1) Talk about sequence of random variables

2) Concentration of (tail) probability (measures)

Multi armed bandits

$\mu(1)$ ... $\mu(k)$

Arm 1 . . . Arm k

P_1 ... P_k

random variable

at round t, choose arm A_t

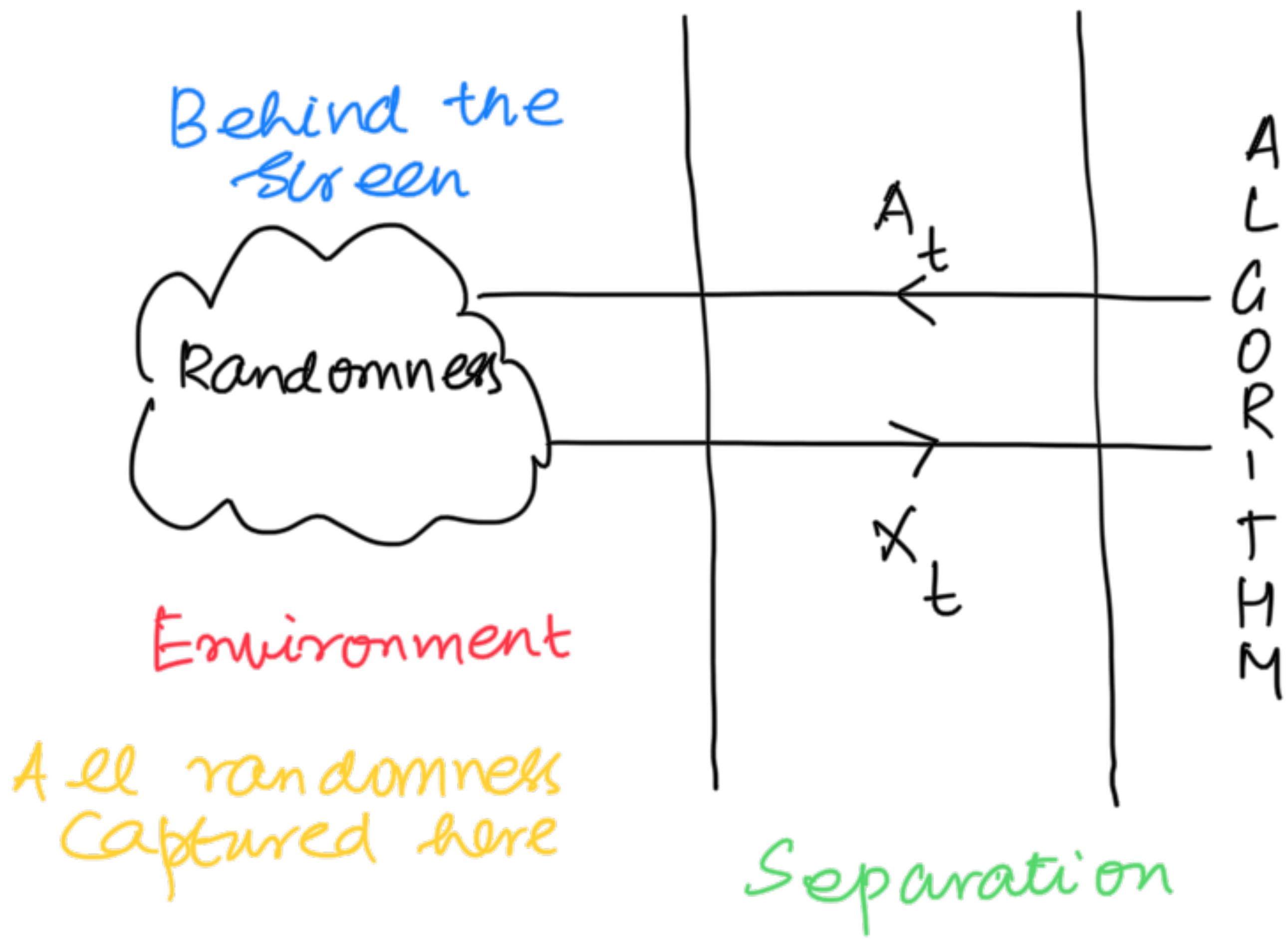
obtain $X_t \sim P_{A_t}$

Goal : Minimise $R_n = \mathbb{E}\left[\sum_{t=1}^{n} (\mu_* - X_t)\right]$

Regret

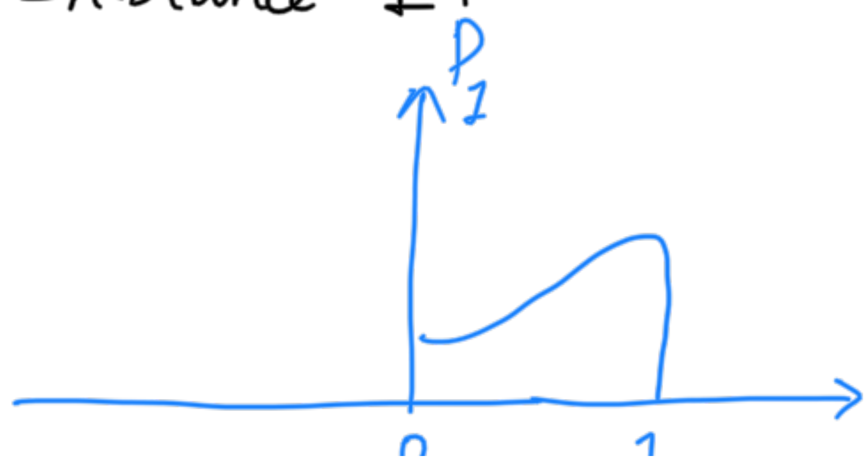
$\mu(a) = \mathbb{E}[X \ldots]$ $\quad \mu_* = \max \mu(a)$

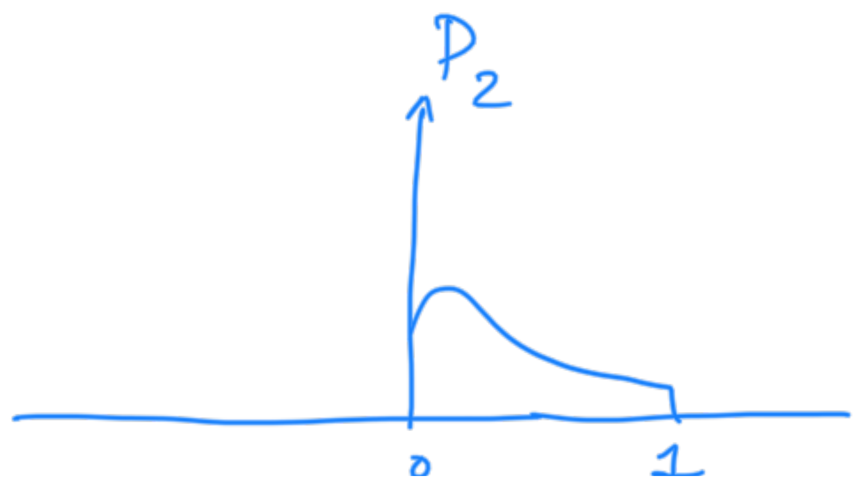
$\mu(a) = \mathbb{E}_{P_a}[X_t]$, $\mu_* = \max_a \mu(a)$



Consider two instance of a 2-armed bandit problem

Problem Instance 1:





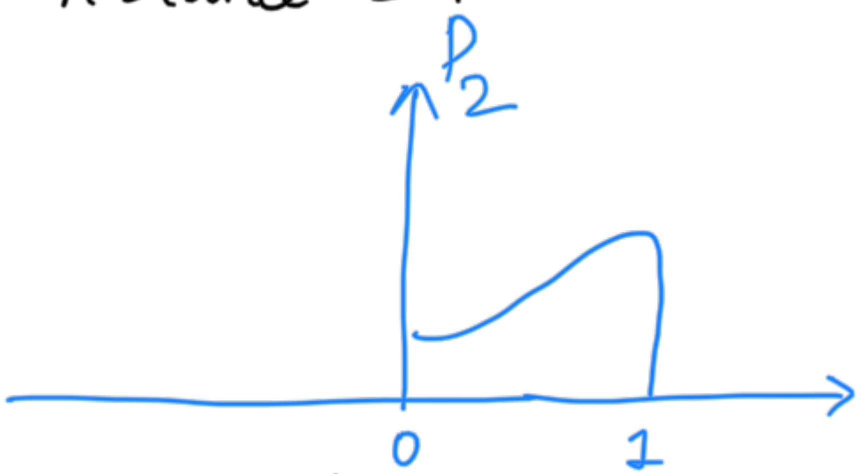
determistic algoritham that keeps interacting with the two arms

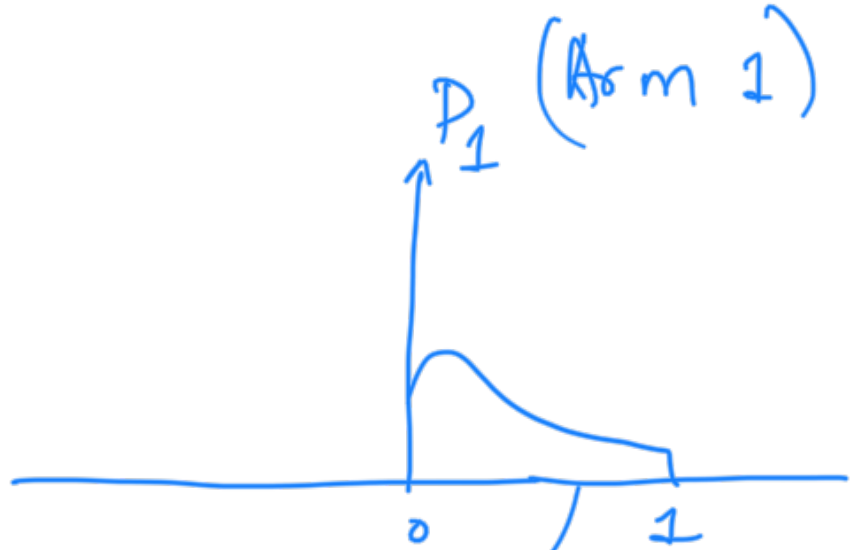
$$A_1 = 1, \quad X_1 = 0.9$$

$$A_2 = 2, \quad X_2 = 0.1$$

$$A_3 = \text{blah}$$

Problem Instance 2 :





$$A_1 = 1, \quad X_1 = 0.9$$

$$A_2 = 2, \quad X_2 = 0.1$$

A_3 = blah ... probability of sequences

A_3 - Bern

probability of ...
changes between

Moral : If the sequence of rewards are same then a deterministic algorithm should behave the same irrespective of the problem instance

$$\Downarrow$$

we want to bucket / bundle all the randomness in one place and look at algorithm as a map that takes in seq of reward outs the decisions.

$$\Downarrow$$

Need for a probability space

(i) can talk about several random variables in a unified manner.

(ii) algorithmic interaction

all randomness resides

Probability Space

$$(\Omega, \mathcal{F}, P)$$

omega script F

Ω : Sample space

$\mathcal{F} \subseteq 2^{\Omega}$ (where 2^{Ω} is the powerset)

Ω

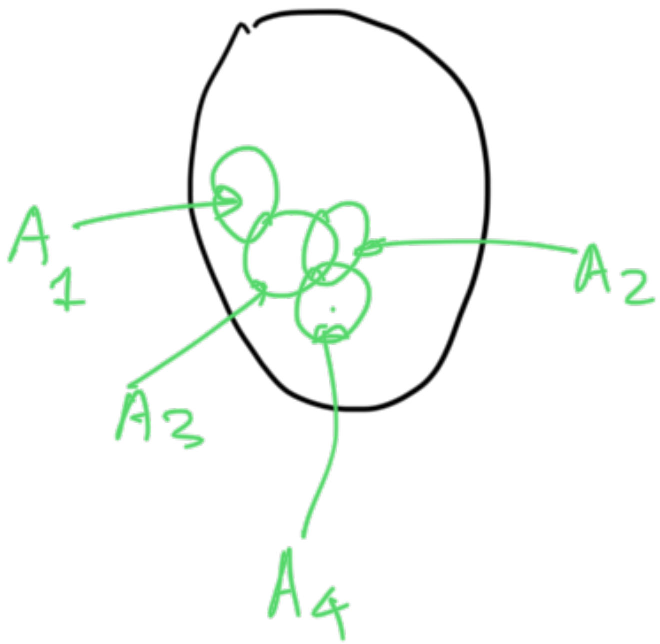
Contains all those sets whose probability we care

$\mathcal{F}$ is a σ- algebra (sigma algebra)

* a collection of subsets of Ω
* if $A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$
* $\{A_i\}_{i \geq 0} \in \mathcal{F} \Rightarrow \bigcup_i A_i \in \mathcal{F}$

$P(\Omega) = 1$

Ω



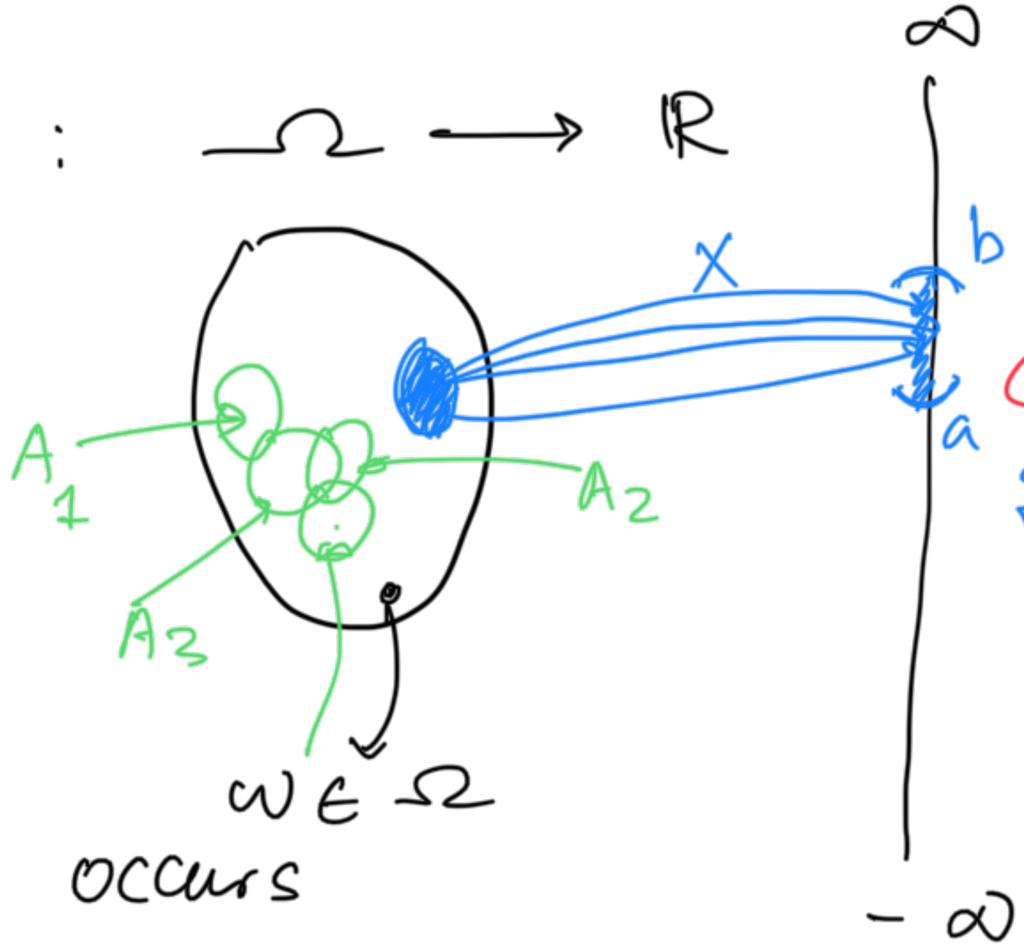
$\text{Prob}(A)$

$\text{Prob}(A^c) = 1 - \text{Prob}(A)$

$\text{Prob}(\bigcup_i A_i)$

real valued random variable

$X : \Omega \longrightarrow \mathbb{R}$



$\text{Prob}(a < X < b)$

(a, b)

$\{\omega : a < X(\omega) < b\}$

$\omega \in \Omega$

random event = occurs

$\text{Prob}(a < X < b) = P(\{w : a < X(w) < b\})$

we ask that $\{w : a < X(w) < b\} \in \mathcal{F}$

for all $a, b > 0$ $\qquad X^{-1}((a,b))$

Let us put $(\Omega, \mathcal{F}, P)$

$\mathcal{F} = 2^{\Omega}$ in couple of examples.

(i) Toss of a coin

$X : \Omega \to \{H, T\}$

$\Omega = \{H, T\}$

$\mathcal{F} = 2^{\Omega} = \{\phi, \{H\}, \{T\}, \{H, T\}\},$

$A \in \mathcal{F}$, $\quad P(A) = \dfrac{|A|}{\quad}$ $\qquad X(w) =$

/ 2

$X(H) = H$
$X(T) = T$

Looks very trivial

(ii) Roll of Dice

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$\mathcal{F} = 2^{\Omega} = \{\phi, \Omega, \{1\}, \{2\}, \ldots \{1,2\}, \ldots \}$$

$$X(1) = 1, \quad X(2) = 2, \ldots, X(6) = 6$$

$$A \in \mathcal{F}, \quad P(A) = \frac{|A|}{6}$$

Model: Nature picks $\omega \in \Omega$

shows us $X(\omega)$

Exercise: $\overset{X_1}{\text{Roll of Dice}}$, $\overset{X_2}{\text{Toss of Coin}}$

$$(\Omega, \mathcal{F}, P)$$

Case 1 * Dice and coin are independent and both fair

Case 2 * Dice is loaded or unfair and Coin is fair (both independent)

$$X_1 : \Omega \to \{1, \dots, 6\}$$

$$X_2 : \Omega \to \{H, T\}$$