UNIVERSIDAD AUTONÓMA DE NUEVO LEÓN

FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA

LEARNING UNIT: **Selected Topics of Optimization**

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TASK: Selective Traveling Salesman Problem

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Content

Introduction

The Selective Traveling Salesman Problem (STSP) is a variant of the Traveling Salesman Problem (TSP) with some additional constraints. When solving the TSP, one focuses on visiting all given places in the least amount of time. In the STSP, a constraint of time (or budget) is also given. This means that not all the places will be visited as usually there will not be enough time for this to happen. Instead, the objective of the STSP is to find a path which may give the highest profit in the given time.

The mathematical description may be written as follows (Gutin & Punnen, 2006):

- For each node i of G, a weight wi is given
- The objective is to find a cycle Y in G such that the sum of the weights of nodes in Y is maximized
- The sum of the costs cij belonging to Y must remain less than or equal to L, where L is a given budget

The objective is, thus, to find a path or cycle that maximizes the profit without surpassing the limit cost.

In this report, a Constructive Heuristic is proposed, as well as an Improving Heuristic by means of Local Search, to solve the STSP. Coupled with that, the Local Search was improved further by implementing a K-best multi-start heuristic and limiting the candidate list with a reactive alpha.

In order to test this heuristic, Sets 1, 2, and 3 from Tsiligirides, the diamond-shaped test problem (set 64), and squared-shaped test problem (set 66) were evaluated (Chao, Golden, & Wasil, 1995).

Constructive Heuristic

A proposed heuristic to solve this problem consists on creating two paths, each of which will evaluate all feasible places to be visited and pick the one with the **best profit-to-distance ratio**. These will be generated in a pseudo-parallel way: the first path will pick an unvisited feasible place, then the second one will pick another place. These paths will then be joined. If by joining them both the result is still a feasible solution, then it will continue expanding the path.

In this constructive heuristic (as well as in the local search) the restricted candidate list consists on only the feasible places, thus the nodes whose selection would outcome in running out of time will be ignored.

Pseudo-Code

Given a set of *Places* to be visited, each with a *Profit*, as well as a given *timeLimit*:

1. Locate the *startingPoint* and the *endingPoint* and store them respectively in the lists *Beginning* and *End*
2. *pendingPlaces* = *Places* \ (*Beginning* U *End*)
3. *noPlaces* is the number of elements in *pendingPlaces*
4. *N* = 1
5. While *N* <= *noPlaces*
    1. Pick a *Place1* from *pendingPlaces* whose ratio *Profit* / *Eucledian(StartingPoint, Place1)* is the *N* best and add it to the end of the list *Beginning*
    2. *M* = 1
    3. While *M* <= *noPlaces*
        1. Pick a *Place2* from *pendingPlaces* whose ratio *Profit* / *Eucledian(Place2, endingPoint)* is the *M* best other than *Place1* and add it to the beginning of the list *End*
        2. If the cost of the path (*Beginning* + *End*) <= *timeLimit*
            1. *startingPoint* = *Place1*
            2. *endingPoint* = *Place2*
            3. Go to line 2
        3. Else
            1. *Beginning* = *Beginning* \ *Place1*
            2. *End* = *End* \ *Place2*
        4. M = M + 1
    4. N = N + 1
6. The solution is the sum of *Profit* of the places in path (*Beginning* + *End*)

Analysis of the results

Total instances	Total time (s)	Average time per instance (s)	Average deviation from best known (%)	Best known solutions
89	0.75	0.0084	-19.66	4

Table 1. Constructive Heuristic Analysis

Solving 89 different solved instances (table 2a, Constructive Heuristic column) with this algorithm took less than a second. All of the answers were feasible solutions. Nonetheless, when comparing best known solutions, only four of the obtained solutions were equal to the best known, the average deviation from it resulting in -19.66%, which meant that there was still room for improvement.

Notes for the Heuristic

Something to note from this heuristic is that it consists on following two paths that may or may not be close to each other. This means that there is a possibility of both paths drifting apart, making the nexus between each other possibly larger than what it could be. Also, there is always a path that has priority over the other, as the second one cannot chose a place already picked in the first path; a future improvement may be to vary this priority so the growing of the path can be more uniform.

Local Search Improving Heuristic

In order to improve the solution, the following local search was proposed; this improving heuristic is divided in two parts: finding more time while conserving or improving the profit by changing nodes (spacing), and adding new nearby nodes (inserting).

For fulfilling the spacing part, the original solution is explored, switching a selected place for another one that has not been already chosen as long as the profit does not lowers; when a neighbor is found, the search starts over, making it a first improvement search. An additional constraint is considered as well: a given radius. This radius is meant for reducing the candidate list, thus reducing the heuristic's time.

For the inserting part, still taking into account the radius for limiting the candidate list, nodes outside the answer provided by the neighborhood will be inserted in the latter as long as the solution remains feasible.

Pseudo-code

Given a set of *Places*, a *Solution* path and its *SProfit* for the STSP:

neighborhood is a list where neighbors will be stored

1. *Radius* = 3 * (*Cost* of *Solution*) / (*noPlaces* of *Solution*)
2. *bestProfit* = *SProfit*
3. *bestSolution* = *Solution*
4. For each *Place* in *bestSolution*
    1. *unvisitedPlaces* = *Places* \ *Solution*
    2. If there is an *unvisitedPlace* whose Eucledian distance to *Place* <= *Radius*
        1. *newSolution* = change *Place* for *unvisitedPlace* in *Solution*
        2. If changing *Place* for *unvisitedPlace* results in a path whose *uCost* <= *timeLimit* and whose *uProfit* >= *SProfit*
            1. *bestSolution* = *newSolution*
            2. *bestProfit* = *uProfit*
            3. Go to line 4
5. *unvisitedPlaces* = *Places* \ *bestSolution*
6. If there is an *unvisitedPlace* whose Eucledian distance to *location* <= *Radius*
    1. If adding the *unvisitedPlace* between *location* and next *location* results in a path whose *cost* <= *timeLimit*
        1. If the *newProfit* of this *newSolution* >= *bestProfit*
            4. *bestSolution* = *newSolution*
            5. *bestProfit* = *newProfit*
5. The improved solution is the *bestProfit*

Analysis of the results

Total instances	Improved instances	Total time (s)	Average time (s)	Average raise (%)	Average deviation from best known (%)	Best known solutions
89	63	14.26	0.1602	5.17	-15.20	11

Table 2. Constructive Heuristic Analysis

Solving the 89 instances (table 2a, Local Search column) with this algorithm took almost 15 seconds, but still taking less than a second per instance. Compared to the Constructive Heuristic, on average the solutions were 5.17% better, and although the best known solutions obtained increased from 4 to 11, the average deviation was -15%, which is closer to those solutions, but still with some areas of opportunity.

Notes

By doing the spacing part of the algorithm the solution ends up being a little less greedy by selecting some nodes which might not be initially considered due to the evaluating function being dependent of the current node.

GRASP

In order to avoid being greedy, as well as to consider other possible solutions, two of the choices were randomized in the constructive heuristic: instead of picking the node with the best profit-to-cost ratio, the first k best options were scrambled, this for both initial and ending paths. After that, the improving heuristic took apart and this process was repeated 1000 times.

Another change that took place was the restricted candidate list (RCL). By using alpha values, it got filtered based on the following formula:

$$RCL = (1 - \alpha) * maxRatio + \alpha * minRatio$$

Where maxRatio is the place with the biggest profit-to-distance ratio and minRatio is the place with the smallest one.

Whereas for solving these instances four values of K were pre-selected (k = 2, k = 3, k = 4, and k = 5) and the 89 instances ran four times, one per each value of K, the alpha value was reactive. Eleven different values of alpha (0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1) started all with the same probability P of 1/11 chances of being selected. After 100 iterations, this probability was updated based on the average profit obtained per each alpha following this formula:

$P_i = \frac{\frac{\hat{Z}_i}{Z}}{\frac{\sum \hat{Z}}{Z}}$, where P_i is the probability of α= i, $\hat{Z}_i$ is the average profit obtained with α= i, $\sum \hat{Z}$ is the sum of all the average profits, and Z is the maximum profit obtained up to that point.

This method gave a higher probability for alphas with good results to be chosen one again while avoiding discarding alphas with lower performance, giving them a chance to be picked once more.

Hence, by having a multi-start and a reactive alpha restricted candidate list, the heuristic becomes a Greedy Randomized Adaptive Search Procedure (GRASP).

Analysis of the results

K	Average rise from Local Search (%)	Average deviation from best know (%)	Best known solutions	Mode of α from improved solutions	Total time (s)	Average time(s)
2	2.15	-13.26	12	0.5	6404.46	71.96
3	**9.94**	**-5.98**	**15**	**0.8**	**6247.98**	**70.20**
4	9.06	-6.91	13	0.8	6262.04	70.36
5	8.93	-7.13	13	0.5	6083.75	68.36
Avrg	7.52	-8.32			6249.5575	70.22
				Total	24998.23	

Table 3. GRASP analyisis

Running the GRASP with four different K's (Tables a1, a2, a3, and a4) gave as a result an average improvement of 7.52% compared to only using the local search. The average deviation from the best known solutions went from -15.20% to -8.32%. Focusing the instances run with K = 3, the deviation gets even closer to the best known, reaching -5.98%, over four times better than only using the constructive heuristic (-19.66%). The majority of the objective functions reached with K = 3 were obtained using an alpha value of α = 0.8. The second best (K = 4) used α = 0.8 mainly as well. These four runs, each with 1001 repetitions per each of the 89 instances, were done in a span of 24998.23 seconds (6.94 hours) using a personal computer whose specs are specified later on, thus having a reactive alpha value was a great advantage in order to avoid repeating the process another time per each alpha per K.

Discussion

It was possible as well to use either K-best or a reactive alpha independently, but it did not throw out such great results.

Heuristic	Average rise from Local Search (%)	Average deviation from best know (%)	Best known solutions	Mode of α from improved solutions	Total time (s)	Average time(s)
K-best	6.22	-9.70	14		6740.46	75.74
Reactive α	9.29	-6.66	**19**	0.6	**5374.64**	**60.39**
K & α	**9.94**	**-5.98**	15	0.8	6247.98	70.20

Table 4. Comparison of K-best & reactive α versus only either of them.

Working with only either K-best or reactive alpha does not give the best result possible. Considering only K = 3 (Table a5) gave as a result a similar amount of best known solution as K & alpha combined, but the gap from the known solution is greater, as well as the computation time.

Using only a reactive alpha did offer best results in terms of reaching more optimal solutions (four more than K & α), as well as a faster computation time. Nonetheless, in the long run a combination of K-best & reactive α gives a higher average improvement percent and on average the results were closer to the best known solutions, as seen in the third column of Table 4.

Conclusion

In order to solve a STSP, a GRASP was proposed which creates two paths simultaneously, joining them at the end of the algorithm. Although it did not always gave the best known solutions obtained in the literature, reaching only 15 of these out of 89, the average results were not too far apart from them (only 5.98% below), the average run time was better. The computation time is slightly better as well (Table a7); whereas for these 89 same instances the best recorded method would solve the same instances in a total computation time of 7,368.15 seconds (2h03min), this method on average does it in 6,249.56 seconds (1h44min), 2.5 times faster.

By combining K-best & reactive α one can obtain results relatively close to the best answers known. Reactive α by itself did have some good standing points, but if time is discarded as a factor, for in another computer it may be lower, the combination of α and K turns out better.

Software & Hardware Specifications

The following are the characteristics in which this heuristic was run:

- Python version 3.7.2
- Computer: Laptop
- Processor: Intel® Core™2 Duo 2.00GHz
- OS: Windows 10 32-bits
- Memory: 3.00GB

Bibliography

Chao, M., Golden, B., & Wasil, E. (1995). A fast and effective heuristic for the orienteering problem. *EUROPEAN JOURNAL OF OPERATIONAL RESEARCH*, 475-489.

Gutin, G., & Punnen, A. (2006). *The Traveling Salesman Problem and Its Variations.* New York: Springer.

Anexes

Table a1

Budget	Constructive Heuristic			Local Search				K-best (2) & α (best result out of 1000)				
	Profit	Cost	Time (s)	Profit	Cost	Time (s)	Rise (%)	Profit	Cost	Time (s)	Rise (%)	α
set_64												
15	36	14.9985	0.06	78	14.901	0.19	53.85	78	14.901	75.106	0	1
20	186	19.799	0.02	294	19.799	0.04	36.73	294	19.799	54.369	0	1
25	282	24.6274	0.06	390	24.6274	0.08	27.69	390	24.6274	63.026	0	1
30	360	29.4558	0.08	468	29.4558	0.11	23.08	468	29.4558	78.302	0	1
35	414	34.7975	0.06	468	34.7975	0.09	11.54	468	34.7975	90.45	0	1
40	540	39.598	0.04	594	39.598	0.07	9.09	594	39.598	107.57	0	1
45	600	44.4264	0.12	606	44.0147	5.16	0.99	690	44.4264	124.15	12.17	0.5
50	678	49.8313	0.07	732	49.8313	0.11	7.38	798	49.2548	125.95	8.27	0.2
55	738	53.7401	0.02	792	54.9117	0.07	6.82	882	54.9117	132.19	10.2	0.9
60	780	59.9049	0.05	822	59.9603	0.15	5.11	972	59.397	137.73	15.43	0.2
65	900	64.2254	0.03	918	64.2254	0.12	1.96	1026	64.2254	141.96	10.53	0
70	1056	69.8823	0.04	1074	69.8823	0.08	1.68	1086	69.8823	141.50	1.1	1
75	1104	74.7107	0.05	1122	74.7107	0.09	1.6	1122	74.7107	145.35	0	1
80	1128	78.3675	0.01	1152	79.7817	0.06	2.08	1176	79.196	144.56	2.04	0.2
set_66												
005	10	4.2361	0.05	10	4.2361	0.05	0.0	10	4.2361	41.1	0	1
010	30	8.8751	0.02	40	9.8929	0.03	25.0	40	9.8929	44.16	0	1
015	75	14.2136	0.05	80	14.639	0.06	6.25	80	14.639	60.551	0	1
020	90	19.3852	0.07	90	19.3852	0.08	0.0	90	19.3852	62.815	0	1
025	175	24.6857	0.03	175	24.6857	0.05	0.0	185	24.6857	73.204	5.41	0.7
030	270	29.8573	0.04	270	29.8573	0.06	0.0	270	29.8573	81.773	0	1
035	405	34.2136	0.01	410	34.639	0.04	1.22	410	34.639	89.554	0	1
040	570	39.3852	0.04	575	39.8106	0.07	0.87	575	39.8106	99.459	0	1
045	620	43.3852	0.04	630	44.2361	0.07	1.59	640	43.3852	107.59	1.56	0
050	660	49.8573	0.06	660	49.8573	0.09	0.0	720	49.8106	113.42	8.33	0.4
055	715	54.2136	0.04	720	54.639	0.07	0.69	750	54.639	124.23	4	0.5
060	720	59.3852	0.02	725	59.8106	0.06	0.69	795	59.8573	131.32	8.81	1
065	830	63.3852	0.01	840	64.2361	0.04	1.19	875	64.6857	133.77	4	0.6
070	855	69.8573	0.04	855	69.8573	0.07	0.0	950	69.9863	138.37	10	0.7
075	910	74.3426	0.02	915	74.768	0.06	0.55	1015	74.6989	143.69	9.85	1
080	985	79.1579	0.03	990	79.5833	0.12	0.51	1145	79.8573	148.68	13.54	0.9
085	1130	84.3294	0.04	1135	84.7549	0.11	0.44	1225	84.6857	147.34	7.35	1
090	1260	88.3294	0.02	1270	89.1803	0.09	0.79	1275	89.7097	150.07	0.39	0.8
095	1310	92.3294	0.04	1330	94.7549	0.23	1.5	1340	94.7549	151.46	0.75	0.5
100	1365	99.1579	0.02	1370	99.5833	0.09	0.36	1410	99.8106	149.44	2.84	0.5
105	1385	104.3294	0.02	1390	104.7549	0.05	0.36	1445	104.6857	148.67	3.81	1
110	1470	109.9863	0.01	1470	109.9863	0.05	0.0	1510	109.8573	146.48	2.65	0.7
115	1530	114.8016	0.01	1530	114.8016	0.05	0.0	1570	113.8573	140.76	2.55	0.1
120	1585	119.1579	0.01	1590	119.5833	0.04	0.31	1605	119.8573	137.20	0.93	1
125	1610	124.3294	0.01	1615	124.7549	0.05	0.31	1640	124.2136	135.39	1.52	0.8

130	1615	128.3294	0.01	1620	128.7549	0.03	0.31	1675	129.3852	132.18	3.28	0
tsiligirides_problem_1												
05	10	4.1426	0.0	10	4.1426	0.01	0.0	10	4.1426	15.414	0	1
10	15	6.8665	0.01	15	6.8665	0.01	0.0	15	6.8665	25.885	0	1
15	25	11.9617	0.01	35	14.8144	0.01	28.57	35	14.8144	25.879	0	1
20	40	17.8032	0.01	40	17.8032	0.01	0.0	40	17.8032	24.632	0	1
25	50	23.0498	0.01	55	24.8689	0.03	9.09	55	24.8689	29.753	0	1
30	65	25.9761	0.01	70	29.784	0.02	7.14	70	29.784	33.801	0	1
35	65	25.9761	0.01	80	34.9059	0.04	18.75	80	34.9059	37.249	0	1
40	75	35.3289	0.01	85	39.7118	0.03	11.76	85	39.7118	36.518	0	1
46	100	44.3893	0.01	100	44.3893	0.02	0.0	100	44.3893	39.584	0	1
50	110	47.8885	0.0	120	49.992	0.02	8.33	120	49.992	40.018	0	1
55	140	54.8292	0.01	140	54.8292	0.02	0.0	140	54.8292	43.358	0	1
60	160	59.9624	0.0	160	59.9624	0.02	0.0	160	59.9624	41.061	0	1
65	195	63.8974	0.0	205	64.8153	0.02	4.88	205	64.8153	42.119	0	1
70	205	68.8381	0.01	210	69.6065	0.02	2.38	210	69.6065	41.92	0	1
73	205	68.8381	0.0	215	72.0543	5.83	4.65	215	72.0543	48.774	0	1
75	210	74.2232	0.01	215	74.9916	0.02	2.33	215	74.9916	43.065	0	1
80	220	79.6536	0.0	220	79.6536	0.01	0.0	230	77.4183	40.279	4.35	0.6
85	240	84.8917	0.0	240	84.8917	0.02	0.0	240	84.8917	41.127	0	1
tsiligirides_problem_2												
15	100	14.3683	0.0	115	14.6676	0.01	13.04	120	14.8972	19.073	4.17	0.4
20	165	19.7149	0.0	165	19.7149	0.0	0.0	180	19.3456	19.803	8.33	0.4
23	200	21.4912	0.0	200	21.4912	0.0	0.0	200	21.4912	19.283	0	1
25	200	21.4912	0.0	200	21.4912	0.0	0.0	215	24.7277	19.873	6.98	0.3
27	230	25.7401	0.0	230	25.7401	0.0	0.0	230	25.7401	19.719	0	1
30	230	25.7401	0.0	230	25.7401	0.0	0.0	245	29.8165	19.917	6.12	0.5
32	260	31.531	0.0	260	31.531	0.0	0.0	260	31.531	19.51	0	1
35	260	31.531	0.0	260	31.531	0.0	0.0	260	31.531	19.749	0	1
38	260	31.531	0.0	260	31.531	0.0	0.0	275	37.0291	21.359	5.45	0.5
40	290	38.7902	0.0	290	38.7902	0.0	0.0	290	38.7902	20.279	0	1
45	340	42.8646	0.0	340	42.8646	0.0	0.0	355	43.2109	20.736	4.23	0.4
tsiligirides_problem_3												
015	160	14.8275	0.0	170	14.846	0.01	5.88	170	14.846	28.996	0	1
020	180	19.6627	0.0	190	19.6812	0.01	5.26	190	19.6812	31.515	0	1
025	220	23.0849	0.0	240	24.4924	0.05	8.33	240	24.4924	36.951	0	1
030	290	28.2481	0.0	320	28.7698	0.01	9.38	320	28.7698	39.303	0	1
035	370	34.7962	0.0	390	34.9334	0.01	5.13	390	34.9334	39.947	0	1
040	370	34.7962	0.0	400	38.0444	0.01	7.5	400	38.0444	45.261	0	1
045	420	43.4546	0.02	450	43.9764	0.03	6.67	450	43.9764	49.082	0	1
050	420	43.4546	0.73	470	49.5738	0.74	10.64	470	49.5738	49.493	0	1
055	440	52.3478	0.03	470	54.9612	0.04	6.38	470	54.9612	49.232	0	1
060	460	55.6265	0.02	490	58.8747	0.03	6.12	490	58.8747	50.451	0	1
065	500	63.4195	0.05	530	63.9413	0.07	5.66	530	63.9413	45.243	0	1
070	530	68.4145	0.03	560	68.9363	0.04	5.36	560	68.9363	48.751	0	1
075	550	72.9938	0.07	580	73.5156	0.09	5.17	580	73.5156	61.973	0	1
080	580	78.8949	0.01	610	79.4166	0.03	4.92	610	79.4166	52.043	0	1

085	600	84.8989	0.16	610	84.9174	0.17	1.64	610	84.9174	50.689	0	1
090	610	89.4648	0.05	640	89.9866	0.06	4.69	640	89.9866	52.48	0	1
095	660	94.6031	0.04	680	94.7403	0.05	2.94	680	94.7403	50.288	0	1
100	660	94.6031	0.06	700	99.8724	0.27	5.71	700	99.8724	52.2	0	1
105	700	103.192	0.05	730	103.7137	0.07	4.11	730	103.7137	50.532	0	1
110	700	103.192	0.09	740	108.4613	0.1	5.41	740	108.4613	55.274	0	1

Table a1. Constructive, improving, and random heuristic where k = 2

Table a2

Budget	Constructive Heuristic			Local Search				K-best (3) & α (best result out of 1000)				
	Profit	Cost	Time (s)	Profit	Cost	Time (s)	Rise (%)	Profit	Cost	Time (s)	Rise (%)	α
set_64												
15	36	14.9985	0.03	78	14.901	0.09	53.85	78	14.901	78.498	0.0	1
20	186	19.799	0.0	294	19.799	0.03	36.73	294	19.799	57.345	0.0	1
25	282	24.6274	0.01	390	24.6274	0.04	27.69	390	24.6274	66.946	0.0	1
30	360	29.4558	0.02	468	29.4558	0.05	23.08	468	29.4558	75.264	0.0	1
35	414	34.7975	0.02	468	34.7975	0.06	11.54	528	34.8608	90.466	11.36	0.8
40	540	39.598	0.01	594	39.598	0.05	9.09	666	39.9411	104.882	10.81	0.6
45	600	44.4264	0.02	606	44.0147	5.14	0.99	780	44.4264	118.269	22.31	0.2
50	678	49.8313	0.01	732	49.8313	0.06	7.38	852	49.2548	121.526	14.08	0.2
55	738	53.7401	0.01	792	54.9117	0.07	6.82	942	53.7401	127.659	15.92	0
60	780	59.9049	0.02	822	59.9603	0.13	5.11	1026	59.397	133.688	19.88	0.1
65	900	64.2254	0.02	918	64.2254	0.07	1.96	1080	64.2254	134.156	15.0	0
70	1056	69.8823	0.01	1074	69.8823	0.08	1.68	1140	69.8823	136.927	5.79	0.2
75	1104	74.7107	0.01	1122	74.7107	0.07	1.6	1176	74.7107	137.636	4.59	0.2
80	1128	78.3675	0.01	1152	79.7817	0.06	2.08	1212	79.5391	138.792	4.95	0.2
set_66												
005	10	4.2361	0.0	10	4.2361	0.01	0.0	10	4.2361	41.198	0.0	1
010	30	8.8751	0.02	40	9.8929	0.05	25.0	40	9.8929	43.766	0.0	1
015	75	14.2136	0.02	80	14.639	0.03	6.25	80	14.639	60.076	0.0	1
020	90	19.3852	0.05	90	19.3852	0.06	0.0	145	19.8704	60.37	37.93	1
025	175	24.6857	0.03	175	24.6857	0.05	0.0	215	24.9592	75.48	18.6	1
030	270	29.8573	0.01	270	29.8573	0.03	0.0	340	29.0420	78.823	20.59	0.9
035	405	34.2136	0.01	410	34.639	0.04	1.22	455	34.2136	91.214	9.89	0.7
040	570	39.3852	0.01	575	39.8106	0.04	0.87	575	39.8106	100.755	0.0	1
045	620	43.3852	0.01	630	44.2361	0.04	1.59	640	43.3852	108.57	1.56	0.2
050	660	49.8573	0.02	660	49.8573	0.05	0.0	705	49.3852	114.911	6.38	0.3
055	715	54.2136	0.01	720	54.639	0.05	0.69	775	54.2136	120.324	7.1	0.8
060	720	59.3852	0.02	725	59.8106	0.07	0.69	865	59.3024	126.699	16.18	0.7
065	830	63.3852	0.01	840	64.2361	0.04	1.19	940	63.3852	131.721	10.64	0.3
070	855	69.8573	0.02	855	69.8573	0.05	0.0	1000	69.8573	134.897	14.5	0.5
075	910	74.3426	0.02	915	74.768	0.06	0.55	1050	73.8573	140.499	12.86	0.3
080	985	79.1579	0.01	990	79.5833	0.07	0.51	1145	79.0420	142.348	13.54	0.5
085	1130	84.3294	0.01	1135	84.7549	0.08	0.44	1225	84.6857	145.035	7.35	0.9
090	1260	88.3294	0.01	1270	89.1803	0.11	0.79	1275	89.042	146.845	0.39	0.4

095	1310	92.3294	0.02	1330	94.7549	0.24	1.5	1340	94.6857	148.136	0.75	0.4
100	1365	99.1579	0.02	1370	99.5833	0.05	0.36	1410	99.8106	147.308	2.84	0.4
105	1385	104.3294	0.02	1390	104.7549	0.05	0.36	1430	104.5381	148.6	2.8	0.3
110	1470	109.9863	0.01	1470	109.9863	0.05	0.0	1510	109.8573	148.08	2.65	0.7
115	1530	114.8016	0.01	1530	114.8016	0.05	0.0	1570	113.8573	148.786	2.55	0.1
120	1585	119.1579	0.01	1590	119.5833	0.04	0.31	1605	119.8573	144.994	0.93	0.9
125	1610	124.3294	0.01	1615	124.7549	0.03	0.31	1640	124.2136	146.019	1.52	0.8
130	1615	128.3294	0.01	1620	128.7549	0.03	0.31	1675	129.3852	143.302	3.28	0
tsiligirides_problem_1												
05	10	4.1426	0.0	10	4.1426	0.0	0.0	10	4.1426	15.309	0.0	1
10	15	6.8665	0.01	15	6.8665	0.01	0.0	15	6.8665	23.364	0.0	1
15	25	11.9617	0.01	35	14.8144	0.01	28.57	35	14.8144	27.07	0.0	1
20	40	17.8032	0.01	40	17.8032	0.01	0.0	55	19.6597	28.159	27.27	1
25	50	23.0498	0.01	55	24.8689	0.03	9.09	70	24.8843	30.289	21.43	0.7
30	65	25.9761	0.01	70	29.784	0.02	7.14	85	29.1845	32.431	17.65	0.9
35	65	25.9761	0.01	80	34.9059	0.04	18.75	125	34.618	35.982	36.0	0.7
40	75	35.3289	0.01	85	39.7118	0.03	11.76	145	39.1214	36.228	41.38	1
46	100	44.3893	0.01	100	44.3893	0.02	0.0	170	45.9055	38.31	41.18	0.8
50	110	47.8885	0.0	120	49.992	0.02	8.33	185	48.3406	38.551	35.14	0.8
55	140	54.8292	0.01	140	54.8292	0.02	0.0	195	54.8653	40.451	28.21	1
60	160	59.9624	0.0	160	59.9624	0.02	0.0	215	59.0619	41.482	25.58	0.8
65	195	63.8974	0.0	205	64.8153	0.02	4.88	240	64.3442	42.978	14.58	0.9
70	205	68.8381	0.0	210	69.6065	0.02	2.38	250	69.1123	42.283	16.0	0.7
73	205	68.8381	0.0	215	72.0543	5.87	4.65	260	72.5821	47.96	17.31	0.7
75	210	74.2232	0.0	215	74.9916	0.02	2.33	265	74.3003	42.539	18.87	0.8
80	220	79.6536	0.0	220	79.6536	0.01	0.0	270	79.3578	43.23	18.52	0.8
85	240	84.8917	0.0	240	84.8917	0.02	0.0	280	83.3107	42.427	14.29	0.9
tsiligirides_problem_2												
15	100	14.3683	0.0	115	14.6676	0.01	13.04	120	14.8972	17.62	4.17	0.3
20	165	19.7149	0.0	165	19.7149	0.0	0.0	190	19.9614	19.906	13.16	0.4
23	200	21.4912	0.0	200	21.4912	0.0	0.0	200	21.4912	19.603	0.0	1
25	200	21.4912	0.0	200	21.4912	0.0	0.0	230	24.7767	19.666	13.04	0.7
27	230	25.7401	0.0	230	25.7401	0.0	0.0	230	25.7401	21.182	0.0	1
30	230	25.7401	0.0	230	25.7401	0.0	0.0	260	29.8655	20.557	11.54	0.9
32	260	31.531	0.0	260	31.531	0.0	0.0	280	31.9245	21.116	7.14	0.9
35	260	31.531	0.0	260	31.531	0.0	0.0	310	33.3384	20.583	16.13	0.6
38	260	31.531	0.0	260	31.531	0.0	0.0	340	37.7806	20.211	23.53	0.4
40	290	38.7902	0.0	290	38.7902	0.0	0.0	360	39.6399	20.812	19.44	0.4
45	340	42.8646	0.0	340	42.8646	0.0	0.0	435	44.4919	20.665	21.84	0.4
tsiligirides_problem_3												
015	160	14.8275	0.0	170	14.846	0.01	5.88	170	14.846	28.259	0.0	1
020	180	19.6627	0.0	190	19.6812	0.01	5.26	190	19.6812	31.686	0.0	1
025	220	23.0849	0.01	240	24.4924	0.05	8.33	250	24.9235	32.966	4.0	0.7
030	290	28.2481	0.0	320	28.7698	0.01	9.38	320	28.7698	34.428	0.0	1
035	370	34.7962	0.0	390	34.9334	0.01	5.13	390	34.9334	36.996	0.0	1
040	370	34.7962	0.0	400	38.0444	0.01	7.5	410	39.8468	38.165	2.44	0.4
045	420	43.4546	0.0	450	43.9764	0.02	6.67	450	43.9764	39.069	0.0	1

050	420	43.4546	0.01	470	49.5738	0.02	10.64	470	49.5738	40.687	0.0	1
055	440	52.3478	0.01	470	54.9612	0.02	6.38	490	54.4473	41.342	4.08	0.5
060	460	55.6265	0.01	490	58.8747	0.02	6.12	530	59.1011	42.18	7.55	0.8
065	500	63.4195	0.01	530	63.9413	0.02	5.66	550	64.6087	44.139	3.64	0.6
070	530	68.4145	0.0	560	68.9363	0.02	5.36	580	69.9399	43.765	3.45	1
075	550	72.9938	0.0	580	73.5156	0.02	5.17	600	74.8935	44.377	3.33	0.9
080	580	78.8949	0.01	610	79.4166	0.02	4.92	640	79.5448	44.491	4.69	0.8
085	600	84.8989	0.0	610	84.9174	0.02	1.64	640	84.5164	45.211	4.69	0.8
090	610	89.4648	0.0	640	89.9866	0.02	4.69	680	88.6772	46.038	5.88	0.6
095	660	94.6031	0.0	680	94.7403	0.02	2.94	690	94.5145	47.121	1.45	0.5
100	660	94.6031	0.0	700	99.8724	0.23	5.71	730	97.9058	46.086	4.11	1
105	700	103.192	0.0	730	103.7137	0.02	4.11	770	104.0594	44.483	5.19	0.5
110	700	103.192	0.0	740	108.4613	0.02	5.41	800	109.9026	44.721	7.5	0.5

Table a2. Constructive, improving, and random heuristic where k = 3

Table a3

Budget	Constructive Heuristic			Local Search				K-best (4) & α (best result out of 1000)				
	Profit	Cost	Time (s)	Profit	Cost	Time (s)	Rise (%)	Profit	Cost	Time (s)	Rise (%)	α
set_64												
15	36	14.9985	0.03	78	14.901	0.08	53.85	78	14.901	90.765	0.0	1
20	186	19.799	0.0	294	19.799	0.03	36.73	294	19.799	58.195	0.0	1
25	282	24.6274	0.01	390	24.6274	0.04	27.69	390	24.6274	65.378	0.0	1
30	360	29.4558	0.02	468	29.4558	0.08	23.08	468	29.4558	85.168	0.0	1
35	414	34.7975	0.02	468	34.7975	0.06	11.54	528	34.8608	91.533	11.36	0.8
40	540	39.598	0.01	594	39.598	0.04	9.09	654	39.9411	100.762	9.17	0.3
45	600	44.4264	0.02	606	44.0147	5.15	0.99	780	44.4264	119.592	22.31	0.2
50	678	49.8313	0.01	732	49.8313	0.06	7.38	852	49.8313	121.437	14.08	0.1
55	738	53.7401	0.01	792	54.9117	0.07	6.82	942	53.7401	125.389	15.92	0.1
60	780	59.9049	0.02	822	59.9603	0.11	5.11	1026	59.397	129.661	19.88	0.1
65	900	64.2254	0.02	918	64.2254	0.07	1.96	1080	64.2254	131.594	15.0	0
70	1056	69.8823	0.01	1074	69.8823	0.06	1.68	1128	69.8823	136.259	4.79	0
75	1104	74.7107	0.01	1122	74.7107	0.06	1.6	1176	74.7107	138.289	4.59	0.2
80	1128	78.3675	0.01	1152	79.7817	0.06	2.08	1212	79.2872	138.133	4.95	0.2
set_66												
005	10	4.2361	0.0	10	4.2361	0.01	0.0	10	4.2361	41.172	0.0	1
010	30	8.8751	0.02	40	9.8929	0.03	25.0	40	9.8929	43.014	0.0	1
015	75	14.2136	0.02	80	14.639	0.03	6.25	80	14.639	61.67	0.0	1
020	90	19.3852	0.03	90	19.3852	0.04	0.0	145	19.8704	61.847	37.93	1
025	175	24.6857	0.03	175	24.6857	0.04	0.0	215	24.9592	72.39	18.6	1
030	270	29.8573	0.01	270	29.8573	0.03	0.0	340	29.042	81.341	20.59	0.9
035	405	34.2136	0.01	410	34.639	0.04	1.22	455	34.2136	89.844	9.89	0.8
040	570	39.3852	0.01	575	39.8106	0.04	0.87	575	39.8106	100.185	0.0	1
045	620	43.3852	0.01	630	44.2361	0.04	1.59	640	43.3852	107.398	1.56	0.1
050	660	49.8573	0.02	660	49.8573	0.05	0.0	705	49.3852	113.688	6.38	0.3
055	715	54.2136	0.01	720	54.639	0.05	0.69	775	54.2136	119.158	7.1	0.5

060	720	59.3852	0.02	725	59.8106	0.06	0.69	840	59.3852	130.381	13.69	0.3
065	830	63.3852	0.01	840	64.2361	0.04	1.19	910	63.3852	130.053	7.69	0.3
070	855	69.8573	0.02	855	69.8573	0.05	0.0	955	69.8704	148.86	10.47	0.4
075	910	74.3426	0.02	915	74.768	0.08	0.55	1050	74.9592	147.631	12.86	0.5
080	985	79.1579	0.01	990	79.5833	0.08	0.51	1145	79.042	144.482	13.54	0.5
085	1130	84.3294	0.01	1135	84.7549	0.14	0.44	1210	84.5963	145.006	6.2	0.1
090	1260	88.3294	0.01	1270	89.1803	0.08	0.79	1270	89.1803	146.785	0.0	1
095	1310	92.3294	0.02	1330	94.7549	0.2	1.5	1330	94.7549	147.673	0.0	1
100	1365	99.1579	0.02	1370	99.5833	0.09	0.36	1375	99.3852	149.08	0.36	0
105	1385	104.3294	0.02	1390	104.7549	0.05	0.36	1425	104.3294	151.502	2.46	0
110	1470	109.9863	0.01	1470	109.9863	0.05	0.0	1480	109.6314	147.84	0.68	0.1
115	1530	114.8016	0.02	1530	114.8016	0.05	0.0	1570	113.8573	147.58	2.55	0
120	1585	119.1579	0.01	1590	119.5833	0.04	0.31	1600	119.7097	145.078	0.62	0.2
125	1610	124.3294	0.01	1615	124.7549	0.03	0.31	1635	123.8573	145.957	1.22	0
130	1615	128.3294	0.01	1620	128.7549	0.03	0.31	1675	129.3852	143.962	3.28	0.1
tsiligirides_problem_1												
05	10	4.1426	0.01	10	4.1426	0.01	0.0	10	4.1426	15.212	0.0	1
10	15	6.8665	0.01	15	6.8665	0.01	0.0	15	6.8665	23.054	0.0	1
15	25	11.9617	0.01	35	14.8144	0.01	28.57	35	14.8144	26.916	0.0	1
20	40	17.8032	0.01	40	17.8032	0.01	0.0	55	19.4026	28.709	27.27	0.7
25	50	23.0498	0.01	55	24.8689	0.03	9.09	70	24.6162	29.869	21.43	0.7
30	65	25.9761	0.01	70	29.784	0.02	7.14	85	29.1845	32.214	17.65	0.8
35	65	25.9761	0.01	80	34.9059	0.07	18.75	120	34.6596	34.318	33.33	0.6
40	75	35.3289	0.01	85	39.7118	0.03	11.76	145	39.8716	36.401	41.38	0.6
46	100	44.3893	0.01	100	44.3893	0.02	0.0	170	45.9055	37.884	41.18	0.9
50	110	47.8885	0.01	120	49.992	0.02	8.33	185	48.3406	38.244	35.14	0.6
55	140	54.8292	0.01	140	54.8292	0.02	0.0	195	54.1132	40.269	28.21	0.7
60	160	59.9624	0.0	160	59.9624	0.02	0.0	215	59.0619	40.449	25.58	0.8
65	195	63.8974	0.0	205	64.8153	0.02	4.88	225	64.769	41.217	8.89	1
70	205	68.8381	0.0	210	69.6065	0.02	2.38	235	68.9048	41.734	10.64	0.9
73	205	68.8381	0.0	215	72.0543	5.91	4.65	245	71.8089	48.209	12.24	0.8
75	210	74.2232	0.0	215	74.9916	0.02	2.33	245	74.5616	42.302	12.24	0.9
80	220	79.6536	0.0	220	79.6536	0.01	0.0	260	79.8368	41.924	15.38	1
85	240	84.8917	0.0	240	84.8917	0.02	0.0	260	84.779	41.883	7.69	0.9
tsiligirides_problem_2												
15	100	14.3683	0.0	115	14.6676	0.01	13.04	120	14.8972	17.535	4.17	0.3
20	165	19.7149	0.0	165	19.7149	0.0	0.0	180	19.3456	19.064	8.33	0.5
23	200	21.4912	0.0	200	21.4912	0.0	0.0	200	21.4912	19.475	0.0	1
25	200	21.4912	0.0	200	21.4912	0.0	0.0	230	24.7767	19.5	13.04	0.4
27	230	25.7401	0.0	230	25.7401	0.0	0.0	230	25.7401	19.24	0.0	1
30	230	25.7401	0.0	230	25.7401	0.0	0.0	250	29.3717	19.525	8.0	0.6
32	260	31.531	0.0	260	31.531	0.0	0.0	270	31.9401	19.628	3.7	0.8
35	260	31.531	0.0	260	31.531	0.0	0.0	310	33.3384	19.862	16.13	0.8
38	260	31.531	0.0	260	31.531	0.0	0.0	340	37.7806	19.991	23.53	0.4
40	290	38.7902	0.0	290	38.7902	0.0	0.0	360	39.6399	19.928	19.44	0.4
45	340	42.8646	0.0	340	42.8646	0.0	0.0	435	44.4919	19.446	21.84	0.4
tsiligirides_problem_3												

015	160	14.8275	0.0	170	14.846	0.01	5.88	170	14.846	27.787	0.0	1
020	180	19.6627	0.0	190	19.6812	0.01	5.26	190	19.6812	31.292	0.0	1
025	220	23.0849	0.0	240	24.4924	0.05	8.33	250	24.9235	32.329	4.0	0.7
030	290	28.2481	0.0	320	28.7698	0.01	9.38	320	28.7698	34.644	0.0	1
035	370	34.7962	0.0	390	34.9334	0.01	5.13	390	34.9334	35.551	0.0	1
040	370	34.7962	0.0	400	38.0444	0.01	7.5	410	39.8468	36.634	2.44	0.4
045	420	43.4546	0.01	450	43.9764	0.02	6.67	450	43.9764	37.336	0.0	1
050	420	43.4546	0.01	470	49.5738	0.02	10.64	470	49.5738	39.412	0.0	1
055	440	52.3478	0.01	470	54.9612	0.02	6.38	490	54.3032	40.982	4.08	0.4
060	460	55.6265	0.01	490	58.8747	0.02	6.12	530	59.7095	41.156	7.55	1
065	500	63.4195	0.01	530	63.9413	0.02	5.66	550	64.6087	41.504	3.64	0.6
070	530	68.4145	0.0	560	68.9363	0.02	5.36	570	69.8391	42.862	1.75	0.9
075	550	72.9938	0.01	580	73.5156	0.02	5.17	600	74.524	43.221	3.33	0.7
080	580	78.8949	0.0	610	79.4166	0.02	4.92	640	79.5448	43.839	4.69	0.9
085	600	84.8989	0.0	610	84.9174	0.02	1.64	650	83.9061	44.432	6.15	0.7
090	610	89.4648	0.0	640	89.9866	0.02	4.69	670	89.8985	44.739	4.48	0.7
095	660	94.6031	0.0	680	94.7403	0.02	2.94	690	93.8778	45.037	1.45	0.6
100	660	94.6031	0.0	700	99.8724	0.22	5.71	720	99.8349	52.551	2.78	0.9
105	700	103.192	0.01	730	103.7137	0.02	4.11	740	104.082	52.603	1.35	0.4
110	700	103.192	0.0	740	108.4613	0.02	5.41	760	109.9835	44.397	2.63	0.8

Table a3. Constructive, improving, and random heuristic where k = 4

Table a4

Budget	Constructive Heuristic			Local Search				K-best (5) & α (best result out of 1000)				
	Profit	Cost	Time (s)	Profit	Cost	Time (s)	Rise (%)	Profit	Cost	Time (s)	Rise (%)	α
set_64												
15	36	14.9985	0.03	78	14.901	0.09	53.85	78	14.901	79.711	0.0	1
20	186	19.799	0.0	294	19.799	0.02	36.73	294	19.799	54.582	0.0	1
25	282	24.6274	0.01	390	24.6274	0.04	27.69	390	24.6274	64.5	0.0	1
30	360	29.4558	0.02	468	29.4558	0.05	23.08	468	29.4558	76.404	0.0	1
35	414	34.7975	0.02	468	34.7975	0.06	11.54	504	34.2843	88.816	7.14	0.5
40	540	39.598	0.01	594	39.598	0.04	9.09	618	39.9411	101.558	3.88	0.3
45	600	44.4264	0.02	606	44.0147	5.15	0.99	768	44.4264	114.954	21.09	0
50	678	49.8313	0.01	732	49.8313	0.06	7.38	852	49.8313	117.586	14.08	0
55	738	53.7401	0.01	792	54.9117	0.07	6.82	942	53.7401	122.922	15.92	0
60	780	59.9049	0.02	822	59.9603	0.11	5.11	1026	59.397	127.469	19.88	0
65	900	64.2254	0.02	918	64.2254	0.07	1.96	1080	64.2254	130.39	15.0	0.1
70	1056	69.8823	0.01	1074	69.8823	0.06	1.68	1134	69.0538	135.603	5.29	0.2
75	1104	74.7107	0.01	1122	74.7107	0.05	1.6	1152	73.6303	141.968	2.6	0.3
80	1128	78.3675	0.01	1152	79.7817	0.07	2.08	1212	78.7107	137.835	4.95	0.2
set_66												
005	10	4.2361	0.0	10	4.2361	0.01	0.0	10	4.2361	40.711	0.0	1
010	30	8.8751	0.02	40	9.8929	0.04	25.0	40	9.8929	45.59	0.0	1
015	75	14.2136	0.02	80	14.639	0.03	6.25	95	14.8751	56.603	15.79	0.8
020	90	19.3852	0.03	90	19.3852	0.04	0.0	145	19.8704	61.65	37.93	1

025	175	24.6857	0.04	175	24.6857	0.06	0.0	215	24.9592	69.976	18.6	0.6
030	270	29.8573	0.01	270	29.8573	0.03	0.0	320	29.8704	78.746	15.62	0.9
035	405	34.2136	0.01	410	34.639	0.05	1.22	455	34.2136	86.78	9.89	0.6
040	570	39.3852	0.01	575	39.8106	0.04	0.87	575	39.8106	96.99	0.0	1
045	620	43.3852	0.01	630	44.2361	0.07	1.59	640	43.3852	105.248	1.56	0
050	660	49.8573	0.02	660	49.8573	0.05	0.0	705	49.3852	110.927	6.38	0.3
055	715	54.2136	0.01	720	54.639	0.05	0.69	775	53.3852	119.495	7.1	0.3
060	720	59.3852	0.02	725	59.8106	0.06	0.69	825	59.8573	123.303	12.12	0.3
065	830	63.3852	0.01	840	64.2361	0.04	1.19	905	64.9592	127.903	7.18	0.5
070	855	69.8573	0.02	855	69.8573	0.06	0.0	965	68.1308	131.536	11.4	0.3
075	910	74.3426	0.02	915	74.768	0.06	0.55	1035	74.6161	136.267	11.59	0.3
080	985	79.1579	0.02	990	79.5833	0.03	0.51	1110	79.5832	53.175	10.81	0
085	1130	84.3294	0.01	1135	84.7549	0.08	0.44	1230	84.2136	143.744	7.72	0.3
090	1260	88.3294	0.01	1270	89.1803	0.08	0.79	1270	89.1803	144.115	0.0	1
095	1310	92.3294	0.02	1330	94.7549	0.2	1.5	1330	94.7549	145.018	0.0	1
100	1365	99.1579	0.02	1370	99.5833	0.05	0.36	1405	99.3024	146.319	2.49	0.2
105	1385	104.3294	0.02	1390	104.7549	0.05	0.36	1440	104.9592	155.402	3.47	0.2
110	1470	109.9863	0.01	1470	109.9863	0.05	0.0	1495	109.7745	149.242	1.67	0.2
115	1530	114.8016	0.01	1530	114.8016	0.05	0.0	1570	113.8573	146.043	2.55	0.1
120	1585	119.1579	0.01	1590	119.5833	0.04	0.31	1600	119.7097	145.767	0.62	0
125	1610	124.3294	0.01	1615	124.7549	0.03	0.31	1635	123.8573	145.399	1.22	0.2
130	1615	128.3294	0.01	1620	128.7549	0.03	0.31	1675	129.3852	144.165	3.28	0
tsiligirides_problem_1												
05	10	4.1426	0.0	10	4.1426	0.01	0.0	10	4.1426	15.361	0.0	1
10	15	6.8665	0.01	15	6.8665	0.01	0.0	15	6.8665	22.566	0.0	1
15	25	11.9617	0.01	35	14.8144	0.01	28.57	35	14.8144	24.4	0.0	1
20	40	17.8032	0.01	40	17.8032	0.01	0.0	55	19.4107	28.297	27.27	0.9
25	50	23.0498	0.01	55	24.8689	0.03	9.09	75	24.7147	29.364	26.67	1
30	65	25.9761	0.01	70	29.784	0.02	7.14	95	29.8865	32.18	26.32	0.7
35	65	25.9761	0.01	80	34.9059	0.04	18.75	115	33.3312	33.868	30.43	0.7
40	75	35.3289	0.01	85	39.7118	0.03	11.76	145	39.1214	35.478	41.38	0.8
46	100	44.3893	0.01	100	44.3893	0.02	0.0	160	45.848	37.67	37.5	0.6
50	110	47.8885	0.0	120	49.992	0.02	8.33	175	49.6759	37.694	31.43	0.6
55	140	54.8292	0.01	140	54.8292	0.02	0.0	185	53.7165	39.965	24.32	0.8
60	160	59.9624	0.0	160	59.9624	0.02	0.0	200	59.8977	40.729	20.0	0.6
65	195	63.8974	0.0	205	64.8153	0.02	4.88	220	63.2484	41.304	6.82	1
70	205	68.8381	0.0	210	69.6065	0.02	2.38	235	69.2237	42.918	10.64	1
73	205	68.8381	0.0	215	72.0543	5.85	4.65	235	71.7319	47.969	8.51	0.6
75	210	74.2232	0.0	215	74.9916	0.02	2.33	235	74.5793	42.006	8.51	0.7
80	220	79.6536	0.0	220	79.6536	0.01	0.0	260	78.2043	43.401	15.38	0.8
85	240	84.8917	0.0	240	84.8917	0.02	0.0	270	83.8766	47.323	11.11	0.7
tsiligirides_problem_2												
15	100	14.3683	0.0	115	14.6676	0.01	13.04	120	14.8972	17.643	4.17	0.3
20	165	19.7149	0.0	165	19.7149	0.0	0.0	190	19.9614	20.029	13.16	0.5
23	200	21.4912	0.0	200	21.4912	0.0	0.0	200	21.4912	20.619	0.0	1
25	200	21.4912	0.0	200	21.4912	0.0	0.0	230	24.8947	21.741	13.04	0.4
27	230	25.7401	0.0	230	25.7401	0.0	0.0	230	25.7401	19.588	0.0	1

30	230	25.7401	0.0	230	25.7401	0.0	0.0	250	29.2991	20.792	8.0	0.5
32	260	31.531	0.0	260	31.531	0.0	0.0	280	31.9245	21.52	7.14	1
35	260	31.531	0.0	260	31.531	0.0	0.0	310	34.0279	21.263	16.13	0.6
38	260	31.531	0.0	260	31.531	0.0	0.0	350	37.6933	21.162	25.71	0.5
40	290	38.7902	0.0	290	38.7902	0.0	0.0	370	39.9472	20.647	21.62	0.5
45	340	42.8646	0.0	340	42.8646	0.0	0.0	430	44.0135	20.397	20.93	0.4
tsiligirides_problem_3												
015	160	14.8275	0.0	170	14.846	0.01	5.88	170	14.846	28.57	0.0	1
020	180	19.6627	0.0	190	19.6812	0.01	5.26	190	19.6812	31.601	0.0	1
025	220	23.0849	0.01	240	24.4924	0.05	8.33	250	24.9235	32.935	4.0	0.6
030	290	28.2481	0.0	320	28.7698	0.01	9.38	320	28.7698	34.921	0.0	1
035	370	34.7962	0.0	390	34.9334	0.01	5.13	390	34.9334	36.317	0.0	1
040	370	34.7962	0.0	400	38.0444	0.01	7.5	410	39.8468	38.51	2.44	0.4
045	420	43.4546	0.0	450	43.9764	0.02	6.67	450	43.9764	39.765	0.0	1
050	420	43.4546	0.01	470	49.5738	0.02	10.64	470	49.5738	39.679	0.0	1
055	440	52.3478	0.01	470	54.9612	0.02	6.38	480	53.9187	40.924	2.08	0.3
060	460	55.6265	0.01	490	58.8747	0.02	6.12	510	58.5842	42.314	3.92	0.5
065	500	63.4195	0.01	530	63.9413	0.02	5.66	550	64.8762	43.433	3.64	0.5
070	530	68.4145	0.0	560	68.9363	0.02	5.36	560	68.9363	45.119	0.0	1
075	550	72.9938	0.0	580	73.5156	0.02	5.17	590	74.2182	44.385	1.69	0.5
080	580	78.8949	0.0	610	79.4166	0.02	4.92	620	79.475	46.996	1.61	0.4
085	600	84.8989	0.0	610	84.9174	0.02	1.64	650	84.0477	44.108	6.15	0.6
090	610	89.4648	0.0	640	89.9866	0.02	4.69	660	89.9141	44.051	3.03	0.5
095	660	94.6031	0.0	680	94.7403	0.02	2.94	690	93.8777	44.837	1.45	0.5
100	660	94.6031	0.01	700	99.8724	0.21	5.71	720	99.6528	45.487	2.78	0.4
105	700	103.192	0.0	730	103.7137	0.02	4.11	750	104.3015	44.049	2.67	0.4
110	700	103.192	0.0	740	108.4613	0.02	5.41	770	109.0126	43.441	3.9	0.4

Table a4. Constructive, improving, and random heuristic where k = 5

Table a5. Heuristic done with only K-best

	CONSTRUCTIVE		LOCAL SEARCH		K-BEST			
Instance	**Profit**	**Cost**	**Profit**	**Cost**	**Profit**	**Cost**	**Rise (%)**	**Total Time (S)**
set_64								
1_15	36	14.99851	78	14.90105	78	14.90105	0.00	37.582
1_20	186	19.79899	294	19.79899	294	19.79899	0.00	61.312
1_25	282	24.62742	390	24.62742	390	24.62742	0.00	68.763
1_30	360	29.45584	468	29.45584	468	29.45584	0.00	69.555
1_35	**414**	**34.7975**	468	**34.86076**	**516**	**34.86076**	**9.30**	**76.873**
1_40	540	39.59798	594	39.59798	594	39.59798	0.00	71.278
1_45	**600**	**44.42641**	606	**44.98975**	**648**	**44.98975**	**6.48**	**71.632**
1_50	**678**	**49.83133**	732	**49.83133**	**750**	**49.92254**	**2.40**	**76.185**
1_55	**738**	**53.74012**	792	**54.91169**	**858**	**54.75096**	**7.69**	**74.469**

1_60	**780**	**59.90492**	822	**59.99613**	**858**	**59.99613**	**4.20**	**75.159**
1_65	**900**	**64.2254**	918	**64.2254**	**930**	**64.2254**	**1.29**	**72.721**
1_70	1056	69.88225	1074	69.88225	1074	69.88225	0.00	71.28
1_75	1104	74.71068	1122	74.71068	1122	74.71068	0.00	70.925
1_80	1128	78.36753	1152	79.78175	1152	79.78175	0.00	71.977
set_66								
1_005	10	4.23607	10	4.92081	10	4.92081	0.00	6.094
1_010	**30**	**8.87511**	40	**9.3541**	**40**	**9.3541**	**0.00**	**53.533**
1_015	**75**	**14.21359**	80	**14.63904**	**120**	**14.98784**	**33.33**	**70.917**
1_020	**90**	**19.38516**	90	**19.38516**	**175**	**19.79592**	**48.57**	**99.476**
1_025	**175**	**24.68573**	175	**24.68573**	**240**	**24.95172**	**27.08**	**76.904**
1_030	**270**	**29.8573**	270	**29.8573**	**355**	**29.46747**	**23.94**	**172.134**
1_035	**405**	**34.21359**	410	**34.63904**	**460**	**34.98784**	**10.87**	**75.55**
1_040	570	39.38516	575	39.81062	575	39.81062	0.00	74.57
1_045	620	43.38516	630	44.23607	630	44.23607	0.00	84.606
1_050	**660**	**49.8573**	660	**49.89292**	**670**	**49.89292**	**1.49**	**79.128**
1_055	715	54.21359	720	54.63904	720	54.63904	0.00	85.647
1_060	**720**	**59.38516**	725	**59.81062**	**730**	**59.8999**	**0.68**	**224.166**
1_065	830	63.38516	840	64.23607	840	64.23607	0.00	81.066
1_070	855	69.8573	855	69.8573	855	69.8573	0.00	92.514
1_075	**910**	**74.34258**	915	**74.76803**	**940**	**74.76803**	**2.66**	**89.809**
1_080	985	79.15786	990	79.58332	990	79.58332	0.00	77.945
1_085	1130	84.32944	1135	84.75489	1135	84.75489	0.00	88.008
1_090	**1260**	**88.32944**	1270	**89.93961**	**1290**	**89.93961**	**1.55**	**80.803**
1_095	**1310**	**92.32944**	1330	**94.75489**	**1340**	**94.75489**	**0.75**	**86.872**
1_100	1365	99.15786	1370	99.58332	1370	99.58332	0.00	86.184
1_105	1385	104.3294	1390	104.7549	1390	104.7549	0.00	86.348
1_110	1470	109.9863	1470	109.9863	1470	109.9863	0.00	82.649
1_115	1530	114.8016	1530	114.8016	1530	114.8016	0.00	91.388
1_120	1585	119.1579	1590	119.5833	1590	119.5833	0.00	80.106
1_125	1610	124.3294	1615	124.7549	1615	124.7549	0.00	102.621
1_130	**1615**	**128.3294**	1620	**129.9863**	**1630**	**129.9863**	**0.61**	**86.731**
tsiligirides_problem_1								
budget_05	10	4.14257	10	4.14257	10	4.14257	0.00	2.372
budget_10	15	6.86652	15	6.86652	15	6.86652	0.00	5.445
budget_15	**25**	**11.96174**	35	**14.81438**	**45**	**14.66316**	**22.22**	**21.43**
budget_20	**40**	**17.8032**	40	**17.8032**	**60**	**18.86413**	**33.33**	**68.133**
budget_25	**50**	**23.04981**	55	**24.33255**	**75**	**24.94093**	**26.67**	**70.836**

budget_30	**65**	**25.97607**	70	**29.78395**	**100**	**29.76345**	**30.00**	**55.194**
budget_35	**65**	**25.97607**	80	**34.07617**	**115**	**34.91906**	**30.43**	**149.53**
budget_40	**75**	**35.3289**	85	**39.80378**	**125**	**39.80006**	**32.00**	**74.692**
budget_46	**100**	**44.38931**	100	**44.11223**	**135**	**45.81761**	**25.93**	**74.151**
budget_50	**110**	**47.88853**	120	**49.99201**	**145**	**49.69365**	**17.24**	**74.123**
budget_55	**140**	**54.82923**	140	**54.80349**	**180**	**53.49589**	**22.22**	**101.829**
budget_60	**160**	**59.9624**	160	**59.9624**	**180**	**59.6412**	**11.11**	**76.572**
budget_65	195	63.89742	205	64.81534	205	64.81534	0.00	74
budget_70	**205**	**68.83805**	210	**69.69652**	**230**	**69.95412**	**8.70**	**85.268**
budget_73	**205**	**68.83805**	215	**72.53083**	**245**	**72.53083**	**12.24**	**85.703**
budget_75	210	74.22316	215	74.99162	215	74.99162	0.00	79.713
budget_80	**220**	**79.65358**	220	**79.65358**	**230**	**78.6165**	**4.35**	**82.542**
budget_85	**240**	**84.89173**	240	**84.94477**	**245**	**84.94477**	**2.04**	**75.678**
tsiligirides_problem_2								
budget_15	100	14.36833	115	14.66788	115	14.66788	0.00	49.738
budget_20	**165**	**19.71491**	165	**19.71491**	**180**	**18.75072**	**8.33**	**116.628**
budget_23	200	21.49123	200	21.49123	200	21.49123	0.00	65.655
budget_25	200	21.49123	200	21.49123	200	21.49123	0.00	66.111
budget_27	230	25.74005	230	25.74005	230	25.74005	0.00	60.373
budget_30	**230**	**25.74005**	230	**25.74005**	**245**	**29.97271**	**6.12**	**63.418**
budget_32	**260**	**31.531**	260	**31.531**	**285**	**31.84162**	**8.77**	**114.886**
budget_35	**260**	**31.531**	260	**31.531**	**285**	**34.43731**	**8.77**	**70.282**
budget_38	**260**	**31.531**	260	**37.59885**	**320**	**37.98905**	**18.75**	**105.198**
budget_40	**290**	**38.79017**	290	**38.79017**	**370**	**39.98152**	**21.62**	**70.609**
budget_45	**340**	**42.86463**	340	**42.86463**	**400**	**44.82036**	**15.00**	**120.061**
tsiligirides_problem_3								
budget_015	160	14.82749	170	14.84595	170	14.84595	**0.00**	38.279
budget_020	180	19.66272	190	19.88578	190	19.88578	**0.00**	49.777
budget_025	220	23.08492	240	24.89034	240	24.89034	**0.00**	58.356
budget_030	290	28.24808	320	28.76984	320	28.76984	**0.00**	65.461
budget_035	370	34.79618	390	34.93342	390	34.93342	**0.00**	61.942
budget_040	**370**	**34.79618**	400	**39.58998**	**420**	**39.58998**	**4.76**	**69.402**
budget_045	420	43.45461	450	43.97637	450	43.97637	**0.00**	64.373
budget_050	420	43.45461	470	49.95868	470	49.95868	**0.00**	70.089
budget_055	440	52.34778	470	54.96116	470	54.96116	**0.00**	67.802
budget_060	460	55.62649	490	58.65152	490	58.65152	**0.00**	68.124
budget_065	500	63.41951	530	63.94127	530	63.94127	**0.00**	67.111
budget_070	530	68.41454	560	68.9363	560	68.9363	**0.00**	64.73

budget_075	550	72.9938	580	73.51556	580	73.51556	**0.00**	60.789
budget_080	580	78.89485	610	79.41661	610	79.41661	**0.00**	63.496
budget_085	600	84.89894	610	84.91741	610	84.91741	**0.00**	67.3
budget_090	610	89.46481	640	89.98657	640	89.98657	**0.00**	69.314
budget_095	660	94.60309	680	94.74034	680	94.74034	**0.00**	71.438
budget_100	660	94.60309	700	99.87906	700	99.87906	**0.00**	74.534
budget_105	700	103.192	730	103.7137	730	103.7137	**0.00**	71.678
budget_110	700	103.192	740	108.4613	740	108.4613	**0.00**	70.818

Table a6. Heuristic done with only alpha reactive

	CONSTRUCTIVE		LOCAL SEARCH		ALPHA REACTIVE				
Instance	Profit	Cost	Profit	Cost	Profit	Cost	Time (s)	Rise (%)	Alpha
set_64									
1_15	36	14.9985	78	14.901	96	14.8284	53.221	18.75	0.4
1_20	186	19.799	294	19.799	294	19.799	50.538	0	1
1_25	282	24.6274	390	24.6274	390	24.6274	57.846	0	1
1_30	360	29.4558	468	29.4558	468	29.4558	64.341	0	1
1_35	414	34.7975	468	34.7975	498	34.8608	75.46	6.02	0.3
1_40	540	39.598	594	39.598	642	39.5294	84.321	7.48	0.1
1_45	600	44.4264	606	44.0147	768	44.4264	96.027	21.09	0
1_50	678	49.8313	732	49.8313	882	49.8313	99.616	17.01	0.1
1_55	738	53.7401	792	54.9117	954	53.7401	106.859	16.98	0.1
1_60	780	59.9049	822	59.9603	1026	59.397	112.152	19.88	0
1_65	900	64.2254	918	64.2254	1110	64.2254	113.158	17.3	0.1
1_70	1056	69.8823	1074	69.8823	1146	69.8823	116.153	6.28	0.1
1_75	1104	74.7107	1122	74.7107	1170	74.7107	119.736	4.1	0
1_80	1128	78.3675	1152	79.7817	1218	79.8637	121.602	5.42	0.1
set_66									
1_005	10	4.2361	10	4.2361	10	4.2361	38.155	0	1
1_010	30	8.8751	40	9.8929	40	9.8929	48.165	0	1
1_015	75	14.2136	80	14.639	115	14.9112	46.463	30.43	0.3
1_020	90	19.3852	90	19.3852	185	19.8106	48.335	51.35	0.4
1_025	175	24.6857	175	24.6857	260	24.5624	54.003	32.69	0.3
1_030	270	29.8573	270	29.8573	350	29.3908	60.09	22.86	0.4
1_035	405	34.2136	410	34.639	420	34.5459	67.678	2.38	0.3
1_040	570	39.3852	575	39.8106	575	39.8106	77.934	0	1
1_045	620	43.3852	630	44.2361	640	43.3852	86.187	1.56	0

1_050	660	49.8573	660	49.8573	700	49.3024	93.234	5.71	0
1_055	715	54.2136	720	54.639	735	54.048	99.529	2.04	0.3
1_060	720	59.3852	725	59.8106	785	59.0735	104.022	7.64	0.3
1_065	830	63.3852	840	64.2361	900	64.5624	107.933	6.67	0.3
1_070	855	69.8573	855	69.8573	925	69.2195	113.386	7.57	0.3
1_075	910	74.3426	915	74.768	995	74.7337	119.82	8.04	0.3
1_080	985	79.1579	990	79.5833	1110	79.5832	121.65	10.81	0
1_085	1130	84.3294	1135	84.7549	1210	84.5963	124.84	6.2	0
1_090	1260	88.3294	1270	89.1803	1280	89.1862	127.443	0.78	0.1
1_095	1310	92.3294	1330	94.7549	1335	94.0975	128.513	0.37	0.1
1_100	1365	99.1579	1370	99.5833	1405	99.3024	131.366	2.49	0.1
1_105	1385	104.3294	1390	104.7549	1440	104.9592	133.931	3.47	0.1
1_110	1470	109.9863	1470	109.9863	1495	109.7745	134.994	1.67	0.1
1_115	1530	114.8016	1530	114.8016	1570	113.8573	134.291	2.55	0
1_120	1585	119.1579	1590	119.5833	1600	119.7097	132.643	0.62	0
1_125	1610	124.3294	1615	124.7549	1635	123.8573	134.835	1.22	0
1_130	1615	128.3294	1620	128.7549	1675	129.3852	132.787	3.28	0.2
tsiligirides_problem_1									
budget_05	10	4.1426	10	4.1426	10	4.1426	14.076	0	1
budget_10	15	6.8665	15	6.8665	15	6.8665	20.792	0	1
budget_15	25	11.9617	35	14.8144	45	14.2636	21.817	22.22	0.7
budget_20	40	17.8032	40	17.8032	60	19.3085	23.698	33.33	0.8
budget_25	50	23.0498	55	24.8689	75	24.9286	25.196	26.67	0.6
budget_30	65	25.9761	70	29.784	90	29.8277	27.569	22.22	0.6
budget_35	65	25.9761	80	34.9059	110	34.6519	29.476	27.27	0.6
budget_40	75	35.3289	85	39.7118	125	39.7941	30.272	32	0.6
budget_46	100	44.3893	100	44.3893	150	45.7965	33.559	33.33	0.6
budget_50	110	47.8885	120	49.992	160	46.7677	33.175	25	0.6
budget_55	140	54.8292	140	54.8292	180	54.5148	35	22.22	0.6
budget_60	160	59.9624	160	59.9624	200	59.8558	36.04	20	0.6
budget_65	195	63.8974	205	64.8153	205	64.8153	37.705	0	1
budget_70	205	68.8381	210	69.6065	215	64.6606	37.877	2.33	0.6
budget_73	205	68.8381	215	72.0543	230	72.5226	44.468	6.52	0.6
budget_75	210	74.2232	215	74.9916	250	74.8017	38.694	14	0.6
budget_80	220	79.6536	220	79.6536	250	79.6395	39.983	12	0.6
budget_85	240	84.8917	240	84.8917	250	84.1147	40.671	4	0.6
tsiligirides_problem_2									
budget_15	100	14.3683	115	14.6676	120	14.8972	16.602	4.17	0.3
budget_20	165	19.7149	165	19.7149	200	19.8795	17.5	17.5	0.5

budget_23	200	21.4912	200	21.4912	210	22.6478	18.228	4.76	0.5
budget_25	200	21.4912	200	21.4912	230	24.7767	18.717	13.04	0.4
budget_27	230	25.7401	230	25.7401	230	25.7401	18.177	0	1
budget_30	230	25.7401	230	25.7401	250	29.9183	18.545	8	0.5
budget_32	260	31.531	260	31.531	270	31.586	18.706	3.7	0.6
budget_35	260	31.531	260	31.531	320	34.9886	19.347	18.75	0.4
budget_38	260	31.531	260	31.531	360	37.8423	19.191	27.78	0.4
budget_40	290	38.7902	290	38.7902	370	39.6592	19.221	21.62	0.4
budget_45	340	42.8646	340	42.8646	435	44.4919	19.3	21.84	0.4
tsiligirides_problem_3									
budget_015	160	14.8275	170	14.846	170	14.846	25.35	0	1
budget_020	180	19.6627	190	19.6812	190	19.6812	28.86	0	1
budget_025	220	23.0849	240	24.4924	250	24.9235	28.893	4	0.6
budget_030	290	28.2481	320	28.7698	320	28.7698	31.01	0	1
budget_035	370	34.7962	390	34.9334	390	34.9334	32.657	0	1
budget_040	370	34.7962	400	38.0444	400	38.0444	33.959	0	1
budget_045	420	43.4546	450	43.9764	450	43.9764	34.911	0	1
budget_050	420	43.4546	470	49.5738	470	49.5738	35.924	0	1
budget_055	440	52.3478	470	54.9612	470	54.9612	36.167	0	1
budget_060	460	55.6265	490	58.8747	510	59.6699	38.096	3.92	0.6
budget_065	500	63.4195	530	63.9413	530	63.9413	38.453	0	1
budget_070	530	68.4145	560	68.9363	560	68.9363	38.902	0	1
budget_075	550	72.9938	580	73.5156	580	73.5156	39.645	0	1
budget_080	580	78.8949	610	79.4166	620	79.1897	39.711	1.61	0.7
budget_085	600	84.8989	610	84.9174	640	84.8034	40.494	4.69	0.4
budget_090	610	89.4648	640	89.9866	670	89.6836	40.258	4.48	0.4
budget_095	660	94.6031	680	94.7403	700	93.1699	40.492	2.86	0.4
budget_100	660	94.6031	700	99.8724	720	99.2335	41.522	2.78	0.4
budget_105	700	103.192	730	103.7137	750	101.3046	40.73	2.67	0.4
budget_110	700	103.192	740	108.4613	760	108.0682	41.743	2.63	0.4

Table a7. Profit and running time of proposed heuristic vs Chao's heuristic

Instance _ budget	K-best (K = 3) & α (best result out of 1000)		Chao		
	Profit	Time (s)	Profit	Time (s)	% gap
set_64_1_15	78	78.498	96	13.01	-18.75
set_64_1_20	**294**	**57.345**	**294**	27.86	**0.00**

set_64_1_25	**390**	**66.946**	**390**	238.9	**0.00**
set_64_1_30	468	75.264	474	74.48	-1.27
set_64_1_35	528	90.466	570	139.86	-7.37
set_64_1_40	666	104.882	714	137.9	-6.72
set_64_1_45	780	118.269	816	204.98	-4.41
set_64_1_50	852	121.526	900	231.57	-5.33
set_64_1_55	942	127.659	984	246.18	-4.27
set_64_1_60	1026	133.688	1044	264.77	-1.72
set_64_1_65	1080	134.156	1116	232.57	-3.23
set_64_1_70	1140	136.927	1176	230.95	-3.06
set_64_1_75	1176	137.636	1224	223.12	-3.92
set_64_1_80	1212	138.792	1272	212.27	-4.72
set_66_1_005	**10**	**41.198**	**10**	1.05	**0.00**
set_66_1_010	**40**	**43.766**	**40**	0.46	**0.00**
set_66_1_015	80	60.076	120	4.33	-33.33
set_66_1_020	145	60.37	195	6.17	-25.64
set_66_1_025	215	75.48	290	73.42	-25.86
set_66_1_030	340	78.823	400	54.82	-15.00
set_66_1_035	455	91.214	460	32.42	-1.09
set_66_1_040	**575**	**100.755**	**575**	98.92	**0.00**
set_66_1_045	640	108.57	650	58.13	-1.54
set_66_1_050	705	114.911	730	68.05	-3.42
set_66_1_055	775	120.324	825	65.23	-6.06
set_66_1_060	865	126.699	915	84.59	-5.46
set_66_1_065	940	131.721	980	82.18	-4.08
set_66_1_070	1000	134.897	1070	119	-6.54
set_66_1_075	1050	140.499	1140	116.7	-7.89
set_66_1_080	1145	142.348	1215	108.93	-5.76
set_66_1_085	1225	145.035	1270	132.45	-3.54
set_66_1_090	1275	146.845	1340	502.41	-4.85
set_66_1_095	1340	148.136	1380	467.13	-2.90
set_66_1_100	1410	147.308	1435	128.56	-1.74

set_66_1_105	1430	148.6	1510	316.3	-5.30
set_66_1_110	1510	148.08	1550	469.94	-2.58
set_66_1_115	1570	148.786	1595	474.64	-1.57
set_66_1_120	1605	144.994	1635	357.98	-1.83
set_66_1_125	1640	146.019	1655	268.86	-0.91
set_66_1_130	1675	143.302	1680	23.05	-0.30
tsiligirides_p_1_b_05	**10**	**15.309**	**10**	0.67	**0.00**
tsiligirides_p_1_b_10	**15**	**23.364**	**15**	0.8	**0.00**
tsiligirides_p_1_b_15	35	27.07	45	2.28	-22.22
tsiligirides_p_1_b_20	55	28.159	65	17.49	-15.38
tsiligirides_p_1_b_25	70	30.289	90	9.01	-22.22
tsiligirides_p_1_b_30	85	32.431	110	31.02	-22.73
tsiligirides_p_1_b_35	125	35.982	135	25.25	-7.41
tsiligirides_p_1_b_40	145	36.228	155	17	-6.45
tsiligirides_p_1_b_46	170	38.31	175	22	-2.86
tsiligirides_p_1_b_50	185	38.551	190	25	-2.63
tsiligirides_p_1_b_55	195	40.451	205	25	-4.88
tsiligirides_p_1_b_60	215	41.482	225	25	-4.44
tsiligirides_p_1_b_65	**240**	**42.978**	**240**	23	**0.00**
tsiligirides_p_1_b_70	250	42.283	260	25	-3.85
tsiligirides_p_1_b_73	260	47.96	265	25	-1.89
tsiligirides_p_1_b_75	265	42.539	270	29	-1.85
tsiligirides_p_1_b_80	270	43.23	280	27	-3.57
tsiligirides_p_1_b_85	280	42.427	285	22	-1.75
tsiligirides_p_2_b_15	**120**	**17.62**	**120**	1.3	**0.00**
tsiligirides_p_2_b_20	190	19.906	200	2.3	-5.00
tsiligirides_p_2_b_23	200	19.603	210	4.5	-4.76
tsiligirides_p_2_b_25	**230**	**19.666**	**230**	6	**0.00**
tsiligirides_p_2_b_27	**230**	**21.182**	**230**	6	**0.00**
tsiligirides_p_2_b_30	260	20.557	265	6	-1.89
tsiligirides_p_2_b_32	280	21.116	300	7	-6.67
tsiligirides_p_2_b_35	310	20.583	320	8	-3.13

tsiligirides_p_2_b_38	340	20.211	360	7	-5.56
tsiligirides_p_2_b_40	360	20.812	395	7	-8.86
tsiligirides_p_2_b_45	435	20.665	450	0.6	-3.33
tsiligirides_p_3_b_015	**170**	**28.259**	**170**	4.4	**0.00**
tsiligirides_p_3_b_020	190	31.686	200	5	-5.00
tsiligirides_p_3_b_025	250	32.966	260	9.4	-3.85
tsiligirides_p_3_b_030	**320**	**34.428**	**320**	10	**0.00**
tsiligirides_p_3_b_035	**390**	**36.996**	**390**	15	**0.00**
tsiligirides_p_3_b_040	410	38.165	430	19	-4.65
tsiligirides_p_3_b_045	450	39.069	470	27	-4.26
tsiligirides_p_3_b_050	470	40.687	520	29	-9.62
tsiligirides_p_3_b_055	490	41.342	550	30	-10.91
tsiligirides_p_3_b_060	530	42.18	580	28	-8.62
tsiligirides_p_3_b_065	550	44.139	610	25	-9.84
tsiligirides_p_3_b_070	580	43.765	640	30	-9.38
tsiligirides_p_3_b_075	600	44.377	670	29	-10.45
tsiligirides_p_3_b_080	640	44.491	710	30	-9.86
tsiligirides_p_3_b_085	640	45.211	740	28	-13.51
tsiligirides_p_3_b_090	680	46.038	770	24	-11.69
tsiligirides_p_3_b_095	690	47.121	790	22	-12.66
tsiligirides_p_3_b_100	730	46.086	800	0.67	-8.75
tsiligirides_p_3_b_105	770	44.483	800	0.6	-3.75
tsiligirides_p_3_b_110	**800**	**44.721**	**800**	0.72	**0.00**

Table a7. Comparison with Chao's solution