when there is a jump) and  $\lambda_i$ , the parameter of the exponential which gives the sojourn time in state  $i$ . The construction of  $Q$ , the sum of the  $i$ -th row is

$$q_{ii} + \sum_{j \neq i} q_{ij} = 0$$

comes from more complex mathematical results which are beyond the scope of this course. By the way, this construction takes a lot of time to be given, needs more sophisticated prerequisites and is useless when it comes to use CTMC in practice.

posted 18 days ago by [laurent\\_decreusefond](#) (Staff)

When you are given a system you want to represent by a CTMC, compute the  $p_{ij}$  and the  $\lambda_i$  from the description of your model and then transform them in the  $q_{ij}$ , which are the mathematical quantities of interest.



$$q_{ij} = p_{ij}\lambda_i, \quad q_{ii} = -\lambda_i$$

and in the reverse way

$$p_{ij} = \frac{q_{ij}}{|q_{ii}|}, \quad \lambda_i = |q_{ii}|$$

posted 18 days ago by [laurent\\_decreusefond](#)(Staff)

Thank you, Laurent.



posted 18 days ago by [gmikawa](#)

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